No cálculo 1

$f(x) = x^2$

$\mathbb{R} \to \mathbb{R}$

No início do cálculo 2

$\vec{r}(t) = \langle t, t^2, t^3 \rangle$

$\mathbb{R} \to \mathbb{R}^3$

Agora

$f(x,y,z) = xy^2 + z$

$\mathbb{R}^3 \to \mathbb{R}$

$\boxed{\text{Área do triângulo? Base vezes altura sobre dois}}$

Verbalmente

$\boxed{A(b,h) = \frac{bh}{2}}$

Algebricamente

| 1 variável | 2 variáveis |
|---|---|
| Área do quadrado. | Área do retângulo |
| $A(L) = L^2$ | $A(b,h) = bh$ |

Idade da Morte (escolaridade, Renda, , , , )

RepresentaÃ§Ã£o em Tabela de uma FunÃ§Ã£o de Duas Vari

Umidade relativa (%)

Temperatura real (°C)

| T \ H | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 |
|---|---|---|---|---|---|---|---|---|---|
| 26 | 28 | 28 | 29 | 31 | 31 | 32 | 33 | 34 | 35 |
| 28 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 30 | 34 | 35 | 36 | 37 | 38 | 40 | 41 | 42 | 43 |
| 32 | 37 | 38 | 39 | 41 | 42 | 43 | 45 | 46 | 47 |
| 34 | 41 | 42 | 43 | 45 | 47 | 48 | 49 | 51 | 52 |
| 36 | 43 | 45 | 47 | 48 | 50 | 51 | 53 | 54 | 56 |

$S(T,H)$

$S(30,60) = 38$

$S(31,60) \approx 40$

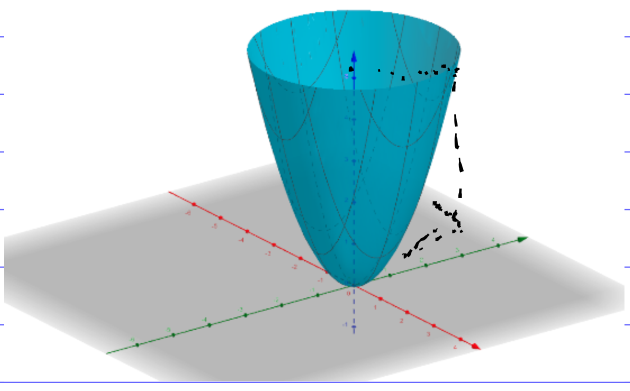
**TABELA 1** Índice de sensação térmica como função da temperatura do ar e velocidade do vento

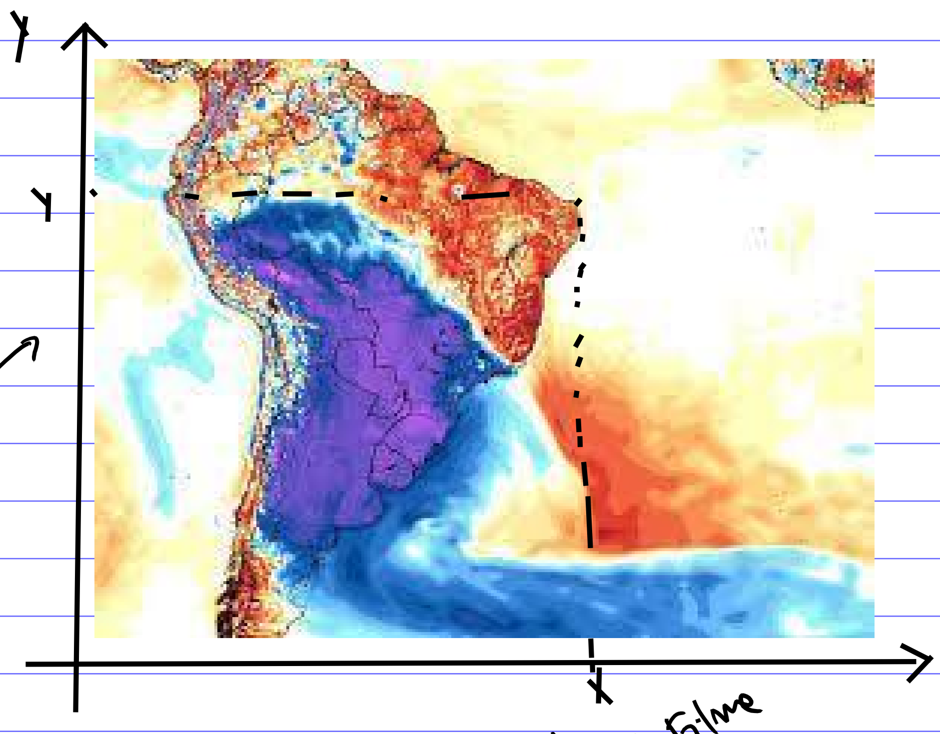
Velocidade do vento (km/h)

