Lecture 10

Stochastic Local Search

(4.8)

Lecture Overview

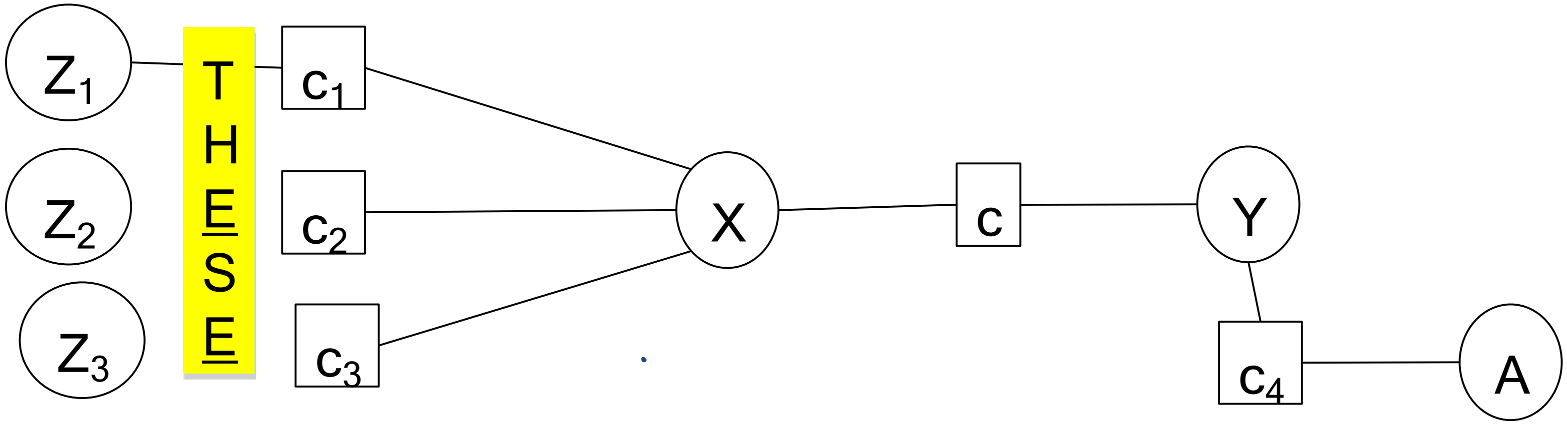
• Recap • Domain Splitting for Arc Consistency • Local Search • Stochastic Local Search (SLS) • Comparing SLS

Arc Consistency Algorithm

- Go through all the arcs in the network
- Make each arc consistent by pruning the appropriate domain, when needed
- Reconsider arcs that could be turned back to being inconsistent by this pruning
- Eventually reach a ‘fixed point’: all arcs consistent

Which arcs need to be reconsidered?

- When AC reduces the domain of a variable X to make an arc ⟨X,c⟩ arc consistent, which arcs does it need to reconsider?



AC does not need to reconsider other arcs

- If arc ⟨Y,c⟩ was arc consistent before, it will still be arc consistent.
  "Consistent before" means each element y_i in Y must have an element x_i in X that satisfies the constraint. Those x_i would not be pruned from Dom(X), so arc ⟨Y,c⟩ stays consistent

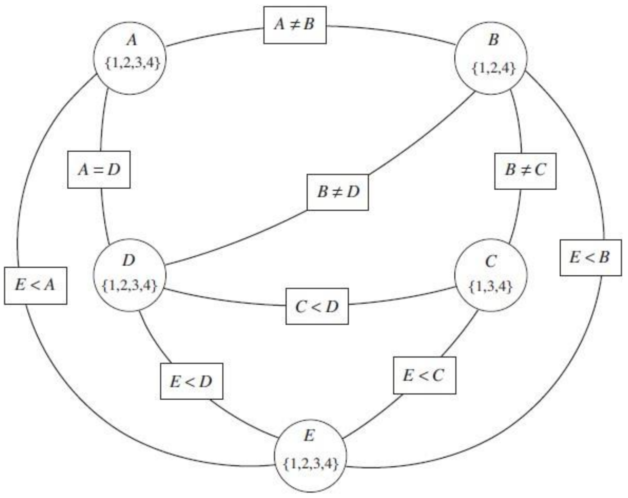
- If an arc ⟨X,c_i⟩ was arc consistent before, it will still be arc consistent

The domains of Z_i have not been touched

- Nothing changes for arcs of constraints not involving X

Arc Consistency Algorithm: Complexity

- Let's determine Worst-case complexity of this procedure (compare with DFS $O(d^n)$)
- let the max size of a variable domain be d
- let the number of variables be n • The max number of binary constraints is ? (n * (n-1)) / 2
- How many times, at worst, the same arc can be inserted in the ToDoArc list? O(d)
- How many steps are involved in checking the consistency of an arc? $O(d^2)$

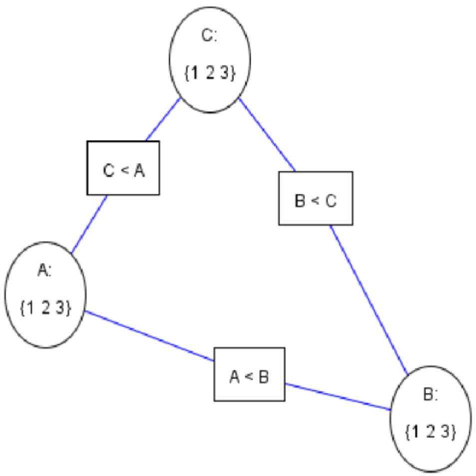


- **Overall complexity:** Overall complexity: $O(n^2d^3)$ $O(n^2d^3)$
- Compare to $O(d^N)$ of DFS. Arc consistency is MUCH faster So did we find a polynomial algorithm to solve CPSs?

No, AC does not always solve the CPS. It is a way to possibly simplify the original CSP and make it easier to solve

Arc Consistency Algorithm: Interpreting Outcomes

- Three possible outcomes (when all arcs are arc consistent):
- Each domain has a single value,
  - ✓ e.g. built-in AISpace example "Scheduling problem 1" ✓ We have: a (unique) solution.



- At least one domain is empty,
  - ✓ We have: No solution! All values are ruled out for this variable.

- e.g. try this graph (can easily generate it by modifying Simple Problem 2)

- Some domains have more than one value,
  - There may be: one solution, multiple ones, or none
  - Need to solve this new CSP (usually simpler) problem:
    - same constraints, domains have been reduced

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- Recap
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- Local Search
- Stochastic Local Search (SLS)
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Search vs. Domain Splitting

- Arc consistency ends: Some domains have more than one value → may or may not have a solution

