$H \longrightarrow$ $\Delta H = 5$

Umidade relativa (%)

$\Delta T = 2$

$T \downarrow$

Temperatura real (°C)

T \ H	40	45	50	55	60	65	70	75	80
26	28	28	29	31	31	32	33	34	35
28	31	32	33	34	35	36	37	38	39
30	34	35	36	37	$\boxed{38}$	40	41	42	43
32	37	38	39	41	42	43	45	46	47
34	41	42	43	45	47	48	49	51	52
36	43	45	47	48	50	51	53	54	56

$$\frac{\partial}{\partial H} S(H,T) = \lim_{\Delta H \to 0} \frac{S(H+\Delta H, T) - S(H,T)}{\Delta H}$$

$$\frac{\partial}{\partial H} S(60,30) = \left.\frac{\partial S}{\partial H}\right|_{(60,30)} = S_H(60,30)$$

Estimativa

$$\frac{\partial}{\partial H} S(H,T) \approx \frac{S(H+\Delta H, T) - S(H,T)}{\Delta H}$$

$$\frac{\partial}{\partial H} S(60,30) \approx \frac{S(60+5,30) - S(60,30)}{5}$$

$$\approx \frac{40 - 38}{5}$$

$$\approx \frac{2}{5} \approx 0.4$$

Vamos melhorar a estimativa.

$$\frac{\partial}{\partial H} S(60,30) \approx \frac{\frac{S(60+5,30) - S(60,30)}{5} + \frac{S(60-5,30) - S(60,30)}{(-5)}}{2}$$

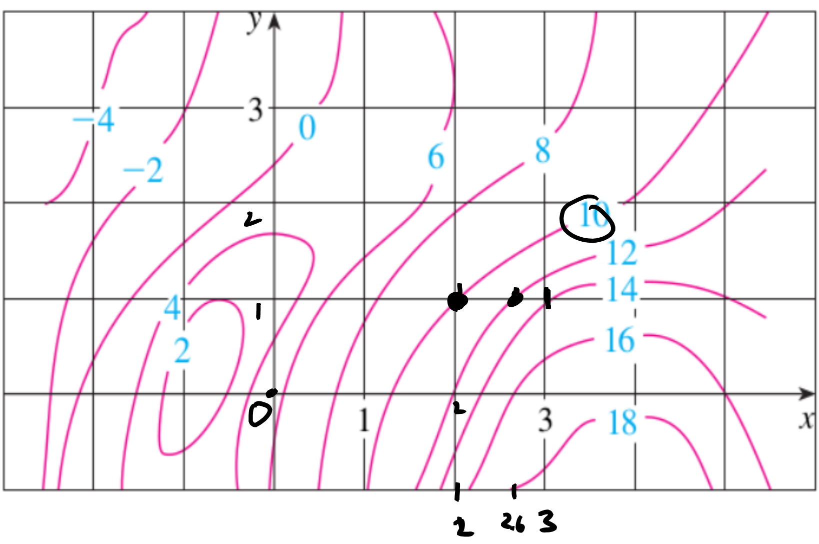
$$\approx \frac{\frac{S(65,30) - S(60,30)}{5} + \frac{S(55,30) - S(60,30)}{-5}}{2}$$

$$\approx \frac{\frac{S(65,30) - \cancel{S(60,30)} - S(55,30) + \cancel{S(60,30)}}{5}}{2}$$

$$\approx \frac{S(65,30) - S(55,30)}{10}$$

$$\approx \frac{40 - 37}{10} = \frac{3}{10} \approx 0{,}337$$

10. Um mapa de contorno de uma função f é apresentado. Utilize-o para estimar $f_x(2, 1)$ e $f_y(2, 1)$.



$$f_x(2,1) = \left.\frac{\partial f}{\partial x}\right|_{(2,1)} \approx \frac{f(2+0.6,1) - f(2,1)}{0.6}$$

$$\approx \frac{f(2.6,1) - f(2,1)}{0.6}$$

$$\approx \frac{12 - 10}{0.6}$$

$$\approx \frac{2}{0.6} = \frac{2}{6/10} = \frac{20}{6} = \frac{10}{3}$$

18. $f(x, t) = \sqrt{x} \ln t$

30. $F(\alpha, \beta) = \int_\alpha^\beta \sqrt{t^3 + 1}\, dt$

$$f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(\sqrt{x} \ln t) = \ln t \frac{\partial}{\partial x}\sqrt{x}$$

