For Every X

An Introduction to Critical Reasoning and Formal Logic

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# Release Notes

## About

As noted on the copyright page, this book is a mashup of two OER reasoning works: *For All X, the Lorain County Remix* by Loftin and *An Introduction to Reasoning* by Woods. We use Woods for a discussion on real-world reasoning (informal logic) and Loftin as an introduction to formal logic.

This book has been created as an OER text because we assert that access to knowledge is a human right and should not be locked behind paywalls or rented out to those who have the privilege to pay. Logic texts are notoriously and prohibitively expensive, which is tragic, given the social value of coherent reasoning in society. We applaud Magnus (creator of the original *For All X*), Loftin, and Woods for working to opening access to quality formal and informal logic works. We also applaud all of the other scholars working hard to make this history and collection of knowledge as accessible as possible. For more on this, research the Open Logic Project.[[3]](#footnote-4)

## Planned updates

Below is the roadmap for future versions.

* Correct minor typography issues, such as styling all variables with italics.
* Adding sections on statistic and scientific reasoning.
* Adding a section on navigating sources in various domains.
* Add a more in-depth discussion of sets, based upon the discussion found in the Open Logic Project.
* Adding more exercises and an exercise answer key, both for student experimentation and evaluation.

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# Part 1: Critical Reasoning

*This portion of the text is based upon* Critical Reasoning *by Cathal Woods.*

## Introduction to Real World Reasoning

### 1 Reasoning & Critical Reasoning

**1.** To reason is to use some propositions as reasons for accepting another proposition. For example: "This cheese is moldy. So, I should not eat it." Here, one proposition is a reason or *premise* and the other is the target or *conclusion*.

**2.** Reasoning produces an *inference*. An inference is the premise(s) and the conclusion put together in a set. When an inference is created by an individual for herself, she is said to be *inferring*. An inference can be presented to an audience in order that the audience will come to believe the target based on the reasons. Presenting an audience with an inference is *arguing.*

**3.** In inferring and arguing, the reasons serve as reasons-for-believing. Reasons can also serve as reasons-which-explain (or more precisely, reasons-which-*causally*-explain). For example: "The game was cancelled because the pitch was frozen.". "The pitch was frozen." is the reason or *cause*, "The game was cancelled." is the target or *effect*.

The propositions expressing the cause(s) and effect form an *explanation*. Explanations are created by reasoning about causes, and, once created, explanations can be offered to an audience in an act of explaining.

**4.** In sum, reasons can be used by a person as reasons-for-believing (which creates an inference), given (to an audience) as reasons-for-believing (as premises in an inference), or given as reasons-which-explain (causes in an explanation).

Although inferring involves *using* reasons (for oneself) to arrive at a target, while arguing and explaining involve *giving* reasons and target (to an audience), for ease of reference we will speak of all three as *reason-giving*. Since both inferring and arguing involve inferences, we can collapse these two into one category. This allows to speak more simply, of inferences and explanations.

**5.** To reason *critically* is to reason about reason-giving. Or, to reason critically is to reason about inferences and explanations. Or, to reason critically is to form an inference which has, as a conclusion, a proposition about whether or not the inference or explanation under consideration is good.

Or, critical reasoning is the evaluation of reason-giving. "Critical" means "to judge". So, we have theater critics, wine critics, livestock judges, and many more. Theater critics judge whether or not the play they are watching is a good play. Wine critics judge whether or not the wine they are tasting is a good wine. Livestock judges judge whether or not the horse, cow, etc. that they are looking at is a good horse, cow, etc. Critical reasoners judge whether or not the inference or explanation is a good one. Critical reasoning is reasoning about reason-giving. Critical reasoning is evaluating reason-giving. Critical reasoning is judging the quality of reason-giving. *Critical reasoning is quality control for reason-giving.*

**6.** All forms of judgment operate by *comparing* the play/wine/horse/inference/ explanation/etc. under consideration to the standards for good plays/wines/horses/ inferences/explanations/etc.

Thus, since critical reasoning is the judgment of reason-giving, becoming a judge of reason-giving requires becoming an expert in the standards of good reason-giving and being able to apply them to the inferences and explanations that you see and hear.

### 2 Real-World

**1.** Why is this part of the book called *real-world* reasoning? The answer is that the examples of reasoning aren't presented in symbols but instead in a natural language, in this case English. This matters because humans who present their reasoning in speech or writing often do so very sloppily. Moreover, because speakers just want to get other people to believe as they do, they sometimes skirt the rules of proper reasoning. Plus, real-world examples are actually about something, and this something might be emotionally charged. Using symbols, by contrast, automatically avoids all of these difficulties.

In some senses, then, it's a lot easier to be a logician (using symbols) than it is to be a critical reasoner (using a natural language): logicians just deal with the relationship between sentences and they don't have to deal with the humans, like critical reasoners do. Most schools and colleges that have a critical reasoning course and a logic course put the critical reasoning course *before* the logic course (probably because humans have a tendency to freak out when they see symbols). But a good case can be made for doing the logic first, because you don't have to deal with the imperfections and can focus solely on the connection between the reasons and the target. Then you can go on to deal with the messy human reasoning.

Some of the chapters in the Real World Reasoning section are explicitly about how humans are poor reasoners in these various ways: *Reason Substitutes* is about how speakers try to avoid giving reasons for their claims; *Problems With Meaning* is about how the meaning of words used is often imprecise; *Ignoring Confidence Indicators* gives you practice in ignoring the words speakers use to tell you that their reasoning is great; *Warrants* is all about how speakers often fail to give complete arguments.

So, the Real World Reasoning section will make you familiar with lots of patterns of good reasoning *and* it will alert you to various ways in which the pieces of reasoning that you encounter "in the wild" go wrong.

### 3 Overview

**1.** Here is a table of all of the parts of Real World Reasoning part of this book. You can refer back to it when reading the discussion that follows.

* Basic Analyzing
* Basic Classifying
* Advanced Classifying
* Advanced Analyzing
* Reasoning Substitutes
* Problems With Meaning
* Evaluating The Reasoning
* Ignoring Confidence Indicators
* Sources/Character/Motives
* Practical Reasoning
* Emotional Reasons
* Syntax & Logic
* Warrants
* Diagramming – Basic
* Diagramming – Complex
* Diagramming – Dialogue 1
* Diagramming – Dialogue 2
* Diagramming – Very Long Passages

**2.** To reason critically is to evaluate an inference (or explanation – see I&S). Before we can evaluate an inference, however, various other tasks are required. The first thing to realize that not all "speech" (which for our purposes will include both speaking and writing) presents reasoning. So, as a very preliminary step, you have to determine whether or not the speaker is presenting a piece of reasoning. We have to be able to distinguish reasoning from non-reasoning. In RW, this is called *Classifying* and there are two chapters on classifying. However, once these chapters are done, the rest of the Real World Reasoning section will present only examples of reasoning. This is highly artificial, of course — in the real world, you do a lot of classifying of speech.

The main lesson in Advanced Classifying is the difference between *explaining* and *reasoning.* This distinction is important if you intend to study Inductive & Scientific Reasoning (I&S). Scientific reasoning is about building explanations (and then using explanations in inferences) and the standards for a quality explanation are different from the standard for a quality inference. If you are not going to do I&S, you might skip Advanced Classifying, as all of the Real World Reasoning section is concerned with inferences and not with explanations.

**3.** If you think we are indeed dealing with a piece of reasoning, you next need to analyze it. To analyze is to break down. In the case of an inference, we break it down into the target claim and the reasons being offered for believing the target claim. There are two chapters on *Analyzing.* (There is also a chapter (called *Reason Substitutes*) about recognizing when a speaker hasn't actually given reasons for her claim. Humans do this kind of thing a lot: they make claims but when asked for reasons they avoid giving them.)

In every exercise set after these chapters, the instructions will begin by asking you to analyze each passage.

**4.** There are two other tasks that you need to do before evaluating a piece of reasoning. Although these two are not strictly 'analyzing' in the sense of 'breaking down', you should consider them as part of the general process of analyzing. If it helps, think of "analyzing" as the broader process of asking "What is the speaker saying?"

One is to get clear on the meaning of each proposition in the passage. If you don't even know what the target proposition means, or what the propositions expressing the reasons mean, you can't think about whether the reasons provide good reason for believing the target. The chapter called *Problems With Meaning* tries to articulate the various kinds of difficulties there can be with meaning but the basic idea is that you should always be sure to ask: What do the propositions involved in this piece of reasoning (both the reasons and the target) mean? (Getting clear on the meaning of each proposition is a necessity if we are to say whether the propositions are true or false, which is one of the tasks of evaluation.)

The other task that needs to be done before evaluating is to think about the structure of the piece of reasoning. The most basic structure is of course the difference between reasons and target. But another basic question you can ask about the structure is: If the speaker presents more than one reason, is she putting the two (or however many reasons there are) reasons together in order to justify believing the target claim, or might she in fact be presenting two pieces of reasoning, thinking that either reason is reason enough to accept the claim? Asking and answering this question is the skill introduced in *Basic Diagramming.* *Diagramming Complex Reasoning* is about how speakers sometimes give reasons for one claim and then go on to use that claim (along with more reasons) for another claim — this is called *extended reasoning*. In the two chapters on *Diagramming Dialogue,* you will extend your diagramming skills so that you can include the (hopefully helpful) thoughts that a second speaker has about the quality of the initial speaker's piece of reasoning. Finally, *Diagramming Very Long Passages* helps you grapple with long passages: long passages might contain multiple reasons, extended reasoning, and objections, and it is a good idea to get a picture of the structure overall before diving into the details. All of the diagramming chapters get you to practice thinking about the structure of the piece of reasoning you are looking at.

**5.** In summary, analyzing — understood broadly as 'clarifying what the speaker is saying' — involves:

1. classifying the speech as reasoning (if it's not reasoning, you'll need other skills to deal with it, not the ones in RW),
2. analyzing it into reasons and target,
3. understanding the meaning of the propositions (and clarifying the meaning of each proposition as needed), and,
4. thinking about the structure of the inference.

The broader purpose of analyzing is to bring out all of the aspects of what the speaker is saying. Only when you have clarified what the speaker's target claim is, what her reasons are, what each proposition means, (and, when we get to Diagramming, how the speaker thinks the reasons work together to support the target) can you begin to evaluate.

When you are confident that you are dealing with an inference, with reasons and target separated, and each one with a clear meaning, and (if there are multiple reasons, or targets, or reasons for and against) the structure clear, you can go on to evaluate it.

**6.** Evaluating involves two steps. As mentioned already, one part of evaluating an inference is to think about the truth of the reasons. But there's nothing in this book about this task, because this task depends on knowing about lots and lots and lots of subjects. In other words, to decide whether or not a proposition is true involves subject-specific knowledge, and this book cannot hope to cover all of the subjects that people talk about. It would take forever: there are millions (billions?) of things that humans reason about and so there are millions of different propositions they might use as reasons. For example, if the speaker is reasoning about the role of the tank in World War I, you're going to need to verify some statements about tanks and military operations in the 1910s. the Real World Reasoning section can't be a big book of information about every subject under (and including) the sun. (That's Wikipedia.)

Thus, when the two criteria of evaluation are introduced (in [Evaluating The Reasoning](#_Evaluating_The_Reasoning)), the emphasis is on the other task, evaluating the strength of the reasoning. As far as evaluating the truth of the reasons is concerned, you can either look up the proposition on the internet, and/or get used to writing "I don't know" over and over. (Admitting ignorance is actually a good thing to practice; humans hate saying "I don't know"!)

However, it *is* important to realize that evaluating the truth of the reasons is a different task from evaluating the degree to which the reasons, if true, would make the target claim true. Humans get the two tasks – evaluating the reasons and evaluating the reasoning – easily confused: they think that if the reasons are true they must support the target claim. Not so! (Also, when they already they already believe the target, they think that the reasons must provide good support for believing it. Not so!) So, the first chapter on evaluation, *Two Criteria,* has you practise thinking about these *two* *different* criteria for a good piece of reasoning and gets you to realize that a piece of reasoning can have true reasons without having strong reasoning, or could have strong reasoning without having true reasons.

Because speakers are often bad at finding reasons that actually support their targets, they often *tell* the audience that the reasons are really great reasons for believing the target. So, there is a section called *Ignoring Confidence* *Indicators* which is about ignoring the words and phrases speakers use to tell you that their target is well-justified by the reasons. By using these words and phrases, speakers are effectively telling you that you don't need to think about whether or not the reasons on offer are good reasons for believing the target. You need to ignore these because *you* should be the boss of your brain; don't let anyone tell you to switch off your critical capacities.

**7.** Once you have mastered the difference between the truth of the reasons and the strength of the reasoning, the next step is to familiarize yourself with the patterns of reasoning that can be used to establish a target claim. If you can match the piece of reasoning to a pattern that you know is good, then you know the reasoning is good.

What's a pattern of reasoning? Here's a quick example (of a type you will see in the chapter [*Syntax & Logic*](#_Syntax_&_Logic)*):* Mammals produce milk for their young. Humans are mammals. So, humans produce milk for their young. This pattern is called "Instantiation" because humans are an instance of mammals and what's true (producing milk for their young) of the general class (mammals) is true of the instance (humans). Don't worry too much about *the name* "Instantiation" (though names make for handy psychological shortcuts); hopefully you recognize *the pattern.* There are a number of chapters in the Real World Reasoning section which will introduce to a variety of patterns – reasoning using sources, character, and motives; reasoning about options (a.k.a. decision-making); reasoning using emotions; and a variety of patterns from grammar and logic. When you know how they *should* go, it is easy to say whether the piece of speech or text you are considering is good.

*All of these patterns will be familiar to you.* You might wonder: If all of these patterns are familiar, what's the point of practicing them? Two answers: one is that speakers are lazy and sloppy and as a result often present incomplete reasons (the example of Instantiation above is actually incomplete), the other is that evaluators are lazy and sloppy and often do an incomplete job of evaluation. You need to learn about the patterns, therefore, so that you can flesh out the reasons that the speaker *should* have included and so that you can do a proper evaluation. On the one hand, if you know the patterns, you will be able to know what pattern the speaker is trying to use even if she doesn't match the pattern completely. And, if you know the patterns, you know when the reasoning is good.

**8.** When you have become familiar with all of these patterns, you will be able to recognize that the imperfect, natural-language, inferences that you find in everyday speech and writing are typically a *partial match* with a pattern you already know. Once you have identified the pattern, you can supply the missing reasons or *warrants* to the speaker's inference*.*

Adding warrants falls under the heading of "analyzing", as you are trying to be clear about the pattern of the speaker's inference. However, by adding warrants so that the passage now matches a pattern, you are making life easy for yourself when it comes to evaluating the inference: since the pattern matches completely, all you have to do is think about the truth of the reasons (including those you added as warrants).

**9.** Diagramming is also part of analysis, as diagramming attempts to show the structure of the speaker's inference. The chapters on diagramming will extend your skills of analyzing (in the form of structural indicator words, extended arguments, coping with long passages) *and* your powers of evaluation (with issues such as evaluating a pile of reasons, evaluating arguments with objections, and evaluating very long passages). [Diagramming Very Long Passages](#_Diagramming_–_Very) is the capstone chapter: you need all the preparation you can get in order to confront long passages, especially long passages from writers that aren't very careful.

**10.** Analyzing and evaluating. Analyzing and evaluating. These are the two central tasks of critical thinking.

The instructions for every exercise in the Real World Reasoning section are "Analyze. Evaluate." As you add more chapters, however, the analyzing and the evaluating become more complex.

|  |
| --- |
| Full procedure for analyzing and evaluating an inference  1. classify the speech as reasoning, 2. analyze it into reasons and target, 3. understand the meaning of the propositions (and clarifying the meaning of each proposition as needed), 4. think about (and diagram) the structure of the inference 5. match the (often partial) inference with one of the patterns you know (and add warrants to complete it) 6. evaluate by asking if the reasons are true |

## Analyzing – Part 1: Basic Analyzing

### 1 Basic Analyzing

**1.** Critical reasoning is the judgment of reason-giving. Reasons can be given either in an inference or in an explanation. Consider the following examples:

1. *Smith and Jones are planning their next meeting. Smith says:* Today is Monday the 5th. So, next Monday is the 12th.
2. *Jack says:* I closed the window because I was cold.

The sentence (a) is an inference: the reason ("Today is Monday the 5th.") is a *reason-for-believing* the target ("Next Monday is the 12th."). On the other hand, (b) is an explanation: the reason ("I [Jack] was cold.") is a *reason-which-explains* the target ("I [Jack] closed the window.").

"Reason" and "Target" are generic words that we can use for both inferences and explanations. "Target" is used for the proposition *for which reasons are given*, whether in an inference or in an explanation. In inferences, the reasons are also called "premises" and the target is also called the "conclusion". In explanations, the reasons are also called "explainers" (or "causes") and the target is also called the "explainee" (or the "effect").

**2.** Inferences and explanations involve at least two *propositions* (a.k.a. statements, claims): there is at least one proposition expressing a reason and one proposition expressing a target.

In (a), an inference, "Today is Monday the 5th." is a proposition and it expresses a reason/premise. "Next Monday is the 12th." is a proposition and it expresses the target/conclusion. In (b), an explanation, "He [Jack] was cold." is a proposition and it expresses a reason/explainer. "Jack closed the window." is a proposition and it expresses the target/explainee. Note that **a proposition is not the same as a sentence**. (a) has two sentences and involves two propositions; (b) has one sentence and involves two propositions.

**3.** In addition to their propositions, (a) and (b) involve *reason conjunctions.* The words "so" in (a) and "because" in (b) are reason conjunctions. Reason conjunctions tell us that the propositions — whether they are in one sentence or more than one — are conjoined together as a reason(s) and a target. We will also call them "flag" words or "indicator" words.

The English language contains a variety of reason conjunctions that speakers can use to conjoin reason(s) and target. Here are some of the most common:

**Since** So

**Given that** Therefore

**Because** Thus

**As a result of** As a result

**Due to** Hence

Here's why That's why

For this reason

The words associated with reason-giving, justifying belief, inferring, and explaining, can all be used a flag words. For example, in the sentence "My reason for thinking that Jack is at home is that his car is outside his house." "My reason for thinking that" indicates that a target is coming up, to be followed by a reason.

Some of these – the ones on the left – conjoin a reason and a target *in a single sentence*. For example:

Jack is getting a drink of water because he is thirsty.

In this sentence, "because" indicates that we are looking at reasons and a target; the reason is "He (Jack) is thirsty." and the target is "Jack is getting a drink of water.". In English, the propositions could also come in reverse order:

Because he is thirsty, Jack is getting a drink of water.

Notice, however, that in both versions it is the reason that follows immediately after the word "because". All of the flag words **in bold** on the left are like this: the reason follows immediately after the flag word or phrase. Here are examples using the other bolded items:

**Since** it is already nine o'clock, I will be late to class.

**Given that** you have admitted breaking the rules, you will be disqualified.

Jack got acid reflux **as a result of** eating too much at dinner.

Jill is running late **due to** her previous meeting going over time.

Each of these examples is a single sentence containing a reason, a target, and a flag phrase; and in each of these examples, the reason follows immediately after the flag word/phrase, whether that word/phrase is at the beginning of the sentence or in the middle. Notice that "eating too much at dinner" and "her previous meeting going over" are not exactly propositions.

In the abstract, the bolded items follow the following patterns:

Since <reason>, <target>.

Given that <reason>, <target>.

<target> because <reason>.

<target> as a result of <reason>.

The last conjunction in the left-hand column, "Here's why", is a bit unusual. It is listed on the left because it conjoins reason(s) and target *in a single sentence*, but it is not in bold because the *target* immediately follows the words "here's why" with the reason being given after a colon. Here is an example:

Here's why you should take a course in critical reasoning: it will help you resist the persuasive power of advertisements.

or in the abstract:

Here's why <target>: <reason>.

The flag words/phrases on the *right* are like "here's why" in that they will be followed by the target but they are different from all of the items on the left because the reason will be in a *separate* sentence. For example:

Jack is thirsty. So, he is getting a drink of water.

or in the abstract:

<reason>. So, <target>.

"So" tells you that the target is coming up and that what you just heard was a reason. But unlike the previous examples, the reason is in a separate sentence. (Note that in English, "so" has other uses besides being a reason conjunction. In the sentence "Jack is so much nicer than Henry.", "so" is not used as a reason conjunction.)

Here are examples using some of the other items on the right:

Smith scored 90 on her test. As a result, she was awarded her license.

Henry admitted stealing the computer. That's why he was fired.

Jack ate too much at dinner. For this reason, he got acid reflux.

**4.** In the real world, lots of reason-giving involves two or more people. Correspondingly, throughout *Real-World Reasoning*,many of the passages will be in the form of a dialogue. In dialogues, one speaker will ask another for reasons and these requests also serve as flag words. Here is an example:

*Smith:* We should leave early for the match tonight.

Jones: Why?

*Smith:* I want to get a good seat.

In this example, Jones's question "Why?" is a request for reasons and so indicates that what Smith just said is the target of her query and that the next thing that Smith says will (hopefully! – see *Reason Substitutes*) be one or more reasons. Jones could have asked for reasons with any number of English-language queries: "Why's that?", "For what reason?", "We should?", "No!", and so on.

**5.** Here are some more examples of reason-giving; as always, the words in italics give the context and are not part of the passage. First:

*Henry gets some bad news:* I failed my logic exam. So, I failed the logic course.

The word "So" in the second sentence connects the two propositions as reason and target. The two propositions are "I (Henry) failed the logic exam." and "I (Henry) failed the logic course.". The first one is the reason and the second one is the target.

Here is an example of a reason and target in a single sentence using "because":

Henry and Smith see Jack zooming down the street in his car. Henry says: Jack is driving at high speed because he is late for work.

Here we have two propositions, "Jack is driving at high speed." and "Jack is late for work.". The word "because" connects the two propositions; the reason immediately follows the word "because".

Here is an example using "since", which puts reason and target in a single sentence, separated by a comma:

*Smith says to Jones:* Since you want to get in shape, you should come jogging with me.

Here we have two propositions. The reason is "You (Jones) want to get in shape." and this appears immediately following the "since". The target is "You (Jones) should come jogging with me (Smith)." and this appears after the comma.

Note that in English, "since" has other uses besides being a reason conjunction. It can describe an extent of time within a proposition. The proposition "Jack has been sick since Thursday." cannot be broken into a reason and a target. "Since" must conjoin propositions if the sentence is to be broken up.

### 2 Conjunctions

**1.** So far, the examples have all had *one* reason for the target. It is possible to have more than one reason. When there are *two or more reasons*, each one might be expressed in its own sentence, or they might be joined together in a single sentence.

The most common way of conjoining two reasons is by putting the word "and" between the two reason propositions. The following example expresses two reasons in its first sentence:

*On a TV finance show:* Baby-boomers are living longer than the elderly ever have and medical care is more expensive than ever before. So, they should save more for retirement.

The first sentence gives two reason propositions: "They (baby-boomers) are living longer than the elderly every have." and "Medical care is more expensive than ever before.".

Like the word "since", the word "and" *doesn't always* conjoin reasons. Consider the following:

*Commenting on a race:* Bolt finished between Gay and Carter. So, he came second.

Because the first sentence makes use of the word "and", it is tempting to think that it contains more than one proposition and, thus, that it can be broken up. This, however, is not the case. After all, the word "and" in this sentence is not conjoining two propositions; it is conjoining the names of two runners. There is no way to break up the sentence into two propositions: the word "between" requires that the subject is between one thing *and* another thing.

When there are more than two reasons, commas might be used between the initial reasons, with "and" occurring only before the final reason. For example:

*Jill is describing a meeting from earlier in the day:* I was there, Smith was there, and Jones was there. So, everyone was present.

There are three reasons in the first sentence: "Jill was there.", "Smith was there.", and "Jones was there.".

|  |
| --- |
| Plain Conjunctions & Reason Conjunctions For our purposes, "and", "neither", "nor", "but", "however", "yet", and "although" (and others) are plain conjunctions: they join two propositions together but without making them into reason(s)-and-target.  "So", "because", and all of the others we have considered in section 1 of this chapter are reason conjunctions. The propositions joined are reason(s) and target. |

**2.** While every sentence composed of two or more propositions conjoined with "and" should be broken up, there are some sentence containing multiple propositions that should *not* be broken up.

First, propositions joined with "or" must be treated as *one* proposition. If you were to separate such sentences into two propositions you would change the meaning of the proposition. For example, "The dog ran either to the left or the right." cannot be rendered as "The dog ran to the left." and "The dog ran to the right." for the original proposition asserts only that the dog took one of the two paths, not that he ran to the left *and* to the right.

|  |
| --- |
| “Or” as a Conjunction In English, "or" in a list of options sometimes functions as a conjunction. For example, "We have tea, coffee, or juice." means "We have tea and we have coffee and we have juice."  Also, "or" can function as a conjunction when a single negation is involved. For example, "Jack did not go to school or to work" means "Jack did not go to school and Jack did not go to work". |

The same problem occurs if you attempt to split up "If …, then …" propositions. For example, "If you are late to the theatre, you won't be admitted until the second act." isn't asserting either "You are late to the theatre." or "You won't be admitted until the second act.", or both of them. Thus, "if …, then …" propositions must *not* be broken up.

(Note also that a conjunction occurring *within* any part of an 'or' sentence or an 'if-then' sentence should not be broken up. For example, "Either Jill will go first, or else Smith and Jones will go first.". The conjunction here is the second part of an "either … or else …" sentence and should be left alone.)

**3.** Now that we have seen both a reason and a target in a single sentence and multiple reasons in a single sentence, you can probably see that both of these things could happen in a single sentence. In other words, a *single* sentence can contain *multiple* reasons *and* a target. Consider the following passage:

*On a cable sports show:* Since Cal Ripken has appeared in 19 All-Star games, was a World Series champion in 1983, and has had his number retired by the Orioles, he deserves a spot in the Hall of Fame.

This is a single sentence, but it contains three reasons and a target. The first word is "since", which tells you to expect reason(s), a comma and a target. And, in fact, there are three reasons, with a comma between the first two reasons and an "and" before the third one. After the third reason, we get a comma and (finally!) the target: "Ripken deserves a spot in the Hall of Fame.".

**4.** Finally, here is an example of a reason *embedded* within a sentence that begins with a flag word ("so") indicating a *target*:

Some people have been able to give up cigarettes by getting serious about their problems and using their willpower. So, since everyone could do this, there's no excuse for anyone who wants to give up cigarettes.

The last sentence begins with "So", which indicates a target, but a "since" immediately follows it and "since" indicates a reason. The reason is "Everyone could do this.". The target is then delivered: "(So) … there is no excuse for anyone who wants to give up cigarettes.".

### 3 Marking Up Passages\*

**1.** We are going to analyze passages that contain reasons by marking them up in the following ways:

1. *put in parentheses* any words/phrases that conjoin reason(s) and target (regular conjunction words like "and" are left alone)
2. *underline* the target
3. *bracket* each proposition used to express a reason
4. *number* the propositions expressing the reasons and the target.
5. (Note that conjoining words/phrases are *not* part of the reasons or target; do not underline them and (if possible) do not bracket them.)

**2.** Here are some examples analyzed according to this four-step procedure:

#### Inferences

(Here's why) 1 you should take a course in critical reasoning: 2 [it will help you resist the persuasive power of advertisements.]

*Henry gets some bad news:* 1 [I failedmy logic exam.] (So,) 2 I failed the logic course.

*Smith says to Jones:* (Since) 1 [you wantto get in shape], 2 you should come jogging with me.

*On a TV finance show:* 1 [Baby-boomers are living longer than the elderly ever have] and 2 [medical care is more expensive than ever before]. (So,) 3 they should save more for retirement.

*Smith:* 1 We should leave early for the match tonight.

*Jones:* (Why?)

*Smith:* 2 [I want to get a good seat.]

*Commenting on a race:* 1 [Bolt finished between Gay and Carter.] (So,) he came second.

*On a cable sports show:* (Since) 1 [Cal Ripken has appeared in 19 All-Star games,] 2 [was a World Series champion in 1983,] and 3 [his number has been retired by the Orioles], 4 he deserves a spot in the Hall of Fame.

1 [Some people have been able to give up cigarettes by getting serious about their problems and using their willpower.] (So), (since) 2 [everyone could do this,] 3 there's no excuse for anyone who wants to give up cigarettes.

#### Explanations

Henry and Smith see Jack zooming down the street in his car. Henry says: 1 Jack is driving at high speed (because) 2 [he is late for work.]

*During a downpour*: 1 It started raining (because)2 [the atmospheric pressure dropped.]

**3.** Note that flag words/phrases are *not* part of the propositions they conjoin. So, do not underline them in the target and, if possible, do not bracket them within the reasons. Here is an example where a flag word is not underlined but occurs in the target:

*A politician on TV:* 1 [It takes a despicable person to politicize the death of a young child.] 2 [Smith has tried to tie young Molly's death to the President's policies.] 3 Smith is (therefore) despicable.

The word "therefore" in the target is not underlined, even though it occurs in the middle of the proposition "Smith is despicable.".

### 4 Summary

#### Procedure For Analyzing Passages (4 Steps)

1. *Put in parentheses* words/phrases that conjoin reason(s) and target.
2. *Underline* the target.
3. *Bracket* each proposition used to express a reason.
4. *Number* the propositions expressing the reasons and the target.

#### (Some) Reason Conjunctions (a.k.a. Flag or Indicator Words/Phrases)

**Bold formatting** indicates that a *reason* follows the flag word/phrase. For all others, *the target* follows.

##### Reasons and Target in Single Sentence

|  |  |
| --- | --- |
| Indicator | Structure |
| **Since** | Since <reason>, <target>. |
| **Given that** | Given that <reason>, <target>. |
| **Because** | <target> because <reason>. |
| **As a result of** | <target> as a result of <reason>. |
| **Due to** | <target>, due to <reason>. |
| Here's why | Here's why <target>: <reason>. |

##### Reasons & Targets in Separate Sentences

|  |  |
| --- | --- |
| Indicator | Structure |
| So | <reason>. So, <target>. |
| Therefore | ‘’ |
| Thus | ‘’ |
| As a result, | ‘’ |
| Hence | ‘’ |
| That's why | ‘’ |
| For this reason | ‘’ |

### 5 Exercises

## Classifying – Part 1: Basic Classifying

### 1 Introduction

**1.** Critical reasoning is the judgment of reason-giving. Not every passage involves reason-giving, even if it contains two or more propositions. Consider the following:

1. *One student speaks to another:* First, the instructor passed out the syllabus. Then he went over some basic points about reason-giving. Then he said we should call it a day.

The passage (a) is a set of propositions describing a temporal sequence of events, and so we might describe the speaker as *narrating* or *reporting*. This passage does *not* involve reasons being presented: none of the propositions provides a reason(s) for another one of the propositions.

### 2 Distinguishing 'Reasons' From 'No Reasons'\*

**1.** When confronted with a passage, you need to classify it as either employing reasons or not employing reasons, or, for short, as "Reasons" or "No Reasons".

The *primary* way of classifying passages as Reasons or No Reasons is by thinking about whether or not some of the propositions can act as reasons for another proposition (which we can call the "target").

In other words, you try to make the best sense you can of what the speaker is attempting to do in the passage and in particular if the propositions can be related to one another as reason(s) for a target. Reasons are used either as reasons-for-believing or as reasons-which-explain. If the speaker is bringing herself or the audience to a new belief or explaining something to the audience, she will give reasons. If, on the other hand, she is just reporting or narrating, none of the propositions will function as reasons for a target.

In passage (a) above, none of the propositions seem to provide reasons for another one of the propositions, and the passage is classified as No Reasons. But the following passages are all examples of reasons-for-a-target:

1. *Henry says to Bill:* The thermometer reads 78 degrees Fahrenheit. So, it is 78 degrees Fahrenheit.
2. *Jill says to Jack:* Jim has been inside all day. So, you should take him for a walk.
3. *Jill says to Jack:* Jim peed in the living-room because he was inside all day.

In (b), the first proposition ("The thermometer reads 78 degrees.") is a reason for believing the second proposition ("It is 78 degrees.").

In (c), the first proposition ("Jim was inside all day.") is a reason for accepting the second proposition ("Jack should take him for a walk.").

In (d), the second proposition ("Jim was inside all day.") is a reason which explains the first proposition ("Jim peed in the living-room.").

Passages (b) and (c) are inferences while (d) is an explanation. Inferences and explanations are discussed in greater depth later

**2.** A *second* way to tell Reasons from No Reasons is to look for the words or phrases that speakers often use to conjoin the reason(s) and the target. We can call these "flag words", as they mark the presence of reason-giving.

Here is a list of common flag words:

Since

So

Given that

Therefore

Because

Thus

As a result of

As a result

Due to

Hence

Here's why

That's why

For this reason

If you see any of these, you know reasons are involved. Note, however, that this is only a partial list. There are lots more words and phrases that will indicate that the speaker is presenting reasons. Note also that a passage might not have flag words yet still contain reasons. For example, "I am so screwed. I just failed my exam." is clearly a reason for a target: "I am so screwed." is the target and "I just failed my exam." is a reason.

The possibility that there might not be any flag words/phrases is why looking for flag words is the second way of telling that reasons are being presented. The first way of spotting reasons, just above, is the primary way.

3. Third, the context, given in italics in front of the passage, might also give you a clue: think about whether or not the context describes a place or situation where people typically give reasons, such as in a debate or on a political talk-show.

4. Fourth, and finally, the precise words used in the passage can indicate the presence of reasons. In particular, a word like "should" in the target will indicate that the speaker is trying to persuade somebody to believe or do something, which typically involves giving reasons.

**5.** There are three steps in the process of *classifying* passages:

1. Read the passage carefully.
2. *Classify* the passage as involving Reasons or No Reasons by (i) identifying propositions that are acting as reasons for a target, (ii) looking for flag words, (iii) paying attention to contextual clues, and (iv) paying attention to the precise words used.
3. *Explain* your classification in writing.

**6.** Here is a passage that has been classified in accordance with the procedure above:

*On a talk-radio sports show:* Cal Ripken has appeared in 19 All-Star games. He was a World Series champion in 1983. His number has been retired by the Orioles. For these reasons, he deserves a spot in the Hall of Fame.

Classifying: Reasons. For three reasons:

1. The words "for these reasons" is a flag-phrase, telling us that the previous sentences were reasons.

The word "deserves" in "Cal Ripken *deserves* a spot in the Hall of Fame." suggests that the speaker is trying to convince the talk show's audience by giving them reasons for accepting his recommendation. The fact that he has accomplished these feats is supposed to be sufficient reason to induct Ripken into the Hall of Fame.

1. Finally, the context – a *talk-radio* sports show – is the kind of forum where people like to make claims and (hopefully) back them up with reasons. Radio can describe sporting action, but a lot of programming is debate.

### 3 Exercises - Classifying

Classify each passage.

#### Sample:

*Jill arrives at the apartment*: Jack's car keys are on the kitchen table and there is music coming from his room. So, Jack is home.

Reasons, because "so" indicates a target and the other two propositions can serve as reasons. Jill just got home, and she is arriving at a new belief, based on the evidence.

#### Problems

1. *A horse-rider pauses to consider the view:* Whose woods these are I think I know. His house is in the village. The woods are really filling up with snow! (Based on Robert Frost's *'Stopping By Woods On A Snowy Evening'*.)
2. *Bill says to Smith:* Foundationalism is false. And if foundationalism is false then coherentism is true. Hence, coherentism is true.
3. *Jones says:* I owe Henry the fiver he loaned me last week. I have just received my pay. So, I'll pay him back the money.
4. *Smith enters an empty room and says:* This room is very stuffy. I should open a window.
5. *Henry says to Bill:* George Foreman says his is the best grill on the market. So, it is the best grill on the market.
6. *Henry is at the store, standing in front of a selection of grills:* I need a grill. Since the Foreman grill is best, I should get that one.
7. *In the New York Times:* Tony Blair has been out of power for almost seven years and remains defined in many Britons' eyes by his support for President George W. Bush and the invasion of Iraq.
8. *The engineer from the power company announces:* The power line fell because the ice that had formed on it weighed a lot.
9. *Your dad says to you:* You must be honest. Lying disrespects other people and you only end up hurting yourself.
10. *In USA Today:* Kaymer took a 5-shot lead into the final round and didn't let anyone get closer than four on a crispy course baked by sun-drenched skies, adding a 1-under par-69 to finish at 9-under 271.
11. *Henry says to Jones:* Bill won the fantasy football league at the office because Tom Brady had three touchdown passes and Gronk had six receptions on Sunday.

## Classifying – Part 2: Advanced Classifying

### 1 Justifying & Explaining

**1.** *Classifying* discusses how to classify passages as involving Reasons or No Reasons. But it is possible to be more precise in classifying passages. One important distinction is between reasons used to justify a new belief and reasons used to explain (specifically, to *causally* explain, that is, making clear the cause of some event).

Belief-justification and explanation are easily confused because English uses the word "reasons" for both reasons-for-believing and reasons-which-explain. When we justify belief, some beliefs (which we call "the reasons") justify believing another belief; when we (causally) explain, some causes (which we also call "the reasons") explain how or why the target event happened. To repeat:

Justifying belief occurs when the reason(s) act as reason(s) for believing something new. That is, the reasons (are supposed to) *justify belief* of the target. The reasons (are supposed to) convince an audience to accept a target.

Explaining occurs when the reason(s) act as reasons for how or why the target comes to be. That is, the reasons *causally explain* the target. The reasons are other states or events and by giving them as *causes* for the target event (the *effect*), they (are supposed to) increase the understanding of the audience.

The crucial difference between justifying and explaining is that justifying a target is what we do when the target hasn't yet been accepted and explaining is what we do when it has been accepted. To put it another way, the point of justification is to add to what is *believed*, while the point of explaining is to help increase *understanding* of what is already believed.

Here is an example; Henry is deciding/has decided to congratulate Jack:

|  |  |
| --- | --- |
| Justifying | Jack did well on the test. So, I will congratulate him. |
| Explaining | I congratulated Jack because he did well on the test. |

The fact that Jack did well on the test gives Henry a reason to congratulate him. And if he is asked why he congratulated Jack, he can explain by pointing to the fact that Jack did well on the test.

Here is another example:

|  |  |
| --- | --- |
| Justifying | Jim is a dog. So, he has a tail. |
| Explaining | Jim has a tail because he is a dog. |

**2.** After we have discussed (in section 2) justifying in the forms of inferring and arguing and become familiar with the words and phrases used when justifying, and (in section 3) become familiar with explaining and its vocabulary, we will return to the task of distinguishing justifying and explaining.

### 2 Inferring & Arguing

**1.** Within justifying belief, we can be even more precise. We can distinguish between doing so by oneself and presenting the justification to others. This is the difference between *inferring* and *arguing*.

In the following example, Holmes is using the fact that the dog did not bark as a reason to arrive at a new belief, that the thief was someone familiar to the dog:

The stable dog did not bark while the horse was being stolen. So, the thief was someone familiar to the dog.

Holmes realizes that the dog did not bark and this gives him a reason-for-believing the further proposition that the dog knew the thief. Since *he himself* is arriving at a new belief, Holmes is *inferring.*

Compare that to the following slightly different scenario, in which Holmes is talking to Watson:

*Holmes:* The thief was known to the dog.

*Watson:* How can you be sure?

*Holmes:* The dog didn't bark during the theft of the horse.

Here, Holmes *presents Watson* with reasons for believing the target, so that *Watson* will follow adopt the target as a new belief. Holmes is *arguing*.

One way to distinguish between inferring and arguing is that inferring is the process of *generating* an inference while arguing is the process of *presenting* an inference to another person. An *inference* is a set of propositions, consisting of the target proposition and the reasons for believing it. When a person infers, she uses the available evidence (the reasons) to arrive at a new belief (the target), thus generating an inference (the reasons and target together), and when a person argues, she presents an inference (reasons and target together) to another person who doesn't (yet) believe the target.

In this book, when we talk about people arguing, we do *not* mean that they are engaged in a heated exchange of opinions. This is an everyday understanding of *argue*: we imagine two people shouting at each other with a certain level of insistence and perhaps anger. Such exchanges, however, rarely involve reasons which justify belief of a target. Instead, speakers simplycontradict one another *without providing reasons* for believing their respective positions.Ideally, however, arguing is an attempt to convince another person of the truth of some proposition by the presentation of reasons-to-believe. (The chapter on *Reason Substitutes* describes some of the ways in which people attempt to shortcut proper reason-giving processes.)

The popular meaning of *argue* does contain some truth insofar as arguing involves non-agreement. People only present inferences to an audience when the audience does not already agree with the target. For example, if speaker A says, "Aesop Rock is the most literate rapper." and B just says, "I agree.", there is no need to present reasons to justify belief of the target. For an argument to take place, then, B must not already agree with the target.