Temperatura real (°C)

| T \ v | 5 | 10 | 15 | 20 | 25 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 4 | 3 | 2 | 1 | 1 | 0 | −1 | −1 | −2 | −2 | −3 |
| 0 | −2 | −3 | −4 | −5 | −6 | −6 | −7 | −8 | −9 | −9 | −10 |
| −5 | −7 | −9 | −11 | −12 | −12 | −13 | −14 | −15 | −16 | −16 | −17 |
| −10 | −13 | −15 | −17 | −18 | −19 | −20 | −21 | −22 | −23 | −23 | −24 |
| −15 | −19 | −21 | −23 | −24 | −25 | −26 | −27 | −29 | −30 | −30 | −31 |
| −20 | −24 | −27 | −29 | −30 | −32 | −33 | −34 | −35 | −36 | −37 | −38 |
| −25 | −30 | −33 | −35 | −37 | −38 | −39 | −41 | −42 | −43 | −44 | −45 |
| −30 | −36 | −39 | −41 | −43 | −44 | −46 | −48 | −49 | −50 | −51 | −52 |
| −35 | −41 | −45 | −48 | −49 | −51 | −52 | −54 | −56 | −57 | −58 | −60 |
| −40 | −47 | −51 | −54 | −56 | −57 | −59 | −61 | −63 | −64 | −65 | −67 |

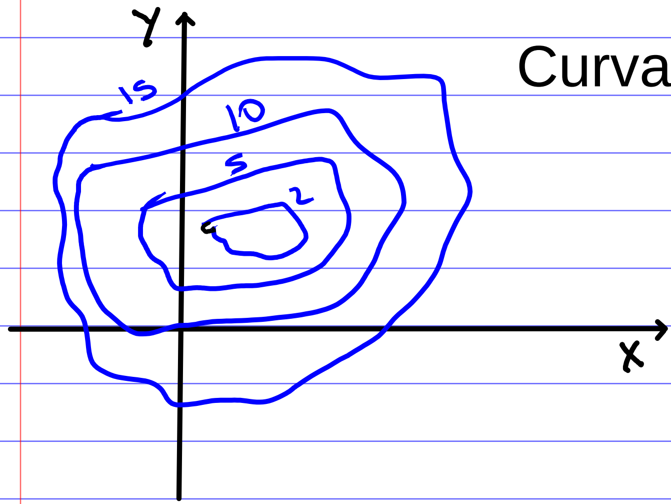
Outra RepresentaÃ§Ã£o GrÃ¡fica de FunÃ§Ã£o de Duas Var

