```
(*starting date: 04 May 2021*)
(*analytical model of Alfven wings when the background field has a flow-
 aligned component, B0=(B0x, 0, B0z), where B0x > 0 and B0z < 0*)
(*This model generates output in TWO different coordinate
 systems: (x,y,z) where z is antiparallel to B0,
and (xcor, ycor=y, zcor), where xcor is along the incident flow direction*)
(*05/07/21: update: equations for magnetic field corrected*)
(*17 June 2021: Adjusted to Triton,*)

(*----------------------------------------------------------------------*)
(*Basic Input Parameters*)
(*----------------------------------------------------------------------*)
RE = 1353 * 10^3;(*moon radius in m*)
B0 = 5.1369 * 10^-9; (*background field magnitude in T*)
theta = 42.94896 * (2 π)/360; (*inclination angle in rd,
as defined in Figure 3 of Neubauer1980,
theta=0 means u0 perpendicular to B0. !TODO: Theta needs to become a free parameter!*)
B0x = 0;
B0y = 0;
B0z = -B0; (*background magnetic field in system (x,y,z)*)
B0xcor = B0x Cos[theta] - B0z Sin[theta];
B0ycor = 0;
B0zcor = B0x Sin[theta] + B0z Cos[theta];
(*background magnetic field in system (xcor,ycor,zcor)*)
n0 = 0.11 * 10^6; (*upstream number density in 1/m^3*)
u0 = 43 * 10^3; (*magnitude of upstream flow velocity in m/s*)
u0xcor = u0;
u0ycor = 0;
u0zcor = 0; (*upstream flow velocity in system (xcor,ycor,zcor)*)
u0x = u0 Cos[theta];
u0y = 0;
u0z = -u0 Sin[theta];(*upstream flow velocity in system (x,y,z)*)
E0 = u0 * B0 * Cos[theta]; (*MAGNITUDE OF convective electric field*)
E0norm = u0 * B0; (*may be needed for normalization purposes*)
mp = 1.6726231 * 10^-27; (*proton mass*)
m = 7.5 * mp; (*upstream ion mass*)
mu0 = 4 * 3.14159265359 * 10^-7; (*magnetic permeability of vacuum*)

(*----------------------------------------------------------------------*)
(*Derived parameters for Alfven wings*)
(*----------------------------------------------------------------------*)
vA = B0/Sqrt[mu0 * n0 * m]; (*Alfven velocity*)
MA = u0 / vA (*Alfvenic Mach number*)
SigmaAn = 1/(mu0 * vA * Sqrt[1 + MA^2 - 2 * MA * Sin[theta]]); (*Alfven conductance, northern wing*)
SigmaAs = 1/(mu0 * vA * Sqrt[1 + MA^2 + 2 * MA * Sin[theta]]);(*Alfven conductance, southern wing*)
```

```
ThetaN = ArcTan[MA Cos[theta] / (1 - MA Sin[theta])] * 360/(2 π)
(*inclination angle against B0 for NORTHERN Alfven wing*)
ThetaS = ArcTan[MA Cos[theta] / (1 + MA Sin[theta])] * 360/(2 π)
(*inclination angle against B0 for NORTHERN Alfven wing*)
SinThetaN = MA Cos[theta] / Sqrt[1 + MA^2 - 2 * MA * Sin[theta]];
CosThetaN = (1 - MA Sin[theta]) / Sqrt[1 + MA^2 - 2 * MA * Sin[theta]];
SinThetaS = MA Cos[theta] / Sqrt[1 + MA^2 + 2 * MA * Sin[theta]];
CosThetaS = (1 + MA Sin[theta]) / Sqrt[1 + MA^2 + 2 * MA * Sin[theta]];
```

$$\text{ThetaN} = \text{ArcTan}\left[\frac{\text{MA Cos[theta]}}{1 - \text{MA Sin[theta]}}\right] * \frac{360}{2\pi}$$

$$\text{ThetaS} = \text{ArcTan}\left[\frac{\text{MA Cos[theta]}}{1 + \text{MA Sin[theta]}}\right] * \frac{360}{2\pi}$$

$$\text{SinThetaN} = \frac{\text{MA Cos[theta]}}{\sqrt{1 + \text{MA}^2 - 2 * \text{MA} * \text{Sin[theta]}}};$$

$$\text{CosThetaN} = \frac{1 - \text{MA Sin[theta]}}{\sqrt{1 + \text{MA}^2 - 2 * \text{MA} * \text{Sin[theta]}}};$$

$$\text{SinThetaS} = \frac{\text{MA Cos[theta]}}{\sqrt{1 + \text{MA}^2 + 2 * \text{MA} * \text{Sin[theta]}}};$$

$$\text{CosThetaS} = \frac{1 + \text{MA Sin[theta]}}{\sqrt{1 + \text{MA}^2 + 2 * \text{MA} * \text{Sin[theta]}}};$$

```
0.348577

18.5011

11.6498

(*-----------------------------------------------------------------*)
(*Ionosphere model, mostly taken from Simon2021 analytical model,
in coordinates (x,y,z)*)
(*Important: need SAME conductance profile in both hemispheres,
otherwise boundary condition from Saur2007 needs to be applied*)
(*-----------------------------------------------------------------*)
```

```
SigmaP = 500;(*exospheric Pedersen conductance,
called SigmaP0 in the 2021 paper, STILL SAME SIGMAP0 IN BOTH HEMISPHERES*)
H = 70 * 10^3; (*exospheric scale height, from Arnold2020b*)
r[x_, y_] = Sqrt[x^2 + y^2]; (*radial distance from z axis in cylindrical coordinates,
NOT radial distance to Europa's surface,
measured perpendicular to the background magnetic field or z axis,
(x,y) dependence NOT needed for merely plotting the exospheric conductance profile*)
R1 = RE ;(*radial boundary of the Europa fluxtube, x^2+y^2 = RE^2*)
R2 =  RE + 5 H;(*outer boundary of region with non-zero transverse conductance,
assumming the atmosphere "ends" at a distance of 5 scale heights. exp(-5)=0.00674*)
betaN = 1/(3 H) ; (*NEW 05/2021: north only*)
betaS =
  1/R1 Log[((SigmaP + SigmaAn) Exp[betaN * R1] + (SigmaAs - SigmaAn)) / (SigmaP + SigmaAs)];
(*given input parameter for conductance profile,
using some multiple of the exospheric scale height H here,
NEW; BETA IS DIFFERENT FOR NORTHERN AND SOUTHERN HEMISPHERES*)
Sigma1n[r_] = (SigmaP + SigmaAn) * Exp[betaN * r] - SigmaAn;
Sigma1s[r_] = (SigmaP + SigmaAs) * Exp[betaS * r] - SigmaAs;
(*Pedersen conductance within the Europa fluxtube r < R1,
05/2021: NOW DIFFERENT FOR NORTH AND SOUTH!!*)
```

