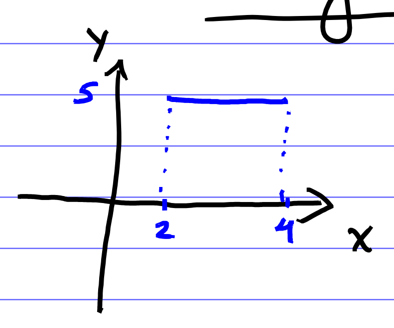
Integrais definidas



$f(x) = 5, \quad 2 \le x \le 4$

$$\int_2^4 f(x)\,dx = 2(5) = 10$$

$$= \lim_{n\to+\infty} \sum_{i=1}^{m} f(x)\Delta x$$

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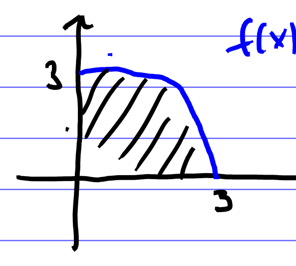


$f(x) = \sqrt{9 - x^2}, \quad 0 \le x \le 3$

$$\int_0^3 f(x)\,dx = \frac{1}{4} A_{\text{círculo}}$$

$$= \frac{1}{4}\pi r^2$$

$$= \frac{1}{4}\pi 9$$

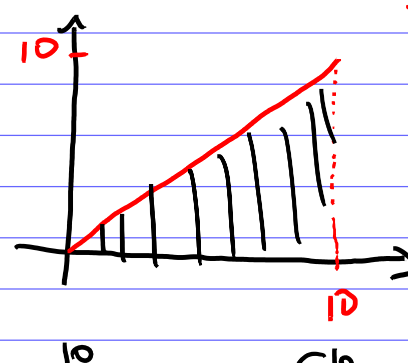
$$= \frac{9\pi}{4}$$

Área sob a curva

"Em baixo"

ESPERA-SE, $\boxed{f(x) \text{ positiva}}$

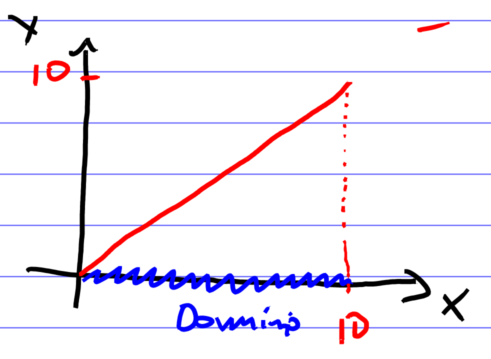
$f(x) = x$



Via Geométrica

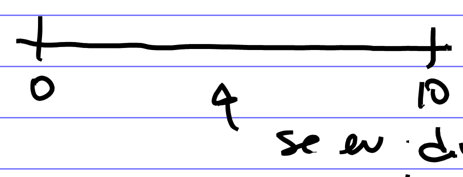
$$\int_0^{10} f(x)\,dx = \int_0^{10} x\,dx = 50$$

Via Definição



Genérico.

$$\int_0^{10} x\,dx = \lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

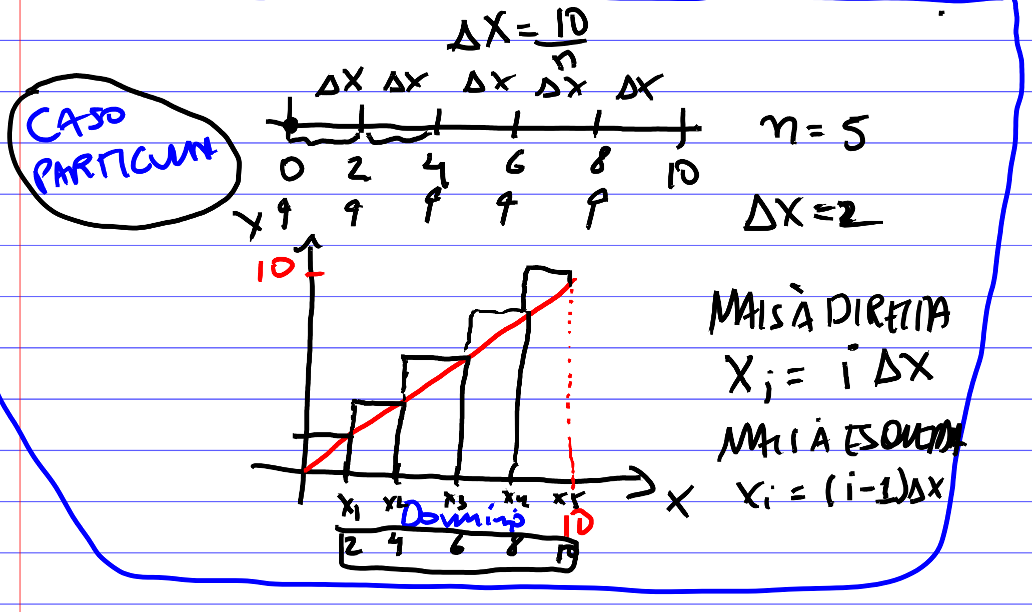


se eu divido em n partes, cada parte tem que tamanho?

