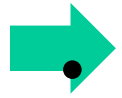


Lecture 23

Planning Under Uncertainty and Decision Networks

Lecture Overview



- Recap
 - Intro to Decision theory
 - Utility and expected utility
 - Decision Networks for Single-stage decision problems

Inference in General

- Y : subset of variables that is queried (e.g. Temperature in previous example)
- E : subset of variables that are observed . $E = e$ ($W = \text{yes}$ in previous example)
- Z_1, \dots, Z_k remaining variables in the JPD (Cloudy in previous example)

We need to compute this **numerator** for each value of Y, y_i

We need to marginalize over all the variables Z_1, \dots, Z_k not involved in the query $P(Y$

$$= y_i, E = e) = \sum_{Z_1} \dots \sum_{Z_k} P(Z_1, \dots, Z_k, Y = y_i, E = e)$$

$$P(Y, E = e) \text{ Def of conditional probability}$$

$$P(Y | E=e) = \frac{P(Y, E=e)}{P(E=e)}$$

$$P(Y, E=e)$$

$$\sum_Y P(Y, E=e)$$

constant

To compute the **denominator**, marginalize over Y

Same value for every ensuring that $P(Y=\sum_Y y P(Y_i))$.

$$\text{Normalization} = \sum_Y P(Y, E=e) = 1$$

- All we need to compute is the numerator: joint probability of the query variable(s) and the evidence!
- **Variable Elimination** is an algorithm that efficiently performs this operation by casting it as operations between **factors** - **introduced next**

Factors

- A factor is a function from a tuple of random variables to the real numbers \mathbb{R}
- We write a factor on variables X_1, \dots, X_j as $f(X_1, \dots, X_j)$

- $P(X_1, X_2)$ is a factor $f(X_1, X_2)$ *Distribution*

- $P(Z | X, Y)$ is a factor $f(Z, X, Y)$ *Set of Distributions*
One for each combination of values for X and Y

- $P(Z=f | X, Y)$ is a factor $f(X, Y)$ *Set of partial Distributions*

| X | Y | Z | val |
|---|---|---|-----|
| t | t | t | 0.1 |
| t | t | f | 0.9 |
| t | f | t | 0.2 |
| t | f | f | 0.8 |
| f | t | t | 0.4 |
| f | t | f | 0.6 |
| f | f | t | 0.3 |
| f | f | f | 0.7 |

- Note: **Factors do not have to sum to one**

- A factor denotes **one** or **more** (**possibly partial**) distributions over the given tuple of variables, e.g.,

Recap: Factors and Operations on Them

If we assign variable $A=a$ in factor $f_7(A,B)$, what is the correct form for the resulting factor? • $f(B)$.

When we assign variable A we remove it from the factor's domain

If we marginalize variable A out from factor $f_7(A,B)$, what is the correct form for the resulting factor? • $f(B)$.

When we marginalize out variable A we remove it from the factor's domain

If we multiply factors $f_4(X,Y)$ and $f_6(Z,Y)$, what is the correct form for the resulting factor?

- $f(X,Y,Z)$
- When multiplying factors, the resulting factor's domain is the union of the multiplicands' domains
- What is the correct form for $\sum_B f_5(A,B) \times f_6(B,C)$

- As usual, product before sum: $\sum_B (f_5(A,B) \times f_6(B,C))$
- Result of multiplication: $f_7(A,B,C)$. Then marginalize out B: $f_8(A,C)$

The variable elimination algorithm,

n

The JPD of a Bayesian network is $P(X_1, \dots, X_n) = \prod P(X_i | pa(X_i))$

Given: $P(Y, E_{\text{observed}_1}, \dots, E_j, Z_1, \dots, Z_k)$ Other variables not involved in the query $i=1$

To compute $P(Y=y_i | E_1=e_1, \dots, E_j=e_j) = \frac{P(Y=y_i, E_1=e_1, \dots, E_j=e_j)}{\sum_y P(Y=y, E_1=e_1, \dots, E_j=e_j)}$

The variable elimination algorithm,

n

The JPD of a Bayesian network is $P(X_1, \dots, X_n) = \prod P(X_i | pa(X_i))$

Given: $P(Y, \text{E}_{observed_1}, \dots, E_j, Z_1, \dots, Z_k)$ Other variables not involved in the query $i=1$

To compute $P(Y=y_i | E_1=e_1, \dots, E_j=e_j) = \frac{P(Y = y_i, E_1 = e_1, \dots, E_j = e_j)}{\sum_{Y=y} P(Y = y, E_1 = e_1, \dots, E_j = e_j)}$

1. Construct a factor for each conditional probability. $P(X_i | pa(X_i)) = f_i(X_i, pa(X_i))$

n

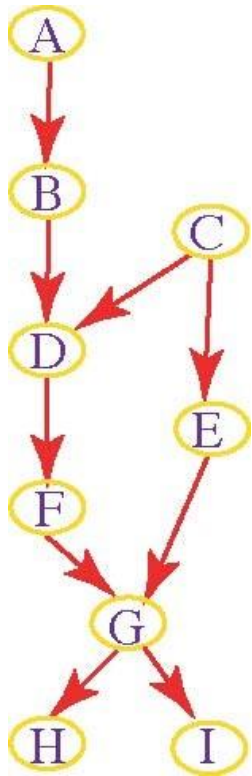
$$P(Y, E_1 = e_1, \dots, E_j = e_j) = \sum_{Z_1} \cdots \sum \prod_{i=1} (f_i)_{E_1=e_1, \dots, E_j=e_j} Z_k$$

2. For each factor, **assign** the observed variables E to their observed values.
3. Given an elimination ordering, decompose sum of products
4. **Sum out** all variables Z_i not involved in the query (one a time)
 - Multiply factors containing Z_i
 - Then marginalize out Z_i from the product
5. Multiply the remaining factors (which only involve Y)
6. **Normalize** by dividing the resulting factor $f(Y)$ by $\sum_y f(Y)$

Step 1: Construct a factor for each cond. probability

Compute $P(G \mid H=h_1)$.

$$P(G, H) = \sum_{A, B, C, D, E, F, I} P(A)P(B/A)P(C)P(D/B, C)P(E/C)P(F/D)P(G/F, E)P(H/G)P(I/G)$$



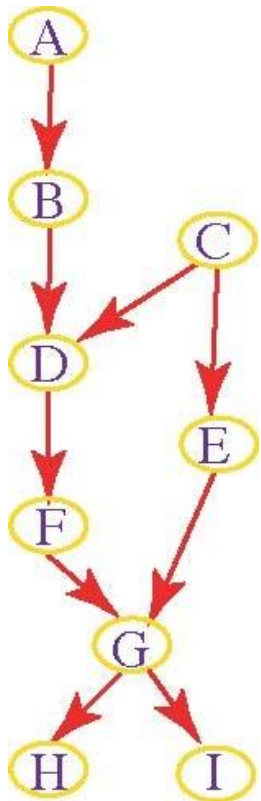
$$P(G,H) = \sum_{A,B,C,D,E,F,I} f_0(A) f_1(B,A) f_2(C) f_3(D,B,C) f_4(E,C) f_5(F, D) f_6(G,F,E) f_7(H,G) f_8(I,G)$$

Step 2: assign to observed variables their observed values.

Compute $P(G \mid H=h_1)$.

Previous state:

$$P(G, H) = \sum_{A, B, C, D, E, F, I} f_0(A) f_1(B, A) f_2(C) f_3(D, B, C) f_4(E, C) f_5(F, D) f_6(G, F, E) f_7(H, G) f_8(I, G)$$





Observe H :

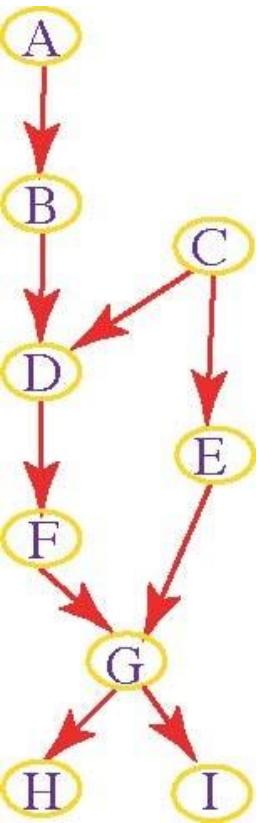
$$P(G, H=h_I) = \sum_{A,B,C,D,E,F,I} f_0(A) f_1(B,A) f_2(C) f_3(D,B,C) f_4(E,C) f_5(F,D) f_6(G,F,E) f_9(G) f_8(I,G)$$

Step 3: Decompose sum of products

Compute $P(G \mid H=h_1)$.

Previous state:

$$P(G, H=h_1) = \sum_{A,B,C,D,E,F,I} f_0(A) f_1(B,A) f_2(C) f_3(D,B,C) f_4(E,C) f_5(F,D) f_6(G,F,E) f_9(G) f_8(I,G)$$



Elimination ordering A, C, E, I, B, D, F :

$$P(G, H=h_1) = f_9(G) \sum_F \sum_D f_5(F, D) \sum_B \sum_I f_8(I, G) \sum_E f_6(G, F, E) \sum_C f_2(C) f_3(D, B, C) f_4(E, C) \sum_A f_0(A) f_1(B, A)$$

Step 4: sum out non query variables (one at a time)

Compute $P(G \mid H=h_1)$.

