GAUSSIAN LASER BEAMS

WEEK 1 INTRO: MEASURING A GAUSSIAN BEAM. CALIBRATING YOUR PHOTODETECTOR

GOALS

In this lab, you will use a lot of equipment and try out some techniques that are generally useful in optics labs and elsewhere. In particular, you will set up a simple optics system for measuring the width of your laser beam and in the process will have to mount and align the laser and optics.

- Proficiency with new equipment
    - Laser: mounting it to table, plugging it in, turning it on.
    - Mounting optics:
        - Mirrors
        - Lenses
        - Post, post holders, bases
    - Aligning optics
        - Mirrors (using two mirrors to adjust a beam to any desired position and angle)
        - Lenses
    - Translation stage
        - Mounting it to the optics table.
        - Mounting optics on it.
        - Reading the micrometer position.
        - Measuring micron-scale displacements.
    - Amplified photodetector
        - Using it.
        - Understanding how it works.
        - Modeling its behavior.
        - Reading the specification/data sheet.
- New skills to apply from Lab Skill Activities
    - Entering data into Mathematica or importing data.
    - Non-linear least-squares fitting.
    - Plotting data and fit function together.
    - Extracting basic fit parameters with standard uncertainties.
- Experimental design
    - Calibration of the photodetector
    - Modeling the photodetector

LAB NOTEBOOK GUIDELINES

The lab notebook will play an important role in this course. You will use your notebook for keeping records of many things including

- Answering pre-lab questions from the lab guide.
- Answering in-lab questions.
- Recording data.
- Including plots of data.
- Analysis and results.
- Diagrams and pictures.
- Procedures of experiments that you design.

The lab notebook will be an important part of your grade because learning to keep a good lab notebook is an important part of your professional development. You may find it helpful to write up many of your notes on the computer, for example, within Mathematica or another program. This is fine. However, before your notebook is turned in, the notes, plots, and analysis should be transferred to the lab notebook by printing and taping the pages or keeping them in a three ring binder. There will also be formal lab reports and oral presentations, but these will be restricted to a limited portion of the experimental work you have conducted in the lab.

DEFINITIONS

Optic – Any optical component that manipulates the light in some way. Examples include lenses, mirrors, polarizing filters, beam splitters, etc.

Optomechanics – This category includes optics mounts and the components to align them. Examples in the lab include post, post holders, bases, lens mounts, adjustable mirror mounts, rotation mounts, and translation stages.

SETTING UP YOUR LASER AND MOUNTING OPTICS

When you start working in the lab you should have an empty optics breadboard. The shelf above the breadboard should have

- An oscilloscope
- A waveform generator
- Triple output DC Power Supply
- Set of ball drivers
- Optics caddy to hold optics already mounted on 0.5" posts.
- Set of 1/4-20 and 8-32 screws, setscrews, washers, and nuts.

Question 1 *Optics lab skills*	a. Get a laser from the cabinets and mount it on your work table. You should use 2" posts and post holders for the laser, which will set the laser at a convenient height for most of the optics labs. b. Each person in your group is responsible for assembling a mounted lens or mirror as shown in Figure 1. In the end, you will need at least 2 mirrors to complete the next task.
Question 2 *Optics lab skills*	**"Walking a beam"** Mount a narrow tube at a random position with a random orientation on your optics breadboard. a. Use only two mirrors to get the beam to pass through the center of your tube. This technique is commonly called "walking a beam." b. Draw a diagram of the configuration of your laser, mirrors, and tube.

MODELING CHARACTERISTICS OF THE PHOTODETECTOR

The goal of this part of the lab is to understand a lot about the specifications given on the datasheet for the Thorlabs PDA36A Switchable Gain Amplified Photodetectors. It is important to realize that data sheets (also called spec sheets or specification sheets) provide a model for the realistic behavior of the device. This model can be tested and improved, a process more commonly called "calibration."

