# Técnica da substituição

a) $\displaystyle\int_0^3 \boxed{2x \cos(x^2)\, dx}$

$u = x^2 \therefore \dfrac{du}{dx} = 2x \therefore du = 2x\,dx$

Quando $x = 0$, $u = 0^2 = 0$
" $x = 3$, $u = 3^2 = 9$

$$= \int_0^9 \cos(u)\, du = \operatorname{sen} u \Big|_0^9 = \operatorname{sen} 9 - \operatorname{sen} 0$$
$$= \operatorname{sen} 9$$

b) $\displaystyle\int_1^2 4x^3 e^{x^4}\, dx$

$u = x^4 \therefore \dfrac{du}{dx} = 4x^3$

$du = 4x^3\, dx$

Quando $x = 1$, $u = 1^4 = 1$
" $x = 2$, $u = 2^4 = 16$

$$\int_1^{16} e^u\, du = e^u \Big|_1^{16}$$
$$= e^{16} - e$$

c) $\displaystyle\int_1^e \boxed{\frac{\ln x}{x}\, dx}$

$u = \ln x \therefore \dfrac{du}{dx} = \dfrac{1}{x} \therefore du = \dfrac{1}{x}\, dx$

Quando $x = 1$, $u = \ln 1 = 0$
" $x = e$, $u = \ln e = 1$

$$\int_0^1 u\, du = \frac{u^2}{2}\Big|_0^1 = \frac{1}{2} - \frac{0}{2} = \frac{1}{2}$$

"A 37 Ã© pra fazer por substituiÃ§Ã£o?" NÃO

**35–40** Calcule a integral, interpretando-a em termos das áreas.

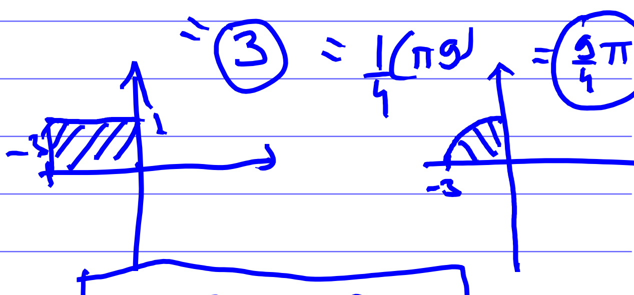
**35.** $\int_{-1}^{2} (1 - x)\, dx$ **36.** $\int_{0}^{9} \left(\frac{1}{3}x - 2\right) dx$

**37.** $\int_{-3}^{0} \left(1 + \sqrt{9 - x^2}\right) dx$ **38.** $\int_{-5}^{5} \left(x - \sqrt{25 - x^2}\right) d$

$$\int_{-3}^{0} \left(1 + \sqrt{9 - x^2}\right) dx$$

$$\int_{-3}^{0} 1\, dx + \int_{-3}^{0} \sqrt{9 - x^2}\, dx$$

$$= 3 \qquad = \frac{1}{4}(\pi 9) = \frac{9}{4}\pi$$




$$\boxed{= 3 + \frac{9}{4}\pi}$$

d) $\displaystyle\int_2^3 x \cos(x^2)\, dx$

$u = x^2 \therefore \dfrac{du}{dx} = 2x \therefore du = 2x\,dx$

$$\boxed{\frac{du}{2} = x\,dx}$$

$$\int_4^9 \cos(u)\, \frac{du}{2}$$

Quando $x = 2$, $u = 4$
$x = 3$, $u = 9$

$$= \frac{1}{2}\int_4^9 \cos(u)\, du = \frac{1}{2} \operatorname{sen} u \Big|_4^9 = \frac{1}{2}\left\{\operatorname{sen} 9 - \operatorname{sen} 4\right\}$$

Outras formas

$$\int \cos(u)\, \frac{du}{2} = \operatorname{sen} u = \operatorname{sen} x^2 \Big|_2^3 = \operatorname{sen} 9 - \operatorname{sen} 4$$

NÃO Recomendado!!!

e) $\displaystyle\int_0^{\pi} \operatorname{sen} x \sqrt{\cos^2 x}\, dx$

# Por partes

$$\int u\,dv = uv - \int v\,du$$

a) $\displaystyle\int_0^{\pi} \underset{u}{\underline{x}}\ \underset{dv}{\underline{\cos x\, dx}}$

$dv = \cos x\, dx \ \therefore\ v = \int dv = \int \cos x\, dx$

$\boxed{u = x}$ $\quad\quad \boxed{v = \operatorname{sen} x}$

$\dfrac{du}{dx} = 1 \ \therefore\ \boxed{du = dx}$

$$\boxed{\int_0^{\pi} x\cos x\, dx = x \operatorname{sen} x \Big|_0^{\pi} - \int_0^{\pi} \operatorname{sen} x\, dx}$$

$$= x \operatorname{sen} x \Big|_0^{\pi} - [-\cos x]\Big|_0^{\pi}$$

$$= [\pi \operatorname{sen} \pi - 0 . \operatorname{sen} 0] - \{[-\cos \pi] - [-\cos 0]\}$$

$$= 0 - \{+1 - [-1]\}$$

$$= -\{1 + 1\} = -2$$

b) $\displaystyle\int_0^{2} x e^{x}\, dx$

a) $\int_0^{\pi/2} \cos^2 x \, dx$

b) $\int_0^{\frac{\pi}{2}} \cos^3 x \, dx$

$= x \Big|_0^{b} - \underbrace{\int_0^{b} 2x \cos(x^2) \, dx}$

$\int_0^2 x^2 \cos(x^2) \, dx$

$u = x^2 \quad \frac{du}{dx} = 2x$

$x = \sqrt{u}$

$\frac{du}{2} = x \, dx$

$\int_0^2 x \, \underline{x} \, \overset{\cos u}{\cos(x^2)} \, \underline{dx}$ $\frac{du}{2}$

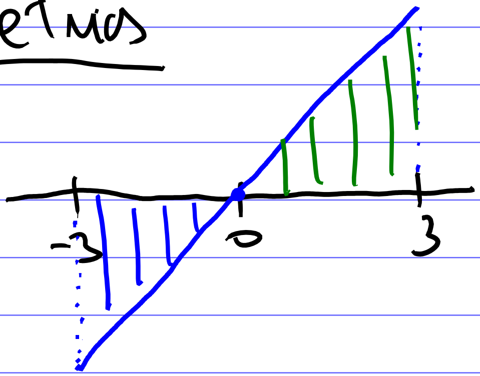
$\int x \cos u \frac{du}{2}$

$\int \sqrt{u} \cos u \frac{du}{2}$

A técnica não ajudou.

# Simetrias

a) $\int_{-3}^{3} x\,dx = 0$




ímpar

$\boxed{f(x) = -f(-x)}$

$f(x) = x$

$f(-x) = -x = -f(x)$ é ímpar

b) $\int_{-3}^{3} x^2\,dx = 2\int_{0}^{3} x^2\,dx$

PAR

$\boxed{f(x) = f(-x)}$

c) $\int_{-\pi/2}^{\pi/2} \cos(x)\,dx = 2\int_{0}^{\pi/2} \cos x\,dx = 2\,\operatorname{sen} x \Big|_{0}^{\pi/2}$

$= 2\cdot\left(\operatorname{sen}\frac{\pi}{2} - \operatorname{sen} 0\right)$

$= 2$

d) $\int_{-\pi/2}^{\pi/2} \operatorname{sen}(x)\,dx = 0$ pois o intervalo é simétrico e a função é ímpar.