


# Lecture 8

## Intro to CSP as Search

# Lecture Overview

- 
- Recap of previous lecture
  - Intro to CSP
  - CSP algorithms using Search
    - Generate and test
    - Graph search
  - Intro to Arc Consistency (time permitting)

# Recap (Must Know How to Fill This)

	Selection	Complete	Optimal	Time	Space
<b>DFS</b>	LIFO	N	N	$O(b^m)$	$O(mb)$
<b>BFS</b>	FIFO	Y	Y	$O(b^m)$	$O(b^m)$
<b>IDS</b>	LIFO	Y	Y	$O(b^m)$	$O(mb)$
<b>LCFS</b>	min cost	Y **	Y **	$O(b^m)$	$O(b^m)$
<b>Best First</b>	min h	N	N	$O(b^m)$	$O(b^m)$
<b>A*</b>	min f	Y**	Y**	$O(b^m)$	$O(b^m)$
<b>B&amp;B</b>	LIFO + pruning	Y**	Y**	$O(b^m)$	$O(mb)$
<b>IDA*</b>	LIFO	Y**	Y**	$O(b^m)$	$O(mb)$

<b>MBA*</b>	min f	Y**	Y**	$O(b^m)$	$O(b^m)$
-------------	-------	-----	-----	----------	----------

\*\* Needs conditions: you need to know what they are<sup>5</sup>

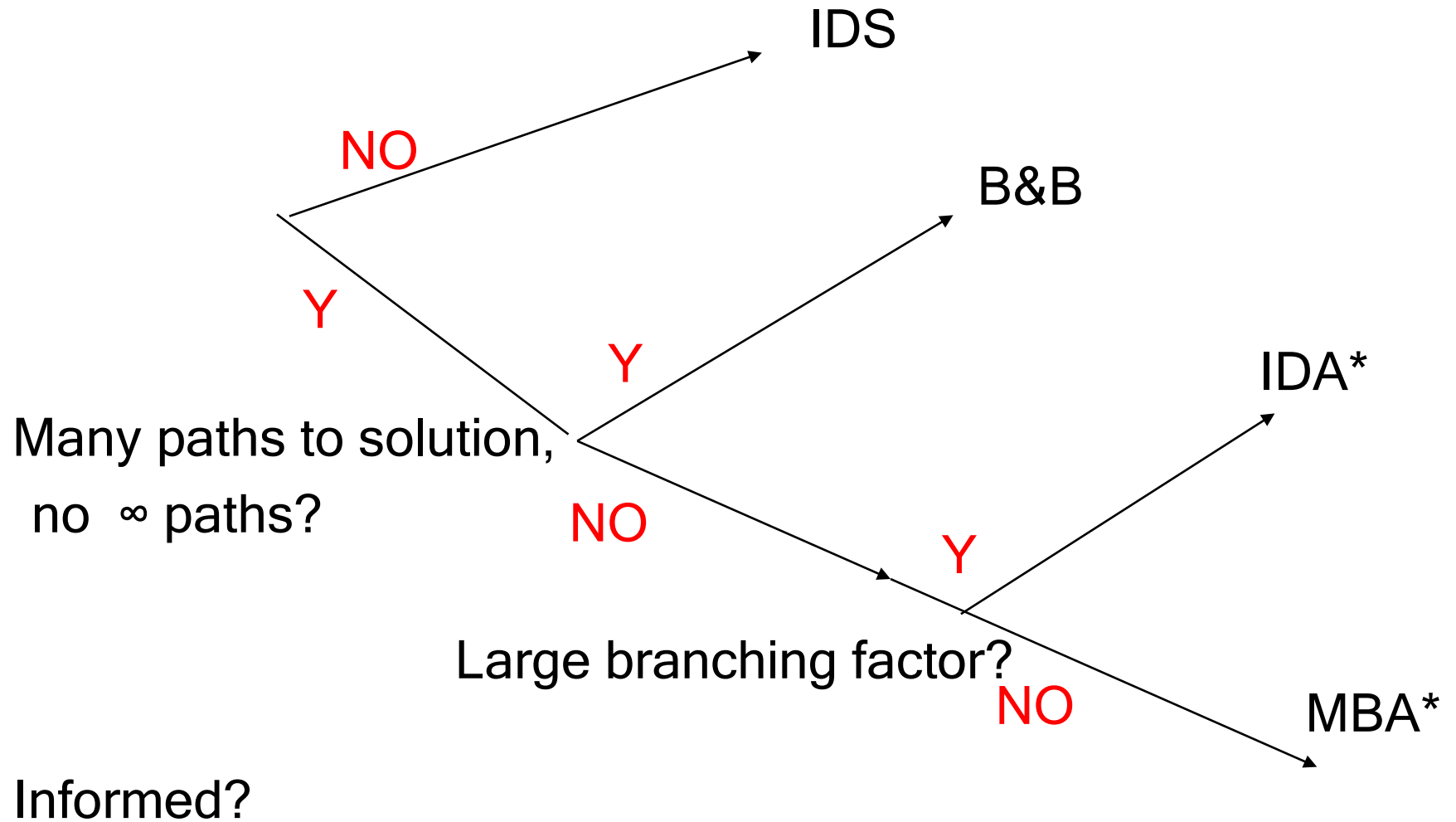
## Algorithms Often Used in Practice

	Selection	Complete	Optimal	Time	Space
<b>DFS</b>	LIFO	N	N	$O(b^m)$	$O(mb)$
<b>BFS</b>	FIFO	Y	Y	$O(b^m)$	$O(b^m)$
<b>IDS</b>	LIFO	Y	Y	$O(b^m)$	$O(mb)$
<b>LCFS</b>	min cost	Y **	Y **	$O(b^m)$	$O(b^m)$

<b>Best First</b>	min h	N	N	$O(b^m)$	$O(b^m)$
<b>A*</b>	min f	Y**	Y**	$O(b^m)$	$O(b^m)$
<b>B&amp;B</b>	LIFO + pruning	Y**	Y**	$O(b^m)$	$O(mb)$
<b>IDA*</b>	LIFO	Y	Y	$O(b^m)$	$O(mb)$
<b>MBA*</b>	min f	Y**	Y**	$O(b^m)$	$O(b^m)$

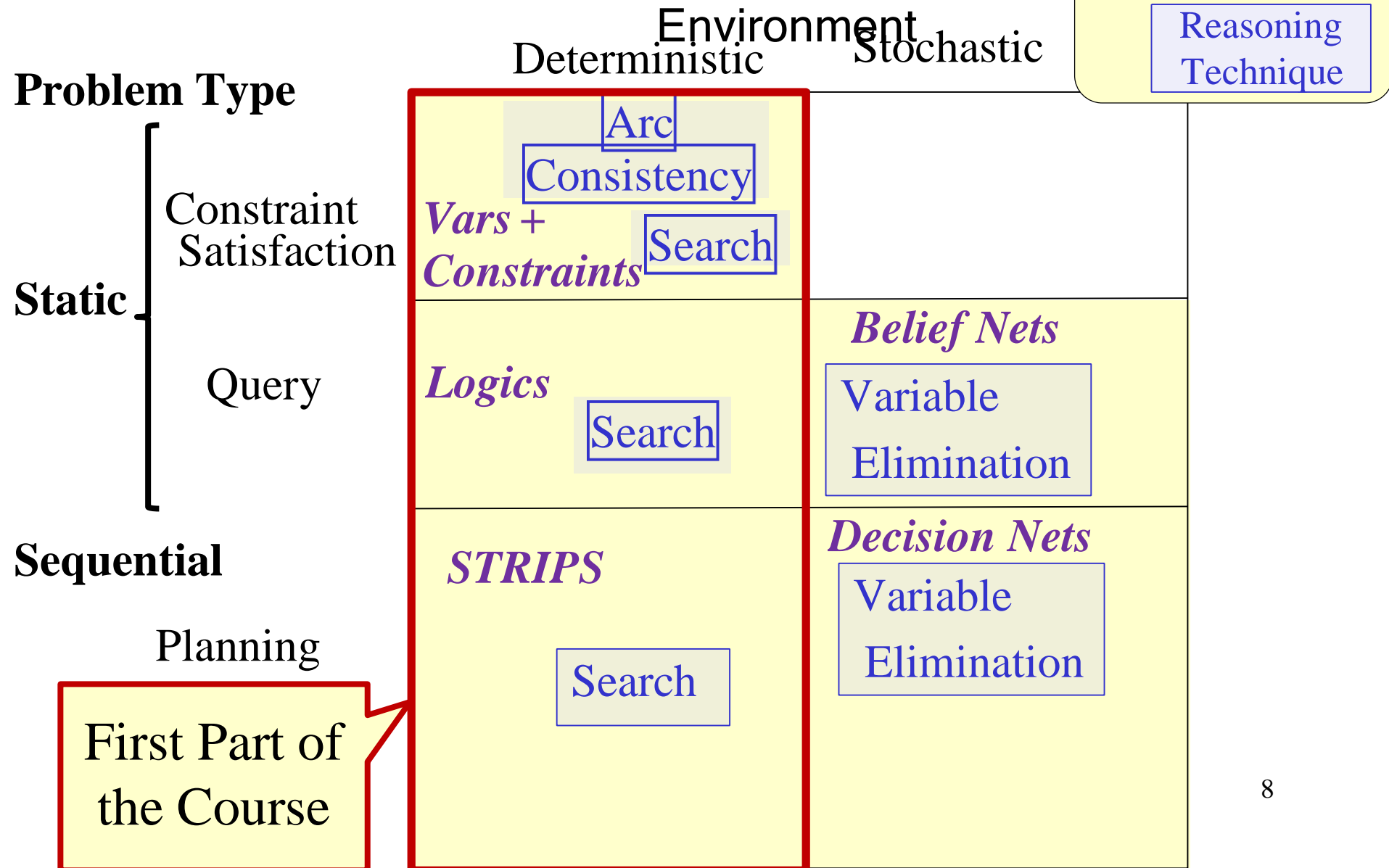
\*\* Needs conditions: you need to know what they are<sup>6</sup>

