1) Qual é o comprimento de arco da curva dada por $r = 1 + \cos(\theta), \quad 0 \le \theta \le \frac{\pi}{3}$?

$$L = \int_{\theta_i}^{\theta_f} \sqrt{r^2 + r'^2}\, d\theta$$

$$r = 1 + \cos\theta \quad \therefore \quad r' = -\operatorname{sen}\theta$$

$$r^2 = 1 + 2\cos\theta + \cos^2\theta \; \Big\{ \quad r'^2 = \operatorname{sen}^2\theta$$

$$L = \int_0^{\pi/3} \sqrt{1 + 2\cos\theta + \underbrace{\cos^2\theta + \operatorname{sen}^2\theta}_{1}}\, d\theta$$

$$L = \int_0^{\pi/3} \sqrt{2 + 2\cos\theta}\, d\theta$$

TÉCNICA: USO DE RELAÇÕES TRIGONOMÉTRICAS

$$\cos^2\varphi = \frac{1 + 1\cos(2\varphi)}{2}$$

$$2\cos^2\varphi = 1 + \cos(2\varphi)$$

$$4\cos^2\varphi = 2 + 2\cos(2\varphi)$$

Escolhas $\rightarrow$ $2\varphi = \theta$

$\varphi = \frac{\theta}{2}$

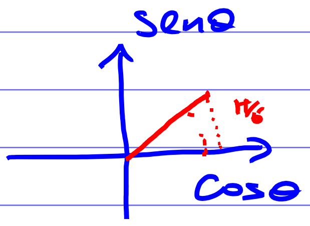
$$4\cos^2\left(\frac{\theta}{2}\right) = 2 + 2\cos(\theta)$$

Atenção

$$\sqrt{x^2} = |x|$$

$$L = \int_0^{\pi/3} \sqrt{4\cos^2\left(\frac{\theta}{2}\right)}\, d\theta$$

$$L = \int_0^{\pi/3} 2\left|\cos\left(\frac{\theta}{2}\right)\right| d\theta$$



Como $\cos\left(\frac{\theta}{2}\right) \geq 0$ p/ $0 < \theta < \frac{\pi}{3}$

reescrevemos

$$L = \int_0^{\pi/3} 2\cos\left(\frac{\theta}{2}\right) d\theta$$

Por substituição simples

$$u = \frac{\theta}{2} \quad \therefore \quad \frac{du}{d\theta} = \frac{1}{2} \quad \therefore \quad 2du = d\theta$$

Quando $\theta = 0$, $u = 0$

" $\theta = \frac{\pi}{3}$, $u = \frac{\pi}{6}$

$$L = \int_0^{\pi/6} 2\cos(u)\, \underbrace{2du}_{d\theta}$$

$$L = \int_0^{\pi/6} 4\cos(u)\, du = 4\operatorname{sen} u \Big|_0^{\pi/6}$$

$$= 4\left(\operatorname{sen}\frac{\pi}{6} - \operatorname{sen} 0\right)$$

$$= 4\left(\frac{1}{2} - 0\right)$$

$$= 2 \text{ u.c.}$$

1) Qual é o valor de k que faz com que a curva dada por $\vec{\mathrm{r}}(\mathrm{t}) = < 3 \cdot \cos(\mathrm{t}), 3 \cdot \sin(\mathrm{t}), \mathrm{k} \cdot \mathrm{t} >$, com $0 \leq \mathrm{t} \leq \pi$ possua comprimento igual a $\sqrt{13}\,\pi$?

