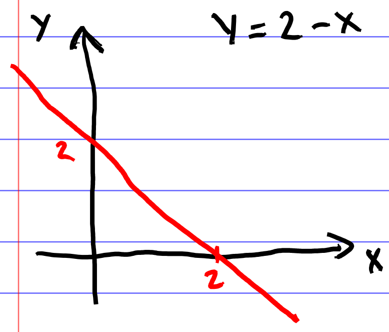
17–18 Encontre uma equação polar para a curva representada pela equação cartesiana dada.

A) **17.** $x + y = 2$ B) **18.** $x^2 + y^2 = 2$

A)



$y = 2 - x$

Convertendo para coordenadas Polares:

$y = 2 - x$

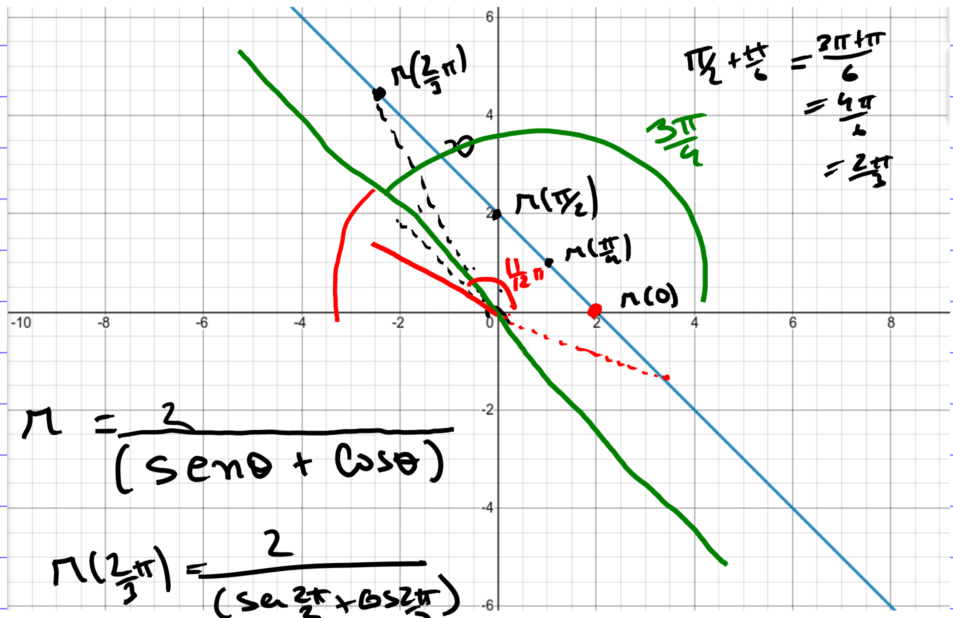
$r\,\mathrm{sen}\,\theta = 2 - r\cos\theta$

$r\,\mathrm{sen}\,\theta + r\cos\theta = 2$

$r(\mathrm{sen}\,\theta + \cos\theta) = 2$

$$\boxed{r = \frac{2}{(\mathrm{sen}\,\theta + \cos\theta)}}$$

https://www.desmos.com/calculator?lang=pt-BR



$$r = \frac{2}{(\mathrm{sen}\,\theta + \cos\theta)}$$

$$r\left(\frac{2}{3}\pi\right) = \frac{2}{(\mathrm{sen}\,\frac{2\pi}{3} + \cos\frac{2\pi}{3})}$$

$$r\left(\frac{2\pi}{3}\right) = \frac{2}{\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)} \approx 5.5$$

$\frac{3}{4}\pi + \frac{\pi}{6}$

$\frac{9\pi + 2\pi}{12} = \frac{11\pi}{12}$

$r\left(\frac{11\pi}{12}\right) \approx -2{,}8$

$$r(\pi) = \frac{2}{(\mathrm{sen}\,\pi + \cos\pi)} = \frac{2}{(0 - 1)} = -2.$$

B) $x^2 + y^2 = 2$

$r^2 = 2 \quad \therefore \quad r = \sqrt{2} \quad$ ou $\quad r = -\sqrt{2}$

OU uma ou outra equação.

