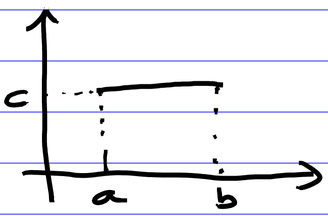
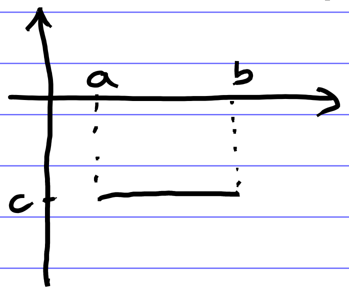
Propriedades da integral definida

1) $\int_a^b c\,dx = c(b-a)$





2) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

3) $\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

4) $\int_a^b (f(x)-g(x))\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

5) $\int_a^a f(x)\,dx = 0$

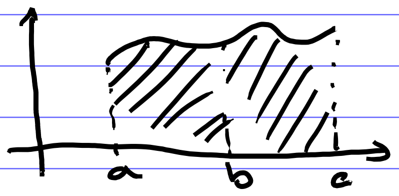
6) $\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$

$$\int_a^b c f(x)\,dx = \lim_{n\to+\infty}\sum_{i=1}^{n} c f(x_i)\Delta x$$

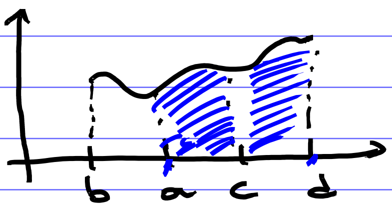
$$= c\lim_{n\to+\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

$$= c\int_a^b f(x)\,dx$$

7) $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$



$\Delta x = x_f - x_i$



$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx + \int_c^d f(x)\,dx = \int_a^d f(x)\,dx$$

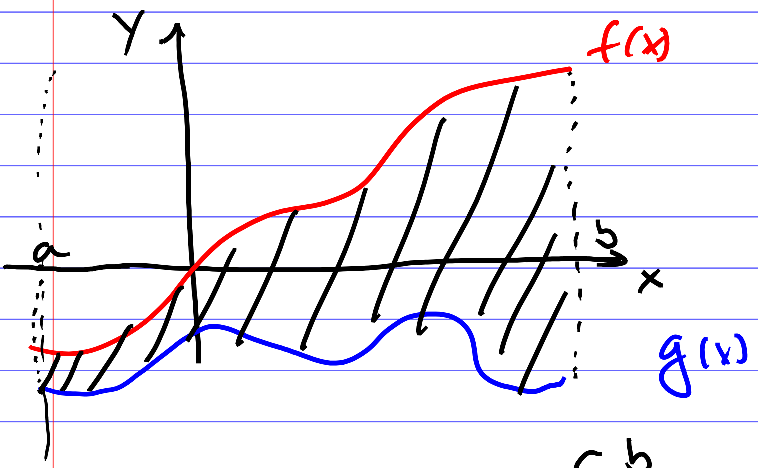
NÃO EXISTE

8) ~~$\int_a^b f(x)g(x)\,dx = \int_a^b f(x)\,dx \cdot \int_a^b g(x)\,dx$~~

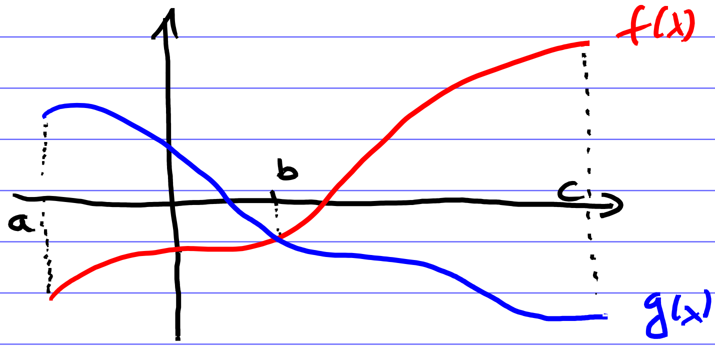
NÃO EXISTE

$$\int x^2\cos x\,dx \neq \int x^2\,dx \int \cos x\,dx$$

Área entre curvas



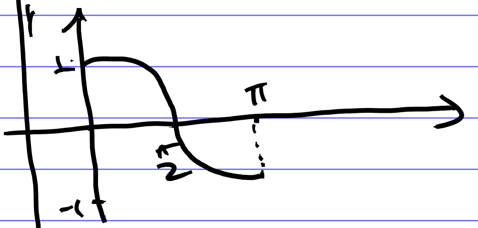
$$A_{\text{entre curvas}} = \int_a^b \left(f(x) - g(x)\right) dx$$