<table>
<tr>
<td>Question 3

Modeing the Measurement System

Math-Physics-Data Connection</td>
<td>Basic function of the amplified photodetector

a. Write an explanation in words that explains how the photodetector converts light into voltage. Use the manufacturers specifications sheet, trustworthy online resources, a book, or other resource as necessary.
b. Draw a diagram explaining the process by which the photodetector converts the light into voltage.
c. How could we measure the conversion factor that converts Watts of light into Volts?
d. What is the conversion of Watts of light into Amps of current for Helium Neon red wavelength (632.8 nm) and for the Frequency doubled Nd:YAG laser (Green laser pointer wavelength, 532 nm)?
    a. How would you convert A/W into electrons per photon?
    b. What is the electron/photon conversion efficiency for the red HeNe and green doubled Nd:YAG lasers?
    c. Is this number less than, equal to, or greater than one? What does this number tell you about how the photodiode works?</td>
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<tr>
<td>Question 4

Modeing the Measurement System

Experimental Design

Math-Physics-Data Connection

Systematic Error Analysis</td>
<td>Calibrating the photodetector offset and gain with the spec sheet.

a. The offset voltage is the output of the photodetector when no light is incident upon the device.
    a. Calibrate the offset of the photodetector as a function of gain setting.
    b. Quantitatively compare it to the specifications given in the table. Is your measured value within the specified range given on the PDA36A photodetector data sheet?
    c. What measures did you take to eliminate stray light? Were your measures sufficient for an accurate calibration?
b. Is it possible to measure the V/A gain for each setting, or can you only measure the change in gain as you switch the settings? Why?
    a. Make a measurement of the gain or relative gain settings for most of the gain settings.
        i. What systematic error sources are of most concern?
    b. Quantitatively compare your results with the range of values given on the data sheet. Do you believe your results provide a more accurate estimate of the photodetector gain than the data sheet? Why or why not?
    c. How do the labeled gain settings 0 dB,…,70 dB relate to the V/A gain? What does a 20 dB gain correspond to in V/A?</td>
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Question 5 *Modeing the Measurement System* *Math-Physics-Data Connection*	**Follow up:** Write a mathematical expression that converts incident power P_{in} to output voltage V and output voltage V to input power P_{in}. Take into account all relevant parameters such as the photodetector gain setting.

MEASURING THE BEAM WIDTH

Note: *Many of the data analysis techniques in this section will use skills from Thursday's Lab Skill Activity.*

The goal of this section is to develop a measurement technique and analysis scheme to measure the width of a beam. The scheme will let you measure the width in one direction. The technique is most useful for beams that are approximately Gaussian profile in intensity. In the second week of the lab you will use this technique to experimentally answer questions about Gaussian beams.

The basic scheme involves measuring the power in the laser beam as the beam is gradually blocked by a razor blade using a setup similar to Figure 2.

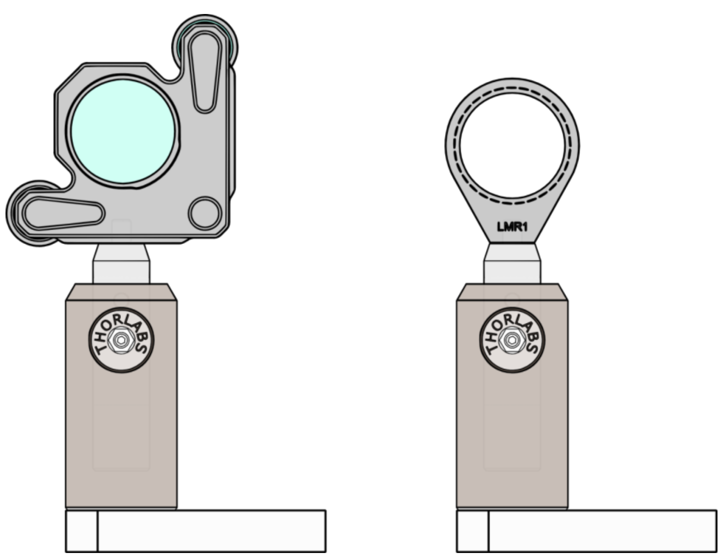

Figure 1 Mounting assemblies for a mirror (left) and a lens (right).