Search in Practice



These are indeed general guidelines, specific problems might yield different choices

# Course Overview





# Standard vs Specialized Search

- We studied general state space search in isolation
- Standard search problem: search in a state space
- **State** is a “black box” - any arbitrary data structure that supports three problem-specific routines:
  - goal test:  $\text{goal}(\text{state})$
  - finding successor nodes:  $\text{neighbors}(\text{state})$
  - if applicable, heuristic evaluation function:  $h(\text{state})$

# Course Overview

## Environment

- We will see more specialized versions of search for various problems

## *Markov Processes*

Value  
Iteration

Problem Type		Representation	
		Deterministic	Stochastic
Static	Constraint Satisfaction	<div>Arc</div> <div>Consistency</div> <i>Vars + Constraints</i> <div>Search</div>	
	Query	<i>Logics</i> <div>Search</div>	<i>Belief Nets</i> <div>Variable Elimination</div>
Sequential		<i>STRIPS</i> <div>Search</div>	<i>Decision Nets</i> <div>Variable Elimination</div>
Planning			

# Course Overview

## Environment

### We will look at Search in Specific R&R Systems

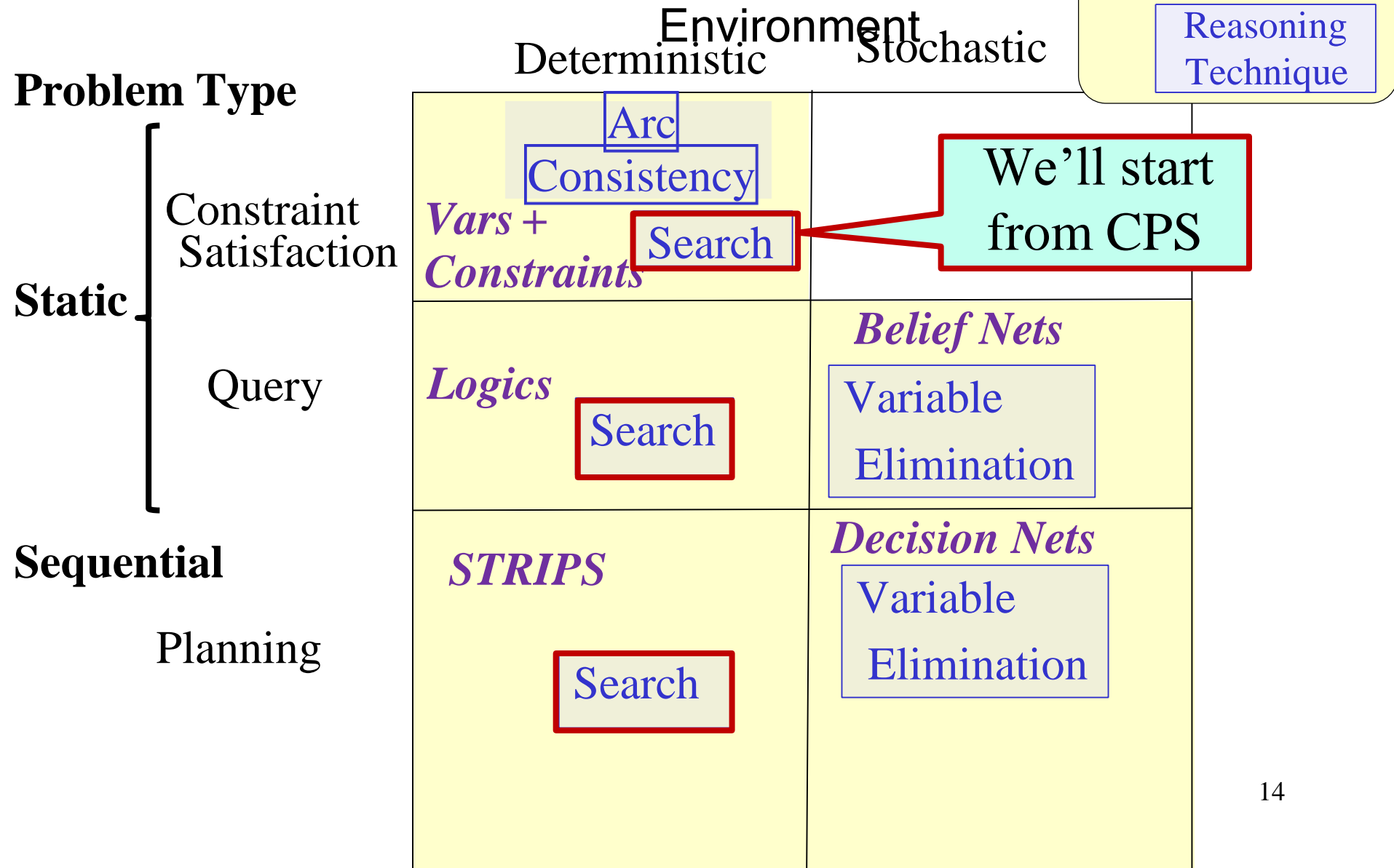
- Constraint Satisfaction Problems (CPS):
  - State
  - Successor function
  - Goal test
  - Solution
- Heuristic function • Query:
  - State
  - Successor function
  - Goal test
  - Solution
  - Heuristic function

### *Markov Processes*

Value Iteration
--------------------

- Planning
- State
- Successor function
- Goal test
- Solution
- Heuristic function

# Course Overview



- Constraint Satisfaction Problems (CPS):
  - State
  - Successor function
  - Goal test
  - Solution
  - Heuristic function

## We will look at Search for CSP

- Query:
  - State
  - Successor function
  - Goal test
  - Solution
  - Heuristic function
- Planning

# Course Overview

## Environment

- State
- Successor function
- Goal test
- Solution
- Heuristic function

*Markov Processes*

Value  
Iteration

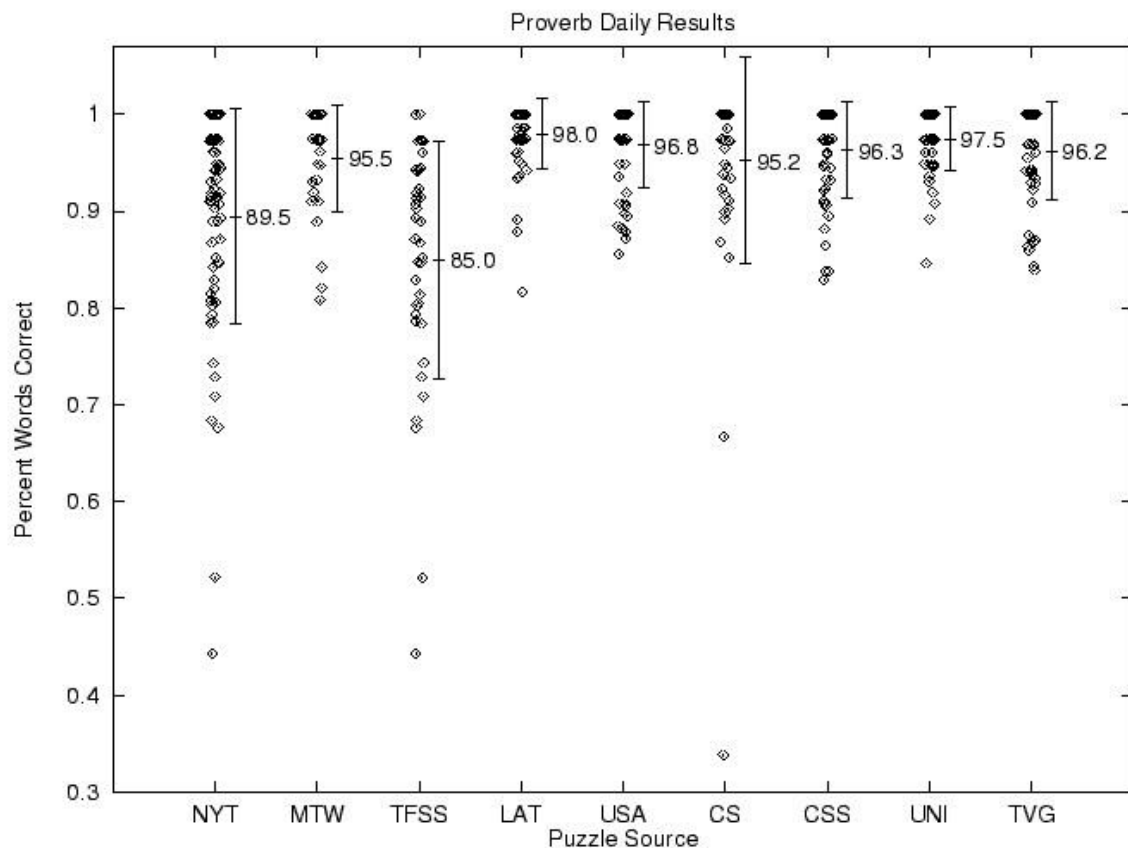


## Daily Puzzles

370 puzzles from 7 sources.

