Derivação implícita

1 Calcule $\dfrac{\partial z}{\partial x}$ para $\underbrace{z + y = x^2}$

Função implícita de 2 variáveis

Vamos explicitar

$$z + y = x^2$$

$$\underbrace{z = x^2 - y}$$

Função explícita de 2 variáveis. z é função de x e y.

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(x^2 - y) = \frac{\partial x^2}{\partial x} - \frac{\partial y}{\partial x}$$

$$= 2x - 0$$

$$\frac{\partial z}{\partial x} = 2x$$

Sem explicitar

$$z + y = x^2$$

$$\frac{\partial z}{\partial \boxed{x}} \left\{ \quad \frac{\partial}{\partial x}(z + y) = \frac{\partial x^2}{\partial x} \right.$$

Se não estiver aqui, é constante.

$$\frac{\partial z}{\partial x} + \frac{\partial y}{\partial x} = \frac{\partial x^2}{\partial x}$$

$$\frac{\partial z}{\partial x} + 0 = 2x$$

$$\frac{\partial z}{\partial x} = 2x$$

E se quiséssemos $\dfrac{\partial x}{\partial y} = ?$

$$z + y = x^2$$

Se for explicitar $x = \sqrt{z + y}$

$$x = -\sqrt{z + y}$$

Derivando implicitamente:

$$\frac{\partial \boxed{x}}{\partial \boxed{y}} \qquad \frac{\partial}{\partial y}(z + y) = \frac{\partial}{\partial y} x^2$$

$$\frac{\partial z}{\partial y} + \frac{\partial y}{\partial y} =$$

$$0 + 1 = 2x \frac{\partial x}{\partial y}$$

$$\frac{\partial x}{\partial y} = \frac{1}{2x}$$

2. Calcule $\frac{\partial z}{\partial x}$ para

$$z^5 + z\cos(xy) + z^2x = 0$$

$$\frac{\partial}{\partial x}\left(z^5 + z\cos(xy) + z^2x\right) = \frac{\partial 0}{\partial x}$$

$$\frac{\partial}{\partial x}(z^5) + \frac{\partial}{\partial x}(z\cos(xy)) + \frac{\partial}{\partial x}(z^2x) = \frac{\partial 0}{\partial x}$$

$$5z^4\cdot\frac{\partial z}{\partial x} + \frac{\partial z}{\partial x}\cdot\cos(xy) + z\frac{\partial}{\partial x}(\cos(xy)) + \frac{\partial z^2}{\partial x}x + z^2\frac{\partial x}{\partial x} = 0$$

$$5z^4\cdot\frac{\partial z}{\partial x} + \frac{\partial z}{\partial x}\cdot\cos(xy) + \underbrace{z\frac{\partial}{\partial x}(\cos(xy))}_{\text{Regra da cadeia}} + \frac{\partial z^2}{\partial x}x + z^2 = 0$$

$$5z^4\cdot\frac{\partial z}{\partial x} + \frac{\partial z}{\partial x}\cdot\cos(xy) + z\left[-\text{sen}(xy)\cdot\frac{\partial(xy)}{\partial x}\right] + 2z\frac{\partial z}{\partial x}x + z^2 = 0$$

$$5z^4\cdot\frac{\partial z}{\partial x} + \frac{\partial z}{\partial x}\cdot\cos(xy) + z\left[-\text{sen}(xy)\cdot y\right] + 2z\frac{\partial z}{\partial x}x + z^2 = 0$$

$$\frac{\partial z}{\partial x}\left(5z^4 + \cos(xy) + 2zx\right) = yz\,\text{sen}(xy) - z^2$$

$$\frac{\partial z}{\partial x} = \frac{yz\,\text{sen}(xy) - z^2}{5z^4 + \cos(xy) + 2zx}$$