Question 6 *Math-Physics-Data Connection*	Suppose a laser beam has a Gaussian intensity profile $I(x,y) = I_{max}e^{-2(x^2+y^2)/w^2}$, and is incident upon a photodiode. What is the expression for the power hitting the photodiode when a portion of the beam is blocked by a razor blade (see Figure 2: Razor blade mounted on a translation stageFigure 2)? a. Draw a diagram showing the beam and the razor. b. Write the mathematical expression for the power incident on the photodiode as a

	function of razor position using the above expression for $I(x, y)$.

Question 7
Statistical Error Analysis
Computer-aided Data Analysis
Modeing the Measurement System
	Before you take data: Create an analysis function to fit a test set of data available on course website.
	a. What is the functional form for your fit function?
	b. Is it a linear or nonlinear fit function? Why?
	c. What are the fit parameters? Why do you need this many?
	d. How do the fit parameters relate to the beam width?
	e. Download the data set from course website.
	    a. Make a plot of the data.
	    b. Make a fit and plot it with the data.
	    c. Check that the fit looks good and you get a beam width of $w = 4.52 \times 10^{-4}$ m

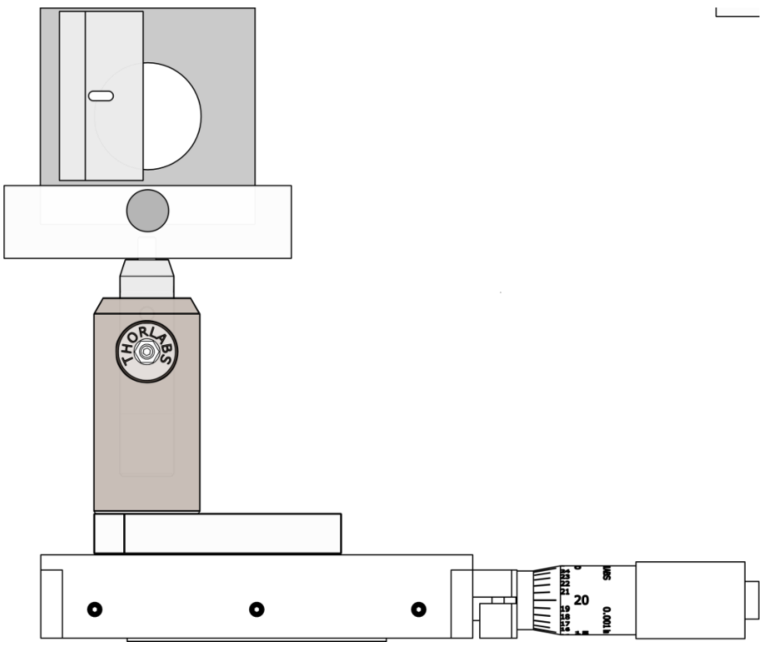

Figure 2: Razor blade mounted on a translation stage

Question 8 *Experimental Design* *Math-Physics-Data Connection* *Lab notebook*	**Build your setup for measuring the beam width of your laser.** a. Draw a detailed schematic of the setup (from the laser all the way to the photodetector. b. After assembling your experiment, but prior to taking a lot of data, how can you quickly determine if the measurement is working? c. Use the measurement scheme to take data of power vs position of the razor. a. Pick at least 2 positions to measure the beam width.

Question 9
Statistical Error Analysis
Experimental Design
	Analysis of the random uncertainty sources
	1. How would you estimate the random uncertainty in the photodetector voltage?
	    a. Estimate the uncertainty (standard deviation, σ_V) for a few different razor positions.
	    b. Does the uncertainty σ_V depend on the razor position? Explain. If it does depend on razor position, when is it largest?
	    c. What are the possible sources of the random fluctuations? Can you do any simple experiments to determine the source of the random fluctuations?
	2. How do you estimate the uncertainty in the position x of the razor σ_X? What is the

	estimate? 3. Uncertainty in the razor position can create fluctuations in the power incident upon the photodetector. Compare the magnitude of the uncertainty in V due to razor position fluctuations and fluctuations from other sources. Which is larger?
Question 10 Statistical Error Analysis Computer-aided Data Analysis	**Analysis of the real data.** Use the analysis procedures verified in Question 7 to find the beam widths for each data set.