Summary statistics:

- ♦ 95.3% words correct (miss three or four words per puzzle)
- ♦ 98.1% letters correct
- ♦ 46.2% puzzles completely correct



P	O	L	O	N	E		P	A	L	O	M	I	N	O
A	S	I	M	O	V		I	S	O	L	A	T	E	D
S	L	E	E	V	E		T	H	W	A	R	T	E	D
T	I	G	G	E	R		C	O	R	N	Y			
A	N	E	A	L	E		A	R	I	D		J	A	M
						E	S	P	I	E	S		L	O
S	E	A	O	T	T	E	R		E	E	N	O	N	
A	B	B	O	T		A	N	A		U	S	A	G	E
B	O	O	Z	E	S		S	N	A	P	S	H	O	T
E	N	V	Y			P	L	I	N	T	H			
R	Y	E				H	I	E	S		T	E	A	S
						K	A	R	E	L		I	M	P
M	A	R	I	N	A	R	A				M	I	A	S
A	B	E	R	D	E	E	N				E	S	C	H
H	H	N	K	Y	A	R	D				S	M	E	A

Source: *Michael Littman*

# Constraint Satisfaction Problems (CSP)

- In a CSP

- state is defined by a set of **variables**  $V_i$  with **values** from domain  $D_i$
- goal test is a set of **constraints** specifying
  1. allowable combinations of values for subsets of variables (hard constraints)
  2. preferences over values of variables (soft constraints)

# Dimensions of Representational Complexity

## (from lecture 2)

- Reasoning tasks (Constraint Satisfaction / Logic&Probabilistic Inference / Planning)
- Deterministic versus stochastic domains

Some other important dimensions of complexity:

- Explicit state or features or relations **Explicit state or features or relations**
- Flat or hierarchical representation

- Knowledge given versus knowledge learned from experience
- Goals versus complex preferences
- Single-agent vs. multi-agent

# Variables/Features and Possible Worlds

- **Variable**: a synonym for **feature**
- We denote variables using capital letters
- Each variable  $V$  has a domain  $\text{dom}(V)$  of possible values
- Variables can be of several main kinds:
  - ✓ Boolean:  $|\text{dom}(V)| = 2$
  - ✓ Finite:  $|\text{dom}(V)|$  is finite
  - ✓ Infinite but discrete: the domain is countably infinite
  - ✓ Continuous: e.g., real numbers between 0 and 1
- **Possible world**:
- **Complete** assignment of values to **each** variable

- This is equivalent to a state as we have defined it so far
  - ✓ Soon, however, we will give a broader definition of state, so it is best to start distinguishing the two concepts.

17

## Example (lecture 2)

### *Mars Explorer Example*

*Weather*

{S, C}

*Temperature*

[-40, 40]

*Longitude*

[0, 359]

*Latitude*

[0, 179]

22

One possible world (state)

$\{S, -30, 320, 210\}$

Number of possible  
exclusive) worlds (states)

(mutually

$$2 \times 81 \times 360 \times 180$$


Product of cardinality of  
each domain

... always exponential in the

number of variables

## Lecture Overview

- Recap of previous lecture

-  CSP: possible worlds, constraints and models
- CSP algorithms using Search
  - Generate and test
  - Graph search
- Intro to Arc Consistency (time permitting)

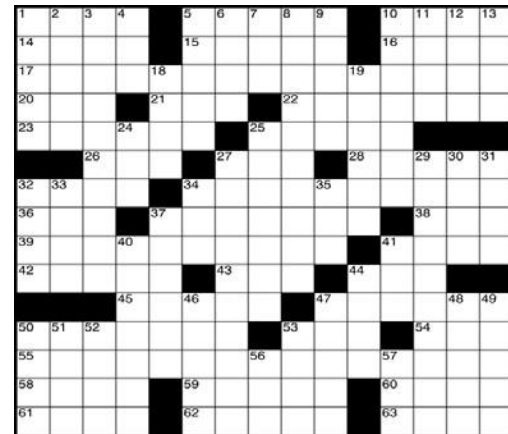


# How many possible worlds?

- **Crossword Puzzle 1:**
- variables are words that have to be filled in
- domains are English words of correct length
- possible worlds: all ways of assigning words

- Number of English words? Let's say 150,000

- Of the right length? Assume for simplicity: 15,000 for each length

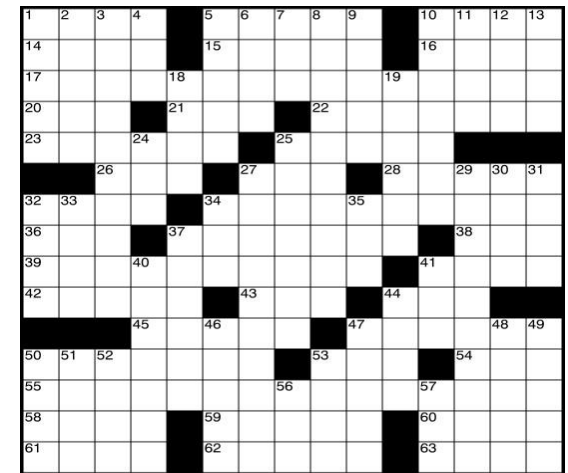


# How many possible worlds?

- Number of words to be filled in? 63
- How many possible worlds? (assume any combination is ok)

## Crossword Puzzle 1:

- variables are words that have to be filled in
  - domains are English words of correct length
  - possible worlds: all ways of assigning words
- 
- Number of English words? Let's say 150,000
  - Of the right length? Assume for simplicity: 15,000 for each length
  - Number of words to be filled in? 63



# How many possible worlds?

- How many possible worlds? (assume any combination is ok)

A.

$$15,000 * 63^{21}$$

B.

$$15,000^{63}$$

C

$$63_{15,000}$$

D.

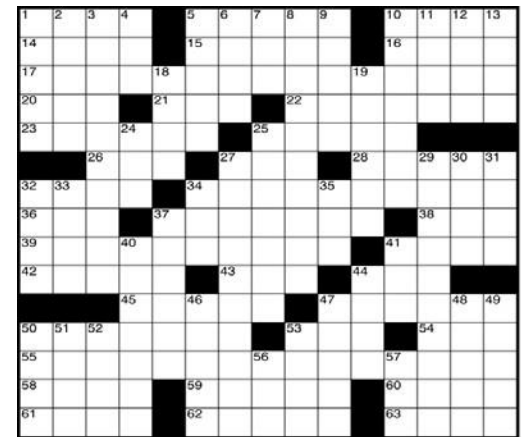
$$1,563^{63}$$

Crossword  
Puzzle:

- variables are words that have to be filled in

- domains are English words of correct length

- possible worlds: all ways of assigning words



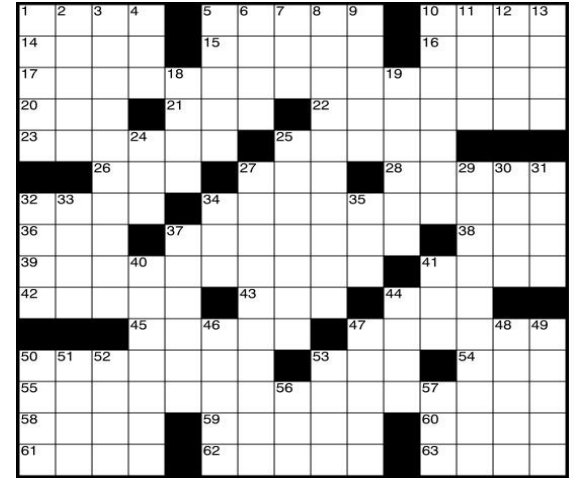
# How many possible worlds?

- Number of English words? Let's say 150,000
- Of the right length? Assume for simplicity: 15,000 for each word
- Number of words to be filled in? 63
- How many possible worlds? (assume any combination is ok)

$$15000^{63}$$

# How many possible worlds?

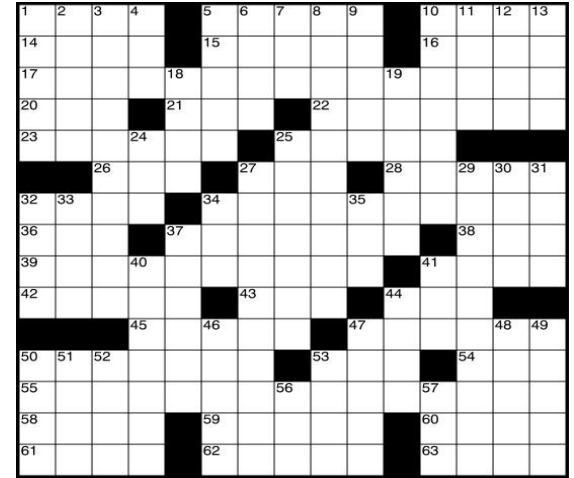
- **Crossword 2:**
- variables are cells (individual squares)
- domains are letters of the alphabet
- possible worlds: all ways of assigning letters to cells



- Number of empty cells?  $15 \times 15 - 32 = 193$
- Number of letters in the alphabet? 26
- How many possible worlds? (assume any combination is ok)

# How many possible worlds?

- **Crossword 2:**
- variables are cells (individual squares)
- domains are letters of the alphabet
- possible worlds: all ways of assigning letters to cells



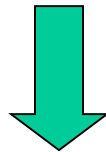
- Number of empty cells?  $15 \times 15 - 32 = 193$
- Number of letters in the alphabet? 26
- How many possible worlds? (assume any combination is ok)

26<sub>193</sub>

- In general: (domain size) <sup>#variables</sup>

## Constraint Satisfaction Problems (CSP)

- Allow for usage of useful **general-purpose algorithms** with more power than standard search algorithms
- They exploit the multi-dimensional nature of the problem and the structure provided by the goal



# How many possible worlds?

set of constraints, \*not\* black box.



