Lecture no. 01

03-03-2025

* Machine Learning:

we make the machine to learn from the data continously. we make the machine to predict the thing that will happen in the future. by the given data. There are many methods that we use for predicting the data.

(i) Supervised learning.
(ii) Unsupervised learning.
(iii) Reinforcement learning.

- Before Applying any algo on to the data, the dataset must be complete.

→ Supervised learning:

where we know what the output is, or the characteristic & feature are known to predict the dataset. e.g.→ predicting the diseases on bases of the data given. Dataset — ML — output

Train
Test

→ Unsupervised learning:

we don't know what the output is, like we have some data to predict and learn about the dataset. but we don't actually know the answer. Dataset — ML — Groups.

→ Reinforcement learning:

There is no dataset in reinforcement learning. There are some reward & punishment. which we make the machine to learn from it.

In RL we make the agent to learn from the environment.

→ Why we need to preprocess the data?

- Data must be complete
- no missing value.
- Data must be acurate.

→ 1st preprosess the data. Secondly check wheather the output is give or not.

- Supervised learning:
  - Dataset given.
  - Features → output.
    x_1, x_2, x_3 → y.

    Feather, no. of legs, Tail etc → Bird, cat, cow etc.

→ If the output is categorical then it is classification. mean the value of output is countable.

- → If the value has only two output like yes, no or 0, 1, then it is Binary classification.
- → If the value is more then two option then it is multiclass classification, multinomial classification.

categorical – classification ← Binary classification
↓ Multiclass classification
(ordinal / multinomial classification).

~~in ordinal~~ (

→ If the output is discrete then it is Regression problem. e.g to predict the house price, etc.

- Linear Regression → value/output will be changed each time.

Algorithms for classification:

- Logistic Regression
- SVM
- KNN
- Decision Tree
- Nave Bayes.

Algorithms for Regression.

- Linear Regression.

- Ordinal Multiclass ~~Regra~~ classification:

  Specific order.

  Strongly Dis Agree, Disagree, Neutral Agree.

  Best, average, worst.

- Multinomial Multiclass classification:

  Non-Specific order.

  → cat, cow, Bird.

  → male, female.

  → Red Blue, white

  → Hot, mild, cold.

Assignment:

Download a dataset from kaggle, UCI.

- Dataset type; no. of rows, volume, feature (15), column rows (1500)

Lecture no. 02

10-03-2024.

* Supervised learning:

→ Classification:

- There must be some input, feature in classification.
- In classification we divide the data into training & testing data.
- lazy learner. That why we call it learner.
- Eager learner.

Input, output.

Training { 70-30, 60-40 } Testing (given)

lazy learner (KNN) Eager learner. (decision Tree, navie bayes, ANN)

→ KNN (k-nearist neighbour), lazy learner.

example: (binary classification.)

Height (cm)	weight (kg)	Tshirt-Size.	Distance	S-D
158	58	M	4.24	9
158	59	M	3.6	6
158	63	M	3.6	6
160	59	M	2.22	3
160	60	M	1.4	1
163	60	M	2.2	4
163	61	M	2	2
160	64	L	3.16	5
163	64	L	3.6	6
165	61	L	4	7
165	62	L	4.1	8
165	65	L	5.6	10

168	62	L	7.06	11
168	63	L	7.28	12
×168	66	L		
×170	63	L		
×170	64	L		
×170.	68	L.		

Q: identify Shirtsize of customer having ht 161 and wt 61kg.

* Step 1: calculate Similarty Based on Distance:

(i) Euclidean formula:

$$d(x,y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

(ii) Manhalan Distance.

$$d(x,y) = \sum_{i=1}^{n} |x_i - y_i|$$

$$d(x,y) = \sqrt{(161-158)^2 + (61-58)^2}$$

(161,61)(158,58).

$$= \sqrt{9+9}$$

$$= \sqrt{18}$$

$$= 4.24.$$

* Step 2: Find K-Nearest Neighbour.

K = 3 => M M M

K = 5 => M M M M L

- Before applying any algo check if the data is valid or invalid. like the attribue in no of washroom in uni is give 2, 2.5 & 3
- if the attribute is given like names of person etc so we cannot convert it into number so here we can drop the attribute.