  A. Apply Depth-First Search with Pruning or

  B. Split the problem in a number of disjoint cases CSP_i: for instance

     CSP with $dom(X) = \{x_1, x_2, x_3, x_4\}$ becomes

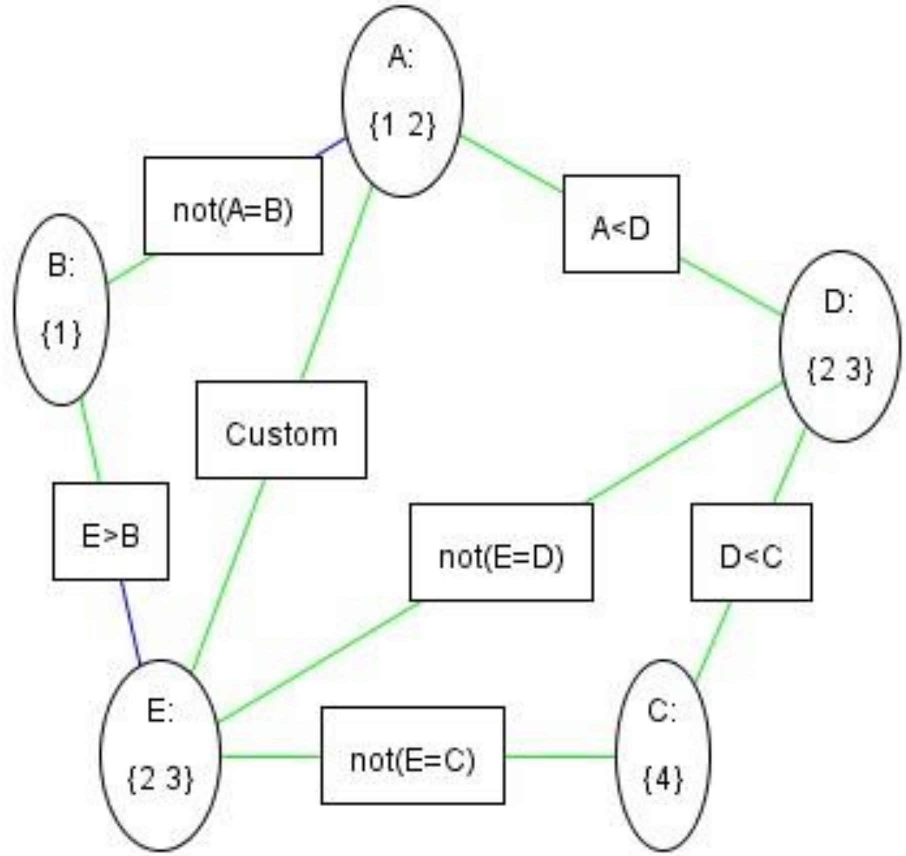
     CSP_1 with $dom(X) = \{x_1, x_2\}$ and
     CSP_2 with $dom(X) = \{x_3, x_4\}$

- Solution to CSP is the union of solutions to CSP_i

Example

Run “Scheduling Problem 2” in AIspace

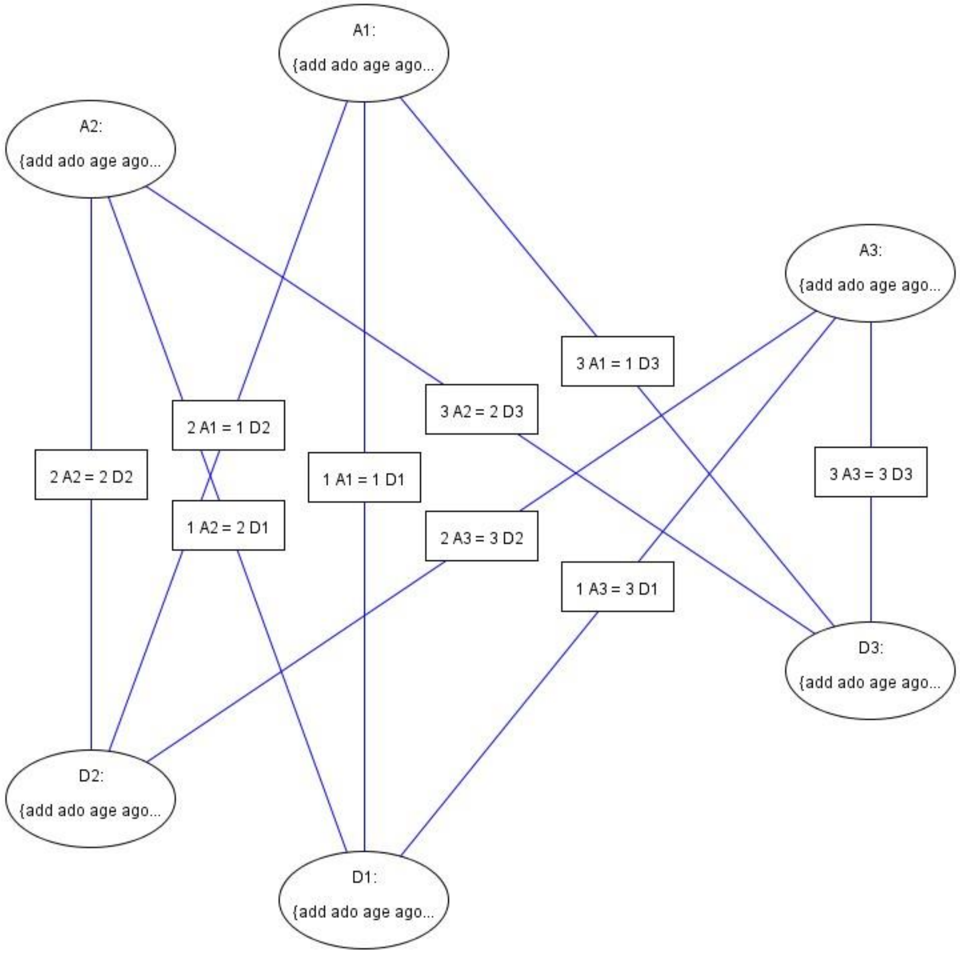
- Try spitting on E (select 4 first, then 2 then 3)


A:
{1 2}
not(A=B)
A<D
B:
{1}
D:
{2 3}
Custom
E>B
not(E=D)
D<C
E:
{2 3}
not(E=C)
C:
{4}

Another Example

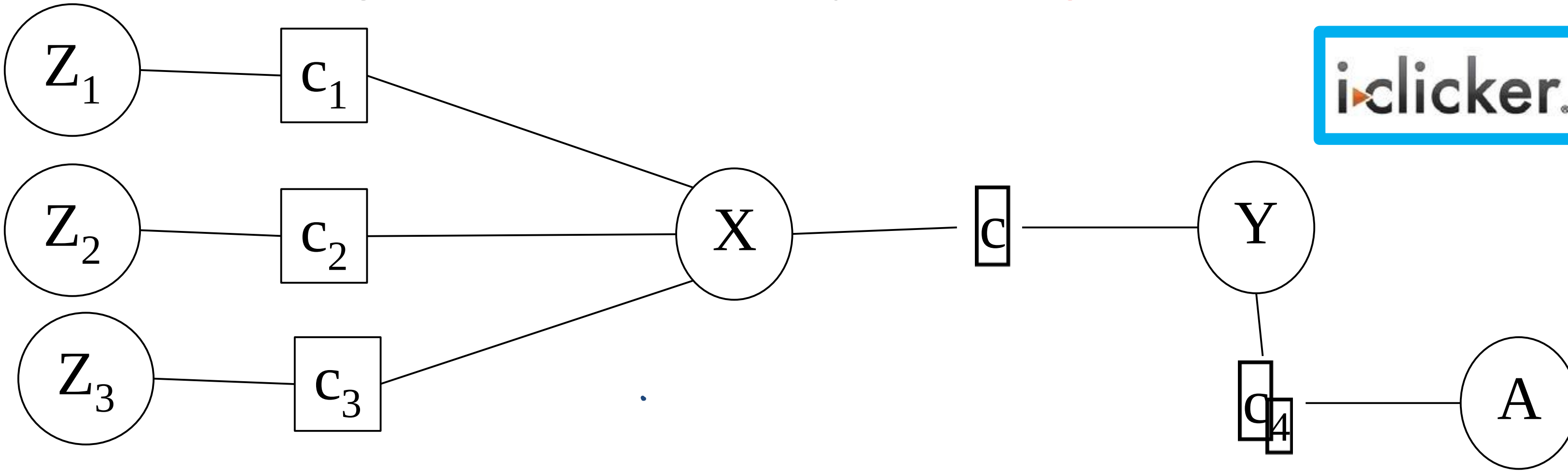
"Crossword 1" in Aispace,

- try splitting on D3 and then A3 (always "select half")



Domain splitting

- For each subCSP, which arcs have to be on the ToDoArcs list when we get the subCSP by splitting the domain of X?