$$\texttt{deltaN} = \frac{\texttt{Log}\left[\frac{\texttt{SigmaP+SigmaAn}}{\texttt{SigmaAn}}\right] + \texttt{betaN} * \texttt{R1}}{\texttt{R2} - \texttt{R1}};$$

$$\texttt{deltaS} = \frac{\texttt{Log}\left[\frac{\texttt{SigmaAn Exp[deltaN*(R2-R1)]+SigmaAs-SigmaAn}}{\texttt{SigmaAs}}\right]}{\texttt{R2} - \texttt{R1}};$$

```
Sigma2n[r_] = SigmaAn * Exp[deltaN * (R2 - r)] - SigmaAn;

Sigma2s[r_] = SigmaAs * Exp[deltaS * (R2 - r)] - SigmaAs;
(*Pedersen conductance outside the Europa fluxtube,
but within the exosphere R1 < r < R2,NEW 05/2021: NOW DIFFERENT FOR NORTH AND SOUTH!!*)
Sigma3[r_] = 0 ; (*Pedersen conductance outside of Europa fluxtube,
R2 < r, always the same in both hemispheres, no N/S splitting required*)
SigmaExon[r_] =
  Piecewise[{{Sigma1n[r], r ≤ R1}, {Sigma2n[r], R1 < r ≤ R2}, {Sigma3[r], R2 < r}}];
SigmaExos[r_] = Piecewise[{{Sigma1s[r], r ≤ R1},
    {Sigma2s[r], R1 < r ≤ R2}, {Sigma3[r], R2 < r}}];
(*putting all three segments together, 05/2021: NOW DIFFERENT EQUATIONS FOR NORTH
  AND SOUTH BUT PROFILES SHOULD STILL LOOK N E A R L Y identical in regions 1 and 2!!*)
```

```
(*----------------------------------------------------------------------*)
(*Ionosphere model, plots and derived parameters*)
(*----------------------------------------------------------------------*)
```

$$\text{AvSigmaPn} = \frac{1}{\text{R2}}\int_0^{\text{R2}} \text{SigmaExon}[r]\, dr$$

```
(*average Pedersen conductivity, northern hemisphere*)
```

$$\text{AvSigmaPs} = \frac{1}{\text{R2}}\int_0^{\text{R2}} \text{SigmaExos}[r]\, dr$$

```
(*average Pedersen conductivity, southern hemisphere*)
AvSigmaPn / AvSigmaPs (*should be as close as possible to 1*)
```

$$\text{alphaN} = \frac{\text{AvSigmaPn}}{\text{AvSigmaPn} + 2 * \text{SigmaAn}}$$ (*average interaction strength,
north !this is only an estimation!*)

$$\text{alphaS} = \frac{\text{AvSigmaPs}}{\text{AvSigmaPs} + 2 * \text{SigmaAs}}$$ (*average interaction strength,
north !this is only an estimation!*)

```
Plot[{SigmaExon[r * RE], SigmaExos[r * RE]}, {r, 0, 1.4}, FrameLabel → {"r (R_E)", "Σ_P (S)"},
 GridLines → Automatic, Frame → True, PlotLabel → "Ionospheric Pedersen Conductance",
 PlotStyle → {{Red, Thick}, {Blue, Thick}}, PlotLegends -> {"north", "south"}]
```

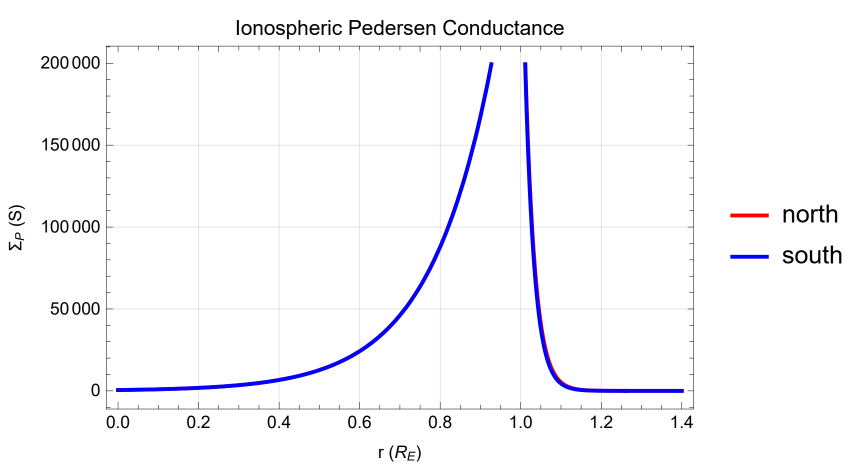
45 475.7

45 190.2

1.00632

0.999647

0.999774



```
(*----------------------------------------------------------------------------------
*)
(*05 May 2021: Potentials, magnetic field and flow velocity, all in system
 (x,y,z)*)
(*----------------------------------------------------------------------------------
*)
(*linear system, northern wing*)
```

