$\xrightarrow{H}$

$\Delta H = 5$

$T \backslash H$	40	45	50	55	60	65	70	75	80
26	28	28	29	31	31	32	33	34	35
28	31	32	33	34	35	36	37	38	39
30	34	35	36	37	38	40	41	42	43
32	37	38	39	41	42	43	45	46	47
34	41	42	43	45	47	48	49	51	52
36	43	45	47	48	50	51	53	54	56

Umidade relativa (%) — Temperatura real (°C) — T

$$\frac{\partial S(H,T)}{\partial H} = \lim_{\Delta H \to 0} \frac{S(H+\Delta H, T) - S(H,T)}{\Delta H}$$

Estimativa

$$\frac{\partial S(H,T)}{\partial H} \approx \frac{S(H+\Delta H, T) - S(H,T)}{\Delta H}$$

$$\frac{\partial S}{\partial H}(60,30) = \left.\frac{\partial S}{\partial H}\right|_{(60,30)} = S_H(60,30)$$

Uma estimativa p/ este derivada é:

$$\frac{\partial S}{\partial H}(60,30) \approx \frac{S(60+\Delta H, 30) - S(60,30)}{\Delta H}$$

Analisando a tabela descubro que o menor ΔH é 5.

$$\frac{\partial S}{\partial H}(60,30) \approx \frac{S(60+5, 30) - S(60,30)}{5}$$

$$\approx \frac{S(65,30) - S(60,30)}{5}$$

$$\approx \frac{40-38}{5} = \frac{2}{5} \approx 0{,}4$$

$$\boxed{\frac{\partial S}{\partial H}(60,30) \approx \frac{2}{5}}$$

Aperfeiçoamento

USAR $\Delta H = 5$ ↓ USAR $\Delta H = -5$ ↓

$$\frac{\partial S}{\partial H}(60,30) \approx \frac{\dfrac{S(60+\Delta H,30) - S(60,30)}{\Delta H} + \dfrac{S(60+\Delta H,30) - S(60,30)}{\Delta H}}{2}$$

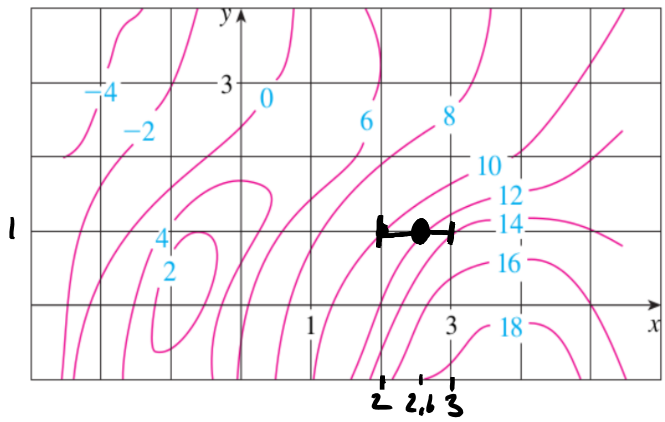
$$\frac{\partial S}{\partial H}(60,30) \approx \frac{\dfrac{S(65,30) - S(60,30)}{5} + \dfrac{S(55,30) - S(60,30)}{(-5)}}{2}$$

$$\approx \frac{\dfrac{2}{5} + \left(\dfrac{37-38}{-5}\right)}{2}$$

$$\approx \frac{\dfrac{2}{5} + \left(+\dfrac{1}{5}\right)}{2} = \frac{\frac{3}{5}}{2} = \frac{3}{10} \approx 0{,}3$$

Continuando estimativas...

10. Um mapa de contorno de uma função f é apresentado. Utilize-o para estimar $f_x(2, 1)$ e $f_y(2, 1)$.



Vamos fazer a estimativa mais simples.

$$f_x(x,y) \approx \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$f_x(2,1) \approx \frac{f(2+0.6, 1) - f(2,1)}{0.6}$$

$$\approx \frac{12-10}{0.6} = \frac{2}{0.6} = \frac{2}{\frac{6}{10}}$$

$$= \frac{20}{6}$$

$$= \frac{10}{3}$$

$$f_x(2,1) \approx \frac{10}{3}$$

Obtenha as derivadas parciais:

