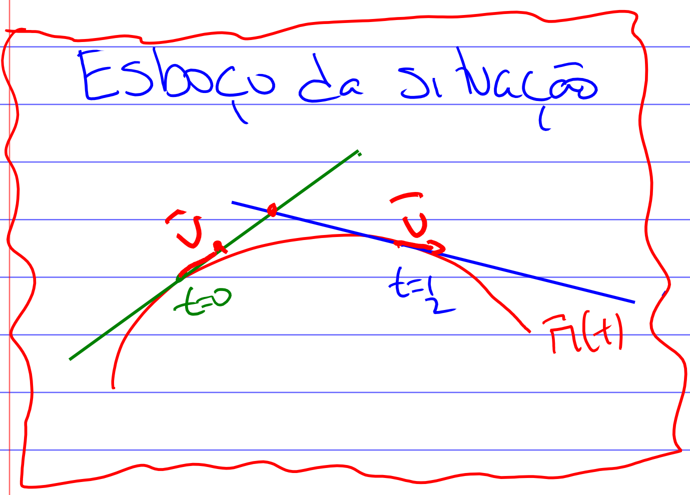
① Determine o ponto de intersecção das retas tangentes à curva

$$\vec{r}(t) = \langle \operatorname{sen}\pi t,\ 2\operatorname{sen}\pi t,\ \cos\pi t \rangle$$

nos pontos $t=0$ e $t=\frac{1}{2}$.



$$\vec{R}(t) = \vec{R}_0 + t\vec{V}$$

$$\vec{V}(t) = \langle \pi\cos\pi t,\ 2\pi\cos\pi t,\ -\pi\operatorname{sen}\pi t \rangle$$

P/ $t=0$

$$\vec{R}(\alpha) = \vec{R}_0 + \alpha\vec{V}$$

$$\vec{R}_0 = \vec{r}(0) = \langle \operatorname{sen} 0,\ 2\operatorname{sen} 0,\ \cos 0 \rangle = \langle 0, 0, 1 \rangle$$

$$\vec{V} = \vec{V}(0) = \langle \pi\cos 0,\ 2\pi\cos 0,\ -\pi\operatorname{sen} 0 \rangle = \langle \pi, 2\pi, 0 \rangle$$

$$\vec{R}(\alpha) = \langle 0, 0, 1 \rangle + \alpha\langle \pi, 2\pi, 0 \rangle \qquad \alpha \in \mathbb{R}$$

$$R(\alpha) = \langle \alpha\pi,\ \alpha 2\pi,\ 1 \rangle$$

P/ $t=\frac{1}{2}$

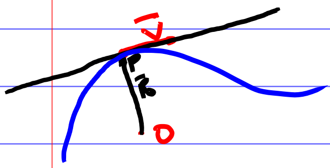
$$\vec{R}(\beta) = \vec{R}_0 + \beta\vec{V}$$

$$\vec{R}_0 = \vec{r}\left(\tfrac{1}{2}\right) = \langle \operatorname{sen}\tfrac{\pi}{2},\ 2\operatorname{sen}\tfrac{\pi}{2},\ \cos\tfrac{\pi}{2} \rangle$$

$$\boxed{\vec{R}_0 = \langle 1, 2, 0 \rangle}$$

$$\vec{V} = \vec{V}\left(\tfrac{1}{2}\right) = \langle \pi\cos\tfrac{\pi}{2},\ 2\pi\cos\tfrac{\pi}{2},\ -\pi\operatorname{sen}\tfrac{\pi}{2} \rangle$$

$$= \langle 0,\ 0,\ -1 \rangle$$



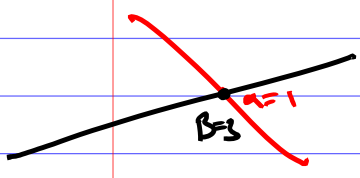
$$\vec{R}(\beta) = \langle 1, 2, 0 \rangle + \beta\langle 0, 0, -\pi \rangle$$

$$\vec{R}(\beta) = \langle 1, 2, -\pi\beta \rangle$$

ONDE ELAS SE CRUZAM?