A. arcs <Z_i, r(Z_i,X)>

B. arcs < Z_i, r(Z_i,X)> and <X, r(Z_i,X)>

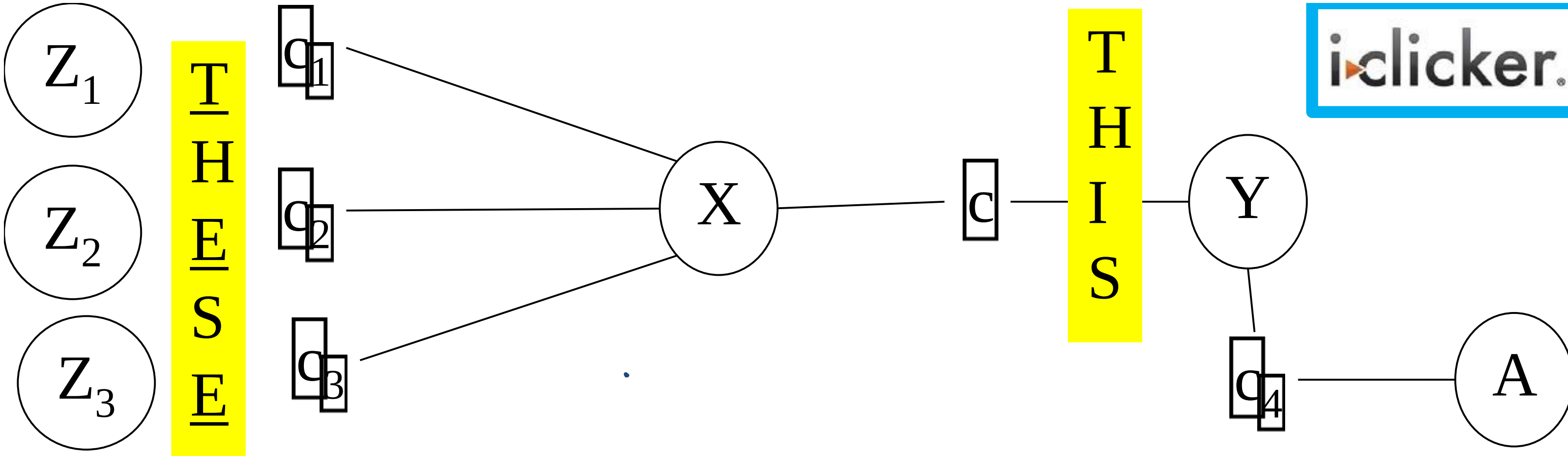
C. arcs < Z_i, r(Z,X)> and <Y, r(X,Y)>

D. All arcs in the figure

E. All 8 arcs related to constraints involving X: (c_1,c_2,c_3,c)

Domain splitting

- For each sub CSP, which arcs have to be on the To Do Arcs list when we get the sub CSP by splitting the domain of X?



C. arcs < Z_i, r(Z,X)> and <Y, r(X,Y)>

Searching by domain splitting

CSP, apply AC

If domains with multiple values

Split on one

CSP$_1$, apply AC CSP$_2$, apply AC

If domains with multiple values If domains with multiple

Split on one values…..Split on one

Another formulation of CSP as search

Arc consistency with domain splitting

- States: vector $(D(V_1), \ldots, D(V_n))$ of remaining domains, with $D(V_i) \subseteq dom(V_i)$ for each V_i

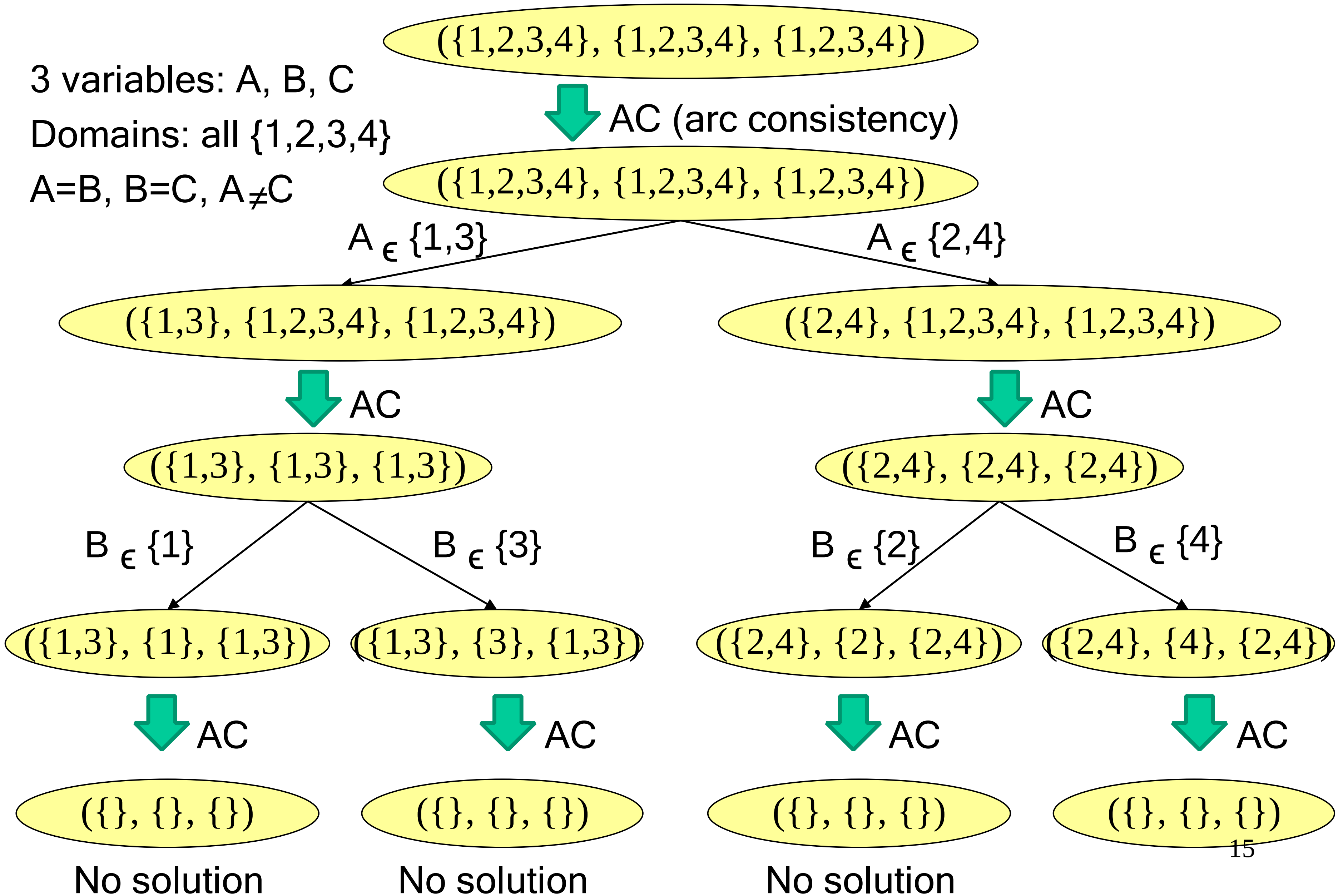
- Start state: vector of original domains ($dom(V_1)$, …, $dom(V_n)$)
- Successor function:
- reduce one of the domains + run arc consistency
- Goal state: vector of unary domains that satisfies all constraints
- That is, only one value left for each variable
- The assignment of each variable to its single value is a model • Solution: that assignment