Elimination order: **A**,C,E,I,B,D,F

Previous state:

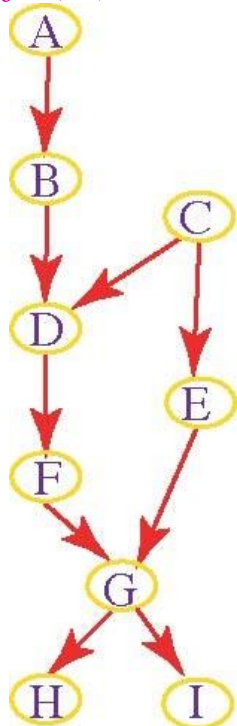
$$P(G, H=h_1) = f_9(G) \sum_F \sum_D f_5(F, D) \sum_B \sum_I f_8(I, G) \sum_E f_6(G, F, E) \sum_C f_2(C) f_3(D, B, C) f_4(E, C) \sum_A f_0(A) f_1(B, A)$$

Eliminate A: perform product and sum out A in

$$P(G, H=h_1) = f_9(G) \sum_F \sum_D f_5(F, D) \sum_B f_{10}(B) \sum_I f_8(I, G) \sum_E f_6(G, F, E) \sum_C f_2(C) f_3(D, B, C) f_4(E, C)$$

Compute

$f_{10}(B)$ does not depend on C, E, or I, so we can push it outside of those sums.



Step 4: sum out non query variables (one at a time)

$$P(G / H=h_1).$$

Elimination order:

A, C, E, I, B, D, F

Previous state:

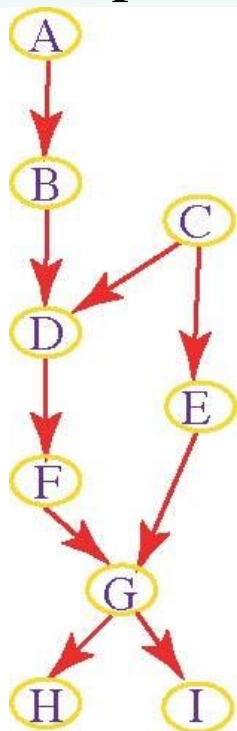
$$P(G, H=h_1) = f_9(G) \sum_F \sum_D f_5(F, D) \sum_B f_{10}(B) \sum_I f_8(I, G) \sum_E f_6(G, F, E) \sum_C f_2(C) f_3(D, B, C) f_4(E, C)$$

Eliminate C: perform product and sum out C in

$$P(G, H=h_1) = f_9(G) \sum_F \sum_D f_5(F, D) \sum_B f_{10}(B) \sum_I f_8(I, G) \sum_E f_6(G, F, E) f_{11}(B, D, E)$$

Step 4: sum out non query variables (one at a time)

Compute $P(G \mid H=h_1)$.



Elimination order: A,C,**E**,I,B,D,F

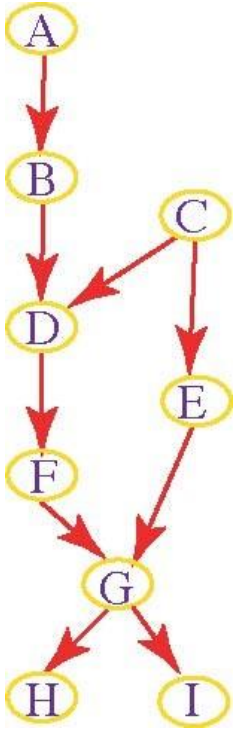
Previous state:

$$P(G, H=h_1) = P(G, H=h_1) = f_9(G) \sum_F \sum_D f_5(F, D) \sum_B f_{10}(B) \sum_I f_8(I, G) \sum_E f_6(G, F, E) f_{11}(B, D, E)$$

Eliminate **E**: perform product and sum out E in

Compute

$$P(G, H=h_1) = P(G, H=h_1) = f_9(G) \sum_F \sum_D f_5(F, D) \sum_B f_{10}(B) f_{12}(B, D, F, G) \sum_I f_8(I, G)$$



Step 4: sum out non query variables (one at a time)

Compute $P(G \mid H=h_1)$.

Elimination order: A,C,E,**I**,B,D,F

Previous state:

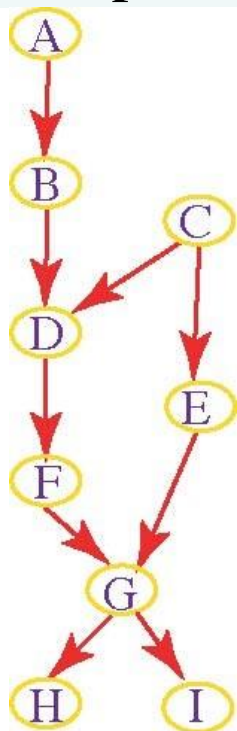
$$P(G, H=h_1) = P(G, H=h_1) = f_9(G) \sum_F \sum_D f_5(F, D) \sum_B f_{10}(B) f_{12}(B, D, F, G) \sum_I f_8(I, G)$$

Eliminate I: perform product and sum out  I in

$$P(G, H=h_1) = P(G, H=h_1) = f_9(G) f_{13}(G) \sum_F \sum_D f_5(F, D) \sum_B f_{10}(B) f_{12}(B, D, F, G)$$

Step 4: sum out non query variables (one at a time)

Compute $P(G \mid H=h_1)$.



Elimination order: A,C,E,I,**B**,D,F

Previous state:

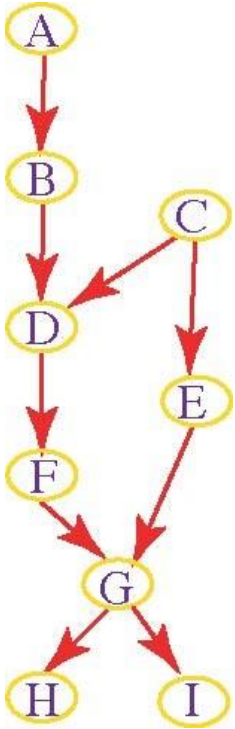
$$P(G, H=h_1) = P(G, H=h_1) = f_9(G) f_{13}(G) \sum_F \sum_D f_5(F, D) \sum_B f_{10}(B) f_{12}(B, D, F, G)$$

Eliminate **B**: perform product and sum out B in

Step 4: sum out non query variables (one at a time)

Compute $P(G \mid H=h_1)$.

$$P(G, H=h_1) = P(G, H=h_1) = f_9(G) f_{13}(G) \sum_F \sum_D f_5(F, D) f_{14}(D, F, G)$$



Elimination order: A,C,E,I,B,**D**,F

Previous state:

$$P(G, H=h_1) = P(G, H=h_1) = f_9(G) f_{13}(G) \sum_F \sum_D f_5(F, D) f_{14}(D, F, G)$$

Step 4: sum out non query variables (one at a time)

Compute $P(G \mid H=h_1)$.

Eliminate D: perform product and sum out D in

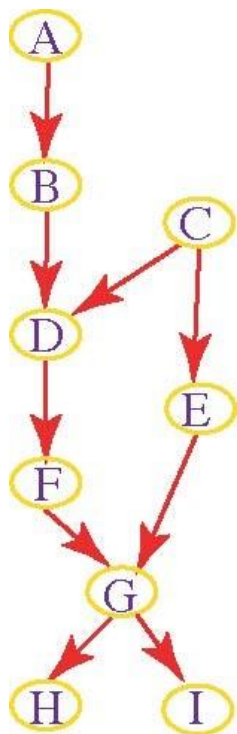
$$P(G, H=h_1) = P(G, H=h_1) = f_9(G) f_{13}(G) \sum_F f_{15}(F, G)$$

Multiply remaining factors (all in G): $P(G, H=h_1) = f_{17}(G)$

Elimination order: A, C, E, I, B, D, F

Previous state:

$$P(G, H=h_1) = P(G, H=h_1) = f_9(G) f_{13}(G) \sum_F f_{15}(F, G)$$



$$P(G = g \mid H = h_1) = \frac{P(G = g, H = h_1)}{P(H = h_1)}$$

$$P(H = h_1)$$

$$\frac{P(G = g, H = h_1)}{P(H = h_1)} = \frac{f_{17}(g)}{P(H = h_1)}$$

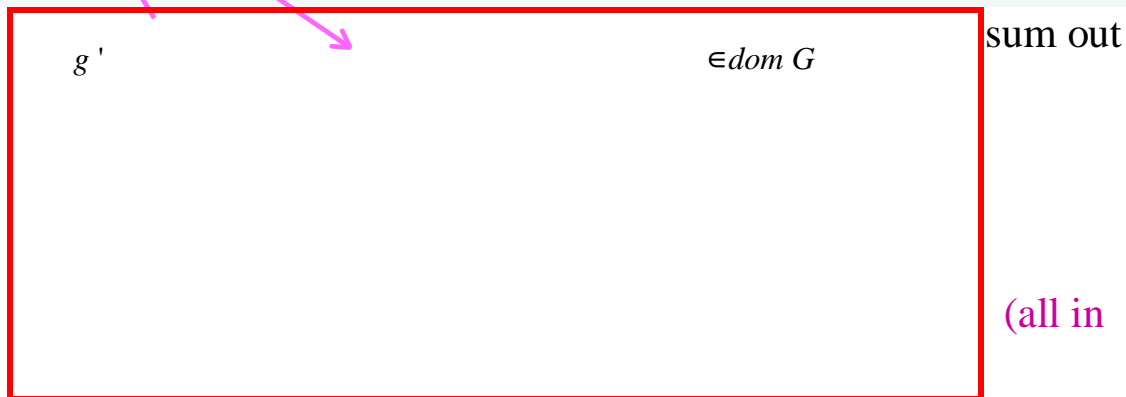
$$= \frac{\sum_{g' \in \text{dom}(G)} P(G = g', H = h_1) f_{17}(g')}{\sum_{g' \in \text{dom}(G)} P(G = g', H = h_1) f_{17}(g')}$$

Step 4: sum out non query variables (one at a time)

Compute $P(G \mid H=h_1)$.