- if the value are discrete and numerical then avoid using decision tree.
- if there is only one attribute in the dataset that is textual then convert it into the numerial value like if age is given like old, young or child then convert it into numbers like 0, 1, 2 where 0 for old, 1 for young and 2 for child

← So the no of wash-R can't be 2.5 so the data is invalid

Lecture no. 03

17-03-2025

* Logistic Regression - Classification:

↓ value will predict the class (ordinal, binary)

→ first we define a treshhold.
like some value so.

→ Treshhold can be multiple.

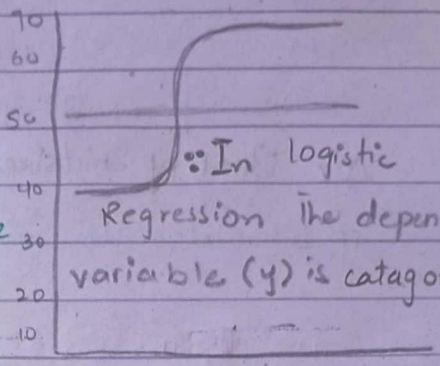
→ Identification of constants.

Formula:

Logistic Regression with one variable or attribute or feature.

$$y = \frac{1}{1+e^{f(x)}}$$

$$= \frac{1}{1+e^{-(\beta_0+\beta_1 x)}}$$



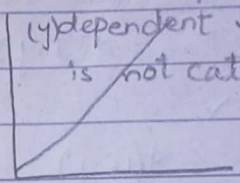
∵ In logistic Regression the dependent variable (y) is catagorial.

make a Sigmoid.

∵ $\beta x \rightarrow x$ is the feature or attribute given.

Linear Regression - Regression

∵ In linear regression (y) dependent variable is not catagorical



make a straight line.

$y = \beta_0 + \beta_1 x$

Logistic Regression with multi Variable.

Formula. $= \frac{1}{1+e^{-(\beta_0+\beta_1 x_1+\beta_2 x_2)}}$

$$= \frac{1}{1+e^{-\beta_0-\beta_1 x_1-\beta_2 x_2}}$$

$\beta_0 = 3$, $\beta_1 = 0.8$

Input x	Output y_A
0.5	0
1.2	0
2.3	0
3.1	1
4.5	1
5.0	1
5.8	1
6.3	1
7.1	1
8.0	1

1) $= \frac{1}{1+e^{-(3+(0.8)(0.5))}}$

$= 0.97$

2) $\frac{1}{1+e^{-(3+(0.8)(1.2))}}$

$= 0.98$

3) $\frac{1}{1+e^{-(3+(0.8)(2.3))}}$

$= 0.99$

4) $= \frac{1}{1+e^{-(3+(0.8)(3.1))}}$

$= 0.99$

(5) $= \frac{1}{1+e^{-(3+(0.8)(4.5))}}$

$= 0.99$

x	y_p	y_p (-3)
0.5	0.97	(0) 0.0691
1.2	0.98	(0) 0.115
2.3	0.992	(0) 0.238
3.1	0.995	(0) 0.372
4.5	0.998	(1) 0.645
5.0	0.999	(1) 0.731
5.8	0.9995	(1) 0.837
6.3	0.996	(1) 0.84
7.1	0.9998	(1) 0.935
8.0	0.9999	(1) 0.967

≈1 all

accuracy → 7/10 = 70% 9/10 = 90

$$\beta_0 = \frac{\sum x^2 \sum y - \sum x \sum xy}{n\sum x^2 - (\sum x^2)}$$

$$\beta_1 = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

Assignment:

1) Write a program in python and read the dataset / display the data

2) Library for logistic regression.
which library is used to import logistic regression.

When the values of β is not known.