There is a wide variety of words that indicate non-agreement. The mildest form of non-agreement is doubt. B might express doubt by saying "It is?", "Really?", or "I am not sure.". A request for reasons, such as "Why do you think that?" also expresses doubt. What we might call "denials", such as "I don't believe it.", "I don't think so.", "I think you're wrong.", or simply "No!", indicate that B not only thinks the target is dubious but in fact thinks that it is false.

All of these forms of non-agreement mean that B has another belief(s) which makes B unwilling to accept the target merely because A has said it. In such cases, he will express doubt and ask for reasons. If he is quite confident that A's position is wrong, he might even ridicule A for her belief by saying things like "You've got to be kidding.", "Rubbish!", "Bullshit!", (or even by attacking A's character, such as the rhetorical question "Are you out of your tiny mind?". See *Sources, Character, Motives*.)

**Can You Argue With Yourself?**

When you acquire new information which leads you to a new target, you are inferring. If you realize that the new target is in tension with a target you currently believe, you can be described as arguing with yourself: you are presenting yourself with reasons for believing a (new) target which you doubt (because you already have a contrary belief).

You are now faced with a decision:

- keep the old target or

- adopt the new one?

You'll probably want to go back and look at the reasons for believing the old target and see which set of reasons is stronger.

**2.** The language associated with inferring, arguing, and inferences is sometimes used by speakers and thus alerts you that reason-giving is going on.

The words *infer* and *argue* themselves might be used by a speaker and thus help you identify that a passage contains reason-giving and more specifically a justification for believing, and more specifically still, that the speaker is inferring or arguing.

Inferring is often done internally, but a person might say out loud "From <this evidence> I *infer* <the new belief>.". Since inferring is a kind of belief-justification and belief-justification is a kind of reason-giving, you can immediately give a complete classification of the passage, as "Reasons – Justifying Belief – Inferring".

Similarly, the word *argue* (as in "I would argue that …") tells you that reason-giving, justifying belief, and arguing are taking place. The target immediately follows the phrase "I will/would argue that …" and then the reason(s) are given. "I will argue that …" means "I will give reasons for believing that …".

Another word for *inference* when in the context of arguing is *argument.* This word is often used when one person is summing up what another person has said, as in, "So your argument is that <target> because <reasons>.".

In any inference, whether it is made by oneself (inferring) or is presented to an audience (arguing), the reason proposition(s) are called the *premise(s)*. The target proposition is called the *conclusion*. The premises justify belief of the conclusion. The word *conclusion* (or *conclude*) tells you that reason-giving, and more specifically justifying, is taking place. Similarly, a speaker might explicitly indicate that she is justifying a belief by saying "My premises are …" (or *justification, grounds*, or *evidence*). She might also say "The reason is …" but you have to be careful with the word *reasons* since people who are offering an explanation will also talk about reasons. The proposition the speaker is trying to get others to believe – the target, the conclusion – which can also be called the *point,* the *contention* (and also, incorrectly and super-confusingly, the *argument*!) and so she might say something like "my conclusion is justified by the reasons …" or "my position is the conclusion of the following inference...".

In any inference, whether it is made by oneself or presented to others, the premises can be said to (be thought to) *justify, support, make likely, imply, establish,* *demonstrate,* or *prove,* the conclusion, and the conclusion is said to *be supported by, be made likely by, be justified by, be implied by, follow from, be derivable from,* or *be established by* the reasons. People who infer can be described as *concluding* or *drawing the conclusion that …*.

**The Language Of Evaluation**

There are also various words which people use to talk about the *quality* of inferences, such as *valid/invalid, sound/unsound, cogent/incogent.* These usually only appear when people are evaluating an inference, and they will almost always appear alongside other reason flag words.

### 4 Explaining

**1.** When people *explain* they give an *explanation.*

An explanation makes something clear. There are various kinds of explanation. In this book we are exclusively concerned with *causal* explanations. A causal explanation is an explanation of *how* or *why* some phenomenon (a.k.a. state of affairs, event) comes to be. Phenomena can be specific or general. For example, "This grass is brown." concerns some specific patch of grass, while "Grass turns brown when deprived of sunlight." is about grass in general. Either can be causally explained.

The explanation can be of a change in the past – how the continents came to be in their current position or how the computer came to be in the basement – or of a change in the future – why the moon will go dark later tonight – or of a change that comes to be repeatedly – why most trees are bare of leaves each winter.

Causal explanations of phenomena in the material world will involve other elements of the natural world. For example, the browning of the grass is explained in terms of lack of sunlight and various properties of grass.

The world of behavior, on the other hand, will be explained in terms of belief and desire. For example, Henry removed the computer because he believed it was scheduled for repair. Since behavior takes place in the material world, however, a single event can involve both kinds of factors. Explaining how the dog got outside might require both a material factor ("Because the door was open.") and explanation in terms of desires and beliefs ("Because he wanted to run around freely.") Both together explain how the dog came to be outside.

**2.** An explanation is sought in response to curiosity. If there is no curiosity, there is no need for an explanation. If A witnesses a traffic accident and says, "I saw a crash on Laskin Road today." and neither A nor his audience is curious about how it happened, A won't need to offer an explanation.

An explanation might be triggered by any expression of curiosity, from intense phrases such as "I'm dying to know why!" or "Wow!", to straightforward requests for an explanation such as "Do you know why that happens?".

**Some More Types of Explanation**

An explanation makes something clear. Beyond this book you will encounter various other kinds of explanation besides causal explanation.

***Instructions*** or "how-to" guides make clear how to produce something or perform some activity. For example, to make clear how to produce tied laces, someone might say "To tie your laces, start by crossing one over the other …". To do the hokey-cokey, "Put your left leg in, your left leg out, …".

***Definitions****:* Defining the meaning of a word is giving an explanation of how to use that word, such as "In Irish, "ríomhaire" means "computer"." or "In English, the word "trustworthy" means "can be relied on as honest".".

***Compositional*** explanations make clear what something is by stating what it is composed of and how those parts are structured, such as "Water is comprised of hydrogen and oxygen atoms in a two-to-one ratio.".

**3.** We end this section by considering the vocabulary of explaining.

Explicit use of the word *explain* or *explanation* (as in "The explanation is simple, …") indicates that explaining, is taking place. The reasons can be said to (be thought to) *explain* or *give an account of* or *give a strong explanation for* some thing, which is *explained by* the reasons.

When giving a causal explanation, the sentences being used to explain arealso known as the *explanans* or the *explainers* or even just the *explanation*, and what is being explained is called the *explainee,* or the *explanandum* or the (target) *state of affairs* or *phenomenon. Cause* and *effect* are also commonly used in the context of causal explanations for the reasons and the target, and the word *because* is often used to join the two into a single English sentence: This happened because that happened.

### 5 Justifying & Explaining (Again)\*

**1.** Distinguishing between justification and causal explanation can be tricky. Flag words are of limited help. Consider the following:

The game is cancelled since it is raining heavily.

"Since" is a flag word which is immediately followed by a reason ("It is raining heavily.") and so we know that the speaker is presenting reasons for a target. But it's hard to say whether the reason is being offered in order to justify a belief or to explain something. In some contexts, this passage couldbe an inference, as though the speaker were saying, "Look! It's raining heavily. I guess the game is (or: will be) cancelled.". The speaker presents the fact that it is raining heavily as a reason which justifies believing that the game is cancelled. Alternatively, in other contexts this passage could be an explanation. It would be an explanation if the audience wants to know why the game has been cancelled and the speaker is presenting the fact that it is raining heavily as the cause of the cancellation.

The *context* of the passage often contains clues. Consider the following scenario:

*Jack is at the breakfast table and shows no sign of hurrying. Jill says:* You should leave now. It's almost nine a.m. and it takes three hours to get there.

In the context described by the words in italics, Jill is best construed as arguing. Jack's inaction suggests that he does not believe that he needs to leave now and so Jill provides reasons that might convince him. Notice that there are no reason flag words or phrases in this example.

The precise language used can also help distinguish between justifying or explaining. For example, the word "should" in the conclusion "You should leave now." suggest that Jill is trying to convince Jack. Words such as "ought" and "should" indicate that the speaker is trying to get the audience to believe something about which they are currently doubtful.

**2.** When deciding between justifying and explaining, the main distinguishing factor is whether or not the target is already accepted by the speaker and the audience (if there is one).

Consider the following pair of passages:

Highway repairs begin downtown today. And a bridge lift is scheduled for the middle of rush hour. I predict that traffic is going to be terrible.

and

Yeah, I know traffic is going to be terrible. It's because repairs begin downtown today. And a bridge lift is scheduled for the middle of rush hour.

The words "I predict" in the first passage suggest the conclusion is a novel belief. It's novel even to the speaker – she is inferring. The second passage starts out with the speaker saying "I know" about what is clearly the target, because of the reasons offered subsequently. In the first, therefore, the speaker is using various pieces information to arrive at a *new* belief, "Traffic is going to be terrible." is true. The second, on the other hand, is an explanation. The speaker already accepts the target and is trying to describe the causal connections between states of affairs; she is not trying to increase her (or anyone else's) store of knowledge.

In dialog, you might get a clue about what the audience believes from how they respond to the initial presentation of the target. Speakers begin arguing in response to doubt or disbelief expressed by a skeptical audience. If the speaker asserts "*Revolver* is the Beatles's best album.",the skeptic might express his doubt or disagreement by saying things such as ""It is?", "Really?", "I doubt it.", "I disagree." or "No way!", or by explicitly requesting reasons with something like "Why do you think that?" (which is different from "Yes. But why is that?" which calls for an explanation). An explanation, on the other hand, is produced in response to an expression of curiosity such as "Do you know how/why …?".

Things can change quickly, from explaining to arguing and back again. Consider the following conversation:

*Jones*: The reservoir is at a low level because of several releases to protect the downstream ecology.

*Smith*: Wait. The reservoir is low?

*Jones*: Yeah. I just walked by there this morning. You haven't been up there in a while?

*Smith*: I guess not.

*Jones*: Yeah, it's because they've been releasing a lot of water to protect the ecology lately.

Jones might initially intend his first sentence as an explanation, but since Smith does not believe the target (that the reservoir's water level is low), he will first have to give her reasons for believing *that* it is low. When challenged, Jones offers evidence from his memory: he saw the reservoir that morning. Once Smith accepts that the water level is low, Jones can restate his explanation (in the last sentence).

When thinking about whether or not the audience already believes the target, keep it in mind that a target can be accepted by the audience on the authority of the text/author *as soon as* it is uttered.

This kind of thing happens frequently. For example, if you are friends with the speaker, or you trust the speaker (on the topic being mentioned), you will often believe the target and take any reasons as explanatory. Consider the following:

*Your friend Bea is on the phone:* Kelly is driving me insane. First she told Michael that I was out when I was right there in my room, and then she ate the leftover food I was keeping for lunch today.

In this passage, you accept "Kelly is driving me insane." as soon as your friend says it. You don't need any additional convincing: if Bea says Kelly is driving her insane, that's enough for you to believe it, and Bea expects you to believe it, too. What then follows is then an explanation of how/why Kelly is driving Bea insane.

To be precise, then, we might modify the question "Does the audience already believe the target?" slightly; it might be more accurate to ask "Does the audience believe the the target *before the reasons are offered?*" If the audience accepts the target as soon as it is uttered, the reasons that follow will be explanatory, not justificatory.

However, if, for some reason, you express a doubt that Kelly is driving your friend insane (perhaps you suspect your friend Bea is being overly dramatic), Bea would have to convince you. She could even use the exact same reasons, but in this context they would serve as reasons-for-believing.

### 6 Explanatory & Non-Explanatory Reasons

The reasons used in an explanation can be used (perhaps with a little rewriting) in an inference. For example, if extremely cold weather in Europe is explained by the movement of air from Siberia, in another context the movement of air from Siberia could be used to infer or to argue that it is or will be extremely cold.

The reverse, however, is not always true: *not all inferences are based on reasons that are also explanatory*, and so not every inference could be reconfigured as an explanation of how or why the conclusion is the case. Compare the following pair of inferences:

Jack says traffic will be bad this afternoon. So, traffic will be bad this afternoon.

vs

Oh no! Highway repairs begin downtown today. And a bridge lift is scheduled for the middle of rush hour. Traffic is going to be terrible!

The reasons in the second inference could (in another context) be used to justify belief of the conclusion could be used in an explanation. Indeed, someone who accepts the target on the basis of these reasons will also have an explanation ready to offer if someone should later ask "Traffic was terrible today! I wonder why?".

This is not true of the first passage: bad traffic is not *explained* by saying "Jack said it would be bad.". The reason "Jack said that traffic would be bad." can only be used to justify the belief "Traffic will be bad."; it cannot be used to explain the bad traffic.

Belief-justification based on an understanding of how the world works is more satisfying than one which appeals to the authority or expertise of others, because we get both a justification and an explanation. Although arguments based on explanatory premises are preferred, we must often rely on other people for our beliefs, because of constraints on our time and access to evidence. But *they* (or at least someone at the beginning of the chain of testimony) should hold the belief on the basis of empirical experience.

Consider another example of inference from a source:

The IPCC, a panel of experts from various countries, has [stated](http://www.ipcc.ch/publications_and_data/ar4/syr/en/spms2.html) that human activity has an impact on climate. So, that's how it is.

In this passage, a speaker provides a reason for believing *that* human activity has an impact on climate. The reason is that an international panel believes so. The speaker provides a premise which might justify adopting the conclusion as a belief. This premise, however, does not explain *why* or *how* human activity impacts climate. It might thus be a justification, but it could not be used as an explanation. If one speaker tells another that something is so because some source says it, you are observing an argument.

Another kind of belief-justification that uses non-explanatory reasons is *wishful thinking*, that is, believing something because it makes you feel good to believe it or that you can avoid feeling bad by not believing it. Consider the following example:

It would be super-depressing to think that Obama has won a second term [as U.S. president]. So, he hasn't won.

This is not a secure way to justify a belief. More importantly for present purposes, even if this is a sufficient reason-for-believing, a reason of the type <the belief would be pleasant or unpleasant to hold> could not be used to expand our understanding of the target (assuming it were true).

The same is true of beliefs you *act upon* because doing so will bring about something beneficial or allow you to avoid something harmful. Consider the following:

If I act in accordance with the belief that Bill is the best basketball player on the team, Jack will be mad at me, which I want him to be. So, Bill is not the best.

and

If I believe that <my country/team/school/etc.> is the greatest, I'll fit in. And I want to fit in*.* So, <my country/team/school/etc.> is the greatest*.*

These reasons might convince you to act as if the belief were true, but they couldn't be used to explain why Bill is not the best player or why <my country> is the greatest.)

Another important type of non-explanatory reasons is called "inference by elimination". Elimination works by listing the available options and then eliminating all but one. The one that remains is then taken to be true. For example, someone might infer from the fact that the dog is either inside or outside and the fact that the dog is not inside that the dog is outside. Elimination of the alternatives might be a fine way to justify belief that the dog is outside but it does not tell us how or why the dog is outside.

The fact that the propositions from an explanation can be used in either an inference or argument can allow audiences to become convinced, even when the speaker thinks he is giving an explanation. Consider the following case:

Bill and Henry have just finished playing basketball.

Bill: Man, I was terrible today.

Henry: I thought you played fine.

Bill: Nah. It's because I have a lot on my mind from work.

Bill and Henry disagree about what is happening — arguing or explaining. Henry doubts Bill's initial statement, which should provoke Bill to argue (i.e. to present reasons for believing). But instead, he appears to plough ahead with his explanation. What Henry can do in this case, however, is take the reason that Bill offers asan explanation (that Bill is preoccupied by issues at work) and use it as a premise in an inference with the conclusion "Bill played terribly.". Perhaps Henry will think (to himself): "It's true that Bill has a lot on his mind from work. And whenever a person is preoccupied, his basketball performance is likely to be degraded. So, perhaps he did play poorly today (even though I didn't notice).".

### 7 Summary

#### Full Procedure For Classifying Passages

1. Read the passage carefully.
2. *Classify* the passage, by
   1. trying to find propositions acting as reasons for a target,
   2. looking for flag words
   3. paying attention to contextual clues,
   4. paying attention to the precise words used, and
   5. paying attention to what *the listener* (if there is one) believes, *classify* the passage as involving Reasons or No Reason, and (if Reasons) as Justifying Belief or as Causally Explaining, and (if Justifying Belief) as Inferring or Arguing.

If you think that the passage is not a piece of reason-giving at all (No Reasons), try to say what the speaker is doing with her words.

Here is Step 2 in the form of a decision-tree:

Reasons

Justifying Belief Causally Explaining

Inferring Arguing

No Reasons

1. Explain your classification in writing.

#### Vocabulary of Reason-Giving vs. Reasoning Conjunctions

|  |  |
| --- | --- |
| (Some) Vocabulary Associated With Reason-Giving | (Some) Reason Conjunctions (a.k.a. Flag or Indicator Words/Phrases) |
| Since | So |
| Given that | Therefore |
| Because | Thus |
| As a result of | As a result |
| Here's why | Hence |
|  | That's why |
|  | For this reason |

#### Vocabulary of Arguing vs. Explaining

|  |  |
| --- | --- |
| Vocabulary of Inferring & Arguing | Vocabulary of Explaining |
| These reasons justify/support the belief … | *These are the reasons that* explain... |
| premise | the cause(s), explainer |
| Conclusion | the effect, explainee, phenomenon |
| I infer … | She explained … |
| inference | explanation |
| I would argue … |  |

### 8 Exercises

Classify each passage.

#### Sample:

*Henry arrives at work late:* Bill is not here. He very rarely arrives late. So, he is not coming in today.

Basic Classifying:

* Reasons because of the flag word "so".

Advanced Classifying:

* Justifying Belief – Inferring, because the conclusion is something that Henry did not already believe – he just arrived at work noted Bill's absence, and drew a conclusion, based on Bill's track record, that he would not come in at all.

1. Jack is reading a popular science magazine. It says: People who rate themselves as "very happy" are less successful financially than those who rate themselves as "moderately happy". He then says to himself, "Huh! It seems that a little unhappiness is financially beneficial."
2. Bill works something out for himself: You have to be smart to understand the rules of Dungeons & Dragons. Most smart people are nerds. So, I bet most people who play D&D are nerds.
3. Henry is lamenting to his friend Bill: I can't stand it any more. I'll tell you why: I'm tired of living all alone. No one ever calls me on the phone. And my landlady tried to hit me with a mop. (Based on Lou Reed's 'I Can't Stand It', from "Lou Reed".)
4. Two teenaged friends are talking. Track what Saida says.

Saida: I can't go to the show tonight.

Jordan: Bummer.

Saida: I know! My mother wouldn't let me go out when I asked.

1. A mother is speaking to her teenage son. Analyze only the mother's words.

Mother: You have to stay in tonight.

Son: OK. But why, mom?

Mother: Because you have to do your homework.

1. An economist reacts to the latest economic news: Any time the public receives a tax rebate, consumer spending increases. The public just received a tax rebate. Therefore, consumer spending will increase.
2. Henry is writing a letter to the newspaper that he hopes will get published for everyone to read: Today's kids are all slackers. American society is doomed.
3. On the ESPN web-site: Duke beat Butler 61-59 for the national championship Monday night. Gordon Hayward's half-court, 3-point heave for the win barely missed to leave tiny Butler one cruel basket short of the Hollywood ending.
4. Two detectives. Analyze what Marple says:

Marple: Henry stole the computer.

Poirot: No!

Marple: His fingerprints were found on it.

1. On Monday, Jack receives a note that his unit ships to Iraq in two days: I was hoping to go to Henry's birthday party next weekend. But I'm shipping out on Wednesday. So, I will miss it.
2. A student is speaking to her instructor: Yes, I was late for class. It is because the battery in my mobile phone ran out.
3. Smith and Jones. Analyze what Smith says:

Smith: There is a lot of positive talk concerning parenthood.

Jones: Yup.

Smith: It's because people tend to think about the positive effects that have a child brings. And they tend to exclude the numerous negatives that it brings.

## Analyzing – Part 2: Advanced Classifying

### 1 Standard Form

**1.** Basic analyzing of passages that contain reason-giving involves the following steps:

1. *Put in parentheses* words/phrases that conjoin reason(s) and target.
2. *Underline* the target.
3. *Bracket* each proposition used to express a reason.
4. *Number* the propositions expressing the reasons and the target.

This chapter adds two more steps to our procedure for analyzing passages. The first of these, Step 5, requires that the reasons and target be written in *standard form.* Putting the reasons and target in standard form involves four things:

* (5a) putting the propositions expressing the reasons and target in a particular kind of format,
* (5b) using only propositions,
* (5c) making sure that there is only one proposition on each line, and
* (5d) writing each proposition"in full"*.*

(5a) Standard form requires that the reason(s) are written one on each line, above a dashed line and the target below that line. The context is removed. The flag words are also removed. (And if you have done *Classifying (2)*, a "J" (for "justifies believing") or an "E" (for "causally explains") can be placed to the left of the dividing line.)

Here is a simple example of a passage that has been analyzed using steps 1-4 and then (5a-d) put in standard form:

*Jack was hoping to go for a walk:* 1 [It is raining.] (So,) 2 I'll stay at home.

1) It is raining.

---------------

2) Jack will stay at home.

The propositions in the standard form get the same number as in the initial analysis. In this case, the target is spoken after the reason and so in the standard form the numbers are sequential. Had the target been spoken first and numbered as proposition (1), it would get the number (1) in the standard form, even though the target is always put below the line in standard form.

### 2 Propositional Content

**1.** The second part of putting reasoning in standard form is to make sure that (5b) there is a proposition on each line. We need to think about propositions versus other kinds of sentence.

There are many things people can do with sentences: they can describe the world, propose a plan of action, make promises, exclaim (in pain, in anger, in surprise, in dismay, in fear, and others), ask questions, and lots of other things. Here are some examples:

* 1. *Jack* s*ays to Jill:* It is a lovely day outside.
  2. *Jill says to Jack:* You should take Jim for a walk in the park.
  3. *Jones says to Smith:* Get me a beer!
  4. *Bill says to Henry:* Is Jack home from Baghdad yet?
  5. *Smith says:* Ouch!
  6. *Jill says:* If only the Lakers would win on Saturday!
  7. *Henry says to Jack:* I apologize for yelling at you.
  8. *Smith says to Jones:* With this ring I thee wed.

In (a), Jack is informing Jill about the weather outside; Jill can consider whether or not this is a true description of (this part of) the world. (a) is a *descriptive proposition*.

In (b), Jill is proposing that a certain action is good to do (in this case, that Jack should take Jim (the Great Dane) to the park); Jack can consider whether or not this is a good thing to do. (b) is a *practical proposition.*

Descriptive and practical propositions are two kinds of proposition. A proposition in general is a sentence offered ("proposed") by a speaker for consideration and adoption by an audience.

**2.** For our purposes, the words "statement", "claim" and "assertion" are all equivalent in meaning to "proposition". However, we do *not* use "sentence" as an equivalent for "proposition": a proposition is a kind of sentence, but there are other kinds of sentences besides propositions, such as (c) through (h) and lots of others.

None of (c) through (h) is a proposition; in each case, the sentence's primary purpose is not to propose a description of the world or to propose a course of action. In each case, the speaker is doing something else. Among the statements listed, (c) is a command, (d) is a question, (e) is Smith exclaiming in pain, (f) is a wish, (g) is an apology, and (h) forms part of a wedding ceremony – if all goes well, two people will become married. (c) through (h) is just a small sample of the things people can do with words.

**3.** Standard form involves propositions and not the other kinds of sentence. And, only a descriptive or practical proposition can be a target, and only a descriptive proposition can be a reason. Does this mean that you will never see any of the other types of sentence in reason-giving passages? Not quite.

In the context of reason-giving, non-propositions have an *implied propositional content*. This means that speakers will sometimes will utter a non-proposition but it is the related proposition we're really going to be interested in.

An implied proposition is the descriptive or practical version of what the speaker is doing with her words. In the following examples, at least one of the sentences in the passage is a non-proposition, but in the standard form it has been written in the form of a proposition:

* 1. *Jill is trying to get Jack to take responsibility for Jim running wild:* 1 [Jack, you let go of the leash]. 2 You are to blame for Jim's escape!

1 Jack let go of the leash.

----------------------------

1. Jack is to blame for Jim's escape.
   1. *A neighboring garden has an unwelcome visitor:* 1 [Jim is trampling on my vegetables!]. 2 Get out of my garden!

1 Jim is trampling on the neighbor's vegetables.

-----------------------------------------------------

2 Jim should get out of the neighbor's garden.

* 1. *Jones is talking Henry:* 1 [I realize now that the computer was scheduled for repair.] 2 I am sorry that I accused you of stealing it.

1 Jones realizes now that the computer was scheduled for repair.

--------------------------------------------------------------------------

2 Jones is sorry that he accused Henry of stealing the computer.

You can think of the implied propositional content as the proposition that the speaker must accept before she goes on to do something with her words (such as blame, apologize, etc):

In (a), the fact that Jack let go of the leash leads Jill to believe that he is to blame, and once she believes this, she can can go on to blame him. She does this with an exclamation "You are to blame!" but we are not interested in the exclamation but in the proposition she believes, which is that Jack is to blame.

In (b), the fact that Jim is trampling on the neighbor's vegetables gives the neighbor reason to believe that it would be good for Jim to get out of the garden, which he then orders Jim to do. We are not interested in what the neighbor says as an order but only as a practical proposition, that Jim should get out of the garden.

In (c), Jones's realization gives him a reason for believing that his accusation was mistaken, and then he goes on to apologize. We are not interested in what Jones says as an apology, only in the statement that he apologizes.

**4.** There is a kind of question that has an implied propositional content. This kind of question is called a *rhetorical question*. A rhetorical question assumes an answer to the question asked. This implied answer is a proposition, and so the question can be understood as this proposition. Consider the following version of a famous inference:

After death, there is no more perception. Pain is only painful when it is perceived. So, why fear death?

The third sentence is a question, but it is a *rhetorical* question. The speaker thinks that the answer to this question is obvious and the speaker wants the audience to think of that answer, rather than the question itself. In this case, the implied answer to the question is something like "There is no reason to fear death.". We analyze as follows:

1 [After death, there is no more perception.] 2 [Pain is only painful when it is perceived.] (So), 3 why fear death?

1 After death, there is no more perception.

2 Pain is only painful when it is perceived.

------------------------------------------------

3 There is no reason to fear death.

The conclusion of the argument just above might alternatively have been presented in the form of a command, "Do not fear death!", which could be understood as the practical proposition "You should not fear death.", like example (b), above.

**5.** Speakers will sometimes put practical propositions in the form of a question. For example, "Can you please pass the salt?" in some contexts is in fact a request to pass the salt, which includes the practical proposition "You should pass the salt." and "What about coming to the game with me this weekend?" includes the practical proposition "You should come to the game with me this weekend.". A question is perceived as being a polite way of suggesting to someone that they do something.

**6.** Finally, a reason or target might not expressed in the form of a proposition but rather as a *verbal noun or a noun phrase*. Compare:

Jack's serving in the Army is a result of his admiration for his father.

with

Jack *is* serving in the Army because he *admires* his father.

The first involves a verbal noun ("Jack's serving in the Army") and a noun phrase "his admiration for his father"); the second turns each of these into a freestanding propositions ("Jack is serving in the Army." and "Jack admires his father."). *As part of Analysis Step 5, we turn such phrasings into propositions.* The first version would be analyzed as follows:

1. Jack's serving in the Army (is a result of) 2 [his admiration for his father.]

2 Jack admires his father.

----------------------------

1 Jack is serving in the Army.

### 3 One Reason Per Line

**1.** Not only must each reason and target be in the form of a proposition but (5c) there must only be one proposition on each line. As in *Analyzing 1*, we will break up any single English sentence that contains multiple pieces of information and express each reason in a separate proposition.

Separating reasons from each other, and separating reasons from target, are as much as we will do in this chapter on the *structure.* The chapters on *Diagramming* are about analyzing the passage for structure.

For now, the passages you see will only have multiple *reasons* in a single sentence. In *Diagramming – Extended*, however, there will be examples with multiple *targets* in a single sentence.

Analyzing 1 noted that conjunctions, such as "and" and "but", or commas, can be used to put multiple reasons in a single sentence. Here is an example:

*Jill is describing a meeting from earlier in the day:* Smith, Jones, and I were there. So, there were at least three students there.

The first sentence should be broken into three. Rewriting propositions (1) and (2) in full in the standard form helps make sense of the brackets around "Smith" and "Jones" and "I were there" in the initial analysis:

*Jill is describing a meeting from earlier in the day:* 1 [Smith], 2 [Jones] and 3 [I were there]. (So,) 4 there were at least three students there.

1 Smith was at the meeting.

2 Jones was at the meeting.

3 Jill was at the meeting.

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4 There were at least three students at the meeting.

We might like to be more specific about "the meeting" – which meeting is she referring to? – but we can't be more specific, unfortunately.

Similarly, the sentence "Jack went to the park with Jim, his leash, a tennis ball, and some treats." would be broken down into four simple propositions: "Jack went to the park with Jim." "Jack went to the park with Jim's leash.", "Jack went to the park with a tennis ball." and "Jack went to the park with some treats.".

**2.** Another way in which multiple pieces of information can be given in a single English sentence is by using words (*relative pronouns*) such as "that", "which", and "who". Consider the sentence "Jack, who is home on leave from the war, is taking Jim for a walk.". This sentence contains two propositions and should be broken up into "Jack is home on leave from the war." and "Jack is taking Jim for a walk.".

### 4 Indexicals

**1.** The sentences that appear in the passage are not always complete. (5d) states that when we put the passage in standard form, each proposition must be written in full (or at least, as fully as possible.) In this section, we consider some simple ways in which propositions can be rewritten in order to make the more complete. A simple test for completeness is this: for each proposition, imagine walking into a room full of strangers and uttering the proposition. Will the people in the room know (as much as is possible) what the proposition means? If not, rewrite the proposition (if possible) to make it more meaningful.

**2.** Propositions might need to be rewritten by making explicit pronouns and other indexicals (pointers). Consider the following version of the willpower inference:

*A letter to the editor:* 1 [Some people have been able to give up cigarettes by using their willpower.] 2 [Everyone can draw on his or her own willpower.] (That's why) 3 anyone who wants to can do it.

The word "it" at the end of the target (3) refer to the action of giving up cigarettes. The speaker can abbreviate the full proposition – "Anybody who wants to give up cigarettes can give up cigarettes." – because she is not talking about giving anything else up. Since she must be referring to giving up cigarettes, it is unnecessary to repeat the same thought twice.

But when we write the reasons and target in standard form, we write each proposition in full. The willpower inference is thus analyzed as follows:

*A letter to the editor:* 1 [Some people have been able to give up cigarettes by using their willpower.] 2 [Everyone can draw on his or her own willpower.] (That's why) 3 it's possible for anyone who wants to do it.

1 Some people have been able to give up cigarettes by using their willpower.

2 Everyone can draw on his or her own willpower.

------------------------------------------------

3 It's possible for anyone who wants to give up cigarettes to give up cigarettes.

In the standard form, the target has been written in full.

There are many ways in which speakers will avoid repeating themselves. Most commonly, look out for sentences containing *pronouns* (such as "I", "you", "they"), *demonstrative adjectives* (such as "this", "those"), or adverbs like "so" and "thusly".

Although it is natural to abbreviate and avoid repetition, when you analyze the passage into separate propositions you will *provide each proposition in full*. We do this because in order to evaluate reason-giving we must know what the propositions say and it is good to be careful: sometimes something important is lost or obscured when propositions are abbreviated.

Making indexicals explicit is only the most basic way of giving more meaning to propositions. There are various other ways in which the meaning of a proposition can be improved – see the separate chapter called *Problems with Meaning*.

### 5 Things To Omit\*

**1.** A passage might contain extra words or sentences that do not play a part in the reason-giving and which need not be included in an analysis of a passage. This section alerts you to various items you can omit in your analysis. As a result, the job of identifying the reasons and the target might be more accurately described as *extracting* the reasons and the target from the passage. Analysis Step 6 is:

6. Explain any omissions.

**2.** If a proposition is repeated in a passage, it gets the same proposition number in both places and is bracketed or underlined in both places but in the standard form it is only included once.

The target in particular often appears more than once, especially when people are arguing, for the sake of emphasis; the additional appearance does not add a new proposition. This is also true for reasons — when a speaker repeats a reason, she does not add any new information and you should use the same number for both appearances.

Consider the following example, which has been analyzed in accordance with all six steps:

*A human resources director at Acme Inc. is arguing with the chief executive:* 1 We should have an affirmative action policy. (Here's why.) 2 [Research has confirmed that employers do not review black job applications as thoroughly as applications from whites.] 3 [This leads black people to invest less in education and training,] 4 [which only reinforces the prejudice of employers.] 4 [Affirmative action counteracts this vicious cycle by acting as an incentive for African-Americans to invest in education.] (So), 1 we should have an affirmative action policy.

2. Research has confirmed that employers do not review black job applications as thoroughly as applications from whites.

3. Not reviewing black job applications as thoroughly as applications from whites leads black people to invest less in education and training.

4. Black people investing less in education and training only reinforces the prejudice of employers.

5. Affirmative action counteracts the vicious cycle described in 2, 3, and 4 by acting as an incentive for African-Americans to invest in education.

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1. Acme Inc. should have an affirmative action policy.

In this example, the (single) target appears twice, at the opening of the inference and at the end, but it is given the same number (1) in the initial analysis and is listed only once in the standard form.

**3.** Words or phrases which comment on the quality of the strength of the support that premises give to the conclusion, or comment on how good the explanation is, are omitted. Consider the following passage:

*Henry is out hiking and sees a cottage in the distance:* There's smoke coming from that chimney, and there would be smoke coming from that chimney if there were a fire in a fireplace in that house. Thus, in all probability, there is a fire in a fireplace in that house.

The phrase "in all probability" is the speaker's comment on the strength of the support that the premises give the conclusion. Such comments are not included in our analysis of the target (though they can be bracketed as flag words) and we add an explanation of the omission:

*Henry is out hiking and sees a cottage in the distance:* 1 [There's smoke coming from that chimney,] and 2 [there would be smoke coming from that chimney if there were a fire in a fireplace in that house.] (Thus) in all probability, 3 there is a fire in a fireplace in that house.

1 There's smoke coming from the chimney the speaker sees.

2 There would be smoke coming from the chimney the speaker sees if there were a fire in a fireplace in the house.

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3 There is a fire in a fireplace in the house the speaker sees.

"In all probability" was omitted because it is a confidence indicator.

Speakers usually add words of confidence, though they could add words of uncertainty. For example, if the target is expressed as "Jack must be at the station." you should rewrite as "Jack is at the station.". Similarly, if the target is expressed as "So these are probably Al's boots." you would omit the "probably".

#### The Problem Of Saying

Sometimes (but not all the time), words that describe the speaker's attitude to what she is saying can be left out.

Speakers will sometimes tell the audience that they personally think or believe what they are saying, in order to emphasize what they are saying. They might even say that they *strongly* believe what they are saying and raise their voices and pound the table. Consider the following:

*Smith is going to play poker with Jack, but Jones has some reservations. Jones says:* Well, I say that Jack is a low-down cheat. So, he will cheat when you play poker with him this weekend.

The fact that *Jones says that* Jack is a low-down cheat is not really the reason for thinking that Jack will cheat this weekend; rather, what's important, as a reason supporting the conclusion, is that *Jack is* a low-down cheat. You can analyze *without including "I say that"*, as follows:

*Smith is going to play poker with Jack, but Jones has some reservations. She says:* I say that 1 [Jack is a low-down cheat.] (So,) 2 he will cheat when you play poker with him this weekend.

1 Jack is a low-down cheat.

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2 Jack will cheat when Smith plays poker with him this weekend.

Similarly, when someone says "I order you to stop playing video games." (rather than simply "Stop playing video games.") we are probably meant to understand this sentence as an order, rather than as a proposition about the person's ordering.

Sometimes, however, the mental attitude *is* relevant. Consider the following explanation:

*Henry is filling Bill in on who was invited and who wasn't:* Jill believes that Jack is a low-down cheat. That's why she did not invite him to play poker this weekend.

In this example, the fact that *Jill believes* something about Jack *is* relevant to the target, since the target is about another mental "event" of Jill's – her decision not to invite Jack. Analyze as follows:

*Henry is filling Bill in on who was invited and who wasn't:* 1 [Jill believes that Jack is a low-down cheat.] (That's why) 2 she did not invite him to play poker this weekend.

1 Jill believes that Jack is a low-down cheat.

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2 Jill did not invite Jack to play poker this weekend.

This issue of whether the mental attitude is important is sometimes called "the problem of saying" even though the problem occurs not just with saying but with believing or anything similar.

**4.** Another source of extraneous verbiage is that speakers might wander off and insert a tangent or parenthetical remark. Consider the following argument:

Potatoes are vegetables. They're my favorite vegetable, in fact. And vegetables are good for you. So, potatoes are good for you.

The fact that potatoes are the speaker's favorite vegetable will be immediately thought to be irrelevant to the support for the conclusion given by the other premises. *If you are confident in this judgment*, you can analyze as follows:

1 [Potatoes are vegetables.] They're my favorite vegetable, in fact. And 2 [vegetables are good for you.] (So,) 3 potatoes are good for you.

If you are not confident, analyze as follows:

1 [Potatoes are vegetables.] 2 [They're my favorite vegetable], in fact. And 3 [vegetables are good for you.] (So,) 4 potatoes are good for you.

Here is another example, this time in a dialogue:

Al, a fireman, has been killed in a fire.

Henry: Although the body is badly burned, I am sure this is the body of my friend Al.

Bill: How do you know?

Henry: These are the boots of his father, which his father gave to him after he stopped working in the coal mines.

Bill: But anyone could have boots like that.

Henry: No. These have a quite distinctive pattern on the sides.

There is clearly reason-giving here: Bill asks for reasons to justify belief of the target claim that the body is Al's body. But what are the reasons? The reason for thinking that the body is Al's is that the boots are so distinctive that they could only be Al's. However, the information that the boots previously belonged to Al's father, who worked as a coal miner, seems irrelevant. If you are confident in this judgment, the relevant parts would simply be: Al wore boots with a distinctive pattern on the sides. This body has boots with that distinctive pattern on the sides. So, this is the body of Al.

**5.** Overall, be cautious when thinking about excluding words or propositions from your analysis. You can discard information only when you are confident that the information is not needed in order to support the conclusion or explain the explainee. When reason-giving is complicated, it can be difficult to tell how, or whether, a proposition is involved. In these cases, it is usually a good practice to include all of the sentences in the passage in your analysis, even though it might turn out that they are unneeded.

Notice that we have strayed into the territory of evaluation, rather than analysis. To take the dialogue about identifying Al by his boots, above, as an example, the reason you might throw out the information that the boots belonged to Al's father is that you are already thinking about how boots might be used to identify a body, and have thus already started thinking about whether the target is worth believing on the basis of one of the reasons.

#### Obviously Bad Reasons

It is possible that a set of propositions with no apparent relation between reasons and target should be understood as reason-giving, if flag words or the context demand it. For example, imagine someone says:

Stocks are up this morning. And so, the Yankees will beat the Red Sox in this afternoon's game.

The flag word "so" indicates a target and that the speaker is reason-giving and that he thinks there is some connection between the first proposition ("Stocks are up this morning.") and the second ("The Yankees will beat the Red Sox in this afternoon's game."), though the mind struggles to understand how "Stocks are up this morning." in any way justifies or explains the proposition "The Yankees will beat the Red Sox in this afternoon's game.". It is possible that the speaker does not understand how to use the word "so". It is also possible, on the other hand, that the speaker sees some connection between the two that the audience does not, and so you stand to learn something from the speaker. You might thus err on the side of caution and take the speaker as being sincere when he uses "so" and treat what he says as reason-giving.

(See also the section on *Irrelevance/Red Herring* in *Reason Substitutes*.)