$$Mn = \left\{\left\{\frac{betaN * R1 - 1 + Exp[-betaN * R1]}{R1}, \frac{deltaN * R1 + 1}{R1}, \frac{-Exp[deltaN * R1]}{R1}, 0\right\}, \left\{0, -\frac{deltaN * R2 + 1}{R2}, \frac{Exp[deltaN * R2]}{R2}, -\frac{1}{R2}\right\}, \left\{\frac{1 - (betaN * R1 + 1) * Exp[-betaN * R1]}{R1^2}, -\frac{1}{R1^2}, -\frac{1}{R1^2} Exp[deltaN * R1] (deltaN * R1 - 1), 0\right\}, \left\{0, \frac{1}{R2^2}, \frac{Exp[deltaN * R2] * (deltaN * R2 - 1)}{R2^2}, \frac{1}{R2^2}\right\}\right\};$$

```
c = {{0}, {E0 * R2}, {0}, {E0}};
MatrixForm[Mn];
{{K1n}, {K3n}, {K4n}, {K5n}} = Inverse[Mn].c;
(*linear system, southern wing*)
```

$$Ms = \left\{\left\{\frac{betaS * R1 - 1 + Exp[-betaS * R1]}{R1}, \frac{deltaS * R1 + 1}{R1}, \frac{-Exp[deltaS * R1]}{R1}, 0\right\}, \left\{0, -\frac{deltaS * R2 + 1}{R2}, \frac{Exp[deltaS * R2]}{R2}, -\frac{1}{R2}\right\}, \left\{\frac{1 - (betaS * R1 + 1) * Exp[-betaS * R1]}{R1^2}, -\frac{1}{R1^2}, -\frac{1}{R1^2} Exp[deltaS * R1] (deltaS * R1 - 1), 0\right\}, \left\{0, \frac{1}{R2^2}, \frac{Exp[deltaS * R2] * (deltaS * R2 - 1)}{R2^2}, \frac{1}{R2^2}\right\}\right\};$$

```
(*same c vector as for northern wing*)
MatrixForm[Ms];
{{K1s}, {K3s}, {K4s}, {K5s}} = Inverse[Ms].c;
```

... Inverse: Result for Inverse of badly conditioned matrix {{4.02398×10^{-6}, 0.0000309993, -4.46292×10^{11}, 0.}, {0., −≪20≫, ≪23≫, -5.87199×10^{-7}}, {≪1≫}, {0., 3.44803×10^{-13}, 4.18514×10^{11}, 3.44803×10^{-13}}} may contain significant numerical errors.

... Inverse: Result for Inverse of badly conditioned matrix {{4.02823×10^{-6}, 0.0000322907, -2.56121×10^{12}, 0.}, {0., −≪24≫, ≪22≫, -5.87199×10^{-7}}, {≪1≫}, {0., 3.44803×10^{-13}, 3.93852×10^{12}, 3.44803×10^{-13}}} may contain significant numerical errors.

```
(*Potentials, their derivatives, magnetic field, and bulk velocity*)
```

$$sinphi[x\_, y\_] = \frac{y}{\sqrt{x^2 + y^2}};$$

$$cosphi[x\_, y\_] = \frac{x}{\sqrt{x^2 + y^2}};$$

```
(*potential psi, northern wing*)
```

$$psi1n[x\_, y\_] = sinphi[x, y] * \frac{1}{r[x, y]} (K1n * (betaN * r[x, y] - 1) + K1n * Exp[-betaN * r[x, y]]);$$