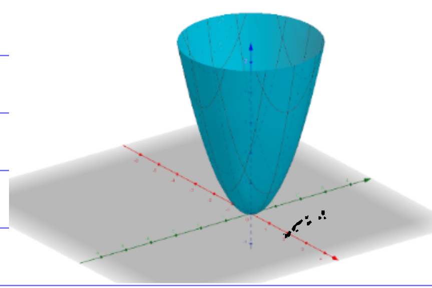


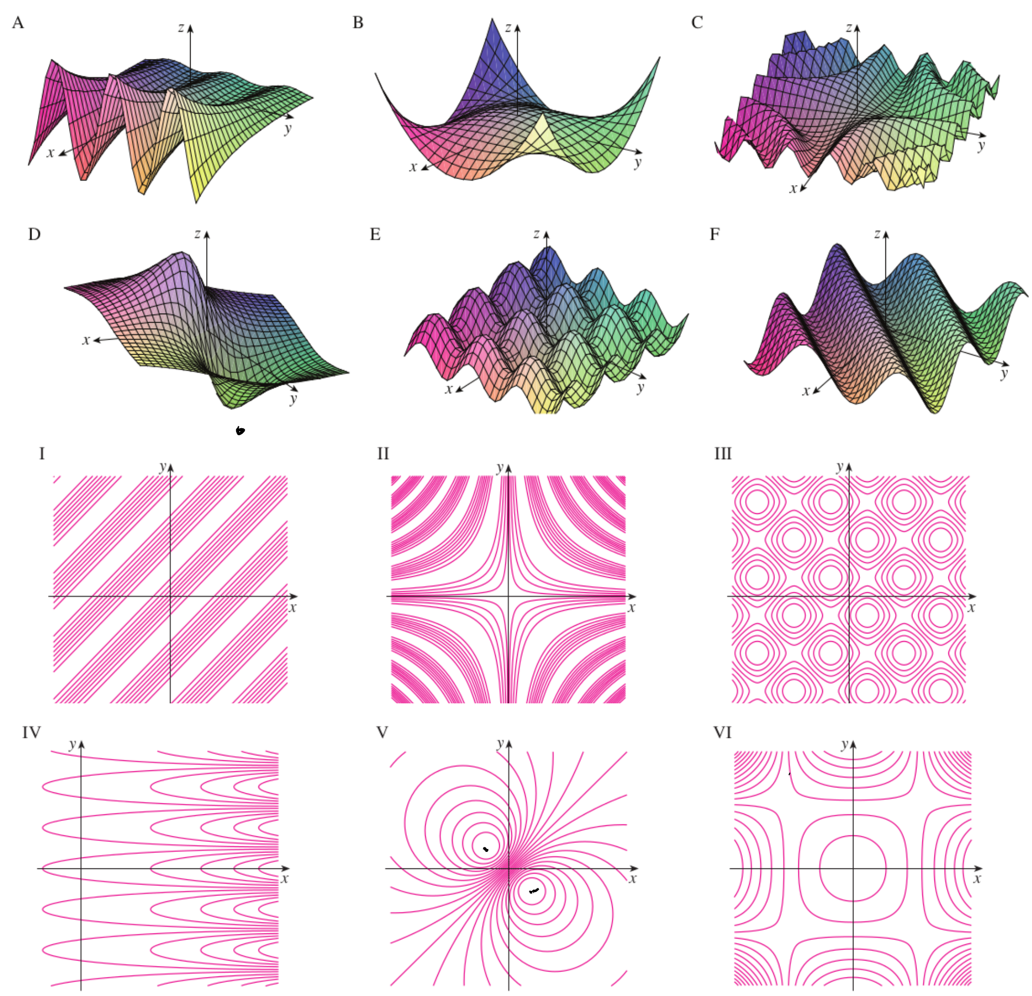



$T(x,y,t) \to$ Filme

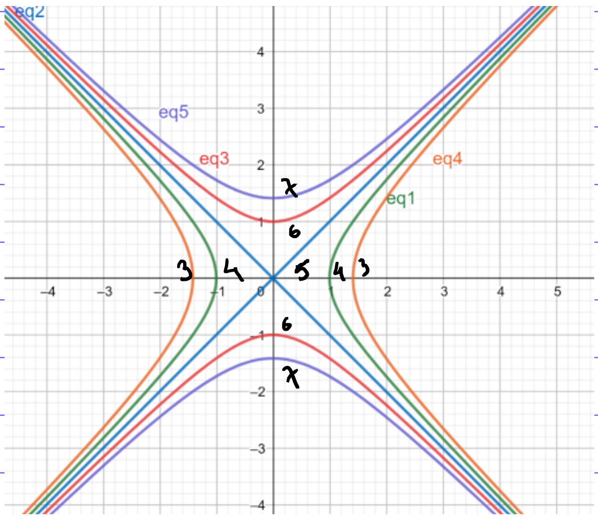
# Curvas de NÃ-vel





# Limites




Umidade relativa (%)

| Temperatura real (°C) — T \ H | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 |
|---|---|---|---|---|---|---|---|---|---|
| 26 | 28 | 28 | 29 | 31 | 31 | 32 | 33 | 34 | 35 |
| 28 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 30 | 34 | 35 | 36 | 37 | 38 | 40 | 41 | 42 | 43 |
| 32 | 37 | 38 | 39 | 41 | 42 | 43 | 45 | 46 | 47 |
| 34 | 41 | 42 | 43 | 45 | 47 | 48 | 49 | 51 | 52 |
| 36 | 43 | 45 | 47 | 48 | 50 | 51 | 53 | 54 | 56 |

a) $\displaystyle \lim_{(x,y)\to(2,-1)} \frac{x^2y + xy^2}{x^2-y^2}$

b) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^2}$

**4.** Verifique se, para a função de produção de Cobb-Douglas

$$P(L, K) = 1{,}01L^{0{,}75}K^{0{,}25}$$