$$R_A(\alpha) = \langle \alpha\pi,\ \alpha 2\pi,\ 1 \rangle$$

$$\vec{R}_B(\beta) = \langle 1, 2, -\pi\beta \rangle$$



$$\begin{cases} \alpha\pi = 1 \rightarrow \alpha = \frac{1}{\pi} \\ \alpha 2\pi = 2 \rightarrow \alpha = \frac{1}{\pi} \\ 1 = -\pi\beta \rightarrow \beta = -\frac{1}{\pi} \end{cases}$$

$$\vec{R}_A\left(\tfrac{1}{\pi}\right) = \langle 1, 2, 1 \rangle$$

$$\vec{R}_B\left(-\tfrac{1}{\pi}\right) = \langle 1, 2, 1 \rangle$$

O ponto é $(1, 2, 1)$.

② Calcule $\int_0^1 \left(\frac{4}{1+t^2}\,\hat{j} + \frac{2t}{1+t^2}\,\hat{k} \right) dt$

1º Integral

$$\int_0^1 \frac{4}{1+t^2}\,dt$$

$\sec^2\theta - tg^2\theta = 1$

$\sec^2\theta = 1 + tg^2\theta$

$t = tg\,\theta$

$\frac{dt}{d\theta} = \sec^2\theta$

$$\int_0^{\frac{\pi}{4}} \frac{4 \cdot \sec^2\theta\, d\theta}{\sec^2\theta}$$

$$= \int_0^{\frac{\pi}{4}} 4\,d\theta = 4\left(\frac{\pi}{4} - 0\right)$$

$$= \pi$$

Quando $t = 0$, $\theta = ?$

$t = tg\,\theta$

$0 = tg\,\theta \therefore \theta = arctg\,0$

$\theta = 0$

Quando $t = 1$, $\theta = ?$

$t = tg\,\theta$

$1 = tg\,\theta \therefore \theta = arctg(1)$

$\theta = \frac{\pi}{4}$

2º Integral

$$\int_0^1 \frac{2t}{1+t^2}\,dt$$

$U = 1+t^2 \therefore \frac{dU}{dt} = 2t \therefore dU = 2t\,dt$

$$= \int_1^2 \frac{dU}{U} = \ln U \Big|_1^2$$

$$= \ln 2 - \ln 1$$

$$= \ln 2$$

Quando $t = 0$, $U = 1$

" $t = 1$, $U = 2$

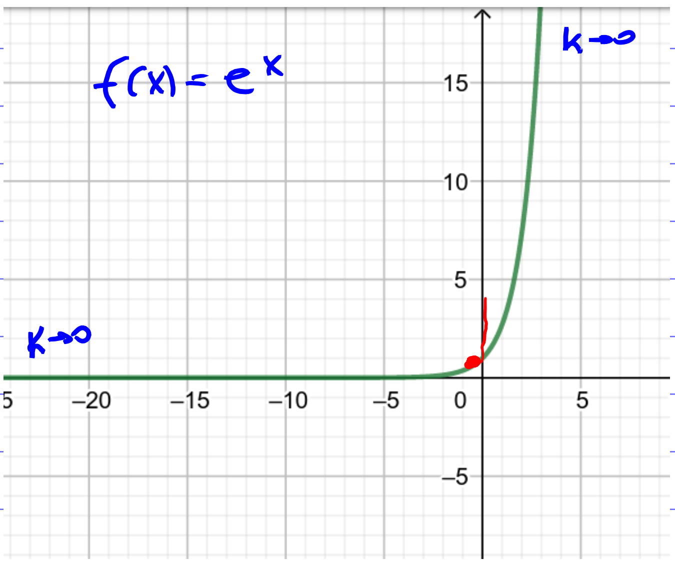
$$\int_0^1 \left(\frac{4}{1+t^2}\,\hat{j} + \frac{2t}{1+t^2}\,\hat{k} \right) dt = \pi\,\hat{j} + \ln 2\,\tilde{k}$$

③ Em que Ponto a Curvatura é máxima?

$Y = e^x$

Vamos calcular o pontos críticos de $K(x)$.

