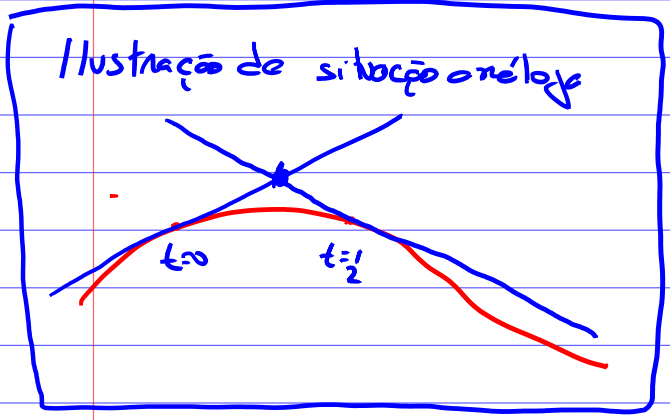
① Determine o ponto de intersecção das retas tangentes à curva

$$\vec{r}(t) = \langle \operatorname{sen} \pi t,\ 2\operatorname{sen} \pi t,\ \cos \pi t \rangle$$

nos pontos $t=0$ e $t=\frac{1}{2}$.

Solução



$$\vec{R}(t) = \vec{R}_0 + t\vec{V}$$

$$\vec{V}(t) = \frac{d\vec{R}(t)}{dt}$$

$$\vec{V}(t) = \langle \pi\cos(\pi t),\ 2\pi\cos(\pi t),\ -\pi\operatorname{sen}(\pi t) \rangle$$

P/ $t=0$

$$\vec{R}_0 = \vec{r}(0) = \langle \operatorname{sen}(\pi 0),\ 2\operatorname{sen}(\pi 0),\ \cos(\pi 0) \rangle$$

$$\bar{R}_0 = \langle 0, 0, 1 \rangle$$

$$\vec{V}(0) = \langle \pi\cos(0),\ 2\pi\cos(0),\ -\pi\operatorname{sen} 0 \rangle$$
$$= \langle \pi,\ 2\pi,\ 0 \rangle$$

$$\bar{R}(\alpha) = \bar{R}_0 + \alpha\bar{V}(0)$$

$$\bar{R}(\alpha) = \langle 0,0,1 \rangle + \alpha\langle \pi, 2\pi, 0 \rangle$$

$$\vec{R}(\alpha) = \langle \alpha\pi,\ \alpha 2\pi,\ 1 \rangle \qquad \alpha \in \mathbb{R}$$

P/ $t=\frac{1}{2}$

$$\vec{R}_0 = \vec{r}\left(\tfrac{1}{2}\right) = \langle \operatorname{sen}\left(\tfrac{\pi}{2}\right),\ 2\operatorname{sen}\left(\tfrac{\pi}{2}\right),\ \cos\left(\tfrac{\pi}{2}\right) \rangle$$
$$= \langle 1, 2, 0 \rangle$$

$$\vec{V}(0) = \langle \pi\cos\left(\tfrac{\pi}{2}\right),\ 2\pi\cos\left(\tfrac{\pi}{2}\right),\ -\pi\operatorname{sen}\tfrac{\pi}{2} \rangle$$
$$= \langle 0,\ 0,\ -\pi \rangle$$

$$\vec{R}(\beta) = \langle 1,2,0 \rangle + \beta\langle 0,0,-\pi \rangle$$

$$\bar{R}(\beta) = \langle 1,\ 2,\ -\pi\beta \rangle$$

Onde elas se cruzam?

$$\vec{R}_A(\alpha) = \langle \alpha\pi,\ \alpha 2\pi,\ 1 \rangle$$

$$\bar{R}_B(\beta) = \langle 1,\ 2,\ -\pi\beta \rangle$$

$$\begin{cases} \alpha\pi = 1 & \Rightarrow \alpha = \frac{1}{\pi} \\ \alpha 2\pi = 2 & \\ 1 = -\beta\pi & \Rightarrow \beta = -\frac{1}{\pi} \end{cases}$$

$$\bar{R}_A\left(\tfrac{1}{\pi}\right) = \langle 1,\ 2,\ 1 \rangle$$

$$\bar{R}_B\left(-\tfrac{1}{\pi}\right) = \langle 1,\ 2,\ 1 \rangle$$

Eles se cruzam em $(1, 2, 1)$

② Calcule $\int_0^1 \left(\frac{4}{1+t^2}\hat{j} + \frac{2t}{1+t^2}\hat{k} \right) dt$