Uma segunda maneira

$x = r\cos\theta \quad y = r\,\mathrm{sen}\,\theta$

$x^2 + y^2 = 2$

$(r\cos\theta)^2 + (r\,\mathrm{sen}\,\theta)^2 = 2$

$r^2\cos^2\theta + r^2\,\mathrm{sen}^2\theta = 2$

$r^2(\cos^2\theta + \mathrm{sen}^2\theta) = 2$

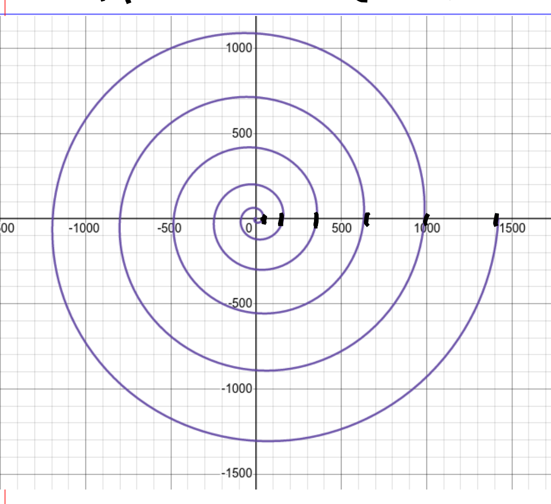
$r^2 = 2$

$r = +\sqrt{2} \quad$ ou $\quad r = -\sqrt{2}$

1–4 Encontre a área da região que é delimitada pelas curvas dadas e está no setor especificado.

1. $r = \theta^2$, $0 \leqslant \theta \leqslant \pi/4$

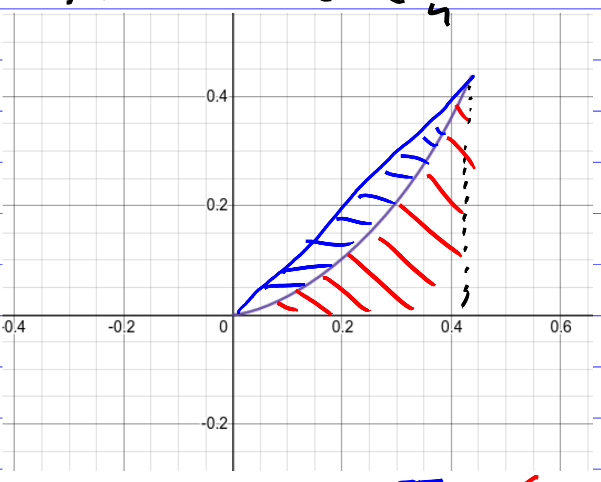
$r = \theta^2 \quad 0 \leq \theta \leq 10\pi$



Direções
$0, 2\pi, 4\pi, 6\pi, 8\pi, 10\pi, 12\pi$

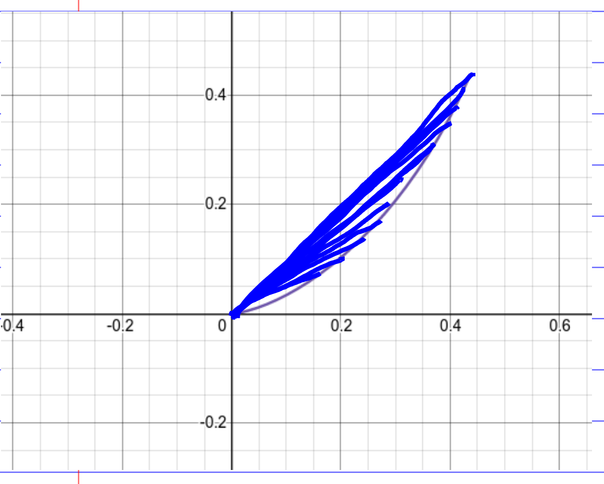
$r(0) = 0$
$r(2\pi) = 4\pi^2$
$r(4\pi) = 16\pi^2$
$r(6\pi) = 36\pi^2$

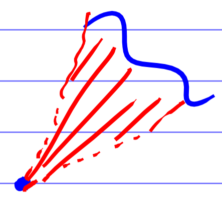
$r = \theta^2 \quad 0 \leq \theta \leq \frac{\pi}{4}$

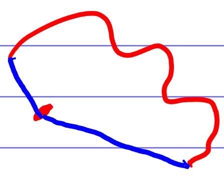


Área sob a curva em coord. cartesian

Não







$$A = \int_{\theta_i}^{\theta_f} \frac{1}{2} r^2 \, d\theta$$

$$A = \int_0^{\frac{\pi}{4}} \frac{1}{2} \theta^4 \, d\theta = \frac{1}{2} \int_0^{\frac{\pi}{4}} \theta^4 \, d\theta$$

$$A = \frac{1}{2} \theta^5 \Big|_0^{\frac{\pi}{4}} = \frac{1}{2} \cdot \frac{\pi^5}{4^5}$$

$$= \frac{1}{2} \frac{\pi^5}{(2^2)^5} = \frac{\pi^5}{2^{11}}$$

45–48 Calcule o comprimento exato da curva polar.

