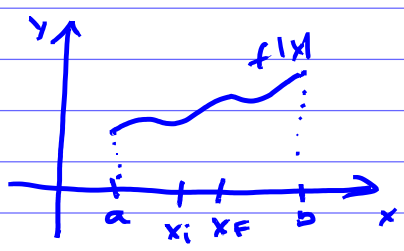
Propriedades de integral definida

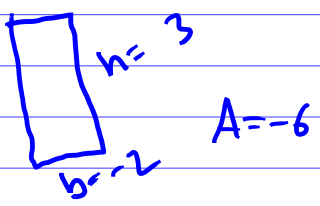
1) $$\int_a^b c\,dx = (b-a)c$$

2) $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$= \lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

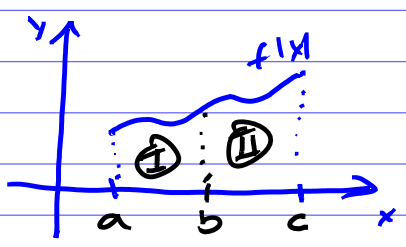


$\Delta x = x_f - x_i$

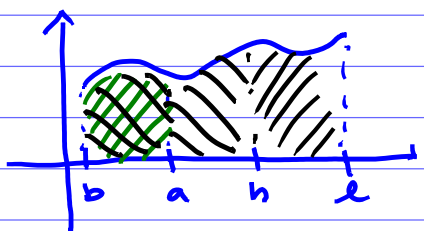


$A = -6$

3) $$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$



$$\int_a^b f(x)\,dx + \int_b^h f(x)\,dx + \int_h^\ell f(x)\,dx = \int_a^\ell f(x)\,dx$$



4) $$\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$$

$$= \lim_{n\to+\infty} \sum_{i=1}^{n} k f(x_i)\,dx = k \lim_{n\to+\infty} \sum_{i=1}^{n} f(x_i)\,dx$$

$$= k\int_a^b f(x)\,dx$$

5) $$\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

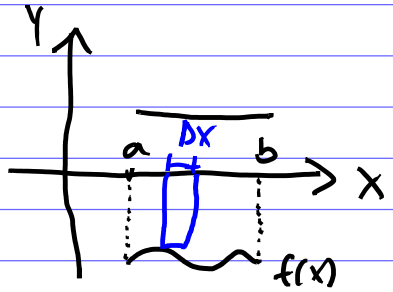
6) $$\int_a^b (f(x)-g(x))\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

7) $$\int_a^a x^2\,dx = 0$$

8) $$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt = \int_a^b f(u)\,du$$

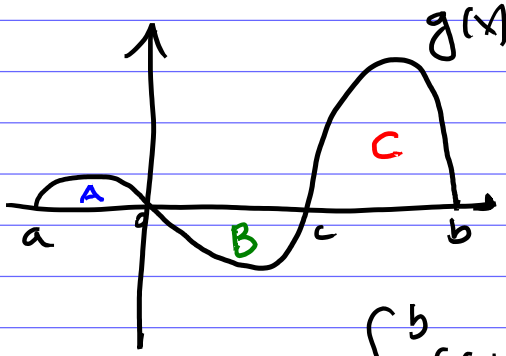
Variáveis ou fictícias.

Exercícios



$$\int_a^b f(x)\,dx < 0$$

$$\int_b^a f(x)\,dx > 0$$



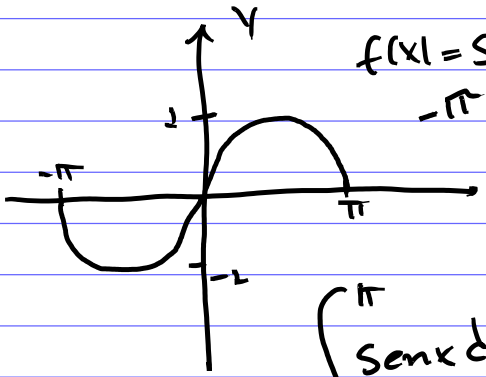
$$\int_a^b f(x)\,dx > 0$$

$$\int_a^b f(x)\,dx = A - B + C$$

A integral é a área Líquida.

Área visível

$$\int_a^0 g(x)\,dx - \int_0^c g(x)\,dx + \int_c^b g(x)\,dx$$

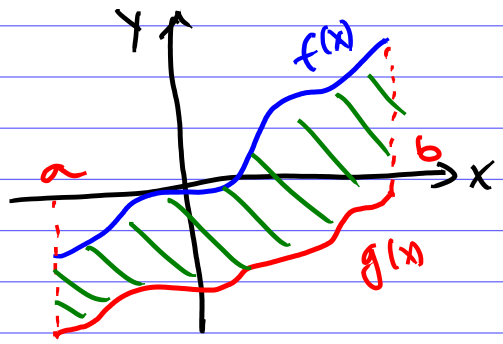


$f(x) = \operatorname{sen} x$

$-\pi \leq x \leq \pi$

$$\int_{-\pi}^{\pi} \operatorname{sen} x\,dx = 0$$

Área entre curvas



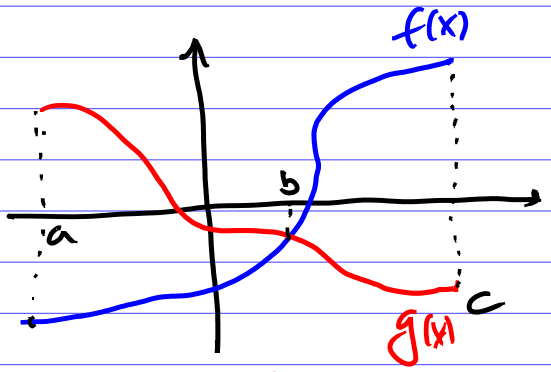
$$A_{\text{entre curvas}} = \int_a^b \left(f(x) - g(x)\right) dx$$

Quando usar as propriedades?

$$\int_a^b (\cos x - \operatorname{sen} x)\, dx = \int_a^b \cos x\, dx - \int_a^b \operatorname{sen} x\, dx$$

$$\int_a^b (2x - x)\, dx = \int_a^b x\, dx$$

R: Depende do contexto.



$$\Rightarrow A_{\text{entre curvas}} = \int_a^b \left(g(x) - f(x)\right) dx + \int_b^c \left(f(x) - g(x)\right) dx$$

$$A_{\text{entre curvas}} = \int_a^c |f(x) - g(x)|\, dx$$

T.F.C 1

① $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

② $\int_a^b f(t)\,dt = F(b) - F(a)$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\frac{d}{dx}\int_3^x \cos t\,dt = \cos x$$

$$\frac{d}{dx}\left(\lim_{n\to\infty}\sum_{i=1}^{n}\cos(t_i)\,\Delta t\right) = \cos x$$

Ex 1:

a) $\frac{d}{dx}\int_3^x \operatorname{sen}^2 t\,dt = \operatorname{sen}^2 x$

b) $\frac{d}{dx}\int_2^x \operatorname{sen} t\cos^2 t\,dt = \operatorname{sen} x\cos^2 x$

c) $\frac{d}{dx}\int_x^2 \operatorname{sen} t\cos^2 t\,dt = -\frac{d}{dx}\int_2^x \operatorname{sen} t\cos^2 t\,dt$

$= -\operatorname{sen} x\cos^2 x$

T.F.C 2.

$F(x)$ é uma derivada de $f(x)$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\int_0^{\pi} \cos(x)\,dx = \operatorname{Sen} x \Big|_0^{\pi} = \operatorname{Sen}\pi - \operatorname{Sen} 0$$

$$= 0 - 0 = 0$$