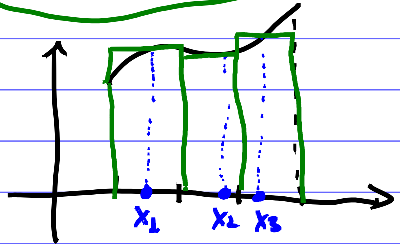
Soma de Riemann

$$\sum_{i=1}^{3} f(x_i)\,\Delta x_i$$

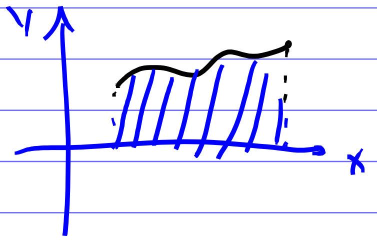


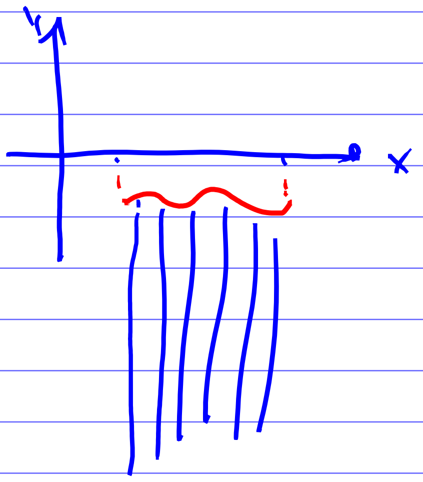
$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\,\Delta x_i$$

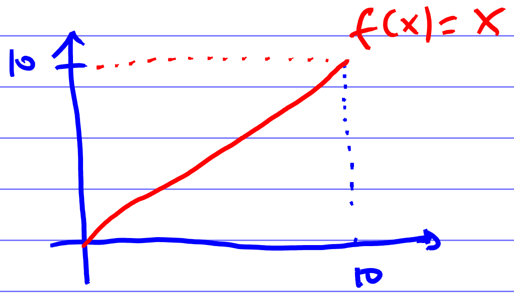
Área sob a curva

Em baixo

f(x) positiva.

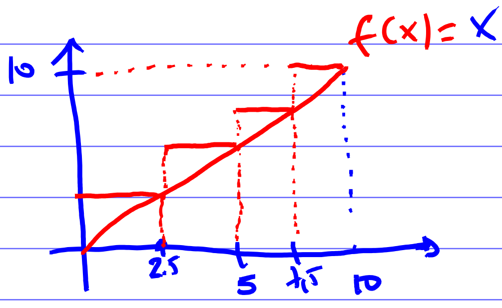






$A_{\text{sob a curva}} = 50$

Como podemos estimar?



$\Delta x = \frac{10}{4} = 2.5$

MAIS À DIREITA.

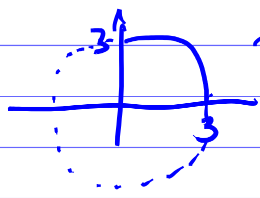
$A \approx f(2.5)\,\Delta x + f(5.0)\,\Delta x + f(7.5)\,\Delta x + f(10)\,\Delta x$

$A \approx (2.5 + 5 + 7.5 + 10)\,\Delta x$

$A \approx 25\,\Delta x$

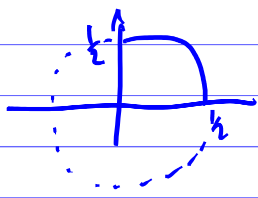
$A \approx 62{,}5$

Por Geometria $\int_0^{10} x\,dx = 50$



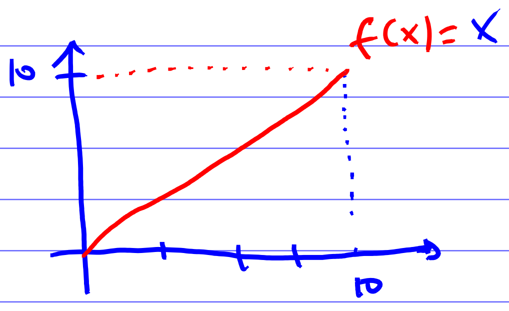
$f(x) = \sqrt{9 - x^2}$

$$\int_0^3 \sqrt{9-x^2}\,dx = \frac{\pi 9}{4}$$



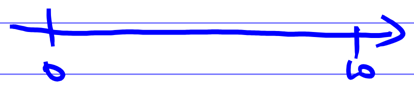
$f(x) = \sqrt{\frac{1}{4} - x^2}$

$$\int_0^{1/2} \sqrt{\frac{1}{4}-x^2}\,dx = \frac{\pi}{4}\left(\frac{1}{4}\right) = \frac{\pi}{16}$$



E a conta exatamente?

