Técnica da substituição

a) $\int_0^3 2x\cos(x^2)\,dx$ $\quad U = x^2 \therefore \frac{dU}{dx} = 2x$ $\therefore dU = 2x\,dx$

Quando $x=0,\ U=0^2=0$
" $x=3,\ U=3^2=9$

$$\int_0^9 \cos(U)\,dU = \operatorname{sen} U\Big|_0^9 = \operatorname{sen} 9 - \operatorname{sen} 0 = \operatorname{sen} 9$$

b) $\int_1^2 4x^3 e^{x^4}\,dx$ $\quad U = x^4 \therefore \frac{dU}{dx} = 4x^3$

$dU = 4x^3\,dx$

Quando $x=1,\ U=1^4=1$
" $x=2,\ U=2^4=16$

$$\int_1^{16} e^U\,dU = e^U\Big|_1^{16} = e^{16} - e^1$$

$\boxed{\int x^2\cos(x^2)\,dx}$

$= \int x\, x\cos(x^2)\,dx$ $\quad U = x^2 \therefore \frac{dU}{dx} = 2x \therefore \frac{dU}{2} = x\,dx$

$\boxed{= \int \sqrt{U}\,\cos(U)\,dU}$

Substituição não foi útil!

c) $\int_1^e \frac{\ln x}{x}\,dx$ $\quad U = \ln x \therefore \frac{dU}{dx} = \frac{1}{x} \therefore dU = \frac{1}{x}dx$

$\boxed{\frac{d\ln x}{dx} = \frac{1}{x}}$

Quando $x=1,\ U=\ln 1=0$
" $x=e,\ U=\ln e=1$

$$\int_0^1 U\,dU = \frac{U^2}{2}\Big|_0^1 = \frac{1^2}{2} - \frac{0^2}{2} = \frac{1}{2}$$

Outros profs podem resolver assim:

$\int_1^e \frac{\ln x}{x}\,dx$ $\quad U = \ln x \therefore dU = \frac{1}{x}dx$

$\int U\,dU = \frac{U^2}{2} = \frac{(\ln x)^2}{2}\Big|_1^e = \frac{(\ln e)^2}{2} - \frac{(\ln 1)^2}{2} = \frac{1}{2} - \frac{0}{2} = \frac{1}{2}$

Não escrevem os limites.

ERRO!!!

$\boxed{\int_1^e U\,dU = \frac{U^2}{2} = \frac{e^2}{2} - \frac{1}{2}}$

d) $\int_2^3 x\cos(x^2)\,dx$ $\quad U = x^2 \therefore \frac{dU}{dx} = 2x \therefore \frac{dU}{2} = x\,dx$

Quando $x=2,\ U=4$
" $x=3,\ U=9$

$$\int_4^9 \frac{\cos U\,dU}{2} = \frac{1}{2}\operatorname{sen} U\Big|_4^9 = \frac{1}{2}(\operatorname{sen} 9 - \operatorname{sen} 4)$$

e) $\int_0^\pi \operatorname{sen} x\sqrt{\cos^2 x}\,dx$

Integração por Partes

a) $\displaystyle\int_0^{\pi} \underbrace{x}_{u} \cdot \underbrace{\cos x\, dx}_{dv}$

$$\int u\,dv = uv - \int v\,du$$

$\boxed{U = x}$

$\dfrac{du}{dx} = 1 \therefore \boxed{dU = dx}$

$dv = \cos x\, dx$

$v = \int dv = \int \cos x\, dx$

$\boxed{v = \operatorname{sen} x}$

$$\int_0^{\pi} x\cos x\, dx = x \operatorname{sen} x \Big|_0^{\pi} - \int_0^{\pi} \operatorname{sen} x\, dx$$

$$= (\pi \operatorname{sen}\pi) - (0 \cdot \operatorname{sen} 0) - [-\cos x]\Big|_0^{\pi}$$

$$= 0 + [\cos x]\Big|_0^{\pi}$$

$$= \cos\pi - \cos 0$$

$$= -1 - 1 = -2$$

ou

$$-[-\cos x]\Big|_0^{\pi}$$

$$-\left([-\cos\pi] - [-\cos 0]\right)$$

$$-\left([+1] - [-1]\right) = -2$$

b) $\displaystyle\int_0^{2} \underbrace{x}_{u}\underbrace{e^x\, dx}_{dv}$

$U = x$

$\dfrac{dU}{dx} = 1 \therefore dU = dx$

$dv = e^x dx$

$v = \int dv$

$v = \int e^x dx$

$v = e^x$

$$\int u\,dv = uv - \int v\,du$$

$$\int_0^2 xe^x dx = xe^x\Big|_0^2 - \int_0^2 e^x dx$$

$$= xe^x\Big|_0^2 - e^x\Big|_0^2$$

$$= [2e^2 - 0] - [e^2 - e^0]$$

$$= 2e^2 - e^2 + 1$$

$$= e^2 + 1$$

Uso de relações trigonométricas

a) $\int_0^{\pi/2} \cos^2 x\, dx = \int_0^{\pi/2} \frac{1+\cos(2x)}{2}\, dx$

$$\boxed{\begin{aligned} \cos^2 x &= \frac{1+\cos(2x)}{2} \\ \operatorname{sen}^2 x &= \frac{1-\cos(2x)}{2} \end{aligned}}$$

$= \int_0^{\pi/2} \frac{1}{2}\, dx + \frac{1}{2}\int_0^{\pi/2} \cos(2x)\, dx$

$= \frac{1}{2}\left(\frac{\pi}{2} - 0\right) + \frac{1}{2}\int_0^{\pi/2} \cos(2x)\, dx$

$= \frac{\pi}{4} + \frac{1}{2}\int_0^{\pi/2} \cos(2x)\, dx$

$u = 2x \therefore \frac{du}{dx} = 2 \therefore \frac{du}{2} = dx$

Qnd $x = 0,\ u = 0$

" $x = \frac{\pi}{2},\ u = \pi$

$= \frac{\pi}{4} + \frac{1}{2}\int_0^{\pi} \cos(u)\, \frac{du}{2}$

$= \frac{\pi}{4} + \frac{1}{4}\int_0^{\pi} \cos(u)\, du$

$= \frac{\pi}{4} + \frac{1}{4} \cdot \operatorname{sen} u \Big|_0^{\pi}$

$= \frac{\pi}{4} + \frac{1}{4}(\operatorname{sen}\pi - \operatorname{sen} 0)$

$= \frac{\pi}{4} + \frac{1}{4}(0-0)$

$= \frac{\pi}{4}$

b) $\int_0^{\frac{\pi}{2}} \cos^3 x\, dx$

Simetrías

$$\int_{-3}^{3} x\,dx$$

$$\int_{-3}^{3} x^2\,dx$$

$$\int_{-\pi/2}^{\pi/2} \cos(x)\,dx$$

$$\int_{-\pi/2}^{\pi/2} \operatorname{sen}(x)\,dx$$