$$A_{\text{entre curvas}} = \int_a^b \left(g(x) - f(x)\right) dx + \int_b^c \left(f(x) - g(x)\right) dx$$

$$A = \int_a^c \left|f(x) - g(x)\right| dx$$

Integrando é função módulo $\displaystyle\int_0^{\pi} |\cos x|\, dx =$



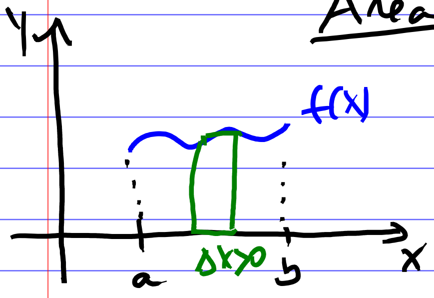
$$= \int_0^{\pi/2} \cos x \, dx + \int_{\pi/2}^{\pi} (-\cos x)\, dx$$

$f(x) = \cos x$

$g(x) = 0$

$$= \int_0^{\pi/2} (\cos x - 0)\, dx + \int_{\pi/2}^{\pi} (0 - \cos x)\, dx$$

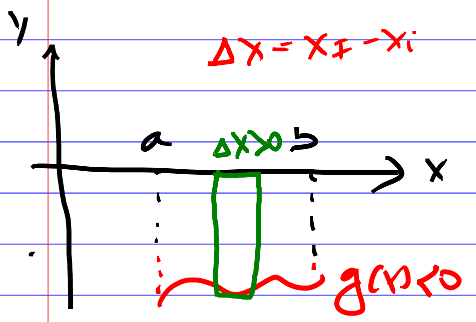
Área líquida



$$\int_a^b f(x)\,dx = \lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

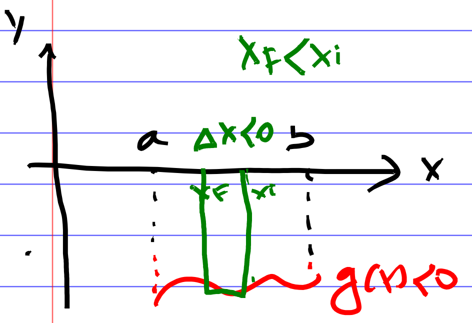
$$\Delta x = x_f - x_i$$

$$\int_a^b f(x)\,dx > 0$$



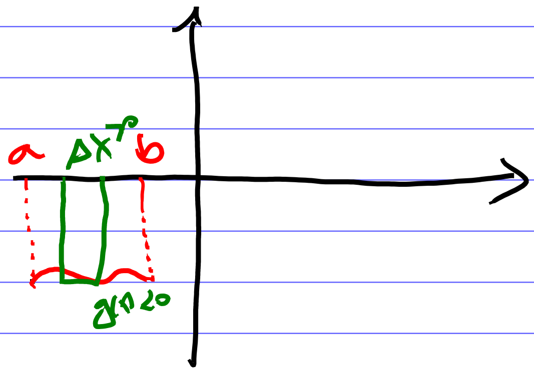
$$\int_a^b g(x)\,dx < 0$$

"Retângulo c/ área negativa"



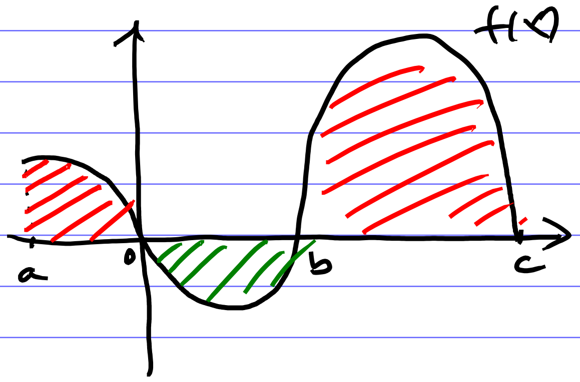
$$\int_b^a g(x)\,dx > 0$$

base é negativa
altura é negativa } → Área positiva.