Arc consistency + domain splitting: example

3 variables: A, B, C

Domains: all {1,2,3,4}

A=B, B=C, A≠C

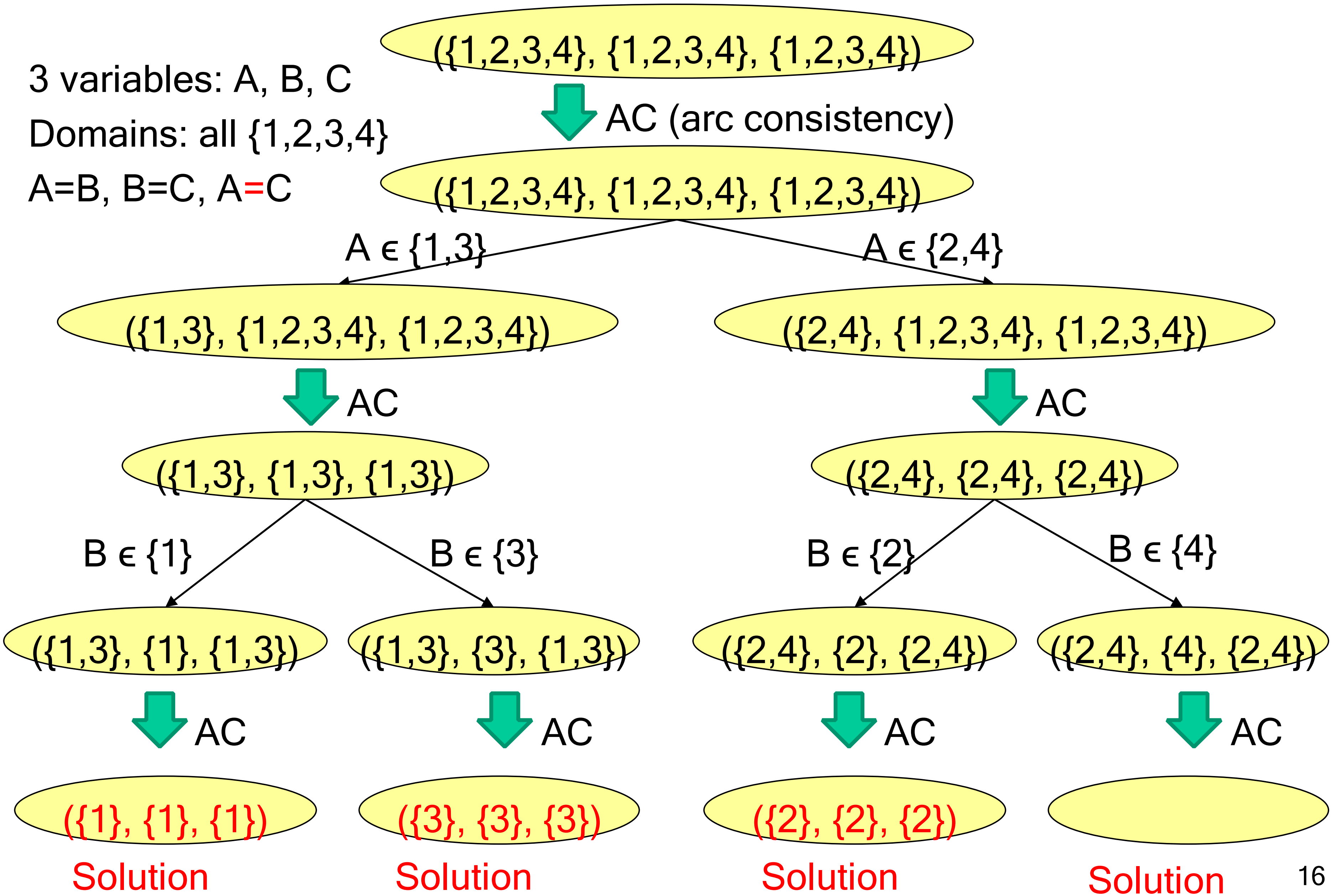


Arc consistency + domain splitting: another example

3 variables: A, B, C

Domains: all {1,2,3,4}

A=B, B=C, A=C



({4}, {4}, {4})

Searching by domain splitting

CSP, apply AC

If domains with multiple values

Split on one

CSP_1, apply AC

CSP_2, apply AC

If domains with multiple values

If domains with multiple

Split on

one

values…..Split on one

How many CSPs do we need to keep around at a time? Assume solution at depth m and b children at each split

A. $O(bm)$ B. $O(b^m)$ B. $O(m^b)$ B. $O(b^2)$

Searching by domain splitting

CSP, apply AC

If domains with multiple values

Split on one

CSP_1, apply AC

If domains with multiple values

CSP_2, apply AC

If domains with multiple

Split on

one

values.....Split on one

How many CSPs do we need to keep around at a time?
Assume solution at depth m and b children at each split

 O(bm): It is DFS

Systematically solving CSPs: Summary

- Build Constraint Network
- Apply Arc Consistency
- One domain is empty →

- Each domain has a single value →
- Split the problem in a number of disjoint cases
- Apply Arc Consistency to each case, and repeat

Limitation of Systematic Approaches

- Some domains have more than one value →
- Apply Depth-First Search with Pruning OR

- Many CSPs (scheduling, DNA computing, etc.) are simply too big for systematic approaches

- If you have 10^5 vars with dom(var_i) = 10^4

Systematic Search	Constraint Network
Branching factor b =	Size =
Solution depth d =	
Complexity =	Complexity of AC =

Limitation of Systematic Approaches

- Many CSPs are simply too big for systematic approaches • If you have 10^5 vars with dom(var_i) = 10^4

Systematic Search	Constraint Network
Branching factor b = 10^4 Solution depth d = 10^5 Time Complexity = $O((10^4)^{105})$	Size = $O(10^5 + 10^5 * 10^5)$ Time Complexity of AC = $O((10_{5*2} * 10_{4*3})$

Learning Goals for CSP

- Define possible worlds in term of variables and their domains
- Compute number of possible worlds on real examples
- Specify constraints to represent real world problems differentiating between:
- Unary and k-ary constraints

- List vs. function format
- Verify whether a possible world satisfies a set of constraints (i.e., whether it is a model, a solution)
- Implement the Generate-and-Test Algorithm. Explain its disadvantages.
- Solve a CSP by search (specify neighbors, states, start state, goal state). Compare strategies for CSP search. Implement pruning for DFS search in a CSP.
- Define/read/write/trace/debug the arc consistency algorithm. Compute its complexity and assess its possible outcomes
- Define/read/write/trace/debug domain splitting and its integration with arc consistency

Lecture Overview

• Recap • Domain Splitting for Arc Consistency • Local Search • Stochastic Local Search (SLS) • Comparing SLS

Local Search: Motivation

- Solving CSPs is NP-hard
  - Search space for many CSPs is huge
  - Exponential in the number of variables
  - Even arc consistency with domain splitting is often not enough

- Alternative: local search
- use algorithms that search the space locally, rather than systematically
- Often finds a solution quickly, but are not guaranteed to find a solution if one exists (thus, cannot prove that there is no solution)

Local Search

- Idea:
- Consider the space of complete assignments of values to variables (all possible worlds)
- Neighbors of a current node are similar variable assignments
- Move from one node to another according to a function that scores how good each assignment is

- Useful method in practice

- Best available method for many constraint satisfaction and constraint optimization problems

Local Search Problem: Definition

Definition: A local search problem consists of a:

CSP: a set of variables, domains for these variables, and constraints on their joint values.

A node in the search space will be a complete assignment to all of the variables.

Neighbor relation: an edge in the search space will exist when the neighbor relation holds between a pair of nodes.

Scoring function: h(n), judges cost of a node (want to minimize)

- E.g. the number of constraints violated in node n.
- E.g. the cost of a state in an optimization context.

Local Search Problem: Example

Definition: A local search problem consists of a:

CSP: a set of variables, $\{V_1 \ldots., V_n\}$, each with domain $Dom(V_i)$, and constraints on their joint values.

A node in the search space will be a complete assignment to all of the variables.