# Constraints

- Constraints are **restrictions** on the values that one or more variables can take
- **Unary constraint**: restriction involving a single variable
- **k-ary constraint**: restriction involving k different variables
  - ✓ We will mostly deal with binary constraints
- Constraints can be specified by
  1. listing all **combinations of valid domain values** for the variables participating in the constraint
  2. giving a **function** that returns true when given values for each variable which satisfy the constraint

# Constraints: Simple Example

- **Unary constraint:**  $V_2 \neq 2 \quad V = \{V_1, V_2\}$ 
  - $\text{dom}(V_1) = \{1, 2, 3\};$
- **k-ary constraint:**
  - $\text{dom}(V_2) = \{1, 2\}$
- binary ( $k=2$ ):  $V_1 + V_2 < 5$
- 3-ary:  $V_1 + V_2 + V_4 < 5$ 

We will mostly deal with binary constraints
- Constraints can be specified by

1. listing all combinations of valid values for the variables participating in the constraint

- for constraint  $V_1 > V_2$  and  $\text{dom}(V_1) = \{1,2,3\}$  and  $\text{dom}(V_2) = \{1,2\}$ :

$V_1$	$V_2$

domain

2. giving a function (predicate) that returns true if given values for each variable satisfy the constraint else false:

## Constraints: Simple Example

- **Unary constraint:**  $V_2 \neq 2$   $V = \{V_1, V_2\}$ 
  - $\text{dom}(V_1) = \{1,2,3\}$ ;
- **k-ary constraint:**
  - $\text{dom}(V_2) = \{1,2\}$
  - ✓ binary ( $k=2$ ):  $V_1 + V_2 < 5$

✓3-ary:  $V_1 + V_2 + V_4 < 5$

We will mostly deal with binary constraints

- Constraints can be specified by

1. listing all combinations of valid domain values for the variables participating in the constraint

- for constraint  $V_1 > V_2$  and  $\text{dom}(V_1) = \{1,2,3\}$  and  $\text{dom}(V_2) = \{1,2\}$ :

$V_1$	$V_2$
2	1
3	1
3	2

returns true if satisfy the

2. giving a function (predicate) that given values for each variable constraint else false:

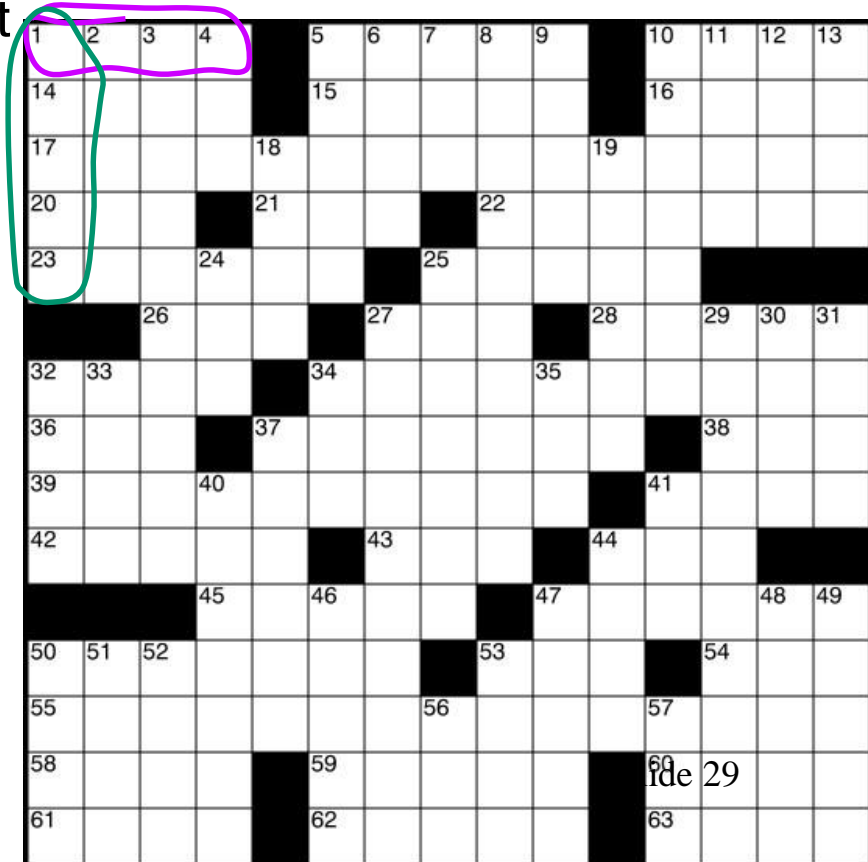
$$V_1 > V_2$$

# Examples

- Crossword Puzzle 1:
- variables are words that have to be filled in

letters at points where they intersect

$v_1[ ]$

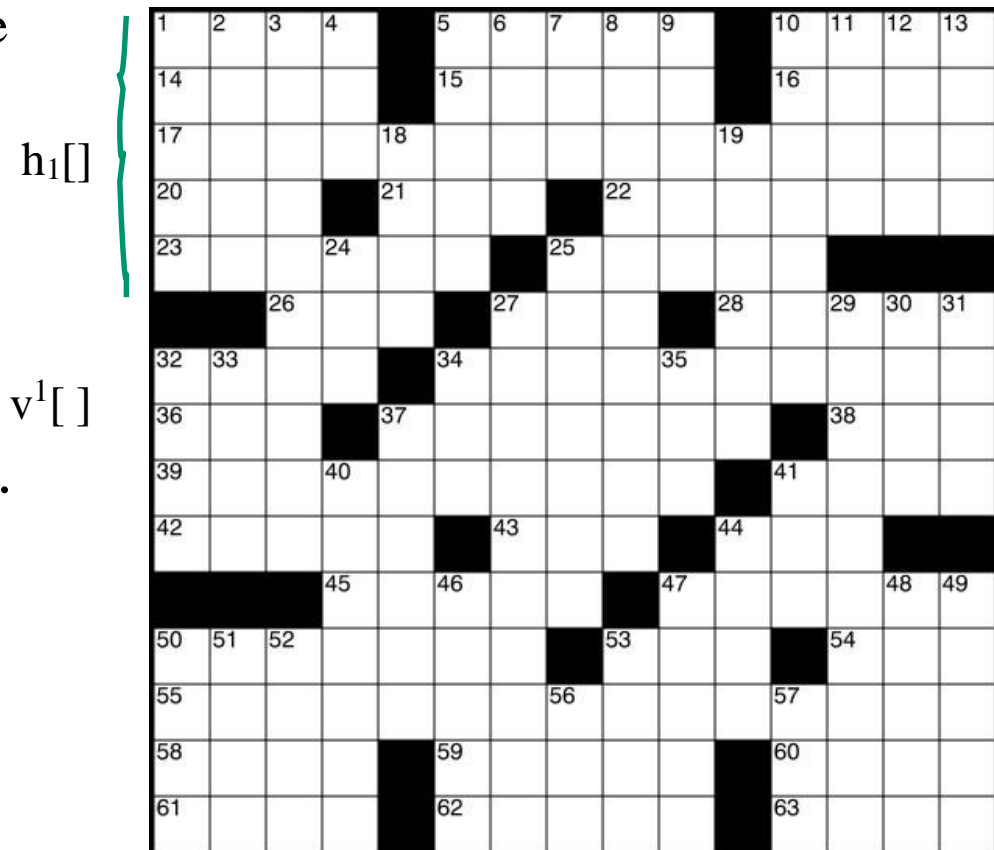


- domains are valid English words
- constraints: words have the same  $h_1[]$

# Examples

- **Crossword Puzzle 1:**
- variables are words that have to be filled in • domains are valid English words
- *constraints:* words have the same letters at points where they intersect

$h_1[0] = v_1[0]$   $h_1[1] = v_2[0]$  .....  
 ~225 constraints



# Example: Map-Coloring

**Variables** WA, NT, Q, NSW, V, SA, T

**Domains**  $D_i = \{\text{red, green, blue}\}$

**Constraints:** adjacent regions must have different colors e.g.,





Or

WA	NT

NT	SA

NT	QU

.....

## Example: Map-Coloring

**Variables** WA, NT, Q, NSW, V, SA, T

**Domains**  $D_i = \{\text{red, green, blue}\}$

**Constraints:** adjacent regions must have different colors



e.g., WA ≠ NT, NT ≠ SA, NT ≠ QU, .....