X	Y	X^2	XY
0.5	0	0.25	0
1.2	0	1.44	0
2.3	0	5.29	0
3.1	1	9.61	3.1
4.5	1	20.25	4.5
5.0	1	25	5.0
5.8	1	33.64	5.8
6.3	1	39.69	6.3
7.1	1	50.41	7.1
8.0	1	64	8.0
$\Sigma x = 43.80$	$\Sigma y = 7$	$\Sigma x^2 = 250$	$\Sigma xy = 39.8$

$\beta_0 = -3$ } 90%.
$\beta_1 = 0.8$ } accuracy

$\beta_0 = 3$ } 70%.
$\beta_1 = 0.8$ } accuracy.

$$\beta_0 = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{n\Sigma x^2 - (\Sigma x)^2}, \quad \beta_1 = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} = 0.157$$

$$= \frac{(250)(7) - (43.80)(39.8)}{10(250) - (43.80)^2}$$

$$= \cancel{0.157}\ 0.006.$$

e.g: If value of y is not given.

$x = 5.2$

$y = ?$

Check the better accuracy (β_0 & β_1)

Then

Put values in formula.

$$\cancel{*}\ Y = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1)}}$$

$$Y = \frac{1}{1 + e^{-(-0.3 + (0.8)(5.2))}}$$

Given a dataset of students containing their study hours to identify whether they pass or fail the exams. If the student studied 3.2 hours for exams. you need to identify whether he/she will pass the exam using the appropriate algo based on the dataset

Identify the choice of algo. Also identify the accuracy of your algorithm.

Students	study hours (x)	Pass(1)/fail(0) (y)
1	1.5	0
2	2.0	0
3	2.5	0
4	3.0	0
5	3.5	1
6	4.0	1
7	4.5	1
8	5.0	1
9	5.5	1
10	6.0	1
	$\Sigma x = 37.5$	$\Sigma y = 6$

x^2	xy
2.25	0
4	0
6.25	0
9	0
12.25	3.5
16	4.0
20.25	4.5
25	5.0
30.25	5.5
36	6.0
$\Sigma x^2 = 161.25$	$\Sigma xy = 28.5$

$$\beta_0 = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{n \Sigma x^2 - (\Sigma x)^2}$$

$$\beta_1 = \frac{n \Sigma xy - \Sigma x \Sigma y}{n \Sigma x^2 - (\Sigma x)^2}$$

$$\beta_0 = \frac{(161.25)(6) - (37.5)(28.5)}{10(161.25) - (37.5)^2} = -0.4909$$

$$\beta_1 = \frac{10(28.5) - (37.5)(6)}{10(161.25) - (37.5)^2} = 0.2909$$

$\beta_0 = -0.4909$, $\beta_1 = 0.2909$

$$y = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1)}}$$

$$y = \frac{1}{1 + e^{-(-0.49 + (0.29)(1.5))}}$$

x	y_p	y
1.5	0.486 ≈ 0	0
2.0	0.522 ≈ 1	0
2.5	0.558 ≈ 1	0
3.0	0.593 ≈ 1	0
3.5	0.628 ≈ 1	1
4.0	0.661 ≈ 1	1
4.5	0.69 ≈ 1	1
5.0	0.72 ≈ 1	1
5.5	0.75 ≈ 1	1
6.0	0.77 ≈ 1	1

accuracy is $7/10 = 70\%$

Lecture no. 04

30-04-2024

★ Regression :-

• Linear Regression

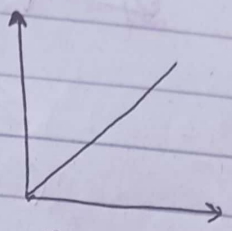
→ Formula:

∵ output can be continous

• Linear Regression with one variable:

$$y_p = \beta_0 + \beta_1 x_1$$

β_0 ↓ Constant, x_1 ↘ attribute.



Linear Regression (straight line).

• Linear Regression with multi variable.

$$y = \beta_0 + \beta_1 x_1 + \dots + \beta_n x_n$$

• e.g:- • Linear Regression for one variable.