1ª integral $\int_0^1 \frac{4}{1+t^2} dt$

$\sec^2\theta - tg^2\theta = 1$

$\sec^2\theta = 1 + tg^2\theta$

$t = tg\,\theta$

$\frac{dt}{d\theta} = \sec^2\theta$

$1+t^2 = 1+tg^2\theta = \sec^2\theta$

$= \int_0^{\frac{\pi}{4}} \frac{4\sec^2\theta\, d\theta}{\sec^2\theta}$

$= \int_0^{\frac{\pi}{4}} 4\, d\theta$

$= 4\left(\frac{\pi}{4} - 0\right)$

$= \pi$

Quando $t=0$, $\theta = ?$

$t = tg\,\theta$

$0 = tg\,\theta$

$\theta = arc\,tg(0) = 0$

Quando $t=1$, $\theta = ?$

$t = tg\,\theta$

$1 = tg\,\theta$

$\theta = arc\,tg(1) = \frac{\pi}{4}$

2ª integral $\int_0^1 \frac{2t}{1+t^2} dt$

$u = 1+t^2 \therefore \frac{du}{dt} = 2t$

$du = 2t\,dt$

Quando $t=0$, $u = 1$

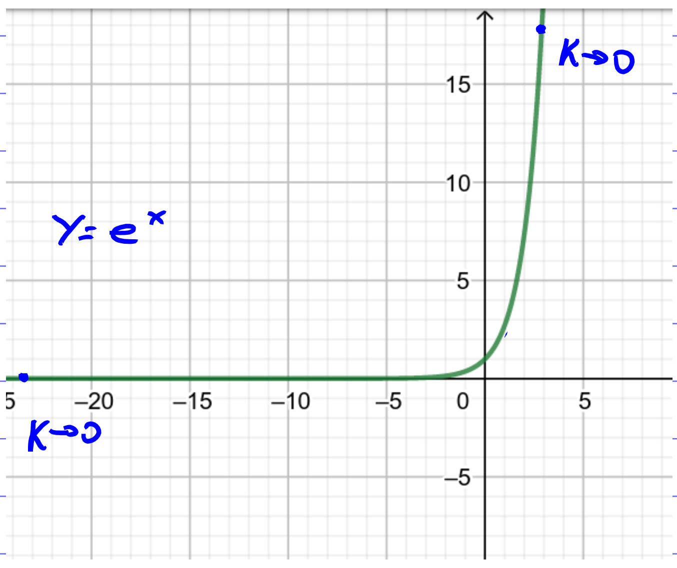
" $t=1$, $u = 2$

$\int_1^2 \frac{du}{u} = \ln u \Big|_1^2 = \ln 2 - \ln 1 = \ln 2$

$\int_0^1 \left(\frac{4}{1+t^2}\hat{j} + \frac{2t}{1+t^2}\hat{k} \right) dt = \pi\hat{j} + \ln 2\,\hat{k}$

③ Em que Ponto a curvatura é máxima?

$Y = e^x$



$$K = \left| \frac{d\vec{T}}{ds} \right|$$

$$K = \frac{\left| \frac{d\vec{T}}{dt} \right|}{|\vec{v}|}$$

$$K = \frac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}\,'|^3}$$

Se eu fosse usar esta fórmula?
$\vec{r}(t) = \langle t, e^t \rangle$

$$k = \frac{f''}{(1+(f')^2)^{3/2}}$$

Vamos usar esta expressão

Solução

$f(x) = e^x$

$f'(x) = e^x$

$f''(x) = e^x$

$$K = \frac{f''}{(1+(f')^2)^{3/2}}$$

$$K = \frac{e^x}{[1+(e^x)^2]^{3/2}}$$

$$K(x) = \frac{e^x}{[1+e^{2x}]^{3/2}}$$

Onde a derivada é nula? (Ponto crítico)

$K'(x) \overset{?}{=} 0$

$$K'(x) = \frac{e^x[1+e^{2x}]^{3/2} - e^x \frac{3}{2}[1+e^{2x}]^{1/2} \cdot e^{2x} \cdot 2}{[1+e^{2x}]^3}$$