### 6 Summary

#### Procedure For Analyzing Passages (6 steps)

*Analyzing* (if you think the passage involves reason-giving)

1. *Put in parentheses* words/phrases that conjoin reason(s) and target.
2. *Underline* the target.
3. *Bracket* each proposition used to express a reason.
   1. (Note that flag words and phrases are *not* part of the reasons or target; do not underline them and (if possible) do not bracket them.)
4. *Number* the propositions expressing the reasons and the target.
5. *Write* the reasons and target in *standard form.* 
   1. Put the reasons above a line, the target below,
   2. Each reason and the target is a proposition,
   3. Only one proposition on each line (break up sentences with conjunctions and relative pronouns such as "who" and "which"),
   4. Write each propositionin full*.*
6. *Explain* any omissions (such as repeated propositions; tangential remarks; phrases expressing the speaker's confidence)

### 7 Exercises

## Reasoning Substitutes

### 1 Introduction

**1.** For many people (or indeed, for all of us a lot of the time) even trivial beliefs and explanations are not a trivial matter because being wrong or ignorant *about anything at all* threatens our sense of self, and protecting our self-esteem is more important to us than finding out the truth or learning from others. It's hard to admit one's shortcomings to oneself; it's even harder to do so to others or in front of others.

We like to be accepted by others. So, when we are challenged to give reasons for our claims, we feel attacked, and we might refuse to give reasons. Or, because having reasons makes us look believable and competent, we might pretend to have reasons for our beliefs. Similarly, we generally like to be the person doing the explaining rather than the person asking for the explanation. The person who can provide an explanation appears knowledgeable and wise, while the person who cannot appears ignorant and useless.

To put it positively, arguing with someone (in a meaningful way) requires open-mindedness and a mutual willingness to be wrong; learning from another person's explanation likewise requires an admission of ignorance and a willingness to learn.

But often the reason-giving process is short-circuited in various ways. In this chapter, we examine the ways in which people avoid the task of providing reasons. These avoidance strategies can collectively be called *reason substitutes*. And they can be (crudely) divided into two types: the speaker refuses to give or listen to reasons and the speaker pretends to give reasons. Or, *avoiding* giving reasons and *appearing* to have reasons. As a critical reasoner, you should try *not* to use any of these strategies for avoiding reason-giving and instead be willing to give and listen to reasons.

Both giving and listening to reasons are hard. Part of the problem humans have with reasoning is that it is more complicated for a person to remember the reasons for her beliefs than to (simply) hold on to the belief, so that when the time comes to justify a belief or give an explanation to others, she does not remember the reasons why she holds the belief. And even if she does vaguely remember the reasons, it can be difficult work to produce them in an organized fashion.

Listeners have a hard time, too. Listening to and tracking reasons and their relationship to the target is hard work. The "sound like..." strategies that follow are often effective because it is just as difficult for listeners as it is for speakers to keep track of and remember the reasons, never mind evaluate them and their connection to the target, but they want to appear to be following the speaker. If the speaker is talking confidently and talking at length, the listener might worry that perhaps *he* is the one who is incompetent.

The other chapters in the Real World Reasoning section will make you familiar with some of the most common ways in which people give reasons for beliefs (and I&S discusses reasons as explanations) so that you can become better at both giving reasons and following other people's reasons.

### 2 Refusals

#### 1. Simple Refusal.

Sometimes, speakers will be frank about their unwillingness to offer reasons or an audience will frankly refuse to listen to them.

A speaker will simply express a belief and, when asked for reasons, indicate that, for her, the belief does not require reasons. She might say "That's just how I feel about it." or "I don't have/need a reason for it; it just is.".

Similarly, the audience might cut off a speaker's attempt to give reasons by saying "Don't even talk to me." or "Nothing you can say will shake my confidence.".

If a speaker is being asked repeatedly for reasons (because the first reasons she offered were unsatisfactory), then she might leave the conversation, saying something like "I'm done trying to convince you." or even "It's too bad you aren't really listening.". Likewise, if a person's belief is being subjected to criticism, she might simply stop listening, saying something like "You can say what you like, but it won't change my mind.". Constant, unwavering conviction is thought by some people to be a valuable trait. It is better, in fact, to proportion the strength of one's convictions to the strength of the evidence.

#### 2. Taboo. (The Subject Is Too Sensitive To Discuss).

One specific way to refuse to talk about a subject is to claim that it is taboo. A subject is taboo when it is *too sensitive* to be discussed. This often happens with topics that have a strong emotional charge, such as issues of patriotism, religion, or sex, and those trying to discuss these subjects (and especially those who ask for reasons for current practices) might be cast as traitors or heretics or perverts. Speakers who think the subject is taboo might react with horror or disgust at any expression of doubt, saying something like "How can you not believe that <the target>?" or "We simply don't talk about that.". (Notice that a taboo is *socially* grounded whereas refusal is personal.) The goal of the speaker is to avoid reason-giving by making it seem that even talking about the topic is disrupting a social convention and thus threatening the enquirer with fear of ostracism from the community. (See the section on using emotions in *Emotional Reasons*.)

#### 3. The Reasons Are Obvious.

Another type of refusal (though one which starts to move us towards "sound like..." strategies) is to assert that there is no need to produce the reasons because the claim is obviously true and the skeptic should already hold the belief and have access to the reasons. This attitude is exhibited by use of phrases such as "It's obvious that …".

A stronger version of this strategy is to add ridicule, by using abusive phrases which attack the person seeking reasons, such as "Only a fool could fail to know that/why …" Such phrases are often accompanied by assertive body language and a strong tone of voice.

Here is an example:

1 LeBron is the best player in basketball. 2 [Even my six-year-old cousin knows that.]

2 Even my six-year-old cousin knows that LeBron is the best player in basketball.

------------------------------------------------------------

1 LeBron is the best player in basketball.

This strategy is effective in practice. The speaker is saying "There *are* reasons." but then declines to give them and suggests that the audience should be able to provide them for himself, or is stupid for not already knowing what the reasons are.

The speaker can also use social pressure by saying something like "Everyone knows …" or "We have always believed …". Implicit in these phrases are a threat that the audience will be at odds with a belief that is widespread in the community or with one which has been held for a long time, or both. (Again, see also the chapter on *Emotional Reasons*.)

#### 4. I Can't Give The Reasons Right Now.

Sometimes a speaker will beg off from giving the reasons, by saying, for example, "There are too many reasons to enumerate …" or "It's too complicated to explain …" or even simply "Unfortunately, I need to get going.".

As with 'the reasons are obvious', this strategy suggests to the audience that *there are reasons* but that some practical consideration prevents the speaker from giving them. This strategy is sometimes combined with verbal abuse and social pressure, though it doesn't have to be – a speaker could sound apologetic: "I'm really sorry that we don't have the time just now to go into this in depth …".

(These next two are perhaps not refusals, but they are included in this section because they both involve thinking that someone else to do the reason-giving.)

#### 5. Shifting the Burden of Proof. (*You* Should Convince Me Otherwise).

Shifting the burden of proof occurs when a person putting forward a proposition insists that the skeptic should provide reason(s) against it, and refuses to offer any reason(s) for the initial proposition.

*Jill:* 1 We should go to Ireland for our summer holiday this year.

*Jack:* (Oh yeah? Why's that?)

*Jill:* Well, 2 [why shouldn't we?]

Perhaps because she cannot produce reasons of her own, Jill claims that it's Jack's job to convince her that they should *not* go to Ireland. In fact, the responsibility lies with her, since she is the one putting forward the new proposition (that they should holiday in Ireland) and there is nothing else in the context that would absolve her from having to give reasons.

In a court of law, the burden of proof lies with the prosecution. The defendant is presumed to be innocent until the case for guilt is made and accepted.

#### 6. Appeal to Ignorance.

Another strategy is to argue that it is okay to believe some proposition because there is no proof of its *falsity*. For example: "No one has yet proven that cigarettes cause lung cancer. So, they do not cause lung cancer.". In other words, the speaker thinks that an *absence* of evidence *against* her believe means that she doesn't need reasons *for* her belief.

Typically, this kind of reasoning is fallaciously used in order to support a belief that the speaker is in some way invested in. It thus often involves wishful thinking and leads to a more serious kind of faulty thinking: raising or lowering the standard for good reasons depending on how the belief (or rejection of the belief) will affect the speaker.

### 3 Sound Like You're Giving Reasons\*

**1.** A second general strategy is for the speaker to *sound as though* she is giving reasons, even as no good reason is offered. The strategies in this section differ from those in the previous section in that the speaker does offer some kind of reason for the target, but the reason is defective in some way. (There's no firm dividing line between the various types of refusal in the previous and the types of sounds-like-reasons in this section.)

#### 2. Imprecisely Point at Reasons.

One strategy is to *point vaguely at* reasons, especially sources, rather than giving the reason itself. Consider the following passage, which has been analyzed according to *Analyzing (1)* and *Analyzing (2)*:

1 People cannot help but try to get an advantage over one another. 2 [This comes from our evolutionary background] and 3 [the competition for mates.]

2 Our evolutionary background is a cause of people having to get an advantage over one another.

3 The competition for mates is a cause of people having to get an advantage over one another.

---------------------------------------------------------------------

1 People cannot help but try to get an advantage over one another.

One problem you might have with this argument or explanation is that (2) refers loosely to "our evolutionary background". But this could mean any number of things. It might simply be what is mentioned explicitly in (3) (competition for mates) or to something else. It would be appropriate to wonder what, precisely, is meant.

Pointing *to a source* is vague if the details of where to find the source making the claim are omitted. For example:

1 [Professor Krugman says that the although the economy has been recovering, it will enter a second or "double-dip" recession.] (So), 2 that's how it will be.

It is not clear how we would go about verifying whether the first proposition is true or false. It would be better if the speaker gave a precise reference, say to Krugman's op-ed in the *New York Times*, November 10, 2009, page 14.

Even more imprecise would be an appeal to an unnamed source, as in phrases such as "experts say" or "everybody knows". Often the speaker does not, in fact, have any specific source in mind, but rather feels so sure of her belief that she assumes that other people must agree with her. Imprecision is a problem discussed at length in [*Problems With Meaning*](#_Problems_With_Meaning)*.*

#### 3. Irrelevance.

Another strategy for *appearing* to be responding to a request for reasons but not actually doing so is to give "reasons" that are irrelevant to the target. Consider the following example.

*Jack:* We should get get another dog.

Jill: Why?

*Jack:* Because if we get a cat it will have to go to the toilet inside the apartment. And cats are often not very social.

Jack has said *something* substantive in response to the request for reasons, but his reasons are all about why he doesn't want to get a cat. But the target he's supposed to be supporting is about why they should get a dog. His reasons are irrelevant.

This strategy is sometimes called ***red herring*.** A red herring is a salted or smoked herring, and using "red herring" as the name of this anti-reasoning strategy might come from a story of dragging a red herring across a trail to distract hounds who were chasing a hare.

#### 4. Repeat The Target As A Reason.

As a speaker, one strategy for appearing to give reasons without actually giving reasons is to repeat one's target in slightly different words. Consider the following example:

*Henry*: 1 LeBron is the best player in basketball.

*Bill*: Oh yeah? Why's that?

*Henry*: 2 [There's simply no one out there who is as good.]

2 There's simply no one out there who is as good.

--------------------------------------------------------

1 LeBron is the best player in basketball.

Although Henry has said *something* in response to a request for reasons, and so appears to co-operating in the reason-giving process, Henry hasn't really given a reason to support his claim. He has said, in effect:

2 LeBron is the best player.

------------------------------

1 LeBron is the best player.

This argument will not be convincing. Henry has not added any reasons for believing the target. Thus, since Bill does not already believe the target (LeBron is the best) the so-called reason won't help convince him of it.

Speakers sometimes repeat the target as a reason but make it stronger or more outrageous. This is just as futile as repeating the target: if the listener is skeptical about the target, he will be even more skeptical about the reason. Consider the following version of the argument about LeBron:

*Henry*: 1 LeBron is the best player in basketball.

*Bill*: Oh yeah? Why's that?

*Henry*: 2 [There's simply no one out there who comes close.]

Henry has said, in effect:

2 LeBron is by far the best player.

--------------------------------------

1 LeBron is the best player.

Well, yes: if he's by far the best player, then he is the best player. But if the goal is to convince Bill that LeBron is the best player, saying that he's by far the best player won't help. If Henry is going to convince Bill, the reasons Henry offers should be reasons Bill will (be more likely to) accept.

For another example, see the advertisement from the movie *Idiocracy.*

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| --- |
|  |
| Figure 1 Brawndo Branding from Idiocracy[[4]](#footnote-5) |

#### Repeating the Target

This is a very basic version of a fallacy called *begging the question* or *circular reasoning.* Begging the question further allows that the target, or some aspect of it, is *assumed* by the reasons; the target (or something close to it) doesn't have to be *explicitly* used as a reason.

#### The Power of Flag Words!

Many of these strategies are often successful because just *sounding* as though one is giving reasons is often enough to satisfy the audience, even though the reasons are not very good at all.

These strategies works better when accompanied by strong gestures and tone, whether these are to reassure the audience ("I'll be happy to give you my reasons …") or to belittle the audience ("*Of course* I have reasons. They are …").

There is some evidence that simply using flag words or phrases convinces audiences. In an experiment conducted by Ellen Langer and colleagues, people standing in a queue for a photocopier were approached by someone hoping to join the queue in front of them. The person joining the queue said one of the following three things:

1. Excuse me, I have five pages. May I use the Xerox machine?
2. Excuse me, I have five pages. May I use the Xerox machine because I have to make some copies?
3. Excuse me, I have five pages. May I use the Xerox machine because I am in a rush?

Remarkably, the rates of acceptance were almost the same for B (93%) and C (94%), even though B does not offer a good reason for jumping the queue. Option B 'repeats the target': someone who wants to join the queue for the photocopier is obviously someone with copies to make. The argument thus begs the question. It is thought that word "because" was somehow sufficient to persuade the people already in line — option A worked (only) 60% of the time.

Thus, it seems that simply using flag words or phrasesmight be enough to satisfy an audience. When a speaker begins a sentence with a phrase such as "Let me tell you the reasons …" or "My reasons are these: …", she makes clear to the audience that the reasons are on the way. But speakers might then go on *not* to provide a reason, or provide information that is irrelevant.

Why might the use of flag words and phrases help convince an audience? The emotion being conveyed is that the speaker is responsible and trustworthy. The listener wants to know that he is not wasting his time. If he is willing to listen, he feels better when he can be sure that there are reasons being offered. Indeed, taking the time to ask for reasons and then spending time listening to whatever the speaker says take up the audience's time and energy. In order to feel feeling that she has wasted her time, the audience might be willing to let the speaker get away without giving reasons or with very poor reasons. In other words, audiences might default to the easier task of making sure that the speaker *sounds as though* she is offering a reason. A speaker, accordingly, might take advantage of this and simply create the impression of giving reasons, which can be done by using flag words and then a proposition. The longer and more complicated this "reason" is, the more convinced the (typical) audience might be that the speaker really does have a reason. (Politicians are experts at this.) As a critical audience, however, you should try to get over the fear that you will sound stupid if you ask *and re-ask* for reasons.

### 4 Summary/Types of Reason Substitute

#### Refusal

1. Simple Refusal
2. Taboo (Too sensitive to talk about)
3. The Reasons Are Obvious
4. I Can't Give the Reasons Right Now
5. Shift the Burden Of Proof (You should convinceme otherwise!)
6. Appeal to Ignorance (No one has proved the opposite …)

#### Sound Like You're Giving Reasons

1. Imprecisely Point At Reasons (especially sources)
2. Irrelevance (a.k.a. Red Herring)
3. Repeat the Target As A Reason (perhaps with a slight variation in wording)

### 5 Exercises

## Problems with Meaning

### 1 Problems with Meaning

**1.** The 6-step process of analysis requires that in standard form the reasons and the target are each a proposition and each written in full (as much as possible). In other words, the process of putting reasons and target into standard form requires that each proposition be as meaningful as possible.

Moreover, one of the criteria of evaluation is that the reasons must be true (or at least, the evaluator must be willing to accept them). But in order to judge whether or not a reason is true, it must first have a clear meaning.

In short, the meaning of the propositions involved must be clear to the person doing the analyzing and evaluating. Different propositions are clear to different people. For example, a piece of technical jargon might be clear to an expert in that subject but might not be clear to a beginner. Thus, the basic question to ask about every proposition you read is, "Do I know what this means?"

The meaning of a *descriptive* proposition (one which describes how things are, such as "The sky is blue.") is clear when you know what it would take for the proposition to be true or false; the meaning of a *practical* proposition (one which proposes what someone should do, such as "I should wear the blue sweater.") is clear if you know exactly what is being proposed.

Knowing what a proposition means is not the same as knowing whether it is true or false. You might not know whether it is true or false that "It is raining in Baltimore right now." or that "Khartoum is the capital of Sudan." but even so, you know what those propositions mean, that is, you know what would make each of them believable.

**2.** The first difficulty in knowing the meaning of a proposition is not recognizing a word it contains at all. When you do the exercises, if you do not recognize a word, don't simply ignore it. Instead, try to work out its meaning from the context or, better, look it up in a dictionary.

Words can also have insufficient meaning, which means that, even if you associate the word with some other things, you do not know what its definition is. For example, you might have heard the word "peristalsis" and you might know that it is a medical word, but in many contexts this will not be enough to understand the proposition.

**3.** There is a variety of problems with meaning, which, speaking generally, all stem from the fact that language is sometimes *imprecise* (or: *vague*).The rest of this chapter surveys some specific categories of imprecision.

### 2 Metaphor And Simile

**1.** In an effort to be entertaining at the same time as being informative, humans often use colorful language when speaking. Unfortunately, as far as knowing the meaning of the proposition is concerned, the color is a distraction.

Here is an example:

*Henry makes a prediction:* Jack won't let let Jim off the leash to chase squirrels because Jack has a heart of stone.

The passage is initially analyzed as follows:

*Henry makes a prediction:* 1 Jack won't let Jim off the leash to chase squirrels (because) 2 [Jack has a heart of stone.]

When we put the passage in standard form, we want each of the propositions to be freestanding and to have a clear meaning. In this passage, the meaning of proposition 1 is clear but proposition (2) is not, because Jack cannot literally have a heart of stone and the speaker does not intend to say that he does. Rather, the speaker is saying something about Jack in a metaphorical (and dramatic) way.

We need to *rewrite* the reason with a plausible translation. While it is easy to spot metaphors and similes, it can be difficult to say exactly what they mean in non-metaphorical terms. Perhaps in this case, the reason is "Jack is mean-spirited.". Our full analysis is as follows:

*Henry makes a prediction:* 1 Jack won't let Jim off the leash to chase squirrels (because) 2 [Jack has a heart of stone.]

2 Jack is mean-spirited.

--------------------------

1 Jack will not let Jim off the leash to chase squirrels.

Here are two more examples:

The new iPhone is flying off the shelves. Visit your local Apple store today!

Life is like a box of chocolates. There's no reason to give up because of one setback.

As before, mobile devices do not literally fly off shelves and life is in many ways not like a box of chocolates. Plausible translations for the reason in each case might be "The new iPhone is selling well.", and "A variety of events happen in the course of life.".

A definite meaning can often be given to metaphors and similes, but this is additional work for the audience. Metaphors and similes perhaps stimulate the brain more than plain language, and they might do so in ways that help an argument be convincing, but they can also make the propositions they appear in difficult to understand.

### 3 Euphemism & Dysphemism

**1.** Some words have positive or negative connotations separate from the literal meaning (if there is any!) of the words.

***Euphemisms*** make something sound better than it really is, such as using "laid off" instead of "fired". ***Dysphemisms***make something sound worse than it is, such as describing an unreliable car as a "death trap". We will expand the meaning of these terms to mean words that emphasize the emotional aspect rather than the descriptive. As such, euphemism and dysphemism will also include ***jargon***and ***buzzwords****.*

**2.** Here are some words/phrases which will stir up positive feelings in most people but which obscure the literal meaning:

family values; moving forward; much-needed change; fulfilling the promise of a generation; working [as in "working Canadians"]; green initiative; unbeatable prices; fuel-injection technician; commodity relocation; freedom fighter; vertically challenged; full-figured; passed on; between jobs; pre-owned; sales associate; executive assistance; downsizing; enhanced interrogation; transfer tubes; creation science; climate change

A "fuel-injection technician" is in fact someone who pumps petrol/gasoline at a filling station, but the words "fuel-injection" and "technician" are both intended to make the audience think that the job is quite sophisticated and perhaps even glamorous!

Here is an example of a phrase used euphemistically:

Lundberg Family has been farming rice in environmentally friendly ways for three generations. Ask for our rice at your local supermarket.

The phrase "environmentally friendly ways" sounds good, but it is not clear what it means, and this is not a problem we can resolve by using a dictionary, since there are a number of different things it might mean. If the company provides more information (perhaps on its web site), we can clarify and rewrite the proposition when writing the inference in standard form, but, if not, the standard form repeats the sentences from the passage and we make a note that there is a problem:

1 [Lundberg Family has been farming rice in environmentally friendly ways for three generations.] 2 Ask for our rice at your local supermarket.

1 Lundberg Family has been farming rice in environmentally friendly ways for three generations.

--------------------------------------------------------------------------

2 You should buy Lundberg Family rice.

In 2, "environment", friendly", and "environmentally friendly ways" are euphemistic; it sounds good but I have no idea in what way their farming methods are friendly towards the environment.

Dysphemisms are used to make things seem worse than they actually are. Consider the following examples:

tree-hugger; snail-mail; death tax; anti-life; grammar Nazi

**3.** Under this category (euphemism) we will also include ***jargon***. Jargon is any term used by specialists communicating with each other, and, because it is understood only by specialists, jargon can be used on *non-*specialists to give the impression of authority or sophistication. For example, in the marketing of food, antioxidants and Omega-3 have been prominent recently. Consumers are encouraged to make purchases based on these features, though it's not likely many consumers know exactly what claims such as "This product contains antioxidants." and "Antioxidants are beneficial to health." mean. Speakers hope that audiences will put aside the fact that they don't understand the terminology and be impressed by the jargon because it is science-y.

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| Figure 2 xkcd: Magnetohydrodynamics[[5]](#footnote-6) |

**4.** Jargon is especially effective if it is also a *buzzword*, a word that has "buzz" (that is, lots of people have heard of it). In the case of "antioxidant", the buzz is positive: people think antioxidants are good. But few people know what it means; most people will know that it has something to do with food and that it's a good thing.

**5.** Buzzwords need not be specialized terms. Many common words acquire a buzz, especially when used by advertisers in connection with products for sale. Examples include "natural", "green", "healthy". This is because the job of a modern advertisement is no longer to inform the audience about the features of the product but instead to get the audience to feel a certain emotion (typically a positive emotion) and associate it with the product. (Many ads are included in the chapter on *Emotional Reasoning*.)

Take a look at the packaging of the snack-food below:

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| Figure 3 Picture of "Baked Vegetable Crisps[[6]](#footnote-7) |

Every word on the packaging for these crisps/chips is euphemistic: "gluten free", "good natured", "selects", "baked", "vegetable", and even "crisps". "Gluten free" is a buzzword not understood by many people, whereas "baked" has an ordinary meaning but, like "gluten free", is also meant to connote greater healthiness (as compared to other cooking methods, such as frying). The company (and by extension, the product) is called "good natured" which literally means "kind" and "friendly" but it makes no literal sense for chips to be called "good natured" (and it is hard to know what it means for a company to be good natured). In any case, the point is rather that the company and its products are "good" and "natural". "Selects" is a euphemism meant to give an impression of quality due to selectiveness. "Vegetable" is perhaps the most plainly descriptive word on the packaging, though it too might suggest health. "Crisps" is significant because the usual word for this type of product in the USA (where this product is sold) is "chips"; because it is unusual, "crisps" more effectively suggests that the product is crisp and fresh.

Like metaphors and similes, euphemisms and dysphemisms are often not vacuous, if a meaning can be given to them, but they add an extra step to the process. When confronted by a passage with a proposition which includes a euphemism or a dysphemism, the proposition must be re-written or a note made pointing out the problem.

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| Figure 4 xkcd: Adjective Foods |

### 4 Comparatives Without Comparisons

**1.** Comparatives (such as "tall*er*", "cheap*er*", and so on) are meaningless if not made explicit. Consider the following:

NEW IMPUNITY CIGARS — smoother by far!

In this example, we do not know what the cigars are smoother than. (We would also want a more precise understanding of "by far" which is a fuzzy term – see subsections 5 and 6, below). In this case, it is not possible to clarify the meaning and so, we include pointing out the problem:

1 NEW IMPUNITY CIGARS — 2 [smoother by far!]

2 Impunity cigars are smoother by far.

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1 You should try Impunity cigars.

In 2, there is nothing to compare the smoothness of the cigars to.

To provide a comparison, advertisers will often say that a product is better than "a leading brand" or "many other brands", but notice: these comparisons themselves need to be made explicit: Which is the leading brand? What other brands is the product being compared to?

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| Figure 5 Comedian Ricky Gervais in a Verizon ad, complaining about a competitor's claim to be "four times better."[[7]](#footnote-8) |

### 5 Weaseling

**1.** An imprecise word (or phrase) can be used to weaken (or 'weasel out of') a definite or striking claim. The speaker’s hope is that the audience will not pay attention to the weasel words — their imprecision means that their meaning is not immediately obvious — and will instead focus only on the rest. For example:

NEW *WEIGHT-AWAY* HELPS YOU LOSE THE POUNDS!!

"Helps" is used here as a weasel word. The speaker is hoping that the audience simply connects the product with "losing weight", perhaps because it is difficult to give a definite meaning to the word "helps". Here is another example:

LOSE UP TO 10 POUNDS A WEEK WITH NEW *WEIGHT-AWAY*!! Try it today!!!

"Up to" is a weasel phrase. The advertiser is hoping that the audience thinks only of "lose" and "10 pounds a week". Plus, "up to" could be anywhere between 0 and 10. Since no precise meaning is given, the brain might instead fix on "10", which is precise (and so easier for the brain to deal with) and optimistic.

When we analyze, we make a note pointing out the problem:

1 [LOSE UP TO 10 POUNDS A WEEK WITH NEW *WEIGHT-AWAY*!!] 2 Try it today!!!

1 A person can lose up to 10 pounds a week by using *Weight-Away*.

-----------------------------------------------------------------------------

2 You should use *Weight-Away*.

In 1, "up to" is a weasel phrase: it is used to weaken "10 pounds" (and it is also imprecise).

A common form of weaseling in advertising is to present a striking claim prominently, and then qualify it in the "fine print" which is, literally, difficult to read. For example:

LOSE 10 POUNDS IN TWO WEEKS WITH NEW *WEIGHT-AWAY*!!\*

(\*In conjunction with a moderate diet and regular exercise.)

As the saying goes: the large print giveth; the small print taketh away. (This specific example might also be interpreted as exploiting an ambiguity in the word "with". See subsections 7 and 8)

### 6 Fuzzy Terms

**1.** A word is *fuzzy* if is understood in terms of something which varies in quantity and no precise quantity is used in the definition.

The word "bald", for example, is fuzzy because knowing whether or not something is bald depends on the number of hairs it has (per square centimeter or other unit of area). It clearly applies to a person who has no hair, and it clearly does not apply to a person who has a full head of hair. But for some people, whether or not it applies is unclear. In short, there are borderline cases and a precise number of hairs would be needed in order to distinguish bald from not-bald.

Fuzzy words involve quantities and lack a precise specification of when the word applies and when it doesn't. For example, "light" and "dark" are fuzzy because they are understood in terms of *how much* light there is. The same is true of "tall" and "short" when these are understood in terms of centimeters or inches.

Many words for quantities are fuzzy, such as "lots", "by far", and many others. When does lots of something become 'not lots'? If you can't tell when a word switches from applying to not applying, it is fuzzy.

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| Figure 6 Tulips by Tulpen[[8]](#footnote-9) |
| When things are fuzzy, it's hard to tell where one ends and another begins. Where does the red shade begin and end in the color wheel on the left? Where does one flower end and another begin in the image on the right? | |

In order to deal with a proposition which contains a fuzzy term, we need to make the term precise. In the case of "bald" (in propositions such as "Jones is bald."), we might say just how many hairs Jones question has, or, more realistically, give at least a more precise statement of the degree of baldness, such as "Jones is totally bald." or "Jones is bald on top, but has hair on the sides." or "Jones is as bald as Winston Churchill was.".

To return to the example above about Impunity cigars, we would add a note pointing out the fuzzy term "by far" in addition to the fact that there is nothing to compare their smoothness to:

1 NEW IMPUNITY CIGARS — 2 [smoother by far!]

2 Impunity cigars are smoother by far.

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1 You should try Impunity cigars.

In 2, there is nothing to compare the smoothness of the cigars to. In 2, "by far" is fuzzy – I don't know when one thing is 'far' smoother than another. Although it is not the focus of the ad, the word "smooth" itself is a fuzzy term: at what degree of roughness does something go from being smooth to not-smooth?

**2.** The continuum fallacy (or sorites paradox) exploits fuzzy terms. It is a form of inference which takes advantage of the fact that fuzzy terms do not have clear dividing lines at any point between the extremes but vary only by degree, in order to argue, fallaciously, that two quite different states share some property. Such arguments have the following general form:

1. There is a continuum c.

2. Some thing on one end of c has property p.

3. Moving one increment along c cannot result in a change from p to not p.

-------------------------------------------------------------------------------------

4. Some thing on the other end of c has p.

The mere fact that there is no sharp line between things having property-p and things not having p is supposed to give us good reason for thinking that there is no difference in terms of p between things on one end of continuum-c and things on the other end — so that since the things on one end of the spectrum have p, so do the things on the other.

Here is an example:

A person having exactly 1 penny is not significantly different in wealth from a person having exactly 2 pennies, and a person having exactly 2 pennies is not significantly different in wealth from a person having exactly 3 pennies,..., and a person having exactly 99,999,999,999 pennies is not significantly different in wealth from a person having exactly 100,000,000,000 pennies. Thus, since a person having exactly 1 penny is not rich, a person having exactly 100,000,000,000 pennies is not rich.

In this passage, the fuzzy term is "wealth" because it depends on a continuum ("c") of the number of pennies owned, while the property ("p") in question is "not rich". So, in standard form, the inference would be:

1. There is a continuum of pennies owned, from one upward.

2. A person with one penny is not rich.

3. Moving one increment (one penny) along the continuum of pennies owned cannot result in a change from 'not rich' to 'rich'.

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4. A person with 1,000,000 pennies is not rich.

There is good reason for thinking the reasons are true, but there is no good reason for thinking the conclusion is true. In fact, there is good reason for thinking the conclusion is false. The reasoning, thus, must be bad.

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| At what point does day become night? "Day" and "night" are both fuzzy words when they are understood in terms of the amount of light present and a precise measurement of when it turns from day to night is lacking. In Judaism the start of evening is determined when one can make out 2 stars in the sky. |

### 7 Ambiguity

**1.** A word is ambiguous when it has multiple precise meanings. The word "pen" is ambiguous: it can be used to refer to a tool for writing, it can be used to refer to an enclosure for animals, and it can be used to refer to a penitentiary. The same goes for "has" in the sentence "Hannibal often has people for dinner.". It can be used to say that Hannibal often has people over to his place for dinner, and it can be used to say that Hannibal often eats people for dinner.

In general terms, a word, phrase, or sentence is ambiguous whenever it has multiple meanings. In order to evaluate a sentence containing an ambiguous word, we must first resolve the ambiguity.

Ambiguities can arise *syntactically* (that is, because of the way in which the words/phrases are placed next to one another). A classic example is

"Wanted: A piano by a local woman with wooden legs.".

Does the woman have wooden legs? Or the piano?

Here are some others:

"Man's arm severed, 3 others critically injured in crash near Midway." (Chicago Sun Times, July 19, 2009)

"The student described how the relationship escalated from Facebook flirtations to sexual intercourse during a courtroom appearance." (Huffington Post, July 16th, 2009).

**2.** *Equivocation.* When an ambiguous word is used with more than one of its meanings in different propositions *in the same inference*, the inference is said to *equivocate*. Equivocation violates the rule that a word or phrase should have the same meaning every time it is used in an inference or explanation. Consider the following argument from the abortion debate:

1 [A fetus is a human being.] 2 [A human being has a right to life.] (So,) 3 a fetus has a right to life.

This argument equivocates, since the phrase "human being" has different meanings in (1) and in (2). In (1) it means a biological human being whereas in (2) it means a person. If we apply either meaning *consistently* throughout the argument, one of the premises will be false. (As discussed in the chapter on *Warrants*, we try to interpret speakers charitably, by not attributing false or controversial claims to them, but in this case, it is impossible to do so for both propositions simultaneously.)

Let's consider each meaning, one at a time. If we take "human being" to mean "biologically human" and replace each occurrence of "human" with it, we get the following inference:

1 A fetus is a biological human.

2 A biological human has a right to life.

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3 A fetus has a right to life.

If we take "human being" to mean "person", we get the following inference:

1 A fetus is a person.

2 A person has a right to life.

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3 A fetus has a right to life.

The second premise in the first argument – "A person has a right to life." – will be thought false by many, and so will the first premise in the second argument – "A fetus is a person.".

In general terms, an inference equivocateswhen a single word, phrase, or proposition disguises different meanings. "Equivocation" literally means "say to be the same", when in fact the same word is being used with a different meaning on each occasion. The word should be replaced in the standard form with the different meanings in order to show the equivocation:

1 [A fetus is a human being.] 2 [A human being has a right to life.] (So,) 3 a fetus has a right to life.

1 A fetus is a biological human.

2 A person has a right to life.

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3 A fetus has a right to life.

The term "human being" is used ambiguously; it means different things in proposition (1) and proposition (2).

### 8 Summary – Problems with Meaning

In the exercises, read every word in every proposition and ask yourself "Do I know what this word means?"

If possible, rewrite any problematic words/phrases in the standard form with a word/phrase that has a clear meaning.

If no clear meaning can be given to a word/phrase, make a note identifying it and explain why it is problematic. Use the following categories:

* Unknown/Insufficient Meaning
* Imprecise Meaning
  + Metaphor & Simile
  + Euphemism & Dysphemism (including *jargon* – specialized terms being used on non-experts – and *buzzwords* – words widely known, with a positive or negative association, but not well-understood)
  + Comparatives Without Comparisons
  + Imprecise Meaning Used For Weaseling
  + Fuzziness (exploited in the Continuum Fallacy)
  + Ambiguity (exploited in the Fallacy Of Equivocation)

A word/phrase can have more than one problem.

### 9 Exercises

## Evaluating The Reasoning

## Ignoring Confidence Indicators

## Sources/Character/Motives

## Practical Reasoning

## Emotional Reasons

## Syntax & Logic

## Warrants

## Diagramming – Basic

## Diagramming – Complex

## Diagramming – Dialogue 1

## Diagramming – Dialogue 2

## Diagramming – Very Long Passages

# Part II: Introduction to Formal Logic

*This section is based upon* For All X, The Lorain County Remix *by Magus and remixed by Loftin.*

## What is Formal Logic?

### 1 Formal as in Concerned with the Form of Things

This part of the book is concerned with formal logic. Formal logic is distinguished from other branches of logic by the way it achieves content neutrality. A feature of logic is that it is neutral about the content of the argument it evaluates. If a kind of argument is strong—say, a kind of statistical argument—it will be strong whether it is applied to sports, politics, science or whatever. Formal logic takes radical measures to ensure **content neutrality**: it removes the parts of a statement that tie it to particular objects in the world and replaces them with abstract symbols.

Consider the two arguments below

|  |  |
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| P1: Socrates is a person.  P2: All persons are mortal.  C: Socrates is mortal. | P1: Socrates is a person.  P2: All people are carrots.  C: Socrates is a carrot. |

These arguments are both valid. In each case, if the premises were true, the conclusion would have to be true. (In the case of the first argument, the premises are actually true, so the argument is sound, but that is not what we are concerned with right now.) What makes these arguments valid is that they are put together the right way. Another way of thinking about this is to say that they have the same logical form. Both arguments can be written like this:

P1: *S* is *M.*

P2: All *M* are *P.*

C: *S* is *P.*

In both arguments *S* stands for Socrates and *M* stands for person. In the first argument, *P* stands for mortal; in the second, *P* stands for carrot. The letters ‘S’, ‘M’, and ‘P’ are **variables**. They are just like the variables you may have learned about in algebra class. In algebra, you had equations like ***y* = 2*x* + 3**, where *x* and *y* were variables that could stand for any number. Just as *x* could stand for any number in algebra, ‘S’ can stand for any name in logic. In fact, this is one of the original uses of variables. Long before variables were used to stand for numbers in algebra, they were used to stand for classes of things, like people or carrots, by Aristotle in his book the *Prior Analytics* (c. 350 BCE). At about the same time, over in India, the ancient grammarian and linguist Panini was also using variables to represent possible sounds that could be used in different forms of a word. Both thinkers introduce their variables fairly causally, as if their readers would be familiar with the idea, so it may be that people prior to them actually invented the variable.

Whoever invented it, the variable was one of the most important conceptual innovations in human history, right up there with the invention of the zero, or alphabetic writing. The importance of the variable for the history of mathematics is obvious. But it was also incredibly important in one of its original fields of application, logic. For one thing, it allows logicians to be more content neutral. We can set aside any associations we have with people, or carrots, or whatever, when we are analyzing an argument. More importantly, once we set aside content in this way, we discover that something incredibly powerful is left over, the logical structure of the sentence itself. This is what we investigate when we study formal logic. In the case of the two arguments above, identifying the logical structure of statements reveals not only that the two arguments have the same logical form, but they have an impeccable logical form. Both arguments are valid, and any other arguments that have this form will be valid.

When Aristotle introduced the variable to the study of logic he used it the way we did in the argument above. His variables stood for names and categories in simple two-premise arguments called syllogisms. The system of logic Aristotle outlined became the dominant logic in the Western world for more than two millennia. It was studied and elaborated on by philosophers and logicians from Baghdad to Paris. The thinkers that carried on Aristotelian tradition were divided by language and religion. They were pagans, Muslims, Jews, and Christians writing typically in Greek, Latin or Arabic. But they were all united by the sense that the tools Aristotle had given them allowed them to see something profound about the nature of reality. They were looking at abstract structures which somehow seemed to be at the foundation of things. As the philosopher and historian of logic Catarina Dutilh Novaes points out, the logic that the thinkers of all these religious traditions were pursuing was formal in that it concerned the *forms* of things **?**. As formal logic evolved, however, the idea of being “formal” would take on an additional meaning.

### 2 Formal as in Strictly Following Rules

Despite its historical importance, Aristotelean logic has largely been superseded. Starting in the 19th century people learned to do more than simply replace categories with variables. They

learned to replicate the whole structure of sentences with a formal system that brought out all sorts of features of the logical form of arguments. The result was the creation of entire artificial languages. An artificial language is a language that was consciously developed by identifiable individuals for some purpose. Esperanto, for instance, is an artificial language developed by Ludwig Lazarus Zamenhof in the 19th century with the hope of promoting world peace by creating a common language for all. J.R.R. Tolkien invented several languages to flesh out the fictional world of his fantasy novels, and even created timelines for their evolution. For Tolkien, the creation of languages was an art form in itself, “An art for which life is not long enough, indeed: the construction of imaginary languages in full or outline for amusement, for the pleasure of the constructor or even conceivably of any critic that might occur” (Tolkein 1931). And it is an art that is really beginning to catch on, especially with Hollywood commissioning languages to be constructed for blockbuster films.

Artificial languages contrast with natural languages, which develop spontaneously and are learned by infants as their first language. Natural languages include all the well-known languages spoken around the world, like English or Japanese or Arabic. It also includes more recently developed languages and evolved spontaneously amongst groups of people. For instance, whenever you put deaf children together, for instance in a boarding school, they will spontaneously develop their own sign language. This phenomenon was important for the development of American Sign Language (ASL) and is part of why ASL counts as a *natural* language. For similar reasons Nicaraguan Sign Language counts as a natural language, even though it emerged very recently—in the late 1970s and 80s, when the new Sandinista government set up schools for the deaf for the first time. Natural languages can also develop by creolization, when languages merge and children grow up speaking the merged language as their first language. Haitian Creole is the most famous example of this.

The languages developed by logicians are artificial, not natural. Their goal is not to promote global harmony, like Zamenhof’s Esperanto. Nor are they creating art for art’s sake, as Tolkein was, although logical languages can have a great deal of beauty. When the languages first started being developed in the late 19th and early 20th centuries, the goal was, in fact, to have a logically pure language, free of the irrationalities the plague natural languages. More specifically, they had two distinct goals: first, remove all ambiguity and vagueness, and second, to make the logical structure of the language immediately apparent, so that the language wore its logical structure on its face, as it were. If such a language could be developed, it would help us solve all kinds of problems. The logician and philosopher Rudolf Carnap, for instance, felt that the right artificial language could simply make philosophical problems disappear.

