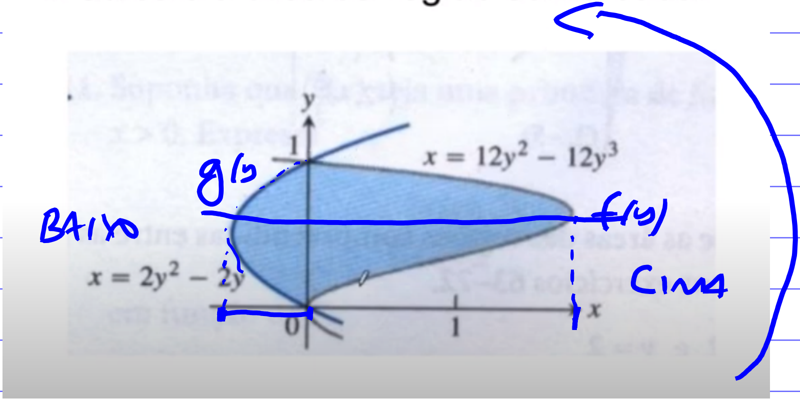
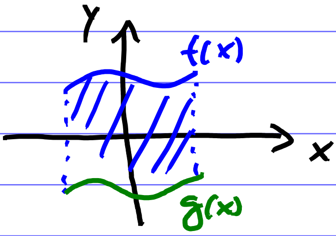
5.2 23

5.3 53

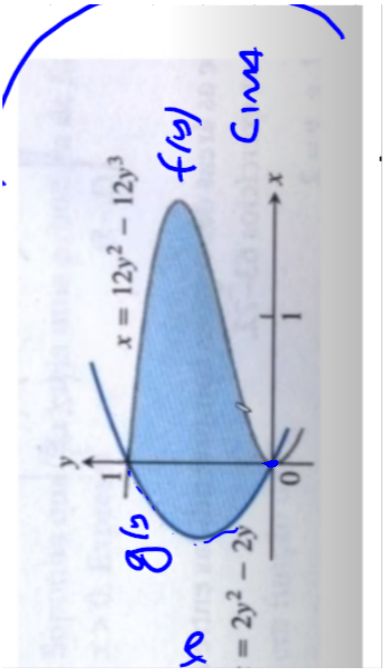
4. Calcule a área da região sombreada:



$$A = \int_a^b \left(f(x) - g(x)\right) dx$$



$$A = \int_a^b \left(f(y) - g(y)\right) dy$$



$$A = \int_0^1 \left(\left[12y^2 - 12y^3\right] - \left[2y^2 - 2y\right]\right) dy$$

4 Teorema Se f for integrável em $[a, b]$, então

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

onde $\quad \Delta x = \dfrac{b-a}{n} \quad$ e $\quad x_i = a + i\,\Delta x$

21–25 Use a forma da definição de integral dada no Teorema 4 para calcular a integral.

21. $\int_{-1}^{5} (1 + 3x)\,dx$

22. $\int_{1}^{4} (x^2 + 2x - 5)\,dx$

23. $\int_{-2}^{0} (x^2 + x)\,dx$

24. $\int_{0}^{2} (2x - x^3)\,dx$

25. $\int_{0}^{1} (x^3 - 3x^2)\,dx$

$$\int_{-2}^{0} (x^2+x)\,dx \qquad \Delta x = \frac{0-(-2)}{n} = \frac{2}{n}$$

$$x_i = -2 + i\frac{2}{n}$$

$$\int_{-2}^{0} (x^2+x)\,dx = \lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$f(x) = x^2 + x$$

$$f(x_i) = x_i^2 + x_i = \left(-2+\frac{i2}{n}\right)^2 + \left(-2+\frac{i2}{n}\right)$$

$$f(x_i) = 4 - \frac{8i}{n} + \frac{4i^2}{n^2} - 2 + \frac{2i}{n}$$

$$f(x_i) = 2 - \frac{6i}{n} + \frac{4i^2}{n^2}$$

$$\int_{-2}^{0} (x^2+x)\,dx = \lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$= \lim_{n\to+\infty} \sum_{i=1}^{n} \left(2 - \frac{6i}{n} + \frac{4i^2}{n^2}\right)\frac{2}{n}$$

$$= \lim_{n\to+\infty} \sum_{i=1}^{n} \left(\frac{4}{n} - \frac{12i}{n^2} + \frac{8i^2}{n^3}\right)$$

$$= \lim_{n\to+\infty} \left(\frac{4}{n}\sum_{i=1}^{n} 1 - \frac{12}{n^2}\sum_{i=1}^{n} i + \frac{8}{n^3}\sum_{i=1}^{n} i^2\right)$$

$$= \lim_{n\to+\infty} \left(\frac{4}{n}\,n - \frac{12}{n^2}\,\frac{n(n+1)}{2} + \frac{8}{n^3}\,\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \lim_{n\to+\infty} \left(4 - 6\cdot\frac{n^2+n}{n^2} + \frac{8}{6}\,\frac{(n^2+n)(2n+1)}{n^3}\right)$$

