```
(* Flow deflection around Europa's Alfven wings, start: 11/21/20 *)

(*--------------------------------------------*)
(*Basic Input Parameters*)
(*--------------------------------------------*)
```

In[ ]:=

```
RE = 1560.8 * 10^3;
B0 = 450 * 10^-9; (*background magnetic field,
southward, center of plasma sheet, from Arnold2020a*)
n0 = 60 * 10^6; (*upstream number density, from Arnold2020, table 1*)
u0 = 100 * 10^3; (*upstream flow velocity 100 km/s, from Arnold2020b *)
E0 = u0 * B0; (*MAGNITUDE OF convective electric field*)
mp = 1.6726231 * 10^-27; (*proton mass*)
m = 18.5 * mp; (*upstream ion mass, from Arnold2020b*)
SigmaP = 30 ;(*exospheric Pedersen conductance,
from table 21.1, page 517, Kivelson2004*)
H = 100 * 10^3; (*exospheric scale height, from Arnold2020b*)
mu0 = 4 * 3.14159265359 * 10^-7 ;(*magnetic permeability of vacuum*)
```

In[ ]:=

```
(*--------------------------------------------*)
(*Derived parameters for Alfven wing*)
(*--------------------------------------------*)
```

$$vA = \frac{B0}{\sqrt{mu0 * n0 * m}}; \text{(*Alfven velocity*)}$$

```
MA = u0 / vA; (*Alfvenic Mach number*)
```

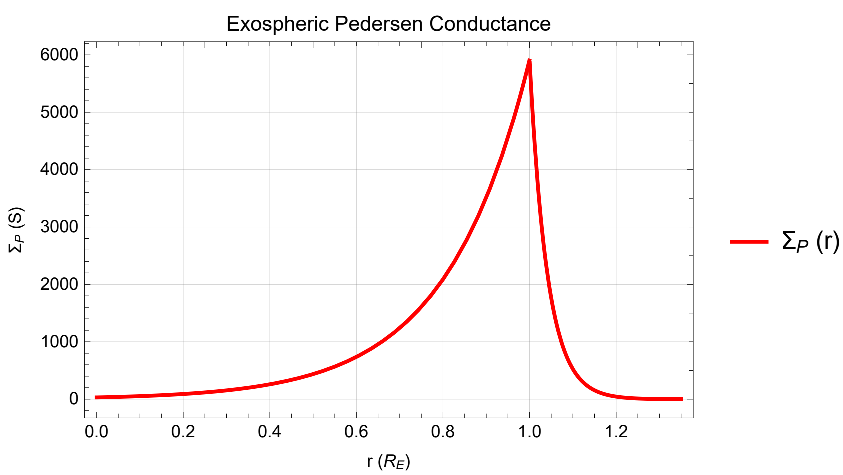
$$SigmaA = \frac{1}{mu0 * vA * \sqrt{1 + MA^2}}; \text{(*Alfven conductance, Equation (5) in Simon2011*)}$$

```
(*--------------------------------------------*)
(*Exospheric conductance profile*)
(*--------------------------------------------*)
```

In[∘]:=
```
r[x_, y_] = Sqrt[x^2 + y^2]; (*radial distance from z axis in cylindrical coordinates,
NOT radial distance to Europa's surface,
(x,y) dependency NOT needed for merely plotting the exospheric conductance profile*)
R1 = RE ;(*radial boundary of the Europa fluxtube, x^2+y^2 = RE^2*)
R2 =  RE + 5 H;(*outer boundary of region with non-zero transverse conductance,
assumimng the atmosphere "ends" at a distance of 5 scale heights. exp(-5)=0.00674*)
beta = 1/(3 H) ;(*given input parameter for conductance profile,
using some multiple of the exospheric scale height H here*)
Sigma1[r_] = (SigmaP + SigmaA) * Exp[beta * r] - SigmaA;
(*Pedersen conductance within the Europa fluxtube r < R1*)
Sigma2[r_] = SigmaA * Exp[(Log[(SigmaP+SigmaA)/SigmaA] + beta * R1)/(R2 - R1) * (R2 - r)] - SigmaA;
(*Pedersen conductance outside the Europa fluxtube,
but within the exosphere R1 < r < R2*)
Sigma3[r_] = 0 ; (*Pedersen conductance outside of Europa fluxtube, R2 < r*)
SigmaExo[r_] =
  Piecewise[{{Sigma1[r], r ≤ R1}, {Sigma2[r], R1 < r ≤ R2}, {Sigma3[r], R2 < r}}];
(*putting all three segments together*)

(*Plot exopspheric conductance profile*)
Plot[SigmaExo[r * RE], {r, 0, 1.35}, FrameLabel → {"r (R_E)", "Σ_P (S)"},
 GridLines → Automatic, Frame → True, PlotLabel → "Exospheric Pedersen Conductance",
 PlotStyle → {Red, Thick}, PlotLegends -> {"Σ_P (r)"}]
```

Out[∘]=



In[∘]:=
```
(*-----------------------------------------------*)
(*Solution for the electric potential psi*)
(*-----------------------------------------------*)
delta = (Log[(SigmaP+SigmaA)/SigmaA] + beta * R1)/(R2 - R1);
(*Exponent of the conductance profile SigmaP= gamma exp(MINUS delta r) for R1 < r < R2*)
```