$$= \ln t \frac{\partial}{\partial x} x^{1/2}$$

$$= \ln t \ \frac{1}{2} x^{-\frac{1}{2}}$$

$$= \frac{\ln t}{2\sqrt{x}}$$

$$f_t = \frac{\partial f}{\partial t} = \frac{\partial}{\partial t}(\sqrt{x} \ln t) = \sqrt{x} \frac{\partial}{\partial t} \ln t$$

$$= \sqrt{x} \frac{1}{t} = \frac{\sqrt{x}}{t}$$

$$F(\alpha, \beta) = \int_\alpha^\beta \sqrt{t^3 + 1}\, dt$$

$$F_\alpha = \frac{\partial F}{\partial \alpha} = \frac{\partial}{\partial \alpha} \int_\alpha^\beta \sqrt{t^3+1}\, dt$$

$$= -\frac{\partial}{\partial \alpha} \int_\beta^\alpha \sqrt{t^3+1}\, dt$$

$$= -\sqrt{\alpha^3 + 1}$$

$$F_\beta = \frac{\partial F}{\partial \beta} = \frac{\partial}{\partial \beta} \int_\alpha^\beta \sqrt{t^3+1}\, dt$$

$$= \sqrt{\beta^3 + 1}$$

15–40 Determine as derivadas parciais de primeira ordem da função.

15. $f(x, y) = y^5 - 3xy$

16. $f(x, y) = x^4y^3 + 8x^2y$

33. $\boxed{w = \ln(x + 2y + 3z)}$

18. $f(x, t) = \sqrt{x}\ln t$

39. $\boxed{u = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}}$

$$w = \ln(x + 2y + 3z)$$

$$\frac{\partial w}{\partial x} = \frac{1}{x+2y+3z} \cdot \frac{\partial}{\partial x}(x+2y+3z)$$

$$= \frac{1}{x+2y+3z}$$

$$\frac{\partial w}{\partial y} = \frac{1}{x+2y+3z} \frac{\partial}{\partial y}(x+2y+3z)$$

$$= \frac{2}{x+2y+3z}$$

$$\frac{\partial w}{\partial z} = \frac{1}{x+2y+3z} \cdot \frac{\partial}{\partial z}(x+2y+3z)$$

$$= \frac{3}{x+2y+3z}$$

$$U = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$$

U é função de n variáveis então admite n derivadas parciais.

$$\frac{\partial U}{\partial x_1} = \frac{1}{2}(x_1^2 + x_2^2 + \cdots + x_n^2)^{-1/2} \cdot \frac{\partial}{\partial x_1}(x_1^2 + x_2^2 + \cdots + x_n^2)$$

$$\frac{\partial U}{\partial x_1} = \frac{1}{2}(x_1^2 + x_2^2 + \cdots + x_n^2)^{-1/2} \cdot 2x_1$$

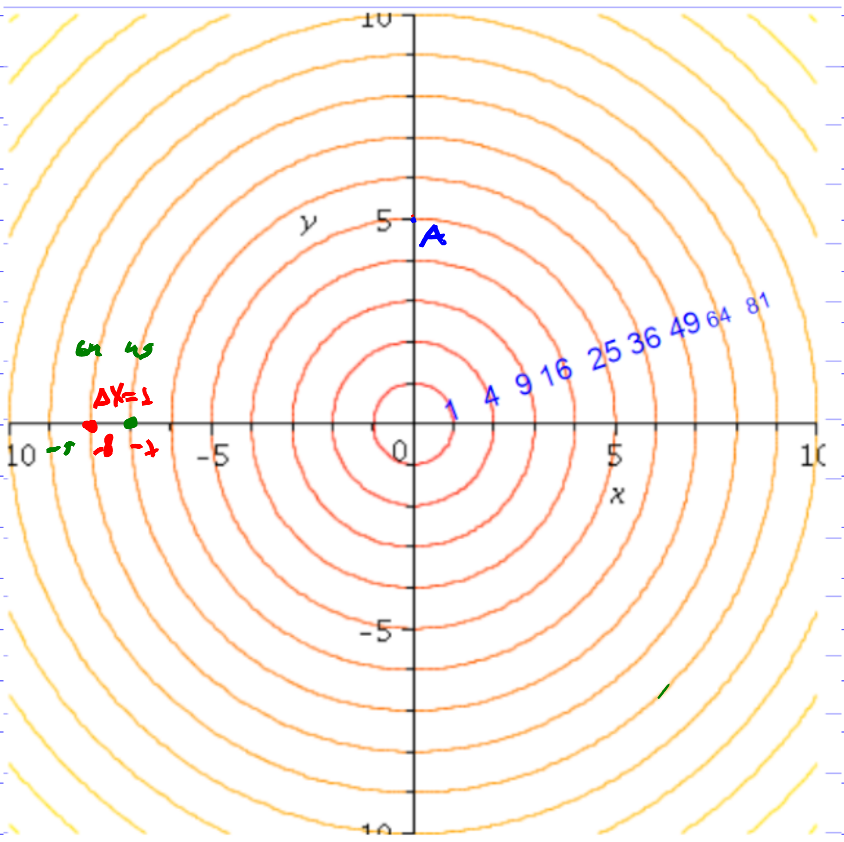
$$\frac{\partial U}{\partial x_1} = \frac{x_1}{\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}}$$

