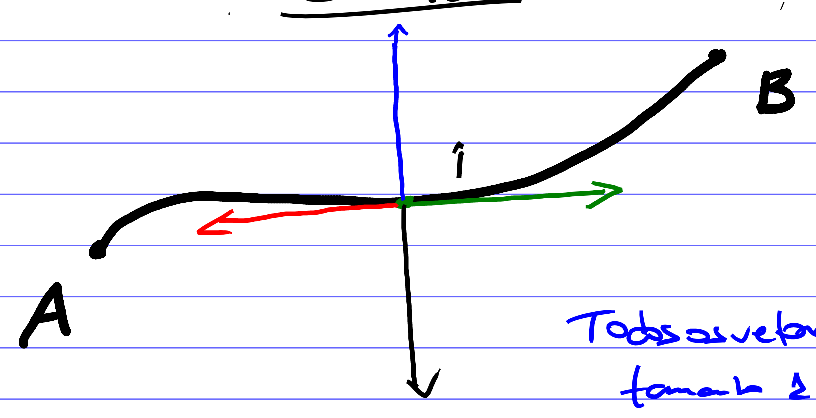
Exercício



Todos os vetores tem tamanho 1.

Para uma partícula que vai de A até B

$\vec{T}$? $\vec{N}$?

Solução: $\vec{T}$ $\vec{N}$

Para uma partícula que vai de B até A?

$\vec{T}$? $\vec{N}$?

Solução: $\vec{T}$ $\vec{N}$

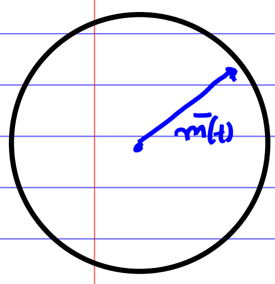
Como calcular $\vec{N}$?

$$\vec{T} = \frac{\vec{r}'}{|\vec{r}'|}$$

Sempre que derivamos uma função vetorial de módulo constante, obtemos um vetor perpendicular a ela.

$$\vec{m}(t) = \langle 3\cos t, 3\,\text{sen}\, t \rangle$$

$$|\vec{m}(t)| = \sqrt{9\cos^2 t + 9\,\text{sen}^2 t} = 3$$



$$|\vec{m}| = \sqrt{\vec{m}\circ\vec{m}} = c$$

$$\vec{m}\circ\vec{m} = c^2$$

$$\frac{d}{dt}(\vec{m}\circ\vec{m}) = \frac{dc^2}{dt}$$

$$\frac{d\vec{m}}{dt}\circ\vec{m} + \vec{m}\circ\frac{d\vec{m}}{dt} = 0$$

$$2\left(\vec{m}\circ\frac{d\vec{m}}{dt}\right) = 0$$

$$\vec{m}\circ\frac{d\vec{m}}{dt} = 0$$

$\vec{m}$ é perpendicular a $\frac{d\vec{m}}{dt}$.

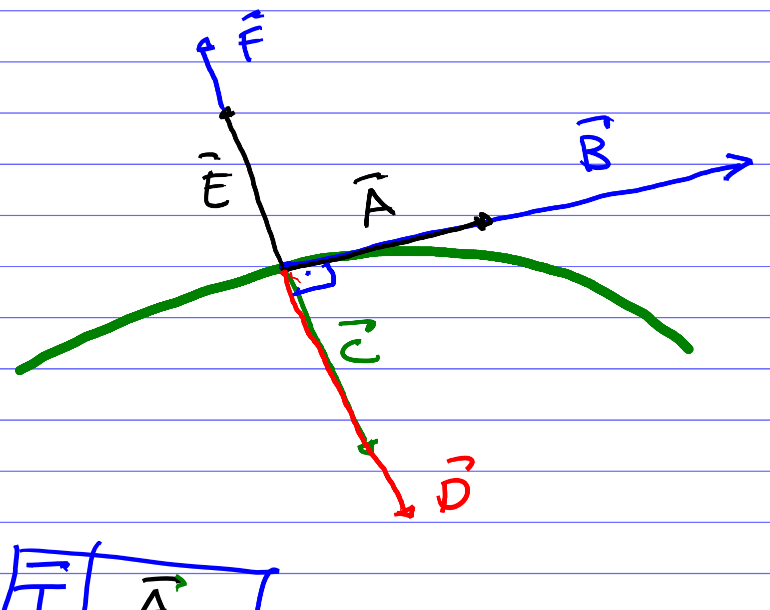
$\vec{T}$ tem módulo constante.

