17–18 Encontre uma equação polar para a curva representada pela equação cartesiana dada.

17. $x + y = 2$ **18.** $x^2 + y^2 = 2$

$y = 2 - x$

$y = 2 - x$

$r \operatorname{sen}\theta = 2 - r\cos\theta$

$r \operatorname{sen}\theta + r\cos\theta = 2$

$r(\operatorname{sen}\theta + \cos\theta) = 2$

$$r = \frac{2}{(\operatorname{sen}\theta + \cos\theta)}$$

$$r(\theta) = \frac{2}{(\operatorname{sen}\theta + \cos\theta)}$$

b) $x^2 + y^2 = 2$

$r^2 = 2 \quad \therefore \quad r = \sqrt{2}$ ou $r = -\sqrt{2}$

Ambas descrevem o mesmo lugar. Use uma ou outra.

<u>2ª maneira</u>

$x = r\cos\theta$ e $y = r\operatorname{sen}\theta$

$x^2 = r^2\cos^2\theta$ $\quad y^2 = r^2\operatorname{sen}^2\theta$

$x^2 + y^2 = 2$

$r^2\cos^2\theta + r^2\operatorname{sen}^2\theta = 2$

$r^2(\cos^2\theta + \operatorname{sen}^2\theta) = 2$

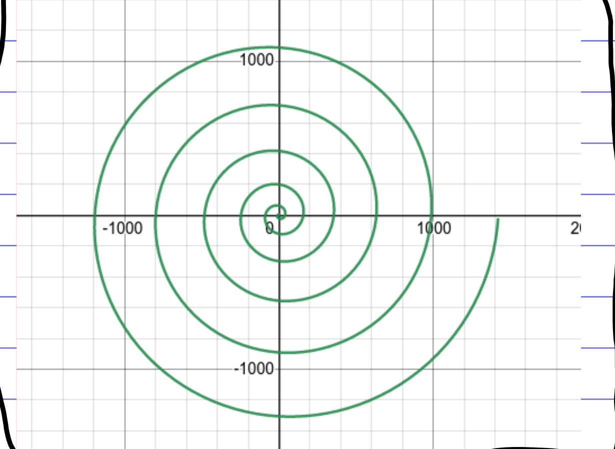
$r^2 = 2 \quad \therefore \quad r = \sqrt{2}$ ou $r = -\sqrt{2}$

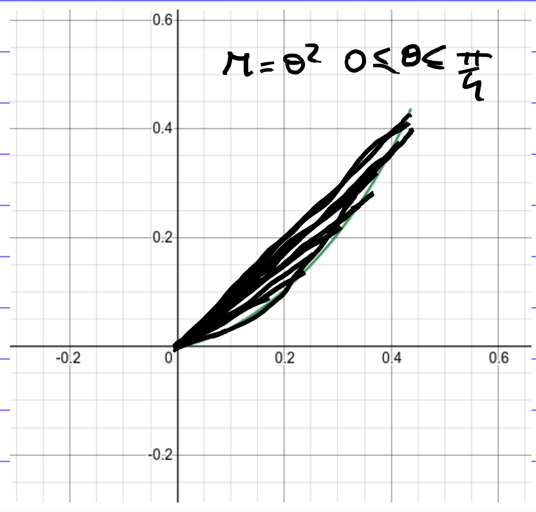
1–4 Encontre a área da região que é delimitada pelas curvas dadas e está no setor especificado.

1. $r = \theta^2, \quad 0 \leqslant \theta \leqslant \pi/4$

Qual é o gráfico de

$r = \theta^2, \quad 0 \leq \theta \leq 12\pi$





θ	r
0	0
$\frac{\pi}{10}$	$\frac{\pi^2}{100}$
$\frac{\pi}{6}$	$\frac{\pi^2}{36}$

$$A = \int_{\theta_i}^{\theta_f} \frac{1}{2} r^2 \, d\theta = \int_0^{\frac{\pi}{4}} \frac{1}{2} \theta^4 \, d\theta$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{4}} \theta^4 \, d\theta$$

$$= \frac{1}{2} \left. \frac{\theta^5}{5} \right|_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2} \left(\frac{\left(\frac{\pi}{4}\right)^5}{5} - \frac{0^5}{5} \right)$$

$$= \frac{1}{10} \left(\frac{\pi^5}{4^5} - 0 \right) = \frac{\pi^5}{4^5 \cdot 10}$$

Cálculo da área de um círculo de raio $r = 3$

$$A = \int_0^{2\pi} \frac{1}{2} 3^2 \, d\theta$$

$$A = \frac{9}{2} \, 2\pi = 9\pi$$

45–48 Calcule o comprimento exato da curva polar.

