

Exponential and Logarithms – IB Exam Practice Questions

1. Solve the following equations.

(a) $\log_x 49 = 2$

(3)

(b) $\log_2 8 = x$

(2)

(c) $\log_{25} x = -\frac{1}{2}$

(3)

(d) $\log_2 x + \log_2(x - 7) = 3$

(5)

(Total 13 marks)

2. Solve $\log_2 x + \log_2(x - 2) = 3$, for $x > 2$.

(Total 7 marks)

3. Let $f(x) = k \log_2 x$.

(a) Given that $f^{-1}(1) = 8$, find the value of k .

(3)

(b) Find $f^{-1}\left(\frac{2}{3}\right)$.

(4)

(Total 7 marks)

4. The population of a city at the end of 1972 was 250 000. The population increases by 1.3% per year.

(a) Write down the population at the end of 1973.

(b) Find the population at the end of 2002.

(Total 6 marks)

5. Let $f(x) = \log_3 \sqrt{x}$, for $x > 0$.

(a) Show that $f^{-1}(x) = 3^{2x}$.

(2)

(b) Write down the range of f^{-1} .

(1)

Let $g(x) = \log_3 x$, for $x > 0$.

(c) Find the value of $(f^{-1} \circ g)(2)$, giving your answer as an integer.

(4)

(Total 7 marks)

6. The number of bacteria, n , in a dish, after t minutes is given by $n = 800e^{0.13t}$.

(a) Find the value of n when $t = 0$.

(2)

(b) Find the rate at which n is increasing when $t = 15$.

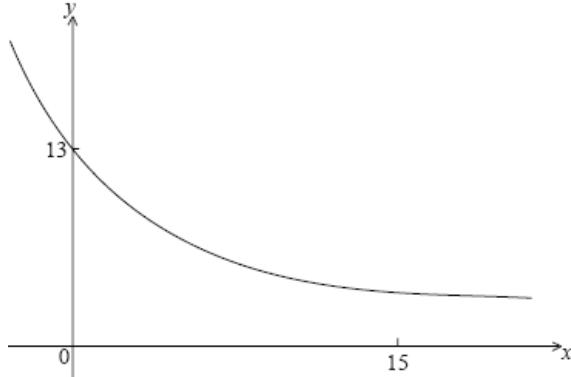
(2)

(c) After k minutes, the rate of increase in n is greater than 10 000 bacteria per minute. Find the least value of k , where $k \in \mathbb{Z}$.

(4)

(Total 8 marks)

7. Let $f(x) = Ae^{kx} + 3$. Part of the graph of f is shown below.



The y -intercept is at $(0, 13)$.

- (a) Show that $A = 10$. (2)

- (b) Given that $f(15) = 3.49$ (correct to 3 significant figures), find the value of k . (3)

- (c) (i) Using your value of k , find $f'(x)$.

- (ii) Hence, explain why f is a decreasing function.

- (iii) Write down the equation of the horizontal asymptote of the graph f . (5)

(Total 10 marks)

- 8.** Let $f(x) = 3 \ln x$ and $g(x) = \ln 5x^3$.

- (a) Express $g(x)$ in the form $f(x) + \ln a$, where $a \in \mathbb{Z}^+$. (4)

- (b) The graph of g is a transformation of the graph of f . Give a full geometric description of this transformation. (3)

(Total 7 marks)

- 9.** Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

- (a) Show that $f(x) = \log_3 2x$. (2)

- (b) Find the value of $f(0.5)$ and of $f(4.5)$.

(3)

The function f can also be written in the form $f(x) = \frac{\ln ax}{\ln b}$.

- (c) (i) Write down the value of a and of b .

- (ii) Hence, in the space below, sketch the graph of f , for $-5 \leq x \leq 5$, $-5 \leq y \leq 5$, using a scale of 1 cm to 1 unit on each axis.

- (iii) Write down the equation of the asymptote.

(6)

- (d) Write down the value of $f^{-1}(0)$.

(1)

The point A lies on the graph of f . At A, $x = 4.5$.

- (e) On your diagram, sketch the graph of f^{-1} , noting clearly the image of point A.

(4)

(Total 16 marks)