In[ ]:=
```
(*Calculting the constants of integration, same nomenclature as in hand-written notes*)
M = {{(beta * R1 - 1 + Exp[-beta * R1])/R1, (delta * R1 + 1)/R1, (-Exp[delta * R1])/R1, 0},
   {0, -(delta * R2 + 1)/R2, Exp[delta * R2]/R2, -1/R2},
   {(1 - (beta * R1 + 1) * Exp[-beta * R1])/R1^2, -1/R1^2, -1/R1^2 Exp[delta * R1] (delta * R1 - 1) , 0},
   {0, 1/R2^2, (Exp[delta * R2]  * (delta * R2 - 1))/R2^2, 1/R2^2}};
c = {{0}, {E0 * R2}, {0}, {E0}};
MatrixForm[M];
{{K1}, {K3}, {K4}, {K5}} = Inverse[M].c;
(*Sin phi and Cos phi*)
sinphi[x_, y_] = y/Sqrt[x^2 + y^2];
cosphi[x_, y_] = x/Sqrt[x^2 + y^2];
(*Solution for the electric potential psi, angle term from equation 19 in Simon2015*)
```

Inverse: Result for Inverse of badly conditioned matrix {{2.69616 × 10^-6, 0.0000161338, −20351.2, 0.}, {0., −0.0000159783, 3.566 × 10^7, −4.85248 × 10^-7}, {3.96484 × 10^-13, −4.10493 × 10^-13, −0.302264, 0.}, {0., 2.35466 × 10^-13, 535.18, 2.35466 × 10^-13}} may contain significant numerical errors.

```
M.Inverse[M]; // MatrixForm ;(*test: M. Inverse[M]= unit matrix*)
```

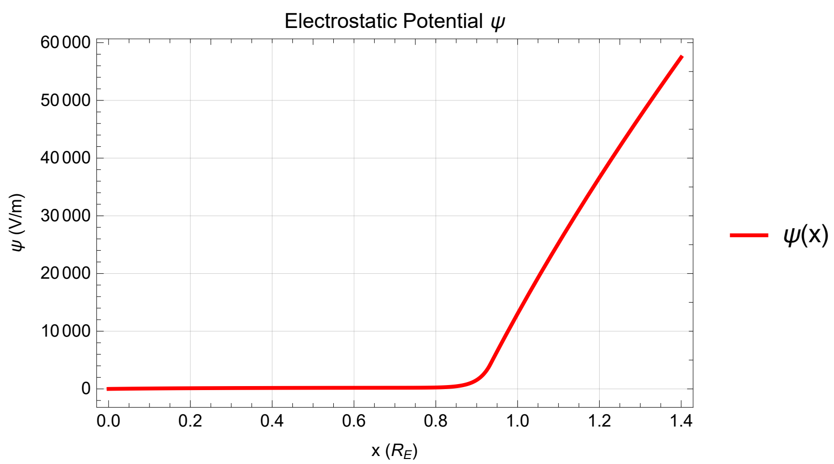
Inverse: Result for Inverse of badly conditioned matrix {{2.69616 × 10^-6, 0.0000161338, −20351.2, 0.}, {0., −0.0000159783, 3.566 × 10^7, −4.85248 × 10^-7}, {3.96484 × 10^-13, −4.10493 × 10^-13, −0.302264, 0.}, {0., 2.35466 × 10^-13, 535.18, 2.35466 × 10^-13}} may contain significant numerical errors.

In[ ]:=
```
psi1[x_, y_] = sinphi[x, y] * (K1 * (beta * r[x, y] - 1) + K1 * Exp[-beta * r[x, y]])/r[x, y];
psi2[x_, y_] = sinphi[x, y] * (-K3 * (delta * r[x, y] + 1) + K4 * Exp[delta * r[x, y]])/r[x, y];
psi3[x_, y_] = sinphi[x, y] * (E0 * r[x, y] + K5/r[x, y]);
psi[x_, y_] = Piecewise[{{0, r[x, y] == 0}, {psi1[x, y], 0 < r[x, y]  ≤ R1},
    {psi2[x, y],  R1 < r[x, y]  ≤ R2}, {psi3[x, y], R2 < r[x, y] }}];
```

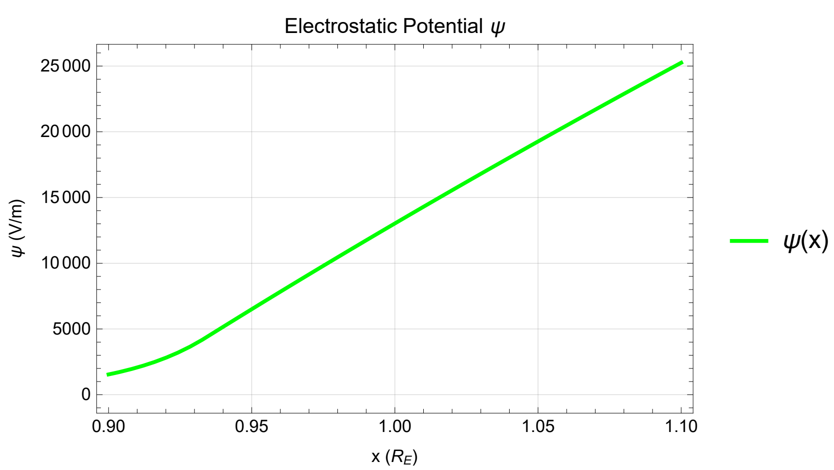
In[ ]:=
```
(*Cut through potential along diagonal axis (x,x) , diagnostic plots*)
psialongdiag[x_] = psi[x, x];
```

