Suppose that $r : [a, b] \to C$ is smooth and f is continuous on r^*. Then

$$\int_r f(z)dz = \int_a^b f(r(t))r'(t)dt$$

ML Inequality: Suppose that $r : [a, b] \to C$is a smooth curve and f is continuous on r^*.If $|f| \leq M$ on r^* and L is the length of r then

$$\left| \int_r f(z)dz \right| \leq ML$$

Cauchy's Theorem for derivatives：Suppose V is an open subset of the plane, $f : V \to C$ is continuous, and there exists an$F : V \to C$such that $f = F'$ in V.Then

$$\int_r f(z)dz = 0$$

for any smooth closed curve r in V

Cauchy's Integral Formula for disks:

$$f(z) = \frac{1}{2\pi i} \int_r \frac{f(w)dw}{w - z}$$

Cauchy Integral Formula for derivatives: Suppose $f \in H(V)$and $\overline{D(z_0, r)} \subset V$ .Define $r : [0, 2\pi] \to V$ by $r(t) = z_0 + re^{it}$

$$f^{(n)}(z) = \frac{n!}{2\pi i} \int_r \frac{f(w)dw}{(w-z)^{n+1}}$$

for all $z \in D(z_0, r)$

Cauchy's Estimates: suppose that f is holomorphic in a neighborhood of the closed disk$\overline{D(z_0,r)}$ and $|f| \leq M$ in $\overline{D(z_0,r)}$ .Then

$$\left| f^{(n)}(z_0) \leq \frac{Mn!}{r^n} \right|$$

Liouville's Theorem: A bounded entire function must be constant.

Fundamental Theorem of Algebra:

Suppose that $f \in H(V)$ and V is connected. If all the derivatives of f vanish at some point of V then f is constant.

suppose that $f \in H(V)$ and f has a zero of order N at $z \in V$.Then there exists $g \in H(V)$ with $g(z) \neq 0$ such that

$$f(w) = (w - z)^N g(w)$$

for all $w \in V$

Maximum Modulus Theorem: Suppose that $f \in H(V)$ and V is connected. Then $|f|$ cannot achieve a (local) maximum in V unless f is constant: If f is nonconstant then for every $a \in V$ and $\delta > 0$ there exists $z \in V$ with $|f(z)| > |f(a)|$ and $|z - a| < \delta$.

Parseval Formula： suppose that the power series

$$f(z)=\sum_{n=0}^{\infty}c_n(z-a)^n$$

converges for $|z-a|<r$ .Then

$$\frac{1}{2\pi}\int_0^{2\pi}\left|f(a+pe^{it})^2\right|dt=\sum_{n=0}^{\infty}|c_n|^2p^{2n}$$

for every $p\in[0,r)$.

洛朗级数 $f(w) = \sum_{n=-\infty}^{\infty} c_n(w-z)^n$ 可以分为两部分：

解析部分 (Analytic Part): $\sum_{n=0}^{\infty} c_n(w-z)^n$。这部分在 z 点是全纯的（如果收敛半径足够大）。

主要部分 (Principal Part): $\sum_{n=-\infty}^{-1} c_n(w-z)^n$。这部分决定了奇点的性质。对于极点，主要部分是有限项的。

可去奇点 (Removable Singularity): 如果洛朗级数中没有负幂次项（即主要部分为零），那么函数在该点可以被定义（或重新定义）为一个全纯函数。这相当于说，在 z 点的奇点“可以被移除”。

极点 (Pole): 如果洛朗级数中只有有限多项负幂次项，并且最高负幂次项的系数不为零，那么函数在该点有一个极点。这个极点的阶数就是最高负幂次的指数（取绝对值）。

本质奇点 (Essential Singularity): 如果洛朗级数中有无限多项负幂次项，那么函数在该点有一个本质奇点。

suppose Ω is simply connected with $1 \in \Omega$,$0 \notin \Omega$.Then in Ω there is a branch of the logarithm $F(z) = log_{\Omega}(z)$ so that

F is holomorphic in Ω

$e^{F(z)} = z$ for all $z \in \Omega$

$F(r) = logr$ whenever r is a real number near 1

Let f be defined in $D(z_0, r) \setminus \{z_0\} = \{z \in \mathbb{C} \mid 0 < |z - z_0| < r\}$ we say f has a pole at z_0 if the function:

If f has a pole at $z_0 \in \Omega$ then in some $D_r(z_0)$ there exists a non-vanishing holomorphic h and a unique $n \in N^+$ s.t.

$$f(z) = \frac{h(z)}{(z - z_0)^n}$$