The languages developed by logicians in the late 19th and early 20th centuries got labeled formal languages, in part because the logicians in question were working in the tradition of formal logic that was already established. A shift began to happen here with the meaning of formal, however, a change which is well documented by Dutilh Novaes **?**. Logicians began to hope that the languages that were being developed were so logical that everything about them could be characterized by a machine. A machine could be used to create sentences in this language, and then again to identify all the valid arguments in this language. This brings out another sense of the word “formal.” As Dutilh Novaes puts it instead of being “formal” in the sense of concerning the forms of things, logic was formal in the sense that it followed rules perfectly precisely. You might compare this to the way a “formal hearing” in a court of law follows the rule of law to the letter.

For the purposes of this textbook, we will say that the core idea of a formal language is that it is an artificial language designed to bring out the logical structure of ideas and remove all the ambiguity and vagueness that plague natural languages like English. We will further add that sometimes, formal languages are languages that can be implemented by a machine. Creating formal languages always involves all kinds of trade offs. On the one hand, we are trying to create a language that makes a logical structure clear and obvious. This will require simplifying things, removing excess baggage from the language. On the other hand, we want to make the language perfectly precise, free of vagueness and ambiguity. This will mean adding complexity to the language. The other thing was that it was very important for the people developing these languages that you be able to prove the all the truths of mathematics in them. This meant that the languages had to have a certain scope.

This was a trade off no logician was ever able to get perfectly correct, because, as it turns out, a logically pure language is impossible. No formal language can do everything that a natural language can do. Logicians became convinced of this, naturally enough, because of a pair of logical proofs. In 1931, the logician Kurt G¨odel showed that you couldn’t do all of mathematics in a consistent logical system, which was enough to persuade most of the logicians engaged in the project to drop it. There is a more general problem with the idea of a purely logical language, though, which is that that many of the features logicians were trying to remove from language were actually necessary to make it function. Arika Okrent puts the point quite well. For Okrent, the failure of artificial languages is precisely what illuminates the virtues of natural language.

[By studying artificial languages we] gain a deeper appreciation of natural language and the messy qualities that give it so much flexibility and power and that a simple communication device. The ambiguity and lack of precision allow to serve as a instrument of thought *formation*, of experimentation and discovery. We don’t know exactly what we mean before we speak; we can figure it out as we go along, We can talk just to talk, to be social, to feel connected, to participate. At the same time natural language still works as an instrument of thought transmission, one that can be *made* extremely precise and reliable when we need it to be, or left loose and sloppy when we can’t spare the time or effort

The languages developed in the late 19th and early 20th centuries had goals that were theoretical, rather than practical. They languages were meant to improve our understanding of the world for the sake of improving our understanding of the world. They failed at this theoretical goal, but they wound up having a practical spin-off of world-historical proportions, which is why formal logic is a thriving discipline to this day. Remember that in this period people started thinking of formal languages as languages that could be implemented mechanically. At first, the idea of a a mechanistic language was a metaphor. The rules that were being followed to the letter were to be followed by a human being actually writing down symbols. This human being was generally referred to as a “computer,” because they were computing things. The world changed when a logician named Alan Turing started using literal machines to be computers.

In the 1930s, Turing developed the idea of a reasoning machine that could compute any function. At first, this was just an abstract idea: it involved an infinite stretch of tape. But during World War II, Turing went to work the British code breaking effort at Bletchley Park. The Nazis encoded messages using a device called the Enigma Machine. The Allies had captured one, but since they settings on the machine were reshuffled for each message, it didn’t do them much good. Turing, together with people like the mathematicians Gordon Welchman and Joan Clarke, managed to build another machine that could test Enigma settings rapidly to identify the configuration being used. People had made computing machines before, but now the science of logic was so much more advanced that they real power of mechanical computing could be exploited. The human computers became the fully programmable machines we know today, and the formal languages logicians created for theoretical reasons came the computer languages the world of the 21st century depends on. (All of this information, plus lots of fascinating pictures and diagrams, is available at www.turing.org.uk.)

### 3 More Logical Notions for Formal Logic

Part [I](#_bookmark2) covered the basic concepts you need to study any kind of logic. When we study formal logic, we will be interested in some additional logical concepts, which we will explain here.

#### Truth values

A truth value is the status of a statement as true or false. Thus the truth value of the sentence “All dogs are mammals” is “True,” while the truth value of “All dogs are reptiles” is false. More precisely, a truth value is the status of a statement with relationship to truth. We have to say this, because there are systems of logic that allow for truth values besides “true” and “false,” like “maybe true,” or “approximately true,” or “kinda sorta true.” For instance, some philosophers have claimed that the future is not yet determined. If they are right, then statements about *what will be the case* are not yet true or false. Some systems of logic accommodate this by having an additional truth value. Other formal languages, so-called paraconsistent logics, allow for statements that are both true *and* false. We won’t be dealing with those in this textbook, however. For our purposes, there are two truth values, “true” and “false,” and every statement has exactly one of these. Logical systems like ours are called bivalent.

#### Tautology, contingent statement, contradiction

In considering arguments formally, we care about what would be true *if* the premises were true. Generally, we are not concerned with the actual truth value of any particular statements— whether they are *actually* true or false. Yet there are some statements that must be true, just as a matter of logic.

Consider these statements:

1. It is raining.
2. Either it is raining, or it is not.
3. It is both raining and not raining.

In order to know if statement (a) is true, you would need to look outside or check the weather channel. Logically speaking, it might be either true or false. Statements like this are called ***contingent***statements.

Statement (b) is different. You do not need to look outside to know that it is true. Regardless of what the weather is like, it is either raining or not. If it is drizzling, you might describe it as partly raining or in a way raining and a way not raining. However, our assumption of bivalence means that we have to draw a line, and say at some point that it is raining. And if we have not crossed this line, it is not raining. Thus, the statement “either it is raining or it is not” is always going to be true, no matter what is going on outside. A statement that has to be true, as a matter of logic is called a **tautology** or logical truth.

You do not need to check the weather to know about statement (c), either. It must be false, simply as a matter of logic. It might be raining here and not raining across town, it might be raining now but stop raining even as you read this, but it is impossible for it to be both raining and not raining here at this moment. The third statement is *logically false*; it is false regardless of what the world is like. A logically false statement is called a **contradiction**.

We have already said that a contingent statement is one that could be true, or could be false, as far as logic is concerned. To be more precise, we should define a contingent statement as a statement that is neither a tautology nor a contradiction. This allows us to avoid worrying about what it means for something to be logically possible. We can just piggyback on the idea of being logically necessary or logically impossible.

A statement might *always* be true and still be contingent. For instance, it may be the case that in no time in the history of the universe was there ever an elephant with tiger stripes. Elephants only ever evolved on Earth, and there was never any reason for them to evolve tiger stripes. The statement “Some elephants have tiger stripes,” is therefore always false. It is, however, still a contingent statement. The fact that it is always false is not a matter of logic. There is no contradiction in considering a possible world in which elephants evolved tiger stripes, perhaps to hide in really tall grass. The important question is whether the statement *must* be true, just on account of logic.

When you combine the idea of tautologies and contradictions with the notion of deductive validity, as we have defined it, you get some curious results. For one thing, any argument with a tautology in the conclusion will be valid, even if the premises are not relevant to the conclusion. This argument, for instance, is valid.

P1: There is coffee in the coffee pot.

P2: There is a dragon playing bassoon on the armoire.

C: All bachelors are unmarried men.

The statement “All bachelors are unmarried men” is a tautology. No matter what happens in the world, all bachelors have to be unmarried men, because that is how the word “bachelor” is defined. But if the conclusion of the argument is a tautology, then there is no way that the premises could be true and the conclusion false. So the argument must be valid.

Even though it is valid, something seems really wrong with the argument above. The premises are not relevant to the conclusion. Each sentence is about something completely different. This notion of relevance, however, is something that we don’t have the ability to capture in the kind of simple logical systems we will be studying. The logical notion of validity we are using here will not capture everything we like about arguments.

Another curious result of our definition of validity is that any argument with a contradiction in the premises will also be valid. In our kind of logic, once you assert a contradiction, you can say anything you want. This is weird, because you wouldn’t ordinarily say someone who starts out with contradictory premises is arguing well. Nevertheless, an argument with contradictory premises is valid.

#### Logically Equivalent and Contradictory Pairs of Sentences

We can also ask about the logical relations *between* two statements. For example:

1. John went to the store after he washed the dishes.
2. John washed the dishes before he went to the store.

These two statements are both contingent, since John might not have gone to the store or washed dishes at all. Yet they must have the same truth value. If either of the statements is true, then they both are; if either of the statements is false, then they both are. When two statements necessarily have the same truth value, we say that they are logically equivalent.

On the other hand, if two sentences must have opposite truth values, we say that they are contradictories. Consider these two sentences

1. Susan is taller than Monica.
2. Susan is shorter or the same height as Monica.

One of these sentences must be true, and if one of the sentences is true, the other one is false. It is important to remember the difference between a single sentence that is a *contradiction* and a pair of sentences that are *contradictory*. A single sentence that is a contradiction is in conflict with itself, so it is never true. When a pair of sentences is contradictory, one must always be true and the other false.

#### Consistency

Consider these two statements:

1. My only brother is taller than I am.
2. My only brother is shorter than I am.

Logic alone cannot tell us which, if either, of these statements is true. Yet we can say that *if* the first statement (a) is true, *then* the second statement (b) must be false. And if (b) is true, then (a) must be false. It cannot be the case that both of these statements are true. It is possible, however that both statements can be false. My only brother could be the same height as I am.

If a set of statements could not all be true at the same time, they are said to be inconsistent. Otherwise, they are consistent.

We can ask about the consistency of any number of statements. For example, consider the following list of statements:

1. There are at least four giraffes at the wild animal park.
2. There are exactly seven gorillas at the wild animal park.
3. There are not more than two Martians at the wild animal park.
4. Every giraffe at the wild animal park is a Martian.

Statements (a) and (d) together imply that there are at least four Martian giraffes at the park. This conflicts with (c), which implies that there are no more than two Martian giraffes there. So the set of statements (a)–(d) is inconsistent. Notice that the inconsistency has nothing at all to do with (b). Statement (b) just happens to be part of an inconsistent set.

Sometimes, people will say that an inconsistent set of statements “contains a contradiction.” By this, they mean that it would be logically impossible for all of the statements to be true at once. A set can be inconsistent even when all of the statements in it are either contingent or tautologous. When a single statement is a contradiction, then that statement alone cannot be true.

### 4 Practice Exercises

#### Part A Label the following tautology, contradiction, or contingent statement.

**Example**:

Caesar crossed the Rubicon.

**Answer**:

Contingent statement.

(The Rubicon is a river in Italy. When General Julius Caesar took his army across it, he was committing to a revolution against the Roman Republic. Since that time, “crossing the Rubicon” has been a expression referring to making an irreversible decision.)

1. Someone once crossed the Rubicon.
2. No one has ever crossed the Rubicon.
3. If Caesar crossed the Rubicon, then someone has.
4. Even though Caesar crossed the Rubicon, no one has ever crossed the Rubicon.
5. If anyone has ever crossed the Rubicon, it was Caesar.

#### Part B Label the following tautology, contradiction, or contingent statement.

1. Elephants dissolve in water.
2. Wood is a light, durable substance useful for building things.
3. If wood were a good building material, it would be useful for building things.
4. I live in a three-story building that is two stories tall.
5. If gerbils were mammals they would nurse their young.

#### Part C Which of the following pairs of statement are logically equivalent?

1. Elephants dissolve in water. If you put an elephant in water, it will disintegrate.
2. All mammals dissolve in water. If you put an elephant in water, it will disintegrate.
3. George Bush was the 43rd president. Barack Obama is the 44th president.
4. Barack Obama is the 44th president. Barack Obama was president immediately after the 43rd president.
5. Elephants dissolve in water. All mammals dissolve in water.

#### Part D Which of the following pairs of statement are logically equivalent?

1. Thelonious Monk played piano. John Coltrane played tenor sax.
2. Thelonious Monk played gigs with John Coltrane. John Coltrane played gigs with Thelonious Monk.
3. All professional piano players have big hands. Piano player Bud Powell had big hands.
4. Bud Powell suffered from severe mental illness. All piano players suffer from severe mental illness.
5. John Coltrane was deeply religious. John Coltrane viewed music as an expression of spirituality.

#### Part E Consider again the statements on p.53:

1. There are at least four giraffes at the wild animal park.
2. There are exactly seven gorillas at the wild animal park.
3. There are not more than two Martians at the wild animal park.
4. Every giraffe at the wild animal park is a Martian.

Now consider each of the following sets of statements. Which are consistent? Which are inconsistent?

1. Statements [(b)](#_bookmark62), [(c)](#_bookmark63), and [(d)](#_bookmark64)
2. Statements [(a)](#_bookmark61), [(c)](#_bookmark63), and [(d)](#_bookmark64)
3. Statements [(a)](#_bookmark61), [(b)](#_bookmark62), and [(d)](#_bookmark64)
4. Statements [(a)](#_bookmark61), [(b)](#_bookmark62), and [(c)](#_bookmark63)

#### Part F Consider the following set of statements.

1. All people are mortal.
2. Socrates is a person.
3. Socrates will never die.
4. Socrates is mortal.

Which combinations of statements form consistent sets? Mark each consistent or inconsistent.

1. Statements [(a)](#_bookmark66), [(b)](#_bookmark67), and [(c)](#_bookmark68)
2. Statements [(b)](#_bookmark67), [(c)](#_bookmark68), and [(d)](#_bookmark69)
3. Statements [(b)](#_bookmark67) and [(c)](#_bookmark68)
4. Statements [(a)](#_bookmark66) and [(d)](#_bookmark69)
5. Statements [(a)](#_bookmark66), [(b)](#_bookmark67), [(c)](#_bookmark68), and [(d)](#_bookmark69)

#### Part G Which of the following is possible? If it is possible, give an example. If it is not possible, explain why.

1. A valid argument that has one false premise and one true premise
2. A valid argument that has a false conclusion
3. A valid argument, the conclusion of which is a contradiction
4. An invalid argument, the conclusion of which is a tautology
5. A tautology that is contingent
6. Two logically equivalent sentences, both of which are tautologies
7. Two logically equivalent sentences, one of which is a tautology and one of which is contingent
8. Two logically equivalent sentences that together are an inconsistent set
9. A consistent set of sentences that contains a contradiction
10. An inconsistent set of sentences that contains a tautology

#### Part H Which of the following is possible? If it is possible, give an example. If it is not possible, explain why.

1. A valid argument, whose premises are all tautologies, and whose conclusion is contingent
2. A valid argument with true premises and a false conclusion
3. A consistent set of sentences that contains two sentences that are not logically equivalent
4. A consistent set of sentences, all of which are contingent
5. A false tautology
6. A valid argument with false premises
7. A logically equivalent pair of sentences that are not consistent
8. A tautological contradiction
9. A consistent set of sentences that are all contradictions

### 5 Key Terms

* Artificial language
* Consistent
* Contingent statement
* Contradiction
* Contradictories
* Formal language
  + Formal logic as concern for logical form
  + Formal logic as strictly following rules Inconsistent
* Logically equivalent
* Natural language
* Tautology
* Truth value

## Categorical Statements

### 1 Quantified Categorical Statements

Earlier we saw that a statement was a unit of language that could be true or false. In this chapter and the next we are going to look at a particular kind of statement, called a quantified categorical statement, and begin to develop a formal theory of how to create arguments using these statements. This kind of logic is generally called “categorical” or “Aristotelian” logic, because it was originally invented by the great logician and philosopher Aristotle in the fourth century bce. This kind of logic dominated the European and Islamic worlds for 20 centuries afterward, and was expanded in all kinds of fascinating ways, some of which we will look at here.

Consider the following propositions:

1. All dogs are mammals.
2. Most physicists are male.
3. Few teachers are rock climbers.
4. No dogs are cats.
5. Some Americans are doctors. (f) Some adults are not logicians.
6. Thirty percent of Canadians speak French.
7. One chair is missing.

These are all examples of quantified categorical statements. A quantified categorical statement is a statement that makes a claim about a certain quantity of the members of a class or group. (Sometimes we will just call these “categorical statements”) Statement [(a)](#_bookmark73), for example, is about the class of dogs and the class of mammals. These statements make no mention of any particular members of the categories or classes or types they are about. The propositions are also *quantified* in that they state *how many* of the things in one class are also members of the other. For instance, statement [(b)](#_bookmark74) talks about *most* physicists, while statement [(c)](#_bookmark75) talks about *few* teachers.

Categorical statements can be broken down into four parts: the quantifier, the subject term, the predicate term, and the copula. The quantifier is the part of a categorical sentence that specifies a portion of a class. It is the “how many” term. The quantifiers in the sentences above are all, most, few, no, some, thirty percent, and one. Notice that the “no” in sentence [(d)](#_bookmark76) counts as a quantifier, the same way zero counts as a number. The subject and predicate terms are the two classes the statement talks about. The subject class is the first class mentioned in a quantified categorical statement, and the predicate class is the second. In sentence [(e)](#_bookmark77), for instance, the subject class is the class of Americans and the predicate class is the class of doctors. The copula is simply the form of the verb “to be” that links subject and predicate. Notice that the quantifier is always referring to the subject. The statement “Thirty percent of Canadians speak French” is saying something about a portion of Canadians, not about a portion of French speakers.

Sentence [(g)](#_bookmark78) is a little different than the others. In sentence [(g)](#_bookmark78) the subject is the class of Canadians and the predicate is the class of people who speak French. That’s not quite the way it is written, however. There is no explicit copula, and instead of giving a noun phrase for the predicate term, like “people who speak French,” it has a verb phrase, “speak French.” If you are asked to identify the copula and predicate for a sentence like this, you should say that the copula is implicit and transform the verb phrase into a noun phrase. You would do something similar for sentence [(h)](#_bookmark79): the subject term is “chair,” and the predicate term is “things that are missing.” We will go into more detail about these issues in Section [4.3](#_bookmark88).

In the previous chapter we noted that formal logic achieves content neutrality by replacing some or all of the ordinary words in a statement with symbols. For categorical logic, we are only going to be making one such substitution. Sometimes we will replace the classes referred to in a quantified categorical statement with capital letters that act as variables. Typically, we will use the letter *S* when referring to the class in the subject term and *P* when referring to the predicate term, although sometimes more letters will be needed. Thus the sentence “Some Americans are doctors,” above, will sometimes become “Some *S* are *P.*” The sentence “No dogs are cats” will sometimes become “No *S* is *P.*”

#### Practice Exercises

##### Part A

For each of the following sentences identify the quantifier, the subject term, the predicate term, and the copula. Some of these are like the example “Thirty percent of Canadians speak French” where the copula is implicit and the predicate needs to be transformed into a noun phrase.

**Example**:

Some dinosaurs had feathers

**Answer**:

Quantifier: Some

Subject term: Dinosaurs Copula: Implicit

Predicate term: Things with feathers

1. Some politicians are not members of tennis clubs.
2. All dogs go to heaven.
3. Most things in the fridge are moldy.
4. Some birds do not fly.
5. Few people have seen Janet relaxed and happy.
6. No elephants are pocket-sized.
7. Two thirds of Americans are obese or overweight.
8. All applicants must submit to a background check.
9. All handguns are weapons.
10. One man stands alone against injustice.

##### Part B

For each of the following sentences identify the quantifier, the subject term, the predicate term, and the copula. Some of these are like the example “Thirty percent of Canadians speak French” where the copula is implicit and the predicate needs to be transformed into a noun phrase.

1. No dog has been to Mars.
2. All human beings are mortal.
3. Some spears are six feet long.
4. Most dogs are friendly
5. Eighty percent of Americans graduate from high school.
6. Few doctors are poor.
7. All squids are cephalopods.
8. No fish can sing.
9. Some songs are sad.
10. Two dogs are playing in the backyard.

### 2 Quantity, Quality, Distribution, and Venn Diagrams

Ordinary English contains all kinds of quantifiers, including the counting numbers themselves. In this chapter and the next, however, we are only going to deal with two quantifiers: “all,” and “some.” We are restricting ourselves to the quantifiers “all” and “some” because they are the ones that can easily be combined to create valid arguments using the system of logic that was invented by Aristotle. We will deal with other quantifiers in chapter in the larger version of the text on induction. There we will talk about much more specific quantified statements, like “Thirty percent of Canadians speak French,” and do a little bit of work with modern statistical methods.

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| Figure 7 Parts of a quantified categorical statement. |

The quantifier used in a statement is said to give the quantity of the statement. Statements with the quantifier “All” are said to be “universal” and those with the quantifier “some” are said to be “particular.”

Here “some” will just mean “at least one.” So, “some people in the room are standing” will be true even if there is only one person standing. Also, because “some” means “at least one,” it is compatible with “all” statements. If I say “some people in the room are standing” it might actually be that *all* people in the room are standing, because if all people are standing, then at least one person is standing. This can sound a little weird, because in ordinary circumstances, you wouldn’t bother to point out that something applies to some members of a class when, in fact, it applies to all of them. It sounds odd to say “*some* dogs are mammals,” when in fact they *all* are. Nevertheless, when “some” means “at least one” it is perfectly true that some dogs are mammals.

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| --- | --- | --- |
| **Mood** | **Form** | **Example** |
| **A** | All *S* are *P* | All dogs are mammals. |
| **E** | No S are *P* | No dogs are reptiles. |
| **I** | Some *S* are *P* | Some birds can fly. |
| **O** | Some *S* are not *P* | Some birds cannot fly. |

Figure 8 The four moods of a categorical statement

In addition to talking about the quantity of statements, we will talk about their quality. The quality of a statement refers to whether the statement is negated. Statements that include the words “no” or “not” are negative, and other statements are affirmative. Combining quantity and quality gives us four basic types of quantified categorical statements, which we call the statement moods or just “moods.” The four moods are labeled with the letters A, E, I, and O. Statements that are universal and affirmative are mood-A statements. Statements that are universal and negative are mood-E statements. Particular and affirmative statements are mood-I statements, and particular and negative statements are mood-O statements.

Aristotle didn’t actually use those letters to name the kinds of categorical propositions. His later followers writing in Latin came up with the idea. They remembered the labels because the “A” and the “I” were in the Latin word “**a**ff **i**rmo,” (“I affirm”) and the “E” and the “O” were in the Latin word “n**e**g**o**” (“I deny”).

The distribution of a categorical statement refers to how the statement describes its subject and predicate class. A term in a sentence is said to be distributed if a claim is being made about the whole class. In the sentence “All dogs are mammals,” the subject class, dogs, is distributed, because the quantifier “All” refers to the subject. The sentence is asserting that every dog out there is a mammal. On the other hand, the predicate class, mammals, is not distributed, because the sentence isn’t making a claim about all the mammals. We can infer that at least some of them are dogs, but we can’t infer that all of them are dogs. So in mood-A statements, only the subject is distributed.

On the other hand, in an I sentence like “Some birds can fly” the subject is not distributed. The quantifier “some” refers to the subject, and indicates that we are not saying something about all of that subject. We also aren’t saying anything about all flying things, either. So in mood-I statements, neither subject nor predicate is distributed.

Even though the quantifier always refers to the subject, the predicate class can be distributed as well. This happens when the statement is negative. The sentence “No dogs are reptiles” is making a claim about all dogs: they are all not reptiles. It is also making a claim about all reptiles: they are all not dogs. So mood-E statements distribute both subject and predicate.

Finally, negative particular statements (mood-O) have only the predicate class distributed. The statement “some birds cannot fly” does not say anything about all birds. It does, however say something about all flying things: the class of all flying things excludes some birds.

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| Figure 9 Euler Circles |

The quantity, quality, and distribution of the four forms of a categorical statement are given in Figure 8 The four moods of a categorical statement. The general rule to remember here is that universal statements distribute the subject, and negative statements distribute the predicate.

In 1880 English logician John Venn published two essays on the use of diagrams with circles to represent categorical propositions. Venn noted that the best use of such diagrams so far had come from the brilliant Swiss mathematician Leonhard Euler (see Figure 9 Euler Circles), but they still had many problems, which Venn felt could be solved by bringing in some ideas about logic from his fellow English logician George Boole. Although Venn only claimed to be building on the long logical tradition he traced, since his time these kinds of circle diagrams have been known as Venn diagrams.

In this section we are going to learn to use Venn diagrams to represent our four basic types of categorical statement. Later in this chapter, we will find them useful in evaluating arguments. Let us start with a statement in mood A: “All *S* are *P.*” We are going to use one circle to represent *S* and another to represent *P.* There are a couple of different ways we could draw the circles if we wanted to represent “All *S* are *P.*” One option would be to draw the circle for *S* entirely inside the circle for *P* , as in Figure 9.

It is clear from Figure 9 that all *S* are in fact *P.* And outside of college logic classes, you may have seen people use a diagram like this to represent a situation where one group is a subclass of another. You may have even seen people call concentric circles like this a Venn diagram. But Venn did not think we should put one circle entirely inside the other if we just want to represent “All *S* is *P.*” Technically speaking Figure 9 shows Euler circles.

Venn pointed out that the circles in Figure 9 don’t just say that “All *S* are *P.*” They also says that “All *P* are *S*” is false. But we don’t necessarily know that if we have only asserted “All *S* are *P.*” The statement “All *S* are *P* ” leaves it open whether the *S* circle should be smaller than or the same size as the *P* circle.

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| --- | --- | --- | --- | --- |
| **Mood** | **Form** | **Quantity** | **Quality** | **Terms Distributed** |
| A | All *S* are *P* | Universal | Affirmative | S |
| E | No *S* are *P* | Universal | Negative | S and P |
| I | Some *S* are *P* | Particular | Affirmative | None |
| O | Some *S* are not *P* | Particular | Negative | P |

Figure 10 Quantity, quality, and distribution.

Venn suggested that to represent just the content of a single proposition, we should **always begin by drawing partially overlapping circles**. This means that we always have spaces available to represent the four possible ways the terms can combine.

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| Figure 11 Areas of a Venn Diagram |

Area 1 represents things that are *S* but not *P* ; area 2, things that are *S* and *P* ; area 3, things that are just *P* ; and area 4 represents things that are neither *S* nor *P.* We can then mark up these areas to indicate whether something is there or could be there.

We shade a region of the diagram to represent the claim that nothing can exist in that region. For instance, if we say “All *S* are *P* ,” we are asserting that nothing can exist that is in the *S* circle unless it is also in the *P* circle. So, we shade out the part of the *S* circle that doesn’t overlap with *P.*

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| Figure 12 All S are P |

If we want to say that something does exist in a region, we put an “x” in it. This is the diagram for “Some *S* are *P”*:

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| Figure 13 Some S are P |

If a region of a Venn diagram is blank, if it is neither shaded nor has an x in it, it could go either way. Maybe such things exist, maybe they do not.

The Venn diagrams for all four basic forms of categorical statements are in Figure [4.4](#_bookmark87). Notice that when we draw diagrams for the two universal forms, A and E, we do not draw any x’s. **For these forms we are only ruling out possibilities, not asserting that things actually exist**. This is part of what Venn learned from Boole, and we will see its importance later in the book.

|  |  |
| --- | --- |
|  |  |
| All S are P | No S are P |
|  |  |
| Some S are P | No S are P |
| Figure 14 Venn Diagrams for the Four Basic Forms of a Categorical Statement | |

Finally, notice that so far, we have only been talking about categorical statements involving the variables *S* and *P.* Sometimes, though, we will want to represent statements in regular English. To do this, we will include a dictionary saying what the variables *S* and *P* represent in this case. For instance, this is the diagram for “No dogs are reptiles.”

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*S*: Dogs

*P* : Reptiles

#### Practice Exercises

##### Part A

Identify each of the following sentences as A, E, I, or O; state its quantity and quality; and state which terms are distributed. Then draw the Venn Diagram for each.

**Example**:

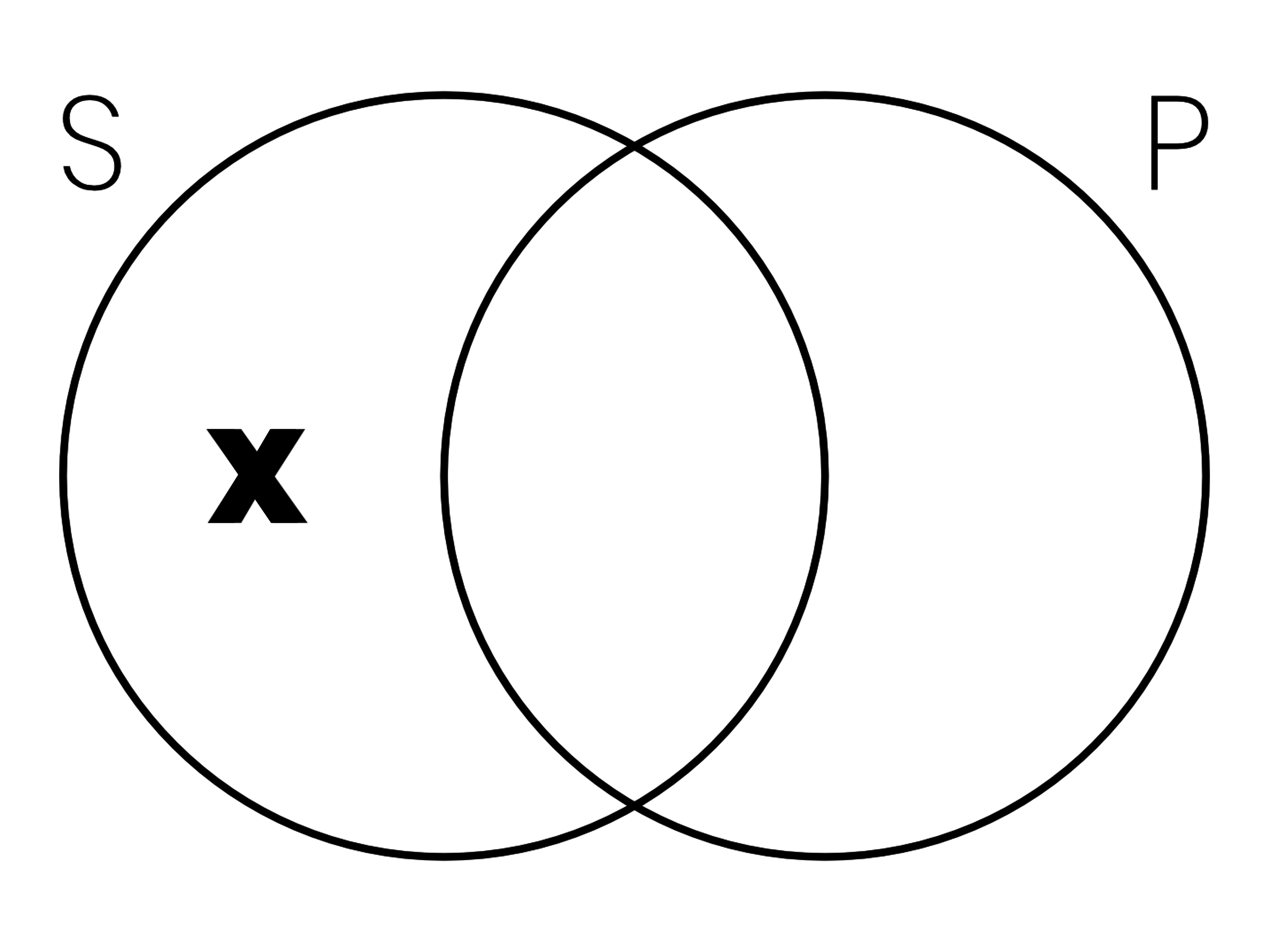
Some dinosaurs are not herbivores

**Answer**:

Form: O

Quantity: particular

Quality: negative



S: Dinosaurs

P : Herbivores

1. All gerbils are rodents.
2. Some planets do not have life.
3. Some manatees are not rappers.
4. All rooms have televisions.
5. All stores are closed.
6. Some dancers are graceful.
7. No extraterrestrials are in Cleveland.
8. Some crates are empty.
9. No customers are mistaken.
10. All cats love catnip.

##### Part B

Identify each of the following sentences as A, E, I, or O; state its quantity and quality; and state which terms are distributed. Then draw the Venn Diagram for each.

1. No appeals are rejected.
2. All bagels are boiled.
3. Some employees are late.
4. All forgeries are discovered eventually.
5. Some shirts are purple.
6. Some societies are matriarchal.
7. No sunflowers are blue.
8. Some appetizers are filling.
9. Some jokes are funny.
10. Some arguments are invalid.

##### Part C

Transform the following sentences by switching their quantity, but not their quality.

**Example**:

Some dogs have fleas.

**Answer**:

All dogs have fleas.

1. Some trees are not evergreen.
2. All smurfs are blue.
3. Some swords are sharp.
4. Some sweaters are not soft.
5. All snails are invertebrates.
6. Some puffins are not large.
7. Some Smurfs are female.
8. All guitars are stringed instruments.
9. No lobsters are extraterrestrial.
10. Some metals are alloys

##### Part D

Transform the following sentences by switching their quality, but not their quantity.

**Example**:

Some elephants are in zoos.

**Answer**:

Some elephants are not in zoos.

1. Some lobsters are white.
2. Some responsibilities are onerous.
3. No walls are bridges.
4. Some riddles are not clever.
5. All red things are colored.
6. All drums are musical instruments.
7. No grandsons are female.
8. Some crimes are felonies.
9. Some airplanes are not commercial.
10. All scorpions are arachnids.

##### Part F

Transform the following sentences by switching both their quality and quantity.

**Example**:

No sharks are virtuous.

**Answer**:

Some sharks are virtuous.

1. No lobsters are vertebrates.
2. Some colors are not pastel.
3. All tents are temporary structures.
4. No goats are bipeds.
5. Some shirts are plaid.
6. No shirts are pants.
7. All ducks are birds.
8. Some possibilities are not likely events.
9. Some raincoats are blue.
10. Some days are holidays.

### 3 Transforming English into Logically Structured English

Because the four basic forms are stated using variables, they have a great deal of generality. We can expand on that generality by showing how many different kinds of English sentences can be transformed into sentences in our four basic forms. We already touched on this a little in section [4.1](#_bookmark72), when we look at sentences like “Thirty percent of Canadians speak French.” There we saw that the predicate was not explicitly a class. We needed to change “speak French” to “people who speak French.” In this section, we are going to expand on that to show how ordinary English sentences can be transformed into something we will call “logically structured English.” logically structured English is English that has been put into a standardized form that allows us to see its logical structure more clearly and removes ambiguity. Doing this is a step towards the creation of formal languages, which we will start doing in Chapter [6](#_bookmark153).

Transforming English sentences into logically structured English is fundamentally a matter of understanding the meaning of the English sentence and then finding the logically structured

English statements with the same or similar meaning. Sometimes this will require judgment calls. English, like any natural language, is fraught with ambiguity. One of our goals with logically structured English is to reduce the amount of ambiguity. Clarifying ambiguous sentences will always require making judgments that can be questioned. Things will only get harder when we start using full blown formal languages in Chapter [6](#_bookmark153), which are supposed to be completely free of ambiguity.

To transform a quantified categorical statement into logically structured English, we have to put all of its elements in a fixed order and be sure they are all of the right type. All statements must begin with the quantifies “All” or “Some” or the negated quantifier “No.” Next comes the subject term, which must be a plural noun, a noun phrase, or a variable that stands for any plural noun or noun phrase. Then comes the copula “are” or the negated copula “are not.” Last is the predicate term, which must also be a plural noun or noun phrase. We also specify that you can only say “are not” with the quantifier “some,” that way the universal negative statement is always phrased “No *S* are *P* ,” not “All S are not *P.*” Taken together, these criteria define the standard form for a categorical statement in logically structured English.

The subsections below identify different kinds of changes you might need to make to put a statement into logically structured English. Sometimes translating a sentence will require using multiple changes.

#### Nonstandard Verbs

In section [4.1](#_bookmark72) we saw that “Some Canadians speak French” has a verb phrase “speaks French” instead of a copula and a plural noun phrase. To transform these sentences into logically structured English, you need to add the copula and turn all the terms into plural nouns or plural noun phrases.

Below are some examples

|  |  |
| --- | --- |
| **English** | **Logically Structured English** |
| No cats bark. | No cats are animals that bark. |
| All birds can fly. | All birds are animals that can fly. |
| Some thoughts should be left unsaid. | Some thoughts are things that should be left unsaid. |

Adding a plural noun phrase means you have to come up with some category, like “people” or “animals.” When in doubt, you can always use the most general category, “things.”

#### Implicit Noun Phrases

Sometimes you just have an adjective for the predicate, and you need to turn it into a noun, as in the examples below.

|  |  |
| --- | --- |
| **English** | **Logically Structured English** |
| Some roses are red. | Some roses are red flowers. |
| Football players are strong. | All football players are strong persons. |
| Some names are hurtful. | Some names are hurtful things. |

Again, you will have to come up with a category for the predicate, and when it doubt, you can just use “things.”

#### Unexpressed Quantifiers

Sometimes categorical generalizations come without an explicit quantifier, which you need to add.

|  |  |
| --- | --- |
| **English** | **Logically Structured English** |
| Boots are footwear. | All boots are footwear. |
| Giraffes are tall. | All giraffes are tall things. |
| A dog is not a cat. | No dogs are cats. |
| A lion is a fierce creature. | All lions are fierce creatures. |

Notice that in the second sentence we had to make two changes, adding both the words “All” and “things.”

In the last two sentences, the indefinite article “a” is being used to create a kind of generic sentence. Not all sentences using the indefinite article work this way. If a story begins “A man is walking down the street,” it is not talking about all men generically. It is introducing some specific man. For this kind of statement, see the subsection on singular propositions. You will have to use your good judgment and understanding of context to know how the indefinite article is being used.

#### Nonstandard Quantifiers

English has many alternate ways of saying “all” and “some.” You need to change these when translating to logically structured English.

|  |  |
| --- | --- |
| **English** | **Logically Structured English** |
| Every day is a blessing. | All days are blessings. |
| Whatever is a dog is not a cat. | No dogs are cats. |
| Not a single dog is a cat. | No dogs are cats. |
| There are Americans that are doctors. | Some Americans are doctors. |
| Someone in America is a doctor. | Some Americans are doctors. |
| At least a few Americans are doctors. | Some Americans are doctors. |
| Not everyone who is an adult is a logician. | Some adults are not logicians. |
| Most people with a PhD in psychology are female. | Some people with a PhD in psychology are female. |
| Among the things that Sylvia inherited was a large mirror | Some things that Sylvia inherited were large mirrors |

Notice in the last case we are losing quite a bit of information when we transform the sentence into logically structured English. “Most” means more that fifty percent, while “some” could be any percentage less than a hundred. This is simply a price we have to pay in creating a standard logical form. As we will see when we move to constructing artificial languages, no logical language has the expressive richness of a natural language.

#### Singular Propositions

Aristotle treated sentences about individual things, like specific people, differently than either general or particular categorical statements. A statement like “Socrates is mortal,” for Aristotle, was neither A, E, I, nor O. We can expand the power of logically structured English by bringing these kind of singular propositions into our system of categorical propositions. Using phrases like “All things identical to... ” we can turn singular terms into general ones.

|  |  |
| --- | --- |
| **English** | **Logically Structured English** |
| Socrates is mortal. | All persons identical with Socrates are mortal. |
| The Empire State Building is tall. | All things identical to The Empire State Building are tall things. |
| Ludwig was not happy. | No persons identical with Ludwig are happy. |
| A man is walking down the street. | Some men are things that are walking down the street. |

#### Adverbs and Pronouns

In English we use specific adverbs like “everywhere” and “always” to create quantified statements about place and time. We can transform these into logically structured English by talking about “all places” or “all times” and things like that. English also has specific pronouns for quantified statements about people or things, such as “everyone” or “nothing.”

|  |  |
| --- | --- |
| **English** | **Logically Structured English** |
| “Whenever you need me, I’ll be there.” – Michael Jackson | All times that you need me are times that I will be there. |
| “We are never, ever, ever getting back together.” – Taylor Swift | No times are times when we will get back together. |
| “Whoever fights with monsters should be careful lest he thereby become a monster.” –Friedrich Nietzsche | All persons who fight with monsters are persons who should be careful lest they become a monster. |
| “What does not destroy me, makes me stronger.”–Friedrich Nietzsche | All things that do not destroy me are things that make me stronger. |

#### Conditional Statements

A conditional is a statement of the form “If... then....” They will become a big focus of our attention starting in the next chapter when we begin introducing modern formal languages. They are not given special treatment in the Aristotelian tradition, however. Instead, where we can, we just treat them as categorical generalizations:

|  |  |
| --- | --- |
| **English** | **Logically Structured English** |
| If something is a cat, then it is a feline. | All cats are feline. |
| If something is a dog, then it’s not a cat. | No dogs are cats. |

#### Exclusive Propositions

Phrases like “only,” “none but,” or “none except” are used in English to create exclusive propositions. They are applied to the predicate term and exclude everything but the predicate term from the subject term.

|  |  |
| --- | --- |
| **English** | **Logically Structured English** |
| Only people over 21 may drink. | All people who drink are over 21. |
| No one, except those with a ticket, may enter the theater. | All people who enter the theater have a ticket. |
| None but the strong survive. | All people who survive are strong people. |

The important thing to see here is that words like “only” are actually modifying the predicate, and not the subject. So, when you translate them into logically structured English, the order of the words often gets turned around. In “only people over 21 may drink” the predicate is actually “people over 21.”