Eliminate F: perform product and
F in

$$P(G, H=h_1) = f_9(G) f_{13}(G) f_{16}(G)$$

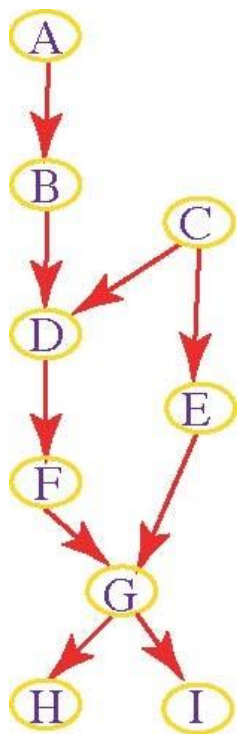


Multiply
remaining factors
G):

$$P(G, H=h_1) =$$

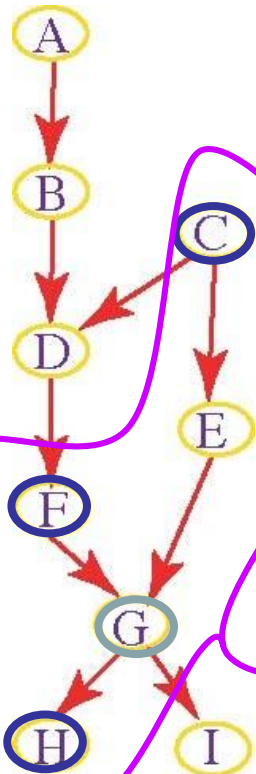
$$f_{17}(G)$$

Normalize



Variable elimination: pruning

- Before running VE, we can prune all variables Z that are conditionally independent of the query Y given evidence E :
 $Z \perp\!\!\!\perp Y \mid E$
 - They cannot change the belief over Y given E !



Thus, if the query is
 $P(G=g \mid C=c_1, F=f_1, H=h_1)$
we only need to consider this subnetwork


- We can also prune unobserved leaf nodes
 - Since they are unobserved and not predecessors of the query nodes, they cannot influence the posterior probability of the query nodes

Complexity of Variable Elimination (VE) (not required)

- The complexity of VE is **exponential** in the maximum number of variables in any factor during its execution
- This number is called the **treewidth of a graph** (along an ordering)

- Elimination ordering influences treewidth
- Finding the best ordering is NP complete
- I.e., the ordering that generates the minimum treewidth
- Heuristics work well in practice (e.g. least connected variables first)
- Even with best ordering, inference is sometimes infeasible
- ✓ In those cases, we need approximate inference. See CS422 & CS540

Lecture Overview

- Recap
-  Intro to Decision theory
- Utility and expected utility

- Decision Networks for Single-stage decision problems

Where are we?

Representation

Reasoning

Environment

Deterministic

Arc

Consistency

Query

Problem Type

Constraint Satisfaction

*Vars**Constraints*+

Search

Stochastic

Search
Technique

This concludes the module
on answering queries in
stochastic environments

Logics

Search

Belief Nets

Variable
Elimination

STRIPS

Decision Nets

Variable
Elimination

Planning

Static

Sequential

Problem Type

Static

Constraint Satisfaction

Query

Sequential

Planning

Deterministic

Stochastic

Representation

Reasoning
Technique

Arc

Consistency

Vars + Constraints

Search

Now we will look at **acting** in stochastic environments

Logics

Search

Belief Nets

Variable
Elimination

STRIPS

Search

Decision Nets

Variable
Elimination

Environment

Decisions Under Uncertainty: Intro

- Earlier in the course, we focused on decision making in deterministic domains
- Planning
- Now we face **stochastic domains**
- so far we've considered how to represent and update beliefs
- what if an agent has to **make decisions** (act) **under uncertainty**?

- Making decisions under uncertainty is important •
We represent the world probabilistically so we can use our beliefs as the basis for making decisions



Decisions Under Uncertainty: Intro

- An agent's decision will depend on
 - What actions are available
 - What beliefs the agent has
 - Which goals the agent has
- Differences between deterministic and stochastic setting •
Obvious difference in representation: need to represent our uncertain beliefs

- **Actions** will be pretty straightforward: represented as **decision variables**
- **Goals** will be interesting: we'll move from all-or-nothing goals to a richer notion:
 - ✓ rating **how happy the agent is** in different situations.
- Putting these together, we'll extend Bayesian Networks to make a new representation called **Decision Networks**

Delivery Robot Example

- Robot needs to reach a certain room
- Robot can go
- the short way - faster but with more obstacles, thus more prone to accidents that can damage the robot and prevent it from reaching the room

- the long way - slower but less prone to accident
- Which way to go? Is it more important for the robot to arrive fast, or to minimize the risk of damage?
- The Robot can choose to wear pads to protect itself in case of accident, or not to wear them. Pads make it heavier, increasing energy consumption
- Again, there is a tradeoff between reducing risk of damage, saving resources and arriving fast
- Possible outcomes
- No pad, no accident 
- Pad, no accident
- Pad, Accident
- No pad,  accident



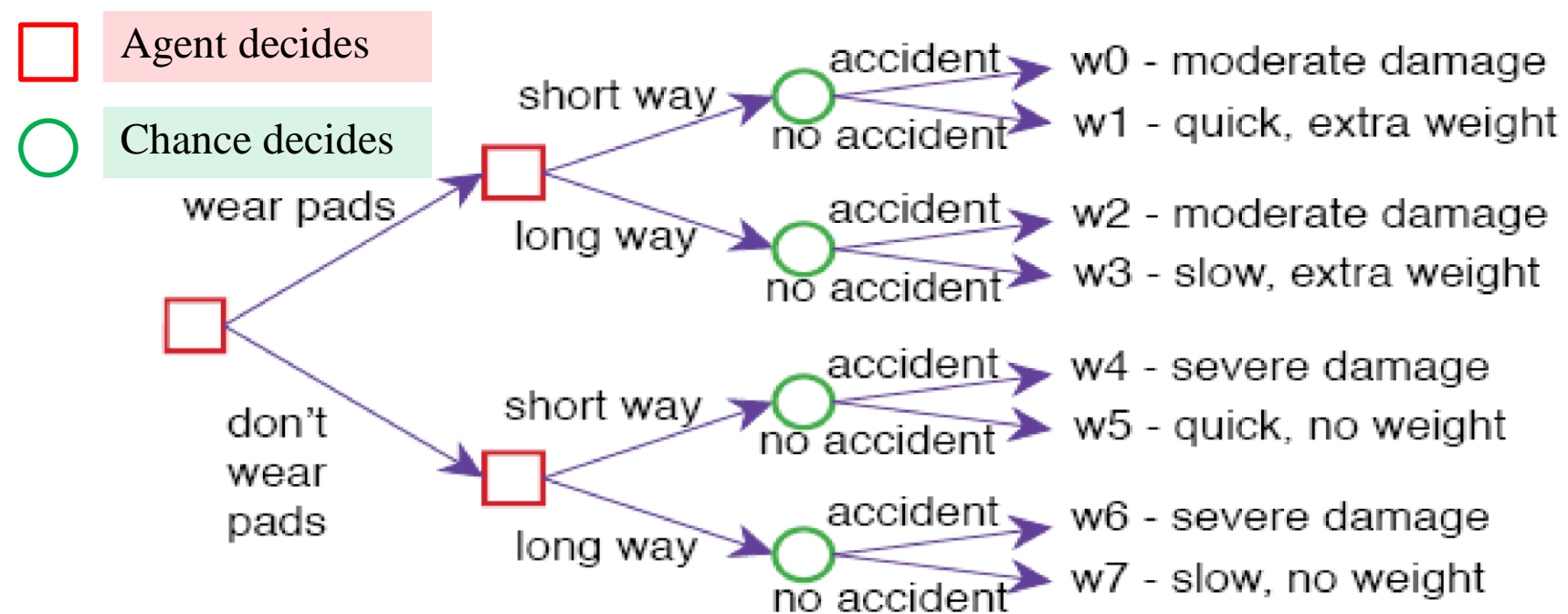
Next

- We'll see how to represent and reason about situations of this nature using **Decision Trees**, as well as
- **Probability** to measure the uncertainty in action outcome
- **Utility** to measure agent's preferences over the various outcomes
- Combined in a measure of **expected utility** that can be used to identify the action with the **best expected outcome**
- Best that an intelligent agent can do when it needs to act in a stochastic environment