In[•]:=
```
Plot[psialongdiag[x * RE], {x, 0, 1.4}, FrameLabel → {"x (R_E)", "ψ (V/m)"},
  GridLines → Automatic, Frame → True, PlotLabel → "Electrostatic Potential ψ",
  PlotStyle → {Red, Thick}, PlotLegends -> {"ψ(x)"}]
Plot[psialongdiag[x * RE], {x, 0.9, 1.1}, FrameLabel → {"x (R_E)", "ψ (V/m)"},
  GridLines → Automatic, Frame → True, PlotLabel → "Electrostatic Potential ψ",
  PlotStyle → {Green, Thick}, PlotLegends -> {"ψ(x)"}]
(*visualizes first "kink" at R1=1  RE*)
Plot[psialongdiag[x * RE], {x, 1.2, 1.4}, FrameLabel → {"x (R_E)", "ψ (V/m)"},
  GridLines → Automatic, Frame → True, PlotLabel → "Electrostatic Potential ψ",
  PlotStyle → {Blue, Thick}, PlotLegends -> {"ψ(x)"}]
(*visualizes second "kink", for these tests located at R2=1.3205  RE = RE+5H*)
```

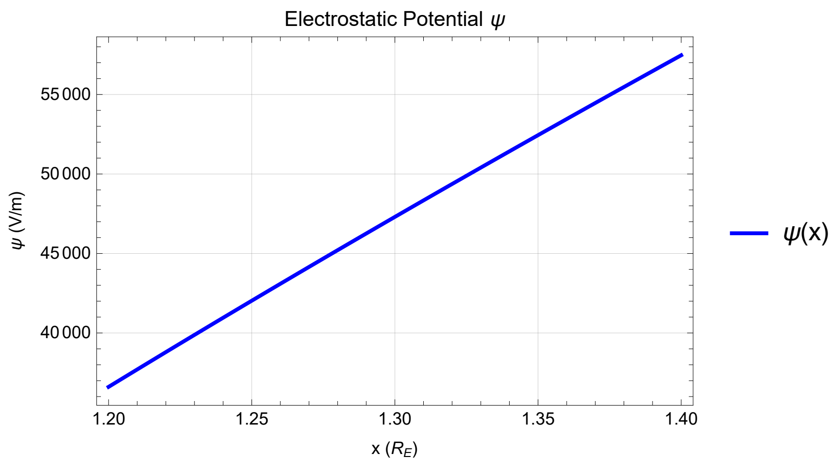
Out[•]=



Out[•]=



Out[•]=

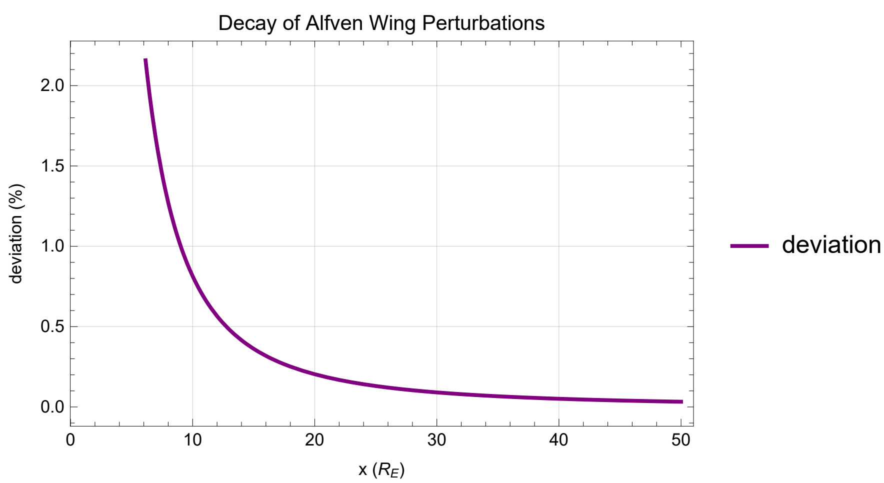



```
(*Now: verify that for large r= sqrt(2) x,
psialongdiag becomes indeed E0 r sin phi = E0 sqrt(2) x * x/sqrt{2}x + E0 x*)
```

In[•]:=

```
deviation[x_] = Abs[(E0 * x - psialongdiag[x])/(E0 * x)] * 100; (*deviation in percent*)
Plot[deviation[x * RE], {x, 1, 50}, FrameLabel → {"x (R_E)", "deviation (%)"},
 GridLines → Automatic, Frame → True, PlotLabel → "Decay of Alfven Wing Perturbations",
 PlotStyle → {Purple, Thick}, PlotLegends -> {"deviation"}]
```

Out[•]=




```
(*---------------------------------------------*)
(*Calculate flow field u near Alfven wings*)
(*---------------------------------------------*)
```

