推导 SE(3) 的指数映射

回答:

左雅可比 $J = \sum_{n=0}^{\infty} \frac{1}{(n+1)!} (\emptyset^{\wedge})^{n}$

因为 $\emptyset^{\wedge} = \theta a^{\wedge}$

$$\sum_{n=0}^{\infty} \frac{1}{(n+1)!} (\emptyset^{\wedge})^{n} = \sum_{n=0}^{\infty} \frac{1}{(n+1)!} (\theta a^{\wedge})^{n+1} / \theta a^{\wedge}$$

$$= \frac{1}{\theta a^{\wedge}} (-I + I + \theta a^{\wedge} + \frac{1}{2!} \theta^{2} (a^{\wedge})^{2} + \frac{1}{3!} \theta^{3} (a^{\wedge})^{3} + \frac{1}{4!} \theta^{4} (a^{\wedge})^{4} + \ldots)$$

$$= \frac{1}{\theta a^{\wedge}} (-I + cos\theta I + (1 - cos\theta) a a^{T} + sin\theta a^{\wedge})$$

$$= \frac{sin\theta}{\theta} I + \left(1 - \frac{sin\theta}{\theta}\right) a a^{T} + \frac{1-cos\theta}{\theta} a^{\wedge}$$