Nneighbour relation: The neighbors of node with assignment

$A= \{V_1 / v_1,\ldots,V_n / v_n\}$ are nodes with assignments that

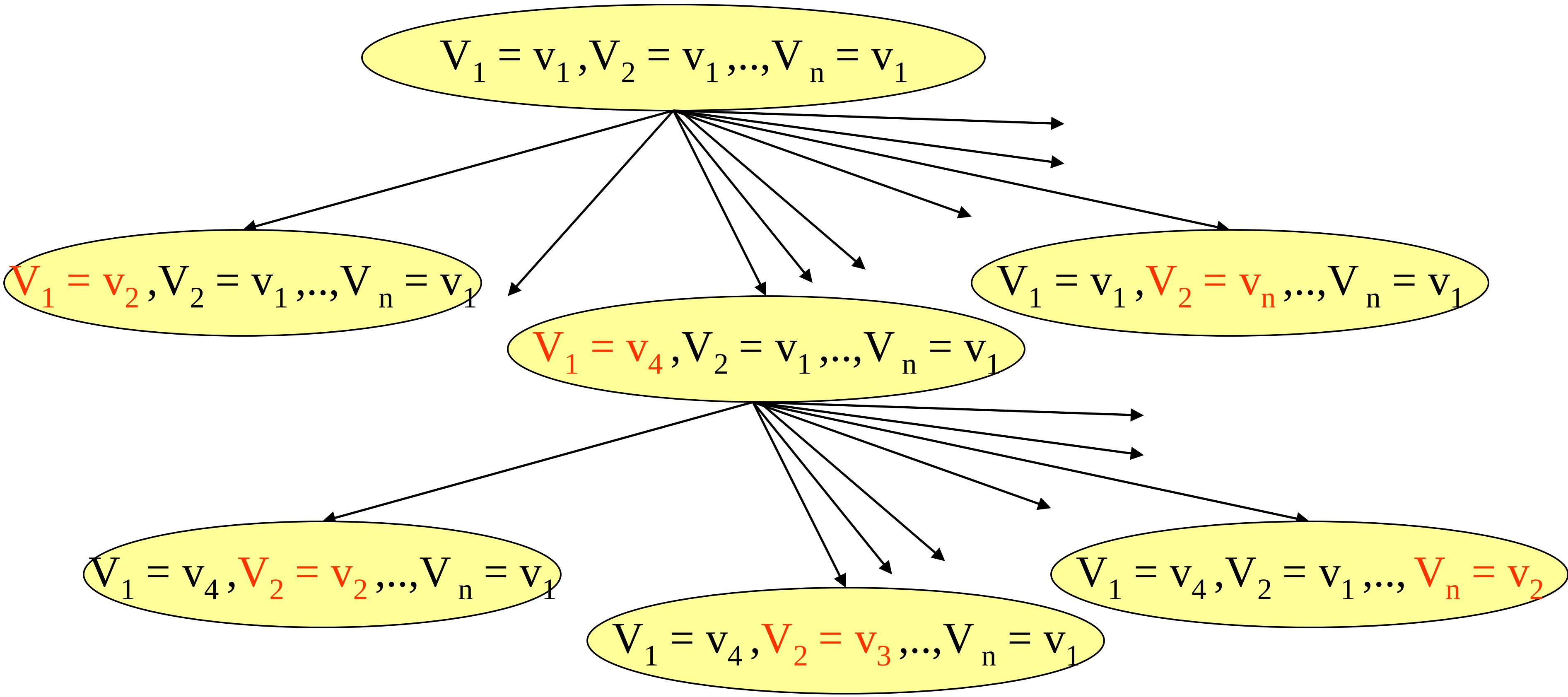
differ from A for one value only

Scoring function: h(n), judges cost of a node (want to minimize) -

E.g. the number of constraints violated in node n.

- E.g. the cost of a state in an optimization context.

Search Space



- Only the current node is kept in memory at each step.
- Very different from the systematic tree search approaches we have seen so far!

- Local search does NOT backtrack! 29

Iterative Best Improvement

- How to determine the neighbor node to be selected?
- Iterative Best Improvement:
- select the neighbor that optimizes some evaluation function
- Which strategy would make sense? Select neighbor with …

A. Maximal number of constraint violations

B. Similar number of constraint violations as current state

C. No constraint violations

D. Minimal number of constraint violations

Iterative Best Improvement

- How to determine the neighbor node to be selected?
- Iterative Best Improvement:
- select the neighbor that optimizes some evaluation function
- Which strategy would make sense? Select neighbour with …

Minimal number of constraint violations

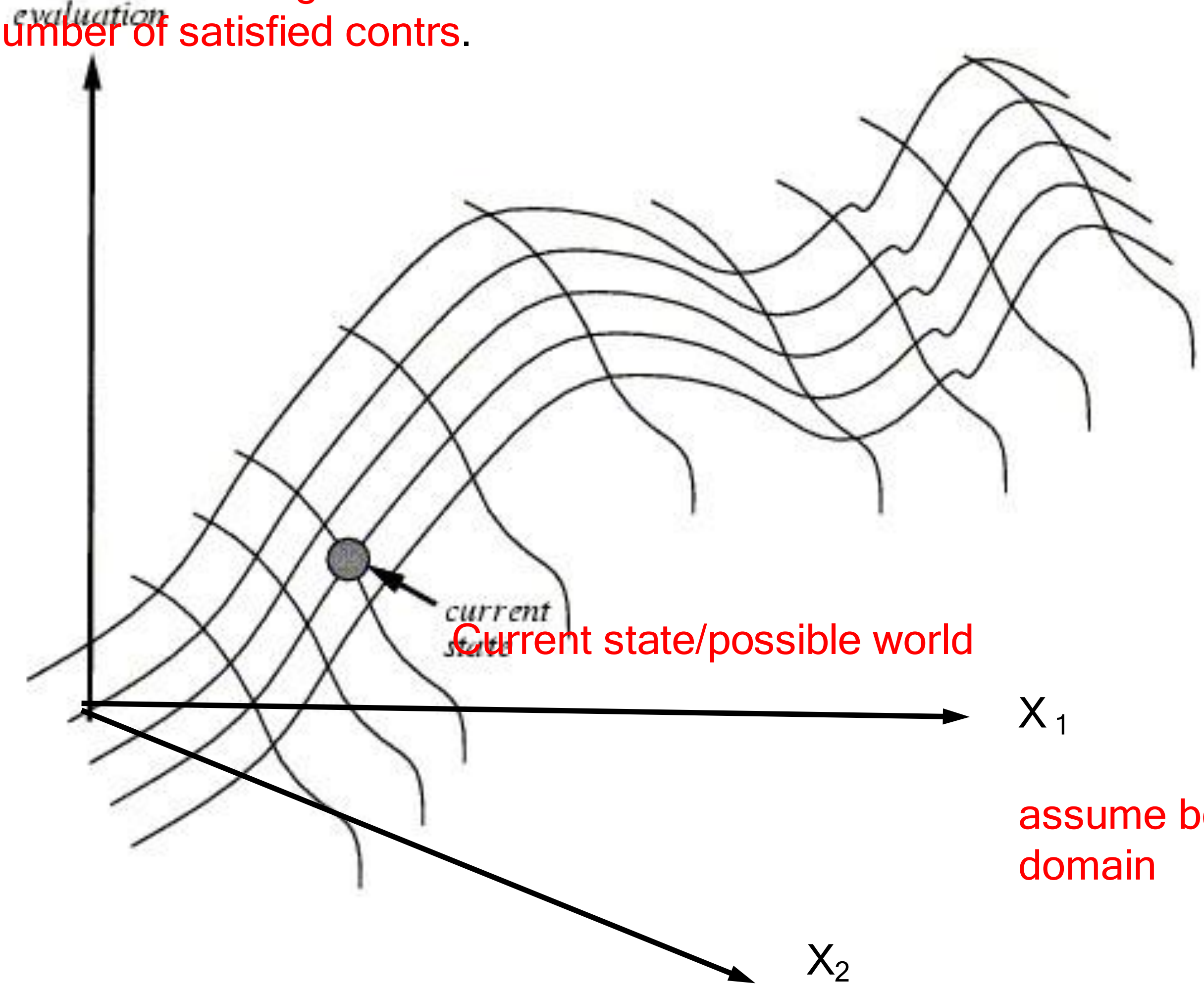
- Evaluation function:
  h(n): number of constraint violations in state n
- Greedy descent: evaluate h(n) for each neighbour, pick the neighbour n with minimal h(n) – minimize the number of unsatisfied constraints
- Hill climbing: equivalent algorithm for maximization problems

- Here: Maximize number of satisfied constraints

Hill Climbing

NOTE: Everything that will be said for Hill Climbing is also true for Greedy Descent

evaluation/scoring function:
number of satisfied contrs.