Predict the obtained marks of a student who studies for 6.5 hr.

x (H-Studies)	Y_A (Marks obtained)	x^2	xy	y_p'	error (E_p) $(Y_A - y_p')$	$(E_p)^2$
1	50	1	50	51.27	1.27	
2	55	4	110	56.11	1.11	
3	65	9	195	60.95	4.05	
4	70	16	280	65.7	4.3	
5	65	25	325	70.63	5.63	
6	75	36	450	75.47	0.47	
7	80	49	560	80.31	0.30	
8	85	64	680	85.15	0.15	
9	90	81	810	89.99	0.01	
10	95	100	950	94.83	0.17	
11	100	121	1100	99.67	0.33	
12	105	144	1260	104.5	0.49	ΣE_p^2
$\Sigma x = 78$	$\Sigma y = 935$	$\Sigma x^2 = 650$	$\Sigma xy = 6770$		$\Sigma(Y_A - y_p) = 18.29$	

$$\beta_0 = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{n(\Sigma x^2) - (\Sigma x)^2}$$

$$\beta_0 = \frac{(650)(935) - (78)(6770)}{12(650) - (78)^2}$$

$$\beta_0 = 46.43$$

$$\beta_1 = \frac{n(\Sigma xy) - \Sigma x \Sigma y}{n(\Sigma x^2) - (\Sigma x)^2}$$

$$\frac{12(6770) - (78)(935)}{12(650) - (78)^2}$$

$$\beta_1 = 4.84$$

$\because$ $y_p = \beta_0 + \beta_1 x_1$

$y_p = 46.43 + 4.84(1) =$

$\because$ $E_{p_1} = (y_A - y_p)$

$E_{p_1} = 50 - 51.27$

* Performance Calculation of Linear Regression:

• Mean Square ERROR: (1st Iteration)

$$\left(\sum^{Ep}(Y_A - Y_P)^2/n\right) \Rightarrow (18.29)^2/12 = 27.87$$

$(\sum |E_p|)^2/n$

$\sum (Y_A - Y_P)^2/n$.

• Root mean Square:

$$\sqrt{\sum (Y_A - Y_P)^2/n} =$$

$$\Rightarrow \sqrt{27.87} = 5.2.$$

Y_p^2	$E_{p_2}^2$ $(Y_A - Y_p^2)$	Y_p^3	$E_{p_3}^2$
50.43	0.43	50.5	0.5
54.43	0.57	55	0
58.43	6.57	59.5	5.5
64.43	7.57	64	6
66.43	1.43	68.5	3.5
70.43	4.50	73	2
74.43	5.57	77.5	2.5
78.43	6.57	82	3
82.43	7.57	86.1	3.5
86.43	8.57	91	4
90.43	9.57	95.5	4.5
94.43	10.57	100	5

$\beta_1 = 4$ ↓ $\sum E_{p_2}^2 =$ 69.49

$\beta_1 = 4.5$ ↓ $\sum E_{p_3}^2 =$ 40.

∵ To find y_p^2, Suppose, any random value near to the β_1, like here we are supposing 4 which is nearest to the value of β_1. After finding the value of E_p^2 the check for the distance b/w actual 'Yp' and new Y_p^2, we can change both β_0 & β_1.

$\beta_1 = 4$.

∵ 'iteration:

we do the iterations for the algo by changing the value of (mean square error), the value of mean square error must be near to 0 it can't be actual value if it is then the calculation are wrong.

→ • mean Square error (for 2nd iteration)

$$\text{Error} = \Sigma (Y_A - Y_p^{32})^2 / n$$
$$= (69.49)^2 / 12$$
$$= 403.2$$

→ • Root mean Square:

$$\sqrt{\Sigma (Y_A - Y_p^2)^2 / n}$$
$$= 20.07$$

→ Mean Square error : (3rd iteration)

$$\text{Error} = \Sigma (Y_A - Y_p^3)^2 / n$$
$$= (40)^2 / 12$$
$$= 133$$

• root mean Square:

$$= \sqrt{\Sigma (Y_A - Y_p^3)^2 / n}$$
$$= \sqrt{133} = 11.5$$

Q: If the student studied for 6.5 h, then how many marks will be obtain.

So, lowes mean square value is 5.2

$$= 46.43 + 4.84 (6.5)$$
$$= 77.89$$

• Linear Regression for multi variable.