$$L = \int_{t_i}^{t_f} |\vec{r}\,'| \, dt$$

$$\vec{r}\,' = < -3 \,\text{sen}\, t, 3\cos t, k >$$

$$|\vec{r}\,'| = \sqrt{9\,\text{sen}^2 t + 9\cos^2 t + k^2}$$

$$|\vec{r}\,'| = \sqrt{9 + k^2}$$

$$L = \int_0^{\pi} \sqrt{9 + k^2} \, dt$$

$$\sqrt{13}\,\pi = \int_0^{\pi} \sqrt{9 + k^2} \, dt$$

$$\sqrt{13}\,\pi = \sqrt{9 + k^2} \int_0^{\pi} dt$$

$$\sqrt{13}\,\pi = \sqrt{9 + k^2}\; \pi$$

$$\sqrt{13} = \sqrt{9 + k^2}$$

$$13 = 9 + k^2$$

$$4 = k^2 \;\therefore\; \boxed{k = 2 \text{ ou } k = -2}$$

1) Qual é o **módulo da componente tangencial** (em $\frac{m}{s^2}$) da aceleração de uma partícula cuja velocidade escalar no instante em questão é 2 m/s, cuja aceleração escalar é $10\ \frac{m}{s^2}$ e cuja curvatura do ponto onde se encontra é $2\ \mathrm{m}^{-1}$?

$$|\vec{a}_T| = ?$$
$$|\vec{v}| = 2\,m/s$$
$$|\vec{a}| = 10\,m/s^2$$
$$k = 2\,m^{-1}$$
$$\Downarrow$$
$$a_N = k\,|\bar{v}|^2$$
$$a_N = 2\cdot 2^2 = 8\,m/s^2$$

$$\boxed{\vec{a} = \frac{d^2s}{dt^2}\vec{T} + k\left(\frac{ds}{dt}\right)^2\vec{N}}$$

$$\vec{a} = \frac{d|\bar{v}|}{dt}\vec{T} + k\,|\bar{v}|^2\,\vec{N}$$

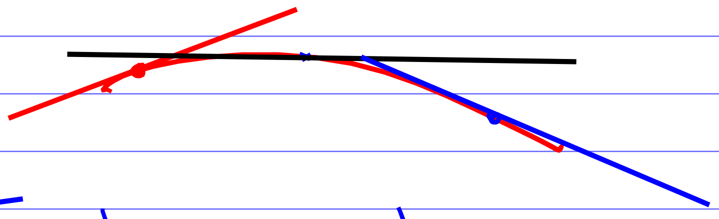
$$\bar{a} = a_T\vec{T} + a_N\vec{N}$$

$$|\bar{a}| = \sqrt{a_t^2 + a_N^2}$$

$$10 = \sqrt{a_t^2 + 8^2}$$

$$100 = a_t^2 + 64$$
$$36 = a_t^2$$
$$\boxed{a_T = 6\,m/s^2}$$

1) Em qual ponto da curva $\vec{r}(t) = <\cos(t), \sin(t), t^2>$ a reta tangente é paralela ao vetor $<-3,\ 0,\ 3\pi\ >$?



Os vetores tangentes à curva são encontrados com $\dfrac{d\vec{r}}{dt}$

$$\frac{d\vec{r}}{dt} = \langle -\text{sen}\,t,\ \cos t,\ 2t \rangle$$

$\vec{A} \times \vec{B} = 0$ se $\vec{A}$ é paralelo a $\vec{B}$

$$\vec{A}\times\vec{B} = \begin{array}{ccccc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ -\text{sen}\,t & \cos t & 2t & -\text{sen}\,t & \cos t \\ -3 & 0 & 3\pi & -3 & 0 \end{array}$$

$$3\cos t\,\hat{k} + 3\pi\,\text{sen}\,t\,\hat{j} + 3\pi\cos t\,\hat{i} - 6t\,\hat{j}$$

$$\vec{A}\times\vec{B} = \langle 3\pi\cos t,\ 3\pi\,\text{sen}\,t - 6t,\ 3\cos t \rangle$$

Para quais t's esse vetor é nulo?

III componente: $3\cos t = 0$

$$t = \frac{\pi}{2},\ \frac{3\pi}{2},\ \frac{5\pi}{2}, \dots$$

$$t = \frac{(2n+1)\pi}{2} \quad n = 0, 1, 2, \dots$$

MAS ELES ATENDEM AS COMPONENTES II e I?