discutida no Exemplo 3, a produção dobrará se as quantidades de trabalho e a de capital investido forem dobradas. Determine se isso também é verdade para uma função de produção genérica

$$P(L, K) = bL^{\alpha}K^{1-\alpha}$$

$$\begin{aligned} P(2L,2K) &= 1.01\,(2L)^{0.75}\,(2K)^{0.25} \\ &= 1.01\; 2^{0.75}\, L^{0.75}\; 2^{0.25}\, K^{0.25} \\ &= 1.01\; 2^{1}\, L^{0.75}\, K^{0.25} \\ &= 2\,(1.01\, L^{0.75}\, K^{0.25}) \\ &= 2\,P(L,K) \end{aligned}$$

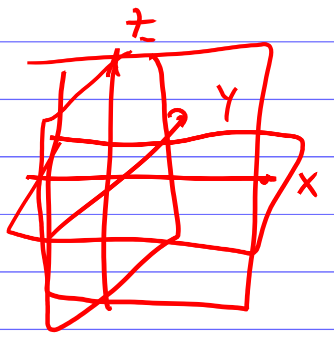
$$\begin{aligned} P(2L,2K) &= b\,(2L)^{\alpha}\,(2K)^{1-\alpha} \\ &= b\,2^{\alpha}L^{\alpha}\; 2^{1-\alpha}\,K^{1-\alpha} \\ &= b\,2^{\alpha}2^{1-\alpha}\; L^{\alpha}K^{1-\alpha} \\ &= b\,2\; L^{\alpha}K^{1-\alpha} \\ &= 2\,P(L,K) \end{aligned}$$

**11.** Seja $f(x, y, z) = \sqrt{x} + \sqrt{y} + \sqrt{z} + \ln(4 - x^2 - y^2 - z^2)$.
(a) Calcule $f(1, 1, 1)$.
(b) Determine o domínio de $f$.

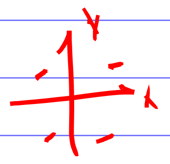
**12.** Seja $g(x, y, z) = x^3 y^2 z \sqrt{10 - x - y - z}$.
(a) Calcule $g(1, 2, 3)$.
(b) Determine o domínio de $g$.

