1. Calcule $\frac{\partial z}{\partial x}$ para $z+y=x^2$

Solução:

Como eu posso explicitar:

$$z = x^2 - y$$

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(x^2-y) = 2x$$

E se não fosse possível explicitar?

$\frac{\partial z}{\partial x} = ?$

$$z+y=x^2$$

$$\frac{\partial (z+y)}{\partial x} = \frac{\partial x^2}{\partial x}$$

Trataremos z e x como variáveis.

O que não for z nem x é constante.

$$\frac{\partial (z+y)}{\partial x} = \frac{\partial x^2}{\partial x}$$

$$\frac{\partial z}{\partial x} + \frac{\partial y}{\partial x} = 2x$$

$$\frac{\partial z}{\partial x} + 0 = 2x \therefore \boxed{\frac{\partial z}{\partial x} = 2x}$$

Exemplo

$$\cos(z^3+z) + z^2 = x^2 - y^4$$

$\frac{\partial z}{\partial x}$ NÃO SABEMOS EXPLICITAR $z=z(x)$

E $\frac{\partial x}{\partial y}$?

$$z+y=x^2$$

Explicitar: $x = +\sqrt{z+y}$

$x = -\sqrt{z+y}$

Fazendo a derivação implícita:

$$\frac{\partial}{\partial y}(z+y) = \frac{\partial x^2}{\partial y}$$

$\boxed{\frac{\partial x}{\partial y}}$

$$\frac{\partial z}{\partial y} + \frac{\partial y}{\partial y} = \frac{\partial x^2}{\partial y}$$

$$0 + 1 = 2x \cdot \frac{\partial x}{\partial y}$$

$$\frac{1}{2x} = \frac{\partial x}{\partial y}$$

$$\boxed{\frac{\partial x}{\partial y} = \frac{1}{2x}}$$

2. Calcule $\dfrac{\partial z}{\partial x}$, $z^5 + z\cos(xy) + z^2 x = 0$

$$\frac{\partial}{\partial x}\left(z^5 + z\cos(xy) + z^2 x\right) = \frac{\partial 0}{\partial x}$$

$$\frac{\partial z^5}{\partial x} + \frac{\partial}{\partial x} z\cos(xy) + \frac{\partial}{\partial x} z^2 x = 0$$

$$5z^4 \cdot \frac{\partial z}{\partial x} + \frac{\partial z}{\partial x} \cdot \cos(xy) + z\frac{\partial}{\partial x}\cos(xy) + \frac{\partial z^2}{\partial x} \cdot x + z^2 \frac{\partial x}{\partial x} = 0$$

$$5z^4 \cdot \frac{\partial z}{\partial x} + \cos(xy)\frac{\partial z}{\partial x} - z\,\mathrm{sen}(xy)\,y + 2z\frac{\partial z}{\partial x}\,x + z^2 = 0$$

$$5z^4\frac{\partial z}{\partial x} + \cos(xy)\frac{\partial z}{\partial x} - zy\,\mathrm{sen}(xy) + 2xz\frac{\partial z}{\partial x} + z^2 = 0$$

$$\frac{\partial z}{\partial x}\left(5z^4 + \cos(xy) + 2xz\right) = -z^2 + yz\,\mathrm{sen}(xy)$$

$$\frac{\partial z}{\partial x} = \frac{yz\,\mathrm{sen}(xy) - z^2}{5z^4 + \cos(xy) + 2xz}$$