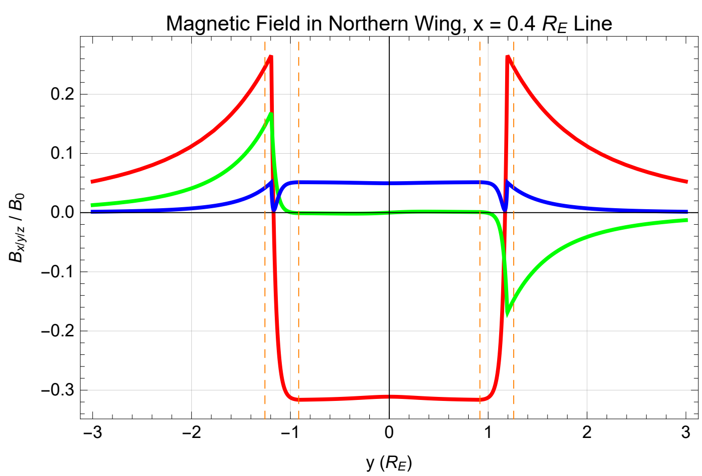
```
psi2n[x_, y_] = sinphi[x, y] *
```

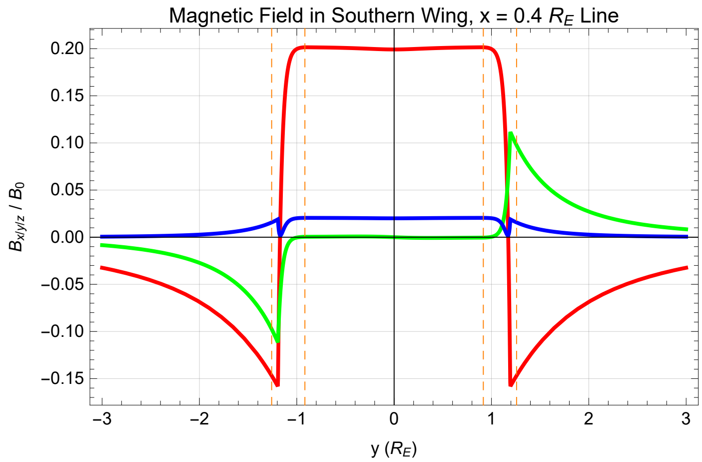
```
    1/r[x, y] (-K3n * (deltaN * r[x, y] + 1) + K4n * Exp[deltaN * r[x, y]]);
psi3n[x_, y_] = sinphi[x, y] * (E0 * r[x, y] + K5n/r[x, y]);
psin[x_, y_] = Piecewise[{{0, r[x, y] == 0}, {psi1n[x, y], 0 < r[x, y]  ≤ R1},
    {psi2n[x, y],  R1 < r[x, y]  ≤ R2}, {psi3n[x, y], R2 < r[x, y] }}];
(*potential psi, southern wing*)
psi1s[x_, y_] =
  sinphi[x, y] * 1/r[x, y] (K1s * (betaS * r[x, y] - 1) + K1s * Exp[-betaS * r[x, y]]);
psi2s[x_, y_] = sinphi[x, y] *
    1/r[x, y] (-K3s * (deltaS * r[x, y] + 1) + K4s * Exp[deltaS * r[x, y]]);
psi3s[x_, y_] = sinphi[x, y] * (E0 * r[x, y] + K5s/r[x, y]);
psis[x_, y_] = Piecewise[{{0, r[x, y] == 0}, {psi1s[x, y], 0 < r[x, y]  ≤ R1},
    {psi2s[x, y],  R1 < r[x, y]  ≤ R2}, {psi3s[x, y], R2 < r[x, y] }}];
(*derivatives of psi, magnetic field, and flow pattern u within the Alfven wings*)
(*derivatives, northern wing*)
dxpsi1n[x_, y_] = cosphi[x, y] * sinphi[x, y] * K1n *
    ((1 - Exp[-betaN * r[x, y]] * (betaN * r[x, y] + 1)) / (r[x, y] * r[x, y])) - sinphi[x, y]/r[x, y] *
    cosphi[x, y] * 1/r[x, y] (K1n * (betaN * r[x, y] - 1) + K1n * Exp[-betaN * r[x, y]]);
dxpsi2n[x_, y_] = cosphi[x, y] * sinphi[x, y] *
    ((K3n + K4n * Exp[deltaN * r[x, y]] * (deltaN * r[x, y] - 1)) / (r[x, y] * r[x, y])) -
    sinphi[x, y]/r[x, y] * cosphi[x, y] *
    1/r[x, y] (-K3n * (deltaN * r[x, y] + 1) + K4n * Exp[deltaN * r[x, y]]);
dxpsi3n[x_, y_] = cosphi[x, y] * sinphi[x, y] * (E0 - K5n/(r[x, y] * r[x, y])) -
    sinphi[x, y]/r[x, y] * cosphi[x, y] * (E0 * r[x, y] + K5n/r[x, y]);
dypsi1n[x_, y_] = sinphi[x, y] * sinphi[x, y] * K1n *
    ((1 - Exp[-betaN * r[x, y]] * (betaN * r[x, y] + 1)) / (r[x, y] * r[x, y])) + cosphi[x, y]/r[x, y] *
    cosphi[x, y] * 1/r[x, y] (K1n * (betaN * r[x, y] - 1) + K1n * Exp[-betaN * r[x, y]]);
dypsi2n[x_, y_] = sinphi[x, y] * sinphi[x, y] *
    ((K3n + K4n * Exp[deltaN * r[x, y]] * (deltaN * r[x, y] - 1)) / (r[x, y] * r[x, y])) +
    cosphi[x, y]/r[x, y] * cosphi[x, y] *
    1/r[x, y] (-K3n * (deltaN * r[x, y] + 1) + K4n * Exp[deltaN * r[x, y]]);
dypsi3n[x_, y_] = sinphi[x, y] * sinphi[x, y] * (E0 - K5n/(r[x, y] * r[x, y])) +
```

```
    cosphi[x, y]                  (            K5n   )
    ------------ * cosphi[x, y] * (E0 * r[x, y] + -------);
      r[x, y]                     (             r[x, y] )
dxpsin[x_, y_] = Piecewise[{{0, r[x, y] == 0}, {dxpsi1n[x, y], 0 < r[x, y] ≤ R1},
    {dxpsi2n[x, y], R1 < r[x, y] ≤ R2}, {dxpsi3n[x, y], R2 < r[x, y] }}];
                                    betaN^2
dypsin[x_, y_] = Piecewise[{{K1n * -------, r[x, y] == 0}, {dypsi1n[x, y], 0 < r[x, y] ≤ R1},
                                       2
    {dypsi2n[x, y], R1 < r[x, y] ≤ R2}, {dypsi3n[x, y], R2 < r[x, y] }}];
(*derivaties, southern wing*)
dxpsi1s[x_, y_] = cosphi[x, y] * sinphi[x, y] * K1s *
    ((1 - Exp[-betaS * r[x, y]] * (betaS * r[x, y] + 1)) / (r[x, y] * r[x, y])) - sinphi[x, y] *
                                                                                   -----------
                                                                                     r[x, y]
                      1
    cosphi[x, y] * -------- (K1s * (betaS * r[x, y] - 1) + K1s * Exp[-betaS * r[x, y]]);
                    r[x, y]
dxpsi2s[x_, y_] = cosphi[x, y] * sinphi[x, y] *
    ((K3s + K4s * Exp[deltaS * r[x, y]] * (deltaS * r[x, y] - 1)) / (r[x, y] * r[x, y])) -
    sinphi[x, y]
    ------------ * cosphi[x, y] *
      r[x, y]
       1
    ------- (-K3s * (deltaS * r[x, y] + 1) + K4s * Exp[deltaS * r[x, y]]);
    r[x, y]
                                                        (          K5s        )
dxpsi3s[x_, y_] = cosphi[x, y] * sinphi[x, y] *         (E0 - --------------- ) -
                                                        (     r[x, y] * r[x, y])
    sinphi[x, y]                  (                K5s   )
    ------------ * cosphi[x, y] * (E0 * r[x, y] + -------);
      r[x, y]                     (               r[x, y])
dypsi1s[x_, y_] = sinphi[x, y] * sinphi[x, y] * K1s *
    ((1 - Exp[-betaS * r[x, y]] * (betaS * r[x, y] + 1)) / (r[x, y] * r[x, y])) + cosphi[x, y] *
                                                                                   ------------
                                                                                     r[x, y]
                      1
    cosphi[x, y] * -------- (K1s * (betaS * r[x, y] - 1) + K1s * Exp[-betaS * r[x, y]]);
                    r[x, y]
dypsi2s[x_, y_] = sinphi[x, y] * sinphi[x, y] *
    ((K3s + K4s * Exp[deltaS * r[x, y]] * (deltaS * r[x, y] - 1)) / (r[x, y] * r[x, y])) +
    cosphi[x, y]
    ------------ * cosphi[x, y] *
      r[x, y]
       1
    ------- (-K3s * (deltaS * r[x, y] + 1) + K4s * Exp[deltaS * r[x, y]]);
    r[x, y]
                                                        (          K5s        )
dypsi3s[x_, y_] = sinphi[x, y] * sinphi[x, y] *         (E0 - --------------- ) +
                                                        (     r[x, y] * r[x, y])
    cosphi[x, y]                  (                K5s   )
    ------------ * cosphi[x, y] * (E0 * r[x, y] + -------);
      r[x, y]                     (               r[x, y])
dxpsis[x_, y_] = Piecewise[{{0, r[x, y] == 0}, {dxpsi1s[x, y], 0 < r[x, y] ≤ R1},
    {dxpsi2s[x, y], R1 < r[x, y] ≤ R2}, {dxpsi3s[x, y], R2 < r[x, y] }}];
                                    betaS^2
dypsis[x_, y_] = Piecewise[{{K1s * -------, r[x, y] == 0}, {dypsi1s[x, y], 0 < r[x, y] ≤ R1},
                                       2
    {dypsi2s[x, y], R1 < r[x, y] ≤ R2}, {dypsi3s[x, y], R2 < r[x, y] }}];
(*magnetic field, northern wing*)
Bxnorth[x_, y_] = -SinThetaN *
    √(B0^2 - mu0^2 * SigmaAn^2 * ((CosThetaN)^2 (dxpsin[x, y])^2 + (dypsin[x, y])^2)) +
   mu0 * SigmaAn * CosThetaN * dypsin[x, y] ;
```