Alguns somatórios de funções polinomiais

1. $\boxed{\sum_{i=m}^{n} 1 = n + 1 - m}$
2. $\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$ (Soma de uma progressão aritmética)
3. $\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ (Número piramidal quadrado)
4. $\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ [3]
5. $\sum_{i=1}^{n} i^4 = 1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$ [3]
6. $\sum_{i=1}^{n} i^5 = 1^5 + 2^5 + 3^5 + \cdots + n^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$ [3]
7. $\sum_{i=1}^{n} i^6 = 1^6 + 2^6 + 3^6 + \cdots + n^6 = \frac{n(n+1)(2n+1)(3n^4+6n^3-3n+1)}{42}$ [3]
8. $\sum_{i=0}^{n} i^p = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^{p} \frac{B_k}{p-k+1}\binom{p}{k}(n+1)^{p-k+1}$

$$= \lim_{n\to+\infty} \left(4 - 6\cdot\frac{n^2+n}{n^2} + \frac{8}{6}\,\frac{(n^2+n)(2n+1)}{n^3}\right)$$

$$= \lim_{n\to+\infty} \left(4 - 6 + \frac{6}{n} + \frac{8}{6}\,\frac{2n^3 + n^2 + 2n^2 + n}{n^3}\right)$$

$$= \lim_{n\to+\infty} \left(-2 + \frac{6}{n} + \frac{8}{6}\left(2 + \frac{1}{n} + \frac{2}{n} + \frac{1}{n^2}\right)\right)$$

$$= \lim_{n\to+\infty} \left(-2 + \cancelto{0}{\frac{6}{n}} + \frac{16}{6} + \cancelto{0}{\frac{8}{6n}} + \cancelto{0}{\frac{16}{6n}} + \cancelto{0}{\frac{8}{6n^2}}\right)$$

$$= -2 + \frac{16}{6} = \frac{-12+16}{6} = \frac{4}{6} = \frac{2}{3}$$

Conferindo via TFC.2

$$\int_{-2}^{0} (x^2+x)\,dx = \left[\frac{x^3}{3} + \frac{x^2}{2}\right]\Bigg|_{-2}^{0}$$

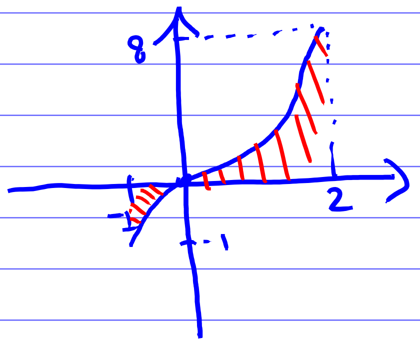
$$= \left[\frac{0}{3} + \frac{0}{2}\right] - \left[\frac{(-2)^3}{3} + \frac{(-2)^2}{2}\right]$$

$$= 0 - \left[\frac{-8}{3} + \frac{4}{2}\right]$$

$$= \frac{8}{3} - 2 = \frac{8-6}{3} = \frac{2}{3}$$

53–54 Calcule a integral e interprete-a como uma diferença de áreas. Ilustre com um esboço.

53. $\int_{-1}^{2} x^3\,dx$ **54.** $\int_{\pi/6}^{2\pi} \cos x\,dx$



$$\int_{-1}^{2} x^3\,dx = \frac{x^4}{4}\Big|_{-1}^{2} = \frac{2^4}{4} - \frac{(-1)^4}{4} = \frac{16}{4} - \frac{1}{4}$$

$$= \frac{15}{4}$$

$$\int_0^2 x^3\,dx - \underbrace{\int_0^{-1} x^3\,dx} = \frac{15}{4}$$

Troquei a ordem dos limites de integraÃ§Ã£o para que a integral represente a Ã¡rea.

6 - Qual é o resultado da operação $\frac{d}{du}\int_0^{u^{\frac{1}{2}}} x^2 \cos\left(x^4\right) dx$?

$$\frac{d}{du}\int_0^{u^{1/2}} x^2 \cos(x^4)\,dx$$

$$= \underbrace{\frac{d}{dw}\int_0^{w} x^2 \cos(x^4)\,dx}_{TFC1} \cdot \frac{dw}{du}$$

$$= w^2 \cos w^4 \cdot \frac{1}{2} u^{-\frac{1}{2}}$$

$$= w^2 \cos w^4 \, \frac{1}{2\sqrt{u}}$$

$$\Rightarrow = \frac{u}{2\sqrt{u}} \cos(u^2) = \frac{\sqrt{u}}{2} \cos(u^2)$$

$f(x) = \cos x^2$

$u = x^2$

$f(u) = \cos u$

$\frac{df}{dx} = ?$

$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$ Regra da cadeia usando a notação de Leibniz