Question 11
Statistical Error Analysis
Experimental Design
	Follow up questions
	a. An alternative method for measuring the width of the Gaussian beam would be to estimate the Gaussian profile from the raw data by taking numerical derivatives.
	    a. Make a plot of the numerically differentiated data to see how well the Gaussian profile shows up. Does it look qualitatively Gaussian? Does it look better or worse than you expected?
	    b. Would there be any disadvantages to fitting the derivative of the data rather than the data itself?
	b. How could you measure the beam width in the vertical direction?
	c. Another method of beam profiling is taking an image of the beam using a camera.
	    a. What advantages could this method have compared to the razor blade method?
	    b. Do you foresee any disadvantages?

WEEK 2: DEVELOPING A QUANTITATIVE MODEL OF THE SPATIAL PROPERTIES OF LIGHT

GOALS

Expand two models of the most frequently used components in the optics experiments. In week 1, we measured the profile of the laser and found it to be Gaussian to a good approximation. However, we don't have any model for how the profile changes as the beam propagates.

Also, we will apply measurement and automation to more rapidly take data. In particular, you will automate two things: the data acquisition, and the fitting and analysis routine. The full set of learning goals includes:

1. Automated data acquisition.
    a. LabVIEW
    b. USB DAQ (NI USB-6009)
2. Automated fitting and analysis of data in Mathematica
3. Using a predictive model of Gaussian laser beams
    a. Contrast Gaussian beams with geometric optics
4. Measure profiles of a Gaussian beam, and extract the Gaussian beam parameters (typically beam waist radius and position).
5. Effect of a lens on Gaussian beams.
    a. Is it still Gaussian?
    b. Does the thin lens equation apply to Gaussian Beams?
    c. What limits the minimum achievable spot size?

PRELAB: INTRODUCTION

Question 12 Reflection on last week's lab	**Answer these before reading ahead in the lab guide based on your experience from last week's lab.** a. Does the beam always stay a Gaussian as it propagates? Why do you think this? b. Does the beam stay Gaussian after it goes through a lens? Why do you think this? c. Does the beam stay Gaussian after it reflects from a mirror? Why do you think this? d. How small does the beam get when it is focused by a lens? Does it focus to a point? Why or why not?

Light is a propagating oscillation of the electromagnetic field. The general principles which govern electromagnetic waves are Maxwell's equations. From these general relations, a vector wave equation can be derived.

$$\nabla^2 \vec{\boldsymbol{E}} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{\boldsymbol{E}}}{\partial t^2} \tag{1}$$

One of the simplest solutions is that of a plane wave propagating in the $\hat{\boldsymbol{z}}$ direction.

$$\vec{\boldsymbol{E}}(x,y,z,t) = E_x \hat{\boldsymbol{x}} \cos(kz - \omega t + \phi_x) + E_y \hat{\boldsymbol{y}}\ \cos\left(kz - \omega t + \phi_y\right) \tag{2}$$

But as the measurements from last week showed, the laser beams are commonly well approximated by a beam shape with a Gaussian intensity profile. Apparently, since these Gaussian profile beams exist, they must be solutions of the wave equation. The next section will discuss how we derive the Gaussian beam electric field, and give a few key results.

PARAXIAL WAVE EQUATION

One important thing to note about the beam output from most lasers is that the width of the beam changes very slowly compared to the wavelength of light. Assume a complex solution, where the beam is propagating in the $\hat{\boldsymbol{z}}$-direction, with the electric field polarization in the $\hat{\boldsymbol{x}}$-direction.

$$\vec{\boldsymbol{E}}(x, y, z, t) = \hat{\boldsymbol{x}}\, A(x, y, z) e^{i(kz - \omega t)} \tag{3}$$

