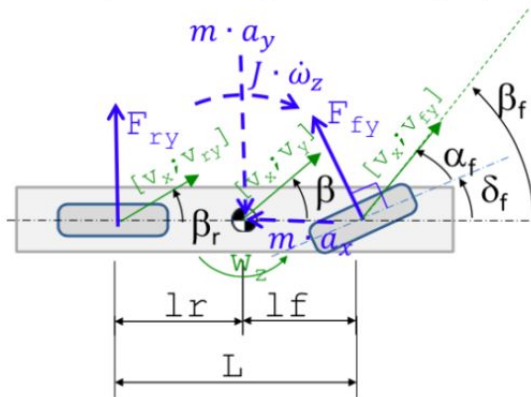


Vehicle dynamics

Bicycle model

Physical model:

- Path radius \gg the vehicle. Then, all forces (and centripetal acceleration) are approximately co-directed.
- Small tyre and vehicle side slip. Then, $\text{angle} = \sin(\text{angle}) = \tan(\text{angle})$.
(Angles are not drawn small, which is the reason why the forces not appear co-linear in figure.)



Mathematical model:

Equilibrium:

$$\begin{aligned} m \cdot (\dot{v}_x - \omega_z \cdot v_y) &\approx F_{fx} + F_{rx}; \text{ where } \dot{v}_x = 0; \\ m \cdot (\dot{v}_y + \omega_z \cdot v_x) &\approx F_{fy} + F_{ry}; \\ J \cdot \dot{\omega}_z &\approx F_{fy} \cdot l_f - F_{ry} \cdot l_r; \end{aligned}$$

$$\text{Constitution: } F_{fy} = -C_f \cdot s_{yf}; \quad F_{ry} = -C_r \cdot s_{yr};$$

Compatibility:

$$\left\{ \begin{aligned} \delta_f + \alpha_f &= \beta_f; \quad \beta_f \approx \frac{v_{fy}}{v_x} = \frac{v_y + l_f \cdot \omega_z}{v_x}; \\ \alpha_r &= \beta_r \approx \frac{v_{ry}}{v_x} = \frac{v_y - l_r \cdot \omega_z}{v_x}; \\ \alpha_f &\approx s_{yf}; \quad \alpha_r \approx s_{yr}; \end{aligned} \right\}$$

Eliminate $F_{fy}, F_{ry}, \alpha_f, \alpha_r, \beta_f, \beta_r$ yields:

$$\begin{aligned} m \cdot \dot{v}_y + \frac{C_f + C_r}{v_x} \cdot v_y + \left(\frac{C_f \cdot l_f - C_r \cdot l_r}{v_x} + m \cdot v_x \right) \cdot \omega_z &\approx C_f \cdot \delta_f; \\ J \cdot \dot{\omega}_z + \frac{C_f \cdot l_f - C_r \cdot l_r}{v_x} \cdot v_y + \frac{C_f \cdot l_f^2 + C_r \cdot l_r^2}{v_x} \cdot \omega_z &\approx 0 \end{aligned}$$

Slip angles

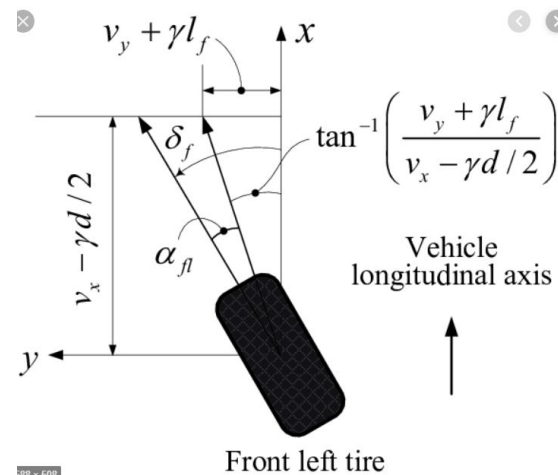
$$\alpha_{frontleft} = \delta - \arctan \left(\frac{v_y + \gamma \cdot \text{wheelbase}_{front}}{v_x - \gamma \cdot \text{track}/2} \right)$$

$$\alpha_{frontright} = \delta - \arctan \left(\frac{v_y + \gamma \cdot \text{wheelbase}_{front}}{v_x + \gamma \cdot \text{track}/2} \right)$$

$$\alpha_{rearleft} = \arctan \left(\frac{v_y - \gamma \cdot \text{wheelbase}_{rear}}{v_x - \gamma \cdot \text{track}/2} \right)$$

$$\alpha_{rearright} = \arctan \left(\frac{v_y - \gamma \cdot \text{wheelbase}_{rear}}{v_x + \gamma \cdot \text{track}/2} \right)$$