Decision Tree for the Delivery Robot Example

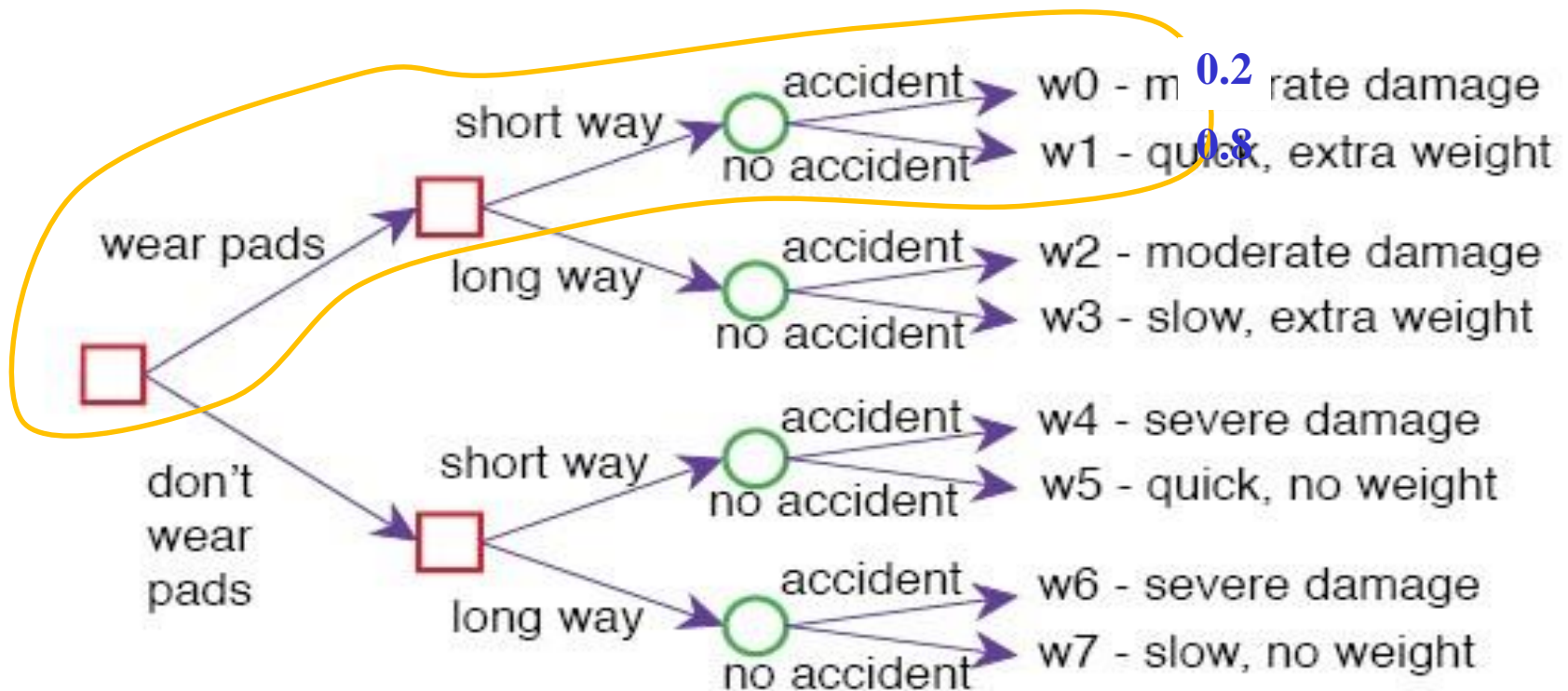
- Decision variable 1: **the robot can choose to wear pads**

- Yes: protection against accidents, but extra weight
- No: fast, but no protection
- **Decision variable 2: the robot can choose the way**
- Short way: quick, but higher chance of accident
- Long way: safe, but slow
- Random variable: is there an accident?



Possible worlds and decision variables

- A **possible world** specifies a value for each random variable and each decision variable
- For each assignment of values to all decision variables:

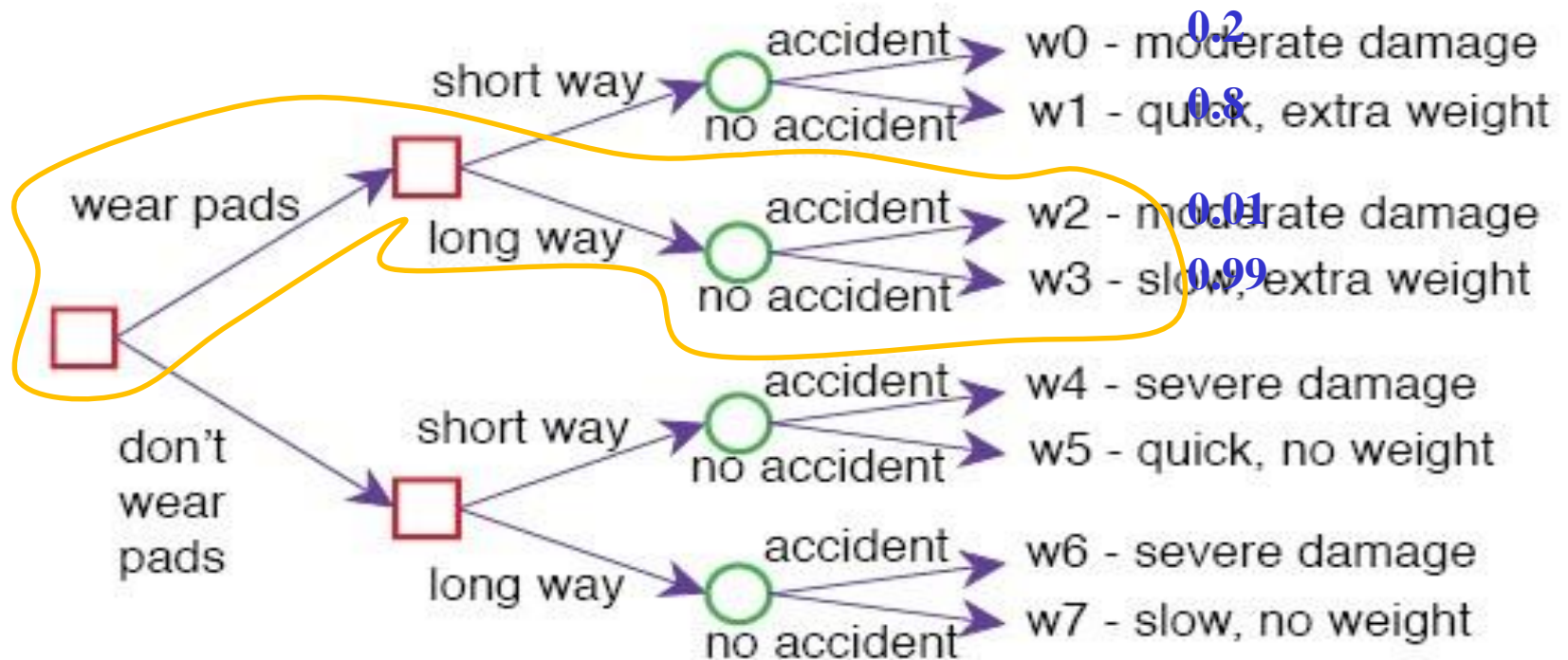


Possible worlds and decision variables

- A **possible world** specifies a value for **each random variable**
 - the probabilities of the worlds satisfying that assignment sum to 1.

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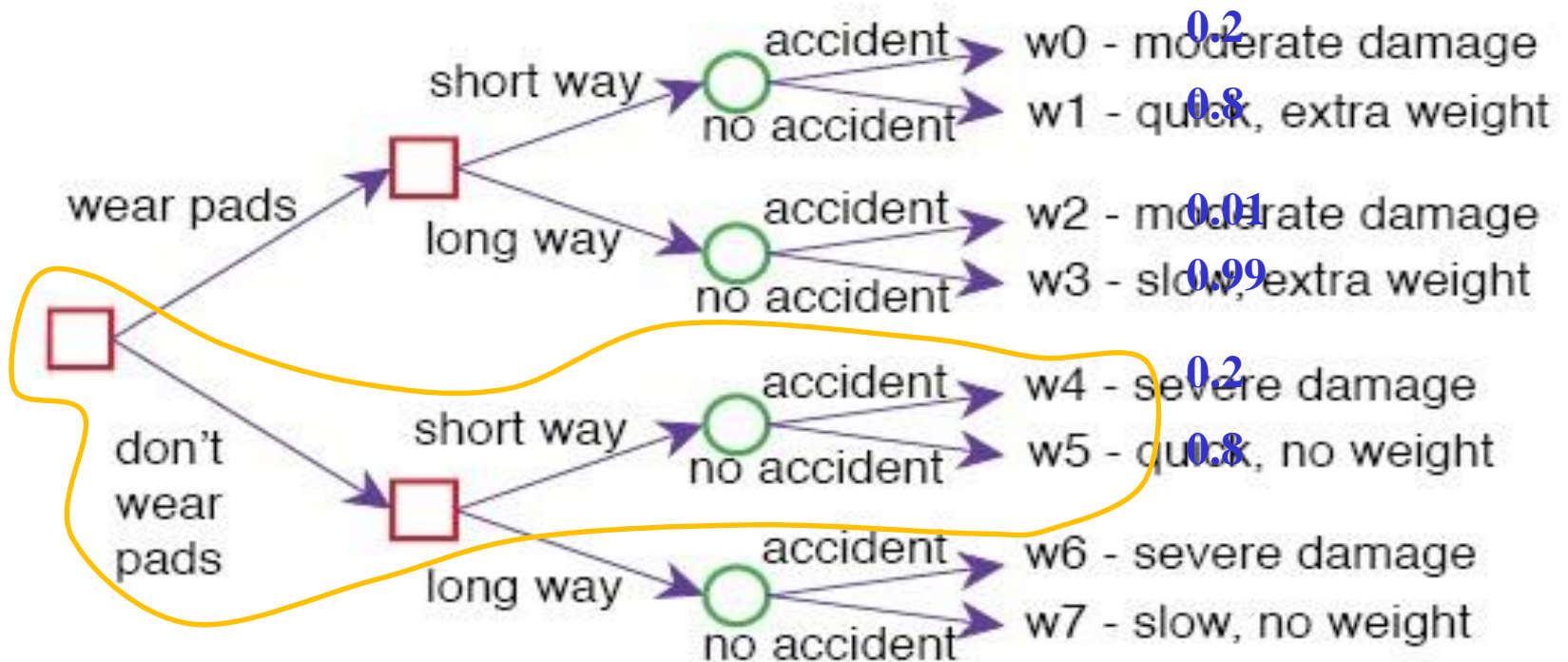