$|\vec{T}| = 1$ Portanto

$\frac{d\vec{T}}{dt}$ é perpendicular a $\vec{T}$

$$\boxed{\vec{N} = \frac{\frac{d\vec{T}}{dt}}{\left|\frac{d\vec{T}}{dt}\right|} = \frac{\vec{T}'}{|\vec{T}'|}}$$

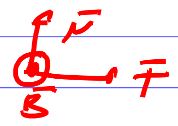
$$\boxed{\vec{T} = \frac{\vec{r}'}{|\vec{r}'|}}$$



$\vec{T}$	$\vec{A}$
$\vec{N}$	$\vec{C}$
$\vec{v}$	$\vec{B}$
$\frac{d\vec{T}}{dt}$	$\vec{D}$

Vetor Binormal

$\vec{B} = \vec{T} \times \vec{N}$



Exercício

Determine os vetores normal e binormal da hélice circular.

$\vec{r}(t) = \cos t\,\hat{i} + \operatorname{sen} t\,\hat{j} + t\,\hat{k}$

Solução

$$\vec{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|} = \frac{\langle -\operatorname{sen} t, \cos t, 1\rangle}{\sqrt{(-\operatorname{sen} t)^2 + (\cos t)^2 + 1}}$$

$$\vec{T} = \frac{1}{\sqrt{2}}\langle -\operatorname{sen} t, \cos t, 1\rangle$$

$$\vec{N} = \frac{\vec{T}\,'}{|\vec{T}\,'|} = \frac{\frac{1}{\sqrt{2}}\langle -\cos t, -\operatorname{sen} t, 0\rangle}{\left|\frac{1}{\sqrt{2}}\langle -\cos t, -\operatorname{sen} t, 0\rangle\right|}$$

$$\vec{N} = \frac{\frac{1}{\sqrt{2}}\langle -\cos t, -\operatorname{sen} t, 0\rangle}{\frac{1}{\sqrt{2}}\left|\langle -\cos t, -\operatorname{sen} t, 0\rangle\right|}$$

$$\vec{N} = \frac{\langle -\cos t, -\operatorname{sen} t, 0\rangle}{\sqrt{\cos^2 t + \operatorname{sen}^2 t + 0^2}}$$

É o menos) aqui, $(-1)^2 = 1$

$$\vec{N} = \langle -\cos t, -\operatorname{sen} t, 0\rangle$$

$$\vec{B} = \vec{T} \times \vec{N} \qquad \boxed{\vec{T} = \frac{1}{\sqrt{2}}\langle -\operatorname{sen} t, \cos t, 1\rangle}$$

$$\vec{B} = \left|\begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ -\frac{1}{\sqrt{2}}\operatorname{sen} t & \frac{1}{\sqrt{2}}\cos t & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\operatorname{sen} t & \frac{1}{\sqrt{2}}\cos t \\ -\cos t & -\operatorname{sen} t & 0 & -\cos t & -\operatorname{sen} t \end{array}\right|$$

$$= \frac{1}{\sqrt{2}}\cos^2 t\,\hat{k} + \frac{1}{\sqrt{2}}\operatorname{sen} t\,\hat{i} + 0\,\hat{j} + 0\,\hat{i} - \frac{1}{\sqrt{2}}\cos t\,\hat{j} + \frac{1}{\sqrt{2}}\operatorname{sen}^2 t\,\hat{k}$$