18. $f(x, t) = \sqrt{x} \ln t$

30. $F(\alpha, \beta) = \int_{\alpha}^{\beta} \sqrt{t^3 + 1}\, dt$

$f(x,t) = \sqrt{x} \ln t$

$$f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(\sqrt{x} \ln t)$$

$$= \ln t \frac{\partial}{\partial x} \sqrt{x}$$

$$= \ln t \frac{\partial}{\partial x} x^{1/2}$$

$$= \ln t \ \frac{1}{2} x^{\frac{1}{2}-1}$$

$$= \ln t \ \frac{1}{2} x^{-\frac{1}{2}}$$

$$= \ln t \ \frac{1}{2} \frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}} \ln t$$

$$\boxed{f_x = \frac{1}{2\sqrt{x}} \ln t}$$

$$f_t = \frac{\partial f}{\partial t} = \frac{\partial}{\partial t}(\sqrt{x} \ln t) = \sqrt{x} \frac{\partial \ln t}{\partial t}$$

$$\boxed{f_t = \frac{\sqrt{x}}{t}}$$

— // —

$$F(\alpha, \beta) = \int_{\alpha}^{\beta} \sqrt{t^3+1}\, dt$$

$$F_\alpha = \frac{\partial}{\partial \alpha} \int_{\alpha}^{\beta} \sqrt{t^3+1}\, dt$$

$$F_\alpha = -\boxed{\frac{\partial}{\partial \alpha} \int_{\beta}^{\alpha} \sqrt{t^3+1}\, dt} \text{ TFC.}$$

$$F_\alpha = -\sqrt{\alpha^3+1}$$

$$F_\beta = \frac{\partial}{\partial \beta} \int_{\alpha}^{\beta} \sqrt{t^3+1}\, dt$$

$$= \sqrt{\beta^3+1}$$

15–40 Determine as derivadas parciais de primeira ordem da função.

15. $f(x, y) = y^5 - 3xy$

16. $f(x, y) = x^4y^3 + 8x^2y$

33. $w = \ln(x + 2y + 3z)$

18. $f(x, t) = \sqrt{x} \ln t$

39. $u = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$

33 $\quad w = \ln(x + 2y + 3z)$

$$\frac{\partial w}{\partial x} = \frac{1}{x + 2y + 3z} \cdot \frac{\partial}{\partial x}(x + 2y + 3z)$$

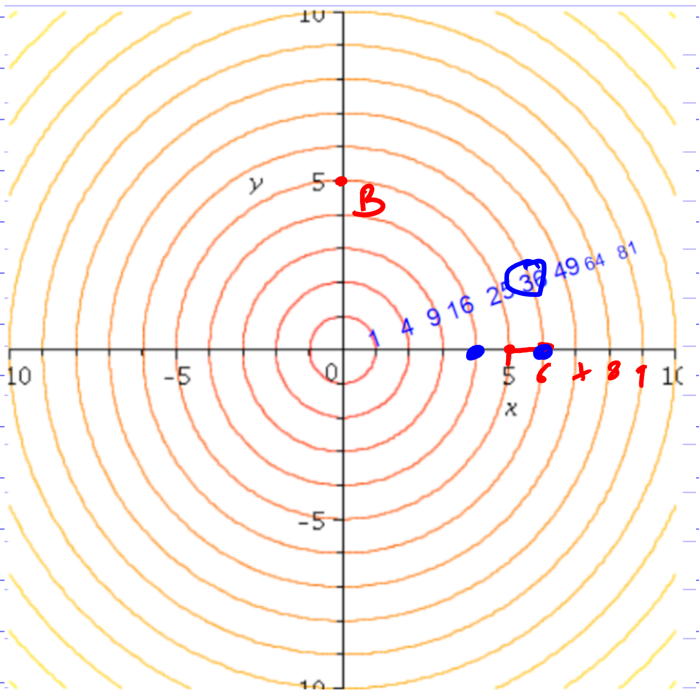
$$\boxed{\frac{\partial w}{\partial x} = \frac{1}{x + 2y + 3z}}$$

$$\frac{\partial w}{\partial y} = \frac{1}{x + 2y + 3z} \cdot \frac{\partial}{\partial y}(x + 2y + 3z)$$

$$\boxed{\frac{\partial w}{\partial y} = \frac{2}{x + 2y + 3z}}$$

$$\frac{\partial w}{\partial z} = \frac{1}{x + 2y + 3z} \cdot \frac{\partial}{\partial z}(x + 2y + 3z)$$

