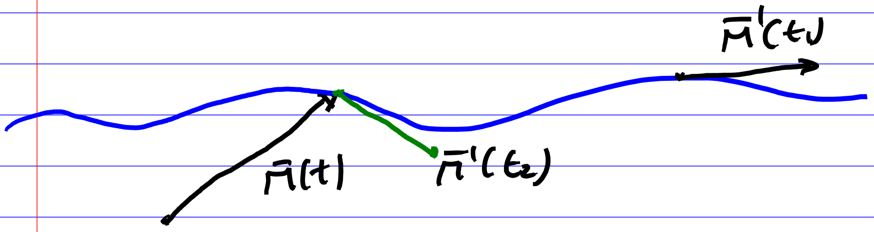
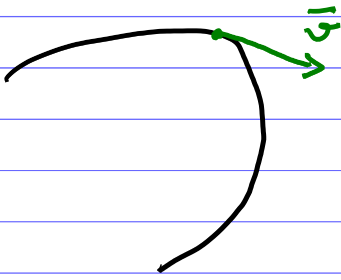



$$\vec{v} = \frac{d\vec{r}(t)}{dt} \quad \therefore \int \vec{v}\,dt = \int d\vec{r}$$

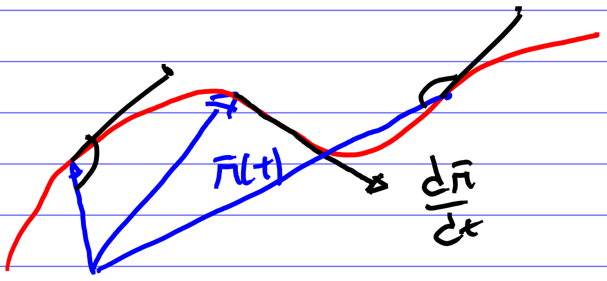
$$\vec{a} = \frac{d\vec{v}}{dt} \qquad \boxed{\int \vec{v}\,dt = \vec{r}}$$




$\vec{r}$ = Posição

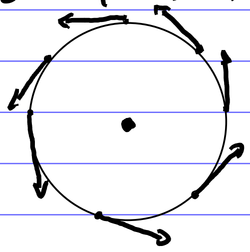
$\vec{r}' = \vec{v}$ = Velocidade.

$\vec{r}'' = \vec{a}$ = aceleração




$$\vec{r} = \langle \cos t, \operatorname{sen} t \rangle$$

$$\frac{d\vec{r}}{dt} = \langle -\operatorname{sen} t, \cos t \rangle$$



$\vec{T}$ vetor tangente unitário.

$$\vec{T} = \frac{\vec{r}'}{|\vec{r}'|} = \frac{\vec{v}}{|\vec{v}|}$$

$$\vec{T} = \frac{\vec{v}}{|\vec{v}|} = \frac{\langle -\operatorname{sen} t, \cos t \rangle}{\sqrt{(-\operatorname{sen} t)^2 + (\cos t)^2}}$$

$$\vec{T} = \langle -\operatorname{sen} t, \cos t \rangle$$

Nesse caso

$\vec{T} = \vec{v}$

coincidência.

**21.** Se $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$, encontre $\mathbf{r}'(t)$, $\mathbf{T}(1)$, $\mathbf{r}''(t)$ e $\mathbf{r}'(t) \times \mathbf{r}''(t)$.

$$\vec{r} = \langle t, t^2, t^3 \rangle$$
$$\vec{r}\,' = \langle 1, 2t, 3t^2 \rangle$$
$$\vec{r}\,'' = \langle 0, 2, 6t \rangle$$

$$\vec{T}(1) = \frac{\vec{r}\,'(1)}{|\vec{r}\,'(1)|} = \frac{\langle 1, 2, 3 \rangle}{\sqrt{1^2+2^2+3^2}}$$

$$= \frac{\langle 1, 2, 3 \rangle}{\sqrt{1+4+9}}$$

OK! $\rightarrow$ $$= \frac{\langle 1, 2, 3 \rangle}{\sqrt{14}}$$

$$= \left\langle \frac{\sqrt{14}}{14}, \frac{2\sqrt{14}}{14}, \frac{3\sqrt{14}}{14} \right\rangle$$

$$\vec{r}\,' = \langle 1, 2t, 3t^2 \rangle$$
$$\vec{r}\,'' = \langle 0, 2, 6t \rangle$$