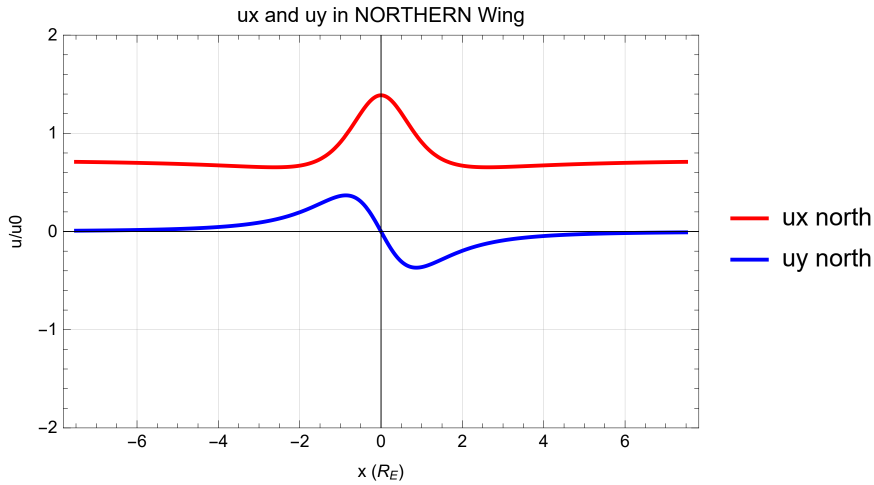
```
Bynorth[x_, y_] = -CosThetaN * mu0 * SigmaAn * dxpsin[x, y];
Bznorth[x_, y_] = -CosThetaN *
    √(B0^2 - mu0^2 * SigmaAn^2 * ((CosThetaN)^2 (dxpsin[x, y])^2 + (dypsin[x, y])^2)) -
   mu0 * SigmaAn * SinThetaN * dypsin[x, y] ;
(*magnetic field, southern wing*)
Bxsouth[x_, y_] = SinThetaS *
    √(B0^2 - mu0^2 * SigmaAs^2 * ((CosThetaS)^2 (dxpsis[x, y])^2 + (dypsis[x, y])^2)) -
   mu0 * SigmaAs * CosThetaS * dypsis[x, y] ;
Bysouth[x_, y_] = CosThetaS * mu0 * SigmaAs * dxpsis[x, y];
Bzsouth[x_, y_] = -CosThetaS *
    √(B0^2 - mu0^2 * SigmaAs^2 * ((CosThetaS)^2 (dxpsis[x, y])^2 + (dypsis[x, y])^2)) -
   SinThetaS * mu0 * SigmaAs * dypsis[x, y] ;
(*velocity field, northern wing*)
uxnorth[x_, y_] = u0x + (Bxnorth[x, y] - B0x)/√(mu0 * n0 * m);
uynorth[x_, y_]  = u0y + (Bynorth[x, y] - B0y)/√(mu0 * n0 * m);
uznorth[x_, y_] = u0z + (Bznorth[x, y] - B0z)/√(mu0 * n0 * m);
(*velocity field, southern wing*)
uxsouth[x_, y_] = u0x - (Bxsouth[x, y] - B0x)/√(mu0 * n0 * m);
uysouth[x_, y_]  = u0y - (Bysouth[x, y] - B0y)/√(mu0 * n0 * m);
uzsouth[x_, y_] = u0z - (Bzsouth[x, y] - B0z)/√(mu0 * n0 * m);
```