#### “The Only”

Sentences with “The only” are a little different than sentences that just have “only” in them. The sentence “Humans are the only animals that talk on cell phones” should be translated as “All animals who talk on cell phones are humans.” In this sentence, “the only” introduces the subject, rather than the predicate.

|  |  |
| --- | --- |
| **English** | **Logically Structured English** |
| Humans are the only animals who talk on cell phones. | All animals who talk on cell phones are human. |
| Shrews are the only venomous mammal in North America. | All venomous mammals in North America are shrews. |

#### Practice Exercises

##### Part A

Transform the following into logically structured English; identify it as A, E, I, or O; and provide the appropriate Venn diagram.

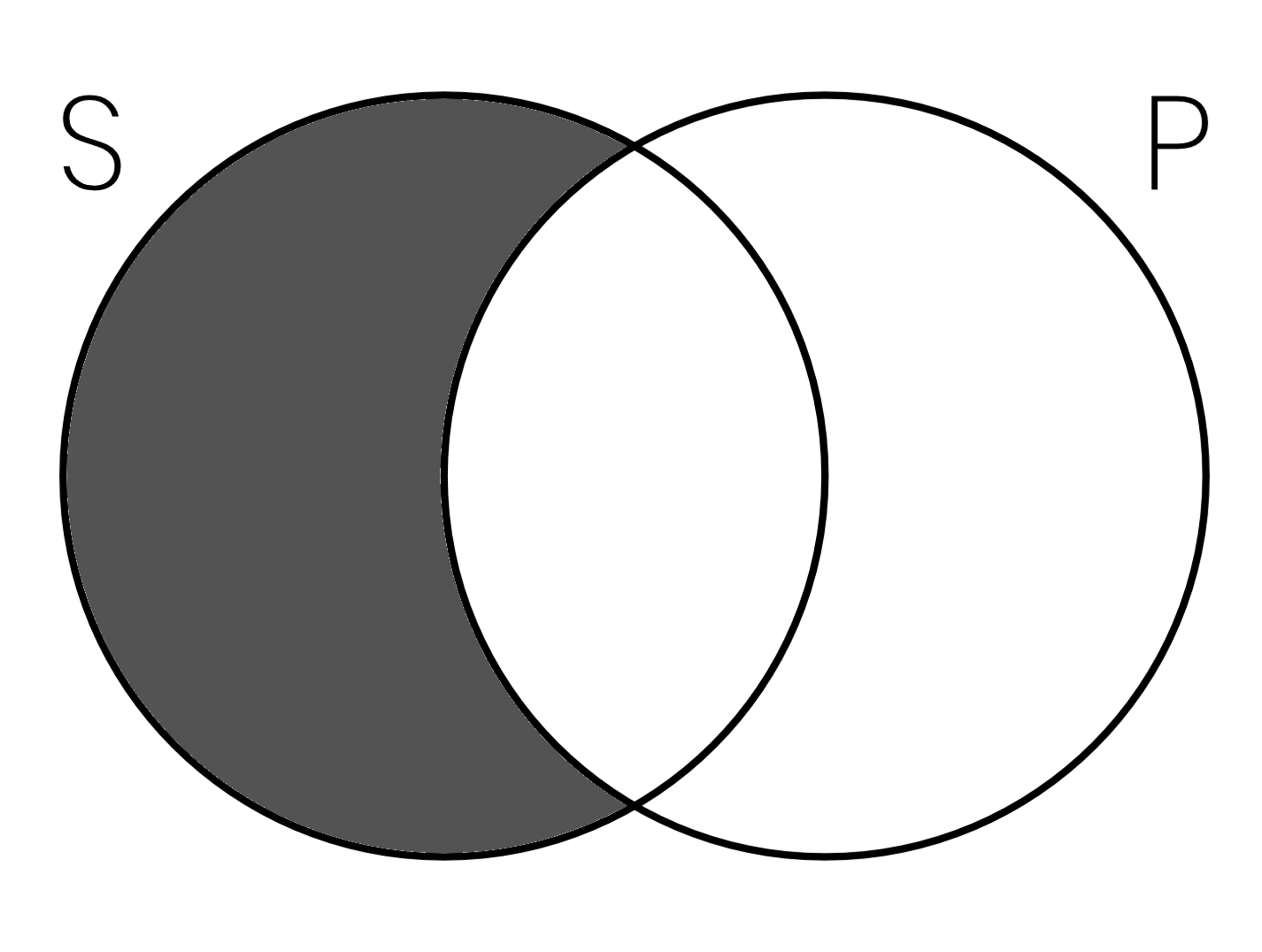
**Example**:

If you can’t stand the heat, get out of the kitchen

**Answer**:

All people who cannot stand the heat are people who should get out of the kitchen.

Form: A



*S*: People who can’t stand the heat

*P* : People who should get out of the kitchen

###### Problems

1. Cats are predators.
2. If something is worth doing, it is worth doing well.
3. Some chimpanzees know sign language.
4. All dogs are loyal.
5. Monotremes are the only egg-laying mammals.
6. Whenever a bell rings, an angel gets its wings.
7. At least one person in this room is a liar.
8. Only natural born citizens can be president of the United States.
9. Gottlob Frege suffered from severe depression.
10. “Anyone who ever had a heart, wouldn’t turn around and break it.” –Lou Reed.

##### Part B

Transform the following into logically structured English, and identify it as A, E, I, or O.

1. If a muffin has frosting, then it is a Wilfred.
2. “Anyone who ever played a part, wouldn’t turn around and hate it.” –Lou Reed.
3. Seahorses are the only fish species in which the male carries the eggs.
4. Seahorses are animals that mate for life.
5. Few dogs are fans of classical music.
6. Ruth Barcan Marcus was a member of the Yale faculty.
7. Only zombies are brain eaters.
8. Some logicians are not mentally ill
9. Some birds eat fish.

##### Part C

Transform the following into logically structured English and identify it as A, E, I, or O. Some problems will require multiple transformations.

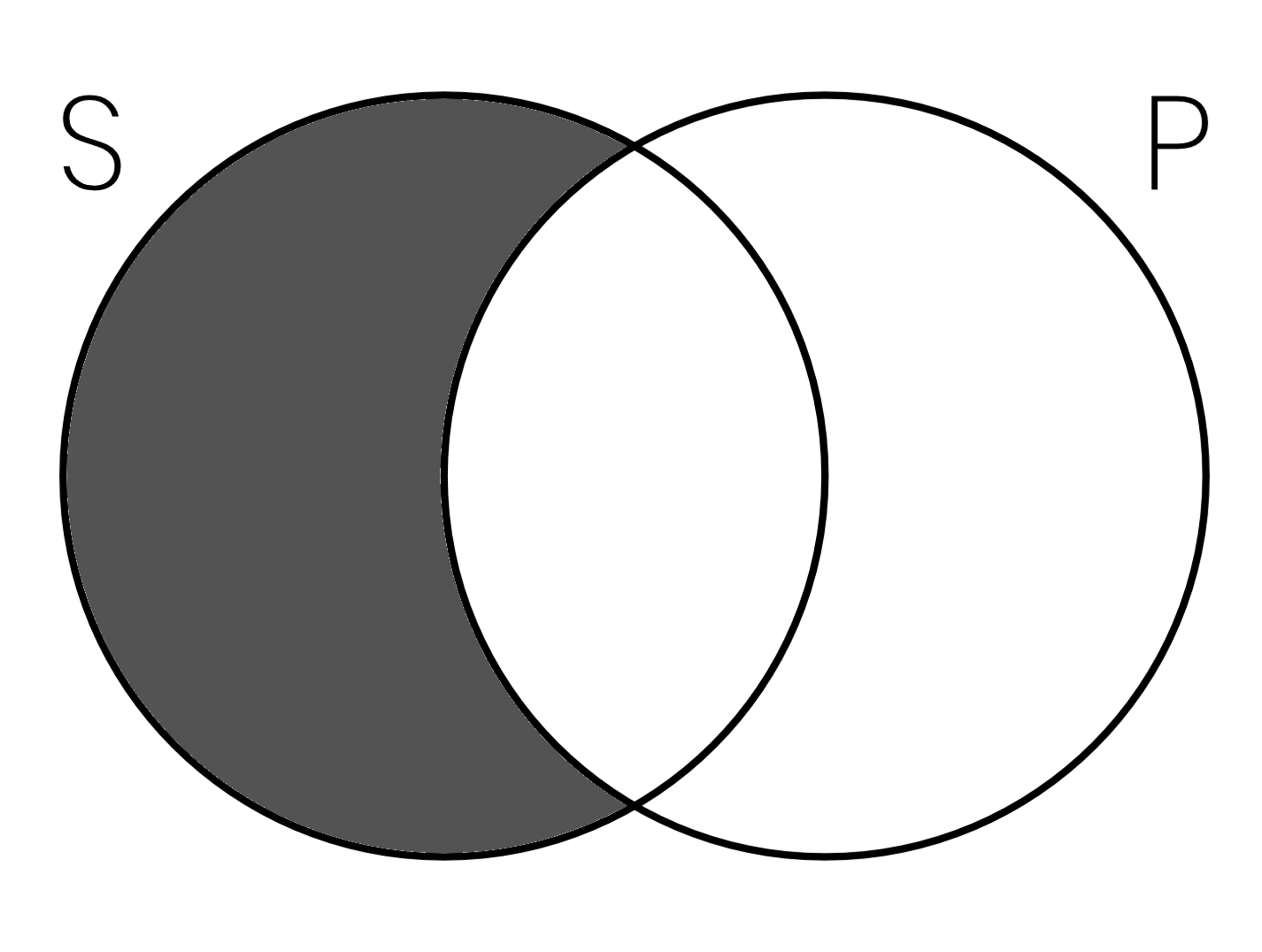
**Example**:

Bertrand Russell was married four times.

**Answer**:

All people who are identical to Bertrand Russell are people who were married four times.

Form: A



*S*: People who are identical to Bertrand Russell

*P* : People who were married four times

1. Many logicians work in computer science.
2. Ludwig Wittgenstein severed in the Austro-Hungarian Army in World War I.
3. Humans can be found everywhere on Earth.
4. Cats are crepuscular.
5. Grover Cleveland was the only president to serve two nonconsecutive terms.
6. Grover Cleveland was the only president named “Grover.”
7. The band’s only singer also plays guitar.
8. If a dog has a collar, it is someone’s pet.
9. Only the basketball players in the class were tall.
10. If you study, then you will pass the test.

##### Part D

Transform the following into logically structured English, and identify it as A, E, I, or O. Some problems will require multiple transformations.

1. People have walked on the moon at least once.
2. Basketball players are tall.
3. Most senior citizens vote.
4. If a bird is a crow, then it is very intelligent.
5. Whoever ate the last cookie is in trouble.
6. “Euclid alone has looked on Beauty bare.” –Edna St. Vincent Millay.
7. If something is a dog, then it is man’s best friend.
8. More than a few students will fail the test.
9. Mercury is the only metal that is liquid at room temperature.
10. Bertrand Russell was married four times.

### 4 Conversion, Obversion, and Contraposition

Now that we have shown the wide range of statements that can be represented in our four standard logical forms A, E, I, and O, it is time to begin constructing arguments with them. The arguments we are going to look at are sometimes called “immediate inferences” because they only have one premise. We are going to learn to identify some valid forms of these one-premise arguments by looking at ways you can transform a true sentence that maintain its truth value.

For instance, “No dogs are reptiles” and “No reptiles are dogs” have the same truth value and basically mean the same thing. On the other hand if you change “All dogs are mammals” into “All mammals are dogs” you turn a true sentence into a false one. In this section we are going to look at three ways of transforming categorical statements—conversion, obversion, and contraposition—and use Venn diagrams to determine whether these transformations also lead to a change in truth value. From there we can identify valid argument forms.

#### Conversion

The two examples in the last paragraph are examples of conversion. conversion is the process of transforming a categorical statement by switching the subject and the predicate. When you convert a statement, it keeps its form—an A statement remains an A statement, an E statement remains an E statement—however it might change its truth value. The Venn diagrams in Figure 14 on page 120 illustrate this.

| **Original** | **Conversion** |
| --- | --- |
| Mood A: All S are P | Mood A: All P are S |
| Mood E: No S are P | Mood E: No P are S. |
| Mood I: Some S are P | Mood I: Some P are S |
| Mood O: Some S are not P | Mood O: Some P are not S |
| Figure 15 Conversions of the Four Basic Forms | |

As you can see, the Venn diagram for the converse of an E statement is exactly the same as the original E statement, and likewise for I statements. This means that the two statements are logically equivalent. Recall that two statements are logically equivalent if they always have the same truth value. In this case, that means that if an E statement is true, then its converse is also true, and if an E statement is false, then its converse is also false. For instance, “No dogs are reptiles” is true, and so is “No reptiles are dogs.” On the other hand “No dogs are mammals” is false, and so is “No mammals are dogs.”

Likewise, if an I statement is true, its converse is true, and if an I statement is false, than its converse is false. “Some dogs are pets” is true, and so is “Some pets are dogs.” On the other hand “Some dogs can fly” is false and so is “Some flying things are dogs.”

The converses of A and O statements are not so illuminating. As you can see from the Venn diagrams, these statements are not identical to their converses. They also don’t contradict their converses. If we know that an A or O statement is true, we still don’t know anything about their converses. We say their truth value is undetermined.

Because E and I statements are logically equivalent to their converses, we can use them to construct valid arguments. An argument is valid if it is impossible for its conclusion to be false whenever its premises are true. Because E and I are logically equivalent to their converses, the two argument forms in **Error! Reference source not found.** are valid.

**Mood O Conversion Argument**

P1) No S are P.

C) No P are S.

**Mood I Conversion Argument**

P1) Some S are P

C) Some P are S.

Figure 16 Valid forms via Conversion

Notice that these are argument forms, with variables in the place of the key terms. This means that these arguments will be valid no matter what; *S* and *P* could be people, or squirrels, or the Gross Domestic Product of industrialized nations, or anything, and the arguments are still valid. While these particular argument forms may seem trivial and obvious, we are beginning to see some of the power of formal logic here. We have uncovered a very general truth about the nature of validity with these two argument forms.

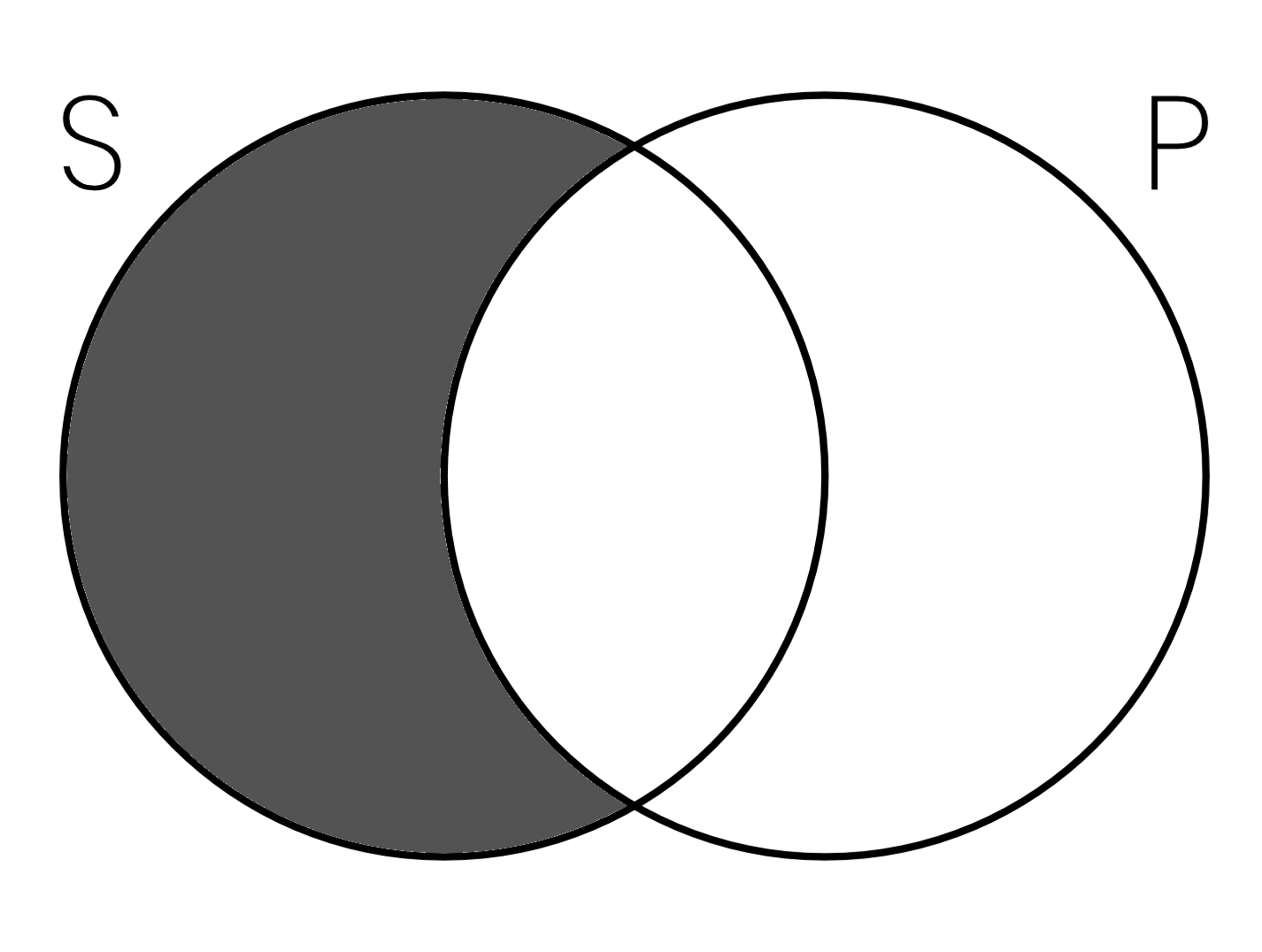
The truth value of the converses of A and O statements, on the other hand, are undetermined by the truth value of the original statements. This means we cannot construct valid arguments from them. Imagine you have an argument with an A or O statement as its premise and the converse of that statement as the conclusion. Even if the premise is true, we know nothing about the truth of the conclusion. So there are no valid argument forms to be found here.

#### Obversion

Obversion is a more complex process. To understand what an obverse is, we first need to define the complement of a class**. The complement of a class is everything that is not in the class**. So the complement of the class of dogs is everything that is not a dog, including not just cats, but battleships, pop songs, and black holes. In English we can easily create a name for the complement of any class using the prefix “non-”. So the complement of the class of dogs is the class of non-dogs. We will use complements in defining both obversion and contraposition.

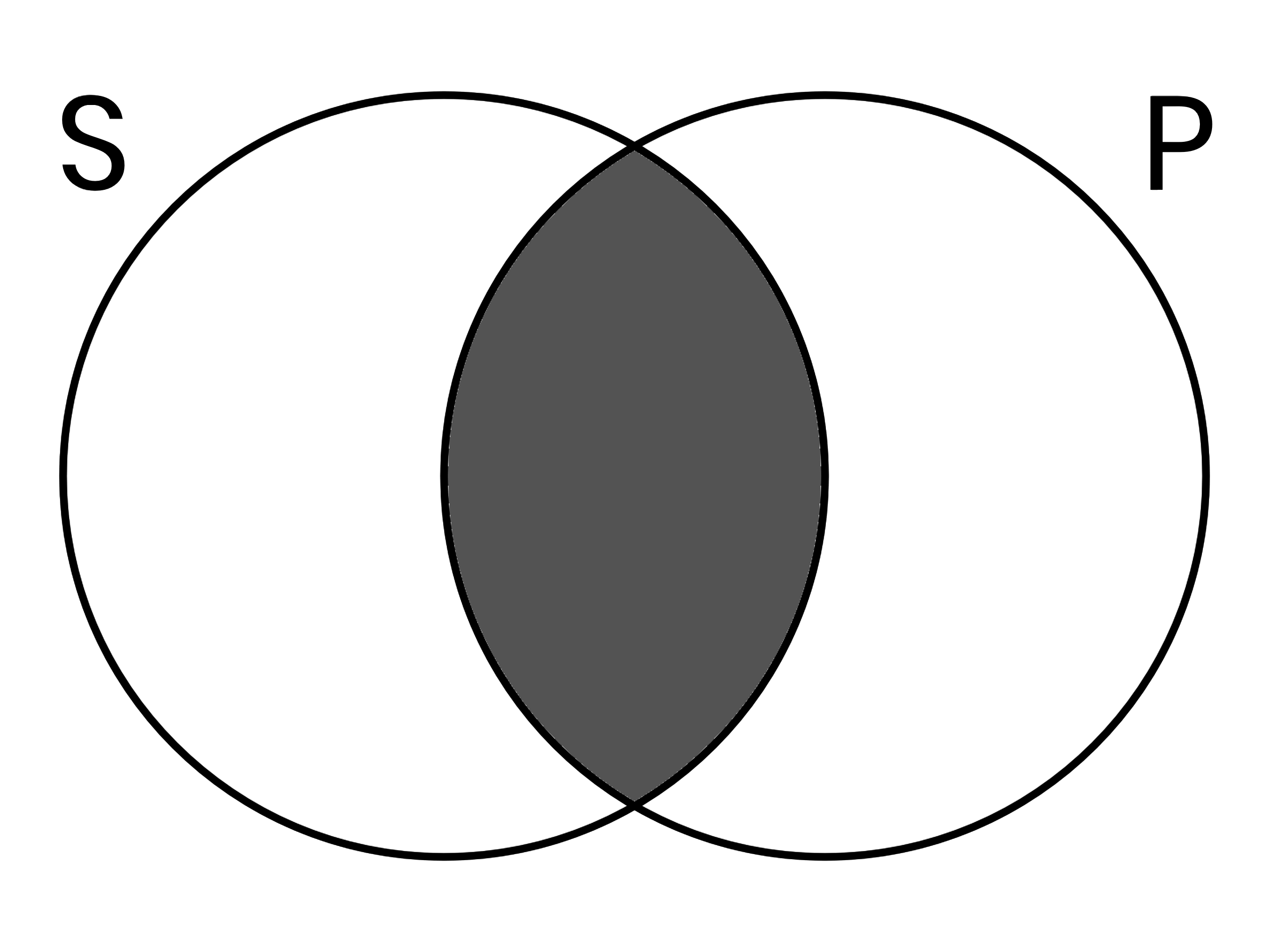
The obversion of a categorical proposition is a new proposition created by changing the quality of the original proposition and switching its predicate to its complement. Obversion is thus a two step process. Take, again, the proposition “All dogs are mammals.” For step 1, we change its quality, in this case going from affirmative to negative. That gives us “No dogs are mammals.” For step 2, we take the complement of the predicate. The predicate in this case is “mammals” so the complement is “non-mammals.” That gives us the obverse “No dogs are non-mammals.”

We can map this process out using Venn diagrams. Let’s start with an A statement.



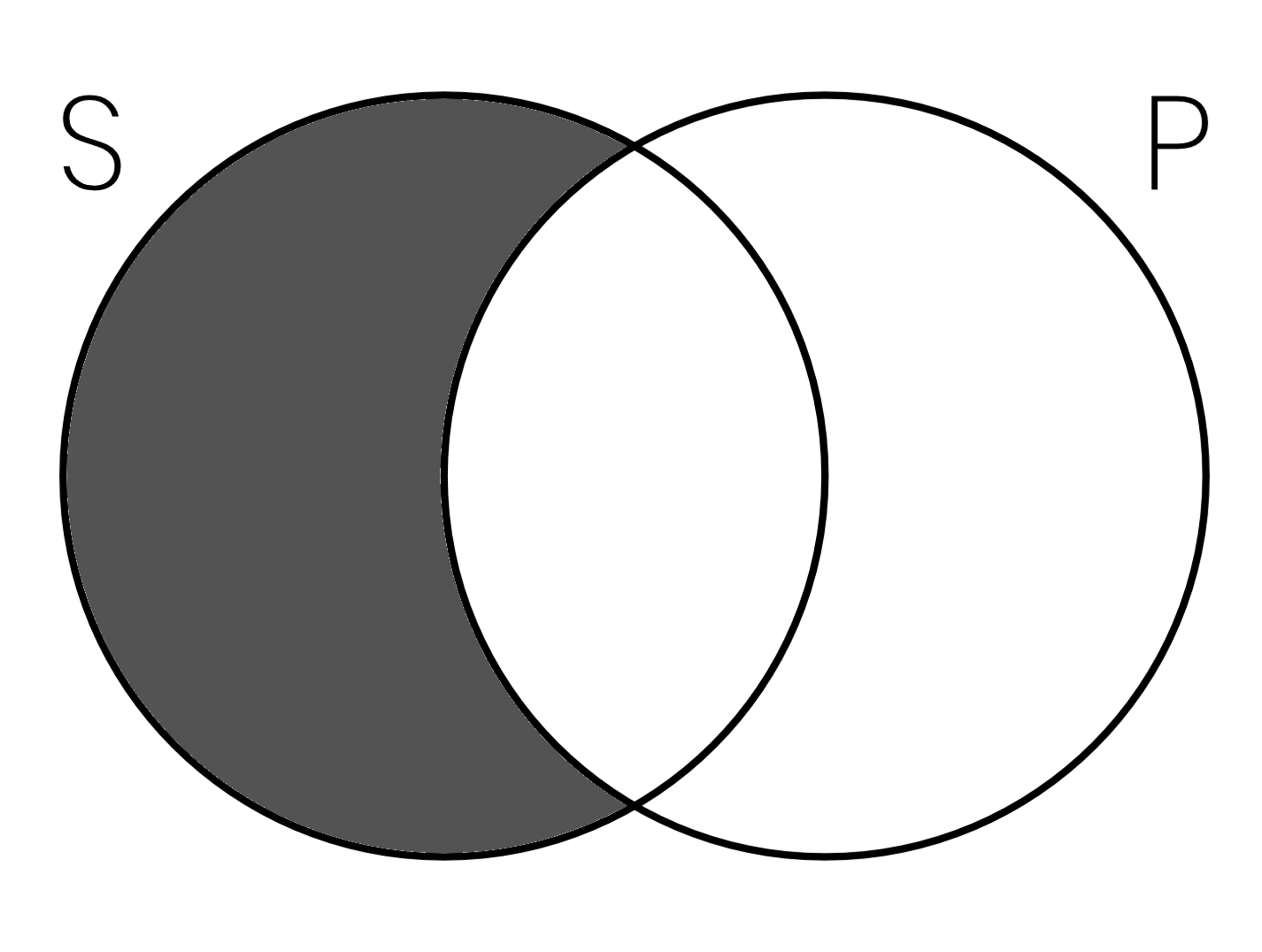
**A**: All *S* are *P.*

Changing the quality turns it into an E statement.



**E**: No *S* are *P.*

Now what happens when we take the complement of *P* ? That means we will shade in all the parts of S that are non-*P* , which puts us back where we started.



E: No S are non-P.

The final statement is logically equivalent to the original A statement.

All S are P ≡ No S are non-P

It has the same form as an E statement, but because we have changed the predicate, it is not logically equivalent to an A statement. As you can see from Figure 17 this is true for all four forms of categorical statement.

| **Original** | **Obverse** |
| --- | --- |
| Mood A: All S are P | Mood E: No S are non-P |
| Mood E: No S are P | Mood A: All S are non-P |
| Mood I: Some S are P | Mood O: Some S are not non-P |
| Mood O: Some S are not P | Mood I: Some S are non-P. |
| Figure 17 Obversions of the Four Basic Forms | |

This in turn gives us four valid argument forms below.

**Mood A**

P1) All S are P.

C) No S are non-P.

**Mood E**

P1) No S are P

C) All S are non-P

**Mood O**

P1) Some S are not non-P

C) Some S are not non-P.

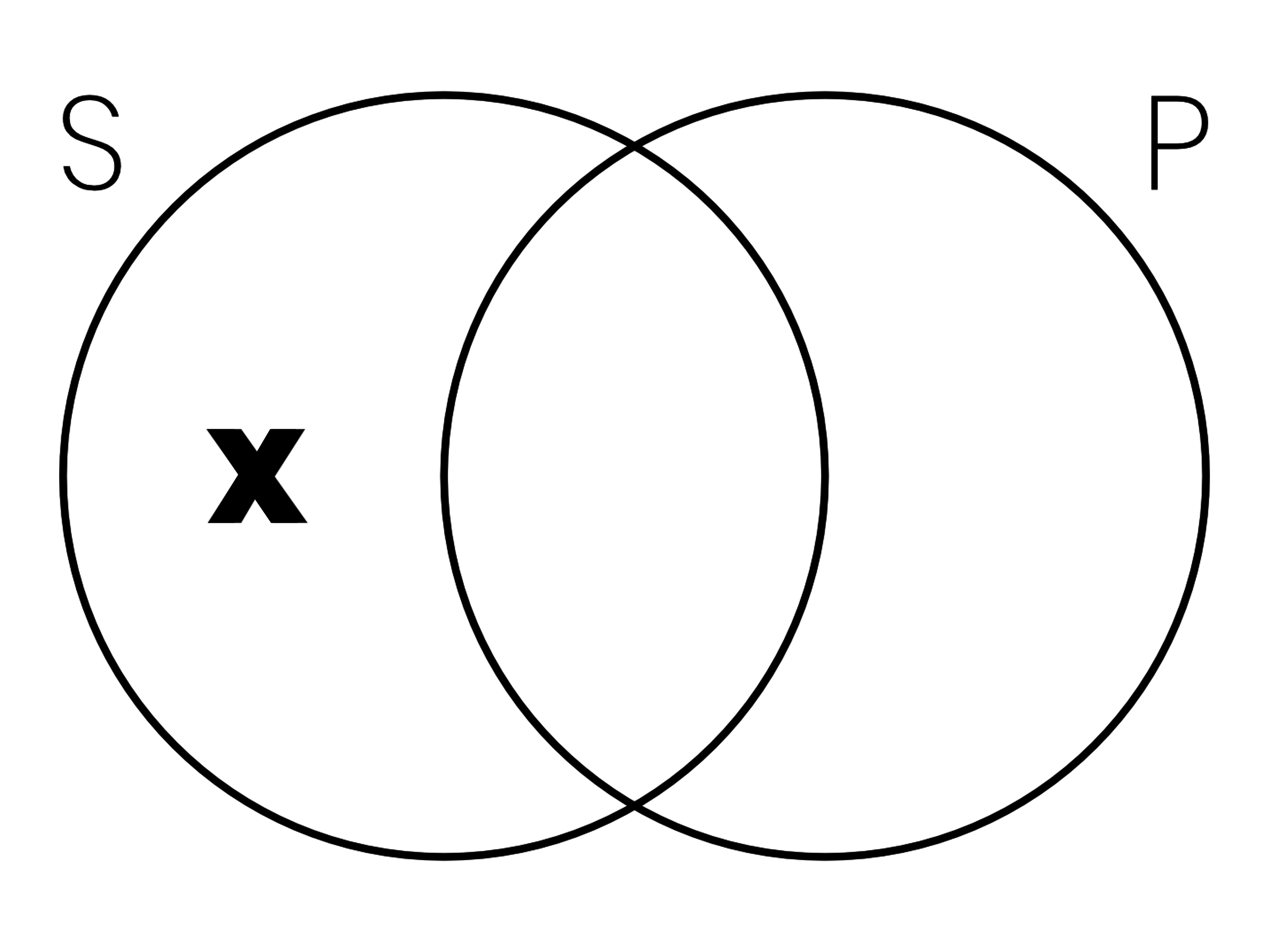
**Mood I**

P1) Some S are not P.

C) Some S are non-P

Figure 18 Valid Arguments via Obversion

One further note on complements. We don’t just use complements to describe sentences that come out of obversion and contraposition. We can also perform these operations on statements that already have complements in them. Consider the sentence “Some *S* are non-*P.*” This is its Venn diagram.



Some *S* are non-*P*

How would we take the obverse of this statement? Step 1 is to change the quality, making it “Some *S* are not non-*P.*” Now how do we take the complement of the predicate? We could write “non-non-*P,*” but if we think about it for a second, we’d realize that this is the same thing as *P.* So we can just write “Some *S* is not *P.*” This is logically equivalent to the original statement, which is what we wanted.

Taking the converse of “Some *S* are non-*P”* also takes a moment of thought. We are supposed to reverse subject and predicate. But does that mean that the “non-” moves to the subject position along with the “*P”*? Or does the “non-” now attach to the *S*? We saw that E and I statements kept their truth value after conversion, and we want this to still be true when the statements start out referring to the complement of some class. This means that the “non-” has to travel with the predicate, because “Some *S* are non-*P* ” will always have the same truth value as “Some non-*P* are *S*.” Another way of thinking about this is that the “non-” is part of the name of the class that forms the predicate of “Some *S* are non-*P.*” The statement is making a claim about a class, and that class happens to be defined as the complement of another class. So, the bottom line is when you take the converse of a statement where one of the terms is a complement, move the “non-” with that term.

#### Contraposition

Contraposition is a two-step process, like obversion, but it doesn’t always lead to results that are logically equivalent to the original sentence. The contrapositive of a categorical sentence is the sentence that results from reversing subject and predicate and then replacing them with their complements. Thus “All *S* are *P* ” becomes “All non-*P* are non-*S*.”

| **Original** | **Contraposition** |
| --- | --- |
| Mood A: All S are P | Mood A: All non-P are non-S |
| Mood E: No S are P | Mood E: No non-P are non-S |
| Mood I: Some S are P | Mood I: Some non-P are non-S |
| Mood O: Some S are not P | Mood O: Some non-P are not non-S |
| Figure 19 Contrapositions the Four Basic Forms | |

Figure 19 shows the corresponding Venn diagrams. In this case, the shading around the outside of the two circles in the contraposed form of E is meant to indicate that nothing can lie outside the two circles. Everything must be *S* or *P* or both. Like conversion, applying contraposition to two of the forms gives us statements that are logically equivalent to the original. This time, though, it is forms A and O that come through the process without changing their truth value.

In this case, the shading of the two circles in the contraposed form of E is meant to indicate that nothing can lie outside the two circles. Everything must be *S* or *P* or both. Like conversion, applying contraposition to two of the forms gives us statements that are logically equivalent to the original. This time, though, it is forms A and O that come through the process without changing their truth value.

**Mood A**

P1) All S are P

C) All non-P are non-S

**Mood O**

P1) Some S are not P

C) Some non-P are not non-S

Figure 20 Valid forms from Contraposition

This then gives us two valid argument forms, shown in Figure [4.10](#_bookmark102). If you have an argument with an A or O statement as its premise and the contraposition of that statement as the conclusion, you know it must be valid. Whenever the premise is true, the conclusion must be true, because the two statements are logically equivalent. On the other hand, if you had an E or an I statement as the premise, the truth of the conclusion is undetermined, so these arguments would not be valid.

#### Evaluating Short Arguments

So far we have seen eight valid forms of argument with one premise: two arguments that are valid by conversion, four that are valid by obversion, and two that are valid by contraposition. As we said, short arguments like these are sometimes called “immediate inferences,” because your brain just flits automatically from the truth of the premises to the truth of the conclusion. Now that we have identified these valid forms of inference, we can use this knowledge to see whether some of the arguments we encounter in ordinary language are valid. We can now tell in a few cases if our brain is right to flit so seamlessly from the premise to the conclusion.

In the real world, the inferences we make are messy and hard to classify. Much of the complexity of this issue is tackled in the parts of the complete version of this text that cover critical thinking. Right now we are just going to deal with a limited subset of inferences: immediate inferences that might be based on conversion, obversion, or contraposition. Let’s start start with the uncontroversial premise “All dogs are mammals.” Can we infer from this that all non-mammals are non-dogs? In canonical form, the argument would look like this.

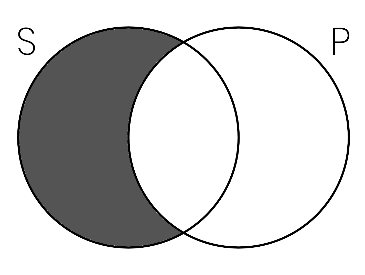
P1: All dogs are mammals

C: All non-mammals are non-dogs.

Evaluating an immediate inference like this is a four-step process. First, identify the subject and predicate classes. Second, draw the Venn diagram for the premise. Third, see if the Venn diagram shows that the conclusion must be true. If it must be, then the argument is valid.

Finally, if the argument is valid, identify the process that makes it valid. (You can skip this step if the argument is invalid.)

For the argument above, the result of the first two steps would look like this:



*S*: Dogs

*P* : Mammals

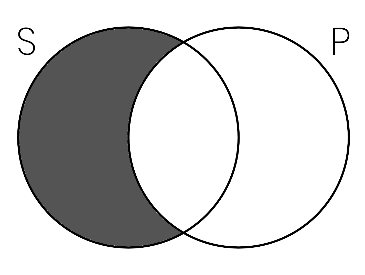
The Venn diagram for the premise shades out the possibility that there are dogs that aren’t mammals. For step three, we ask, does this mean the conclusion must be true? In this case, it does. The same shading implies that everything that is not a mammal must also not be a dog. In fact, the Venn diagram for the premise and the Venn diagram for the conclusion are the same. So the argument is valid. This means that we must go on to step four and identify the process that makes it valid. In this case, the conclusion is created by reversing subject and predicate and taking their complements, which means that this is a valid argument by contraposition.

Now, remember what it means for an argument to be valid. As we said on page [28](#_bookmark36), an argument is valid if it is impossible for the premises to be true and the conclusion false. This means that we can have a valid argument with false premises, so long as it is the case that *if* the premises were true, the conclusion would have to be true. So if the argument above is valid, then so is this one:

P1: All dogs are reptiles.

C: All non-reptiles are non-dogs.

The premise is now false: all dogs are not reptiles. However, *if* all dogs were reptiles, then it would also have to be true that all non-reptiles are non-dogs. The Venn diagram works the same way.



*S*: Dogs

*P* : Reptiles

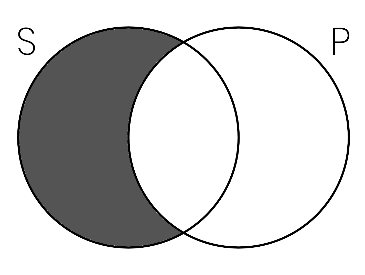
The Venn diagram for the premise still matches the Venn diagram for the conclusion. Only the labels have changed. The fact that this argument form remains true even with a false premise is just a variation on a theme we saw in Figure [2.7](#_bookmark43) when we saw a valid argument (with false premises) for the conclusion “Socrates is a carrot.” So arguments by transposition, just like any argument, can be valid even if they have false premises. The same is true for arguments by conversion and obversion.

Arguments like these can also be invalid, even if they have true premises and a true conclusion. Remember that A statements are not logically equivalent to their converse. So this is an invalid argument with a true premise and a false conclusion:

P1: All dogs are mammals.

C: All mammals are dogs.