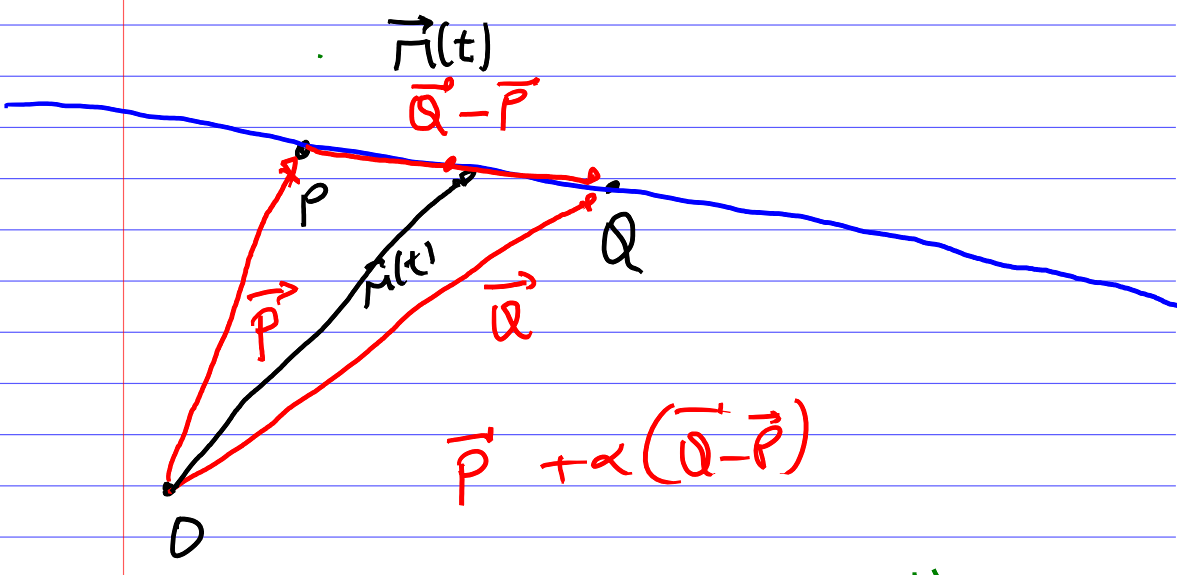
$$\vec{r}\,' \times \vec{r}\,'' = \left| \begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 1 & 2t & 3t^2 & 1 & 2t \\ 0 & 2 & 6t & 0 & 2 \end{array} \right|$$

$$= 0\hat{k} - 6t^2\hat{i} - 6t\hat{j} + 12t^2\hat{i} + 0\hat{j} + 2\hat{k}$$

$$\vec{r}\,' \times \vec{r}\,'' = 6t^2\hat{i} - 6t\hat{j} + 2\hat{k}$$

**17–20** Encontre uma equação vetorial e equações paramétricas para o segmento de reta que liga $P$ e $Q$.

**17.** $P(0, 0, 0)$, $Q(1, 2, 3)$ **18.** $P(1, 0, 1)$, $Q(2, 3, 1)$




$$\vec{R}(\alpha)= \vec{P}+\alpha(\vec{Q}-\vec{P})$$

$$\vec{R}(t)= \vec{P}+t(\vec{Q}-\vec{P})$$

Eq. vetorial da reta

a) $P(0,0,0)$ $Q(1,2,3)$

$$\vec{R}(\alpha)= \vec{P}+\alpha(\vec{Q}-\vec{P})$$

$$\vec{R}(\alpha)= \langle 0,0,0\rangle + \alpha(\langle 1,2,3\rangle - \langle 0,0,0\rangle)$$

$$\vec{R}(\alpha)= \alpha\langle 1,2,3\rangle$$

b) $P(1,0,1)$ $Q(2,3,1)$

$$\vec{R}(\alpha)= \vec{P}+\alpha(\vec{Q}-\vec{P})$$

$$\vec{R}(\alpha)= \langle 1,0,1\rangle + \alpha(\langle 2,3,1\rangle - \langle 1,0,1\rangle)$$

$$\vec{R}(\alpha)= \langle 1,0,1\rangle + \alpha(\langle 1,3,0\rangle)$$

$$\boxed{\vec{R}(\alpha)= \vec{R}_0+\alpha(\vec{P}-\vec{R}_0)}$$

# Exercício

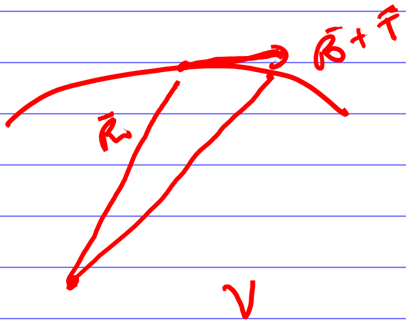
1. Calcule o vetor tangente unitário para o movimento descrito por

$$\vec{r}(t) = \langle \cos t, \text{sen}\, t, t \rangle$$

Solução:

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \frac{\langle -\text{sen}\, t, \cos t, 1 \rangle}{\sqrt{\text{sen}^2 t + \cos^2 t + 1}}$$

$$= \frac{\langle -\text{sen}\, t, \cos t, 1 \rangle}{\sqrt{2}}$$




https://geogebra.org/3d/tqu3teud