Exemplo mais simples

$$z\cos(xz) = y$$

$$\frac{\partial z}{\partial x} = ?$$

$$\frac{\partial}{\partial x}\left(z\cos(xz)\right) = \frac{\partial y}{\partial x}$$

$$\frac{\partial}{\partial x}\left(z\cos(xz)\right) = 0$$

$$\frac{\partial z}{\partial x}\cdot\cos(xz) + z\frac{\partial}{\partial x}\cos(xz) = 0$$

$$\frac{\partial z}{\partial x}\cdot\cos(xz) + z\left[-\operatorname{sen}(xz)\cdot\frac{\partial}{\partial x}(xz)\right] = 0$$

$$\frac{\partial z}{\partial x}\cdot\cos(xz) + z\left(-\operatorname{sen}(xz)\left[\frac{\partial x}{\partial x}\cdot z + x\frac{\partial z}{\partial x}\right]\right) = 0$$

$$\frac{\partial z}{\partial x}\cdot\cos(xz) + z\left(-\operatorname{sen}(xz)\left[z + x\frac{\partial z}{\partial x}\right]\right) = 0$$

$$\frac{\partial z}{\partial x}\cdot\cos(xz) + -z^2\operatorname{sen}(xz) - xz\operatorname{sen}(xz)\frac{\partial z}{\partial x} = 0$$

$$\frac{\partial z}{\partial x}\left(\cos(xz) - xz\operatorname{sen}(xz)\right) = z^2\operatorname{sen}(xz)$$

$$\frac{\partial z}{\partial x} = \frac{z^2\operatorname{sen}(xz)}{\cos(xz) - xz\operatorname{sen}(xz)}$$

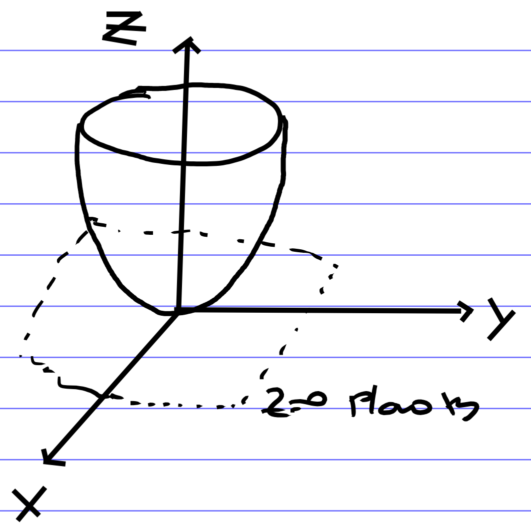
Equação do plano tangente

Se uma superfície é dada como função de duas variáveis $f(x,y)$ podemos calcular o plano tangente à essa superfície no ponto (x_0, y_0, z_0) com o uso da expressão

$$z = z_0 + \left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)} (x - x_0) + \left.\frac{\partial f}{\partial y}\right|_{(x_0,y_0)} (y - y_0)$$

Exercício

1. Calcule a eq. do plano tangente à superfície $f(x,y) = x^2 + y^2$ nos pontos $(0,0,0)$ e $(1,3,10)$



Para o ponto $(0,0,0)$

$$z = z_0 + \left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)} (x - x_0) + \left.\frac{\partial f}{\partial y}\right|_{(x_0,y_0)} (y - y_0)$$

$$(x_0, y_0, z_0) = (0,0,0)$$

$$\left.\frac{\partial f}{\partial x}\right|_{(0,0)} = 2x\Big|_{(0,0)} = 0$$

$$\left.\frac{\partial f}{\partial y}\right|_{(0,0)} = 2y\Big|_{(0,0)} = 0$$

$$z = 0 + 0.(x-0) + 0(y-0)$$

$$z = 0$$

P/ o ponto $(1,3,10)$

$$(x_0, y_0, z_0) = (1,3,10)$$

$$\left.\frac{\partial f}{\partial x}\right|_{(1,3)} = 2x\Big|_{(1,3)} = 2$$

$$\left.\frac{\partial f}{\partial y}\right|_{(1,3)} = 2y\Big|_{(1,3)} = 6$$

$$z = 10 + 2(x-1) + 6(y-3)$$

$$z = 10 + 2x - 2 + 6y - 18$$

$$\boxed{z = 2x + 6y - 10}$$

https://www.geogebra.org/3d/ndvczbr8