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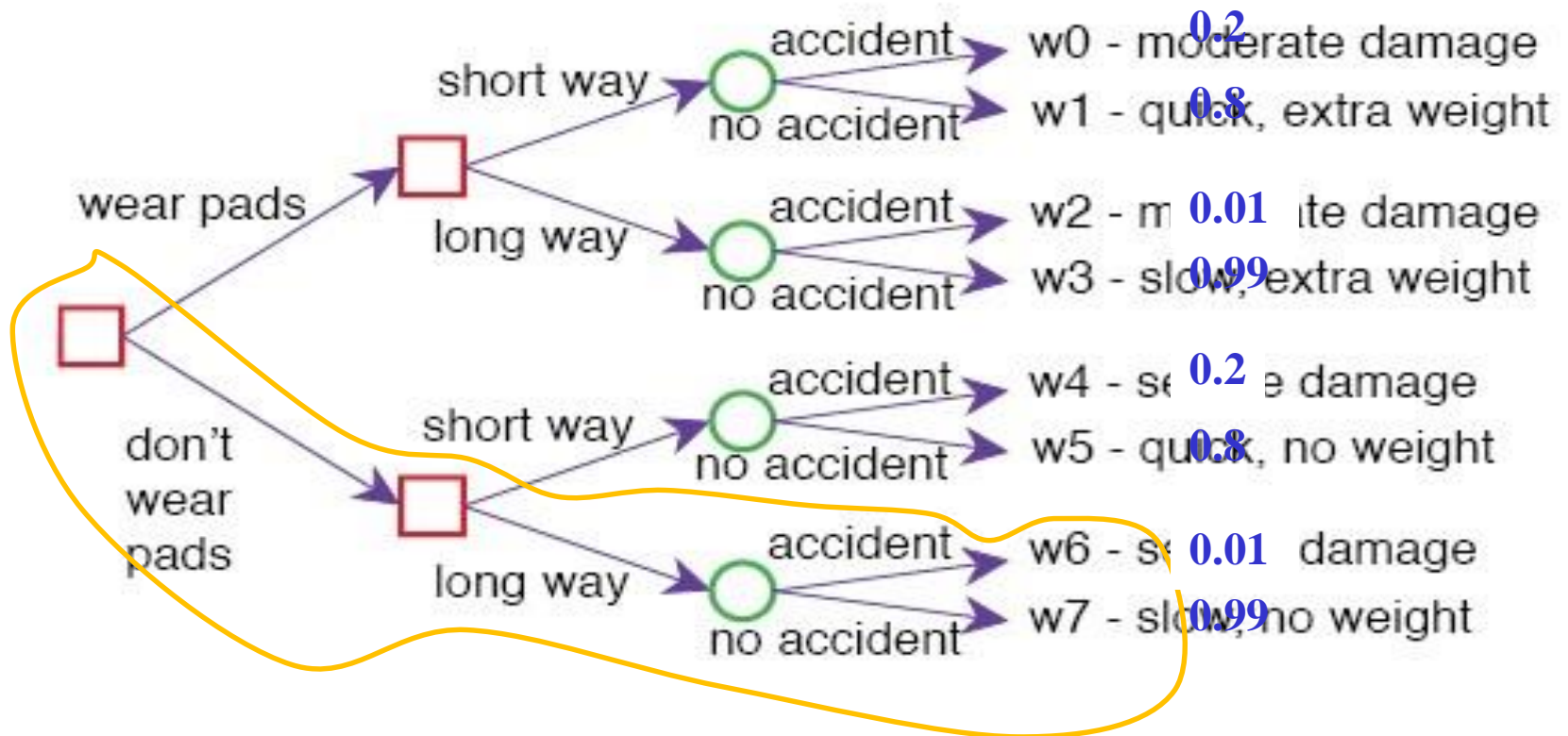


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Possible worlds and decision variables

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Lecture Overview

- Recap
- Intro to Decision theory
- ➔ • Utility and expected utility
- Decision Networks for single-stage decision problems

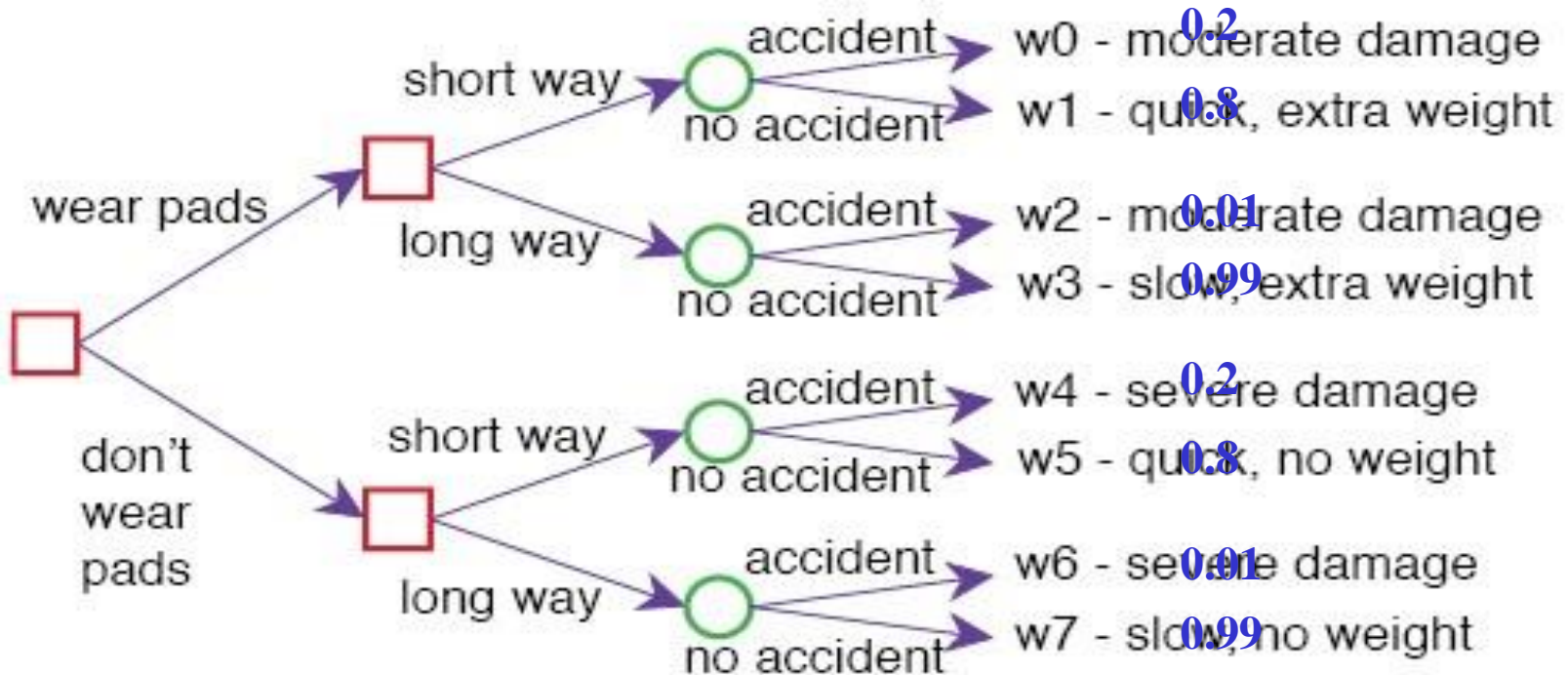
Utility

- **Utility**: a measure of desirability of possible worlds to an agent
- Let U be a real-valued function such that $U(w)$ represents an agent's degree of preference for world w
- Expressed by a number in $[0, 100]$

Utility for the Robot Example

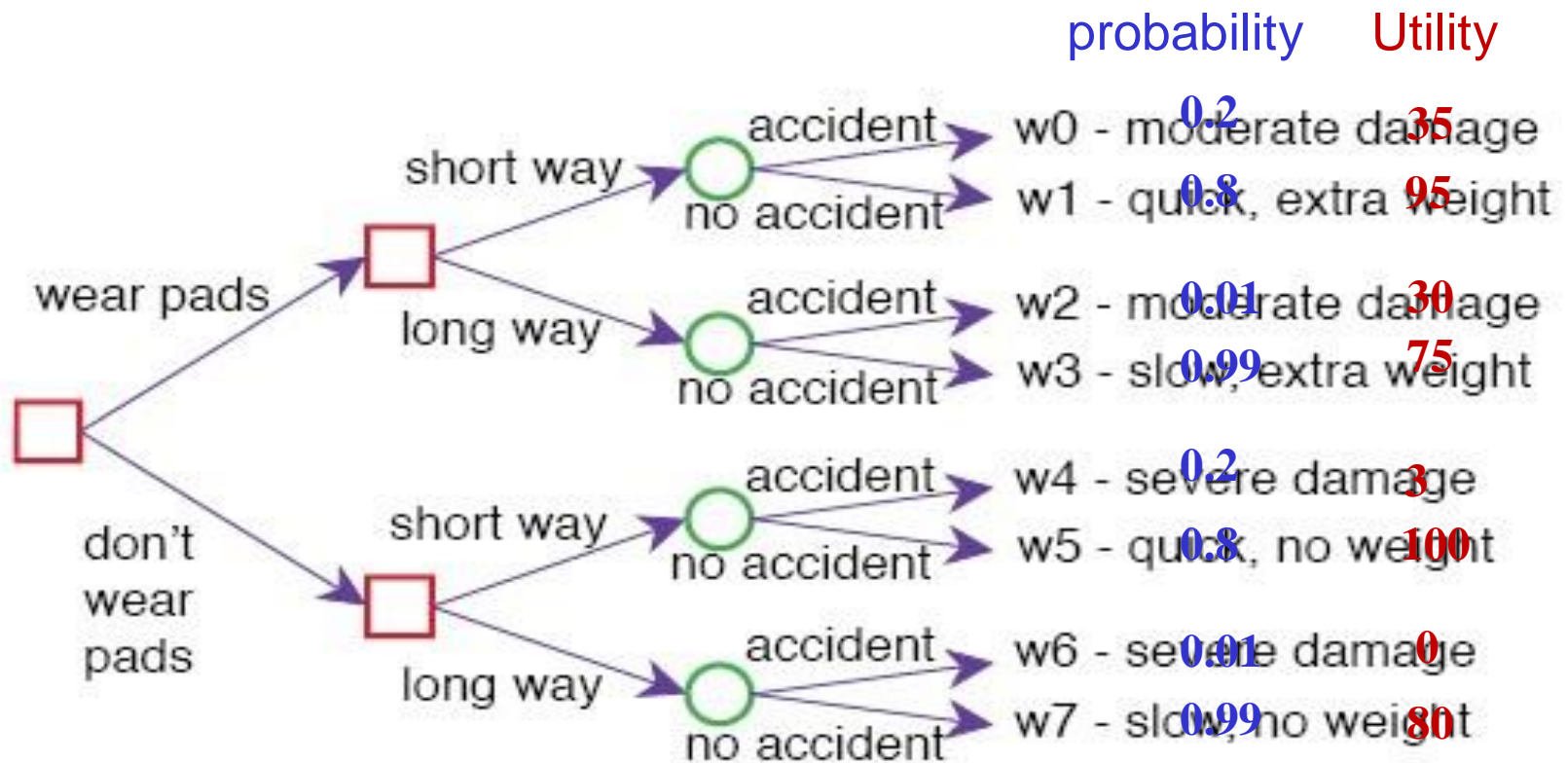
- Which would be a reasonable utility function for our robot?
- Which are the best and worst scenarios? probability

Utility



Utility for the Robot Example

- Which would be a reasonable utility function for our robot?