$K'(x) = 0$

$$e^x[1+e^{2x}]^{3/2} - e^x \frac{3}{2}[1+e^{2x}]^{1/2} \cdot e^{2x} \cdot 2 = 0$$

$$e^x[1+e^{2x}]^{3/2} = e^{3x}\, 3\, [1+e^{2x}]^{1/2}$$

$$\frac{e^x[1+e^{2x}]^{3/2}}{[1+e^{2x}]^{1/2}} = e^{3x}\, 3$$

$$e^x[1+e^{2x}] = e^{3x}\, 3$$

$$[1+e^{2x}] = e^{2x}\, 3$$

$$1 + e^{2x} = 3e^{2x}$$

$$1 = 2e^{2x}$$

$$\frac{1}{2} = e^{2x}$$

$$\ln\left(\tfrac{1}{2}\right) = \ln(e^{2x})$$

$$\ln(2^{-1}) = 2x$$

$$-\ln 2 = 2x \quad \therefore$$

$$x = -\tfrac{1}{2}\ln 2$$

$$Y = e^{(-\frac{1}{2}\ln 2)}$$

$$Y = e^{\ln(2^{-\frac{1}{2}})}$$

$$Y = 2^{-1/2}$$

$$Y = \frac{1}{\sqrt{2}}$$

P.C: $\left(-\tfrac{1}{2}\ln 2, \tfrac{1}{\sqrt{2}}\right)$

(4) Calcule o comprimento exato da curva polar

$r = 5^{\theta} \quad 0 \leq \theta \leq 2\pi$

$$L = \int_{\theta_i}^{\theta_f} \sqrt{r^2 + r'^2}\, d\theta \qquad r = 5^{\theta} \therefore r' = 5^{\theta} \ln 5$$

$$\therefore (r')^2 = 5^{2\theta} (\ln 5)^2$$

$$L = \int_0^{2\pi} \sqrt{5^{2\theta} + 5^{2\theta} (\ln 5)^2}\, d\theta$$

$$L = \int_0^{2\pi} \sqrt{5^{2\theta} [1 + (\ln 5)^2]}\, d\theta$$

$$L = \int_0^{2\pi} 5^{\theta} \sqrt{1 + (\ln 5)^2}\, d\theta$$

$$L = \sqrt{1 + (\ln 5)^2} \int_0^{2\pi} 5^{\theta}\, d\theta$$

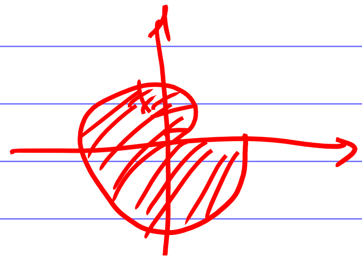
$$L = \sqrt{1 + (\ln 5)^2} \left[\frac{5^{\theta}}{\ln 5} \right] \Bigg|_0^{2\pi}$$

$$L = \sqrt{1 + (\ln 5)^2} \left(\frac{5^{2\pi}}{\ln 5} - \frac{5^{0}}{\ln 5} \right)$$

$$L = \frac{\sqrt{1 + (\ln 5)^2}}{\ln 5} (5^{2\pi} - 1) \quad \text{u.c.}$$

5) Calcule a área da região delimitada por

$r=\sqrt{\theta}$ $0\leq\theta\leq 2\pi$.



$$A=\int_{\theta_i}^{\theta_f}\frac{1}{2}r^2\,d\theta=\int_0^{2\pi}\frac{1}{2}\left(\sqrt{\theta}\right)^2 d\theta$$

$$A=\int_0^{2\pi}\frac{1}{2}\,\theta\,d\theta=\frac{1}{2}\,\frac{\theta^2}{2}\Big|_0^{2\pi}$$

$$=\frac{1}{4}\left(4\pi^2\right)$$

$$=\pi^2$$

Questão 28. pág 762

Curva

$x = \operatorname{sen} t$, $y = \cos t$, $z = \operatorname{sen}^2 t$

Superfícies

① $z = x^2$

$x^2 + y^2 = 1$ ②

NA SUPERFÍCIE ①

$$z = x^2$$

$$(\operatorname{sen}^2 t) = (\operatorname{sen} t)^2$$

Essa identidade é verdadeira?

Sim.

ENTÃO A CURVA ESTÁ CONTIDA EM ①

NA SUPERFÍCIE ②

$$x^2 + y^2 = 1$$

$$(\operatorname{sen} t)^2 + (\cos t)^2 = 1$$

Essa identidade é verdadeira?

SIM

ENTÃO A CURVA ESTÁ CONTIDA EM ②

Como ela estÃ¡ contida na superfÃ-cie 1 e na superfÃ-cie 2, ela Ã© UMA curva de inteseÃ§Ã£o entre as superfÃ-cies. Podem exis