$K'(x) = 0$



Como calcular a curvatura

$$K = \left|\frac{d\vec{T}}{ds}\right| \quad \text{Definição}$$

$$K = \frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right|$$

$$K = \frac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}|^3}$$

Se eu decidir empregar alguma dessas três expressões, eu devo parametrizar a curva:

$$\vec{r}(t) = \langle t, e^t \rangle$$

$$K = \frac{f''}{(1+(f')^2)^{3/2}}$$

$f = e^x$

$f' = e^x$

$f'' = e^x$

$$K(x) = \frac{e^x}{(1+e^{2x})^{3/2}}$$

Procurando o Ponto crítico.

$K'(x) = 0$

$$\frac{e^x(1+e^{2x})^{3/2} - e^x \frac{3}{2}(1+e^{2x})^{1/2} \cdot e^{2x} \cdot 2}{(1+e^{2x})^3} = 0$$

$$e^x(1+e^{2x})^{3/2} - e^x \frac{3}{2}(1+e^{2x})^{1/2} \cdot e^{2x} \cdot 2 = 0$$

$$e^x(1+e^{2x})^{3/2} = e^{3x}\, 3\,(1+e^{2x})^{1/2}$$

$$\frac{(1+e^{2x})^{3/2}}{(1+e^{2x})^{1/2}} = \frac{e^{3x}}{e^x} \cdot 3$$

$$1+e^{2x} = e^{2x} \cdot 3$$

$$1 = 2e^{2x}$$

$$\frac{1}{2} = e^{2x} \quad \therefore \quad \ln\left(\frac{1}{2}\right) = \ln e^{2x}$$

$$\ln(2^{-1}) = 2x$$

$$\boxed{x = -\frac{1}{2}\ln 2}$$

$$Y = e^x \quad \therefore \quad Y = e^{-\frac{1}{2}\ln 2}$$

$$Y = e^{\ln 2^{(-\frac{1}{2})}}$$

$$Y = 2^{-\frac{1}{2}}$$

$$Y = \frac{1}{\sqrt{2}}$$

$$\left(-\frac{1}{2}\ln 2, \frac{1}{\sqrt{2}}\right)$$

(1) Calcule o comprimento exato da curva polar

$r = 5^{\theta} \quad 0 \leq \theta \leq 2\pi$

$$L = \int_{\theta_i}^{\theta_f} \sqrt{r^2 + (r')^2}\, d\theta$$

$$r = 5^{\theta} \qquad r' = 5^{\theta} \cdot \ln 5$$

$$r^2 = 5^{2\theta} \qquad (r')^2 = 5^{2\theta}(\ln 5)^2$$

$$L = \int_0^{2\pi} \sqrt{5^{2\theta} + 5^{2\theta}(\ln 5)^2}\, d\theta$$

$$L = \int_0^{2\pi} \sqrt{5^{2\theta}(1 + [\ln 5]^2)}\, d\theta$$

$$L = \int_0^{2\pi} 5^{\theta} \sqrt{1 + (\ln 5)^2}\, d\theta$$

$$L = \sqrt{1 + (\ln 5)^2} \int_0^{2\pi} 5^{\theta}\, d\theta$$

$$L = \sqrt{1 + (\ln 5)^2} \left. \frac{5^{\theta}}{\ln 5} \right|_0^{2\pi}$$

$$\boxed{L = \frac{\sqrt{1 + (\ln 5)^2}}{\ln 5} \left(5^{2\pi} - 1\right)}$$

(5) Calcule a área da região delimitada por

$r = \sqrt{\theta} \quad 0 \leq \theta \leq 2\pi.$

$$A = \int_{\theta_i}^{\theta_f} \frac{1}{2} r^2 d\theta = \int_0^{2\pi} \frac{1}{2} (\sqrt{\theta})^2 d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} \theta \, d\theta$$

$$= \frac{1}{2} \cdot \frac{\theta^2}{2} \Big|_0^{2\pi}$$

$$= \frac{1}{2} \left(\frac{4\pi^2}{2} - \frac{0}{2} \right)$$

$$= \pi^2 \quad u.a$$