$$\boxed{\frac{\partial w}{\partial z} = \frac{3}{x + 2y + 3z}}$$



$$\left.\frac{\partial f}{\partial x}\right|_B = 0$$

$$\left.\frac{\partial f}{\partial y}\right|_B > 0$$

$$f(5,0) = 25 \qquad \left.\frac{\partial f}{\partial x}\right|_{(5,0)} > 0$$

$$\left.\frac{\partial f}{\partial x}\right|_{(5,0)} \approx \frac{\dfrac{f(5+1,0) - f(5,0)}{1} + \dfrac{f(5-1,0) - f(5,0)}{(-1)}}{2}$$

$$\approx \frac{f(6,0) - \cancel{f(5,0)} - f(4,0) + \cancel{f(5,0)}}{2}$$

$$\approx \frac{f(6,0) - f(4,0)}{2}$$

$$\approx \frac{36 - 16}{2} = 10$$

$$\left.\frac{\partial f}{\partial x}\right|_{(5,0)} \approx 10$$

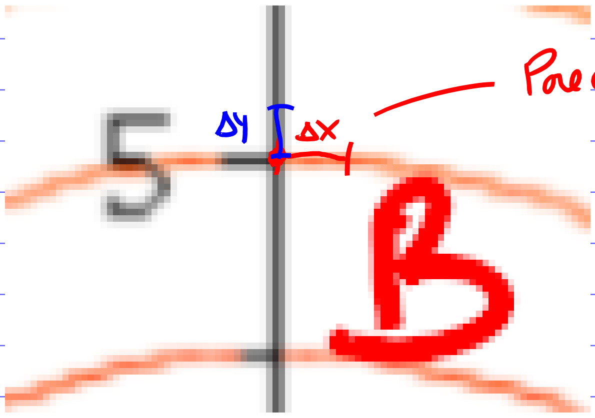
Comparando com o valor exato.

A expressão p/ a função é:

$$f(x,y) = x^2 + y^2$$

$$\left.\frac{\partial f}{\partial x}\right|_{(5,0)} = \left.\frac{\partial}{\partial x}(x^2+y^2)\right|_{(5,0)} = \left.2x\right|_{(5,0)}$$

$$= 10$$



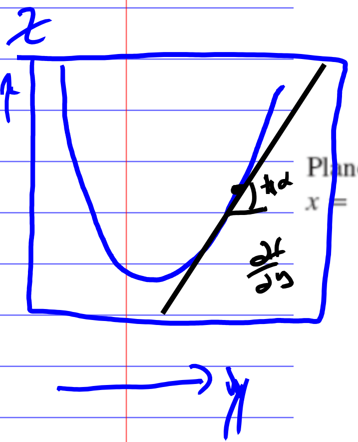
Parece que não sai da função.

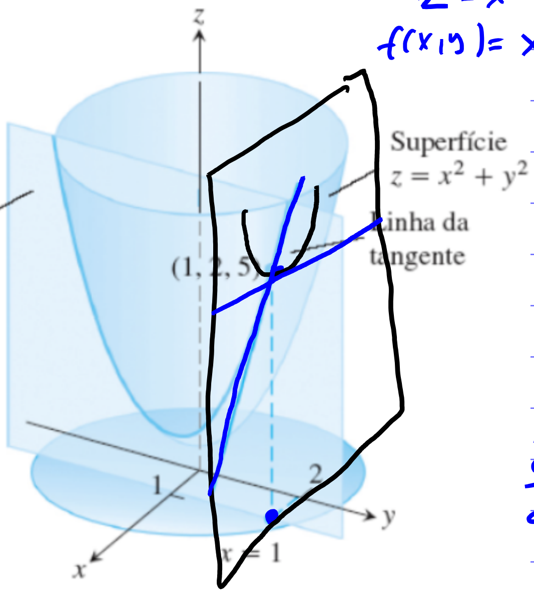
$$\left.\frac{\partial f}{\partial x}\right|_B = 0$$

$$\left.\frac{\partial f}{\partial y}\right|_B > 0$$

$z = x^2 + y^2$

$f(x,y) = x^2 + y^2$





$\boxed{\begin{array}{c} f(1,2) = 1^2 + 2^2 \\ z = 5 \end{array}}$

$\left.\frac{\partial f}{\partial y}\right|_{(1,2)}$

FIGURA 14.16 A tangente à curva de interseção do plano $x = 1$ e da superfície $z = x^2 + y^2$ no ponto (1, 2, 5) (Exemplo 5).