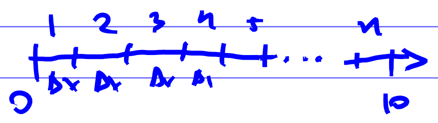
$$\int_0^{10} x\,dx = \lim_{n \to +\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$



$$\Delta x = \frac{\text{Comprimento do domínio}}{n}$$

$$\Delta x = \frac{10}{n}$$

Quem é x_i?



O ponto mais à direita

$x_1 = \Delta x$

$x_2 = 2\Delta x$

$x_3 = 3\Delta x$

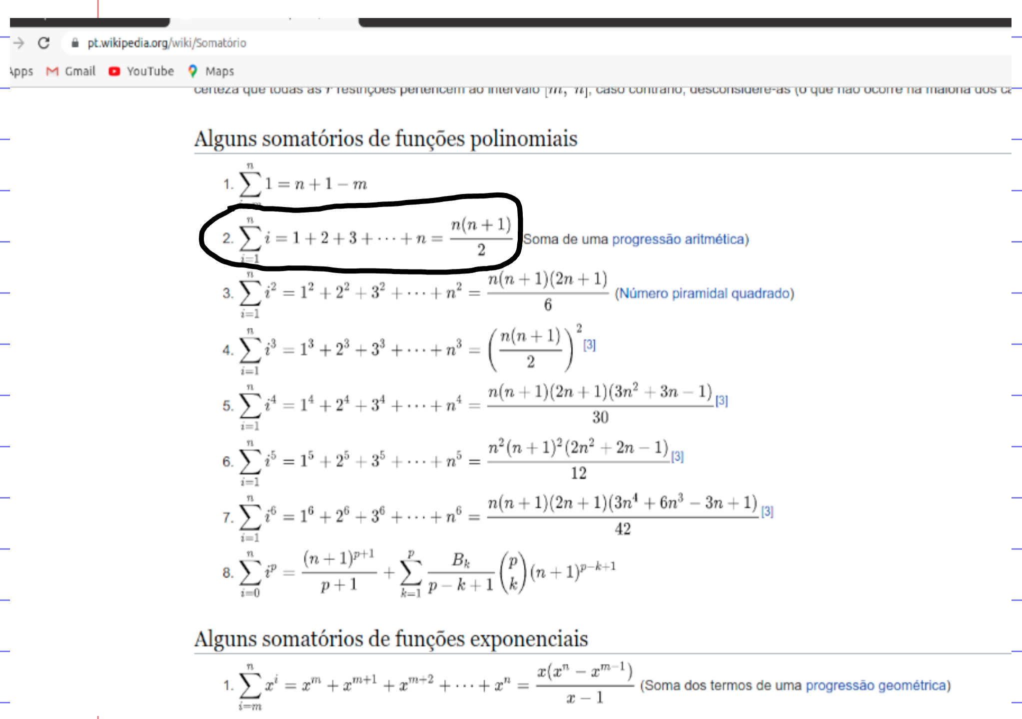
$\vdots$

$x_i = i\,\Delta x = i\,\frac{10}{n}$

$x_i = \frac{10i}{n}$

$$\int_0^{10} x\,dx = \lim_{n \to +\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n \to +\infty} \sum_{i=1}^{n} \frac{10i}{n}\,\frac{10}{n}$$

$$= 100 \lim_{n \to +\infty} \frac{1}{n^2} \sum_{i=1}^{n} i$$



pt.wikipedia.org/wiki/Somatório

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Alguns somatórios de funções polinomiais

1. $\sum_{i=m}^{n} 1 = n + 1 - m$
2. $\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$ (Soma de uma progressão aritmética)
3. $\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ (Número piramidal quadrado)
4. $\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ [3]
5. $\sum_{i=1}^{n} i^4 = 1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$ [3]
6. $\sum_{i=1}^{n} i^5 = 1^5 + 2^5 + 3^5 + \cdots + n^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$ [3]
7. $\sum_{i=1}^{n} i^6 = 1^6 + 2^6 + 3^6 + \cdots + n^6 = \frac{n(n+1)(2n+1)(3n^4+6n^3-3n+1)}{42}$ [3]
8. $\sum_{i=0}^{n} i^p = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^{p} \frac{B_k}{p-k+1}\binom{p}{k}(n+1)^{p-k+1}$