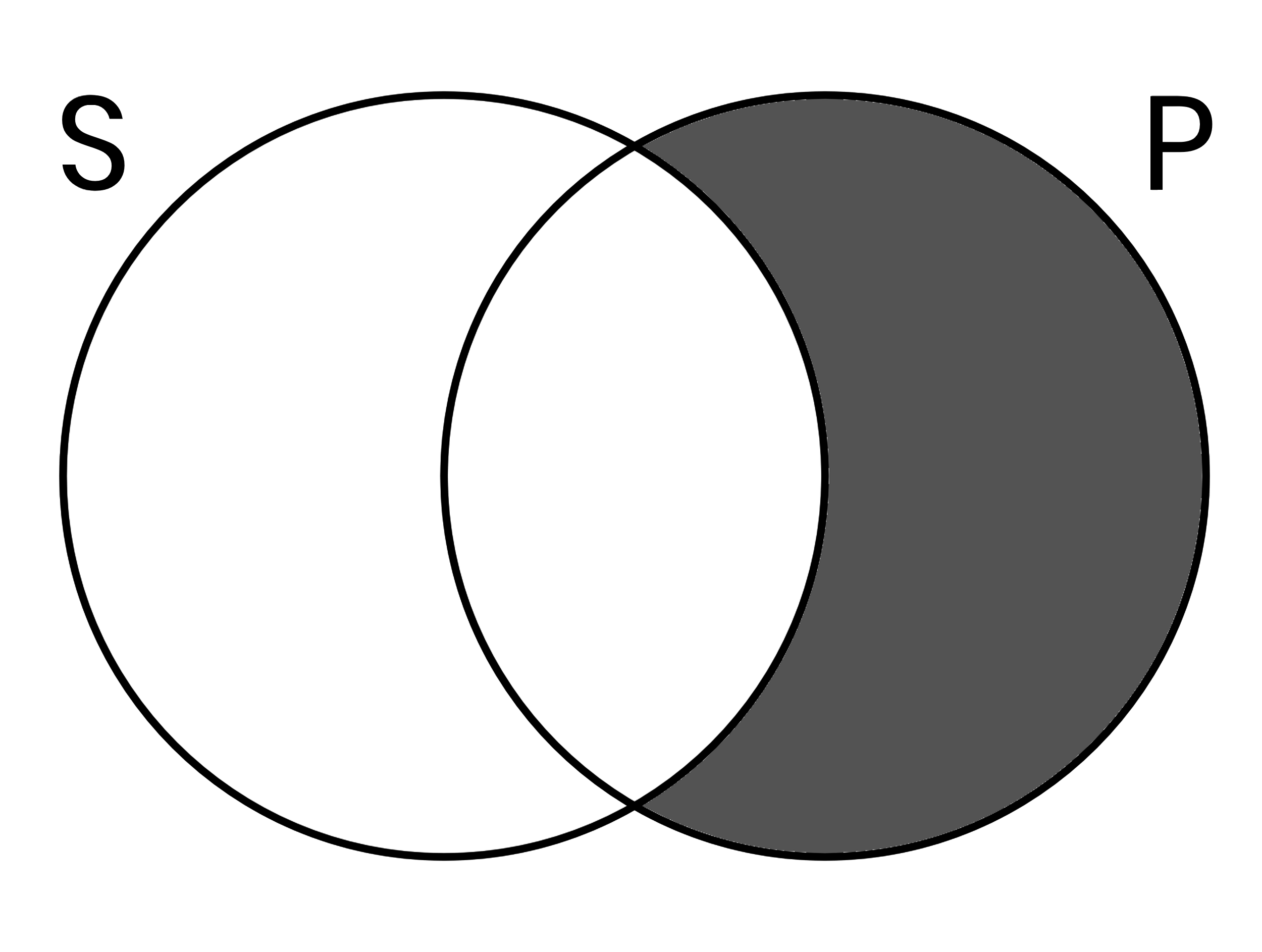
Our Venn diagram test shows that this is invalid. Steps one and two give us this for the premise:



*S*: Dogs

*P* : Mammals

But this is the Venn diagram for the conclusion:



*S*: Dogs

*P* : Mammals

This is an argument by conversion on a mood-A statement, which is invalid. The argument

remains invalid, even if we substitute in a predicate where the conclusion happens to be true. For instance this argument is invalid.

P1: All dogs are *Canis familiaris*.

C: All *Canis familiaris* are dogs.

The Venn diagrams for the premise and conclusion of this argument will be just like the ones for the previous argument, just with different labels. So even though the argument has a true premise and a true conclusion, it is still invalid, because it is possible for an argument of this form to have a true premise and a false conclusion. This is an unreliable argument form that just happened, in this instance, not to lead to a false conclusion. This again is just a variation on a theme we saw in Chapter [2](#_bookmark30), in Figure [2.3](#_bookmark39), when we saw an invalid argument for the conclusion that Paris was in France.

#### Practice Exercises

#### Part A

For each sentence, write the converse, obvserse, or contrapostive as directed.

**Example**:

Write the contrapositive of “Some sentences are categorical."

**Answer**:

Some non-categorical things are non-sentences.

1. Write the converse of “No weeds are benign."
2. Write the converse of “Some minds are not closed."
3. Write the contraposition of “Some dentists are underpaid."
4. Write the converse of “All humor is good."
5. Write the contraposition of “No organizations are self-sustaining."
6. Write the obverse of “Some dogs have fleas."
7. Write the converse of “Some things that have fleas are dogs."
8. Write the obverse of “No detectives are uniformed."
9. Write the converse of “No monkeys are well-behaved."
10. Write the contraposition of “No donkeys are obedient."

#### Part B

The first two columns in the table below give you a statement and a truth value for that statement. The next column gives an operation that can be performed on the statement in the first column, and the final two columns give the new statement and its truth value.

The first row is completed, as an example, but after that there are blanks. In problems 1-5 you must fill in the new statement and its truth value, and in problems 6-10 you must fill in the operation and the final truth value. If the truth value of the resulting statement cannot be determined from the original one, write a “?" for “undetermined."

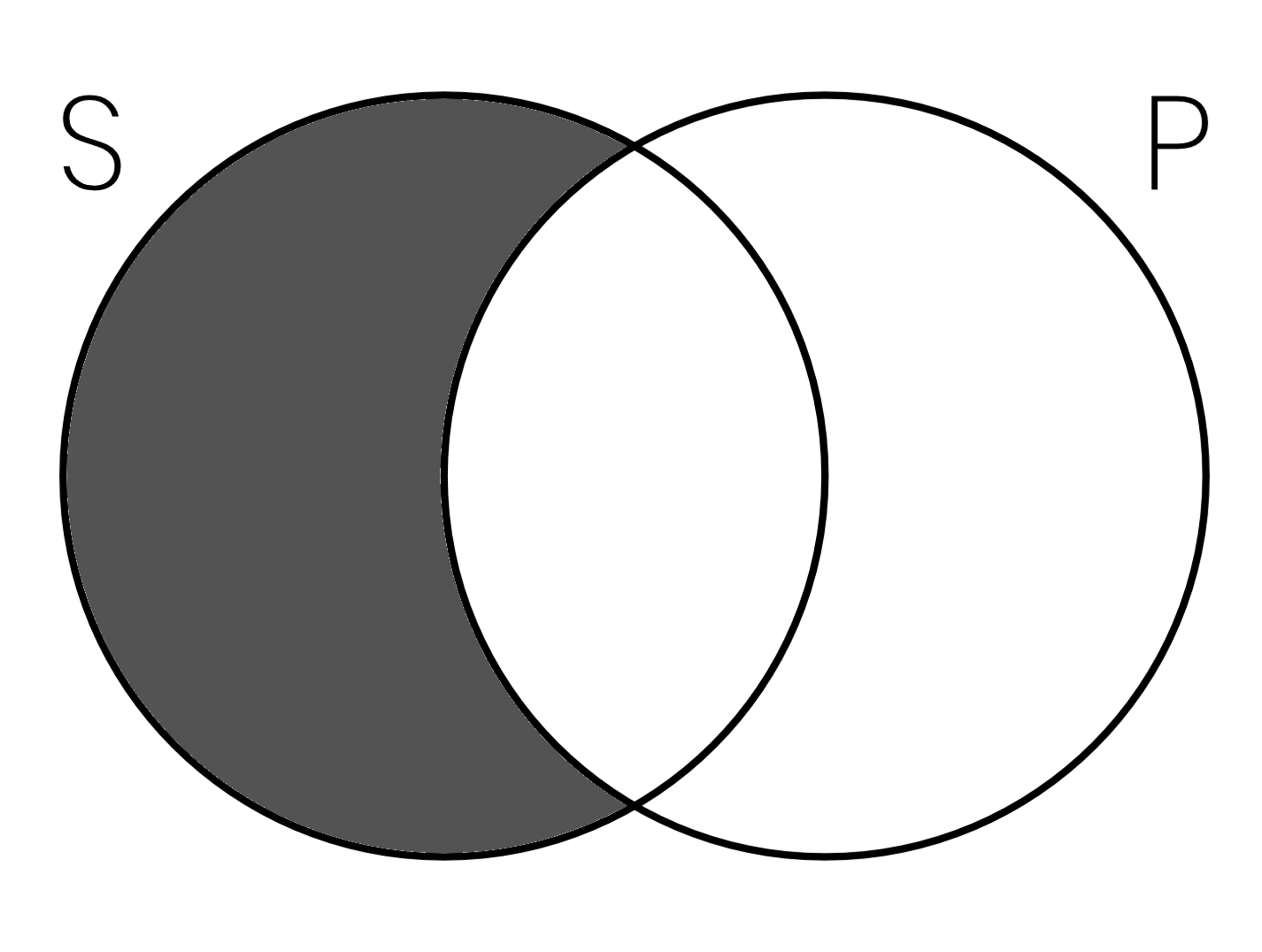
You can check your work with Venn diagrams, or by identifying the logical form of the original statement and seeing if it is one where the named operation changes the truth value.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Given Statement** | **T/F** | **Operation** | **New Statement** |  | **T/F** |
| *EG* | *All S are P* | *F* | *Conv.* | *All S are P* |  | *?* |
| 1 | Some *S* are *P* | F | Obverse |  |  |  |
| 2 | Some non-*S* are *P* | F | Converse |  |  |  |
| 3 | All *S* are *P* | F | Contrap. |  |  |  |
| 4 | Some *S* are *P* | F | Contrap. |  |  |  |
| 5 | Some *S* are non-P | T | Obverse |  |  |  |
| 6 | All *S* are non-*P* | T |  | All P are non-S |  |  |
| 7 | Some non-*S* are not P | T |  | Some P are not non-S |  |  |
| 8 | Some *S* are not P | F |  | Some P are not S |  |  |
| 9 | All non-*S* are *P* | T |  | No non-S are non-P |  |  |
| 10 | No non-*S* are non-P | T |  | All non-S are non-non-P |  |  |

**Part E** Determine whether the following arguments are valid by drawing a Venn diagram for the premise. If they are valid, say whether they are valid by conversion, obversion, or contraposition.

**Example 1**: All swans are white. Therefore, no swans are non-white.

**Answer**:



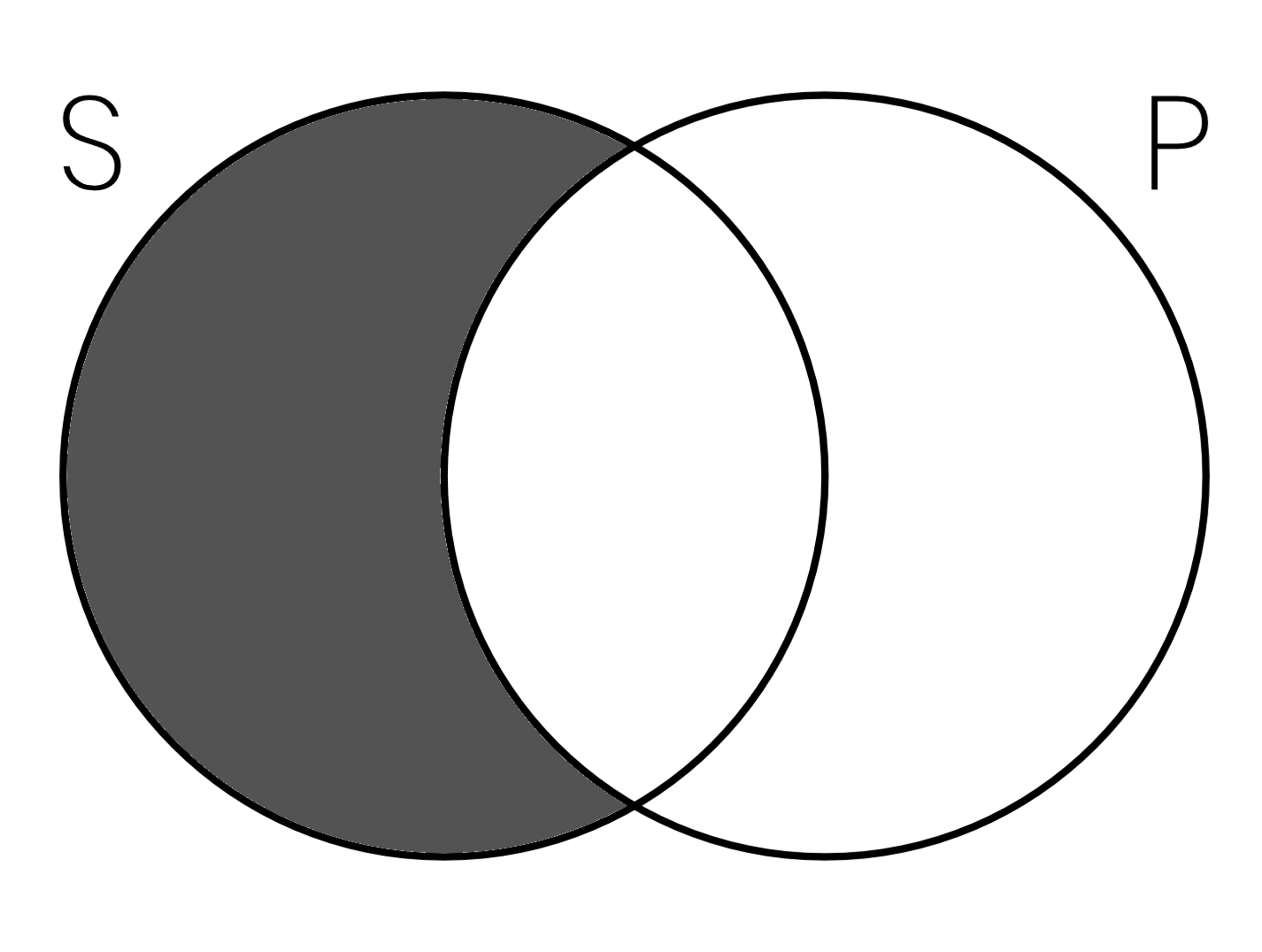
S: Swans

P : White things.

Valid, because the Venn diagram for the premise also makes the conclusion true. Obversion.

**Example 2**: Some dogs are pets. Therefore, some non-pets are non-dogs.

**Answer**:



*S*: Pets

*P* : Dogs.

Invalid, because the Venn diagram for the premise doesn’t make the conclusion true.

1. Some smurfs are not blue. Therefore, some blue things are not smurfs.
2. All giraffes are majestic. Therefore, all non-majestic things are non-giraffes.
3. Some roosters are not pets. Therefore, some pets are not roosters.
4. No anesthesiologists are doctors. Therefore, no doctors are anesthesiologists.
5. All penguins are flightless. Therefore, all flightless things are penguins.
6. No kisses are innocent. Therefore, no non-innocent things are non-kisses.
7. All operas are sung. Therefore, all sung things are operas.
8. Some shopping malls are not abandoned. Therefore, some shopping malls are non-abandoned.
9. Some great-grandfathers are not deceased. Therefore, some non-deceased people are not non-great-grandfathers.
10. Some boats are not seaworthy. Therefore, some boats are non-seaworthy.

### 7 The Traditional Square of Opposition

We have seen that conversion, obversion, and contraposition allow us to identify some valid

one-premise arguments. There are actually more we can find out there, but investigating them is a bit more complicated. The original investigation made by the Aristotelian philosophers made an assumption that logicians no longer make. To help you understand all sides of the issue, we will begin by looking at things in the traditional Aristotelian fashion, and then in the next section move on to the modern way of looking at things.

When Aristotle was first investigating these four kinds of categorical statements, he noticed they conflicted with each other in different ways. If you are just thinking casually about it, you might say that “No *S* is *P* ” is somehow “the opposite” of “All *S* is *P.*” But isn’t the real “opposite” of “All *S* is *P* ” actually “Some *S* is not *P* ”?

Aristotle, in his book *On Interpretation* (c. 350 BCE), notes that the real opposite of A is O, because one must always be true and the other false. If we know that “All dogs are mammals” is true, then we know “some dog is not a mammal” is false. On the other hand, if “All dogs are

mammals” is false then “some dog is not a mammal” must be true. Back on page [52](#_bookmark56) we said that when two propositions must have opposite truth values they are called contradictories. Aristotle noted that A and O sentences are contradictory in this way. Forms E and I also form a contradictory pair. If “Some dogs are mammals” then “No dogs are mammals” is false, and if “Some dogs are mammals” is false, then “No dogs are mammals” is true.

Mood-A and mood-E statements are opposed to each other in a different way. Aristotle claimed that they can’t both be true, but could both be false. Take the statements “All dogs are strays” and “No dogs are strays.” We know that they are both false, because some dogs are strays and others aren’t. However, it is also clear that they could not both be true. When a pair of statements cannot both be true, but might both be false, the Aristotelian tradition says they are contraries. Aristotle’s idea of a pair of contraries is really just a specific case of a set of sentences that are *inconsistent*, an idea that we looked at in Chapter [3](#_bookmark49). (See page [3.3](#_bookmark58))

These distinctions, plus a few other comments from Aristotle, were developed by his later followers into an idea that came to be known as the square of opposition. The square of opposition is simply the diagram you see in Figure [4.11](#_bookmark105). It is a way of representing the four basic propositions and the ways they relate to one another. As we said before, this way of picturing the proposition turned out to make a problematic assumption. To emphasize that this is no longer the way logicians view things, we will call this diagram the traditional square of opposition.

The traditional square of opposition begins by picturing a square with A, E, I, and O at the four corners. The lines between the corners then represent the ways that the kinds of propositions can be opposed to each other. The diagonal lines between A and O and between E and I represent contradiction. These are pairs of propositions where one has to be true and the other false. The line across the top represents contraries. These are propositions that Aristotle thought could not both be true, although they might both be false.

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| Figure 21 Traditional Square of Opposition |

In Figure 21 we have actually drawn each relationship as a pair of lines, representing the kinds of inferences you can make in that relationship. **Contraries** cannot both be true. So, we know that if one is true, the other must be false. This is represented by the two lines going from a T to an F. Notice that there aren’t any lines here that point from an F to something else. This is because you can’t infer anything about contrary statements if you just know that one is false. For the **contradictory** statements, on the other hand, we have drawn double-headed arrows. This is because we know both that the truth of one statement implies that the other is false and that the falsity of one statement implies the truth of the other.

Contraries and contradictories just give us the diagonal lines and the top line of the square. There are still three other sides to investigate. Form I and form O are called **subcontraries**. In the traditional square of opposition, their situation is reversed from that of A and E. Statements of forms A and E cannot both be true, but they can both be false. Statements of forms I and O cannot both be false, but they can both be true. Consider the sentences “Some people in the classroom are paying attention” and “Some people in the classroom are not paying attention.” It is possible for them both to be true. Some people are paying attention and some aren’t. But the two sentences couldn’t both be false. That would mean that everyone in the room was neither paying attention nor not paying attention. But they have to be doing one or the other!

This means that there are two inferences we can make about subcontraries. We know that if I is false, O must be true, and vice versa. This is represented in Figure 21 by arrows going from Fs on one side to Ts on the other. This is reversed from the way things were on the top of the square with the contraries. Notice that this time there are no arrows going away from a T. This is because we can’t infer anything about subcontraries if all we know is that one is true

The trickiest relationship is the one between universal statements and their corresponding particulars. We call this **subalternation**. Both of the statements in these pairs could be true, or they could both be false. However, in the traditional square of opposition, if the universal statement is true, its corresponding particular statement must also be true. For instance, “All dogs are mammals” implies that some dogs are mammals. Also, if the particular statement is false, then the universal statement must also be false. Consider the statement “Some dinosaurs had feathers.” If that statement is false, if no dinosaurs had feathers, then “All dinosaurs have feathers” must also be false. Something like this seems to be true on the negative side of the diagram as well. If “No dinosaurs have feathers” is true, then you would think that “some dinosaurs do not have feathers” is true. Similarly, if “some dinosaurs do not have feathers” is false, then “No dinosaurs have feathers” cannot be true either.

In our diagram for the traditional square of opposition, we represent subalternation by a downward arrow for truth and an upward arrow for falsity. We can infer something here if we know the top is true, or if we know the bottom is false. In other situations, there is nothing we can infer.

Note, by the way, that the language of subalternation works a little differently than the other relationships. With contradiction, we say that each sentence is the “contradictory” of the other. The relationship is symmetrical. With subalternation, we say that the particular sentence is the “subaltern” of the universal one, but not the other way around.

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| Figure 22 One of the earliest surviving versions of the square of opposition, from a 9th century manuscript of a commentary by Apuleius of Madaura on Aristotle’s On Interpretation. Digital image from www.logicmuseum.com, curated by Edward Buckner. |

People started using diagrams like this as early as the second century ce to explain Aristotle’s ideas in *On Interpretation* (See Parsons). Figure 22 shows one of the earliest surviving versions of the square of opposition, from a 9th century manuscript of a commentary on Aristotle attributed to the Roman writer Apuleius of Madaura. Although this particular manuscript dates from the 9th century, the commentary itself was written in the 2nd century, and copied by hand many times over before this one was made. Figure 23 shows a later illustration of the square, from a 16th century book by the Scottish philosopher and logician Johannes de Magistris.

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|  |
| Figure 23 A 16th century illustration of the square of opposition from *Summulae Logicales* by Johannes de Magistris, digital image by Peter Damian and uploaded to Wikimedia Commons. Public Domain-U.S. |

As with the processes of conversion, obversion, and contraposition, we can use the traditional square of opposition to evaluate arguments written in canonical form. It will help us here to introduce the phrase “It is false that” to some of our statements, so that we can make inferences from the truth of one proposition to the falsity of another. This, for instance, is a valid argument, because A and O statements are contradictories.

P1: All humans are mortal.

C: It is false that some human is not mortal.

The argument above is an immediate inference, like the arguments we saw in the previous section, because it only has one premise. It is also similar to those arguments in that the conclusion is actually logically equivalent to the premise. This will not be the case for all immediate inferences based on the square of opposition, however. This is a valid argument, based on the subaltern relationship, but the premise and the conclusion are not logically equivalent.

P1: It is false that some humans are dinosaurs.

C: It is false that all humans are dinosaurs.

#### Practice Exercises

##### Part A

For each pair of sentences say whether they are contradictories, contraries, subcontraries, or one is the subaltern of the other.

**Example**:

Some peppers are spicy.

No peppers are spicy.

**Answer**:

Contradictory

1. No quotations are spurious.
2. Some quotations are not spurious.
3. Some children are not picky eaters. All children are picky eaters.
4. Some joys are not fleeting. Some joys are fleeting.
5. All fires are hot.
6. Some fires are not hot.
7. Some diseases are not fatal. No diseases are fatal.
8. Some planets are not habitable. Some planets are habitable.
9. Some toys are plastic. No toys are plastic.
10. No transfats are healthy. All transfats are healthy.
11. No superheroes are invincible. Some superheroes are invincible.
12. Some villains are deplorable. Some villains are not deplorable.

##### Part C

For each sentence write its contradictory, contrary, subcontrary, or the corresponding sentence in subalternation as directed.

**Example**:

Write the subcontrary of “Some jellyfish sting.”

**Answer**:

Some jellyfish do not sting.

1. Write the contrary of “No hashtags are symbols.”
2. Write the contradictory of “All elephants are social.”
3. Write the subcontrary of “Some children are well behaved.”
4. Write the contradictory of “All eggplants are purple.”
5. Write the sentence that “Some guitars are electric” is a subaltern of.
6. Write the contradictory of “Some arches are not crumbling.”
7. Write the contrary of “No resolutions are unsatisfying.”
8. Write the contradictory of “All flags are flying.”
9. Write the subaltern of “No pains are chronic.”
10. Write the contradictory of “No puffins are mammals.’

**Part E**

Given a sentence and its truth value, evaluate the truth of a second sentence, according to the traditional square of opposition. If the truth value cannot be determined, just write “undetermined.”

**Example**:

If “Some *S* are *P* ” is true, what is the truth value of “Some *S* are not *P* ”?

**Answer**:

Undetermined

1. If “Some S are not P ” is true, what is the truth value of “All S are P ”?
2. If “Some S are not P ” is false, what is the truth value of “Some S are P ”?
3. If “All S are P ” is true, what is the truth value of “No S are P ”?
4. If “Some S are not P ” is false, what is the truth value of “No S are P ”?
5. If “No S are P ” is true, what is the truth value of “Some S are not P ”?
6. If “Some S are not P ” is true, what is the truth value of “All S are P ”?
7. If “Some S are P ” is true, what is the truth value of “All S are P ”?
8. If “All S are P ” is false, what is the truth value of “Some S are P ”?
9. If “No S are P ” is false, what is the truth value of “All S are P ”?
10. If “No S are P ” is true, what is the truth value of “Some S are P ”?

**Part G** Evaluate the following arguments using the traditional square of opposition. If the argument is valid, say which relationship in the square of opposition makes it valid.

**Example**:

No *S* are *P.*Therefore, some *S* are not *P.*

**Answer**:

Valid, because the conclusion is the subaltern of the premise.

1. No *S* are *P.* Therefore, it is false that some *S* are *P.*
2. It is false that no *S* are *P.* Therefore, it is false that all *S* are *P.*
3. All *S* are *P.* Therefore, it is false that no *S* are *P.*
4. It is false that no *S* are *P.* Therefore, it is false that some *S* are not *P.*
5. It is false that all *S* are *P.* Therefore, some *S* are not *P.*
6. It is false that no *S* are *P.* Therefore, some *S* are *P.*
7. It is false that all *S* are *P.* Therefore, some *S* are *P.*
8. Some *S* are *P.* Therefore, all *S* are *P.*
9. It is false that some *S* are *P.* Therefore, some *S* are *P.*
10. It is false that some *S* are not *P.* Therefore, it is false that no *S* are *P.*

### 8 Existential Import and the Modern Square of Opposition

The traditional square of opposition seems straightforward and fairly clever. Aristotle made an interesting distinction between contraries and contradictories, and subsequent logicians developed it into a nifty little diagram. So why did we have to keep saying things like “Aristotle thought” and “according to the traditional square of opposition.” What is wrong here?

The traditional square of opposition goes awry because it makes assumptions about the existence of the things being talked about. Remember that when we drew the Venn diagram for “All *S* are *P* ,” we shaded out the area of *S* that did not overlap with *P* to show that nothing could exist there. We pointed out, though, that we did not put a little x in the intersection between *S* and *P.* *Statements of the form A ruled out the existence of one kind of thing, but they did not assert the existence of another.*The A proposition, “All dogs are mammals,” denies the existence of any dog that is not a mammal, but *it does not assert the existence of some dog that is a mammal*. But why not? Dogs obviously do exist.

The problem comes when you start to consider categorical statements about things that don’t exist, for instance “All unicorns have one horn.” This seems like a true statement, but unicorns don’t exist. Perhaps what we mean by “All unicorns have one horn” is that *if* a unicorn existed, *then* is would have one horn. But if we interpret the statement about unicorns that way, shouldn’t we also interpret the statement about dogs that way? Really all we mean when we say “All dogs are mammals” is that if there were dogs, then they would be mammals. *It takes an extra assertion to point out that dogs do, in fact, exist*.

The issue we are discussing here is called **existential import**. A sentence is said to have existential import if it asserts the existence of the things it is talking about. Figure 24 shows the two ways you could draw Venn diagrams for an A statement, with the x, as in the traditional interpretation, and without, as in our interpretation. If you interpret A statements in the traditional way, they are always false when you are talking about things that don’t exist. So, “All unicorns have one horn” is false in the traditional interpretation.

|  |  |
| --- | --- |
|  |  |
| Without existential import (Modern). | With existential import (Traditional). |
|  |  |
| Figure 24 Interpretations of A: “All S are P.” | |

On the other hand, in the modern interpretation all statements about things that don’t exist are true. “All unicorns have one horn” simply asserts that there are no multi-horned unicorns, and this is true because there are no unicorns at all. We call this vacuous truth. Something is vacuously true if it is true simply because it is about things that don’t exist. Note that *all* statements about nonexistent things become vacuously true if you assume they have no existential import, even a statement like “All unicorns have more than one horn.” A statement like this simply rules out the existence of unicorns with one horn or fewer, and these don’t exist because unicorns don’t exist. This is a complicated issue that will come up again starting in Chapter [6](#_bookmark153) when we consider conditional statements. For now just assume that this makes sense because you can make up any stories you want about unicorns.

Any statement can be read with or without existential import, even the particular ones. Consider the statements “Some unicorns are rainbow colored” and “Some unicorns are not rainbow colored.” You can argue that both of these statements are true, in the sense that if unicorns existed, they could come in many colors. If you say these statements are true, however, you are assuming that particular statements do not have existential import. As Terence Parsons points out, you can change the wording of particular categorical statements in English to make them seem like they do or do not have existential import. “Some unicorns are not rainbow colored” might have existential import, but “not every unicorn is rainbow colored” doesn’t seem to.

So what does this have to do with the square of opposition? A lot of the claims made in the traditional square of opposition depend on assumptions about which statements have existential import. For instance, Aristotle’s claim that contrary statements cannot both be true requires that A statements have existential import. Think about the sentences “All dragons breathe fire” and “no dragons breathe fire.” If the first sentence has no existential import, then both sentences could actually be true. They are both ruling out the existence of certain kinds of dragons and are correct because no dragons exist.

In fact, the entire traditional square of opposition falls apart if you assume that all four forms of a categorical statement have existential import. Parsons shows how we can derive a contradiction in this situation. Consider the I statement “Some dragons breathe fire.” If you interpret it as having existential import, it is false, because dragons don’t exist. But then its contradictory statement, the E statement “No dragons breathe fire” must be true. And if that statement is true, and has existential import, then its subaltern, “Some dragon does not breathe fire” is true. But if it has existential import, it can’t be true, because dragons don’t exist. In logic, the worst thing you can ever do is contradict yourself, but that is what we have just done. So we have to change the traditional square of opposition.

The way some textbooks talk about the problem, you’d think that for two thousand years logicians were simply ignorant about the problem of existential import and thus woefully confused about the square of opposition, until finally George Boole wrote *The Laws of Thought* and found the one true solution to the problem. In fact, there was an extensive discussion of existential import from the 12th to the 16th centuries, mostly under the heading of the “supposition” of a term. Very roughly, we can say that the supposition of a term is the way it refers to objects, or what we now call the “denotation” of the term. So in “All people are mortal” the supposition of the subject term is all of the people out there in the world. Or, as the medievals sometimes put it, the subject term “supposits” all the people in the world.

At least some medieval thinkers had a theory of supposition that made the traditional square of opposition work. Terrance Parsons has argued for the importance of one solution, found

most clearly in the writings of William of Ockham. Under this theory, affirmative forms A and I had existential import, but the negative forms E and O did not. We would say that a statement has existential import if it would be false whenever the subject or predicate terms refer to things that don’t exist. To put the matter more precisely, we would say that the statement would be false whenever the subject or predicate terms “fail to refer.” Linguistic philosophers these days prefer say that a term “fails to refer” rather than saying that it “refers to something that doesn’t exist,” because referring to things that don’t exist seems impossible.

In any case, Ockham describes the supposition of affirmative propositions the same way we would describe the reference of terms in those propositions. Again, if the proposition supposes the existence of something in the world, the medievals would say it “supposits.” Ockham says “In affirmative propositions a term is always asserted to supposit for something. Thus, if it supposits for nothing the proposition is false” (1974). On the other hand, failure to refer or to supposit actually supports the truth of negative propositions: “in negative propositions the assertion is either that the term does not supposit for something or that it supposits for something of which the predicate is truly denied. Thus a negative proposition has two causes of truth” (1974).

So, for Ockham, affirmative statements about nonexistent objects are false. “All unicorns have one horn” and “Some unicorns are rainbow colored” are false, because there are no unicorns.

Negative statements, on the other hand, are vacuously true. “No unicorns are rainbow colored” and “No unicorns have one horn” are both true. There are no rainbow colored unicorns out there, and no one horned unicorns out there, because there are no unicorns out there. The O statement “Some unicorns are not rainbow colored” is also vacuously true. This might be harder to see, but it helps to think of the statement as saying “It is not the case that every unicorn is rainbow colored.”

This way of thinking about existential import leaves the traditional square of opposition intact, even in cases where you are referring to nonexistent objects. Contraries still cannot both be true when you are talking about nonexistent objects, because the A proposition will be false, and the E vacuously true. “All dragons breathe fire” is false, because dragons don’t exist, and “No dragons breathe fire” is vacuously true for the same reason. Similarly, subcontraries cannot both be false when talking about dragons and whatnot, because the I will always be false and the O will always be true. You can go through the rest of the relationships and show that similar arguments hold.

Boole proposed a different solution, which is now taken as the standard way to do things. Instead of looking at the division between positive and negative statements, Boole looked at the division between singular and universal propositions. The universal statements A and E do not have existential import, but the particular statements I and O do have existential import. Thus, all particular statements about nonexistent things are false and all universal statements about nonexistent things are vacuously true.

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| Figure 25 The modern square of opposition |

John Venn was building on the work of George Boole. His diagrams avoided the problems that Euler had by using a Boolean interpretation of mood-A statements, where they really just assert that something is impossible. In fact, the whole system of Venn diagrams embodies Boole’s assumptions about existential import, as you can see in Figure 14Figure 25. The particular forms I and O have you draw an x, indicating that something exists. The other two forms just have us shade in regions to indicate that certain combinations of subject and predicate are impossible. Thus, A and E statements like “All dragons breathe fire” or “No dragons are friendly” can be true, even though no dragons exist.

Venn diagrams doesn’t even have the capacity to represent Ockham’s understanding of existential import. We can represent A statements as having existential import by adding an x, as we did on the right-hand side of Figure 24. However, we have no way to represent the O form without existential import. We have to draw the x, indicating existence. We don’t have a way of representing O form statements about nonexistent objects as vacuously true.

The Boolean solution to the question of existential import leaves us with a greatly restricted form of the square of opposition. Contrary statements are both vacuously true when you refer to nonexistent objects, because neither have existential import. Subcontrary statements are both false when you refer to nonexistent objects, because they do have existential import.

Finally, the subalterns of vacuously true statements are false, while on the traditional square of opposition they had to be true. The only thing remaining from the traditional square of opposition is the relationship of contradiction, as you can see in Figure 25.

#### Practice Exercises

##### Part A

Evaluate each of the following arguments twice. First, evaluate it using Ockham’s theory of existential import, where positive statements have existential import and negative ones do not. If the argument is valid, state which relationship makes it valid (contradictories, contraries, etc.)

Second, evaluate the argument using Boole’s theory, where particular statements have existential import and universal statements do not. If the premise of the argument is just a categorical statement, rather than a claim that a categorical statement is false, draw the Venn diagram for the premise.

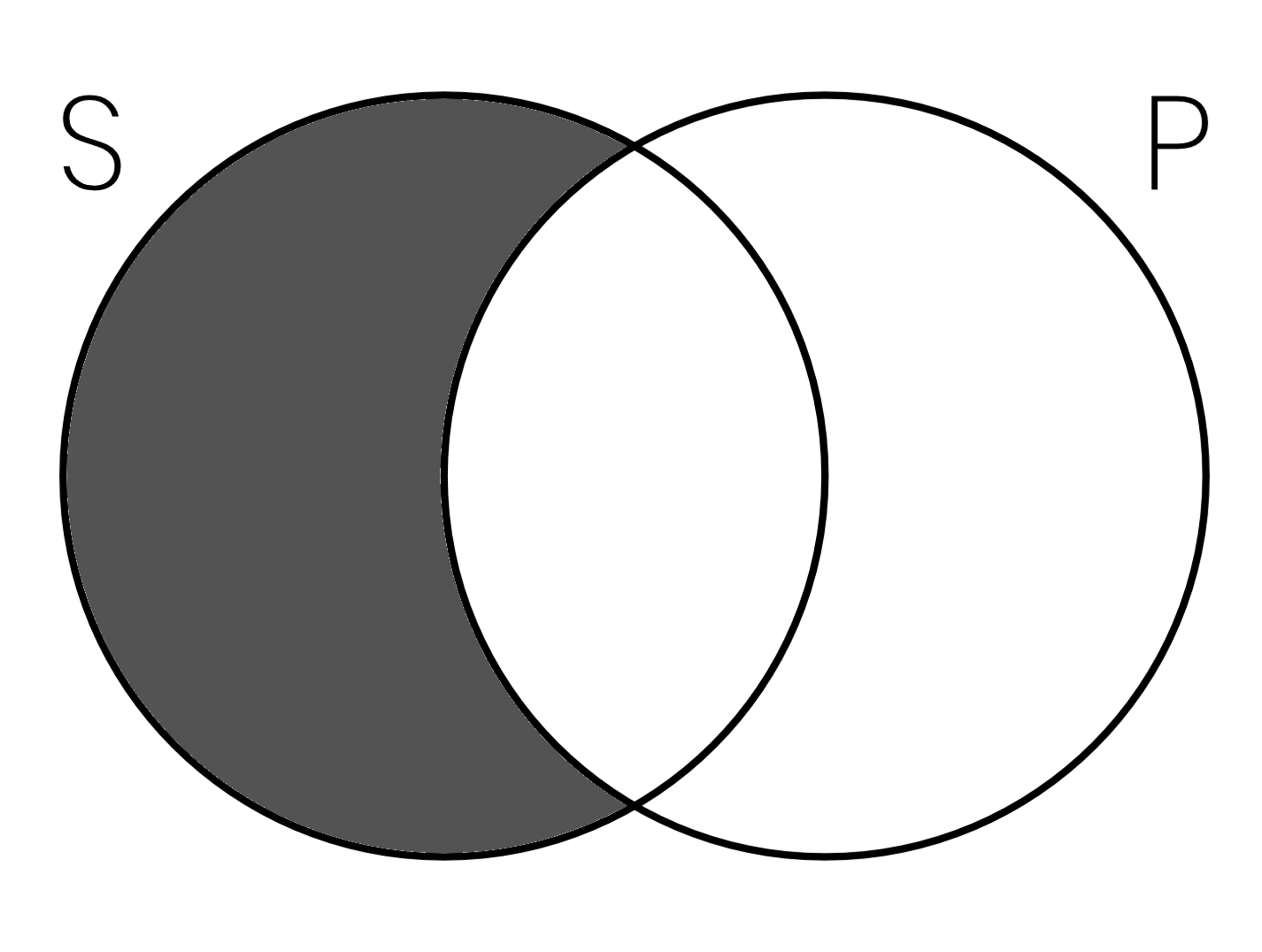
**Example 1:**

All *S* are *P.* Therefore, it is false that no *S* are *P.*

**Answer**:

Ockham: Valid. Contraries.

Boole: Invalid



The overlap between *S* and *P* is neither shaded out nor has an “x” in it, so “No *S* are *P* ” could either be true or false.

**Example 2:**

It is false that all *S* are *P.* Therefore, some *S* are not *P.*

**Answer:**

Ockham: Valid. Contradictories

Boole: Valid.

1. It is false that some *S* are *P.* Therefore, it is false that all *S* are *P.*
2. All *S* are *P.* Therefore, some *S* are not *P.*
3. It is false that some *S* are *P.* Therefore no *S* are *P.*
4. No *S* are *P.* Therefore, it is false that all *S* are *P.*
5. Some *S* are not *P.* Therefore, it is false that all *S* are *P.*
6. It is false that some *S* are not *P.* Therefore, it is false that no *S* are *P.*
7. It is false that some *S* are *P.* Therefore, it is false that no *S* are *P.*
8. It is false that some *S* are *P.* Therefore, some *S* are not *P.*
9. Some *S* are *P.* Therefore, it is false that some *S* are not *P*
10. Some *S* are *P.* Therefore all *S* are *P.*

### 9 Key Terms

* Affirmative
* Complement
* Contradictories
* Contraposition
* Contraries
* Converse
* Copula
* Distribution
* Existential import
* Logically structured
* English Mood-A statement
* Mood-E statement
* Mood-I statement
* Mood-O statement
* Negative
* Obverse
* Particular
* Predicate class
* Quality
* Quantifted categorical statement
* Quantifter
* Quantity
* Square of opposition
* Standard form categorical statement
* Subalternation
* Subcontraries
* Subject class
* Truth value
* Universal
* Vacuous truth
* Venn diagram

## Sentential Logic

This chapter introduces a logical language called SL. It is a version of *sentential logic*, because the basic units of the language will represent statements, and a statement is usually given by a complete sentence in English.

### 1 Sentence Letters

In SL, capital letters, called sentence letters are used to represent simple statements. Considered only as a symbol of SL, the letter *A* could mean any statement, so it is a variable.[[9]](#footnote-10) So when translating from English into SL, it is important to provide a symbolization key, or dictionary. The symbolization key provides an English language sentence for each sentence letter used in the symbolization.

Consider this argument:

P1) There is an apple on the desk.

P2) If there is an apple on the desk, then Jenny made it to class.

C) *∴*Jenny made it to class.