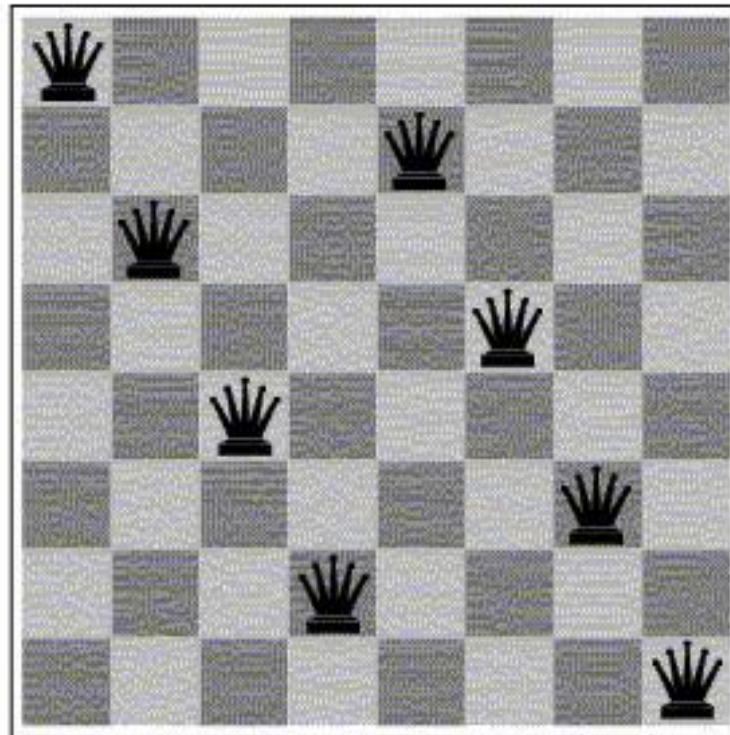
Or

WA	NT	NT	SA	NT	QU
Red	Green	Red	Green	Red	Green
Red	Bue	Red	Bue	Red	Bue
Green	Red	Green	Red	Green	Red
Green	Blue	Green	Blue	Green	Blue
Blue	Red	Blue	Red	Blue	Red
Blue	Green	Blue	Green	Blue	Green

.....

- ## Example: Eight Queen problem

Eight Queen problem: place 8 queens on a chessboard so that no queen can attack the others



## Example 2: 8 queens

- **Variables:**  $V_1, \dots, V_n$ .
- **Constraints:** No queens can be in the same row, column or diagonals

33

$V_i$  = Row occupied by the  $i^{\text{th}}$  queen in the  $i^{\text{th}}$  column

- **Domains:**  $D_{V_i} = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- **Constraints:** Two queens cannot be on the same row, column or on the same diagonal

## Example 2: 8 queens

- **Variables:**  $V_1, \dots, V_n$ .

Slide 34

$V_i$  = Row occupied by the  $i^{\text{th}}$  queen in the  $i^{\text{th}}$  column

- **Domains:**  $D_{V_i} = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- **Constraints:** Two queens cannot be on the same row, column or on the same diagonal
- We can specify the constraints by enumerating explicitly, for each pair of columns, which positions are allowed. Ex:  
 $\text{Constr}(V_1, V_2) =$

## Example 2: 8 queens

- **Variables:**  $V_1, \dots, V_n$ .

Slide 36

$V_i$  = Row occupied by the  $i^{\text{th}}$  queen in the  $i^{\text{th}}$  column

- **Domains:**  $D_{V_i} = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- **Constraints:** : Two queens cannot be on the same row or on the same diagonal
- We can specify the constraints by enumerating explicitly, for each pair of columns, what positions are allowed. Ex:  
 $\text{Constr}(V_1, V_2) = \{(1, 3), (1, 4), \dots, (1, 8)\}$

## Example 2: 8 queens

- Variables:  $V_1, \dots, V_n$ .

$(2,4)(2,5)\dots$

$(8,1)\dots (8,5), (8,6)\}$





# Constraints: one more concept

- Constraints are restrictions on the values that one or more variables can take
- Unary constraint: restriction involving a single variable
- k-ary constraint: restriction involving k different variables
  - ✓ We will mostly deal with binary constraints
- Constraints can be specified by
  1. listing all combinations of valid domain values for the variables participating in the constraint
  2. giving a function that returns true when given values for each variable which satisfy the constraint

- A possible world **satisfies** a set of constraints
  - if the values for the variables involved in **each constraint** are consistent with that constraint, i.e.
    1. They are elements of the list of valid domain values
    2. Function for that constraint returns true for those values

38

## Constraint Satisfaction Problems (CSPs): Definitions

Definition:

A **constraint satisfaction problem (CSP)** consists of:

- a set of **variables  $V$**
- a **domain**  $\text{dom}(V)$  for each variable
- a set of **constraints  $C$**

Definition:

A **model** of a CSP is an assignment of values to all of its variables (i.e., a **possible world**) that **satisfies** all of its constraints.

39

Simple example:

- $V = \{V_1\}$  All models for this CSP:
  - $\text{dom}(V_1) = \{1,2,3,4\}$
- $C = \{C_1, C_2\}$ 
  - $C_1: V_1 \neq 2$
  - $C_2: V_1 > 1$

## Constraint Satisfaction Problems (CSPs): Definitions

Definition:

A **constraint satisfaction problem (CSP)** consists of:

- a set of **variables**  $V$
- a **domain**  $\text{dom}(V)$  for each variable
- a set of **constraints**  $C$

Definition:

A **model** of a CSP is an assignment of values to all of its variables (i.e., a **possible world**) that **satisfies** all of its constraints.

Simple example:

- $V = \{V_1\}$  All models for this CSP:
  - $\text{dom}(V_1) = \{1,2,3,4\}$   $V_1 = 3$
- $C = \{C_1, C_2\}$   $V_1 = 4$ 
  - $C_1: V_1 \neq 2$
  - $C_2: V_1 > 1$

# Models and Possible Worlds

Definition:

A **model** of a CSP is an assignment of values to all of its variables (i.e. **a possible world**) that **satisfies** all of its constraints.

Another example:

- $V = \{V_1, V_2\}$
- $\text{dom}(V_1) = \{1, 2, 3\}$
- $\text{dom}(V_2) = \{1, 2\}$
- $C = \{C_1, C_2, C_3\}$

- $C_1: V_2 \neq 2$
- $C_2: V_1 + V_2 < 5$
- $C_3: V_1 > V_2$

How many models do we have?

A	None
B	1
C	2
D	3
E	4

# Models and Possible Worlds

Definition:

A **model** of a CSP is an assignment of values to all of its variables (i.e. **a possible world**) that **satisfies** all of its constraints.

$V = \{V_1, V_2\}$

Possible worlds for this CSP:

- $\text{dom}(V_1) = \{1, 2, 3\}$      $\{V_1=1, V_2=1\}$

- $\text{dom}(V_2) = \{1, 2\}$      $\{V_1=1, V_2=2\}$

- $C = \{C_1, C_2, C_3\}$

$\{V \mid V_1=2, V_2=1\}$  (a model)

- $C_1: V_2 \neq 2$      $\{V_1=2, V_2=2\}$



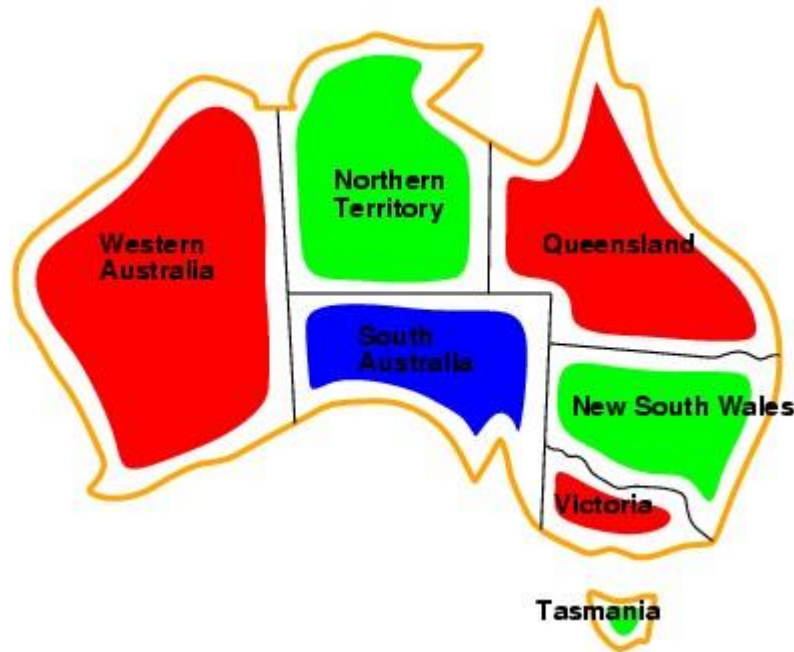
- $C_2: V_1 + V_2 < 5 \quad \{V_1=3, V_2=1\}$  (a model)
- $C_3: V_1 > V_2 \quad \{V_1=3, V_2=2\}$

end<sup>42</sup>

## Example: Map-Coloring

Definition:

A **model** of a CSP is an assignment of values to all of its variables (i.e. **a possible world**) that **satisfies** all of its constraints.



