Substituição trigonométrica

$$\int_0^1 x^3\sqrt{1-x^2}\,dx$$

$$x = \operatorname{sen}\theta$$

$$\frac{dx}{d\theta} = \cos\theta \therefore dx = \cos\theta\,d\theta$$

Quando $x=0$, $\theta = ?$

$$x = \operatorname{sen}\theta$$

$$0 = \operatorname{sen}\theta$$

$$\theta = \operatorname{arc\,sen} 0$$

$$\boxed{\theta = 0}$$

Quando $x=1$, $\theta = ?$

$$x = \operatorname{sen}\theta$$

$$1 = \operatorname{sen}\theta$$

$$\theta = \operatorname{arc\,sen}(1)$$

$$\boxed{\theta = \frac{\pi}{2}}$$

1ª técnica Sub trigonométrica.

$$\int_0^1 x^3\sqrt{1-x^2}\,dx = \int_0^{\pi/2} \operatorname{sen}^3\theta\sqrt{1-\operatorname{sen}^2\theta}\cos\theta\,d\theta$$

$$= \int_0^{\pi/2} \operatorname{sen}^3\theta\sqrt{\cos^2\theta}\cos\theta\,d\theta$$

$$= \int_0^{\pi/2} \operatorname{sen}^3\theta\,|\cos\theta|\cos\theta\,d\theta$$

MAS $0 \le \theta \le \frac{\pi}{2}$, $\cos\theta \ge 0$

$$= \int_0^{\pi/2} \operatorname{sen}^3\theta\cos\theta\cos\theta\,d\theta$$

$$= \int_0^{\pi/2} \operatorname{sen}^3\theta\cos^2\theta\,d\theta$$

2ª técnica
USO DE RELAÇÕES TRIGONOMÉTRICAS

$$= \int_0^{\pi/2} \operatorname{sen}^2\theta\operatorname{sen}\theta\cos^2\theta\,d\theta$$

3ª técnica

$$= \int_0^{\pi/2} (1-\cos^2\theta)\operatorname{sen}\theta\cos^2\theta\,d\theta$$

Quando $\theta = 0$, $u = \cos 0$, $u = 1$
e $\theta = \frac{\pi}{2}$, $u = \cos\frac{\pi}{2} = 0$

$u = \cos\theta \therefore \frac{du}{d\theta} = -\operatorname{sen}\theta$

$-du = \operatorname{sen}\theta\,d\theta$

$$\int_1^0 (1-u^2)\,u^2\,(-du) = \int_0^1 (1-u^2)\,u^2\,du$$

$$= \int_0^1 (u^2 - u^4)\,du$$

$$= \left[\frac{u^3}{3} - \frac{u^5}{5}\right]\Big|_0^1$$

$$= \left(\frac{1}{3} - \frac{1}{5}\right) - \left(\frac{0}{3} - \frac{0}{5}\right)$$

$$= \frac{5-3}{15} = \frac{2}{15}$$

Atenção

Ex: $f(x) = \sqrt{x}$

$f(25) = 5$

Uma equação

$x^2 = 25$

$x = 5$ ou $x = -5$

$$\boxed{\sqrt{x^2} = |x|}$$

UMA FUNÇÃO

(2) $\int_0^a \frac{dx}{(a^2+x^2)^{3/2}} =$

$\cos^2\theta + \operatorname{sen}^2\theta = 1$

$\operatorname{tg}^2\theta + 1 = \sec^2\theta$

$a > 0$

$x = a\operatorname{tg}\theta$

$a^2 + x^2 = a^2 + a^2\operatorname{tg}^2\theta = a^2\sec^2\theta$

$\frac{dx}{d\theta} = a\sec^2\theta$

$dx = a\sec^2\theta\, d\theta$

Quando $x = 0$, $\theta = ?$

$x = a\operatorname{tg}\theta$

$0 = a\operatorname{tg}\theta$

$\operatorname{tg}\theta = 0$

$\theta = \operatorname{arctg}(0)$

$\theta = 0$

Quando $x = a$, $\theta = ?$

$x = a\operatorname{tg}\theta$

$a = a\operatorname{tg}\theta$

$\operatorname{tg}\theta = 1$

$\theta = \operatorname{arctg}(1)$

$\theta = \frac{\pi}{4}$

$$\int_0^{\frac{\pi}{4}} \frac{a\sec^2\theta\, d\theta}{(a^2\sec^2\theta)^{3/2}}$$

$$= \int_0^{\frac{\pi}{4}} \frac{a\sec^2\theta\, d\theta}{(a\sec\theta)^3} = \int_0^{\frac{\pi}{4}} \frac{1}{a^2\sec\theta}\, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{a^2}\cos\theta\, d\theta$$

$$= \frac{1}{a^2}\operatorname{sen}\theta\Big|_0^{\frac{\pi}{4}}$$

$$= \frac{1}{a^2}\left(\operatorname{sen}\frac{\pi}{4} - \operatorname{sen}0\right)$$

$$= \frac{1}{a^2}\left(\frac{\sqrt{2}}{2} - 0\right)$$

$$= \frac{\sqrt{2}}{2a^2}$$

Subs. trigonométricas

$\cos^2\theta + \operatorname{sen}^2\theta = 1$

$\operatorname{tg}^2\theta + 1 = \sec^2\theta$

$\sqrt{a^2 - x^2}$ $\quad x = a \operatorname{sen}\theta$

$\sqrt{a^2 + x^2}$ $\quad x = a \operatorname{tg}\theta$

$\sqrt{x^2 - a^2}$ $\quad x = a \sec\theta$