$$= \left\langle \frac{1}{\sqrt{2}}\operatorname{sen} t, -\frac{1}{\sqrt{2}}\cos t, \frac{1}{\sqrt{2}}\cos^2 t + \frac{1}{\sqrt{2}}\operatorname{sen}^2 t \right\rangle$$

$$= \left\langle \frac{1}{\sqrt{2}}\operatorname{sen} t, -\frac{1}{\sqrt{2}}\cos t, \frac{1}{\sqrt{2}} \right\rangle$$

$$\vec{B} = \frac{1}{\sqrt{2}}\langle \operatorname{sen} t, -\cos t, 1\rangle$$

$$\boxed{\vec{T} = \frac{1}{\sqrt{2}}\langle -\operatorname{sen} t, \cos t, 1\rangle}$$

$$\boxed{\vec{N} = \langle -\cos t, -\operatorname{sen} t, 0\rangle}$$

$$\boxed{\vec{B} = \frac{1}{\sqrt{2}}\langle \operatorname{sen} t, -\cos t, 1\rangle}$$

https://www.geogebra.org/3d/w6xf8yrm

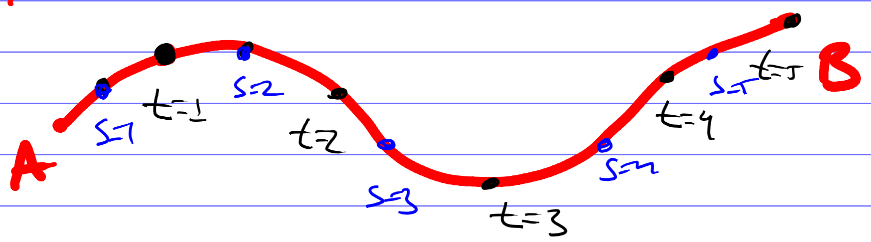
Reparametrização

$$\vec{r}(t) = \langle t, t^2, t^3 \rangle$$

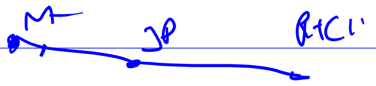
$$\vec{r}(u) = \langle u, u^2, u^3 \rangle$$

ReparametrizaÃ§Ã£o sem alteraÃ§Ã£o de significado

$\vec{r}(t)$



S = Parâmetro comprimento de arco.



$\vec{r}(s)$

$$\vec{r}(t) = \langle \cos t, \operatorname{sen} t, t \rangle$$

$$L = \int_{t_i}^{t_f} |\vec{r}\,'(t)|\, dt$$

Parâmetro comprimento do arco.

$$S = \int_0^t |\vec{r}\,'(\tau)|\, d\tau$$

Se a trajetória é dada por f(x)

$$\boxed{L = \int_a^b \sqrt{1 + (f')^2}\, dx}$$

$$\vec{r}(\tau) = \langle \cos\tau, \operatorname{sen}\tau, \tau \rangle$$

$$\vec{r}\,'(\tau) = \langle -\operatorname{sen}\tau, \cos\tau, 1 \rangle$$

$$|\vec{r}\,'(\tau)| = \sqrt{\operatorname{sen}^2\tau + \cos^2\tau + 1} = \sqrt{2}$$

$$S = \int_0^t \sqrt{2}\, dt$$

$$S = \sqrt{2}\, t \quad \therefore \quad t = \frac{S}{\sqrt{2}}$$

$$\vec{r}(t) = \langle \cos t, \operatorname{sen} t, t \rangle$$

$$\vec{r}(s) = \left\langle \cos\left(\frac{s}{\sqrt{2}}\right), \operatorname{sen}\left(\frac{s}{\sqrt{2}}\right), \frac{s}{\sqrt{2}} \right\rangle$$

Reparametrização