11 - a $f(1,1,1) = \sqrt{1} + \sqrt{1} + \sqrt{1} + \ln(4-1-1-1)$
$= 1 + 1 + 1 + \ln(1)$
$f(1,1,1) = 3$

b Domínio



D: $x \geq 0,\ y \geq 0,\ z \geq 0$



$4 - x^2 - y^2 - z^2 > 0$
$-x^2 - y^2 - z^2 > -4$
$x^2 + y^2 + z^2 < 4$

$(1,-1,1)$

D: $\{(x,y,z) \in \mathbb{R}^3 \;/\; x^2+y^2+z^2 < 4 \;\text{e}\; x \geq 0, y \geq 0, z \geq 0\}$

12

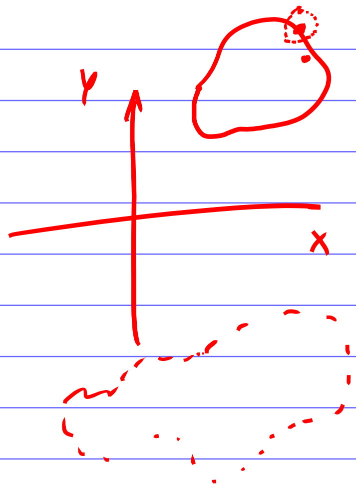
$g(x,y,z) = x^3y^2z\sqrt{10-x-y-z}$

$g(1,2,3) = 1^3\,2^2\,3\,\sqrt{10-1-2-3}$
$= 12\sqrt{4}$
$= 24$

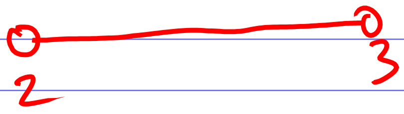
$10 - x - y - z \geq 0$
$10 \geq x+y+z$
$\boxed{x+y+z \leq 10}$

D: $\{(x,y,z) \in \mathbb{R}^3 \;/\; x+y+z \leq 10\}$

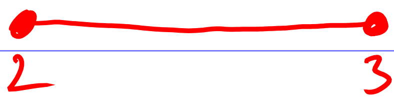
$g(7,8,9) \nexists$ pois $7+8+9 > 10$



$]2,3[$



$[2,3]$



$-\frac{1}{2} - 1 = \frac{-1-2}{2}$

$\vec{r}(t) = \langle t^2, \sqrt{t^2+1}, t+t^3 \rangle$

$\vec{v}(t) = \langle 2t, (t^2+1)^{-\frac{1}{2}} \cdot t, 1+3t^2 \rangle$

$\vec{a}(t) = \langle 2, -\frac{1}{2}(t^2+1)^{-\frac{3}{2}} \cdot 2t \cdot t + (t^2+1)^{-\frac{1}{2}}, 6t \rangle$

$\vec{a}(2) = \langle 2, -\frac{1}{2}(5)^{-3/2} \cdot 8 + (5)^{-1/2}, 12 \rangle$

$\vec{a}(2) = \langle 2, \frac{-8}{2 \cdot 5^{3/2}} + \frac{1}{\sqrt{5}}, 12 \rangle$

$\vec{a}(2) = \langle 2, \frac{-8}{2 \cdot 5\sqrt{5}} + \frac{1}{\sqrt{5}}, 12 \rangle$

$\vec{a}(2) = \langle 2, \frac{-8+10}{10\sqrt{5}}, 12 \rangle$

$\vec{a}(2) = \langle 2, \frac{1}{5\sqrt{5}}, 12 \rangle$

$\vec{a}(2) = \langle 2, \frac{\sqrt{5}}{5\sqrt{5}\sqrt{5}}, 12 \rangle$

$a(2) = \langle 2, \frac{\sqrt{5}}{25}, 12 \rangle$

Qual é a aceleração no instante t=2 de uma partícula cujo movimento é descrito por $\vec{r}(t) = < t^2, \sqrt{t^2+1}, t+t^3 >$?

A) $< 2, 4, 5 >$

✓ B) $< 2, \frac{\sqrt{5}}{25}, 12 >$