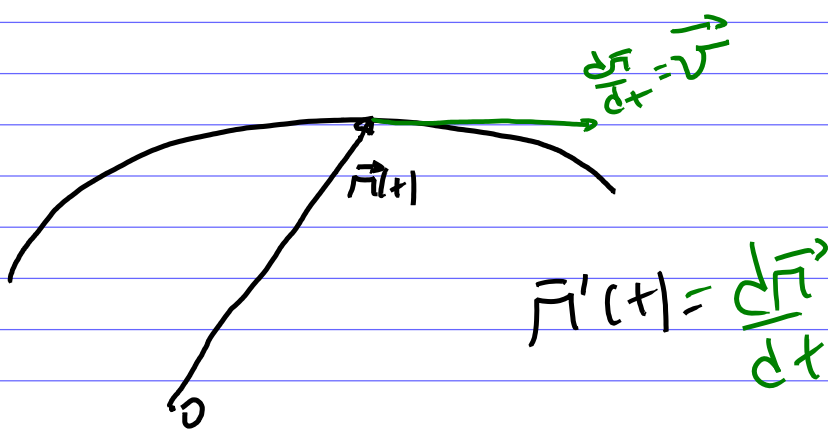


$$\vec{r}(t) = \langle \cos t, \sin t, t \rangle$$



$$\vec{T} = \frac{\frac{d\vec{r}}{dt}}{\left| \frac{d\vec{r}}{dt} \right|} = \frac{\vec{v}}{|\vec{v}|}$$

Vetor tangente unitário.

## Exercício

Calcule o vetor tangente unitário.

$$\vec{r}(t) = \langle t, t^2, 2 \rangle$$

x y

$$\begin{cases} x=t \\ y=t^2 \end{cases} \Rightarrow y=x^2$$

Eq. vet. tang.

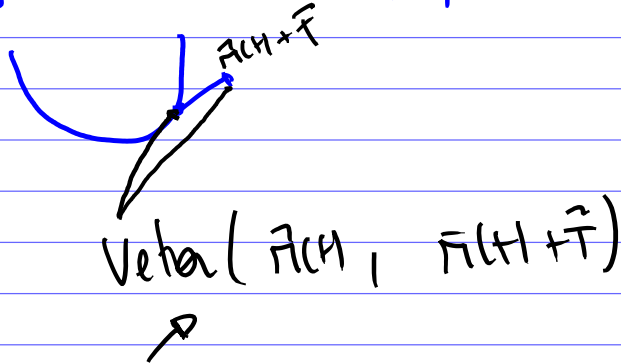
$$\vec{r}'(t) = \langle 1, 2t, 0 \rangle$$

$$|\vec{r}'(t)| = \sqrt{1^2 + (2t)^2 + 0^2} = \sqrt{1 + 4t^2}$$

$$\vec{T} = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \frac{\vec{r}'}{|\vec{r}'|} = \frac{\langle 1, 2t, 0 \rangle}{\sqrt{1 + 4t^2}}$$

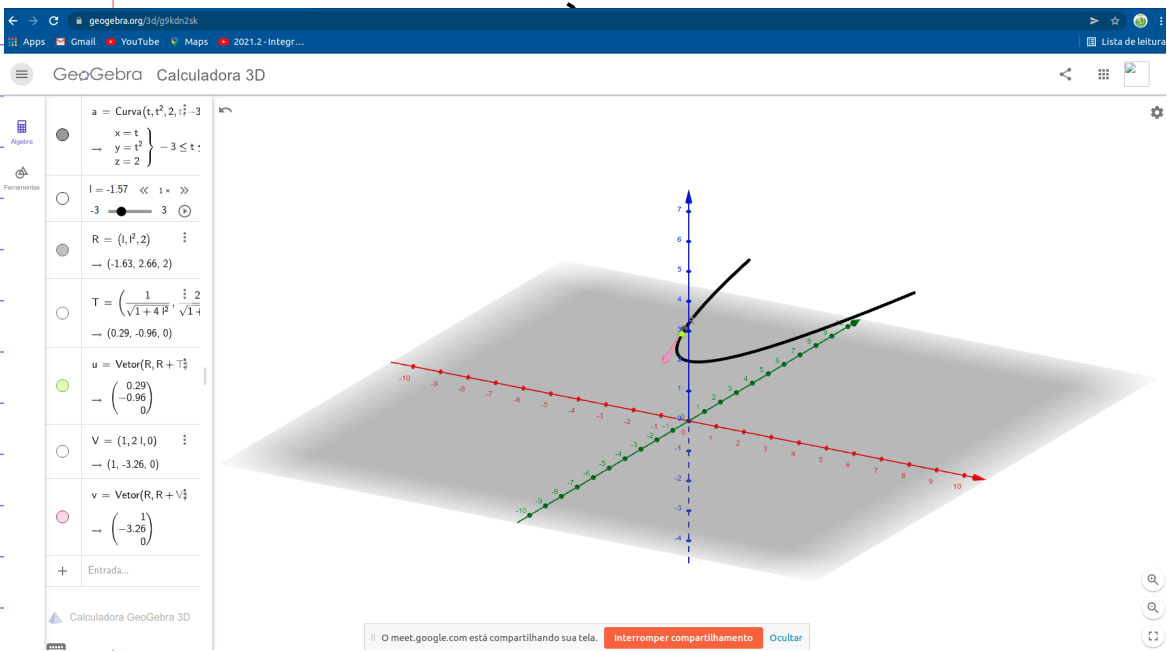
Não precisamos indicar o tempo todo que  $\vec{r}$  é função de  $t$