Utility: Simple Goals

- How can the simple (boolean) goal “reach the room” be specified? **B.**

| Which way | Accident | Wear Pads | Utility |
|-----------|----------|-----------|---------|
| long | true | true | 0 |
| long | true | false | 0 |
| long | false | true | 0 |
| long | false | false | 0 |
| short | true | true | 0 |
| short | true | false | 0 |
| short | false | true | 100 |
| short | false | false | 90 |

| Which way Pads | Accident | Wear | Utility |
|-------------------|----------|------|---------|
|-------------------|----------|------|---------|

| Which way | Accident | Wear Pads | Utility | long |
|----------------|----------|-----------|---------|------|
| long | true | true | 0 | |
| long | true | false | 0 | |
| long | false | true | 100 | |
| long | false | false | 100 | |
| short | true | true | 0 | |
| short | true | false | 0 | |
| short | false | true | 100 | |
| short | false | false | 100 | |
| long | true | 0 long | true | |
| false | 0 long | false | true | |
| 0 long | false | false | 0 | |
| short | true | true | 0 short | |
| true | false | 0 short | false | |
| true 100 short | false | false | false | 0 |

D. Not possible

Utility

- **Utility**: a measure of desirability of possible worlds to an agent • Let U be a real-valued function such that $U(w)$ represents an agent's degree of preference for world w
- Expressed by a number in $[0, 100]$
- Simple goals can still be specified
- Worlds that satisfy the goal have utility 100 • Other worlds have utility 0

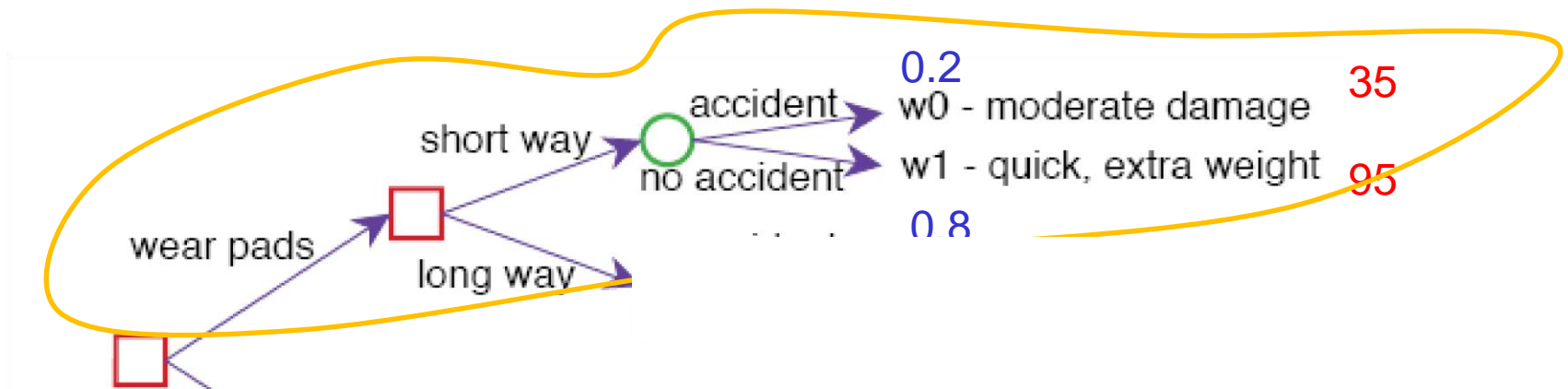
e.g., goal “reach the room”

Optimal decisions: combining Utility and Probability

| Which way Pads | Accident | Wear | Utility |
|-------------------|----------|-------|---------|
| long | true | true | 0 |
| long | true | false | 0 |
| long | false | true | 100 |
| long | false | false | 100 |
| short | true | true | 0 |
| short | true | false | 0 |
| short | false | true | 0 |
| short | false | false | 100 |
| | | | 100 |

- Each set of decisions defines a **probability distribution** over possible outcomes
- Each outcome has a **utility**
- For each set of decisions, we need to know their **expected utility**

- the value for the agent of achieving a certain **probability distribution** over outcomes (possible worlds)
- The **expected utility of a set of decisions** is obtained by
- weighting the utility of the relevant possible worlds by their probability.



- We want to find the decision with **maximum expected utility**

Expected utility of a decision

- The **expected utility** of a **specific decision** $D = d$ is indicated as $E(U | D = d)$ and it is computed as follows

$$P(w_1) \times U(w_1) + P(w_2) \times U(w_2) + P(w_3) \times U(w_3) + \dots + P(w_n) \times U(w_n)$$

Where

- $w_1, w_2, w_3, \dots, w_n$ are all the **possible worlds** in which **d is true**
- $P(w_1), P(w_2), \dots, P(w_n)$ are their probabilities
- $U(w_1), U(w_2), \dots, U(w_n)$ are the corresponding utilities for w_1, \dots, w_n

That is,

- for each possible world w_i in which the decision d is true, multiply the probability of that world and its utility $P(w_i) \times U(w_i)$

- sum all these products together This notation indicates all the

possible worlds in which d is true

In a formula

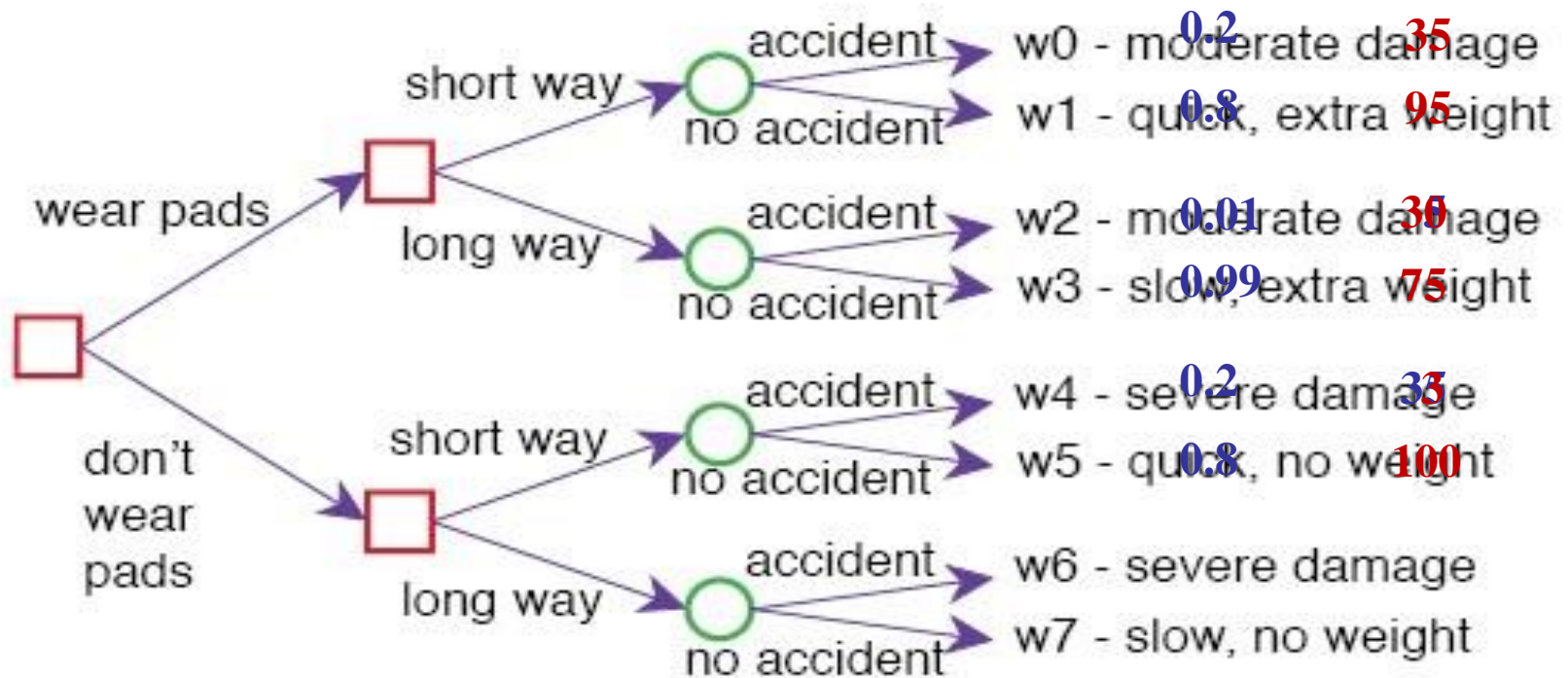
$$E(U \mid D = d_i) = \sum_{w \models (D = d_i)} P(w) \times U(w)$$

Example of Expected Utility

- The **expected utility** of decision $D = d$ is

$$E(U \mid D = d) = \sum_{w \models (D = d)} P(w) U(w) = P(w_1) \times U(w_1) + \dots + P(w_n) \times U(w_n)$$

- What is the **expected utility** of Wearpads=yes, Way=short ?