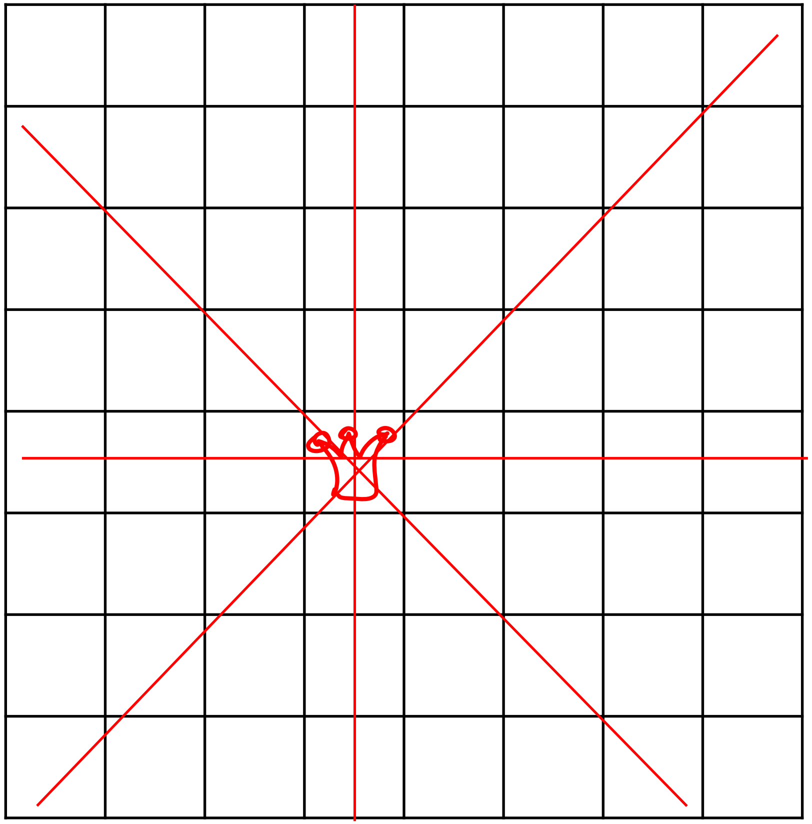
Current state/possible world

assume both vars have integer domain

Example: N-Queens

- Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal (i.e attacking each other)

- Positions a queen can attack



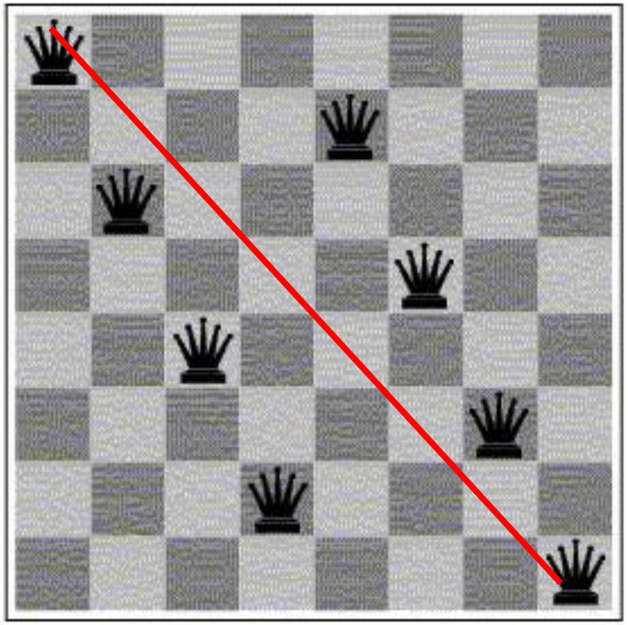
Example: N-queen as a local search problem

CSP: N-queen CSP

- One variable per column; domains {1,…,N} => row where the queen in the i^{th} column seats;
- Constraints: no two queens in the same row, column or diagonal

Neighbour relation: value of a single column differs

Scoring function: number of constraint violations (i.e., number of attacks)



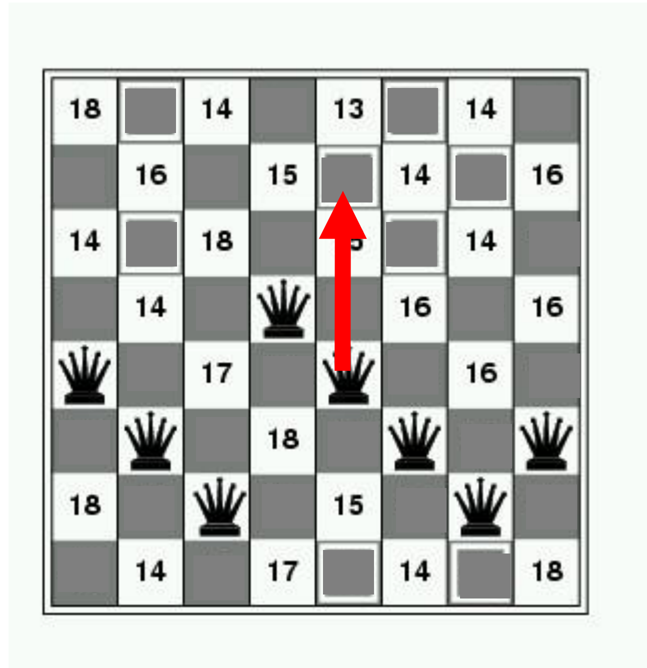
Example: Greedy descent for N-Queen

For each column, assign randomly each queen to a row (a number between 1 and N) Repeat

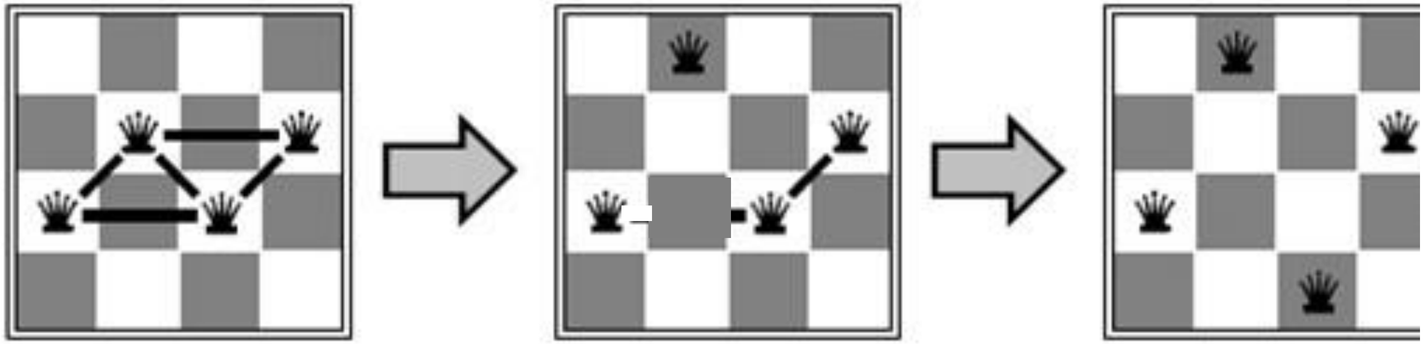
- For each column & each number: Evaluate how many constraint violations changing the assignment to that number would yield

- Choose the column and number that leads to the fewest violated constraints; change it

