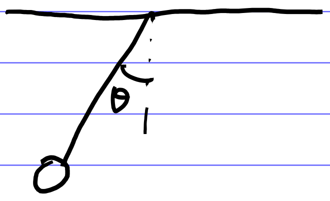
Aproximação Linear



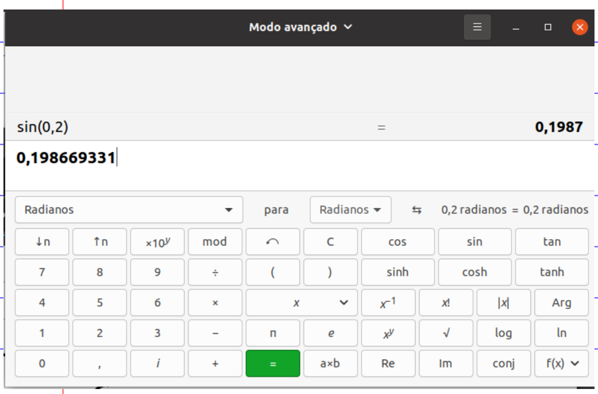
$f(x) = \operatorname{sen} x \approx x$

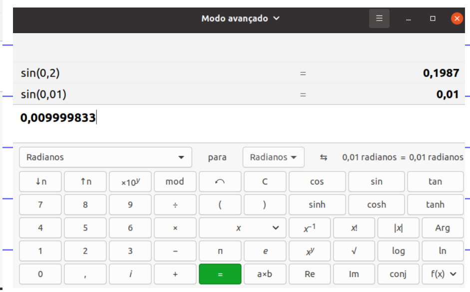
Nas vizinhanças de $x \approx$

$\operatorname{sen}(0.01) \approx 0.01$

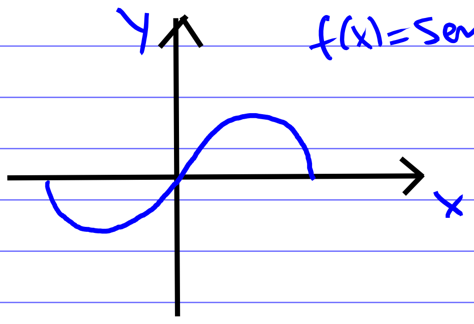
$\operatorname{sen}(0.2) \approx 0.2$

Em rad.





A Aprox. Linear consiste em substituir a função por uma eq. da reta ou plano.



$f(x) = \operatorname{sen} x$

Eq. da reta tangente à função $f(x,y)$ no ponto (x_0, y_0)

$$y = y_0 + \frac{df}{dx}(x - x_0)$$

$(x_0, y_0) = (0,0)$ $\quad f = \operatorname{sen} x \therefore \frac{df}{dx} = \cos x$

$$\left.\frac{df}{dx}\right|_{(0,0)} = \cos 0 = 1$$

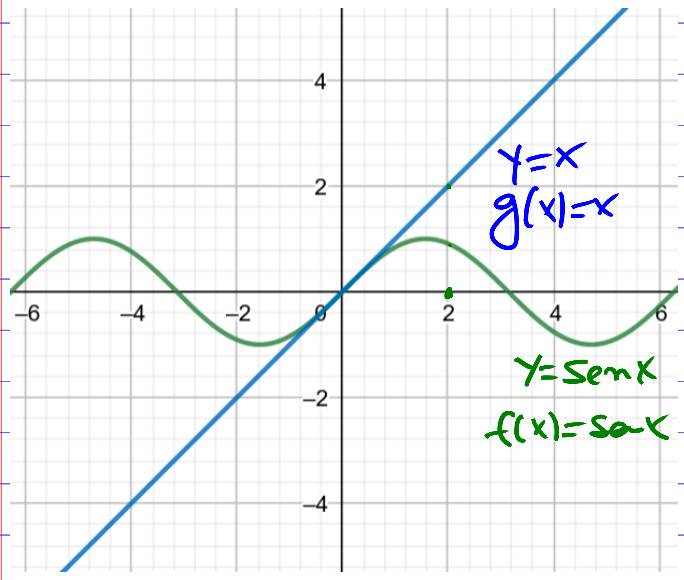
$y = 0 + 1(x - 0)$

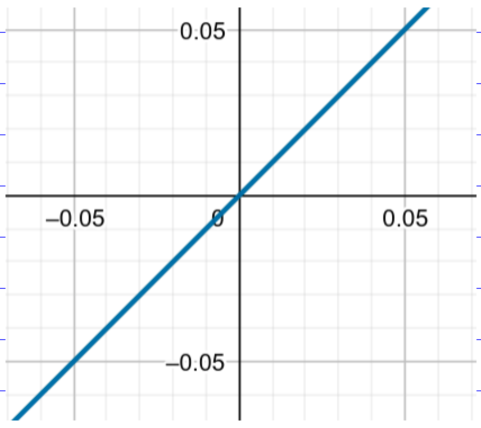
$y = x$

Eq. da reta tangente

$g(x) = x$

$g(x)$ é a aproximação linear p/ $f(x)$ entorno de $x=0$.





Aproximação linear de uma função com mais de uma variável.

Se forem duas variáveis, calcula-se o plano tangente.