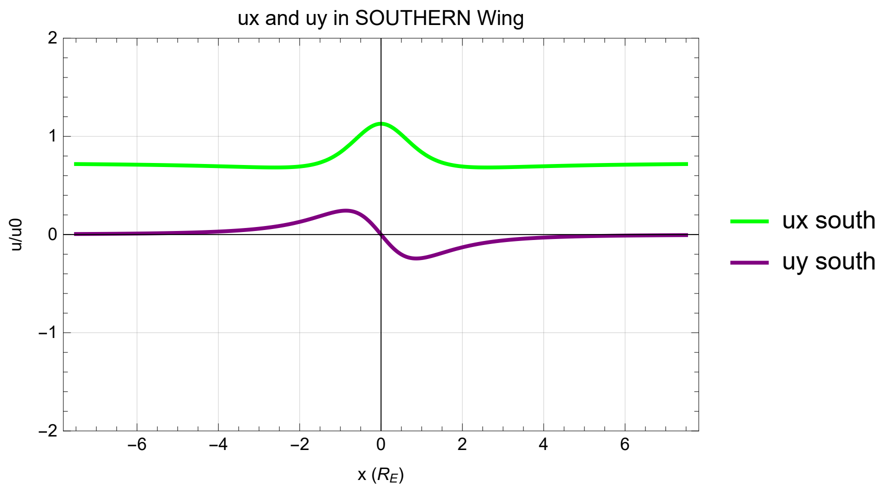
```
(*--------------------------------------------------------------------------------------*)
(*a few plots from Simon2021, to make sure that this all works*)
(*--------------------------------------------------------------------------------------*)
(*magnetic field, see figure 6 from Simon2021 northern and southern wing
 should be somewhat different now*)
Plotmagnorth = Plot[{Bxnorth[0.4 RE, y * RE] / (B0),
    Bynorth[0.4 RE, y * RE] / (B0), Bznorth[0.4 RE, y * RE] / (B0) + B0 / B0},
   {y, -3, 3}, FrameLabel → {"y (R_E)", "B_x/y/z / B_0"}, GridLines → Automatic,
   Frame → True, PlotLabel → "Magnetic Field in Northern Wing, x = 0.4 R_E Line",
   PlotStyle → {{Red, Thick}, {Green, Thick}, {Blue, Thick}},
   Epilog → {Directive[Dashed, Orange], InfiniteLine[{{-0.9165, -0.3}, {-0.9165, 0.3}}],
     InfiniteLine[{{0.9165, -0.3}, {0.9165, 0.3}}],
     InfiniteLine[{{1.258, -0.3}, {1.258, 0.3}}], InfiniteLine[
      {{-1.258, -0.3}, {-1.258, 0.3}}]} (*,PlotLegends ->{"B_x","B_y","B_z"}*)]
Plotmagsouth = Plot[{Bxsouth[0.4 RE, y * RE] / (B0), Bysouth[0.4 RE, y * RE] / (B0),
    Bzsouth[0.4 RE, y * RE] / (B0) + B0 / B0}, {y, -3, 3},
   FrameLabel → {"y (R_E)", "B_x/y/z / B_0"}, GridLines → Automatic, Frame → True,
   PlotLabel → "Magnetic Field in Southern Wing, x = 0.4 R_E Line",
   PlotStyle → {{Red, Thick}, {Green, Thick}, {Blue, Thick}},
   Epilog → {Directive[Dashed, Orange], InfiniteLine[{{-0.9165, -0.3}, {-0.9165, 0.3}}],
     InfiniteLine[{{0.9165, -0.3}, {0.9165, 0.3}}],
     InfiniteLine[{{1.258, -0.3}, {1.258, 0.3}}],
     InfiniteLine[{{-1.258, -0.3}, {-1.258, 0.3}}]}(*,PlotLegends ->{"B_x","B_y","B_z"}*)]
```

