We borrow some function names from R to indicate the corresponding probability functions ***dmultinom*** (for the multinomial distribution) , and a density function ***ddirichlet*** (for the Dirichlet distribution).

We let Dir be the Dirichlet distribution normalizer constant, defined by

$$\begin{aligned} Dir(a_1, a_2, a_3, a_4) &= \frac{Gamma(a_1)Gamma(a_2)Gamma(a_3)Gamma(a_4)}{Gamma\left(\sum_i a_i\right)} \\ &= \frac{(a_1-1)!(a_2-1)!(a_3-1)!(a_4-1)!}{\left(\sum_i a_i - 1\right)!} \end{aligned}$$

where $Gamma$ is the standard gamma function, with the convenient property $Gamma(n) = (n-1)!$.

$$\begin{aligned} &m_{Split} = \Pr(\mathbf{n}|\phi = Split) = \int_{\mathbf{p}} \Pr(\mathbf{n}|\mathbf{p}, \phi = Split) \Pr(\mathbf{p}) \\ &= \int_{\mathbf{p}} dmultinom(\mathbf{n}, \mathbf{p}) \cdot ddirichlet(\mathbf{p}, \mathbf{a}) \\ &= \int_{\mathbf{p}} \left(\tfrac{n!}{n_{RD}! n_{ND}! n_{RL}! n_{NL}!} p_{RD}^{n_{RD}} \cdot p_{ND}^{n_{ND}} \cdot p_{RL}^{n_{RL}} \cdot p_{NL}^{n_{NL}} \right) \cdot \tfrac{p_{RD}^{a_{RD}-1} \cdot p_{ND}^{a_{ND}-1} \cdot p_{RL}^{a_{RL}-1} \cdot p_{NL}^{a_{NL}-1}}{Dir(\mathbf{a})} \end{aligned}$$

For the original data, this is

$$\begin{aligned} m_{Split} &= \frac{100!}{3!2!5!90!} Dir(4,3,6,91) Dir^{-1}(1,1,1,1) \\ &= \frac{100!}{103!} 3! \\ &= 5.654477 \times 10^{-6} \end{aligned}$$