Exemplo: Calcule a aproximação linear para a função $\boxed{f(x,y) = \sqrt{xy}}$ em torno do ponto $(1,2)$

Solução: Eq. do plano tangente.

$$\boxed{z = z_0 + \left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)} (x - x_0) + \left.\frac{\partial f}{\partial y}\right|_{(x_0,y_0)} (y - y_0)}$$

$$\frac{\partial f}{\partial x} = \frac{\partial (xy)^{1/2}}{\partial x} = \frac{1}{2}(xy)^{-1/2} \cdot y = \frac{y}{2\sqrt{xy}}$$

$(x_0, y_0) = (1,2)$

$$\left.\frac{\partial f}{\partial x}\right|_{(1,2)} = \left.\frac{y}{2\sqrt{xy}}\right|_{(1,2)} = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\frac{\partial f}{\partial y} = \frac{\partial (xy)^{1/2}}{\partial y} = \frac{1}{2}(xy)^{-1/2} \cdot x = \frac{x}{2\sqrt{xy}}$$

$$\left.\frac{\partial f}{\partial y}\right|_{(1,2)} = \left.\frac{x}{2\sqrt{xy}}\right|_{(1,2)} = \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{2}}{4}$$

$z = \sqrt{xy}$, pois $f(x,y) = \sqrt{xy}$

$z_0 = \sqrt{x_0 y_0}$

$z_0 = \sqrt{2}$

$$z = z_0 + \left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)} (x - x_0) + \left.\frac{\partial f}{\partial y}\right|_{(x_0,y_0)} (y - y_0)$$

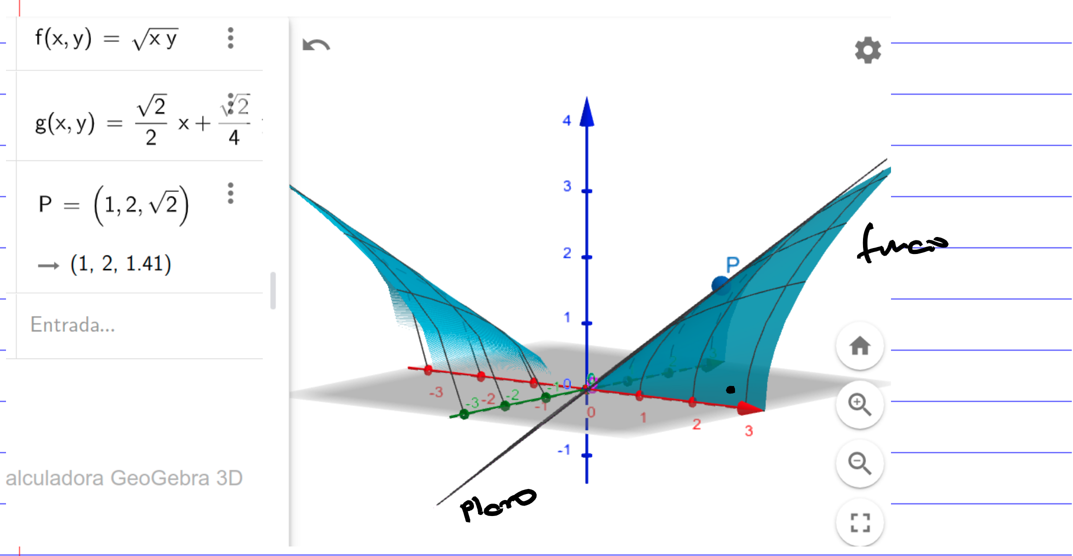
$$z = \sqrt{2} + \frac{\sqrt{2}}{2}(x - 1) + \frac{\sqrt{2}}{4}(y - 2)$$

$$z = \cancel{\sqrt{2}} + \frac{\sqrt{2}}{2}x - \cancel{\frac{\sqrt{2}}{2}} + \frac{\sqrt{2}}{4}y - \cancel{\frac{\sqrt{2}}{2}}$$

$\boxed{z = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{4}y}$ Eq. do plano tangente

A aproximação linear

$\boxed{g(x,y) = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{4}y}$



https://www.geogebra.org/3d/hxzcjtwq

Notações

Derivada de x em relação a t:

$\dot{x}$ Notação de Newton

$\frac{dx}{dt}$ Notação de Leibniz

x' Notação de Lagrange

$$y = \operatorname{sen} x^2$$

$$\frac{dy}{dx} = ?$$

$$u = x^2$$

$$y = \operatorname{sen} u$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \cos u \cdot 2x$$
$$= \cos(x^2)\, 2x$$

Outro exemplo:

$$y = \cos(e^{x^2})$$

$$\frac{dy}{dx}$$

$$u = x^2$$

$$y = \cos e^u$$

$$w = e^u$$

$$y = \cos w$$

$$\frac{dy}{dx} = \frac{dy}{dw} \cdot \frac{dw}{du} \cdot \frac{du}{dx}$$

$$= -\operatorname{sen} w \cdot e^u \cdot 2x$$

$$= -2x\, e^{x^2} \operatorname{sen}(e^{x^2})$$

$$f(x,y) = xy^2$$

$$x = \cos t \quad y = e^t$$

$$\frac{df}{dx} = ?$$

$$\frac{df}{dx} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

$$\frac{df}{dx} = y^2(-\operatorname{sen} t) + 2xy\, e^t$$

$$= -e^{2t} \operatorname{sen} t + 2 \cos t\, e^{2t}$$

Qual é a expressão para $\frac{df}{dt}$ quando $f = f(x,y,z)$ e $x = x(t)$, $y = y(t)$ e $z = z(t)$

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial z} \frac{dz}{dt}$$

Qual é a expressão para $\frac{\partial f}{\partial u}$ quando $f = f(x,y,z)$ e $x = x(u,w)$, $y = y(u,w)$, $z = z(u,w)$

$$\frac{\partial f}{\partial u} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial u} + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial u}$$