Alguns somatórios de funções exponenciais

1. $\sum_{i=m}^{n} x^i = x^m + x^{m+1} + x^{m+2} + \cdots + x^n = \frac{x(x^n - x^{m-1})}{x-1}$ (Soma dos termos de uma progressão geométrica)

$$= 100 \lim_{n \to +\infty} \frac{1}{n^2} \sum_{i=1}^{n} i$$

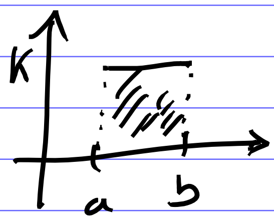
$$= 100 \lim_{n \to +\infty} \frac{1}{n^2} \cdot \frac{n(n+1)}{2}$$

$$= \frac{100}{2} \lim_{n \to +\infty} \frac{(n^2+n)}{n^2}$$

$$= 50 \lim_{n \to +\infty} \left(1 + \frac{1}{n}\right)$$

$$= 50\,(1) = 50$$

Propiedades



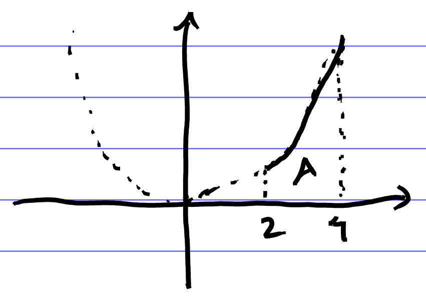
$$\int_a^b K\,dx = K(b-a)$$

$$\int_2^7 5\,dx = 5(7-2)$$
$$= 25$$

$$\int_{27}^{38} 375\,dx = 375(38-27)$$

$$\int_{-3}^{-7} |-8|\,dx = (-8)[(-7)-(-3)]$$

Outro exemplo: $\int_2^4 x^2\,dx$



$$A = \int_2^4 x^2\,dx = \lim_{n\to+\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x$$

$i = 1 \quad 2 \quad 3 \quad \cdots \quad n$

$2 \quad x_1 \quad x_2 \quad \cdots \quad 4$

$$\Delta x = \frac{\text{Comprimento}}{n} = \frac{(4-2)}{n} = \frac{2}{n}$$

MAIS À DIREITA:

$$x_1 = 2 + \Delta x$$
$$x_2 = 2 + 2\Delta x$$
$$x_3 = 2 + 3\Delta x$$
$$\vdots$$
$$x_i = 2 + i\Delta x$$

$$f(x) = x^2$$
$$\boxed{f(x_i) = x_i^2 = (2+i\Delta x)^2}$$

$$\int_2^4 x^2\,dx = \lim_{n\to+\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$= \lim_{n\to+\infty}\sum_{i=1}^{n}(2+i\Delta x)^2\cdot\Delta x$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$= \lim_{n\to+\infty}\sum_{i=1}^{n}(2+i\Delta x)^2\cdot\Delta x$$

$$= \lim_{n\to+\infty}\sum_{i=1}^{n}\left(4 + 4i\Delta x + i^2\Delta x^2\right)\cdot\Delta x$$

$$= \lim_{n\to+\infty}\left(\sum_{i=1}^{n}4\Delta x + \sum_{i=1}^{n}4\cdot i\,\Delta x^2 + \sum_{i=1}^{n}i^2\Delta x^3\right)$$

$$\Delta x = \frac{2}{n}$$

$$= \lim_{n\to+\infty}\left(\sum_{i=1}^{n}4\left(\frac{2}{n}\right) + \sum_{i=1}^{n}4\cdot i\left(\frac{4}{n^2}\right) + \sum_{i=1}^{n}i^2\left(\frac{8}{n^3}\right)\right)$$

$$= \lim_{n\to+\infty}\left(\frac{1}{n}\sum_{i=1}^{n}8 + \frac{16}{n^2}\sum_{i=1}^{n}i + \frac{8}{n^3}\sum_{i=1}^{n}i^2\right)$$

$$= \lim_{n\to+\infty}\left(\frac{1}{n}(8n) + \frac{16}{n^2}\,\frac{n(n+1)}{2} + \frac{8}{n^3}\,\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \lim_{n\to+\infty}\left(8 + 8\cdot\frac{n^2+n}{n^2} + \frac{8}{n^3}\cdot\frac{(n^2+n)(2n+1)}{6}\right)$$

$$= \lim_{n\to+\infty}\left\{8 + 8\left(1+\frac{1}{n}\right) + \frac{8}{6}\left(\frac{2n^3+n^2+2n^2+n}{n^3}\right)\right\}$$

$$= \lim_{n\to+\infty}\left\{8 + 8 + \frac{8}{n} + \frac{8}{6}\left(2 + \frac{1}{n} + \frac{2}{n} + \frac{1}{n^2}\right)\right\}$$

$$= \lim_{n\to+\infty}\left\{16 + \cancel{\frac{8}{n}} + \frac{8}{3} + \cancel{\frac{8}{6n}} + \cancel{\frac{16}{6n}} + \cancel{\frac{8}{6n^2}}\right\}$$

$$= 16 + \frac{8}{3} = \frac{56}{3}$$