In[•]:=

```
(*first
 step: need derivatives of the potential psi. Must be implemented with great care,
since we encounter a 0/0 situation at r=
 0 in both cases that mathematica can not handle.*)
dxpsi1[x_, y_] =
  cosphi[x, y] * sinphi[x, y] * K1 * (1 - Exp[-beta * r[x, y]] * (beta * r[x, y] + 1))/(r[x, y] * r[x, y]) -
```

```
    sinphi[x, y] / r[x, y] * cosphi[x, y] * (K1 * (beta * r[x, y] - 1) + K1 * Exp[-beta * r[x, y]]) / r[x, y];
dxpsi2[x_, y_] = cosphi[x, y] * sinphi[x, y] *
    (K3 + K4 * Exp[delta * r[x, y]] * (delta * r[x, y] - 1)) / (r[x, y] * r[x, y]) -
    sinphi[x, y] / r[x, y] * cosphi[x, y] * (-K3 * (delta * r[x, y] + 1) + K4 * Exp[delta * r[x, y]]) / r[x, y];
dxpsi3[x_, y_] = cosphi[x, y] * sinphi[x, y] * (E0 - K5 / (r[x, y] * r[x, y])) -
    sinphi[x, y] / r[x, y] * cosphi[x, y] * (E0 * r[x, y] + K5 / r[x, y]);
dypsi1[x_, y_] = sinphi[x, y] * sinphi[x, y] * K1 *
    (1 - Exp[-beta * r[x, y]] * (beta * r[x, y] + 1)) / (r[x, y] * r[x, y]) +
    cosphi[x, y] / r[x, y] * cosphi[x, y] * (K1 * (beta * r[x, y] - 1) + K1 * Exp[-beta * r[x, y]]) / r[x, y];
dypsi2[x_, y_] = sinphi[x, y] * sinphi[x, y] *
    (K3 + K4 * Exp[delta * r[x, y]] * (delta * r[x, y] - 1)) / (r[x, y] * r[x, y]) +
    cosphi[x, y] / r[x, y] * cosphi[x, y] * (-K3 * (delta * r[x, y] + 1) + K4 * Exp[delta * r[x, y]]) / r[x, y];
dypsi3[x_, y_] = sinphi[x, y] * sinphi[x, y] * (E0 - K5 / (r[x, y] * r[x, y])) +
    cosphi[x, y] / r[x, y] * cosphi[x, y] * (E0 * r[x, y] + K5 / r[x, y]);
dxpsi[x_, y_] = Piecewise[{{0, r[x, y] == 0}, {dxpsi1[x, y], 0 < r[x, y] ≤ R1},
    {dxpsi2[x, y], R1 < r[x, y] ≤ R2}, {dxpsi3[x, y], R2 < r[x, y] }}];
dypsi[x_, y_] = Piecewise[{{K1 * beta^2 / 2, r[x, y] == 0}, {dypsi1[x, y], 0 < r[x, y] ≤ R1},
    {dypsi2[x, y], R1 < r[x, y] ≤ R2}, {dypsi3[x, y], R2 < r[x, y] }}];
(*magnetic field components near the Alfven wings,
including the background field AND the Alfvenic perturbations. Taken from the Anti-
 Hall paper Simon2011, equations (6)-(8)*)
(*northern wing*)
Bxnorth[x_, y_] = 1 / Sqrt[MA^2 + 1] *
   (-MA * Sqrt[B0^2 - mu0^2 * SigmaA^2 * (1 / (MA^2 + 1) (dxpsi[x, y])^2 + (dypsi[x, y])^2)] +
     mu0 * SigmaA * dypsi[x, y]);
Bynorth[x_, y_] = - 1 / Sqrt[MA^2 + 1] * mu0 * SigmaA * dxpsi[x, y];
Bznorth[x_, y_] =
```

```
1/Sqrt[MA^2 + 1] * (- Sqrt[B0^2 - mu0^2 * SigmaA^2 * (1/(MA^2 + 1) (dxpsi[x, y])^2 + (dypsi[x, y])^2)] -
    MA * mu0 * SigmaA * dypsi[x, y]);
(*southern wing*)
Bxsouth[x_, y_] = 1/Sqrt[MA^2 + 1] *
  (MA * Sqrt[B0^2 - mu0^2 * SigmaA^2 * (1/(MA^2 + 1) (dxpsi[x, y])^2 + (dypsi[x, y])^2)] -
    mu0 * SigmaA * dypsi[x, y]);
Bysouth[x_, y_] = 1/Sqrt[MA^2 + 1] * mu0 * SigmaA * dxpsi[x, y];
Bzsouth[x_, y_] =
  1/Sqrt[MA^2 + 1] * (- Sqrt[B0^2 - mu0^2 * SigmaA^2 * (1/(MA^2 + 1) (dxpsi[x, y])^2 + (dypsi[x, y])^2)] -
    MA * mu0 * SigmaA * dypsi[x, y]);
```

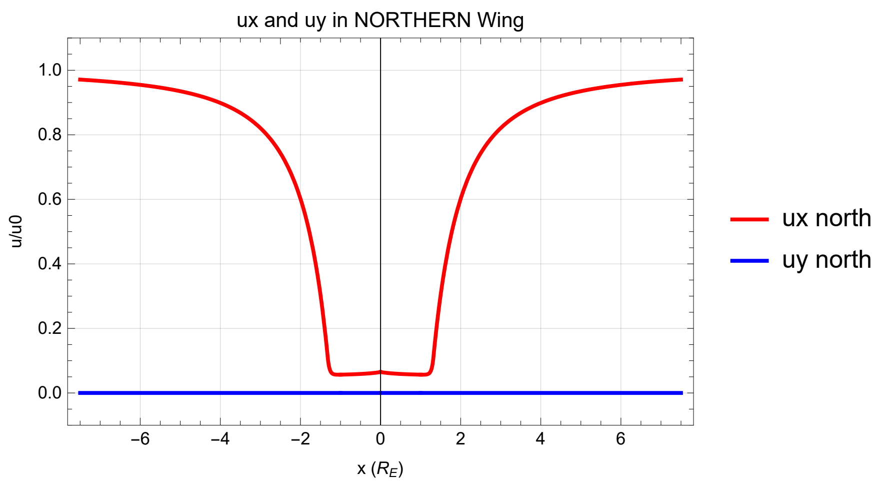
In[ ]:=
```
(*------------------------------------------*)
(*12/01/20: now move on to calculate the flow field u(x,y),
using invariance of the Alfven characteristics*)
(*------------------------------------------*)

(*northern wing*)
uxnorth[x_, y_] = u0 + Bxnorth[x, y]/Sqrt[mu0 * n0 * m];
uynorth[x_, y_] = + Bynorth[x, y]/Sqrt[mu0 * n0 * m];
uznorth[x_, y_] = + (Bznorth[x, y] + B0)/Sqrt[mu0 * n0 * m];
(*warning: the background field points in NEGATIVE z direction,
therefore it's a PLUS B0 in the expression for uz*)
(*southern wing*)
uxsouth[x_, y_] = u0 - Bxsouth[x, y]/Sqrt[mu0 * n0 * m];
uysouth[x_, y_] = - Bysouth[x, y]/Sqrt[mu0 * n0 * m];
uzsouth[x_, y_] = - (Bzsouth[x, y] + B0)/Sqrt[mu0 * n0 * m];
```