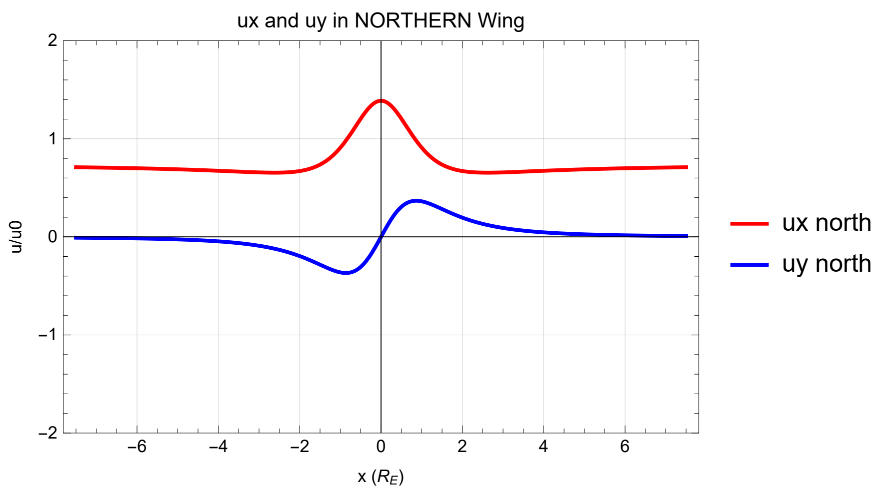


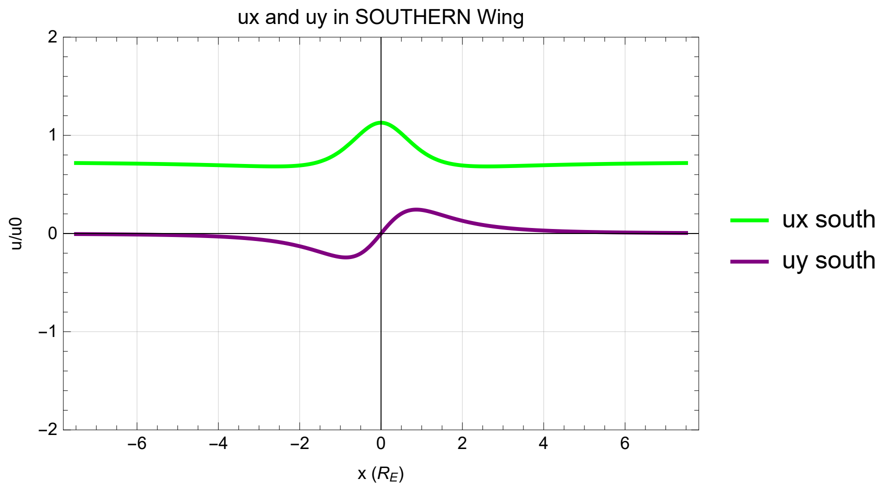



```
(*velocity field, figure 2 of Simon2021*)
uxnorthshiftpos[x_] = uxnorth[x, 1.5 RE] / u0;
(*cut at y=1.5 RE, i.e., displaced TOWARD Jupiter*)
uynorthshiftpos[x_] = uynorth[x, 1.5 RE] / u0;
uxsouthshiftpos[x_] = uxsouth[x, 1.5 RE] / u0;
uysouthshiftpos[x_] = uysouth[x, 1.5 RE] / u0;
Plot[{uxnorthshiftpos[x * RE], uynorthshiftpos[x * RE]}, {x, -7.5, 7.5},
 FrameLabel → {"x (RE)", "u/u0"}, GridLines → Automatic, Frame → True,
 PlotLabel → "ux and uy in NORTHERN Wing", PlotStyle → {{Red, Thick}, {Blue, Thick}},
 PlotLegends -> {"ux north", "uy north"}, PlotRange -> {-2, 2}]
Plot[{uxsouthshiftpos[x * RE], uysouthshiftpos[x * RE]}, {x, -7.5, 7.5},
 FrameLabel → {"x (RE)", "u/u0"}, GridLines → Automatic, Frame → True,
 PlotLabel → "ux and uy in SOUTHERN Wing", PlotStyle → {{Green, Thick}, {Purple, Thick}},
 PlotLegends -> {"ux south", "uy south"}, PlotRange -> {-2, 2}]
uxnorthshiftneg[x_] = uxnorth[x, -1.5 RE] / u0; (*cut at y=-1.5 RE,
i.e., displaced AWAY FROM Jupiter*)
uynorthshiftneg[x_] = uynorth[x, -1.5 RE] / u0;
uxsouthshiftneg[x_] = uxsouth[x, -1.5 RE] / u0;
uysouthshiftneg[x_] = uysouth[x, -1.5 RE] / u0;
Plot[{uxnorthshiftneg[x * RE], uynorthshiftneg[x * RE]}, {x, -7.5, 7.5},
 FrameLabel → {"x (RE)", "u/u0"}, GridLines → Automatic, Frame → True,
 PlotLabel → "ux and uy in NORTHERN Wing", PlotStyle → {{Red, Thick}, {Blue, Thick}},
 PlotLegends -> {"ux north", "uy north"}, PlotRange -> {-2, 2}]
Plot[{uxsouthshiftneg[x * RE], uysouthshiftneg[x * RE]}, {x, -7.5, 7.5},
 FrameLabel → {"x (RE)", "u/u0"}, GridLines → Automatic, Frame → True,
 PlotLabel → "ux and uy in SOUTHERN Wing", PlotStyle → {{Green, Thick}, {Purple, Thick}},
 PlotLegends -> {"ux south", "uy south"}, PlotRange -> {-2, 2}]
```