$t = \frac{\pi}{2} \rightarrow$

$3\pi\,\text{sen}\,t - 6t$

$3\pi\,\text{sen}\frac{\pi}{2} - 6\frac{\pi}{2}$

$3\pi - 3\pi$ OK

$t = \frac{3\pi}{2} \rightarrow 3\pi\,\text{sen}\left(\frac{3\pi}{2}\right) - 6\left(\frac{3\pi}{2}\right)$ X

$3\pi(-1) - 9\pi \neq 0$

$t = \frac{5\pi}{2} \rightarrow 3\pi\,\text{sen}\left(\frac{5\pi}{2}\right) - 6\left(\frac{5\pi}{2}\right)$ X

$3\pi(1) - 15\pi \neq 0$

O ponto da curva é

$$\vec{r}\left(\frac{\pi}{2}\right) = \left\langle \cos\frac{\pi}{2},\ \text{sen}\frac{\pi}{2},\ \left(\frac{\pi}{2}\right)^2 \right\rangle$$

$$= \left\langle 0,\ 1,\ \frac{\pi^2}{4} \right\rangle$$

O ponto é: $\left(0, 1, \frac{\pi^2}{4}\right)$

1) Qual é o vetor binormal no ponto $t = \pi$ para a trajetória descrita por $\vec{r}(t) = <\cos(t), t, \sin(t)>$?

a) $< 0, \frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2} >$
b) $< 0, \frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2} >$
c) $< \frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}, 0 >$
d) $< \frac{-1}{2}, \frac{-\sqrt{2}}{2}, \frac{1}{2} >$
e) $< \frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}, 0 >$
f) $< 1, 0, 0 >$
g) $< 0, 1, 0 >$
h) $< 0, 0, 1 >$

$\vec{r}(t) = <\cos t, t, \operatorname{sen} t>$

$\vec{B} = \vec{T} \times \vec{N}$

$\vec{T} = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \frac{<-\operatorname{sen} t, 1, \cos t>}{\sqrt{\operatorname{sen}^2 t + 1 + \cos^2 t}}$

$\vec{T} = \frac{<-\operatorname{sen} t, 1, \cos t>}{\sqrt{2}}$

$\vec{N} = \frac{\vec{T}'}{|\vec{T}'|}$ então $\vec{T}' = \frac{1}{\sqrt{2}} <-\cos t, 0, -\operatorname{sen} t>$

$|\vec{T}'| = \frac{1}{\sqrt{2}} \sqrt{\cos^2 t + 0^2 + \operatorname{sen}^2 t}$

$|\vec{T}'| = \frac{1}{\sqrt{2}}$

$\vec{N} = \frac{\frac{1}{\sqrt{2}} <-\cos t, 0, -\operatorname{sen} t>}{\frac{1}{\sqrt{2}}}$

$\boxed{\vec{N} = <-\cos t, 0, -\operatorname{sen} t>}$

$\vec{T}(\pi) \frac{<-\operatorname{sen}\pi, 1, \cos\pi>}{\sqrt{2}} = \frac{<0, 1, -1>}{\sqrt{2}}$

$\vec{N}(\pi) = <-\cos\pi, 0, -\operatorname{sen}\pi> = <1, 0, 0>$

$$\vec{T}(\pi) \times \vec{N}(\pi) = \left| \begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 1 & 0 & 0 & 1 & 0 \end{array} \right.$$

$-\frac{1}{\sqrt{2}}\hat{k}$ $\qquad$ $-\frac{1}{\sqrt{2}}\hat{j}$

$<0, \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}>$

$= <0, \frac{-\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}>$

1) Qual das seguintes funções vetoriais é perpendicular à própria derivada?

a) $< \cos\left(t^2\right), \sin\left(t^2\right),\ 1 >$
b) $< \cos\left(t^2\right), \sin\left(t^2\right), t >$
c) $< \cos(t), \sin(t), t >$
d) $< 5, 3, t >$
e) $< \cos(t), t, t^2 >$
f) $< \cos(t), \sin(t),\ e^t >$

1) Qual é a expressão que fornece corretamente a área de interseção entre os dois círculos da figura? (A área hachurada).