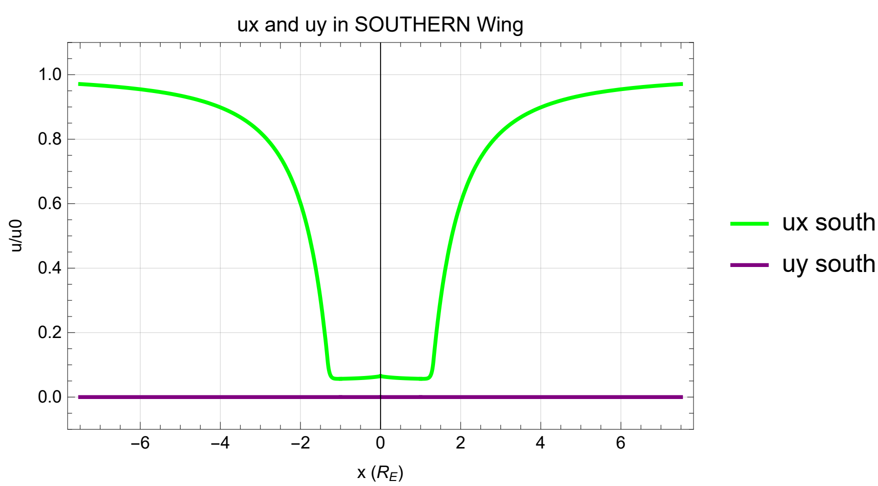
In[∘]:=

```
(*Validation: Plot components of flow field*)
uxnorthalongx[x_] = uxnorth[x, 0] / u0; (*cut along x axis*)
uynorthalongx[x_] = uynorth[x, 0] / u0;
uxsouthalongx[x_] = uxsouth[x, 0] / u0;
uysouthalongx[x_] = uysouth[x, 0] / u0;
Plot[{uxnorthalongx[x * RE], uynorthalongx[x * RE]}, {x, -7.5, 7.5},
 FrameLabel → {"x (R_E)", "u/u0"}, GridLines → Automatic, Frame → True,
 PlotLabel → "ux and uy in NORTHERN Wing", PlotStyle → {{Red, Thick}, {Blue, Thick}},
 PlotLegends -> {"ux north", "uy north"}, PlotRange -> {-0.1, 1.1}]
Plot[{uxsouthalongx[x * RE], uysouthalongx[x * RE]}, {x, -7.5, 7.5},
 FrameLabel → {"x (R_E)", "u/u0"}, GridLines → Automatic, Frame → True,
 PlotLabel → "ux and uy in SOUTHERN Wing", PlotStyle → {{Green, Thick}, {Purple, Thick}},
 PlotLegends -> {"ux south", "uy south"}, PlotRange -> {-0.1, 1.1}]
```