The basic idea is that the spatial pattern of the beam, described by the function $A(x, y, z)$, does not change much over a wavelength. In the case of the HeNe laser output, the function $A(x, y, z)$ is a Gaussian profile that changes its width as a function of z. If we substitute the trial solution in Eq. (3) into the wave equation in Eq. (1) we get

$$\hat{\boldsymbol{x}}\left[\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2}\right) + 2ik\frac{\partial A}{\partial z} - k^2 A\right] e^{i(kz-\omega t)} = \hat{\boldsymbol{x}}\mu_0\epsilon_0 A(-\omega^2) e^{i(kz-\omega t)} \tag{4}$$

This can be simplified recognizing that $k^2 = \omega^2/c^2 = \mu_0\epsilon_0\omega^2$, where the speed of light is related to the permeability and permittivity of free space by $c = (\mu_0\epsilon_0)^{-1/2}$. Also, the $\hat{\boldsymbol{x}}e^{i(kz-\omega t)}$ term is common to both sides and can be dropped, which results in

$$\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2}\right) + 2ik\frac{\partial A}{\partial z} = 0 \tag{5}$$

So far we have made no approximation to the solution or the wave equation, but now we apply the assumption that $\partial A(x, y, z)/\partial z$ changes slowly over a wavelength $\lambda = 2\pi/k$, so we neglect the term

$$\left|\frac{\partial^2 A}{\partial z^2}\right| \ll \left|2k\frac{\partial A}{\partial z}\right| \tag{6}$$

And finally, we get the paraxial wave equation

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + 2ik\frac{\partial A}{\partial z} = 0 \tag{7}$$

One set of solutions to the paraxial wave equation are Gauss-Hermite beams, which have an intensity profiles that look the pictures in Fig. 2. These are the same solutions as for the quantum simple harmonic oscillator, a topic that could be further explored as a final project.

The simplest of these solutions is the Gaussian beam, which has an electric field given by

$$\vec{\boldsymbol{E}}(x,y,z,t) = \vec{\boldsymbol{E}}_0 \frac{w_0}{w(z)} \exp\left(-\frac{x^2+y^2}{w^2(z)}\right) \exp\left(ik\frac{x^2+y^2}{2R(z)}\right) e^{-i\zeta(z)} e^{i(kz-\omega t)} \tag{8}$$

Where $\vec{\boldsymbol{E}}_0$ is a time-independent vector (orthogonal to propagation direction $\hat{\boldsymbol{z}}$) whole magnitude denotes the amplitude of the laser's electric field and the direction denotes the direction of polarization. The beam radius $w(z)$ is given by

$$w(z) = w_0\sqrt{1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2} \tag{9}$$

$R(z)$, the radius of curvature of the wavefront, is given by

$$R(z) = z\left(1+\left(\frac{\pi w_0^2}{\lambda z}\right)^2\right) \tag{10}$$

And the Guoy phase is given by

$$\zeta(z) = \arctan\frac{\pi w_0^2}{z\lambda} \tag{11}$$

The remarkable thing about all these equations is that only two parameters need to be specified to give the whole beam profile: the wavelength λ and the beam waist w_0, which is the narrowest point in the beam profile. There is a more general set of Hermite Gaussian modes which are shown in Figure 3. The laser cavity typically produces the (0,0) mode shown in the upper left corner, but an optical cavity can also be used to create these other modes shapes – a topic that can be explored in the final projects.