Exemplo mais simples

$$z\cos(xz) = y$$

$$\frac{\partial z}{\partial x} = ? \qquad \frac{\partial}{\partial x}\left(z\cos(xz)\right) = \frac{\partial y}{\partial x}$$

$$\frac{\partial z}{\partial x} \cdot \cos(xz) + z\frac{\partial}{\partial x}\cos(xz) = 0$$

$$\cos(xz)\frac{\partial z}{\partial x} + z\left(-\mathrm{sen}(xz) \cdot \frac{\partial}{\partial x}(xz)\right) = 0$$

$$\cos(xz)\frac{\partial z}{\partial x} + z\left(-\mathrm{sen}(xz)\left[\frac{\partial x}{\partial x} \cdot z + x\frac{\partial z}{\partial x}\right]\right) = 0$$

$$\cos(xz)\frac{\partial z}{\partial x} + z\left(-\mathrm{sen}(xz)\left[z + x\frac{\partial z}{\partial x}\right]\right) = 0$$

$$\cos(xz)\frac{\partial z}{\partial x} + z\left(-\mathrm{sen}(xz)\,z - \mathrm{sen}(xz)\,x\frac{\partial z}{\partial x}\right) = 0$$

$$\cos(xz)\frac{\partial z}{\partial x} - z^2\,\mathrm{sen}(xz) - xz\,\mathrm{sen}(xz)\frac{\partial z}{\partial x} = 0$$

$$\frac{\partial z}{\partial x}\left(\cos(xz) - xz\,\mathrm{sen}(xz)\right) = z^2\,\mathrm{sen}(xz)$$

$$\frac{\partial z}{\partial x} = \frac{z^2\,\mathrm{sen}(xz)}{\cos(xz) - xz\,\mathrm{sen}(xz)}$$

Onde está o erro?

$$\frac{\partial}{\partial x}\left(z\cos(xz)\right) = \frac{\partial y}{\partial x}$$

$$\frac{\partial z}{\partial x}\cos(xz) + z\frac{\partial}{\partial x}\left(\cos(xz)\right) = 0$$

$$\cos(xz)\frac{\partial z}{\partial x} - z\,\mathrm{sen}(xz) \cdot \frac{\partial}{\partial x}(xz) = 0$$

$$\cos(xz)\frac{\partial z}{\partial x} - z\,\mathrm{sen}(xz)\left[\cdot\frac{\partial x}{\partial x} z + x\frac{\partial z}{\partial x}\right] = 0$$

$$\cos(xz)\frac{\partial z}{\partial x} - z\,\mathrm{sen}(xz) \cdot z + x\frac{\partial z}{\partial x} = 0$$

TA AQU O ERRO!

$$\frac{\partial z}{\partial x}\left(\cos(xz) + x\right) = z^2\,\mathrm{sen}(xz)$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{z^2\,\mathrm{sen}(xz)}{\cos(xz) + x}}$$

Plano tangente

Se a superfície é dada por uma função explícita de duas variáveis $f(x,y)$ o plano tangente a essa superfície no ponto (x_0,y_0,z_0) é dado por:

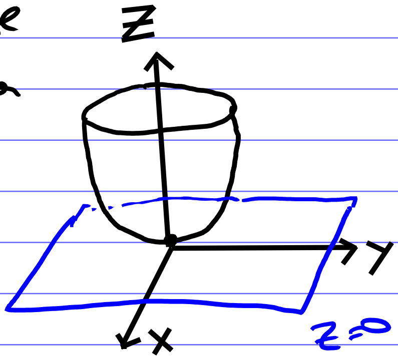
$$z = z_0 + \left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)} (x - x_0) + \left.\frac{\partial f}{\partial y}\right|_{(x_0,y_0)} (y - y_0)$$

Exercício

Calcule o plano tangente à superfície dada por

$f(x,y) = x^2 + y^2$

no ponto $(0,0)$.



$$z = z_0 + \left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)} (x - x_0) + \left.\frac{\partial f}{\partial y}\right|_{(x_0,y_0)} (y - y_0)$$

$x_0 = 0$

$y_0 = 0$

$z_0 = x_0^2 + y_0^2 = 0$ pois $z = x^2 + y^2$

$$z = 0 + \left. 2x \right|_{(0,0)} (x - 0) + \left. 2y \right|_{(0,0)} (y - 0)$$

$$\boxed{z = 0}$$