X_1	X_2	Y	Yp^1	Ep^1	Yp^2	Ep^2
1	0.10	1.5	1.6	0.1	1.4	0.1
2	0.12	2	2.68	0.68	2.4	0.4
3	0.14	3.5	3.71	0.21	3.4	0.1
4	1.16	4.0	4.74	0.74	4.4	0.4
5	0.18	5.5	5.77	0.27	5.4	0
6	0.20	6.0	6.8	0.8	6.4	0.5

Suppose the value of β_0 & β_1 & β_2

$\beta_0 = 0.5$, $\beta_1 = 1$, $\beta_2 = 1.5$

∴ After find the values of β's calculate mean square error and root mean error. Then change the value of β_2.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$
$$= 0.5 + (1)(1) + 1.5(0.10)$$
$$= 1.6 \quad \text{so on}$$

$$Ep^1 = (Y_A - Yp^1)$$
$$= 1.5 - 1.6$$
$$= 0.1 \quad \text{so on.}$$

∴ Change the value of $\beta_0 = 0.25$

mean square root: $\Sigma (Y_A - Yp^{*1})^2 / n$

$$= (1.5)^2/6$$
$$= 0.375$$

Root mean error = $\sqrt{\Sigma (Y_A - Yp^1)^2/n}$

$$= \sqrt{(0.375)}$$
$$= 0.6$$

Formula of B_0 for multi-variables.

$$B_0 = \frac{\Sigma(x_1x_2)^2 \Sigma y - \Sigma(x_1x_2) \Sigma(x_1x_2) y}{n \Sigma(x_1x_2)^2 - (\Sigma x_1x_2)^2}$$

Lecture no. 05

Performance Matrics of Classification

Accuracy: • Prediction must be correct
• 8/12 • TP + TN / TP + TN + FN + FP

Precision: • TP / TP + FP.

Recall: • $\frac{TP}{TP+FN}$

F_1 Square:
• $2 \times \frac{\text{Precision} \times \text{recall}}{\text{Precision} + \text{recall}}$.

Dataset:

S.no		Actual(Y_p)	Predicted (Y_p)	
1	A_1	Positive	P	TP
2	A_2	Positive	N	FN
3	A_3	negative	N	TN
4	A_4	Positive	P	TP
5	A_5	negative.	P	FP.
6	A_6	negative	N	TN
7	A_7	Positive	P	TP
8	A_8	(P)	P	TP
9	A_9	N	N	TN
10	A_{10}	N	P	FP.
11	A_{11}	P	N	FN
12	A_{12}	N	N	TN

• True positive
Actual P, predicted P
• True negative
Actual N, predicted N
• False positive
Actual N, Predicted P
• False negative.
Actual P, Predicted N.

Confusion Matrix:
2×2 matrix.

		Actual	
		P	N
Predicted	P	TP = 4	FP = 2
	N	FN = 2	TN = 4

Precision:

Proportion of all correctly predicted +ve cases out of all predicted +ve cases.

Recall:

Proportion of Correctly predicted +ve cases out of all actual +ve cases.

F1 - Square:

Hormonic mean of precision and recall. It balances both metrics especially useful when classes are imbalanced.

3x3 Matrix. If there are 3x3 matrix.

	cat	Dog	Rabbit
cat :	Tp (1)	2	5
Dog :	3	Tp (4)	2
Rabbit :	1	6	Tp (10)

T-N : not cat -ve, no cat -ve

T-P : cat +ve, cat +ve

F-P : (other) no cat -ve, cat +ve

F-N : cat +ve, (other) no cat -ve

Confusion-Matrix w.r.t cat Predicted.

	cat	others
cat	1	7 (2+5)
others	4 (3+1)	22

4+2+6+10.

C.M w.r.t Dog predicted:

		Predicted: Dog	Predicted: others	
actual	Dog	4	5 (3+2)	
	others	8 (6+2)	17	(1+5+1+10)

C.M w.r.t Rabbit predicted

		Predicted: Rabbit	Predicted: others	
actual	Rabbit	10 (1+2+3+4)	7 (6+1)	
	others	7 (5+2)	10 (1+2+3+4)	

- To check the performance of Regression there is a suitable method named as root mean square & mean Square error.
- To check the performance of classification there is a suitable method named as Confusion matrix.