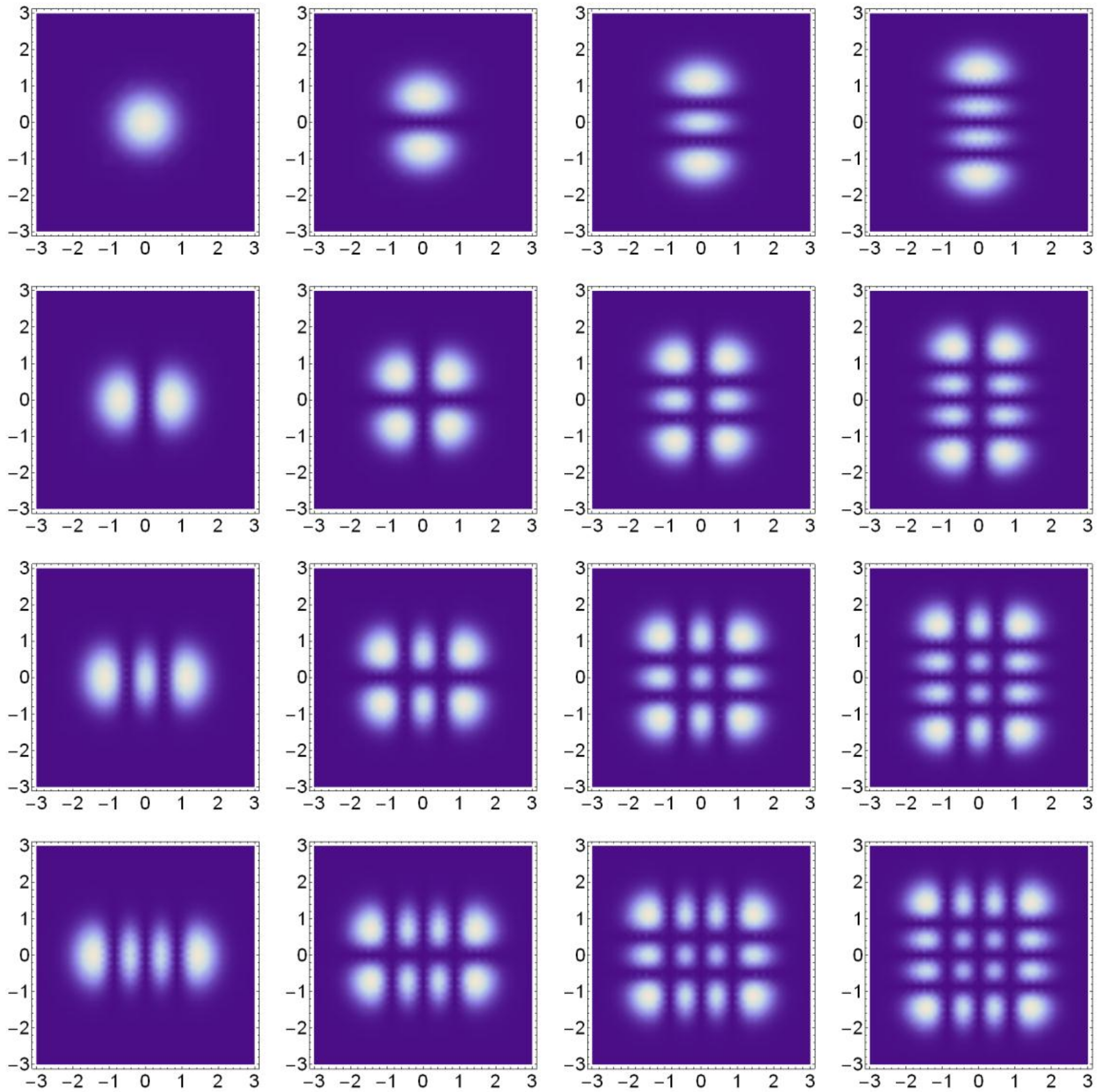

Figure 3 Intensity distributions for the lowest order Gauss-Hermite solutions to the paraxial wave equation. The axes are in units of the beam width, w.

MORE PRELAB: TRYING OUT THE GAUSSIAN BEAM MODEL

Question 13 *Math-Physics-Data Connection*	a. In week 1 of the lab, we assumed the intensity profile of the Gaussian beam was given by $I(x,y) = I_{max}e^{2(x^2+y^2)/w^2}$. The equation for the electric field of the Gaussian Beam in Eq. (8) looks substantially more complicated. How are the expressions for electric field and intensity related? Is Eq. (8) consistent with the simple expression for intensity $I(x,y) = I_{max}e^{2(x^2+y^2)/w^2}$? b. What happens to the radius of curvature of the wavefront, $R(z)$, near $z = 0$? Does this make sense? c. What happens to the radius of curvature of the wavefront $R(z)$, for $z \gg \pi w^2/\lambda$? Does this make sense?
Question 14 *Math-Physics-*	The Gaussian beam equations given in Eqs. (8)-(11) assume the beam comes to its narrowest width (called the beam waist) at $z = 0$. How would you rewrite these four equations assuming