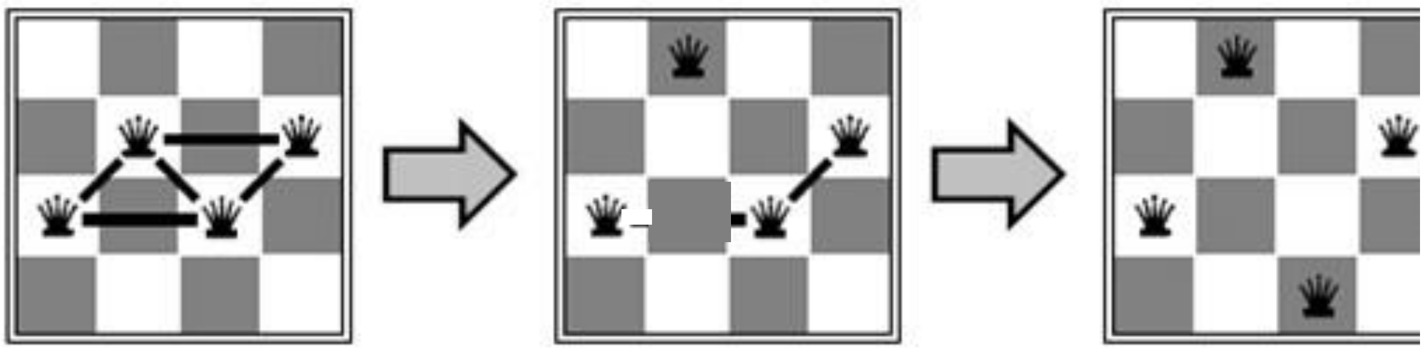
Until solved



Each cell lists h (i.e. #constraints unsatisfied) if you move the queen in that column into the cell



h = 5 h = ? h = ?



h = 5 h = ? h = ?

General Local Search Algorithm

```
Procedure Local-Search(V,dom,C)
        Inputs
                V: a set of variables
                dom: a function such that dom(X) is the domain of variable X
                C: set of constraints to be satisfied
        Output     complete assignment that satisfies the constraints
        Local
                A[V] an array of values indexed by V
        repeat
                for each variable X do
                        A[X] ←a random value in dom(X);

                while (stopping criterion not met & A is not a satisfying assignment)
                        Select a variable Y and a value V∈dom(Y)
                        Set A[Y] ←V

                if (A is a satisfying assignment) then
                        return A

```

Random initialization

Local search step

```
until termination
```

General Local Search Algorithm

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Procedure Local-Search(V,dom,C)
    Inputs
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        while (stopping criterion not met & A is not a satisfying assignment)
            Select a variable Y and a value V dom(Y)
            Set A[Y] ←V