This is obviously a valid argument in English. In symbolizing it, we want to preserve the structure of the argument that makes it valid. What happens if we replace each sentence with a letter? Our symbolization key would look like this:

**A:** There is an apple on the desk.

**B:** If there is an apple on the desk, then Jenny made it to class.

**C:** Jenny made it to class.

We would then symbolize the argument in this way:

P1) A

P2) B

C) ∴ C

There is no necessary connection between some sentence *A*, which could be any statement, and some other sentences *B* and *C*, which could also be anything. The structure of the argument has been completely lost in this translation.

The important thing about the argument is that the second premise is not merely *any statement*, logically divorced from the other statement in the argument. The second premise contains the first premise and the conclusion *as parts*. Our symbolization key for the argument only needs to include meanings for *A* and *C*, and we can build the second premise from those pieces. So we symbolize the argument this way:

P1) A

P2) If A, then C.

C) ∴ C

This preserves the structure of the argument that makes it valid, but it still makes use of the English expression “If*. . .* then*. . .*.” Although we ultimately want to replace all of the English expressions with logical notation, this is a good start.

The individual sentence letters in SL are called atomic sentences, because they are the basic building blocks out of which more complex sentences can be built. We can identify atomic sentences in English as well. An atomic sentence is one that cannot be broken into parts that are themselves sentences. “There is an apple on the desk” is an atomic sentence in English, because you can’t find any proper part of it that forms a complete sentence. For instance, “an apple on the desk” is a noun phrase, not a complete sentence. Similarly, “on the desk” is a prepositional phrase, and not a sentence, and “is an” is not any kind of phrase at all. This is what you will find no matter how you divide “There is an apple on the desk.” On the other hand, you can find two proper parts of “If there is an apple on the desk, then Jenny made it to class” that are complete sentences: “There is an apple on the desk” and “Jenny made it to class.” As a general rule, we will want to use atomic sentences in SL (that is, the sentence letters) to represent atomic sentences in English. Otherwise, we will lose some of the logical structure of the English sentence, as we have just seen.

There are only 26 letters of the alphabet, but there is no logical limit to the number of atomic sentences. We can use the same letter to symbolize different atomic sentences by adding a subscript, a small number written after the letter. We could have a symbolization key that looks like this:

**A**1**:** The apple is under the armoire.

**A**2**:** Arguments in SL always contain atomic sentences.

**A**3**:** Adam Ant is taking an airplane from Anchorage to Albany.

**...**

**A**294**:** Alliteration angers otherwise affable astronauts.

Keep in mind that each of these is a different sentence letter. When there are subscripts in the symbolization key, it is important to keep track of them.

### 2 Sentential Connectives

Logical connectives are used to build complex sentences from atomic components. In SL, our logical connectives are called sentential connectives because they connect sentence letters. There are five sentential connectives in SL. This table summarizes them, and they are explained below.

|  |  |  |
| --- | --- | --- |
| **Symbol** | **Name** | **Meaning** |
| ~ | Negation | It is not the case that |
| & | Conjunction | Both… and… |
| ∨ | Disjunction | Either… or… |
| → | Conditional | If … then … |
| ↔ | Biconditional | …. if and only if …. |

#### Negation

Consider how we might symbolize these sentences:

1. Mary is in Barcelona.
2. Mary is not in Barcelona.
3. Mary is somewhere other than Barcelona.

In order to symbolize sentence [1](#_bookmark156), we will need one sentence letter. We can provide a symbolization key:

**B:** Mary is in Barcelona.

Note that here we are giving *B* a different interpretation than we did in the previous section. The symbolization key only specifies what *B* means *in a specific context*. It is vital that we continue to use this meaning of *B* so long as we are talking about Mary and Barcelona. Later, when we are symbolizing different sentences, we can write a new symbolization key and use *B* to mean something else.

Now, sentence [1](#_bookmark156) is simply *B*. Sentence [2](#_bookmark157) is obviously related to sentence [1](#_bookmark156): it is basically [1](#_bookmark156) with a “not” added. We could put the sentence partly our symbolic language by writing “Not *B*.” This means we do not want to introduce a different sentence letter for [2](#_bookmark157). We just need a new symbol for the “not” part. Let’s use the symbol ‘∼,’ which we will call negation. Now we can translate ‘Not *B*’ to ∼B.

Sentence [3](#_bookmark158) is about whether or not Mary is in Barcelona, but it does not contain the word “not.” Nevertheless, it is obviously logically equivalent to sentence [2](#_bookmark157). They both mean: It is not the case that Mary is in Barcelona. As such, we can translate both sentence [2](#_bookmark157) and sentence [3](#_bookmark158) as ∼B.

A sentence can be symbolized as ∼Aif it can be paraphrased in English as “It is not the case that A.”

Consider these further examples:

1. The widget can be replaced if it breaks.
2. The widget is irreplaceable.
3. The widget is not irreplaceable.

If we let *R* mean “The widget is replaceable”, then sentence [4](#_bookmark159) can be translated as *R*.

What about sentence [5](#_bookmark160)? Saying the widget is irreplaceable means that it is not the case that the widget is replaceable. So even though sentence [5](#_bookmark160) is not negative in English, we symbolize it using negation as ∼R.

Sentence [6](#_bookmark161) can be paraphrased as “It is not the case that the widget is irreplaceable.” Using negation twice, we translate this as ~R. The two negations in a row each work as negations, so the sentence means “It is not the case that it is not the case that *R*.” If you think about the sentence in English, it is logically equivalent to sentence [4](#_bookmark159). So when we define logical equivalence in SL, we will make sure that *R* and ∼∼Rare logically equivalent.

More examples:

1. Elliott is happy.
2. Elliott is unhappy.

If we let *H* mean “Elliot is happy”, then we can symbolize sentence [7](#_bookmark162) as *H*.

However, it would be a mistake to symbolize sentence [8](#_bookmark163) as ~H. If Elliott is unhappy, then he is not happy—but sentence [8](#_bookmark163) does not mean the same thing as “It is not the case that Elliott is happy.” It could be that he is not happy but that he is not unhappy either. Perhaps he is somewhere between the two. In order to symbolize sentence [8](#_bookmark163), we would need a new sentence letter.

For any sentence A: If A is true, then ~Ais false. If ~Ais true, then Ais false. Using T for true and F for false, we can summarize this in a **characteristic truth table**for negation:

|  |  |
| --- | --- |
| A | ~A |
| T | F |
| F | T |

We will discuss truth tables at greater length in the next chapter.

#### Conjunction

Consider these sentences:

1. Adam is athletic.
2. Barbara is athletic.
3. Adam is athletic, and Barbara is also athletic.

We will need separate sentence letters for [9](#_bookmark164) and [10](#_bookmark165), so we define this symbolization key:

**A:** Adam is athletic.

**B:** Barbara is athletic.

Sentence [9](#_bookmark164) can be symbolized as *A*. Sentence [10](#_bookmark165) can be symbolized as *B*.

Sentence [11](#_bookmark166) can be paraphrased as “*A* and *B*.” In order to fully symbolize this sentence, we need another symbol. We will use & . We translate “*A* and *B*” as *A* & *B*. The logical connective & is called the conjunction, and *A* and *B* are each called conjuncts.

Notice that we make no attempt to symbolize “also” in sentence [11](#_bookmark166). Words like “both” and “also” function to draw our attention to the fact that two things are being conjoined. They are not doing any further logical work, so we do not need to represent them in SL.

Some more examples:

1. Barbara is athletic and energetic.
2. Barbara and Adam are both athletic.
3. Although Barbara is energetic, she is not athletic.
4. Barbara is athletic, but Adam is more athletic than she is.

Sentence [12](#_bookmark167) is obviously a conjunction. The sentence says two things about Barbara, so in English it is permissible to refer to Barbara only once. It might be tempting to try this when translating the argument: Since *B* means “Barbara is athletic”, one might paraphrase the sentences as “*B* and energetic.” This would be a mistake. Once we translate part of a sentence as *B*, any further structure is lost. *B* is an atomic sentence; it is nothing more than true or false.

Conversely, “energetic” is not a sentence; on its own it is neither true nor false. We should instead paraphrase the sentence as “*B* and Barbara is energetic.” Now we need to add a sentence letter to the symbolization key. Let *E* mean “Barbara is energetic.” Now the sentence can be translated as B & E.

A sentence can be symbolized as *A* & *B* if it can be paraphrased in English as ‘Both *A*, and *B*.’ Each of the conjuncts must be a sentence.

Sentence [13](#_bookmark168) says one thing about two different subjects. It says of both Barbara and Adam that they are athletic, and in English we use the word “athletic” only once. In translating to SL, it is important to realize that the sentence can be paraphrased as, “Barbara is athletic, and Adam is athletic.” This translates as B & A.

Sentence [14](#_bookmark169) is a bit more complicated. The word “although” sets up a contrast between the first part of the sentence and the second part. Nevertheless, the sentence says both that Barbara is energetic and that she is not athletic. In order to make each of the conjuncts an atomic sentence, we need to replace “she” with “Barbara.”

So we can paraphrase sentence [14](#_bookmark169) as, “*Both* Barbara is energetic, *and* Barbara is not athletic.” The second conjunct contains a negation, so we paraphrase further: “*Both* Barbara is energetic *and it is not the case that* Barbara is athletic.” This translates as E & ∼B.

Sentence [15](#_bookmark170) contains a similar contrastive structure. It is irrelevant for the purpose of translating to SL, so we can paraphrase the sentence as “*Both* Barbara is athletic, *and* Adam is more athletic than Barbara.” (Notice that we once again replace the pronoun “she” with her name.) How should we translate the second conjunct? We already have the sentence letter A which is about Adam’s being athletic and B which is about Barbara’s being athletic, but neither is about one of them being more athletic than the other. We need a new sentence letter. Let R mean “Adam is more athletic than Barbara.” Now the sentence translates as B & R.

Sentences that can be paraphrased “A, but B” or “Although A, B” are best symbolized using conjunction A & B.

It is important to keep in mind that the sentence letters A, B, and R are atomic sentences. Considered as symbols of SL, they have no meaning beyond being true or false. We have used them to symbolize different English language sentences that are all about people being athletic, but this similarity is completely lost when we translate to SL. No formal language can capture all the structure of the English language, but as long as this structure is not important to the argument there is nothing lost by leaving it out.

For any sentences *A* and *B*, A & Bis true if and only if both *A* and *B* are true. We can summarize this in the characteristic truth table for conjunction:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| A | B | A | & | B |
| T | T |  | T |  |
| T | F |  | F |  |
| F | T |  | F |  |
| F | F |  | F |  |

**Conjunction is symmetrical** because we can swap the conjuncts without changing the truth value of the sentence. Regardless of what *A* and *B* are, *A* & *B* is logically equivalent to *B* & *A*.

#### Disjunction

Consider these sentences:

1. Either Denison will play golf with me, or he will watch movies.
2. Either Denison or Ellery will play golf with me.

For these sentences we can use this symbolization key:

**D:** Denison will play golf with me.

**E:** Ellery will play golf with me.

**M:** Denison will watch movies.

Sentence [16](#_bookmark171) is “Either D or M.” To fully symbolize this, we introduce a new symbol. The sentence becomes D∨ M. The connective is called disjunction, and D and M are called disjuncts.

Sentence [17](#_bookmark172) is only slightly more complicated. There are two subjects, but the English sentence only gives the verb once. In translating, we can paraphrase it as “Either Denison will play golf with me, or Ellery will play golf with me.” Now it obviously translates as D ∨ E.

A sentence can be symbolized as A∨ Bif it can be paraphrased in English as “Either Aor B.” Each of the disjuncts must be a sentence.

##### Exclusive vs Inclusive ORs

Sometimes in English, the word “or” excludes the possibility that both disjuncts are true. This is called an exclusive or. An ***exclusive or***is clearly intended when a restaurant menu says, “Entrees come with either soup or salad.” You may have soup; you may have salad; but, if you want *both* soup *and* salad, then you will have to pay extra.

At other times, the word “or” allows for the possibility that both disjuncts might be true. This is probably the case with sentence [17](#_bookmark172), above. I might play with Denison, with Ellery, or with both Denison and Ellery. Sentence [17](#_bookmark172) merely says that I will play with *at least* one of them. This is called an inclusive or.

The symbol ∨ represents an **inclusive or**. So, D∨ E is true if D is true, if E is true, or if both D and E are true. **It is false only if both D and E are false.** We can summarize this with the characteristic truth table for disjunction:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| A | B | A | ∨ | B |
| T | T |  | T |  |
| T | F |  | T |  |
| F | T |  | T |  |
| F | F |  | F |  |

Like conjunction, **disjunction is symmetrical**. A ∨ B is logically equivalent to B ∨ A. These sentences are somewhat more complicated:

1. S1 Either you will not have soup, or you will not have salad.
2. S2 You will have neither soup nor salad.
3. S3 You get either soup or salad, but not both.

We let S1 mean that you get soup and S2 mean that you get salad.

Sentence [18](#_bookmark173) can be paraphrased in this way: “Either *it is not the case that* you get soup, or *it is not the case that* you get salad.” Translating this requires both disjunction and negation. It becomes ∼ S1 ∨ ∼ S2.

Sentence [19](#_bookmark174) also requires negation. It can be paraphrased as, “*It is not the case that* either you get soup or you get salad.” We need some way of indicating that the negation does not just negate the right or left disjunct, but rather negates the entire disjunction. In order to do this, we put parentheses around the disjunction: “It is not the case that (S1 ∨ S2).” This becomes simply ∼( S1 ∨ S2).

Notice that the parentheses are doing important work here. The sentence ~ S1 ∨ S2 would mean “Either you will not have soup, or you will have salad.”

Sentence [20](#_bookmark175) is an *exclusive or*. Although ∨ is an inclusive or, we can symbolize an exclusive or in SL. We just need more than one connective to do it. We can break the sentence into two parts.

The first part says that you get one or the other. We translate this as (S1 ∨ S2). The second part says that you do not get both. We can paraphrase this as “It is not the case both that you get soup and that you get salad.” Using both negation and conjunction, we translate this as ∼( S1 & S2). Now we just need to put the two parts together. As we saw above, “but” can usually be translated as a conjunction. Sentence [20](#_bookmark175) can thus be translated as [(S1 ∨ S2) & ∼( S1 & S2)].

#### Conditional

For the following sentences, let *R* mean “You will cut the red wire” and *B* mean “The bomb will explode.”

1. If you cut the red wire, then the bomb will explode.
2. The bomb will explode only if you cut the red wire.

Sentence [21](#_bookmark176) can be translated partially as “If R, then B.” We will use the symbol → to represent logical entailment. Sentence [21](#_bookmark176) then becomes R → B. The connective is called a conditional. The sentence on the left-hand side of the conditional (R in this example) is called the antecedent. The sentence on the right-hand side (B) is called the consequent.

Sentence [22](#_bookmark177) is also a conditional. Since the word “if” appears in the second half of the sentence, it might be tempting to symbolize this in the same way as sentence [21](#_bookmark176). That would be a mistake.

The conditional R → B says that if R were true, then B would also be true. It does not say that you cutting the red wire is the only way that the bomb could explode. Someone else might cut the wire, or the bomb might be on a timer. The sentence R → B does not say anything about what to expect if R is false. Sentence [22](#_bookmark177) is different. It says that the only conditions under which the bomb will explode involve you having cut the red wire; i.e., if the bomb explodes, then you must have cut the wire. As such, sentence 22 should be symbolized as B → R.

It is important to remember that the connective says only that, if the antecedent is true, then the consequent is true. It says nothing about the *causal* connection between the two events. Translating sentence [22](#_bookmark177) as B → R does not mean that the bomb exploding would somehow have caused you cutting the wire. Both sentence [21](#_bookmark176) and [22](#_bookmark177) suggest that, if you cut the red wire, you cutting the red wire would be the cause of the bomb exploding. They differ on the *logical* connection. If sentence [22](#_bookmark177) were true, then an explosion would tell us—those of us safely away from the bomb—that you had cut the red wire. Without an explosion, sentence [22](#_bookmark177) tells us nothing.

The paraphrased sentence “A only if B” is logically equivalent to “If A, then B.”

“If A, then B” means that if A is true, then so is B. So we know that if the antecedent A is true but the consequent B is false, then the conditional “If A then B” is false. What is the truth value of “If A, then B” under other circumstances? Suppose, for instance, that the antecedent A happened to be false. “If A, then B” would then not tell us anything about the actual truth value of the consequent B, and it is unclear what the truth value of “If A, then B” would be.

In English, the truth of conditionals often depends on what *would* be the case if the antecedent *were true*—even if, as a matter of fact, the antecedent is false. This poses a problem for translating conditionals into SL. Considered as sentences of SL, *R* and *B* in the above examples have nothing intrinsic to do with each other. In order to consider what the world would be like if *R* were true, we would need to analyze what *R* says about the world. Since *R* is an atomic symbol of SL, however, there is no further structure to be analyzed. When we replace a sentence with a sentence letter, we consider it merely as some atomic sentence that might be true or false.

In order to translate conditionals into SL, we will not try to capture all the subtleties of the English language “If*. . .*, then*. . .*.” Instead, the symbol → will be what logicians call a **material conditional**. This means that when A is false, the conditional A →B is automatically true, regardless of the truth value of B. If both A and B are true, then the conditional A →B is true.

In short, A → B is false if and only if A is true and B is false. We can summarize this with a characteristic truth table for the conditional.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| A | B | A | → | B |
| T | T |  | T |  |
| T | F |  | F |  |
| F | T |  | T |  |
| F | F |  | T |  |

**The conditional is asymmetrical.** You cannot swap the antecedent and consequent without changing the meaning of the sentence, because *A* →*B* and *B* →*A* are not logically equivalent.

Not all sentences of the form “If*. . .*, then*. . .*” are conditionals. Consider this sentence:

1. If anyone wants to see me, then I will be on the porch.

When I say this, it means that I will be on the porch, regardless of whether anyone wants to see me or not—but if someone did want to see me, then they should look for me there. If we let *P* mean “I will be on the porch,” then sentence [23](#_bookmark178) can be translated simply as *P.*

**Biconditional**

Consider these sentences:

1. The figure on the board is a triangle only if it has exactly three sides.
2. The figure on the board is a triangle if it has exactly three sides.
3. The figure on the board is a triangle if and only if it has exactly three sides.

Let T mean “The figure is a triangle” and S mean “The figure has three sides.” Sentence [24](#_bookmark179), for reasons discussed above, can be translated as T → S.

Sentence [25](#_bookmark180) is importantly different. It can be paraphrased as “If the figure has three sides, then it is a triangle.” So it can be translated as S → T .

Sentence [26](#_bookmark181) says that T is true if and only if S is true; we can infer S from T , and we can infer T from S. This is called a biconditional, because it entails the two conditionals S → T and T → S. We will use ↔ to represent the biconditional; sentence [26](#_bookmark181) can be translated as S ↔ T.

We could abide without a new symbol for the biconditional. Since sentence [26](#_bookmark181) means “T →S and S → T ,” we could translate it as (T → S) & (S → T ). We would need parentheses to indicate that (T → S) and (S → T ) are separate conjuncts; the expression T → S & → T would be ambiguous.

Because we could always write (A →B) & (B → A) instead of A ↔ B, we do not strictly speaking *need* to introduce a new symbol for the biconditional. Nevertheless, logical languages usually have such a symbol. SL will have one, which makes it easier to translate phrases like “if and only if.”

A ↔ B is true if and only if A and B have the same truth value. This is the characteristic truth table for the biconditional:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| A | B | A | ↔ | B |
| T | T |  | T |  |
| T | F |  | F |  |
| F | T |  | F |  |
| F | F |  | T |  |

### 3 More Complicated Translations

Back in section 4.3, we saw that the system of categorical logic we were studying at the time could actually represent a large range of sentences in ordinary English, even though it only had the quantifiers “All" and “some" plus negation. In this section, we will see that something similar happens with SL. There is actually a lot we can cover, even though we only have five connectives.

#### Combining Connectives

In our system of categorical logic, we just had four kinds of sentences — A, E, I, and O — and if we wanted to combine them, the only way to do that would be to form a syllogism. In SL, we can combine an unlimited number of connectives together into a single sentence to express complicated ideas that couldn’t be represented by Aristotelean logic.

Consider the English sentence “If it is not raining, we will have a picnic." There are two aspects of this sentence we will want to represent with sentential connectives in SL, the “if. . . then. . . " structure and the negation in the first part of the sentence. The rest of the sentence can be represented by these sentence letters

A: It is raining.

M: We will have a picnic.

We can then translate the whole sentence into SL like this: ∼A → B. We can make sentences as complicated as we want this way, even to the point where the equivalent English sentence would be impossible to follow. The sentence ∼(P & Q) → [(R ∨ S) ↔ ∼(T & U)] is perfectly acceptable in SL, even if any English sentence it translates into would be a monster. This is part of the power of a complete formal language like SL, but it is also why arguments in SL begin to resemble abstract math equations.

Although sentences in SL can be as long as you like, you can’t just combine symbols any old way. There is a specific set of rules you have to follow.

The fact that we can write these more complicated sentences means we can actually do without some of the connectives we have given ourselves in SL. For instance, we don’t really need the biconditional. Any sentence of the form A ↔ B is going to be equivalent to the sentence (A → B) & (B → A). This just follows from the way we defined the biconditional earlier. Nevertheless, tradition and convenience mandate that we give the biconditional a separate symbol.

#### Unless

Because our connectives can be put together in different ways, some English sentences can be represented equally well by multiple sentences in SL. English sentences involving the word “unless" are a case in point.

1. Unless you wear a jacket, you will catch cold.
2. You will catch cold unless you wear a jacket.

These are basically two different version of the same English sentence. The only difference is that in one case, the “unless" clause comes first, and in the other it comes second. Let J mean “You will wear a jacket" and let C mean “You will catch a cold." We can paraphrase sentence 27 as “Unless J, C." This means that if you do not wear a jacket, then you will catch cold. With this in mind, we might translate it as ∼J → C. It also means that if you do not catch a cold, then you must have worn a jacket; with this in mind, we might translate it as ∼C → J.

Which of these is the correct translation of sentence 27? Both translations are correct, because the two translations are logically equivalent in SL. Sentence 28, in English, is logically equivalent to sentence 27. So, it also can be translated as either ∼J → D or ∼D → J.

When symbolizing sentences like sentence 27 and sentence 28, it is easy to get turned around. We have two different versions of the English sentence and two different versions of the sentence in SL. The important thing to see here is that none of these sentences are equivalent to J → ∼D. The negated statement must be the antecedent to the conditional.

If this is too many options to keep track of, there is a simpler alternative. It turns out that any “unless" statement is actually equivalent to an “or" statement. Both statements 27 and 28 mean that you will wear a jacket or — if you do not wear a jacket — then you will catch a cold. So, we can translate them as J ∨ D. (You might worry that the “or" here should be an *exclusive or.* However, the sentences do not exclude the possibility that you might *both* wear a jacket *and* catch a cold; jackets do not protect you from all the possible ways that you might catch a cold.)

If a sentence can be paraphrased as “Unless A, B," then it can be symbolized as A∨B.

#### Only

Earlier, we saw that the word “only" could reverse the meaning of a statement in Mood A. “All dogs are mammals" means something different than “Only dogs are mammals," the first one is true but the second one is false. Something similar happens with conditional statements in SL. For the following sentences, let R mean “You will cut the red wire" and B mean “The bomb will explode."

1. If you cut the red wire, then the bomb will explode.
2. The bomb will explode only if you cut the red wire.

Sentence 29 can be translated partially as “If R, then B." Sentence 30 is also a conditional. Since the word “if" appears in the second half of the sentence, it might be tempting to symbolize this in the same way as sentence 29. That would be a mistake. The conditional R → B says that if R were true, then B would also be true. It does not say that you cutting the red wire is the *only* way that the bomb could explode. Someone else might cut the wire, or the bomb might be on a timer. The sentence R ! B does not say anything about what to expect if R is false. Sentence 30 is different. It says that the only conditions under which the bomb will explode involve you having cut the red wire; i.e., if the bomb explodes, then you must have cut the wire. As such, sentence 30 should be symbolized as B → R. It is important to remember that the connective → says only that, if the antecedent is true, then the consequent is true. It says nothing about the *causal* connection between the two events. Translating sentence 30 as B → R does not mean that the bomb exploding would somehow have caused you cutting the wire. Both sentence 29 and 30 suggest that, if you cut the red wire, you cutting the red wire would be the cause of the bomb exploding. They differ on the logical connection. If sentence 30 were true, then an explosion would tell us — those of us safely away from the bomb — that you had cut the red wire. Without an explosion, sentence 30 tells us nothing.

The paraphrased sentence “A only if B" is logically equivalent to “If A, then B."

Things can get a bit more complicated, because English also allows you to reverse the order of the clauses. Think about this sentence

1. The bomb will explode, if you cut the red wire

This is just sentence 29 with the order of the clauses reversed, so it still means R → B. Changing the order of the English clauses does not change the sentence in SL, but adding the word “only" does. If this gets confusing, just remember this rule:

“If. . . " introduces the **antecedent**. “Only if. . . " introduces the **consequent**.

Because “if" and “only if" have opposite meanings, when we put them together, we get the biconditional. Consider these sentences:

1. The figure on the board is a triangle only if it has exactly three sides.
2. The figure on the board is a triangle if it has exactly three sides.
3. The figure on the board is a triangle if and only if it has exactly three sides.

Let T mean “The figure is a triangle" and S mean “The figure has three sides." Sentence 32, for reasons discussed above, can be translated as T → S. Sentence 33 is importantly different. It can be paraphrased as “If the figure has three sides, then it is a triangle." So it can be translated as S → T. Sentence 34 says that T is true *if and only if S* is true; we can infer S from T, and we can infer T from S. In other words, 26 is equivalent to T → S and S → T, which is the same as T ↔ S.

A final way to think about the way “only" effects a conditional sentence is to think about the difference between necessary and sufficient conditions. In a way, the terms are pretty much self-explanatory. Nevertheless, it is really easy to get them confused, to the extent that even professional logicians and trained philosophers can get them mixed up.

A **necessary condition** is one that is needed for something else to be true, just like the name says. Having gas in the tank is a *necessary* condition for the car to move. It just doesn’t go anywhere without gas. However, having gas in the tank isn’t *all you need* to get the car moving. You also have to put the key in the ignition and turn it.

A **sufficient condition**, on the other hand, is *all you need* for something else to be true. If something is a dog, that is a *sufficient* condition for it to be a mammal. Once you know Wilfred (Figure 26) is a dog, you have enough information to infer that he is a mammal. Being a dog is not a necessary condition for being a mammal however. You can also be a mammal being a cat, or a human, or a wombat.

|  |
| --- |
|  |
| Figure 26 This is Wilfred. The fact that he is a dog is a sufficent condition for him to be a mammal. He can also lick his nose. |

The conditional symbol in SL represents a sufficient condition, at least when read forward. That is, the antecedent is a sufficient condition for the consequent. If you have the antecedent, that is all you need to know to infer the consequent. So if *D* is “Wilfred is a dog" and *M* is “Wilfred is a mammal, then D → Cis true. Being a dog is sufficient for being a mammal. As it turns out, if the relationship is sufficient going one direction, it is necessary going the other. So being a mammal is a necessary condition for being a mammal. If Wilfred weren’t a mammal, there would be no way for her to be a dog. Figure 27 shows this relationship.

|  |
| --- |
|  |
| Figure 27 The Antecedent of a material condition is a sufficient condition for the consequent, while the consequent is a necessary condition for the antecedent. |

#### Combining negation with conjunction and disjunction

Tricky things happen when you combine a negation with a conjunction or disjunction, so it is worth taking a closer look here. Consider these sentences

1. (S1)27. Either you will not have soup, or you will not have salad.
2. (S2)28. You will have neither soup nor salad.

We let S1 mean that you get soup and S2 mean that you get salad. Sentence 35 can be paraphrased in this way: “Either *it is not the case that* you get soup, or *it is not the case that* you get salad." Translating this requires both disjunction and negation. It becomes ∼S1 ∨ ∼S2.

Sentence 36 also requires negation. It can be paraphrased as, “*It is not the case that* either you get soup or you get salad." We need some way of indicating that the negation does not just negate the right or left disjunct, but rather negates the entire disjunction. In order to do this, we put parentheses around the disjunction: “It is not the case that (S1 ∨ S2)." This becomes simply ~(S1 ∨ S2). Notice that the parentheses are doing important work here. The sentence ~S1 ∨ S2 would mean “Either you will not have soup, or you will have salad." Something similar happens with negation and conjunction. Consider these sentences

1. You can’t have soup and you can’t have salad.
2. You can’t have both soup and salad.

In sentence 37, the two parts of the sentence are negated individually. We would translate it into SL like this: ∼S1 & ∼S2. In sentence 38, the negation applies to soup and salad taken together. You are allowed to have soup only, or salad only. You just can’t have both together. We would translate sentence 38 like this: ~(S1 ∨ S2). You can combine disjunction, conjunction, and negation to represent the exclusive or, as in this sentence.

1. You get either soup or salad, but not both.

Earlier, on page 172, we said that the ∨ in SL represented an “inclusive Or”. It said “this or that or both." If we want to represent an exclusive or, we need to combine disjunction, conjunction and negation. We can break the sentence into two parts. The first part says that you get one or the other. We translate this as (S1 ∨ S2). The second part says that you do not get both. We can paraphrase this as “It is not the case both that you get soup and that you get salad." Using both negation and conjunction, we translate this as ~(S1 ∨ S2). Now we just need to put the two parts together. As we saw above, “but" can usually be translated as a conjunction. Sentence 39 can thus be translated as (S1 ∨ S2) & ∼(S1 & S2).

### 4 Recursive Syntax for SL

The previous two sections gave you a rough, informal sense of how to create sentences in SL. If I give you an English sentence like “Grass is either green or brown,” you should be able to write a corresponding sentence in SL: “A∨ B.” In this section we want to give a more precise definition of a sentence in SL. When we defined sentences in English, we did so using the concept of truth: Sentences were units of language that can be true or false. (See page 104.)In SL, it is possible to define what counts as a sentence without talking about truth. Instead, we can just talk about the structure of the sentence. This is one respect in which a formal language like SL is more precise than a natural language like English.

The structure of a sentence in SL considered without reference to truth or falsity is called its **syntax**. More generally syntax refers to the study of the properties of language that are there even when you don’t consider meaning. Whether a sentence is true or false is considered part of its meaning. In this chapter, we will be giving a purely syntactical definition of a sentence in SL. The contrasting term is **semantics** the study of aspects of language that relate to meaning, including truth and falsity. (The word “semantics” comes from the Greek word for “mark”)

If we are going to define a sentence in SL just using syntax, we will need to carefully distinguish SL from the language that we use to talk about SL. When you create an artificial language like SL, the language that you are creating is called the object language. The language that we use to talk about the object language is called the metalanguage. Imagine building a house.

The object language is like the house itself. It is the thing we are building. While you are building a house, you might put up scaffolding around it. The scaffolding isn’t part of the house. You just use it to build the house. The metalanguage is like the scaffolding.

The object language in this chapter is SL. For the most part, we can build this language just by talking about it in ordinary English. However, we will also have to build some special scaffolding that is not a part of SL, but will help us build SL. Our metalanguage will thus be ordinary English plus this scaffolding.

An important part of the scaffolding are the **metavariables** These are the fancy script letters we have been using in the characteristic truth tables for the connectives: A, B, C, etc. These are letters that can refer to any sentence in SL. They can represent sentences like Por Q, or they can represent longer sentences, like (((A∨ B) & G) → (P ↔ Q)). Just as the sentence letters *A*, *B*, etc. are variables that range over any English sentence, the metavariables A, B, etc. are variables that range over any sentence in SL, including the sentence letters A, B, etc.

|  |  |
| --- | --- |
| **Element** | **Symbols** |
| **Sentence Letters** | A,B,C,….Z  A1, B1, Z1, A2,…A25,…J342,…. |
| **Connectives** | ~, &, ∨, → , ↔ |
| **Parentheses** | ( , ) and [,] |
| Figure 28 Basic elements of SL | |

As we said, in this chapter we will give a syntactic definition for “sentence of SL.” The definition itself will be given in mathematical English, the metalanguage. Figure 28 gives the basic elements of SL.

Most random combinations of these symbols will not count as sentences in SL. Any random connection of these symbols will just be called a “**string**” or “**expression**” *Random strings only become meaningful sentences when they are structured according to the rules of syntax.* We saw from the earlier two sections that individual sentence letters, like Aand G13 counted as sentences. We also saw that we can put these sentences together using connectives so that ~Aand ~G13 is a sentence. The problem is, we can’t simply list all the different sentences we can put together this way, because there are infinitely many of them. Instead, we will define a sentence in SL by specifying the process by which they are constructed.

∼ ∼

Consider negation: Given any sentence Aof SL, Ais a sentence of SL. It is important here that Ais not the sentence letter *A*. Rather, it is a metavariable: part of the metalanguage, not the object language. Since Ais not a symbol of SL, ~Ais not an expression of SL. Instead, it is an expression of the metalanguage that allows us to talk about infinitely many expressions of SL: all of the expressions that start with the negation symbol.

We can say similar things for each of the other connectives. For instance, if Aand Bare sentences of SL, then (A& B) is a sentence of SL. Providing clauses like this for all of the connectives, we arrive at the following formal definition for a sentence of SL:

1. Every atomic sentence is a sentence.
2. If Ais a sentence, then ∼Ais a sentence of SL.
3. If Aand Bare sentences, then (A& B) is a sentence.
4. If Aand B are sentences, then (A∨ B) is a sentence.
5. If Aand Bare sentences, then (A → B) is a sentence.
6. If Aand Bare sentences, then (A↔ B) is a sentence.
7. All and only sentences of SL can be generated by applications of these rules.

We can apply this definition to see whether an arbitrary string is a sentence. Suppose we want to know whether or not ~~~K is a sentence of SL. Looking at the second clause of the definition, we know that ~~~K is a sentence if ~~K is a sentence. So now we need to ask whether or not ~~K is a sentence. Again, looking at the second clause of the definition, ~~K is a sentence if ~K is. Again, ~K is a sentence if K is a sentence. Now K is a sentence letter, an atomic sentence of SL, so we know that K is a sentence by the first clause of the definition. So for a compound formula like ~~~K, we must apply the definition repeatedly. Eventually we arrive at the atomic sentences from which the sentence is built up.

Definitions like this are called recursive. **Recursive definitions** begin with some specifiable base elements and define ways to indefinitely compound the base elements. Just as the recursive definition allows complex sentences to be built up from simple parts, you can use it to decompose sentences into their simpler parts. To determine whether or not something meets the definition, you may have to refer back to the definition many times. Recursive definitions are also sometimes called “inductive definitions”.

We are now in a position to define what it means for a system of logic to be a system of sentential logic. A **sentential logic** is a system of logic in which statements can be defined using a recursive definition with only sentences in the base class. This book defines on system of sentential logic, which we call SL. Other books use other systems.

When you use a connective to build a longer sentence from shorter ones, the shorter sentences are said to be in the **scope of the connective**. So in the sentence (A & B) → C, the scope of the connective → includes (A & B) and C. In the sentence ∼(A & B) the scope of the ∼ is (A & B). On the other hand, in the sentence ∼A & B the scope of the ∼ is just A.

The last connective that you add when you assemble a sentence using the recursive definition is the **main connective** of that sentence. For example: The main logical operator of ~(E ∨ (F → G) is negation, ~. The main logical operator of (~E ∨ (F → G)) is disjunction, . The main connective of any sentence will have all the rest of the sentence in its scope.

Because statement in our language is defined recursively, we can say it is “uniquely readable." **Unique readability** is a property of formal languages which is present when each well-formed formula can only be constructed in a single way. Every process of building up a sentence recursively yields a unique sentence, and every sentence is the product of a unique process of recursive definitions. This means that in an important sense our language SL is free of ambiguity, which is a key goal in the construction of any formal language. Every sentence in SL will have a unambiguous main connective and every connective in a sentence will have an unambiguous scope.

This makes logicians happy.

### 5 Notational conventions

A sentence like (Q & R) must be surrounded by parentheses, because we might apply the definition again to use this as part of a more complicated sentence. If we negate (Q & R), we get (Q & R). If we just had Q & R without the parentheses and put a negation in front of it, we would have ~Q & R. It is most natural to read this as meaning the same thing as (~Q & R), something very different than ~(Q & R). The sentence ~(Q & R) means that it is not the case that both Q and R are true; Q might be false or R might be false, but the sentence does not tell us which. The sentence (~Q & R) means specifically that Q is false and that R is true. As such, parentheses are crucial to the meaning of the sentence.

So, strictly speaking, Q & R without parentheses is *not* a sentence of SL. When using SL, however, we will often be able to relax the precise definition so as to make things easier for ourselves. We will do this in several ways.

First, we understand that Q & Rmeans the same thing as (Q & R). As a matter of convention, we can leave off parentheses that occur *around the entire sentence*.

Second, it can sometimes be confusing to look at long sentences with many nested pairs of parentheses. We adopt the convention of using square brackets [ and ] in place of parentheses.

There is no logical difference between (P ∨ Q) and [P ∨ Q], for example. The unwieldy sentence

(((H → I) ∨ (I → H)) & (J ∨ K))

could be written in this way:

[(H → I) ∨ (I → H)] & (J ∨ K)

Third, we will sometimes want to translate the conjunction of three or more sentences. For the sentence “Alice, Bob, and Candice all went to the party,” suppose we let A mean “Alice went,” B mean “Bob went,” and C mean “Candice went.” The definition only allows us to form a conjunction out of two sentences, so we can translate it as (A & B) & C or as A & (B & C). There is no reason to distinguish between these, since the two translations are logically equivalent.

There is no logical difference between the first, in which (A & B) is conjoined with C, and the second, in which A is conjoined with (B & C). So, we might as well just write A & B & C. As a matter of convention, we can leave out parentheses when we conjoin three or more sentences.

Fourth, a similar situation arises with multiple disjunctions. “Either Alice, Bob, or Candice went to the party” can be translated as (A ∨ B) ∨ C or as A ∨ (B ∨ C). Since these two translations are logically equivalent, we may write A ∨ B ∨ C.

These latter two conventions only apply to multiple conjunctions or multiple disjunctions. If a series of connectives includes both disjunctions and conjunctions, then the parentheses are essential; as with (A & B) ∨ C and A & (B ∨ C). The parentheses are also required if there is a series of conditionals or biconditionals; as with (A → B) → C and A ↔ (B ↔ C).

We have adopted these four rules as notational conventions, not as changes to the definition of a sentence. Strictly speaking, A ∨ B ∨ C is still not a sentence. Instead, it is a kind of shorthand. We write it for the sake of convenience, but we really mean the sentence (A ∨ (B ∨ C)).

If we had given a different definition for a sentence, then these could count as sentences. We might have written rule 3 in this way: “If A, B, . . . Z are sentences, then (A & B & . . . & Z), is a sentence .” This would make it easier to translate some English sentences, but would have the cost of making our formal language more complicated. We would have to keep the complex definition in mind when we develop truth tables and a proof system. We want a logical language that is expressively simple and allows us to translate easily from English, but we also want a formally simple language. Adopting notational conventions is a compromise between these two desires.

### 6 Practice Exercises

#### Part A

Using the symbolization key given, translate each English-language sentence into SL.

**M:** Those creatures are men in suits.

**C:** Those creatures are chimpanzees.

**G:** Those creatures are gorillas.

1. Those creatures are not men in suits.
2. Those creatures are men in suits, or they are not.
3. Those creatures are either gorillas or chimpanzees.
4. Those creatures are not gorillas, but they are not chimpanzees either.
5. Those creatures cannot be both gorillas and men in suits.
6. If those creatures are not gorillas, then they are men in suits
7. Those creatures are men in suits only if they are not gorillas.
8. Those creatures are chimpanzees if and only if they are not gorillas.
9. Those creatures are neither gorillas nor chimpanzees.
10. Unless those creatures are men in suits, they are either chimpanzees or they are gorillas.

#### Part B

Using the symbolization key given, translate each English-language sentence into SL.

**A:** Mister Ace was murdered.

**B:** The butler did it.

**C:** The cook did it.

**D:** The Duchess is lying.

**E:** Mister Edge was murdered.

**F:** The murder weapon was a frying pan.

1. Either Mister Ace or Mister Edge was murdered.
2. If Mister Ace was murdered, then the cook did it.
3. If Mister Edge was murdered, then the cook did not do it.
4. Either the butler did it, or the Duchess is lying.
5. The cook did it only if the Duchess is lying.
6. If the murder weapon was a frying pan, then the culprit must have been the cook.
7. If the murder weapon was not a frying pan, then the culprit was neither the cook nor the butler.
8. Mister Ace was murdered if and only if Mister Edge was not murdered.
9. The Duchess is lying, unless it was Mister Edge who was murdered.
10. Mister Ace was murdered, but not with a frying pan.
11. The butler and the cook did not both do it.
12. Of course the Duchess is lying!