Out[∘]=

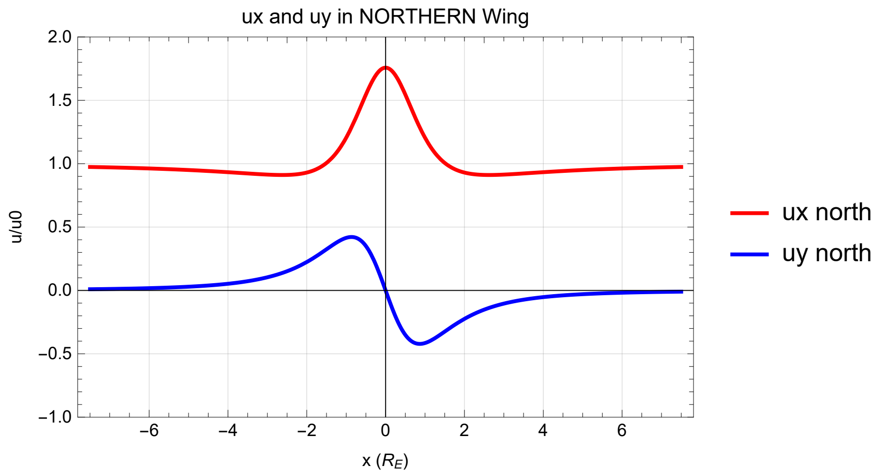


Out[∘]=

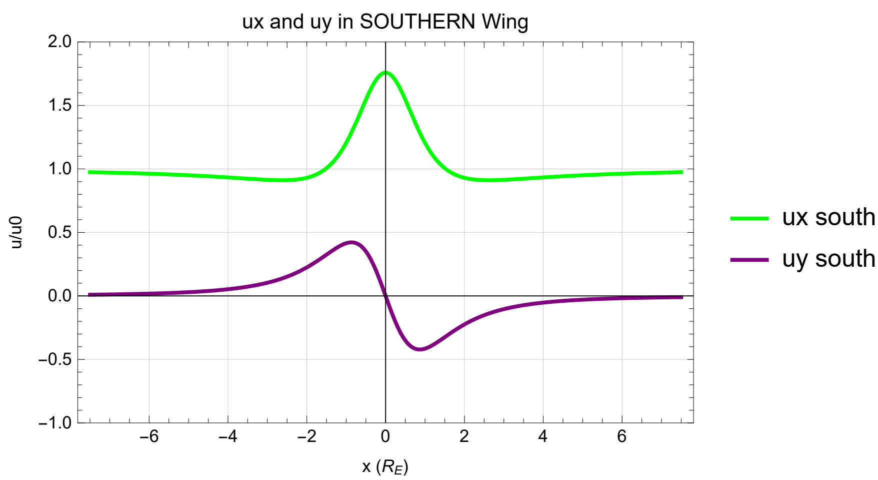


In[∘]:=
```
uxnorthshiftpos[x_] = uxnorth[x, 1.5 RE] / u0;
(*cut at y=1.5 RE, i.e., displaced TOWARD Jupiter*)
uynorthshiftpos[x_] = uynorth[x, 1.5 RE] / u0;
uxsouthshiftpos[x_] = uxsouth[x, 1.5 RE] / u0;
uysouthshiftpos[x_] = uysouth[x, 1.5 RE] / u0;
Plot[{uxnorthshiftpos[x * RE], uynorthshiftpos[x * RE]}, {x, -7.5, 7.5},
 FrameLabel → {"x (RE)", "u/u0"}, GridLines → Automatic, Frame → True,
 PlotLabel → "ux and uy in NORTHERN Wing", PlotStyle → {{Red, Thick}, {Blue, Thick}},
 PlotLegends -> {"ux north", "uy north"}, PlotRange -> {-1, 2}]
Plot[{uxsouthshiftpos[x * RE], uysouthshiftpos[x * RE]}, {x, -7.5, 7.5},
 FrameLabel → {"x (RE)", "u/u0"}, GridLines → Automatic, Frame → True,
 PlotLabel → "ux and uy in SOUTHERN Wing", PlotStyle → {{Green, Thick}, {Purple, Thick}},
 PlotLegends -> {"ux south", "uy south"}, PlotRange -> {-1, 2}]
```

Out[∘]=

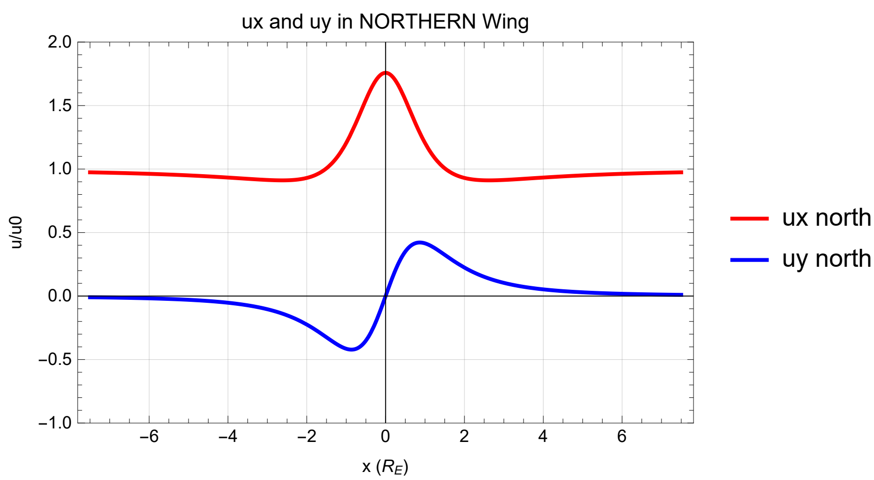


Out[∘]=

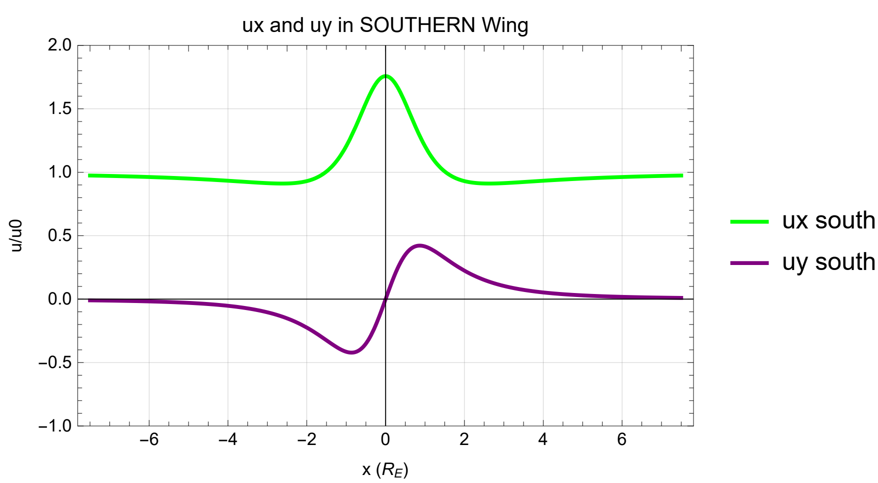


In[•]:=
```
uxnorthshiftneg[x_] = uxnorth[x, -1.5 RE] / u0;
(*cut at y=-1.5 RE, i.e., displaced AWAY FROM Jupiter*)
uynorthshiftneg[x_] = uynorth[x, -1.5 RE] / u0;
uxsouthshiftneg[x_] = uxsouth[x, -1.5 RE] / u0;
uysouthshiftneg[x_] = uysouth[x, -1.5 RE] / u0;
Plot[{uxnorthshiftneg[x * RE], uynorthshiftneg[x * RE]}, {x, -7.5, 7.5},
 FrameLabel → {"x (RE)", "u/u0"}, GridLines → Automatic, Frame → True,
 PlotLabel → "ux and uy in NORTHERN Wing", PlotStyle → {{Red, Thick}, {Blue, Thick}},
 PlotLegends -> {"ux north", "uy north"}, PlotRange -> {-1, 2}]
Plot[{uxsouthshiftneg[x * RE], uysouthshiftneg[x * RE]}, {x, -7.5, 7.5},
 FrameLabel → {"x (RE)", "u/u0"}, GridLines → Automatic, Frame → True,
 PlotLabel → "ux and uy in SOUTHERN Wing", PlotStyle → {{Green, Thick}, {Purple, Thick}},
 PlotLegends -> {"ux south", "uy south"}, PlotRange -> {-1, 2}]
```