A. 7 B. 83 C. 76 D. 157.55

Probability Utility $E[U|D]$

0.01350 0.99 80

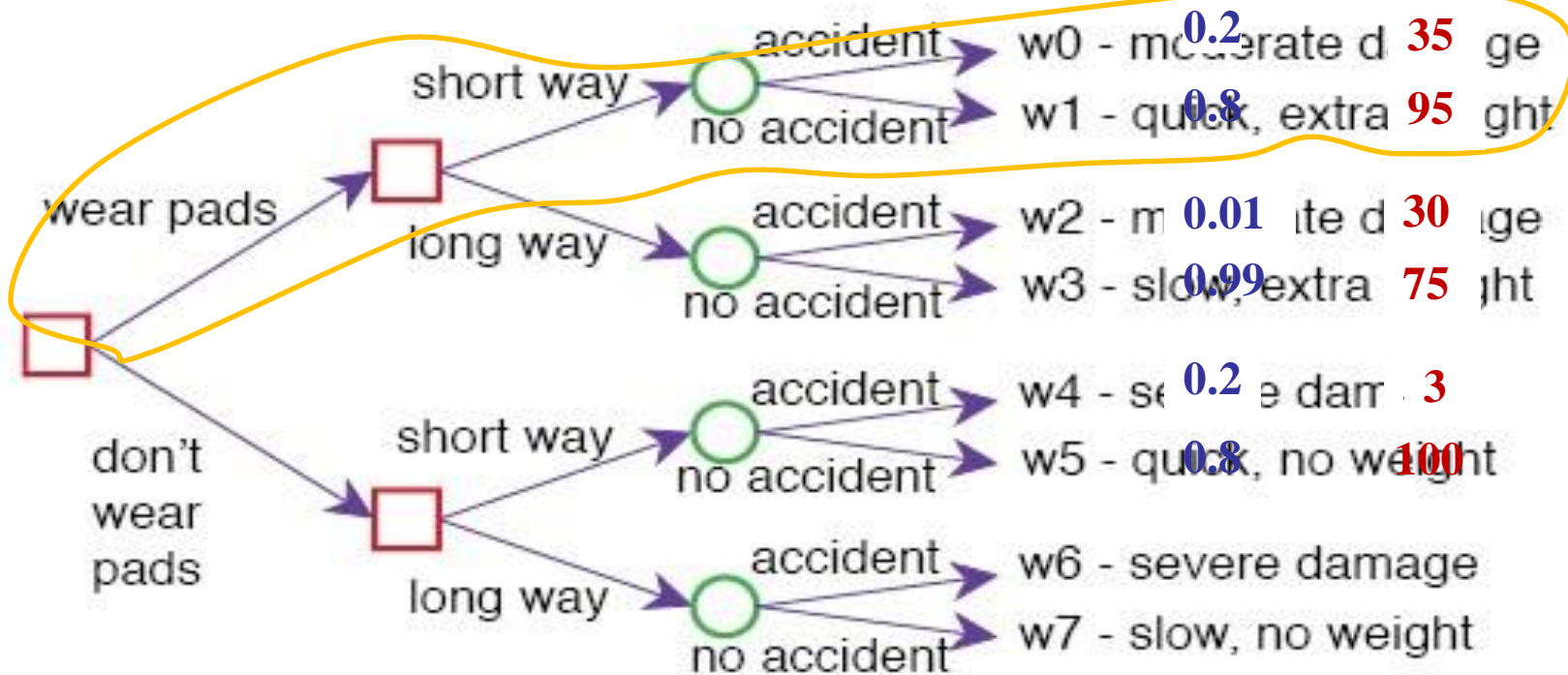
Expected utility of a decision

- The **expected utility** of decision $D = d$ is

$$E(U \mid D = d) = \sum_{w \models (D=d)} P(w) U(w) = P(w_1) \times U(w_1) + \dots + P(w_n) \times U(w_n)$$

- $0.2 * 35 + 0.8 * 95 = 83$

Probability Utility E[U|D]



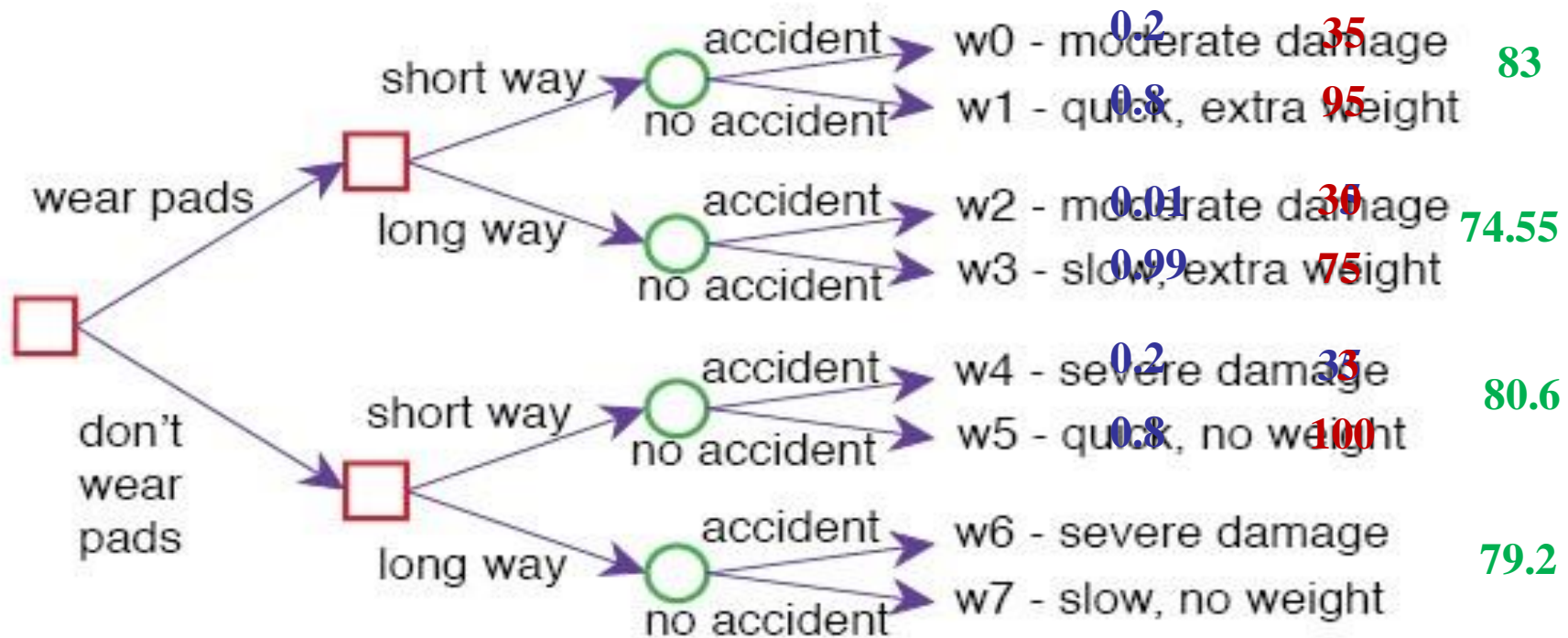
- What is the **expected utility** of Wearpads=yes, Way=short?

0.01350 0.99 80

Expected utility of a decision

- The **expected utility of** decision $D = d$ is


$$E(U \mid D = d) = \sum_{w \models (D = d)} P(w) U(w) = P(w_1) \times U(w_1) + \dots + P(w_n) \times U(w_n)$$



Probability Utility $E[U|D]$

0.01350 0.99 80

Lecture Overview

- Recap
- Intro to Decision theory
- Utility and expected utility
-  • Decision Networks for Single-stage decision problems

Single Action vs. Sequence of Actions

- Single Action (aka One-Off Decisions)
 - One or more **primitive** decisions that can be treated as a single macro decision to be **made before acting**
 - E.g., “WearPads” and “WhichWay” can be combined into macro decision (WearPads, WhichWay) with domain $\{\text{yes, no}\} \times \{\text{long, short}\}$
- Sequence of Actions (Sequential Decisions)
 - Repeat:
 - ✓ make observations
 - ✓ decide on an action
 - ✓ carry out the action

- Agent has to take actions not knowing for sure what the future brings
 - ✓ This is fundamentally different from everything we've seen so far
 - ✓ Planning was sequential, but agent could still think first and then act