p/ x_2

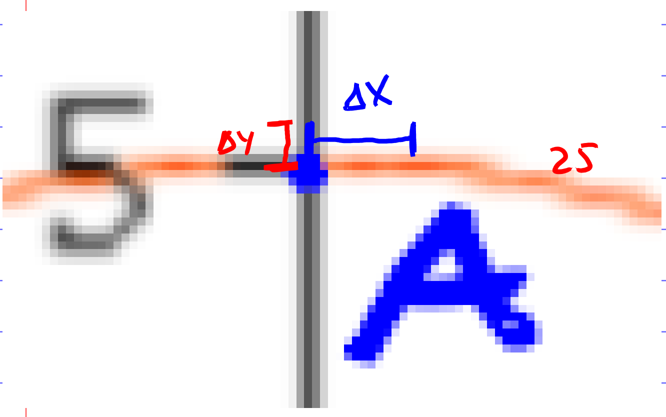
$$\frac{\partial U}{\partial x_2} = \frac{x_2}{\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}}$$

Não é difícil perceber que

$$\boxed{\frac{\partial U}{\partial x_i} = \frac{x_i}{\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}}}$$



$$\left.\frac{\partial f}{\partial x}\right|_{A} = 0 \qquad\qquad \left.\frac{\partial f}{\partial y}\right|_{A} > 0$$



Vamos estimar

$$\left.\frac{\partial f}{\partial x}\right|_{(-8,0)} \approx \frac{\dfrac{f(-8+1,0)-f(-8,0)}{1} + \dfrac{f(-8-1,0)-f(-8,0)}{(-1)}}{2}$$

$$\approx \frac{\dfrac{f(-8+1,0)-f(-8,0)}{1} \; - f(-8-1,0)+f(-8,0)}{2}$$

$$\approx \frac{f(-8+1,0)-\cancel{f(-8,0)} \; - f(-8-1,0)+\cancel{f(-8,0)}}{2}$$

$$\approx \frac{f(-7,0)-f(-9,0)}{2}$$

$$\approx \frac{49-81}{2}$$

$$\approx \frac{-32}{2}$$

$$\approx -16$$

Por questões didáticas vamos comparar com o valor exato.

$$f(x,y) = x^2+y^2$$

$$\left.\frac{\partial f}{\partial x}\right|_{(-8,0)} = \left.\frac{\partial}{\partial x}(x^2+y^2)\right|_{(-8,0)}$$

$$= \left.2x\right|_{(-8,0)} = -16$$

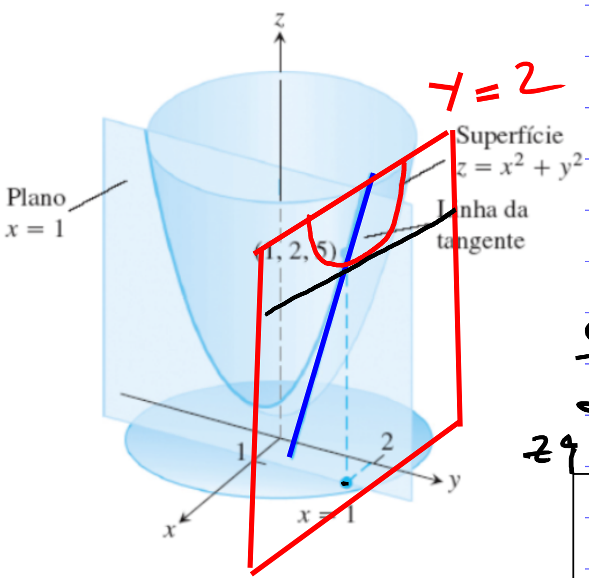


FIGURA 14.16 A tangente à curva de interseção do plano $x = 1$ e da superfície $z = x^2 + y^2$ no ponto (1, 2, 5) (Exemplo 5).

$$\left.\frac{\partial f}{\partial y}\right|_{(1,2)}$$