#### Part C

Using the symbolization key given, translate each English-language sentence into SL.

**E**1**:** Ava is an electrician.

**E**2**:** Harrison is an electrician.

**F**1**:** Ava is a firefighter.

**F**2**:** Harrison is a firefighter.

**S**1**:** Ava is satisfied with her career.

**S**2**:** Harrison is satisfied with his career.

1. Ava and Harrison are both electricians.
2. If Ava is a firefighter, then she is satisfied with her career.
3. Ava is a firefighter, unless she is an electrician.
4. Harrison is an unsatisfied electrician.
5. Neither Ava nor Harrison is an electrician.
6. Both Ava and Harrison are electricians, but neither of them find it satisfying.
7. Harrison is satisfied only if he is a firefighter.
8. If Ava is not an electrician, then neither is Harrison, but if she is, then he is too.
9. Ava is satisfied with her career if and only if Harrison is not satisfied with his.
10. If Harrison is both an electrician and a firefighter, then he must be satisfied with his work.
11. It cannot be that Harrison is both an electrician and a firefighter.
12. Harrison and Ava are both firefighters if and only if neither of them is an electrician.

#### Part D

Using the symbolization key given, translate each English-language sentence into SL.

**J**1**:** John Coltrane played tenor sax.

**J**2**:** John Coltrane played soprano sax.

**J**3**:** John Coltrane played tuba

**M**1**:** Miles Davis played trumpet

**M**2**:** Miles Davis played tuba

1. John Coltrane played tenor and soprano sax.
2. Neither Miles Davis nor John Coltrane played tuba.
3. John Coltrane did not play both tenor sax and tuba.
4. John Coltrane did not play tenor sax unless he also played soprano sax.
5. John Coltrane did not play tuba, but Miles Davis did.
6. Miles Davis played trumpet only if he also played tuba.
7. If Miles Davis played trumpet, then John Coltrane played at least one of these three instruments: tenor sax, soprano sax, or tuba.
8. If John Coltrane played tuba then Miles Davis played neither trumpet nor tuba.
9. Miles Davis and John Coltrane both played tuba if and only if Coltrane did not play tenor sax and Miles Davis did not play trumpet.

#### Part E

Give a symbolization key and symbolize the following sentences in SL.

1. Alice and Bob are both spies.
2. If either Alice or Bob is a spy, then the code has been broken.
3. If neither Alice nor Bob is a spy, then the code remains unbroken.
4. The German embassy will be in an uproar, unless someone has broken the code.
5. Either the code has been broken or it has not, but the German embassy will be in an uproar regardless.
6. Either Alice or Bob is a spy, but not both.

#### Part F

Give a symbolization key and symbolize the following sentences in SL.

1. If Gregor plays first base, then the team will lose.
2. The team will lose unless there is a miracle.
3. The team will either lose or it won’t, but Gregor will play first base regardless.
4. Gregor’s mom will bake cookies if and only if Gregor plays first base.
5. If there is a miracle, then Gregor’s mom will not bake cookies.

#### Part G

For each argument, write a symbolization key and translate the argument as well as possible into SL.

1. If Dorothy plays the piano in the morning, then Roger wakes up cranky. Dorothy plays piano in the morning unless she is distracted. So if Roger does not wake up cranky, then Dorothy must be distracted.
2. It will either rain or snow on Tuesday. If it rains, Neville will be sad. If it snows, Neville will be cold. Therefore, Neville will either be sad or cold on Tuesday.
3. If Zoog remembered to do his chores, then things are clean but not neat. If he forgot, then things are neat but not clean. Therefore, things are either neat or clean—but not both.

#### Part H

For each argument, write a symbolization key and translate the argument as well as possible into SL. The part of the passage in italics is there to provide context for the argument, and doesn’t need to be symbolized.

1. It is going to rain soon. I know because my leg is hurting, and my leg hurts if its going to rain.
2. *Spider-man tries to figure out the bad guys plan.* If Doctor Octopus gets the uranium, he will blackmail the city. I am certain of this because if Doctor Octopus gets the uranium, he can make a dirty bomb, and if he can make a dirty bomb, he will blackmail the city.
3. *A westerner tries to predict the policies of the Chinese government.* If the Chinese government cannot solve the water shortages in Beijing, they will have to move the capital. They dont want to move the capital. Therefore they must solve the water shortage. But the only way to solve the water shortage is to divert almost all the water from the Yangzi river northward. Therefore the Chinese government will go with the project to divert water from the south to the north.

### 7 Key Terms

* Antecedent
* Atomic sentence
* Biconditional
* Conditional
* Conjunct
* Conjunction
* Consequent
* Disjunct
* Disjunction
* Main connective
* Metalanguage
* Metavariables
* Negation
* Object language
* Recursive definition
* Scope
* Semantics
* Sentence letter
* Sentence of SL
* Sentential connective
* Symbolization key
* Syntax

## Truth Tables

This chapter introduces a way of evaluating sentences and arguments of SL called the truth table method. As we shall see, the truth table method is **semantic**because it involves one aspect of the meaning of sentences, whether those sentences are true or false. Semantics is the study of aspects of language related to meaning, including truth and falsity. Although it can be laborious, the truth table method is a purely mechanical procedure that requires no intuition or special insight. Later, we will provide a parallel semantic method for QL; however, this method will not be purely mechanical.

### 1 Basic Concepts

A formal language is built from two kinds of elements: logical symbols and nonlogical symbols. **Logical symbols** have their meaning fixed by the formal language. In SL, the logical symbols are the sentential connectives and the parentheses. When writing a symbolization key, you are not allowed to change the meaning of the logical symbols. You cannot say, for instance, that the symbol will mean “not” in one argument and “perhaps” in another. The ~ symbol always means logical negation. It is used to translate the English language word “not”, but it is a symbol of a formal language and is defined by its truth conditions.

The **nonlogical symbols** are defined simply as all the symbols that aren’t logical. The nonlogical symbols in SL are the sentence letters. When we translate an argument from English to SL, for example, the sentence letter *M* does not have its meaning fixed in advance; instead, we provide a symbolization key that says how *M* should be interpreted in that argument. When translating from English to a formal language, we provided symbolization keys which were interpretations of all the nonlogical symbols we used in the translation.

In logic, when we study artificial languages, we investigate their semantics by providing an interpretation of the nonlogical symbols. An interpretation is a way of setting up a correspondence between elements of the object language and elements of some other language or logical structure. The symbolization keys we defined earlier are a sort of interpretation.

The truth table method will also involve giving an interpretation of sentences. We will not be concerned with what the individual sentence letters mean. We will only care whether they are true or false. We can do this, because of the way that the meaning of larger sentences is generated by the meaning of their parts.

Any nonatomic sentence of SL is composed of atomic sentences with sentential connectives. The truth value of the compound sentence depends only on the truth value of the atomic sentences that it comprises. In order to know the truth value of (D ↔ E), for instance, you only need to know the truth value of D and the truth value of E. Connectives that work in this way are called truth functional. More technically, we define a **truth-functional connective** as an operator that builds larger sentences out of smaller ones, and fixes the truth value of the resulting sentence based only on the truth value of the component sentences.

Because all of the logical symbols in SL are truth functional, the only aspect of meaning we need to worry about in studying the semantics of SL is truth and falsity. If we want to know about the truth of the sentence A & B, the only thing we need to know is whether A and B are true. It doesn’t actually matter what else they mean. So, if A is false, then A & B is false no matter what false sentence A is used to represent. It could be “I am the Pope” or “Pi is equal to 3.19.” The larger sentence A & B is still false. So to give an interpretation of sentences in SL, all we need to do is create a **truth assignment**. A truth assignment is a function that maps the sentence letters in SL onto our two truth values. In other words, we just need to assign Ts and Fs to all our sentence letters.

It is worth knowing that most languages are not built only out of truth functional connectives. In English, it is possible to form a new sentence from any simpler sentence X by saying “It is possible that X .” The truth value of this new sentence does not depend directly on the truth value of *X* . Even if *X* is false, perhaps in some sense *X could* have been true—then the new sentence would be true. Some formal languages, called ***modal logics***, have an operator for possibility. In a modal logic, we could translate “It is possible that X ” as X. However, the ability to translate sentences like these comes at a cost: The ◇ operator is not truth-functional, and so modal logics are not amenable to truth tables.

### 2 Complete Truth Tables

In the last chapter we introduced the characteristic truth tables for the different connectives. To put them all in one place, the truth tables for the connectives of SL are repeated in below.

|  |  |
| --- | --- |
| A | ~A |
| T | F |
| F | T |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| A | B | A *&* B | A *∨* B | A *→* B | A *↔* B |
| T | T | T | T | T | T |
| T | F | F | T | F | F |
| F | T | F | T | T | F |
| F | F | F | F | T | T |

Figure 29 The characteristic truth tables for the connectives of SL.

Notice that when we did this, we listed all the possible combinations of truth and falsity for the sentence letters in these basic sentences. Each line of the truth table is thus a *truth assignment* for the sentence letters used in the sentence we are giving a truth table for. Thus, one line of the truth table is all we need to give an *interpretation* of the sentence, and the full table gives all the possible interpretations of the sentence.

The truth value of sentences that contain only one connective is given by the characteristic truth table for that connective. The characteristic truth table for conjunction, for example, gives the truth conditions for any sentence of the form (*A* & B). Even if the conjuncts Aand Bare long, complicated sentences, the conjunction is true if and only if both Aand B are true.

A straightforward way to determine the number of rows for the T/F values in a truth table is to take two the power of the number of sentence letters in the expression. So, if there are 3 sentence letters, we would cube 2 (23) to get 8 rows.

# of Rows = 2n, where n= # of sentence letters in the expression.

Consider the sentence (H & I) → H. We consider all the possible combinations of true and false for H and I, which gives us four rows. We then copy the truth values for the sentence letters and write them underneath the letters in the sentence.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| H | I | (H | & | I) | → | H |
| T | T | T |  | T |  | T |
| T | F | T |  | F |  | T |
| F | T | F |  | T |  | F |
| F | F | F |  | F |  | F |

Now consider the **subsentence** H & I. This is a conjunction A & B with H as A and with I as B. H and I are both true on the first row. Since a conjunction is true when both conjuncts are true, we write a T underneath the conjunction symbol. We continue for the other three rows and get this:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| H | I | (H | & | I) | → | H |
| T | T | T | T | T |  | T |
| T | F | T | F | F |  | T |
| F | T | F | F | T |  | F |
| F | F | F | F | F |  | F |

The entire sentence is a conditional A → B with (H & I) as A and with H as B. On the second row, for example, (H & I) is false and H is true. Since a conditional is true when the antecedent is false, we write a T in the second row underneath the conditional symbol. We continue for the other three rows and get this:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| H | I | (H | & | I) | → | H |
| T | T | T | T | T | T | T |
| T | F | T | F | F | T | T |
| F | T | F | F | T | T | F |
| F | F | F | F | F | T | F |

The column of Ts underneath the conditional tells us that the sentence (H & I) → H is true regardless of the truth values of H and I. They can be true or false in any combination, and the compound sentence still comes out true. It is crucial that we have considered all of the possible combinations. If we only had a two-line truth table, we could not be sure that the sentence was not false for some other combination of truth values.

Most of the columns underneath the sentence are only there for bookkeeping purposes. When you become more adept with truth tables, you will probably no longer need to copy over the columns for each of the sentence letters. In any case, the truth value of the sentence on each row is just the column underneath the *main connective* of the sentence, in this case, the column underneath the conditional.

A **complete truth table** is a table that gives all the possible interpretations for a sentence or set of sentences in SL. It has a row for all the possible combinations of T and F for all of the sentence letters. The size of the complete truth table depends on the number of different sentence letters in the table. A sentence that contains only one sentence letter requires only two rows, as in the characteristic truth table for negation. This is true even if the same letter is repeated many times, as in this sentence:

[(C ↔ C) → C] & ∼(C → C).

The complete truth table requires only two lines because there are only two possibilities: *C* can be true, or it can be false. A single sentence letter can never be marked both T and F on the same row. The truth table for this sentence looks like this:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| C | [(C | ↔ | C) | → | C] | & | ~ | (C | → | C) |
| T | T | T | T | T | T | F | F | T | T | T |
| F | F | F | F | F | F | F | F | F | T | F |

Looking at the column underneath the main connective, we see that the sentence is false on both rows of the table; i.e., it is false regardless of whether C is true or false.

A straightforward way to determine the number of rows for the T/F values in a truth table is to take two the power of the number of sentence letters in the expression. So, if there are 3 sentence letters, we would cube 2 (23) to get 8 rows.

**Remember, for total number of rows in a Truth Table:**

# of Rows = 2n, where n= # of sentence letters in the expression.

A sentence that contains two sentence letters requires four lines for a complete truth table, as in the characteristic truth tables and the table for (H & I) → I. A sentence that contains three sentence letters requires eight lines. For example: A complete truth table for a sentence that contains four different sentence letters requires 16 lines. For five letters, 32 lines are required. For six letters, 64 lines, and so on.

In order to fill out the reference columns all possible combinations of T/F values, follow the steps below.

1. The right-most reference column is first filled out with rows that alternate Ts and Fs.
2. The next column over alternates sets of two Ts and two Fs.
3. For the third column from the right, you have sets of 4 Ts and 4 Fs.
4. This continues until you reach the leftmost column which will always have all Ts in the top half and Fs in the bottom half.

 Each space over from the right will have alternating sets whose length is equal to the 2x, where x is equal to the number of columns to the right you have moved from the initial rightward column.

**Remember, for the sequences of T/F in Reference Columns**

# of alternating T/F = 2x, where x = # of columns over from the right most column.

**Right-Most Column**

Will always have alternating T/F values

**Left-Most Column**

Will always have the top half of the rows be Ts and the bottom half be Fs.

#### **Practice** **Exercises**

##### Part A

Identify the main connective in each sentence.

**Example:**

(A → C) & ~D

**Answer**

“&”

1. ~(A ∨~B)
2. ~(A ∨ ~B) ∨ ~(A & D)
3. [(A → B) ∨ ~(A & D)] → E
4. [(A → B) & C] ↔ [A ∨ (B & C)]
5. ~~~[A ∨ (B & (C ∨ D))]

##### Part B

Assume A, B, and C are true, and X, Y, and Z are false and evaluate the truth of each sentence using a one-line truth table.[[10]](#footnote-11)

**Example**

(A & ~X) ↔ (B ∨ Y)

**Answer**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| (A | & | ~ | X) | ↔ | (B | ∨ | Y) |
| T | T | T | F | T | T | T | F |

True ( the connector ↔ in the statement is the main connector and its truth value is “true”)

1. ~((A & B) → X)
2. (Y ∨Z) ↔ (~X ↔ B)
3. [(X → A) ∨ (A → X)] & Y
4. (X → A) ∨(A → X)
5. [A & (Y & Z)] ∨ A

##### Part C

Write complete truth tables for the following sentences and mark the column that represents the possible truth values for the whole sentence.

**Example**

D → (D & (~F ∨ F))

**Answer**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| D | F | D | → | (D | & | (~ | F | ∨ | F)) |
| T | T | T | T | T | T | F | T | T | T |
| T | F | T | T | T | T | T | F | T | F |
| F | T | F | T | F | F | F | T | T | T |
| F | F | F | T | F | F | T | F | T | F |

1. ~(S ↔ (P → S))
2. ~[(X & Y) ∨ (X ∨ Y)]
3. (A → B) ↔ (~B ↔ ~A)
4. [C ↔ (D ∨ E)] & ~C
5. ~(G & (B & H)) ↔ (G ∨ (B ∨ H)

### 3 Using Truth Tables

A complete truth table shows us every possible combination of truth assignments on the sentence letters. It tells us every possible way sentences can relate to truth. We can use this to discover all sorts of logical properties of sentences and sets of sentences.

#### **Tautologies, contradictions, and contingent sentences**

We defined a tautology as a statement that must be true as a matter of logic, no matter how the world is. A statement like “Either it is raining or it is not raining” is always true, no matter what the weather is like outside. Something similar goes on in truth tables. With a complete truth table, we consider all of the ways that the world might be. Each line of the truth table corresponds to a way the world might be. This means that if the sentence is true on every line of a complete truth table, then it is true as a matter of logic, regardless of what the world is like.

We can use this fact to create a test for whether a sentence is a tautology: if the column under the main connective of a sentence is a T on every row, the sentence is a tautology. Not every tautology in English will correspond to a tautology in SL. The sentence “All bachelors are unmarried” is a tautology in English, but we cannot represent it as a tautology in SL, because it just translates as a single sentence letter, like B. On the other hand, if something is a tautology in SL, it will also be a tautology in English. No matter how you translate A → A, if you translate the As consistently, the statement will be a tautology.

Rather than thinking of complete truth tables as an imperfect test for the English notion of a tautology, we can define a separate notion of a tautology in SL based on truth tables. A statement is a **semantic tautology** in SL if and only if the column under the main connective in the complete truth table for the sentence contains only Ts. This is actually the semantic definition of a tautology in SL, because it uses truth tables. Later we will create a separate, syntactic definition and show that it is equivalent to the semantic definition. We will be doing the same thing for all the concepts defined in this section.

Conversely, we defined a **contradiction** as a sentence that is false no matter how the world is. This means we can define a semantic contradiction in SL as a sentence that has only Fs in the column under them main connective of its complete truth table. Again, this is the semantic definition of a contradiction.

Finally, a sentence is **contingent** if it is sometimes true and sometimes false. Similarly, a sentence is semantically contingent in SL if and only if its complete truth table for has both Ts and Fs under the main connective.

From the truth tables in the previous section, we know that (H & I) → H is a tautology, that [(C ↔ C) → C] & ~(C → C) is a contradiction, and that M & (N ∨ P ) is contingent.

#### **Logical** **equivalence**

Two sentences are logically equivalent in English if they have the same truth value as a matter of logic (p. [52](https://macc0-my.sharepoint.com/personal/henryi_macc_edu/Documents/Projects/OER%20Textbooks/PHI151%20OER%20Texts/forallx_macc.docx#_bookmark56)). Once again, we can use truth tables to define a similar property in SL: Two sentences are **semantically logically equivalent** in SL if they have the same truth value on every row of a complete truth table.

Consider the sentences ~(A ∨ B) and~A & ~B. Are they logically equivalent? To find out, we construct a truth table.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | B | ~ | (A | ∨ | B) | ~ | A | & | ~ | B |
| T | T | F | T | T | T | F | T | F | F | T |
| T | F | F | T | T | F | F | T | F | T | F |
| F | T | F | F | T | T | T | F | F | F | T |
| F | F | T | F | F | F | T | F | T | T | F |

Look at the columns for the main connectives; negation for the first sentence, conjunction for the second. On the first three rows, both are F. On the final row, both are T. Since they match on every row, the two sentences are logically equivalent.

#### **Consistency**

A set of sentences in English is consistent if it is logically possible for them all to be true at once. This means that a sentence is **semantically consistent** in SL if and only if there is at least one line of a complete truth table on which all of the sentences are true. It is semantically inconsistent otherwise.

Consider the three sentences A → B, B → C, and C → A. Since we are considering them as a set, we will place curly brackets — { , } — around them, as is done in set theory: { A → B, B → C, C → A}. The conditionals in this set form a little loop, but it is possible for all the sentences to be true at the same time, as this truth table shows.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | → | B | B | → | C | C | → | A |
| T | T | T | T | T | T | T | T | T |
| T | T | T | T | F | F | F | T | T |
| T | F | F | F | T | T | T | T | T |
| T | F | F | F | T | F | F | T | T |
| F | T | T | T | T | T | T | F | F |
| F | T | T | T | F | F | F | T | F |
| F | T | F | F | T | T | T | F | F |
| F | T | F | F | T | F | F | T | F |

#### **Validity**

Logic is the study of argument, so the most important use of truth tables is to test the validity of arguments. An argument in English is valid if it is logically impossible for the premises to be true and for the conclusion to be false at the same time. So we can define an argument as semantically valid in SL if there is no row of a complete truth table on which the premises are all marked “T” and the conclusion is marked “F.” An argument is invalid if there is such a row.

Consider this argument:

P1) ∼L → (J ∨ L)

P2) ∼L

C) ∴ J

Is it valid? To find out, we construct a truth table.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **J** | **L** | **~** | **L** | **→** | **(J** | **∨** | **L)** | **~** | **L** | **J** |
| T | T | F | T | T | T | T | T | T | T | T |
| T | F | T | F | T | T | T | F | F | F | T |
| F | T | F | T | T | F | T | T | T | T | F |
| F | F | T | F | F | F | F | F | F | F | F |

Yes, the argument is valid. The only row on which both the premises are T is the second row, and on that row the conclusion is also T.

In Chapters 1 and 2 we used the three dots ∴to represent an inference in English. We used this symbol to represent any kind of inference. The truth table method gives us a more specific notion of a valid inference. We will call this semantic entailment and represent it using a new symbol, ⊨, called the “**double turnstile**.” The ⊨ is like the ∴, except for arguments verified by truth tables. When you use the double turnstile, you write the premises as a set, using curly brackets, { and } , which mathematicians use in set theory. The argument above would be written as follows.

{∼L → (J ∨ L), ∼L} ⊨ J

More formally, we can define the double turnstile this way: A1 . . . An ⊨ B if and only if there is no truth value assignment for which A1 . . . An are true and B is false. Put differently, it means that B is true for any and all truth value assignments for which A1 . . . An are true.

We can also use the double turnstile to represent other logical notions. Since a tautology is always true, it is like the conclusion of a valid argument with no premises. The string ⊨ C means that C is true for all truth value assignments. This is equivalent to saying that the sentence is entailed by anything. We can represent logical equivalence by writing the double turnstile in both directions: A & B For instance, if we want to point out that the sentence A & B is equivalent to B & A we would write this: A & B ⫤ ⊨ B & A.

#### **Practice** **Exercises**

If you want additional practice, you can construct truth tables for any of the sentences and arguments in the exercises for the previous chapter.

##### Part A

Determine whether each sentence is a tautology, a contradiction, or a contingent sentence, using a complete truth table.

**Example**

(A → B) ∨ (B → A)

**Answer**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | B | (A | → | B) | ∨ | (B | → | A) |
| T | T | T | T | T | T | T | T | T |
| T | F | T | F | F | T | F | T | T |
| F | T | F | T | T | T | T | F | F |
| F | F | F | T | F | T | F | T | F |

Tautology, because all of the values under the main connector (∨) are true.

1. A → A
2. C → ~C
3. (A ↔ B) ↔ ~(A ↔ ~B)
4. [(~A ∨ A ) ∨ B] → B
5. [(A ∨ B) & ~A] & (B → A)

##### Part B

Determine whether each the following statements are equivalent using complete truth tables. If the two sentences really are logically equivalent, write ”Logically equivalent.” Otherwise write, ”Not logically equivalent.”

**Example**

A ∨ B ⫤ ⊨ ~A → B

**Answer**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | B | A | ∨ | B | ~ | A | → | B |
| T | T | T | T | T | F | T | T | T |
| T | F | T | T | F | F | T | T | F |
| F | T | F | T | T | T | F | T | T |
| F | F | F | F | F | T | F | F | F |

The statements are logically equivalent. The main connectors for each statement have identical T/F values for each row of the complete truth table.

1. A ⫤ ⊨ ~A
2. A & ~A ⫤ ⊨ ~B ↔ B
3. [(A ∨ B) ∨ C] ⫤ ⊨ [A ∨ (B ∨ C)]
4. A ∨(B & C) ⫤ ⊨ (A ∨ B) & (A ∨ C)
5. [A & (A & B)] → B ⫤ ⊨ A → B

##### Part C

Determine whether each set of sentences is consistent or inconsistent using a complete truth table.

**Example**

{~(A ∨ B), ~A ∨ B, A ∨ ~B}

**Answer**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | B | ~ | (A | ∨ | B | ~ | A | ∨ | B | A | ∨ | ~ | B |
| T | T | F | T | T | T | F | T | T | T | T | T | F | T |
| T | F | F | T | T | F | F | T | F | F | T | T | T | F |
| F | T | F | F | T | T | T | F | T | T | F | F | F | T |
| F | F | T | F | F | F | T | F | T | F | F | T | T | F |

The set of sentences are consistent, because there is at least one row (highlighted) where each of the main connectors (highlighted) have a true value (intersections of highlighted rows and columns).

1. { A & ~B, ~(A → B), B → A }
2. { A ∨ B, A → ~A, B → ~B }
3. {~(~A ∨ B), A → ~C, A → (B → C)}
4. { A → B, A & ~B }
5. { A → (B → C), (A → B) → C, A → C}

##### Part D

Determine whether each argument is valid or invalid, using a complete truth table.

**Example**

{ A ∨ B, C → A, C → B} ⊨ C

**Answer**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | B | C | A | ∨ | B, | C | → | A, | C | → | B, | C |
| T | T | T | T | T | T | T | T | T | T | T | T | T |
| T | T | F | T | T | T | F | T | T | F | T | T | F |
| T | F | T | T | T | F | T | T | T | T | F | F | T |
| T | F | F | T | T | F | F | T | T | F | T | F | F |
| F | T | T | F | T | T | T | F | F | T | T | T | T |
| F | T | F | F | T | T | F | T | F | F | T | T | F |
| F | F | T | F | F | F | T | F | F | T | F | F | T |
| F | F | F | F | F | F | F | T | F | F | T | F | F |

The argument is invalid because it is possible for the premises to be true and the conclusion be false. The second row shows this.

1. A → A ⊨ A
2. A → B, B ⊨ A
3. A ↔ B, B ↔ C ⊨ A ↔ C
4. A → B, A → C ⊨ B → C
5. A → B, B → A ⊨ A ↔ B

### 4 Partial Truth Tables

In order to show that a sentence is a tautology, we need to show that it is T on every row. So we need a complete truth table. To show that a sentence is *not* a tautology, however, we only need one line: a line on which the sentence is F. Therefore, in order to show that something is not a tautology, it is enough to provide a one-line *partial truth table*—regardless of how many sentence letters the sentence might have in it.

#### Showing an Expression is *not* a Tautology

Consider, for example, the sentence (*U* & *T* ) → (*S* & *W* ). We want to show that it is *not* a tautology by providing a partial truth table. We fill in F for the entire sentence.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **S** | **T** | **U** | **W** | **(U** | **&** | **T)** | **→** | **(S** | **&** | **W)** |
|  |  |  |  |  |  |  | F |  |  |  |

The main connective of the sentence is a conditional. In order for the conditional to be false, the antecedent must be true (T) and the consequent must be false (F). So we fill these in on the table:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **S** | **T** | **U** | **W** | **(U** | **&** | **T)** | **→** | **(S** | **&** | **W)** |
|  |  |  |  |  | T |  | F |  | F |  |

In order for the (*U* & *T* ) to be true, both *U* and *T* must be true.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **S** | **T** | **U** | **W** | **(U** | **&** | **T)** | **→** | **(S** | **&** | **W)** |
|  | T | T |  | T | T | T | F |  | F |  |

Now we just need to make (*S* & *W* ) false. To do this, we need to make at least one of *S* and *W* false. We can make both *S* and *W* false if we want. All that matters is that the whole sentence turns out false on this line. Making an *arbitrary* decision,[[11]](#footnote-12) we finish the table in this way:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **S** | **T** | **U** | **W** | **(U** | **&** | **T)** | **→** | **(S** | **&** | **W)** |
|  | T | T |  | T | T | T | F |  | F |  |

*S T U W*

F T T F

(*U*

T

&

T

*T* ) → (*S*

T **F**

F

& *W* )

F F

Showing that something is a contradiction requires a complete truth table. Showing that something is *not* a contradiction requires only a one-line partial truth table, where the sentence is true on that one line.

A sentence is contingent if it is neither a tautology nor a contradiction. So showing that a sentence is contingent requires a *two-line* partial truth table: The sentence must be true on one line and false on the other. For example, we can show that the sentence above is contingent with this truth table:

*S T U W*

F T T F

F T F F

(*U*

T F

&

T F

*T* ) →

T

T

(*S* & *W* )

**F** F F F

**T** F F F

Note that there are many combinations of truth values that would have made the sentence true, so there are many ways we could have written the second line.

Showing that a sentence is *not* contingent requires providing a complete truth table, because it requires showing that the sentence is a tautology or that it is a contradiction. If you do not know whether a particular sentence is contingent, then you do not know whether you will need a complete or partial truth table. You can always start working on a complete truth table. If you complete rows that show the sentence is contingent, then you can stop. If not, then complete the truth table. Even though two carefully selected rows will show that a contingent sentence is contingent, there is nothing wrong with filling in more rows.

|  |  |  |
| --- | --- | --- |
| Property | **Present** | **Absent** |
| tautology | complete truth table | one-line partial truth table |
| contradiction | complete truth table | one-line partial truth table |
| contingent | two-line partial truth table | complete truth table |
| equivalent | complete truth table | one-line partial truth table |
| consistent | one-line partial truth table | complete truth table |
| valid | complete truth table | one-line partial truth table |

Table 7.2: Complete or partial truth tables to test for different properties

Showing that two sentences are logically equivalent requires providing a complete truth table. Showing that two sentences are *not* logically equivalent requires only a one-line partial truth table: Make the table so that one sentence is true and the other false.

Showing that a set of sentences is consistent requires providing one row of a truth table on which all of the sentences are true. The rest of the table is irrelevant, so a one-line partial truth table will do. Showing that a set of sentences is inconsistent, on the other hand, requires a complete truth table: You must show that on every row of the table at least one of the sentences is false.

Showing that an argument is valid requires a complete truth table. Showing that an argument is *invalid* only requires providing a one-line truth table: If you can produce a line on which the premises are all true and the conclusion is false, then the argument is invalid.

Table [7.2](https://macc0-my.sharepoint.com/personal/henryi_macc_edu/Documents/Projects/OER%20Textbooks/PHI151%20OER%20Texts/forallx_macc.docx#_bookmark199) summarizes when a complete truth table is required and when a partial truth table will do.

### 5 Expressive Completeness

We could leave the biconditional ( ) out of the language. If we did that, we could still write “A B” so as to make sentences easier to read, but that would be shorthand for

(A B) & (B A). The resulting language would be formally equivalent to SL, since A B and (A B) & (B A) are logically equivalent in SL. If we valued formal simplicity over expressive richness, we could replace more of the connectives with notational conventions and still have a language equivalent to SL.

There are a number of equivalent languages with only two connectives. You could do logic with only the negation and the material conditional. Alternately you could just have the negation and the disjunction. You will be asked to prove that these things are true in the last problem set. You could even have a language with only one connective, if you designed the connective right. The ***Sheffer stroke***is a logical connective with the following characteristic truth table:

*A* T T F F

*B* T F T F

*A*|*B*

F

T T T

The Sheffer stroke has the unique property that it is the only connective you need to have a complete system of logic. You will be asked to prove that this is true in the last problem set also.

### 6 Practice Exercises

If you want additional practice, you can construct truth tables for any of the sentences and arguments in the exercises for the previous chapter.

##### Part A

Determine whether each sentence is a tautology, a contradiction, or a contingent sentence. Justify your answer with a complete or partial truth table where appropriate.

1. *A* → ∼*A*
2. *A* → (*A* & (*A* ∨ *B*))
3. (*A* → *B*) ↔ (*B* → *A*)
4. *A* → ∼(*A* & (*A* ∨ *B*))
5. ∼*B* → [(∼*A* & *A*) ∨ *B*]
6. ∼(*A* ∨ *B*) ↔ (∼*A* & ∼*B*)
7. [(*A* & *B*) & *C*] → *B*

∼Σ(*C* ∨ *A*) ∨ *B*Σ Σ

1. (*A* & *B*)] → [(*A* & *C*) ∨ (*B* & *D*)]

##### Part B

Determine whether each sentence is a tautology, a contradiction, or a contingent sentence. Justify your answer with a complete or partial truth table where appropriate.

1. ∼(*A* ∨ *A*)
2. (*A* → *B*) ∨ (*B* → *A*)
3. [(*A* → *B*) → *A*] → *A*
4. ∼[(*A* → *B*) ∨ (*B* → *A*)]
5. (*A* & *B*) ∨ (*A* ∨ *B*)
6. ∼(*A* & *B*) ↔ *A*
7. *A* → (*B* ∨ *C*)
8. (*A* & ∼*A*) → (*B* ∨ *C*)
9. (*B* & *D*) ↔ [*A* ↔ (*A* ∨ *C*)]
10. ∼[(*A* → *B*) ∨ (*C* → *D*)]

##### Part C

Determine whether each the following statements of equivalence are true or false using complete truth tables. If the two sentences really are logically equivalent, write ”Logically equivalent.” Otherwise write, ”Not logically equivalent.”

1. A ∼A
2. A → A A ↔ A
3. A & (B & C) A & ∼A
4. A & ∼A ∼B ↔ B
5. ∼(A → B) ∼A → ∼B
6. A ↔ B ∼[(A → B) → ∼(B → A)]
7. (A & B) → (∼A ∨ ∼B) ∼(A & B)
8. [(A ∨ B) ∨ C] [A ∨ (B ∨ C)]
9. (Z & (∼R → O)) ∼(R → ∼O)

##### Part D

Determine whether each the following statements of equivalence are true or false using complete truth tables. If the two sentences really are logically equivalent, write ”Logically equivalent.” Otherwise write, ”Not logically equivalent.”

1. A A ∨ A
2. A A & A
3. A ∨ ∼B A → B
4. (A → B) (∼B → ∼A)
5. ∼(A & B) ∼A ∨ ∼B
6. ((U → (X ∨ X)) ∨ U ) ∼(X & (X & U ))
7. ((C & (N ↔ C)) ↔ C) (∼∼∼N → C)
8. [(A ∨ B) & C] [A ∨ (B & C)]
9. ((L & C) & I) L ∨ C

##### Part E

Determine whether each set of sentences is consistent or inconsistent. Justify your answer with a complete or partial truth table where appropriate.

1. {*A* → *A*, ∼*A* → ∼*A*, *A* & *A*, *A* ∨ *A*}
2. {*A* → ∼*A*, ∼*A* → *A*}
3. {*A* ∨ *B*, *A* → *C*, *B* → *C*}
4. {*A* ∨ *B*, *A* → *C*, *B* → *C*, ∼*C*}
5. {*B* & (*C* ∨ *A*), *A* → *B*, ∼(*B* ∨ *C*)}
6. {(*A* ↔ *B*) → *B*, *B* → ∼(*A* ↔ *B*), *A* ∨ *B*}
7. {*A* ↔ (*B* ∨ *C*), *C* → ∼*A*, *A* → ∼*B*}
8. {*A* ↔ *B*, ∼*B* ∨ ∼*A*, *A* → *B*}
9. {*A* ↔ *B*, *A* → *C*, *B* → *D*, ∼(*C* ∨ *D*)}
10. {∼(*A* & ∼*B*), *B* → ∼*A*, ∼*B* }

##### Part F

Determine whether each set of sentences is consistent or inconsistent. Justify your answer with a complete or partial truth table where appropriate.

1. {*A* & *B*, *C* → ∼*B*, *C*}
2. {*A* → *B*, *B* → *C*, *A*, ∼*C*}
3. {*A* ∨ *B*, *B* ∨ *C*, *C* → ∼*A*}
4. {*A*, *B*, *C*, ∼*D*, ∼*E*, *F* }
5. {*A* & (*B* ∨ *C*), ∼(*A* & *C*), ∼(*B* & *C*)}
6. {*A* → *B*, *B* → *C*, ∼(*A* → *C*)}

##### Part G

Determine whether each argument is valid or invalid. Justify your answer with a complete or partial truth table where appropriate.

1. A → (A & ∼A) ∼A
2. A ∨ B, A → B, B → A A ↔ B
3. A ∨ (B → A) ∼A → ∼B
4. A ∨ B, A → B, B → A A & B
5. (B & A) → C, (C & A) → B (C & B) → A
6. ∼(∼A ∨ ∼B), A → ∼C A → (B → C)
7. A & (B → C), ∼C & (∼B → ∼A) C & ∼C
8. A & B, ∼A → ∼C, B → ∼D A ∨ B
9. A → B (A & B) ∨ (∼A & ∼B)
10. ∼A → B,∼B → C,∼C → A ∼A → (∼B ∨ ∼C)

##### Part H

Determine whether each argument is valid or invalid. Justify your answer with a complete or partial truth table where appropriate.

1. A ↔ ∼(B ↔ A) A
2. A ∨ B, B ∨ C, ∼A B & C
3. A → C, E → (D ∨ B), B → ∼D (A ∨ C) ∨ (B → (E & D))
4. A ∨ B, C → A, C → B A → (B → C)
5. A → B, ∼B ∨ A A ↔ B

##### Part I

Answer each of the questions below and justify your answer.

1. Suppose that A and B are logically equivalent. What can you say about A ↔ B?
2. Suppose that (A & B) C is contingent. What can you say about the argument “A, B, ∴ C ”?
3. Suppose that {A, B, C } is inconsistent. What can you say about (A & B & C )?
4. Suppose that A is a contradiction. What can you say about the argument {A, B} C ?
5. Suppose that C is a tautology. What can you say about the argument {A, B} C ”?
6. Suppose that A and B are not logically equivalent. What can you say about (A ∨ B)?

##### Part J

1. In section 3.5, we said that you could have a language that only used the negation and the material conditional. Prove that this is true by writing sentences that are logically equivalent to each of the following using only parentheses, sentence letters, negation (∼), and the material conditional (→).
   1. A ∨ B
   2. A & B
   3. A ↔ B
2. We also said in section 3.5 that you could have a language which used only the negation and the disjunction. Show this: Using only parentheses, sentence letters, negation (∼), and disjunction (∨), write sentences that are logically equivalent to each of the following.
   1. *A* & *B*
   2. *A* → *B*
   3. *A* ↔ *B*
3. Write a sentence using the connectives of SL that is logically equivalent to (*A*|*B*).
4. Every sentence written using a connective of SL can be rewritten as a logically equivalent sentence using one or more Sheffer strokes. Using only the Sheffer stroke, write sentences that are equivalent to each of the following.
   1. ∼*A*
   2. (*A* & *B*)
   3. (*A* ∨ *B*)
   4. (*A* → *B*)
   5. (*A* ↔ *B*)

### 7 Key Terms

* Complete truth table
* Interpretation
* Logical symbol Nonlogical symbol
* Semantically consistent in SL
* Semantically contingent in SL
* Semantically logically equivalent in SL
* Semantically valid in SL
* Semantic contradiction in SL
* Semantic tautology in SL
* Truth-functional connective
* Truth assignment

## Next Chapter!

1. For more detail, see the license page: < <https://creativecommons.org/licenses/by-nc-sa/4.0/> >. [↑](#footnote-ref-2)
2. For more detail, see the license page: < <https://creativecommons.org/licenses/by-nc-sa/3.0/> >. [↑](#footnote-ref-3)
3. For more, see the Open Logic homepage: < <http://openlogicproject.org/> >. [↑](#footnote-ref-4)
4. brainfuzz, *Idiocracy - Brawndo*, accessed May 10, 2018, https://www.youtube.com/watch?v=VBML3VpbuYQ. [↑](#footnote-ref-5)
5. Randall Munroe, “Xkcd: Magnetohydrodynamics,” XKCD, accessed May 10, 2018, https://web.archive.org/web/20180510191804/https://xkcd.com/1851/. [↑](#footnote-ref-6)
6. “Good Natured Baked Multigrain/Veg-Able Crisps - Check out the Flavors!,” accessed May 10, 2018, https://web.archive.org/web/20180510191539/http://www.herrsstore.com/allnagonabam.html. [↑](#footnote-ref-7)
7. Joe Gonzalez, *VERIZON 4 Times Better 30 v3 8 VO 8*, accessed May 10, 2018, https://www.youtube.com/watch?v=k6VguYqQgpE. [↑](#footnote-ref-8)
8. Gerhard Richter, *Tulips*, 1995, Oil on Canvas, 36 cm x 41 cm, 1995, https://www.gerhard-richter.com/en/art/paintings/photo-paintings/flowers-40/tulips-8113/. [↑](#footnote-ref-9)
9. Formally, variables should be typed in italics. However, at the time of printing this was not possible. It will be corrected in later versions of this text. [↑](#footnote-ref-10)
10. You only need one line because we are giving you the T/F value of each sentence letter. [↑](#footnote-ref-11)
11. Note that we could have said that…. **Finish.** [↑](#footnote-ref-12)