Out[•]=

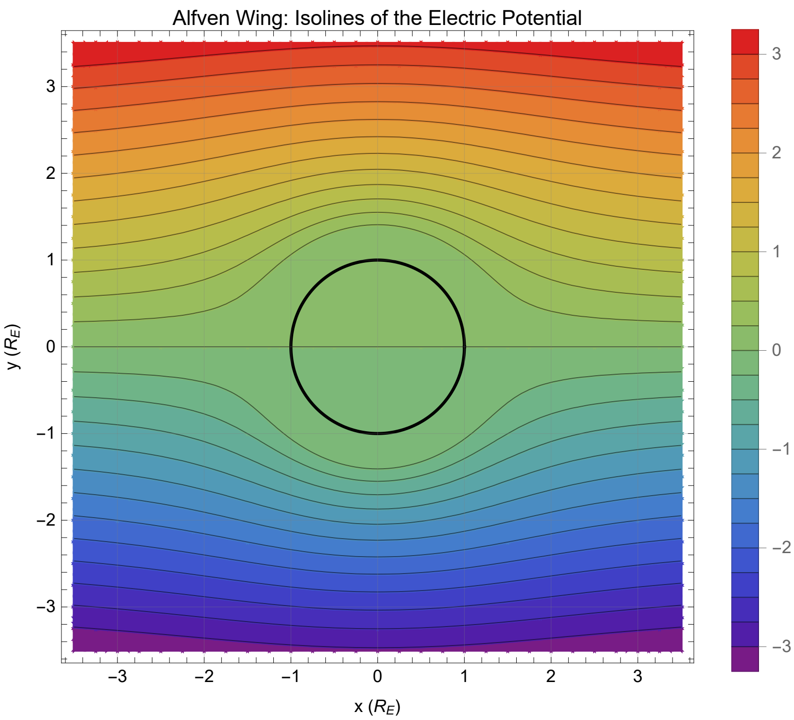


Out[•]=



In[◦]:=
```
(*plot the flow field in both hemispheres. The electric field is
 perpendicular to the flow, and also to the isolines of the electric
 potential. So a visulaization of psi will do the job, as the psi=
 const lines are identical to the stream lines of u. For this plot the potential is
  normalized to the undisturbed electric potential E0 RE at the surface of Europa.*)
Plot1 = ParametricPlot[{Cos[t], Sin[t]}, {t, 0, 2 * Pi}, AspectRatio -> Automatic,
   AxesLabel → {"x (R_E)", "y (R_E)"}, PlotStyle → {Black, thick, solid}];
Plot2 = ContourPlot[psi[x * RE, y * RE] / (E0 * RE), {x, -3.5, 3.5}, {y, -3.5, 3.5},
   Frame → True, GridLines → Automatic, FrameLabel → {"x (R_E)", "y (R_E)"},
   Contours → 25, ColorFunction → "Rainbow", PlotLegends → Automatic];
Plot3 = Show[Plot2, Plot1, PlotLabel -> "Alfven Wing: Isolines of the Electric Potential"]
```

Out[◦]=



In[◦]:=
```
(*----------------------------------------------------------*)
(*12/04/2020: calculate surface flux of the thermal plasma flow
  along Europa's ramside. Radial unit vector: (+cos phi, +sin phi,0).  *)
(*----------------------------------------------------------*)
(*first: confirm north-south symmetry of the flow deflection pattern*)
uxnorth[x, y] - uxsouth[x, y] // FullSimplify
uynorth[x, y] - uysouth[x, y] // FullSimplify
```

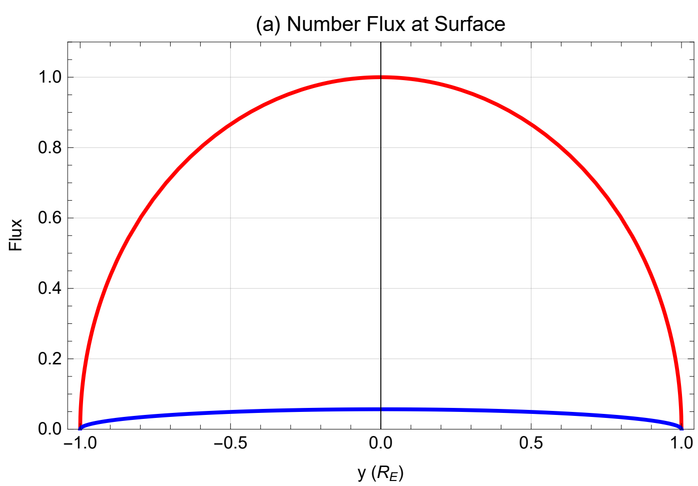
Out[◦]= 0.

Out[◦]= 0.