Optimal single-stage decision

Given a single (macro) decision variable D

- the agent can choose $D=d_i$ for any value $d_i \in \text{dom}(D)$

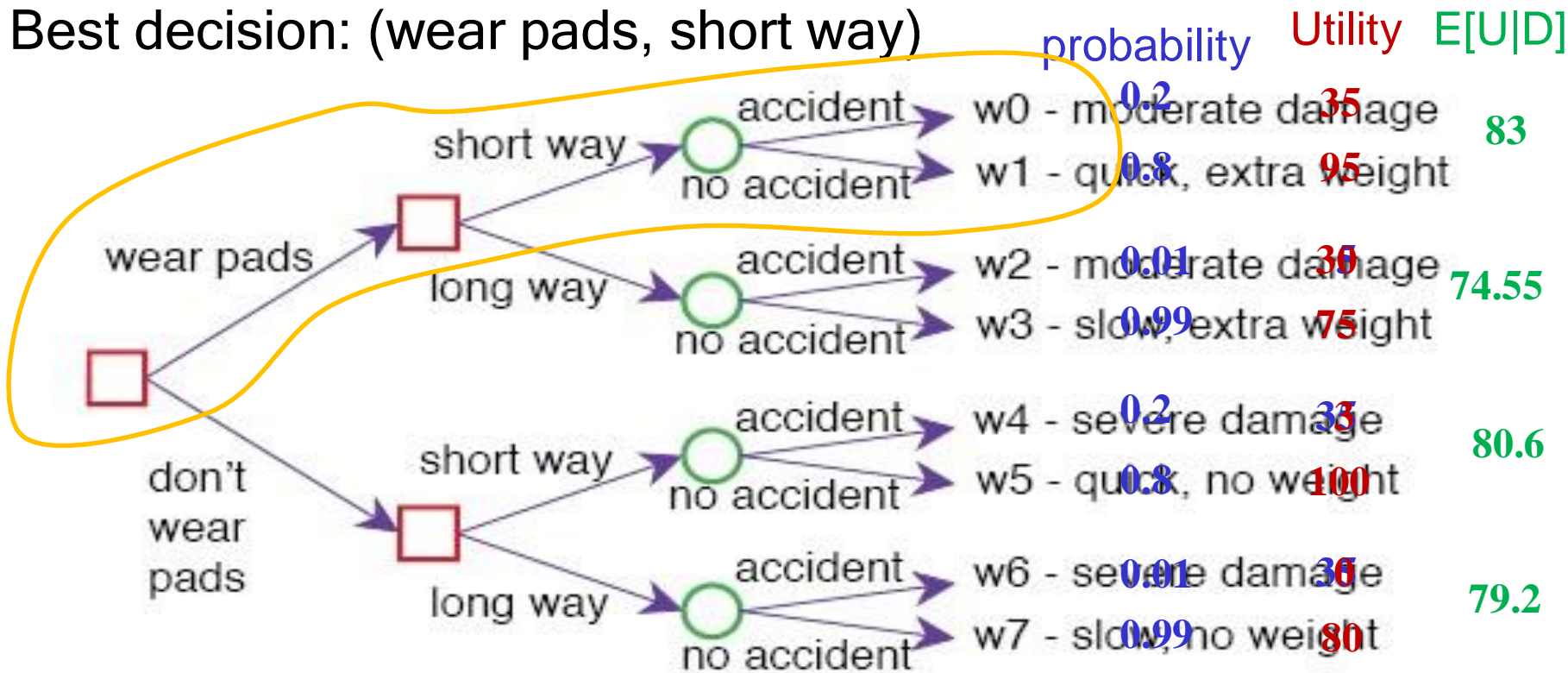
Definition (optimal single-stage decision)

An **optimal single-stage decision** is the decision $D=d_{\max}$ whose expected value is maximal:

$$d_{\max} \in \operatorname{argmax}_{d_i \in \operatorname{dom}(D)} E[U|D=d_i]$$

Optimal decision in robot delivery example

Best decision: (wear pads, short way)



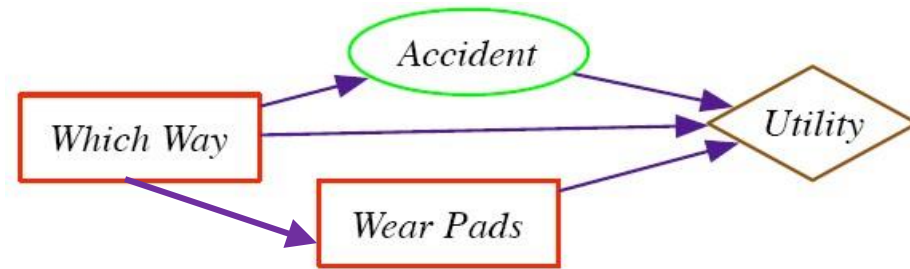
Definition (optimal single-stage decision)

An **optimal single-stage decision** is the decision $D=d_{\max}$ whose expected value is maximal:

$$d_{\max} \in \operatorname{argmax}_{d_i \in \operatorname{dom}(D)} E[U|D=d_i]$$

Conditional

Single-Stage decision networks



Extend belief networks

Random variables: same as in Bayesian networks

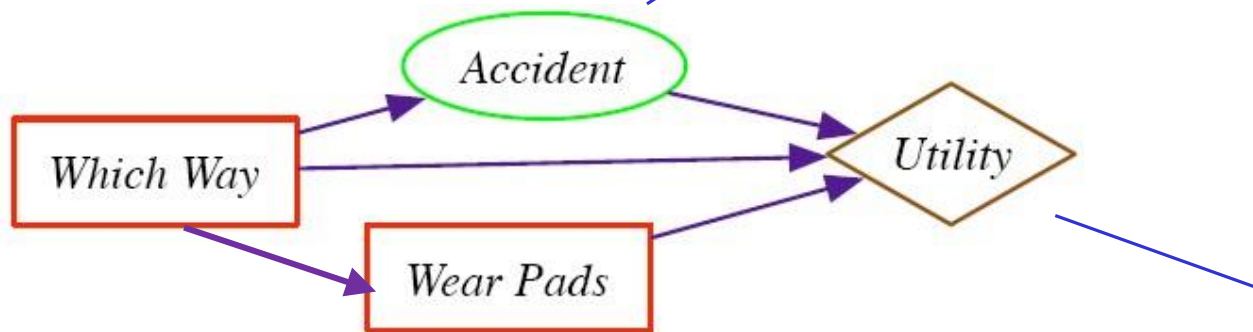
- drawn as an ellipse
- Arcs into the node represent probabilistic dependence
- random variable is conditionally independent of its non-descendants given its parents

Decision nodes, that the agent chooses the value for

- Parents: only other decision nodes allowed
 - ✓ represent information **available when the decision is made**

- Domain is the set of possible actions
- Drawn as a rectangle Exactly one utility node
- Parents: all random & decision variables on which the utility depends
- Specifies a utility for each instantiation of its parents
- Drawn as a diamond

Explicitly shows dependencies.
E.g., which variables affect the
probability of an accident and the
agent's **utility**?



Example Decision Network

| Which Way W | Accident A | P(A W) |
|-------------|------------|--------|
| long | true | 0.01 |
| long | false | 0.99 |
| short | true | 0.2 |
| short | false | |

| Which way Pads | Accident | Wear | Utility |
|-------------------|----------|-------|---------|
| long | true | true | 30 |
| long | true | false | 0 |
| long | false | true | 75 |
| long | false | false | 80 |
| short | true | true | 35 |
| short | true | false | 3 |
| short | false | true | 95 |
| short | false | false | 100 |

0.8

Which Way

t
f

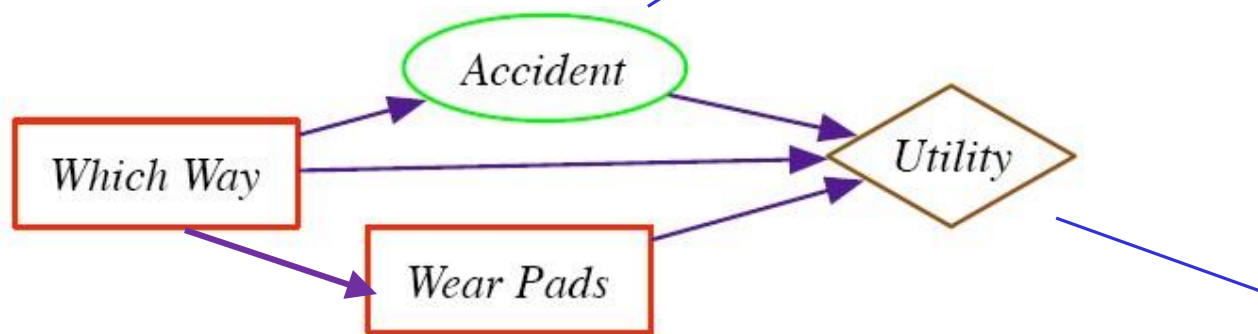
Decision nodes simply list the available decisions.

Example Decision Network

| | | |
|-------------|------------|----------|
| Which Way W | Accident A | $P(A W)$ |
|-------------|------------|----------|



Explicitly shows dependencies.
E.g., which variables affect the
probability of an accident and the
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| Which way Pads | Accident | Wear | Utility |
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| long | true | true | 30 |
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| long | false | true | 75 |
| long | false | false | 80 |
| short | true | true | 35 |
| short | true | false | 3 |
| short | false | true | 95 |
| short | false | false | 100 |

| | | | |
|-------|------|-------|------|
| long | true | false | 0.01 |
| long | true | false | 0.99 |
| short | | | 0.2 |
| short | | | 0.8 |

Decision nodes simply list the available decisions.

Applet for Bayesian and Decision Networks

| |
|-----------|
| Which Way |
| t |
| f |

The Belief and Decision Networks we have seen previously allows you to load predefined Decision networks for various domains and run queries on them.



Select one of the available examples via “File -> Load Sample Problem For Decision Networks

- Choose any of the examples below the blue line in the list that appears
- Right click on a node to perform any of these operations
- View the CPT/Decision table/Utility table for a chance/decision/utility node
- Make an observation for a chance variable (i.e., set it to one of its values)

- Query the current probability distribution for a chance node given the observations made
- A dialogue box will appear the first time you do this. Select “Always brief” at the bottom, and then click “Brief”.
- To compute the optimal decision (policy) click on the “Optimize Decision” button in the toolbar and select Brief in the dialogue box that will appear
- To see the actual policy, view the decision table for each decision node in the network

See available help pages and video tutorials for more details on how to use the Bayes applet (<http://www.aispace.org/bayes/>) Slide 51

Learning Goals for Decision Under Uncertainty so Far

- Compare and contrast stochastic single-stage (one-off) decisions vs. multistage (sequential) decisions
- Define a Utility Function on possible worlds
- Define and compute optimal one-off decisions
- Represent one-off decisions as single stage decision networks
- Compute optimal decisions by Variable Elimination