        if (A is a satisfying assignment) then
```

Random initialization

Local Search Step

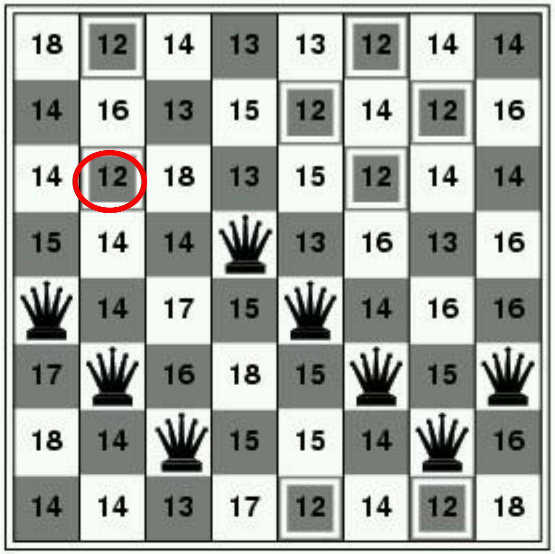
Based on local information.

18: return A E.g., for each neighbour evaluate how many constraints are unsatisfied. 19:

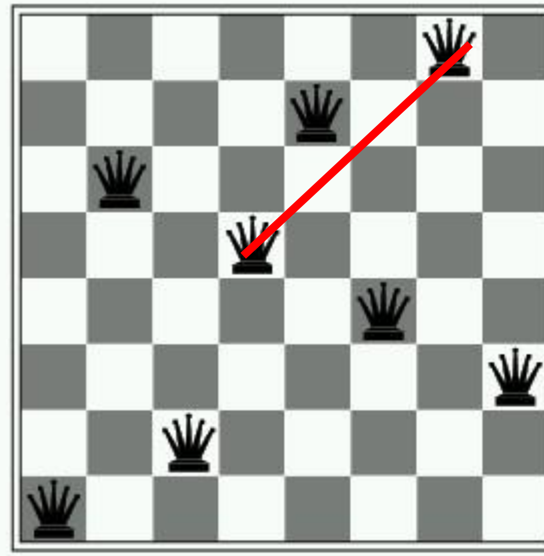
20: until termination

Greedy descent: select Y and V to minimize #unsatisfied constraints at each step

Example: N-Queens



5 steps



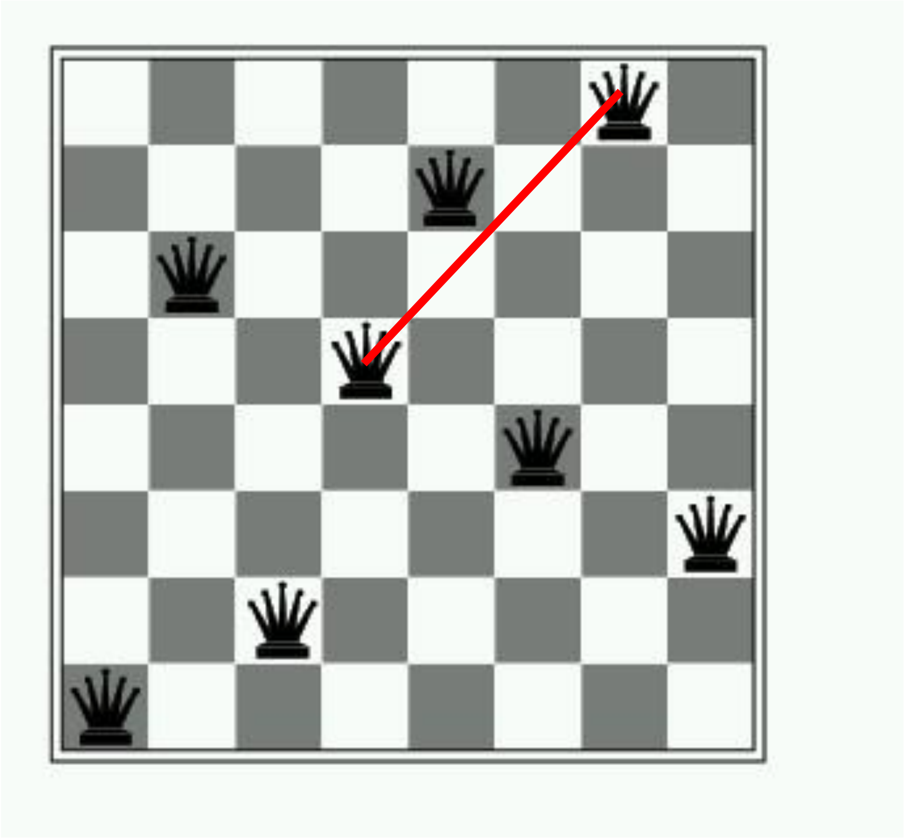
h = 17

h = 1

Each cell lists h (i.e. #constraints unsatisfied) if you move the queen from that column into the cell

Example: N-Queens

- Which move should we pick in this situation?

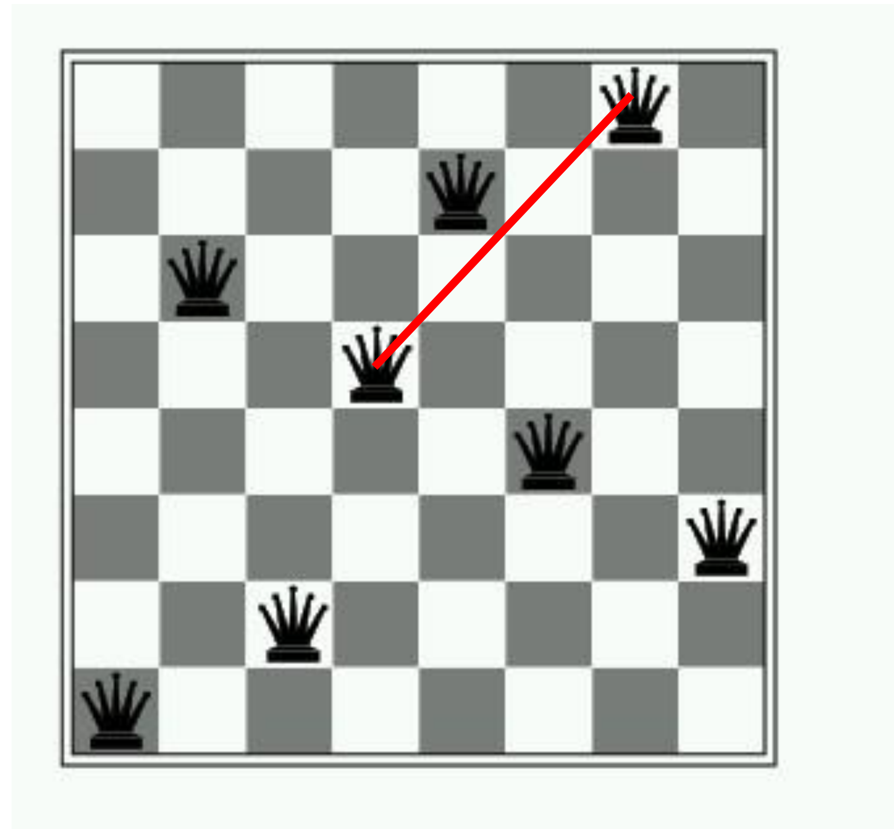


The problem of local minima

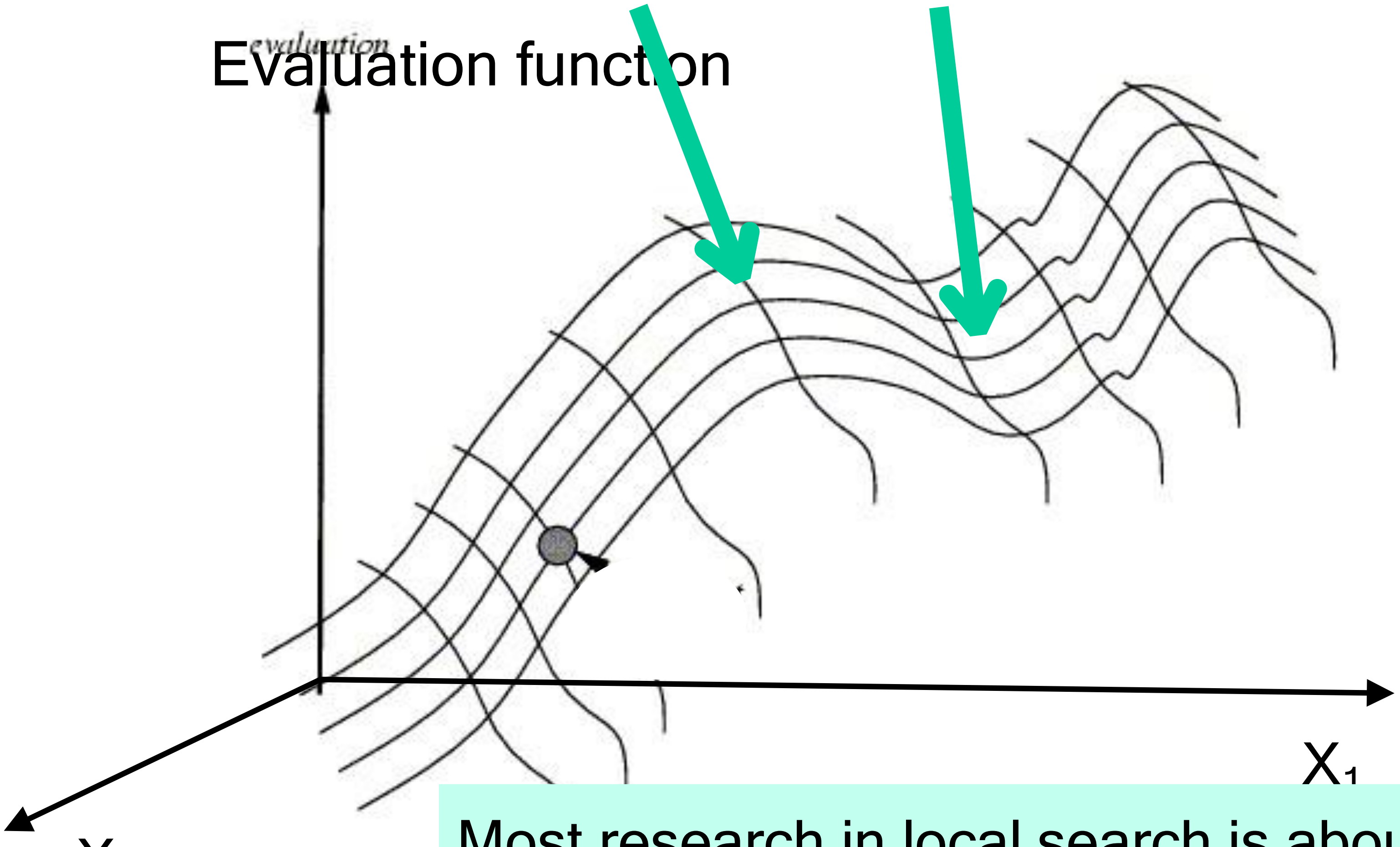
- Which move should we pick in this situation?
  - Current cost: h=1 - No single move can improve on this
  - In fact, every single move only makes things worse ($h \geq 2$)
- Locally optimal solution •

Since we are minimizing:

local minimum



Problems with Iterative Best Improvement



Most research in local search is about finding effective mechanisms for escaping

It gets misled by locally optimal points

- (Local Maxima/ Minima)

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Stochastic Local Search

- We will use greedy steps to find local minima
- Move to neighbour with best evaluation function value

- We will use randomness to escape local minima

Stochastic Local Search (SLS) for CSPs

- Start node: random assignment
- Goal: assignment with zero unsatisfied constraints
- Heuristic function h: number of unsatisfied constraints
  - Lower values of the function are better

- Stochastic local search is a mix of:
  - Greedy descent: move to neighbor with lowest h
  - Random walk: take some random steps
  - i.e., move to a neighbour with some randomness

- Random restart: reassigning values to all variables

General SLS Algorithm

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        dom: a function such that dom(X) is the domain of variable X
        C: set of constraints to be satisfied
    Output    complete assignment that satisfies the constraints
    Local
        A[V] an array of values indexed by V
    repeat
        for each variable X do
            A[X] ← a random value in dom(X);
restart
        while (stopping criterion not met & A is not a satisfying assignment)
            Select a variable Y and a value V ∈ dom(Y)
```

Random

Extreme case 1: random sampling.

Restart at every step: Stopping criterion is “true”

```
                Set A[Y] ←V

        if (A is a satisfying assignment) then
                return A

    until termination
```

General SLS Algorithm

```
Procedure Local-Search(V,dom,C)
    Inputs
        V: a set of variables
        dom: a function such that dom(X) is the domain of variable X
        C: set of constraints to be satisfied
    Output   complete assignment that satisfies the constraints
    Local
        A[V] an array of values indexed by V
    repeat
        for each variable X do
            A[X] ←a random value in dom(X);
```

Random

Extreme case 2: greedy descent
Select the neighbor with best h value (select at random among neighbors with same h)

∈

```
restart
        while (stopping criterion not met & A is not a satisfying assignment)
            Select a variable Y and a value V dom(Y)
            Set A[Y] ←V

        if (A is a satisfying assignment) then
            return A

    until termination
```

Tracing SLS algorithms in AIspace

- Let's look at these algorithms in AIspace:
  - Greedy Descent
  - Random Sampling
- Simple scheduling problem 2 in AIspace:

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Stochastic Local Search Based CSP Solver

version 4.6.0