• Linear Regression for multi-variable:

example:

X_1	X_2	Y	X_1^2	X_2^2	X_1Y	X_2Y	Y_{P1}
1	0.10	1.6	1	0.01	1.6	0.16	30
2	0.12	2.68	4	0.014	5.36	0.32	44.86
3	0.14	3.71	9	0.019	9.51	0.51	59.12
4	0.16	4.74	16	0.025	18.96	0.75	73.68
5	0.18	5.74	25	0.032	28.85	1.03	88.24
6	0.20	6.8	36	0.04	40.8	1.36	
$\Sigma x_1 =$	$\Sigma x_2 =$	$\Sigma y =$	$\Sigma x_1^2 =$	$\Sigma x_2^2 =$	$\Sigma x_1 y =$	$\Sigma x_2 y =$	
21	0.9	25.3	91	0.14	105.19	4.13	

$$\beta_0 = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{n \Sigma x^2 - (\Sigma x)^2}, \quad \beta_1 = \frac{n \Sigma xy - \Sigma x \Sigma y}{n(\Sigma x_1^2) - (\Sigma x_1)^2}$$

$$\beta_0 = \frac{(0.9)(25.3) - (21)(105.19)}{6(91) - (21)^2}, \quad = \frac{6(105.19) - (91)(25.3)}{6(91) - (21)^2}$$

$$\therefore \beta_0 = -20.8 \quad , \quad \beta_1 = -15.9$$

$$\beta_2 = \frac{n \Sigma x_2 y - \Sigma x_2 \Sigma y}{n(\Sigma x_2^2) - (\Sigma x_2)^2}$$

$$\beta_2 = \frac{6(4.13) - (0.9)(25.3)}{6(0.14) - (0.9)^2} \Rightarrow \beta_2 = 67$$

$$y_{P1} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

$$= -20.8 + (-15.9)(1) + (67)(0.10)$$

$$= -30$$

Lecture no.06

Unsupervised Learning:

* clustering:

(i) Partitioning:
K-means

(ii) ~~Hierarchical~~ Hierarchical: (Bottom-up approach)
Aggecometius

(iii) Density Based:
DB-Scan.

k-mean: (Partition)

example:

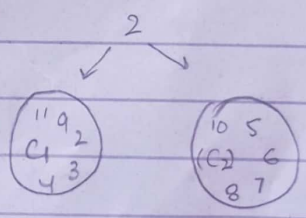
1st iteration.

ID	X_1	X_2		
1	1.0	1.5	C_1	I_2
2	1.2	1.3	C_1	C_1
3	0.8	1.6	C_1	C_1
4	1.1	1.0	C_1	C_1
5	3.5	4.0	C_2	C_2
6	3.8	3.7	C_2	C_2
7	3.6	4.1	C_2	C_2
8	3.9	3.8	C_2	C_2
9	1.3	1.2	C_1	C_1
10	3.7	4.2	C_2	C_2
11	1.0	1.4	C_1	C_1
12	3.6	3.9		

Centroid C_1 I_1 — Centroid C_2

(centroid C_1 (1.06, 1.3) C_2 (3.68, 3.95).

$\min(d(C_1,2), d(C_2,2))$



$C_1 = 2, 3, 4, 9, 11$

$C_2 = 5, 6, 7, 8, 10$

$$d(C_1, 2) = \sqrt{(1.0-1.2)^2 - (1.5-1.3)^2} =$$

$$d(C_2, 2) = \sqrt{(3.6-1.2)^2 - (3.9-1.3)^2} =$$

Calculating Centriod for Iteration 2.

C_1

$$C_1 x_1 = \frac{1.0+1.2+0.8+1.1+1.3+1.0}{6} = 1.066$$

$$C_1 x_2 = \frac{1.5+1.3+1.6+1.0+1.2+1.4}{6} = 1.3$$

C_2

$$C_2 x_1 = \frac{3.5+3.8+3.6+3.9+3.7+3.6}{6} = 3.68$$

$$C_2 x_2 = \frac{4.0+3.7+4.1+3.8+4.2+3.9}{6} = 3.95$$

Hiraschical Clustering:

* Aggecomerative:

Example:

ID	x_1	x_2
P_1	0.40	0.53
P_2	0.22	0.38
P_3	0.35	0.32
P_4	0.26	0.19
P_5	0.08	0.41
P_6	0.45	0.30

Iteration 1.