ux and uy in NORTHERN Wing
u/u0
x (R_E)
ux north
uy north

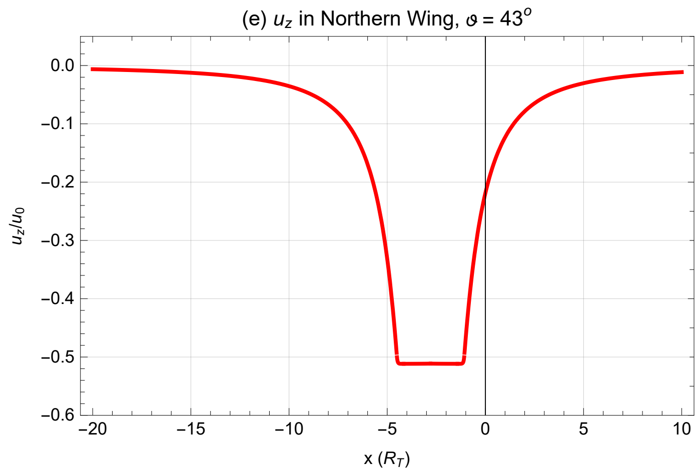

ux and uy in SOUTHERN Wing
u/u0
x (R_E)
ux south
uy south

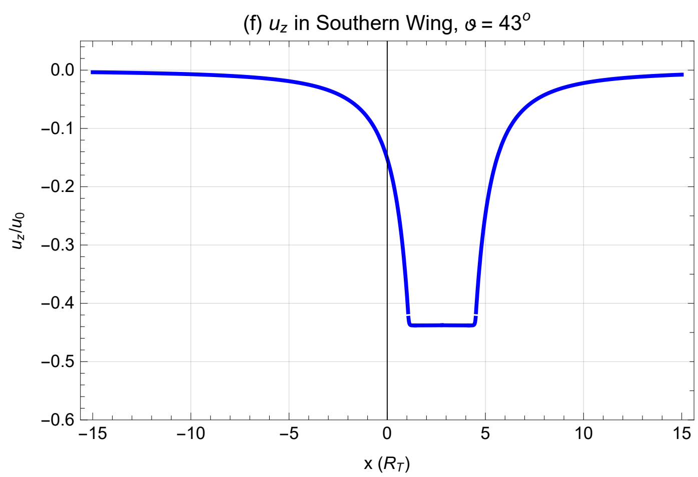

ux and uy in NORTHERN Wing
u/u0
x (R_E)
ux north
uy north


ux and uy in SOUTHERN Wing
u/u0
x (R_E)
2
1
0
−1
−2
−6
−4
−2
0
2
4
6
ux south
uy south

```
(*-----------------------------------------------------------------------------------
                                          ---*)
(*so far, everything is given in coordinate system (x,y,z) where z is
 antiparallel to the background field. Now we need to transform into
 system (xcor,ycor,zcor) where xcor is along the coritation direction*)
(*-----------------------------------------------------------------------------------
                                          ---*)
(*position vectors*)
x[xcor_, ycor_, zcor_] = xcor * Cos[theta] + zcor * Sin[theta];
y[xcor_, ycor_, zcor_] = ycor;
z[xcor_, ycor_, zcor_] = -xcor * Sin[theta] + zcor * Cos[theta];
(*z is actually not even needed,
since in the (x,y,z) system the solution is constant in z direction,
i.e., there are no z dependencies in any quantity*)
(*magnetic field components in northern wing*)
Bxnorthcor[xcor_, ycor_, zcor_] =
  Bxnorth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]] * Cos[theta] -
   Bznorth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]] * Sin[theta];
Bynorthcor[xcor_, ycor_, zcor_] = Bynorth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]];
Bznorthcor[xcor_, ycor_, zcor_] =
  Bxnorth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]] * Sin[theta] +
   Bznorth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]] * Cos[theta];
(*magnetic field components in southern wing*)
Bxsouthcor[xcor_, ycor_, zcor_] =
  Bxsouth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]] * Cos[theta] -
   Bzsouth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]] * Sin[theta];
Bysouthcor[xcor_, ycor_, zcor_] = Bysouth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]];
Bzsouthcor[xcor_, ycor_, zcor_] =
  Bxsouth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]] * Sin[theta] +
   Bzsouth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]] * Cos[theta];
(*bulk velocity in northern wing*)
uxnorthcor[xcor_, ycor_, zcor_] =
  uxnorth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]] * Cos[theta] -
   uznorth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]] * Sin[theta];
uynorthcor[xcor_, ycor_, zcor_] = uynorth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]];
uznorthcor[xcor_, ycor_, zcor_] =
  uxnorth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]] * Sin[theta] +
   uznorth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]] * Cos[theta];
(*bulk velocity in southern wing*)
uxsouthcor[xcor_, ycor_, zcor_] =
  uxsouth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]] * Cos[theta] -
   uzsouth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]] * Sin[theta];
uysouthcor[xcor_, ycor_, zcor_] = uysouth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]];
uzsouthcor[xcor_, ycor_, zcor_] =
  uxsouth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]] * Sin[theta] +
   uzsouth[x[xcor, ycor, zcor], y[xcor, ycor, zcor]] * Cos[theta];
```

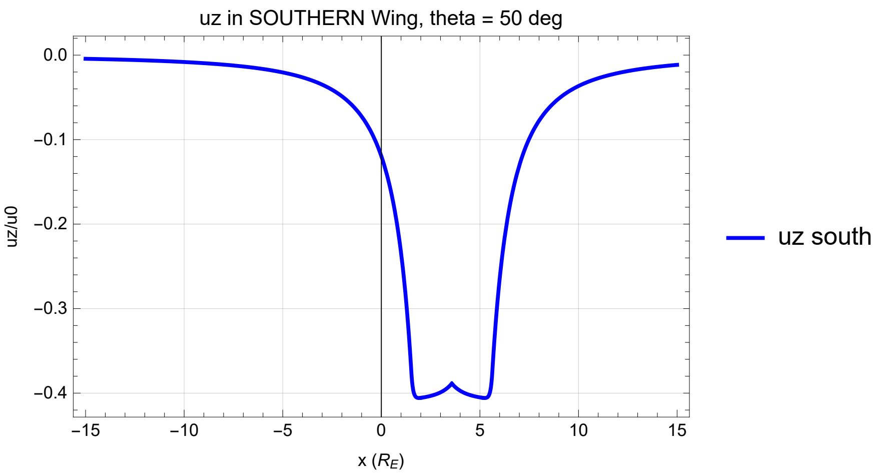
```
(*-----------------------------------------------------------------------------*)
(*now: plotting the results in system (xcor,ycor, zcor)*)
(*-----------------------------------------------------------------------------*)
Plot1 = Plot[uznorthcor[xcor * RE, 0, 3 * RE] / u0, {xcor, -20, 10},
  FrameLabel → {"x (R_T)", "u_z/u_0"}, GridLines → Automatic, Frame → True,
  PlotLabel → "(e) u_z in Northern Wing, ϑ = 43°", PlotStyle → {{Red, Thick}}
  (*,PlotLegends ->{"uz north"}*), PlotRange → {-0.6, 0.05}]
Plot2 = Plot[uzsouthcor[xcor * RE, 0, -3 * RE] / u0, {xcor, -15, 15},
  FrameLabel → {"x (R_T)", "u_z/u_0"}, GridLines → Automatic, Frame → True,
  PlotLabel → "(f) u_z in Southern Wing, ϑ = 43°", PlotStyle → {{Blue, Thick}},
  PlotRange → {-0.6, 0.05} (*,PlotLegends ->{"uz south"}*)]
Export["uz_north43.pdf", Plot1]
Export["uz_south43.pdf", Plot2]
```





uz_north43.pdf

uz_south43.pdf


uz in SOUTHERN Wing, theta = 50 deg
uz/u0
0.0
−0.1
−0.2
−0.3
−0.4
−15
−10
−5
0
5
10
15
x (R_E)
uz south