$$\int_a^b g(x)\,dx < 0$$

$$\int_b^a g(x)\,dx > 0$$

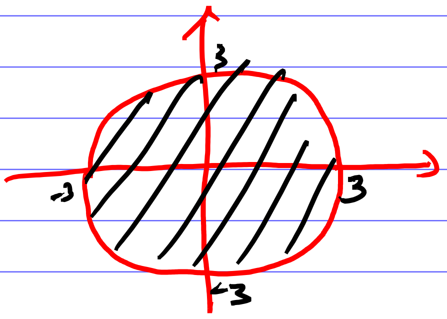


$\int_a^c f(x)\,dx$ é positiva

$\int_a^c f(x)\,dx$ fornece a área hachurada?

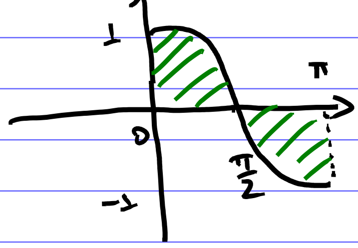
NÃO

Integral fornece área líquida.



$$A = \int_{-3}^{3} \left(\underbrace{\sqrt{9-x^2}}_{\text{cima}} - \left(\underbrace{-\sqrt{9-x^2}}_{\text{baixo}} \right) \right) dx$$

$f(x) = \cos x$



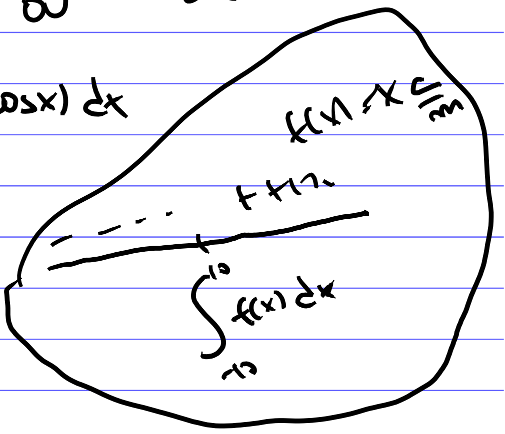
$$\int_0^{\pi} \cos x \, dx = 0$$

E se eu quisesse calcular a área verde?

$$\int_0^{\pi} |\cos x| \, dx$$

ou

$$= \int_0^{\pi/2} (\cos x - 0)\,dx + \int_{\pi/2}^{\pi} (0 - \cos x)\,dx$$



TFC.1

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\frac{d}{dx}\int_3^x \cos t\,dt = \cos x$$

$$\frac{d}{dx}\int_3^x e^t\,dt = e^x$$

$$\frac{d}{dx}\boxed{\int_5^x e^t \cos t\, \text{sen}^2 t\,dt} = e^x \cos x\, \text{sen}^2 x$$

Função de x

$$\frac{d}{dx}\boxed{\int_5^8 e^t \cos t\, \text{sen}^2 t\,dt} = 0$$

constante

$$\frac{d}{dx}\int_x^5 e^t \cos t\, \text{sen}^2 t\,dt$$

$$= -\frac{d}{dx}\int_5^x e^t \cos t\, \text{sen}^2 t\,dt$$

$$= -e^x \cos x\, \text{sen}^2 x$$

TFC 2

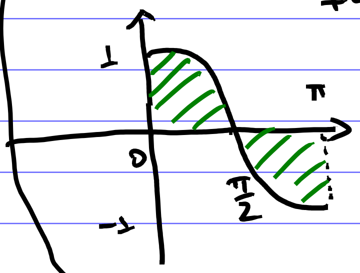
$F(x)$ é antiderivada de $f(x)$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\int_0^\pi \cos x\,dx = \operatorname{sen} x \Big|_0^\pi$$

$$= \operatorname{sen}\pi - \operatorname{sen} 0$$
$$= 0 - 0 = 0$$

$f(x) = \cos x$



$$\int_0^\pi \cos x\,dx = 0$$

$$\int_0^1 x\,dx = \frac{x^2}{2}\Big|_0^1 = \frac{1^2}{2} - \frac{0^2}{2} = \frac{1}{2}$$