$\Delta x = \frac{10}{n}$

E como eu calculo x_i?

Vamos analisar um caso em particular.



Continuando...

$\therefore x_i = i\Delta x$

$x_i = i\frac{10}{n}$

$$\int_0^{10} x\,dx = \lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to+\infty} \sum_{i=1}^{n} f(i\Delta x)\frac{10}{n}$$

$f(x) = x \therefore f(i\Delta x) = i\Delta x$

$$= \lim_{n\to+\infty} \sum_{i=1}^{n} i\Delta x \cdot \frac{10}{n}$$

$$= \lim_{n\to+\infty} \sum_{i=1}^{n} i\frac{10}{n} \cdot \frac{10}{n}$$

$$= \lim_{n\to+\infty} \sum_{i=1}^{n} i\frac{100}{n^2}$$

$$= \lim_{n\to+\infty} \frac{100}{n^2} \sum_{i=1}^{n} i$$

$$= \lim_{n\to+\infty} \frac{100}{n^2} \left[\frac{n(n+1)}{2}\right]$$

pt.wikipedia.org/wiki/Somatório

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certeza que todas as r restrições pertencem ao intervalo [m, n], caso contrário, desconsidere-as (o que não ocorre na maioria dos ca

Alguns somatórios de funções polinomiais

1. $\sum_{i=m}^{n} 1 = n + 1 - m$
2. $\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$ (Soma de uma progressão aritmética)
3. $\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ (Número piramidal quadrado)
4. $\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ [3]
5. $\sum_{i=1}^{n} i^4 = 1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$ [3]
6. $\sum_{i=1}^{n} i^5 = 1^5 + 2^5 + 3^5 + \cdots + n^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$ [3]
7. $\sum_{i=1}^{n} i^6 = 1^6 + 2^6 + 3^6 + \cdots + n^6 = \frac{n(n+1)(2n+1)(3n^4+6n^3-3n+1)}{42}$ [3]
8. $\sum_{i=0}^{n} i^p = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^{p} \frac{B_k}{p-k+1}\binom{p}{k}(n+1)^{p-k+1}$

Alguns somatórios de funções exponenciais

1. $\sum_{i=m}^{n} x^i = x^m + x^{m+1} + x^{m+2} + \cdots + x^n = \frac{x(x^n - x^{m-1})}{x-1}$ (Soma dos termos de uma progressão geométrica)

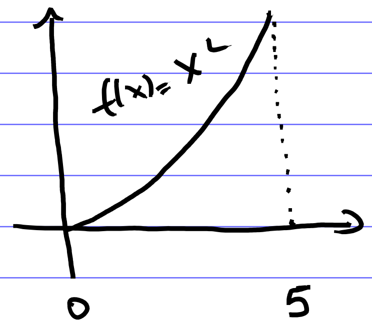
$$= \lim_{n\to+\infty} \frac{100}{n^2}\left[\frac{n^2+n}{2}\right]$$

$$= \lim_{n\to+\infty} \frac{100}{2} \cdot \frac{n^2+n}{n^2}$$

$$= \lim_{n\to+\infty} 50 \cdot \left(1 + \frac{1}{n}\right)$$

$$= 50 \lim_{n\to+\infty} \left(1 + \frac{1}{n}\right)$$

$$= 50 \cdot 1 = 50$$



$$\int_0^5 x^2 dx = \lim_{n \to +\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$\Delta x = \frac{\text{comprimento do domínio}}{n}$$

$$\Delta x = \frac{5}{n}$$

$$x_i = i \Delta x$$
$$x_i = i \frac{5}{n}$$

$$\int_0^5 x^2 dx = \lim_{n \to +\infty} \sum_{i=1}^{n} f\left(\frac{5i}{n}\right) \frac{5}{n}$$

$$f(x) = x^2$$

$$= \lim_{n \to +\infty} \sum_{i=1}^{n} \left(\frac{5i}{n}\right)^2 \frac{5}{n}$$

$$= \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{25 i^2}{n^2} \cdot \frac{5}{n}$$

$$= \lim_{n \to +\infty} \frac{125}{n^3} \sum_{i=1}^{n} i^2$$

$$= 125 \lim_{n \to +\infty} \frac{1}{n^3} \sum_{i=1}^{n} i^2$$

$$= 125 \lim_{n \to +\infty} \frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{125}{6} \lim_{n \to +\infty} \cdot \frac{(n^2+n)(2n+1)}{n^3}$$

$$= \frac{125}{6} \lim_{n \to +\infty} \cdot \frac{2n^3 + n^2 + 2n^2 + n}{n^3}$$

$$= \frac{125}{6} \cdot \lim_{n \to +\infty} \left(2 + \cancel{\frac{1}{n}} + \cancel{\frac{2}{n}} + \cancel{\frac{1}{n^2}}\right)$$

$$= \frac{250}{6} = \frac{125}{3}$$