WA = red, NT = green, Q = red, NSW = green, V = red, SA = blue, T = green

43

## Scope of a constraint

Definition:

The **scope** of a constraint is the set of variables that are involved in the constraint

- Examples:
- $V_2 \neq 2$  has scope  $\{V_2\}$
- $V_1 > V_2$  has scope  $\{V_1, V_2\}$
- $V_1 + V_2 + V_4 < 5$  has scope  $\{V_1, V_2, V_4\}$
- Number of variables in the scope of a k-ary constraint is k

# Finite Constraint Satisfaction Problem: Definition

Definition:

A **finite constraint satisfaction problem** is a CSP with a finite set of variables and a finite domain for each variable.

- We will only study finite CSPs here
- but many of the techniques carry over to countably infinite and continuous domains.

# Constraint Satisfaction Problems: Variants

- We may want to solve the following problems with a CSP:
- determine whether or not a model **exists**
- **find** a model
- **find all** of the models
- **count** the number of models
- find the **best** model, given some measure of model quality
  - ✓ this is now an optimization problem

- determine whether some **property of the variables** holds in all models

46

## Solving Constraint Satisfaction Problems

- Even the simplest problem of determining whether or not a model exists in a general CSP with finite domains is **NPhard**
- There is no known algorithm with worst case polynomial runtime.
- We can't hope to find an algorithm that is polynomial for all CSPs.
- However, we can try to:

- find efficient (polynomial) **consistency algorithms** that reduce the size of the search space
- **identify special cases** for which algorithms are efficient
- work on **approximation algorithms** that can find good solutions quickly, even though they may offer no theoretical guarantees • find algorithms that are fast on **typical** (not worst case) cases

47

## Lecture Overview

- Recap of previous lecture

- CSP: possible worlds, constraints and models
- CSP algorithms using Search
  - ➡
    - Generate and test
    - Graph search
- Intro to Arc Consistency (time permitting)



# Generate and Test (GT) Algorithms

- Systematically check all possible worlds - Possible worlds: cross product of domains  $\text{dom}(V_1) \times \text{dom}(V_2) \times \dots \times \text{dom}(V_n)$  • Generate and Test:
  - **Generate** possible worlds one at a time - **Test** constraints for each one.

Example: 3 variables A,B,C

# Generate and Test (GT) Algorithms

```
For a in dom(A)
  For b in dom(B)
    For c in dom(C)
      if {A=a, B=b, C=c} satisfies all constraints
        return {A=a, B=b, C=c}
fail
```

49

```
For a in dom(A)
  For b in dom(B)
    For c in dom(C)
      if {A=a, B=b, C=c} satisfies all constraints
        return {A=a, B=b, C=c}
fail
```

# Generate and Test (GT) Algorithms

- If there are  $k$  variables, each with domain size  $d$ , and there are  $c$  constraints, the complexity of Generate &

iclicker.

A.  $O(ckd)$

Test

C.  $O(cd^k)$

B.  $O(ck^d)$

D.  $O(d^{ck})$

50

- If there are  $k$  variables, each with domain size  $d$ , and there are  $c$  constraints, the complexity of Generate & Test is

$O(cd^k)$

# Generate and Test (GT) Algorithms

- There are  $d^k$  possible worlds
- For each one need to check  $c$  constraints

51

- Need to find a better way, that exploits the “insights” that we have on the goal expressed in terms of constraints
- Why does GT fail to do this?

# Generate and Test (GT) Algorithms

52

- Need to find a better way, that exploits the “insights” that we have on the goal expressed in terms of constraints
- Why does GT fail to do this?
- It checks constraints only after a full assignment of values to variables has been made
- Fails to leverage the modular/explicit nature of the goal

# Generate and Test (GT) Algorithms

53

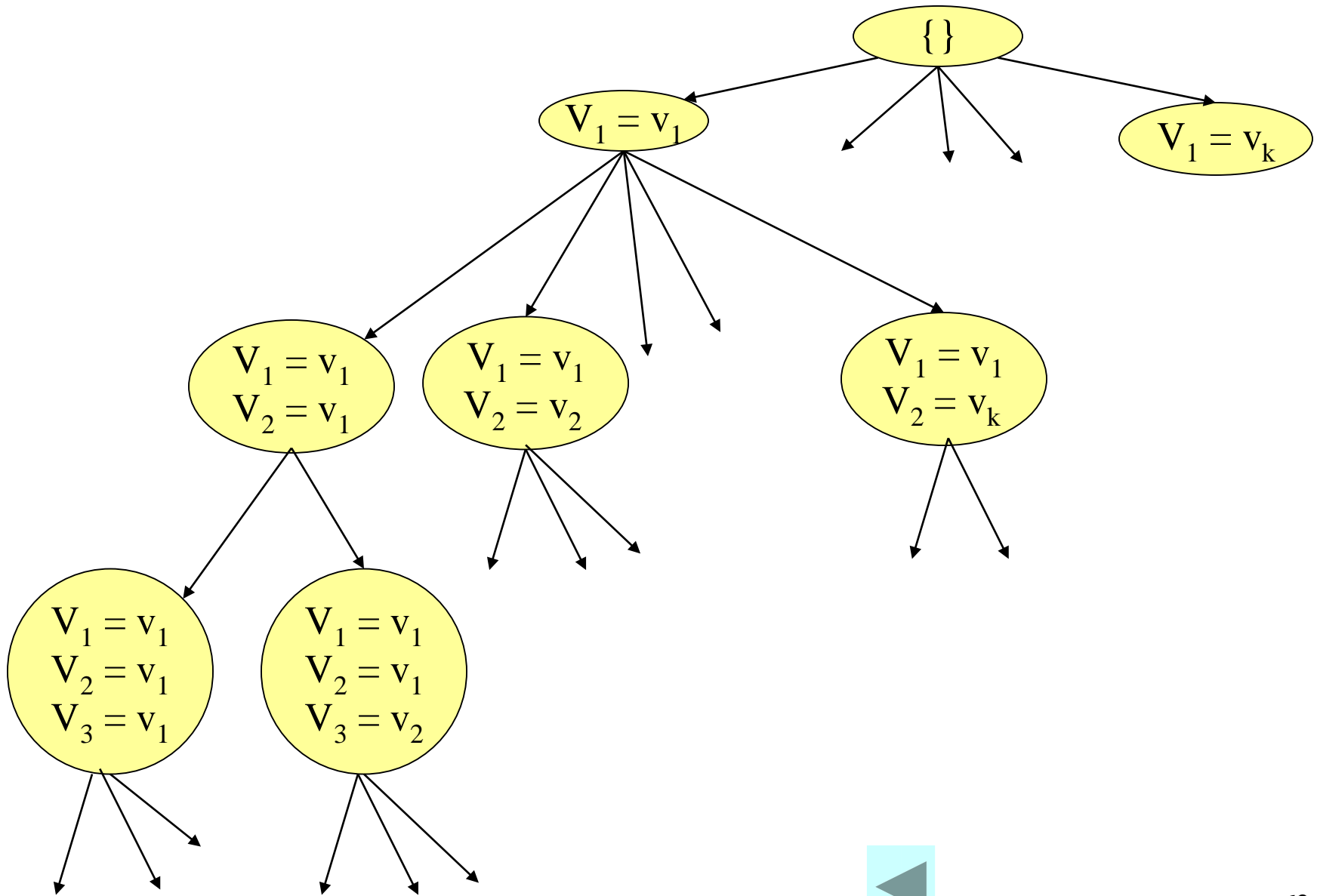
# CSP as a Search Problem: one formulation

- **States**: partial assignments of values to variables
- **Start state**: empty assignment
- **Successor function**: states with the next variable assigned
- Follow a total order of the variables  $V_1, \dots, V_n$  • A state assigns values to the first  $k$  variables:  
$$\checkmark \{V_1 = v_1, \dots, V_k = v_k\}$$
- Neighbors of node  $\{V_1 = v_1, \dots, V_k = v_k\}$ : nodes  $\{V_1 = v_1, \dots, V_k = v_k, V_{k+1} = x\}$  for each  $x \in \text{dom}(V_{k+1})$

- **Goal states**: complete assignments of values to variables that satisfy all constraints
- That is, **models**
- **Solution**: assignment (the path does not matter)



# **CSP as a Search Problem: one formulation**



Which search algorithm would be most appropriate for this formulation of CSP?