	P_1	P_2	P_3	P_4	P_5	P_6
P_1	●					
P_2	0.23	●				
P_3	0.22	0.16	●			
P_4	0.37	0.19	0.13	●		
P_5	0.34	0.14	0.28	0.23	●	
P_6	0.24	0.24	0.10 (min)	0.22	0.39	●

P_1.

$$d(P_1, P_2) = \sqrt{(0.40 - 0.22)^2 + (0.53 - 0.38)^2} = 0.2$$

$$d(P_1, P_3) = \sqrt{(0.40 - 0.35)^2 + (0.53 - 0.32)^2} = 0.21$$

$$d(P_1, P_4) = \sqrt{(0.40 - 0.26)^2 + (0.53 - 0.19)^2} = 0.36$$

$$d(P_1, P_5) = \sqrt{(0.40 - 0.08)^2 + (0.53 - 0.41)^2} = 0.34$$

$$d(P_1, P_6) = \sqrt{(0.40 - 0.45)^2 + (0.53 - 0.30)^2} = 0.23$$

P_2

$d(P_2, P_3) = \sqrt{(0.22 - 0.35)^2 + (0.38 - 0.32)^2} = 0.16$

$d(P_2, P_4)$

$d(P_2, P_5)$

$d(P_2, P_6)$:

P_3.

$d(P_3, P_4)$

$d(P_3, P_5)$

$d(P_3, P_6)$.

P_4:

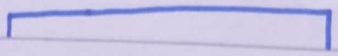
$d(P_4, P_5)$

$d(P_4, P_6)$

P_5:

$d(P_5, P_6)$:

Close clusters:



Iteration 2.

	P_1	P_2	P_3P_6	P_4	P_5
P_1	0				
P_2	0.23	0			
P_3P_6	0.22	0.14	0		
P_4	0.37	0.19	0.13	0	
P_5	0.34	0.14	0.28	0.23	0

$\min((P_3P_6), P_1) = \min(d(P_3, P_1), d(P_6, P_1))$

$= \min(0.22, 0.24)$

$= 0.22$

$\min((P_3P_6), P_2) = \min(d(P_3, P_2), d(P_6, P_2))$

$= \min(0.14, 0.24)$

0.14

$\min((P_3P_6), P_4) = \min(d(P_3, P_4), d(P_6, P_4))$

$= \min(0.13, 0.22)$

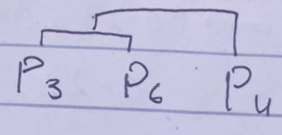
$= 0.13$

$\min((P_3P_6), P_5) = \min(d(P_3P_5), d(P_6, P_5))$

$= \min(0.28, 0.39)$

0.28

Close clusters:



Iteration 3

	P_1	P_2	$P_3P_6P_4$	P_5
P_1	0			
P_2	0.23	0		
$P_3P_6P_4$	0.22	0.14	0	
P_5	0.34	0.14	0.23	0

$\min(d(P_3P_6P_4), P_1) = \min(d(P_3, P_1), d(P_6, P_1))$

$= \min(0.22, 0.37)$

$= 0.22$

$\min(d(P_3P_6P_4)P_2) = \min(d((P_3P_6)P_2), d(P_4, P_2))$

$= \min(0.14, 0.19)$

$= 0.14$

$\min(d(P_3P_6P_4), P_5) = \min(d((P_3P_6), P_2), d(P_4, P_5))$

$= \min(0.28, 0.23)$

$= (0.23)$

	P_1	P_2P_5	$P_3P_6P_4$
P_1	0		
P_2P_5	0.23	0	
$P_3P_6P_4$	0.22	0.14	0

$\min(d(P_2, P_5), P_1) = \min((P_2, P_1), (P_5, P_1))$

$\min(0.23, 0.34)$

$= 0.23$

$d((p_3 p_6, p_4), p_2, p_5) = \min((p_3, p_6 p_4, p_2), (p_3, p_6, p_4), p_5$

$= (0.14, 0.23)$

$= 0.14.$

Close clusters.