Data Connection	the beam waist occurs at a different position $z = z_w$?
Question 15 *Math-Physics-Data Connection*	There is a simple prediction that can be seen from equation 9, that the angle of divergence θ for $z \gg \pi w_0^2/\lambda$ is given by $$\theta \equiv \frac{dw}{dz} \approx \frac{\lambda}{\pi w_0}$$ Plot $w(z)$ for various beam sizes, and verify this inverse relationship between θ and w_0 using your plots.
Question 16 *Statistical Error Analysis Computer-aided Data Analysis*	a. Write a function to fit the following data set available on the course website. i. What is the functional form for your fit function? ii. Is it a linear or nonlinear fit function? Why? b. Assuming a HeNe wavelength of $\lambda = 632.8$ nm, you should get that a beam waist of $w_0 = 93.9(\pm 0.1) \times 10^{-6}$ m and occurs at a position $z_w = 0.3396 \pm 0.0003$ m.

AUTOMATION OF THE MEASUREMENT AND ANALYSIS

In this lab, you will use LabVIEW and your NI USB-6009 data acquisition card. The Lab skill activities on LabVIEW cover how to interface LabVIEW with the USB-6009 and to acquire readings and save them to a file. The Lab Skill Activities on Mathematica cover how to import data. Last week you should have developed analysis routines that fit beam profiles and extract the beam width w. This part of the lab is going to help us reason through the process of automating the experiment.

Question 17 Engineering Design	a. In week one how long did the total process of data taking through analysis take to make a measurement of the beam width w? b. In this lab you may have to take 20-30 beam profiles in order to measure w. How long would this take with your current method? c. What are the most time consuming portions of the process? Which parts of the process would benefit from automation? d. Implement the automation in LabVIEW and Mathematica using the basic LabVIEW data acquisition VI provided to the class. e. Before you go on, make sure the automated acquisition and analysis routine gives the same result as the method you used last week. f. How long does your new measurement method take? 2-3 minutes per w measurement is pretty optimal.

THE EXPERIMENT

The Gaussian beam model of light is useful because it often describes the beam of light created by lasers. This section will test the validity of the model for our HeNe laser beam. Also, the effect of a lens on a Gaussian beams will be tested, and the Gaussian beam model will be compared with predictions from the simpler ray theory. Lastly, the Gaussian Beam theory can be used to describe the minimum possible focus size for a beam and a lens.

Question 18 *Experimental Design*	**Measuring the beam profile of your HeNe laser.** There is a straight-forward reason that a HeNe laser should produce a Gaussian beam. The laser

Math-Physics-Data Connection *Statistical Error Analysis*	light builds up between two mirrors, and the electromagnetic mode that best matches the shape of the mirrors is the Gaussian beam. a. Considering Eq. (8)-(11), which apsects of the Gaussian beam model can you test? Are there any parts of the model you cannot test? b. Measure the beam width w versus distance from the laser. How did you decide what positions z to measure the width at? c. Fit the data to $w(z)$, the predicted expression for a Gaussian beam given in Eq. 8. d. What is the value of the beam waist w_0? Where does the beam waist z_w occur relative to the laser?
Question 19 *Experimental Design*	**How does a lens change a Gaussian beam?** Pick any non-compound lens (excludes the fancy camera lenses). Design and carry out an experiment to quantitatively answer the following questions. a. Does the beam retain a Gaussian profile after the lens? b. What is the new beam waist w_0 and where does it occur? c. What factors affect the beam profile after the lens? d. Does the measured $w(z)$ match the Gaussian beam prediction given in Eq. (9)?
Question 20 Math-Physics-Data Connection Experimental Design Systematic Error Analysis	**Quantitatively modeling the effect of a lens** One of the simplest ways to model the effect of a lens is the thin lens equation, which is based on a ray model of light. (see Figure 4) $$\frac{1}{S_1}+\frac{1}{S_2}=\frac{1}{f}$$ a. Redraw Figure 4 to show how it would change when the light is modeled as a Gaussian beam, rather than rays. In particular, where should the beam waists occur? What determines the relative width of the beam waist? b. Experimentally test the accuracy of the thin lens equation for the imaging of Gaussian beams.
Question 21 *Engineering Design*	**Design Challenge:** An application of the Gaussian beam model. This series of questions will be directed at answering one question: *What is the smallest size you can focus your HeNe down to?* a. Thinking back to the previous questions, what factors affect the size of the beam waist? b. How can these parameters be optimized to minimize the beam waist? What is the smallest spot size you predict can be achieved using a standard 1" diameter lens with a focal length between 25 mm and 500 mm? c. What position resolution will you need when profiling the beam (x-y plane) near the waist? d. What position resolution will you need in the z-direction when profiling the beam near the waist? e. Create the smallest beam waist w_0 possible, and compare your prediction in (b) with your measurement. f. Does the Gaussian beam model seem sufficient to describe the beam? a. If not, what possible assumptions of the model broke down?