- A. Depth First Search
- B. BFS
- C. A \*
- D. IDS

# Which search algorithm would be most appropriate for this formulation of CSP?

## A. Depth First Search

- the search tree is always **finite** and has **no cycles**
- If there are  $n$  variables every solution is at **depth  $n$** .
- Possibly very large branching factor  $b$

# Dealing with complexity

- CSP problems can be huge -  
Thousands of variables
- Exponentially more search states
  - Exhaustive search is typically infeasible
- Many algorithms exploit the structure provided by the goal  $\Rightarrow$  set of constraints, \*not\* black box

# Backtracking algorithms

- Explore search space via DFS but evaluate each constraint as soon as all its variables are bound.
- Any partial assignment that does not satisfy the constraint can be pruned.
- Example:
  - 3 variables A, B, C each with domain {1,2,3,4}

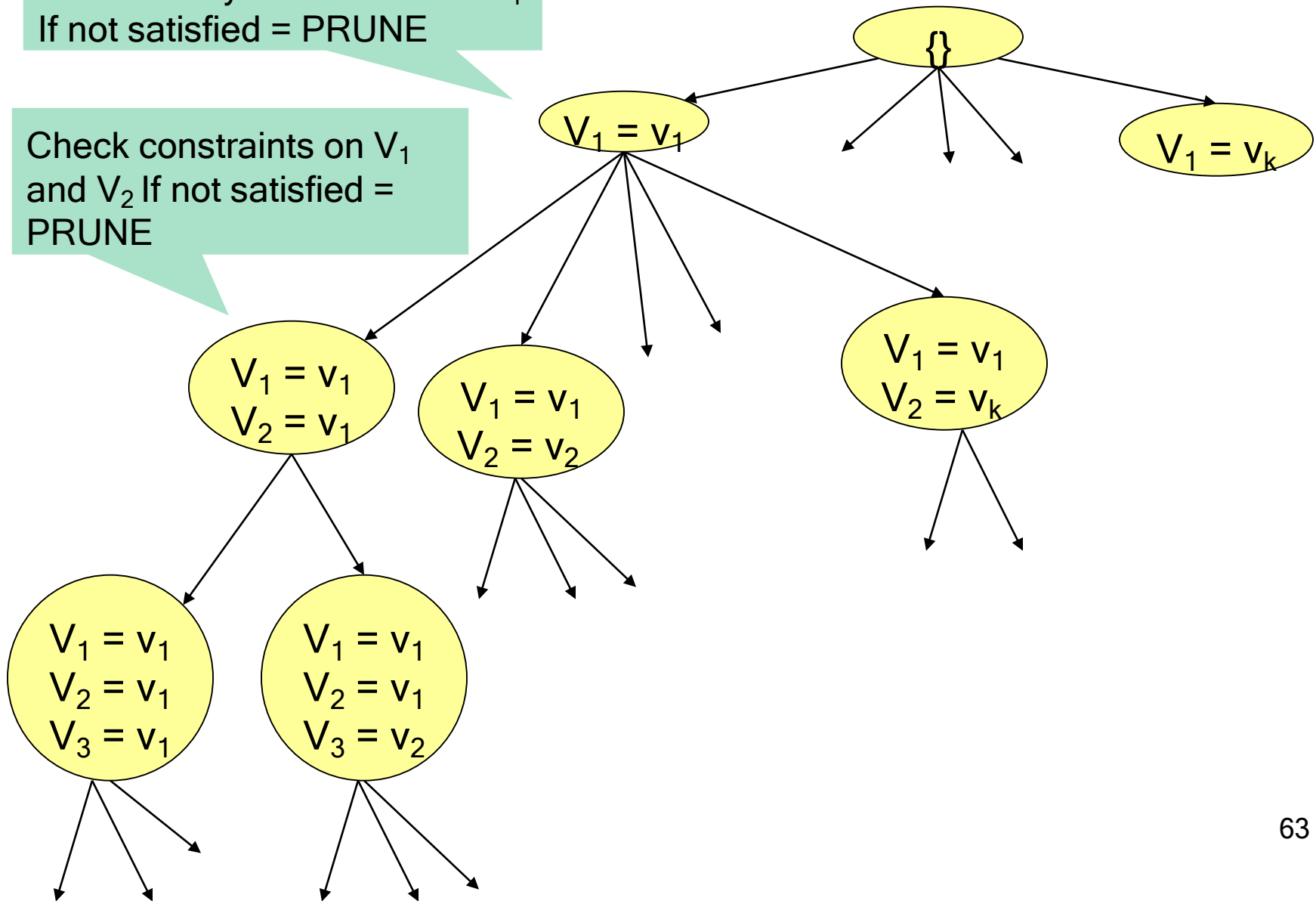
- $\{A = 1, B = 1\}$  is inconsistent with constraint  $A \neq B$   
regardless of the value of the other variables  $\Rightarrow$  Fail!  
Prune!

# CSP as Graph Searching



Check unary constraints on  $V_1$   
If not satisfied = PRUNE

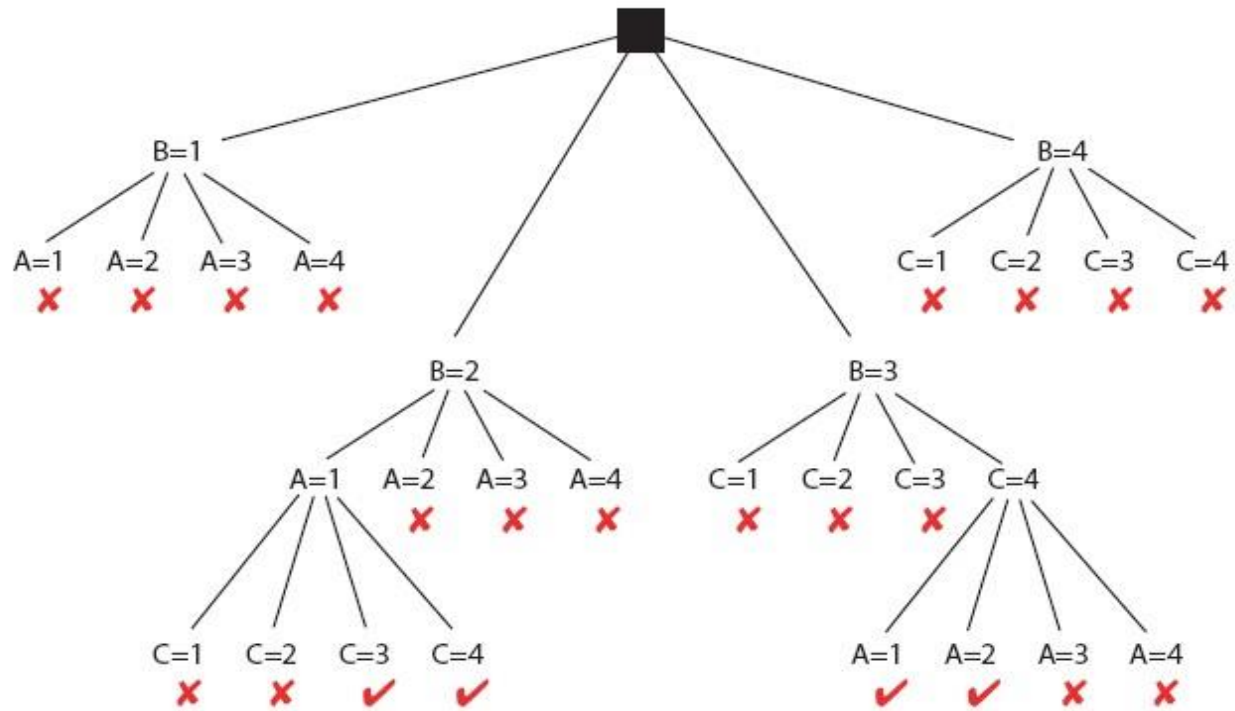
Check constraints on  $V_1$   
and  $V_2$  If not satisfied =  
PRUNE



# Solving CSPs by DFS: Example

- Variables: A,B,C
- Domains: {1, 2, 3, 4}
- Constraints:  $A < B$ ,  $B < C$

Good ordering, lots of pruning happens right away

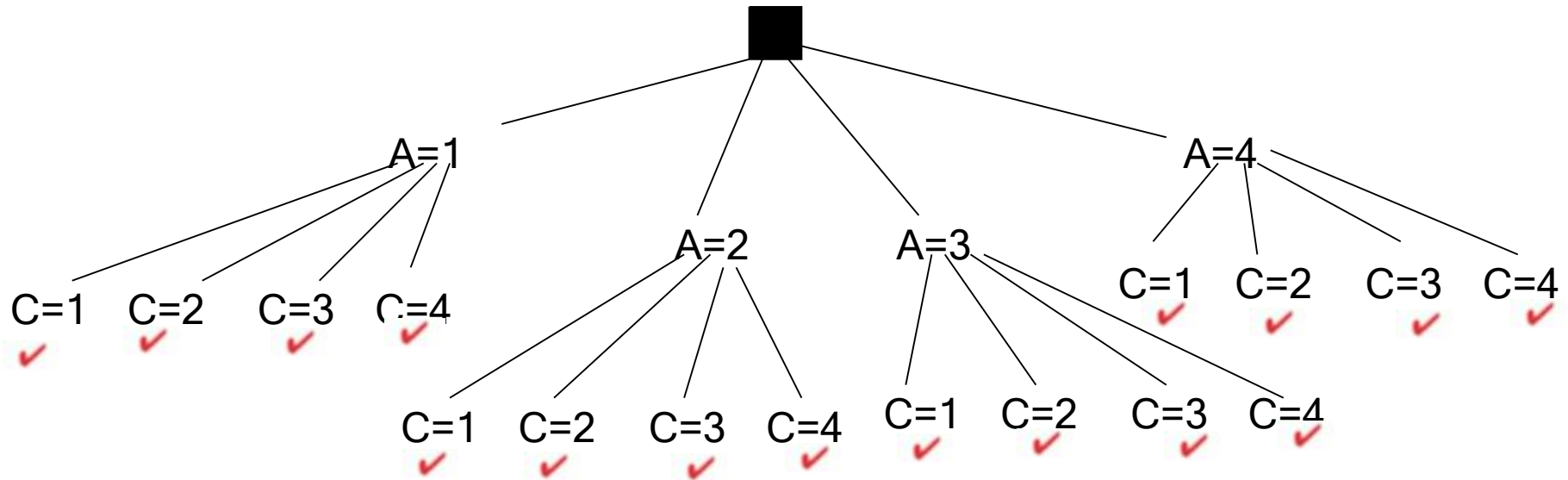


# Solving CSPs by DFS: Example Efficiency

- Variables: A,B,C
- Domains: {1, 2, 3, 4}
- Constraints:  $A < B$ ,  $B < C$

Much worse ordering,  
keeps more branches  
around

**Problem?** Performance heavily depends on  
the order in which variables are considered



63

**EXAMPLE :**  $D_A = D_B = D_C = \{1,2,3,4\}$   $(A \neq B) \wedge (B \neq C) \wedge (C < D) \wedge (A=D) \wedge (B \neq D) \wedge (E < A) \wedge (E < B) \wedge (E < C) \wedge (E < D)$