No geogebra, Vetor( $P_{ini}, P_{fin}$ )



Instrução para desenhar o vetor tangente unitário  $\vec{T}$  na curva deixada por  $\vec{r}(t)$

<https://www.geogebra.org/3d/g9kdn2sk>



21. Se  $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ , encontre  $\mathbf{r}'(t)$ ,  $\mathbf{T}(1)$ ,  $\mathbf{r}''(t)$  e  $\mathbf{r}'(t) \times \mathbf{r}''(t)$ .

$$\vec{r}(t) = \langle t, t^2, t^3 \rangle \quad \text{Posição}$$

$$\vec{r}'(t) = \langle 1, 2t, 3t^2 \rangle \quad \text{velocidade}$$

$$\vec{T}(1) = \frac{\vec{r}'(1)}{|\vec{r}'(1)|} = \frac{\langle 1, 2, 3 \rangle}{\sqrt{1+4+9}}$$

$$\vec{T}(1) = \frac{\langle 1, 2, 3 \rangle}{\sqrt{14}}$$

$$\vec{r}''(t) = \langle 0, 2, 6t \rangle \quad \text{Aceleração}$$

$$\vec{r}' \times \vec{r}'' = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix}$$

$$= 0\hat{k} - 6t^2\hat{i} - 6t\hat{j} + 12t^2\hat{i} + 0\hat{j} + 2\hat{k}$$

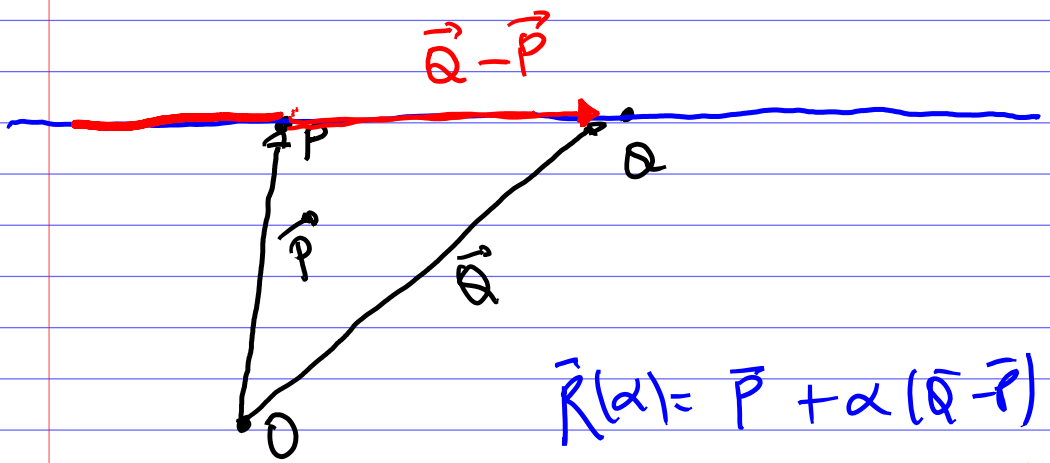
$$\vec{r}' \vee \vec{r}'' = 6t^2\hat{i} - 6t\hat{j} + 2\hat{k}$$

17-20 Encontre uma equação vetorial e equações paramétricas para o segmento de reta que liga  $P$  e  $Q$ .

17.  $P(0, 0, 0)$ ,  $Q(1, 2, 3)$

18.  $P(1, 0, 1)$ ,  $Q(2, 3, 1)$

$$\vec{R}(t) = \vec{P} + t(\vec{Q} - \vec{P})$$



$$\vec{R}(\alpha) = \vec{P} + \alpha(\vec{Q} - \vec{P})$$

$$R(t) = P + \alpha(Q - P)$$

$$\begin{array}{l} P(0,0,0) \\ Q(1,2,3) \end{array} \left| \begin{array}{l} P(1,0,1) \\ Q(2,3,1) \end{array} \right.$$

a)  $\vec{R}(\alpha) = \langle 0, 0, 0 \rangle + \alpha(\langle 1, 2, 3 \rangle - \langle 0, 0, 0 \rangle)$

$$\vec{R}(\alpha) = \alpha \langle 1, 2, 3 \rangle$$

Eq. vetorial

$$x = \alpha$$

$$y = 2\alpha$$

$$z = 3\alpha$$

Eq. Paramétricas

b)  $\vec{R}(\alpha) = \langle 1, 0, 1 \rangle + \alpha(\langle 2, 3, 1 \rangle - \langle 1, 0, 1 \rangle)$

$$\vec{R}(\alpha) = \langle 1, 0, 1 \rangle + \alpha \langle 1, 3, 0 \rangle$$

$$\vec{R}(\alpha) = \langle 1 + \alpha, 3\alpha, 1 \rangle$$

Eq. vetorial

$$x = 1 + \alpha$$

$$y = 3\alpha$$

$$z = 1$$

Eq. Paramétricas

<https://www.geogebra.org/3d/zq78zxdf>