$\gamma = \text{yaw velocity}$



Aero

$$F_{downForce} = 0.5 * A_{front} * C_L * \rho * V^2$$

$$F_{drag} = 0.5 * A_{front} * C_D * \rho * V^2$$

Load transfer

Cornering

$$F_{outer} = \frac{m \cdot g}{2} + \frac{a_y \cdot CGH \cdot m}{track}$$

$$F_{inner} = \frac{m \cdot g}{2} - \frac{a_y \cdot CGH \cdot m}{track}$$

Braking

$$F_{front} = m \cdot g \cdot WD_{front} + \frac{a_x \cdot CGH \cdot m}{wheelbase}$$

$$F_{rear} = m \cdot g \cdot WD_{rear} - \frac{a_x \cdot CGH \cdot m}{wheelbase}$$

Accelerations

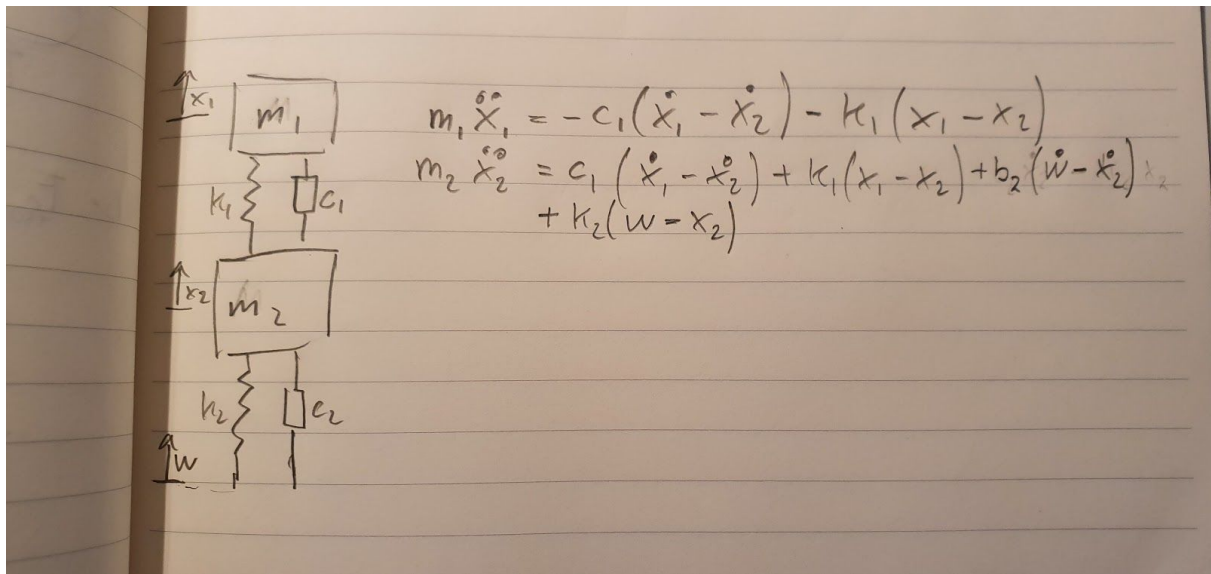
Circular motion

$$a_y = \frac{V^2}{R}$$

Unsprung Mass

Springs

Quantity	In Parallel	In Series
Equivalent spring constant is that	$k_{eq} = k_1 + k_2$	$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$
Equivalent compliance	$\frac{1}{c_{eq}} = \frac{1}{c_1} + \frac{1}{c_2}$	$c_{eq} = c_1 + c_2$
Deflection (elongation)	$x_{eq} = x_1 = x_2$	$x_{eq} = x_1 + x_2$
Force	$F_{eq} = F_1 + F_2$	$F_{eq} = F_1 = F_2$
Stored energy	$E_{eq} = E_1 + E_2$	$E_{eq} = E_1 + E_2$



Ratios

$$IR = \frac{\text{SpringDisplacement}}{\text{WheelDisplacement}}.$$

$$MR = \frac{\text{WheelDisplacement}}{\text{SpringDisplacement}}.$$

$$\text{Wheelrate} = \text{Springrate} * IR^2.$$

$$\text{Wheelrate} = \text{Springrate} / MR^2.$$