A=1 B=1 failure

B=2 C=1 D=1 failure

D=2 failure

D=3 failure

D=4 failure

C=2 failure

C=3 D=1 failure

D=2 failure

D=3 failure

D=4 failure

C=4 D=1 failure

D=2 failure

D=3 failure

D=4 failure



**EXAMPLE :**  $D_A = D_B = D_C = \{1,2,3,4\}$   $(A \neq D) \wedge (E < A) \wedge (E < B) \wedge (E < C) \wedge (E < \neq B) \wedge (B \neq C) \wedge (C < D) \wedge (A = D) \wedge (B = D)$

A=1 D=1 C= 1  
failure  
C=2  
failure  
C=3 failure  
C=4 failure

D=2 failure  
D=3 failure  
D=4 failure

D=2 failure  
D=3 failure  
D=4 failure

D=2 failure  
D=3 failure  
D=4 failure

D=3 failure  
D=4 failure

A=1 B=1 failure



B=2 C=1 D=1 failure

C=4 D=1 failure

D=2 failure

C=2 failure

C=3 D=1 failure

•  
•

## Selecting variables in a smart way

- Backtracking relies on one or more **heuristics** to select which variables to consider next.
  - E.g, variable involved in the largest number of constraints:

•  
•  
•

- Can also be smart about which values to consider first
- But we will look at an alternative approach that can do much better

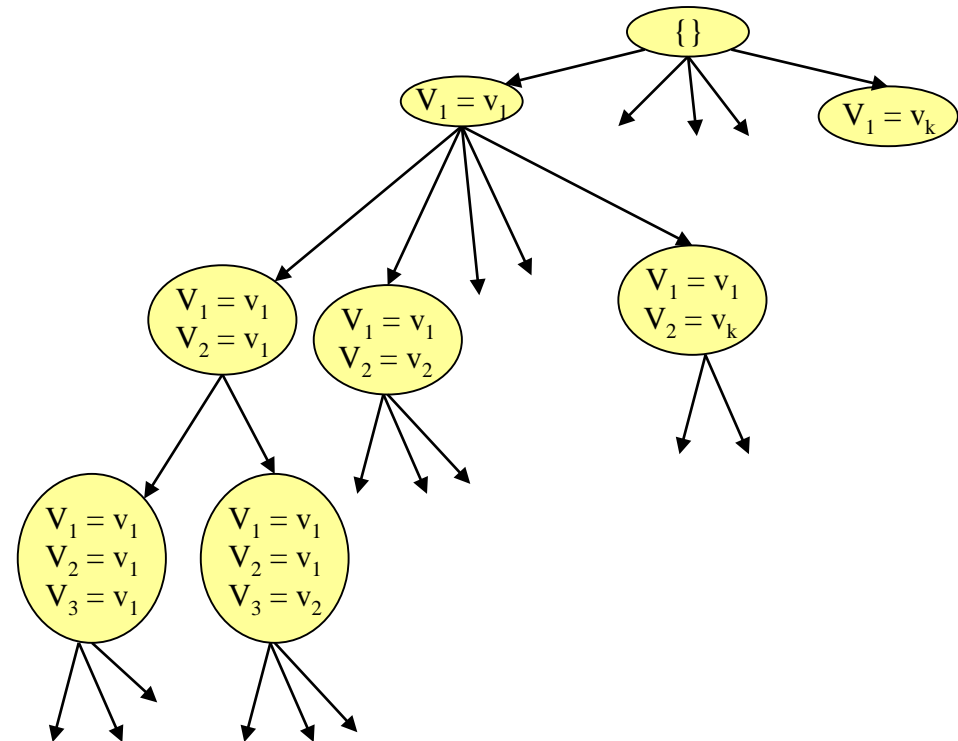
# Meaning of 'heuristic' in CSP as search

- This is a **different use of the word 'heuristic'** 'from the definition we have given in the search module
- Qualifications of 'heuristic' still true in the context of CSP as search
  - Can be computed cheaply during the search
  - Provides guidance to the search algorithm
- Qualification that is not true anymore in this context
  - 'Estimate of the distance to the goal'

- Both meanings are used frequently in the AI literature.
- In general
- ‘heuristic’ means ‘serves to discover’: goal-oriented.
- Does not mean ‘unreliable’!

# Standard Search vs. Specific R&R systems

- Constraint Satisfaction (Problems):
- **State**: assignments of values to a subset of the variables
- **Successor function**: assign values to a “free” variable
- **Goal test**: all variables assigned a value and all constraints satisfied?
- **Solution**: possible world that satisfies the constraints
- **Heuristic function**: none (all solutions at the same distance from start)
- Planning :
- **State**
- **Successor function**
- **Goal test**

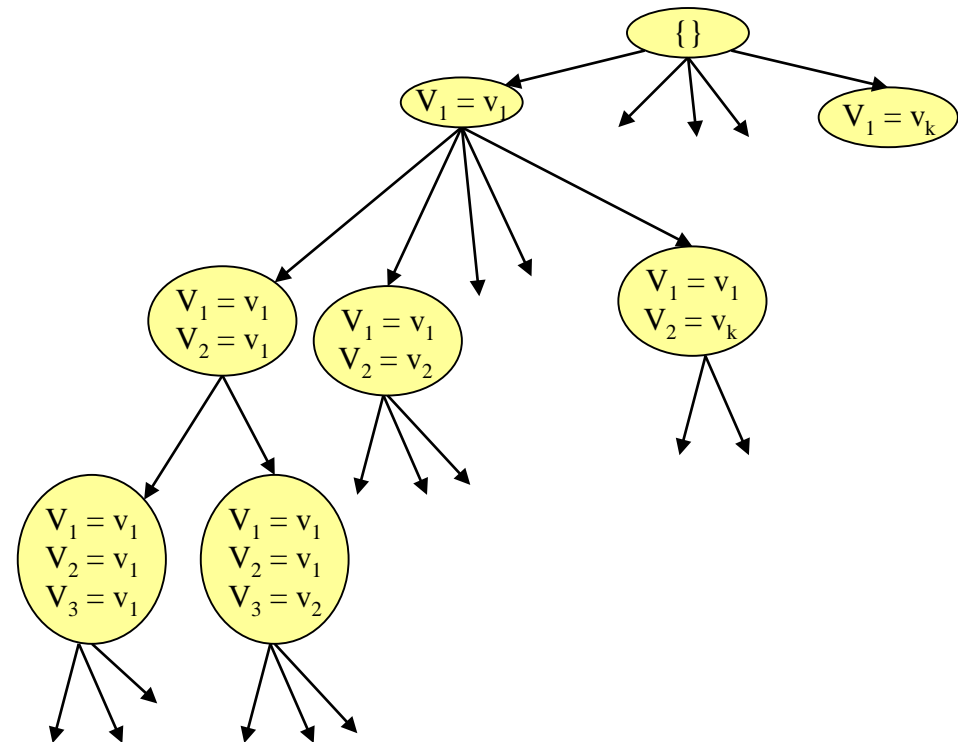


- Solution
- Heuristic function
- Inference
- State
- Successor function
- Goal test
- Solution
- Heuristic function

## Standard Search vs. Specific R&R systems

- Constraint Satisfaction (Problems):
- **State:** assignments of values to a subset of the variables
- **Successor function:** assign values to a “free” variable
- **Goal test:** all variables assigned a value and all constraints satisfied?
- **Solution:** possible world that satisfies the constraints

- **Heuristic function:** none (all solutions at the same distance from start)
- **Planning:**
- **State**
- **Successor function**
- **Goal test**
- **Solution**
- **Heuristic function**
- **Inference**
- **State**
- **Successor function**
- **Goal test**
- **Solution**
- **Heuristic function**



# Learning Goals for CSP

- Define possible worlds in term of variables and their domains
- Compute number of possible worlds on real examples
- Specify constraints to represent real world problems differentiating between:
  - Unary and k-ary constraints
  - List vs. function format
- Verify whether a possible world satisfies a set of constraints (i.e., whether it is a model, a solution)
- Implement the **Generate-and-Test** Algorithm. Explain its disadvantages.



- Solve a **CSP by search** (specify neighbors, states, start state, goal state).
- Compare strategies for CSP search.
- Implement pruning for DFS search in a CSP.