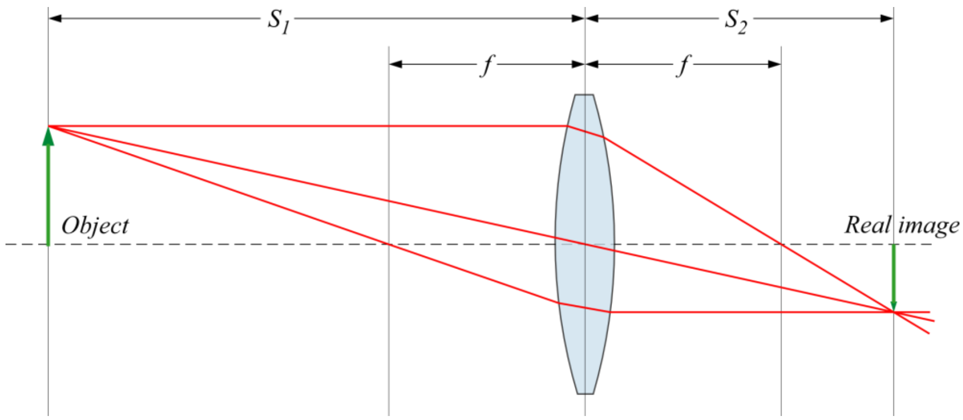

Figure 4 Diagram showing the focusing of light by a thin lens in the ray approximation. The diagram identifies the quantities in the thin lens equation: image distance, object distance, and focal length.

OTHER RANDOM STUFF

There is a disassembled Helium Neon Laser which can be seen as a demonstration.

PROJECT IDEAS

1. Predicting the behavior of complex optical systems using ABCD matrices to transform Gaussian Beams.
2. Build an optical cavity. Study the coupling of light into the cavity, and spatial filtering into different TEM modes. Replicate the awesome pictures.
3. Analogy between paraxial wave equation in free space and 2D Schrodinger wave equation. Solving the Schrodinger equation optically. Adding a potential. Tunneling. Etc.
4. Using a translatable, rotatable slit to map out the beam profile of a funky pattern using the Radon transform, which is used in reconstructing CT scans. Perhaps there is some better application of tomography also.

FORMAL WRITE UP OR PRESENTATION

The lab notebook should contain all of your work done in answering the questions in the lab guide. The formal report will be restricted to a limited portion of your experimental work. In this lab, make a formal presentation of your findings from Question 21 on diffraction limited focusing. This was likely the most challenging measurement in the lab. Make a compelling case that you have understood the theory behind diffraction limited focusing, and that you have also taken a good measurement of the beam waist complete with an analysis of the uncertainties. Make a concise and compelling argument, using graphs, diagrams, and equations when necessary.

REFERENCES

1. http://people.seas.harvard.edu/~jones/ap216/lectures/ls_1/ls1_u3/ls1_unit_3.html (Gaussian Beam theory)
2. http://en.wikipedia.org/wiki/Gaussian_beam