45. $r = 2\cos\theta, \quad 0 \leqslant \theta \leqslant \pi$

46. $r = 5^\theta, \quad 0 \leqslant \theta \leqslant 2\pi$

a) $$L = \int_{\theta_i}^{\theta_f} \sqrt{r^2 + r'^2} \, d\theta$$

$$L = \int_0^{\pi} \sqrt{(2\cos\theta)^2 + (-2\,\text{sen}\,\theta)^2} \, d\theta$$

$$L = \int_0^{\pi} \sqrt{4\cos^2\theta + 4\,\text{sen}^2\theta} \, d\theta$$

$$L = \int_0^{\pi} \sqrt{4} \, d\theta$$

$$L = \sqrt{4}(\pi - 0) = \sqrt{4}\,\pi = 2\pi$$

b) $$L = \int_{\theta_i}^{\theta_f} \sqrt{r^2 + r'^2} \, d\theta$$

$$r = 5^\theta \quad \therefore \quad r^2 = (5^\theta)^2 = 5^{2\theta}$$

$$r = 5^\theta \quad \therefore \quad r' = 5^\theta \cdot \ln 5 \quad \therefore (r')^2 = 5^{2\theta} (\ln 5)^2$$

$$L = \int_0^{2\pi} \sqrt{5^{2\theta} + 5^{2\theta} (\ln 5)^2} \, d\theta$$

$$L = \int_0^{2\pi} \sqrt{5^{2\theta} (1 + [\ln 5]^2)} \, d\theta$$

$$L = \int_0^{2\pi} 5^\theta \sqrt{1 + [\ln 5]^2} \, d\theta$$

$$L = \sqrt{1 + [\ln 5]^2} \int_0^{2\pi} 5^\theta \, d\theta$$

$$L = \sqrt{1 + (\ln 5)^2} \cdot \left. \frac{5^\theta}{\ln 5} \right|_0^{2\pi}$$

$$L = \sqrt{1 + (\ln 5)^2} \cdot \left(\frac{5^{2\pi}}{\ln 5} - \frac{5^0}{\ln 5} \right)$$

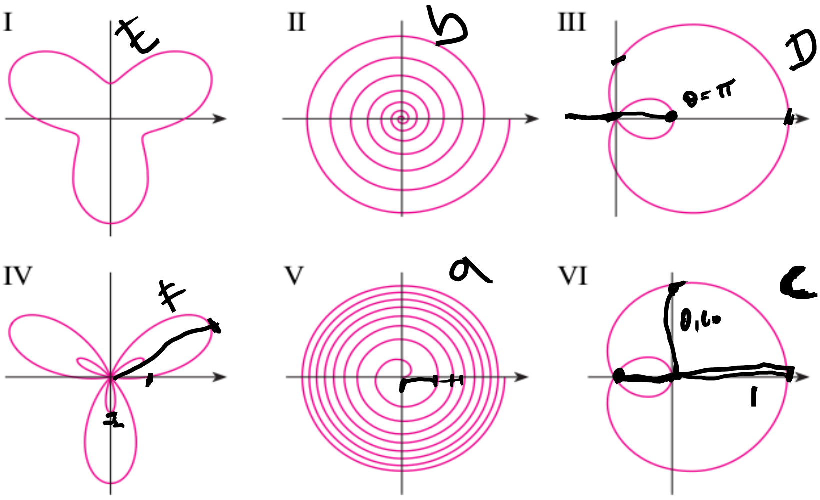
$$L = \sqrt{1 + (\ln 5)^2} \cdot \left(\frac{5^{2\pi} - 1}{\ln 5} \right) \quad \text{u.c.}$$

54. Associe as curvas polares com seus respectivos gráficos I–VI. Dê razões para suas escolhas. (Não use uma ferramenta gráfica.)

(a) $r = \sqrt{\theta}$, $0 \leqslant \theta \leqslant 16\pi$ (b) ~~$r = \theta^2$~~, $0 \leqslant \theta \leqslant 16\pi$

(c) $r = \cos(\theta/3)$ (d) $r = 1 + 2\cos\theta$ $r(0) = 3$ | $r(\frac{\pi}{2}) = 1$

(e) $r = 2 + \text{sen}\, 3\theta$ (f) $r = 1 + 2\,\text{sen}\, 3\theta$ $r(0) = 1$ | $r(\frac{\pi}{2}) = -1$



21–26 Encontre uma equação polar para a curva representada pela equação cartesiana dada.

21. $y = 2$ **22.** $y = x$

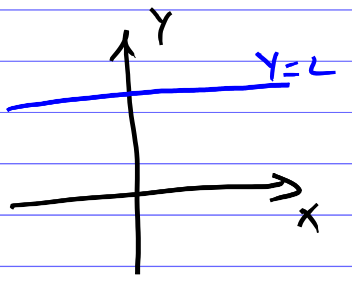
23. $y = 1 + 3x$ **24.** $4y^2 = x$

25. $x^2 + y^2 = 2cx$ **26.** $xy = 4$

21. $y = 2$

$r \operatorname{sen}\theta = 2$

$$\boxed{r = \frac{2}{\operatorname{sen}\theta}}$$



23. $y = 1 + 3x$

$r \operatorname{sen}\theta = 1 + 3r\cos\theta$

$r \operatorname{sen}\theta - 3r = 1$

$r(\operatorname{sen}\theta - 3\cos\theta) = 1$

$$r = \frac{1}{\operatorname{sen}\theta - 3\cos\theta}$$

22. $y = x$

$r \operatorname{sen}\theta = r\cos\theta$

$\operatorname{sen}\theta = \cos\theta$

$$\boxed{\theta = \frac{\pi}{4}}$$