45. $r = 2\cos\theta$, $0 \leqslant \theta \leqslant \pi$ **46.** $r = 5^\theta$, $0 \leqslant \theta \leqslant 2\pi$

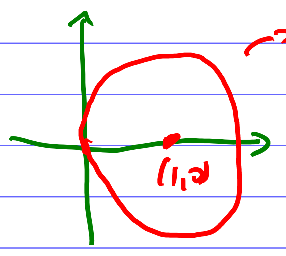
a) $$L = \int_{\theta_i}^{\theta_f} \sqrt{r^2 + r'^2} \, d\theta$$

$r = 2\cos\theta \quad \therefore \quad r^2 = 4\cos^2\theta$
$r' = -2\sin\theta$
$(r')^2 = 4\sin^2\theta$

$$L = \int_0^{\pi} \sqrt{4\cos^2\theta + 4\sin^2\theta} \, d\theta$$

$$L = \int_0^{\pi} \sqrt{4} \, d\theta = 2\int_0^{\pi} d\theta$$

$$L = 2\pi$$



→ círculo de raio 1

b) $r = 5^\theta$

$r^2 = 5^{2\theta}$ $\quad r' = 5^\theta \cdot \ln 5$
$(r')^2 = 5^{2\theta}(\ln 5)^2$

$$L = \int_0^{2\pi} \sqrt{r^2 + (r')^2} \, d\theta$$

$$L = \int_0^{2\pi} \sqrt{5^{2\theta} + 5^{2\theta}(\ln 5)^2} \, d\theta$$

$$L = \int_0^{2\pi} \sqrt{5^{2\theta}(1 + (\ln 5)^2)} \, d\theta$$

$$L = \sqrt{1 + (\ln 5)^2} \int_0^{2\pi} 5^\theta \, d\theta$$

$$L = \sqrt{1 + (\ln 5)^2} \; \frac{5^\theta}{\ln 5} \Big|_0^{2\pi}$$

$$L = \sqrt{1 + (\ln 5)^2} \left(\frac{5^{2\pi}}{\ln 5} - 0 \right)$$

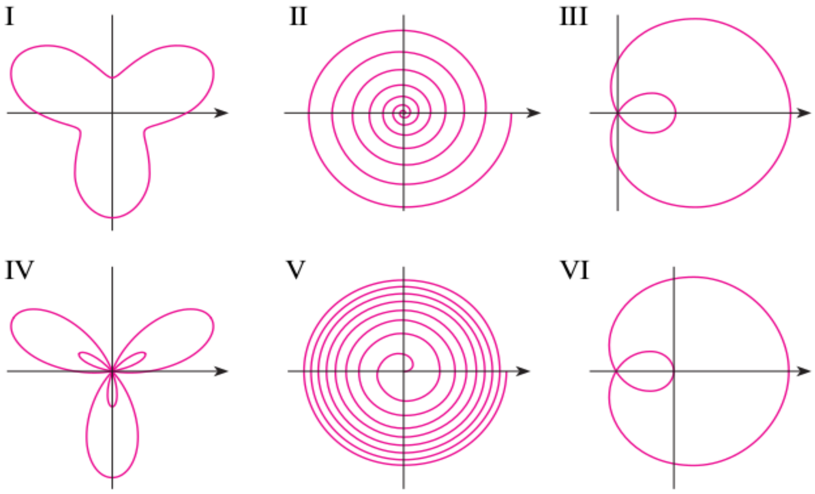
$$\boxed{L = \sqrt{1 + (\ln 5)^2} \; \frac{5^{2\pi}}{\ln 5}}$$

54. Associe as curvas polares com seus respectivos gráficos I–VI. Dê razões para suas escolhas. (Não use uma ferramenta gráfica.)

(a) $r = \sqrt{\theta}, \quad 0 \leqslant \theta \leqslant 16\pi$ (b) $r = \theta^2, \quad 0 \leqslant \theta \leqslant 16\pi$

(c) $r = \cos(\theta/3)$ (d) $r = 1 + 2\cos\theta$

(e) $r = 2 + \text{sen } 3\theta$ (f) $r = 1 + 2 \text{ sen } 3\theta$



21–26 Encontre uma equação polar para a curva representada pela equação cartesiana dada.

21. $y = 2$ **22.** $y = x$

23. $y = 1 + 3x$ **24.** $4y^2 = x$

25. $x^2 + y^2 = 2cx$ **26.** $xy = 4$

$r = \cos\theta + 2 \operatorname{sen}\theta$

$r^2 = r\cos\theta + 2r\operatorname{sen}\theta$

$x^2 + y^2 = x + 2y$

$x^2 - x + y^2 - 2y = 0$

$x^2 - x + \frac{1}{4} - \frac{1}{4} + \quad + y^2 - 2y + 1 - 1 = 0$

$(x - \frac{1}{2})^2 + (y - 1)^2 = 1 + \frac{1}{4}$

$(x - \frac{1}{2})^2 + (y - 1)^2 = \frac{5}{4}$

$(x - x_0)^2 + (y - y_0)^2 = R^2$

centro em $(x_0, y_0) = (\frac{1}{2}, 1)$

$R^2 = \frac{5}{4}$

$R = \frac{\sqrt{5}}{2}$