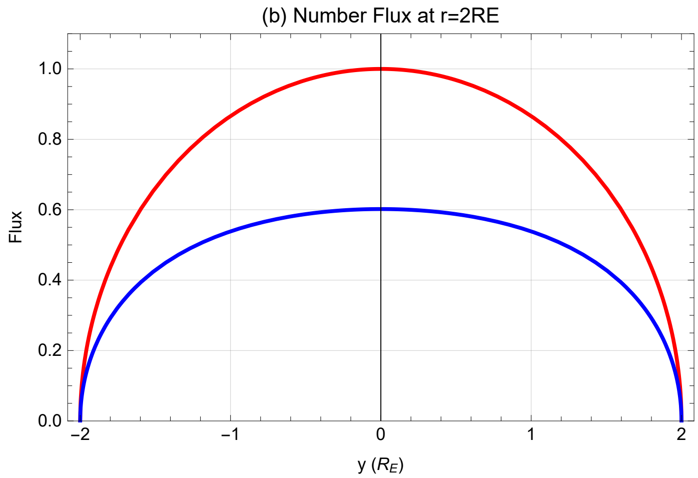
```
In[ ]:= (*Surface NUMBER flux*)
flux[x_, y_] = Abs[n0 * (cosphi[x, y] * uxnorth[x, y] + sinphi[x, y] * uynorth[x, y])];
(*flux along the equator*)
eqflux[y_] = flux[√(RE^2 - y^2), y];
(*flux along a circle of radius lam(bda) RE*)
eqfluxabove[y_, lam_] = flux[√((lam * RE)^2 - y^2), y];
(*standard "bullseye flux *)
bullseyeflux[x_, y_] = Abs[n0 * u0 * cosphi[x, y]];
(*bullseye flux along equator*)
eqbullseyeflux[y_] = bullseyeflux[√(RE^2 - y^2), y];
(*bullseye flux along a circle of radius lam(bda) RE*)
eqbullseyefluxabove[y_, lam_] = bullseyeflux[√((lam * RE)^2 - y^2), y];

In[ ]:= PlotA = Plot[{eqbullseyeflux[y * RE] / (n0 * u0), eqflux[y * RE] / (n0 * u0)},
  {y, -1, 1}, FrameLabel → {"y (RE)", "Flux"}, GridLines → Automatic,
  Frame → True, PlotLabel → "(a) Number Flux at Surface",
  PlotStyle → {{Red, Thick}, {Blue, Thick}}, PlotRange -> {0, 1.1}]
PlotB = Plot[{eqbullseyefluxabove[y * RE, 2] / (n0 * u0), eqfluxabove[y * RE, 2] / (n0 * u0)},
  {y, -2, 2}, FrameLabel → {"y (RE)", "Flux"}, GridLines → Automatic,
  Frame → True, PlotLabel → "(b) Number Flux at r=2RE",
  PlotStyle → {{Red, Thick}, {Blue, Thick}}, PlotRange -> {0, 1.1}]
PlotC = Plot[{eqbullseyefluxabove[y * RE, 3] / (n0 * u0), eqfluxabove[y * RE, 3] / (n0 * u0)},
  {y, -3, 3}, FrameLabel → {"y (RE)", "Flux"}, GridLines → Automatic,
  Frame → True, PlotLabel → "(c) Number Flux at r=3RE",
  PlotStyle → {{Red, Thick}, {Blue, Thick}}, PlotRange -> {0, 1.1}]
PlotD = Plot[{eqbullseyefluxabove[y * RE, 4] / (n0 * u0), eqfluxabove[y * RE, 4] / (n0 * u0)},
  {y, -4, 4}, FrameLabel → {"y (RE)", "Flux"}, GridLines → Automatic,
  Frame → True, PlotLabel → "(d) Number Flux at r=4RE",
  PlotStyle → {{Red, Thick}, {Blue, Thick}}, PlotRange -> {0, 1.1}]
```

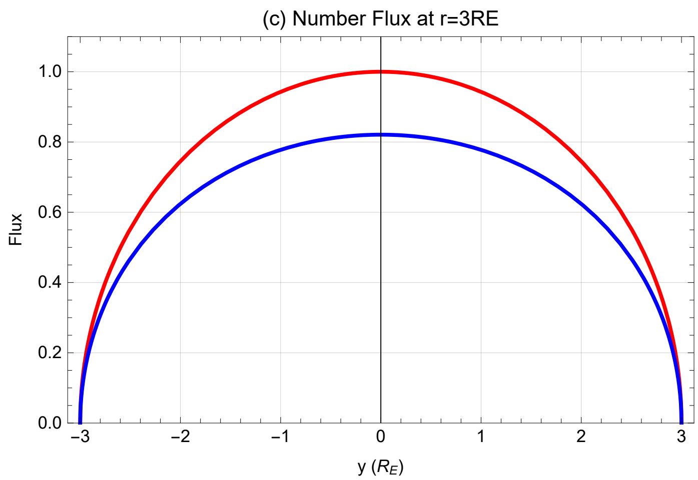
Out[ ]=



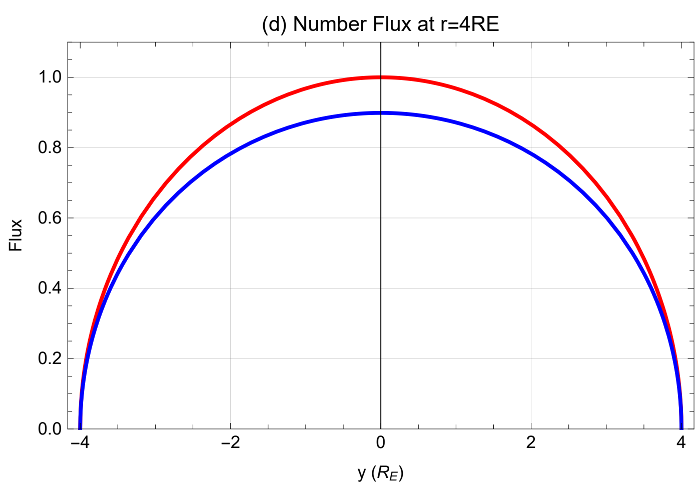
Out[∘]=


(b) Number Flux at r=2RE
Flux
1.0
0.8
0.6
0.4
0.2
0.0
−2
−1
0
1
2
y (R_E)

Out[∘]=

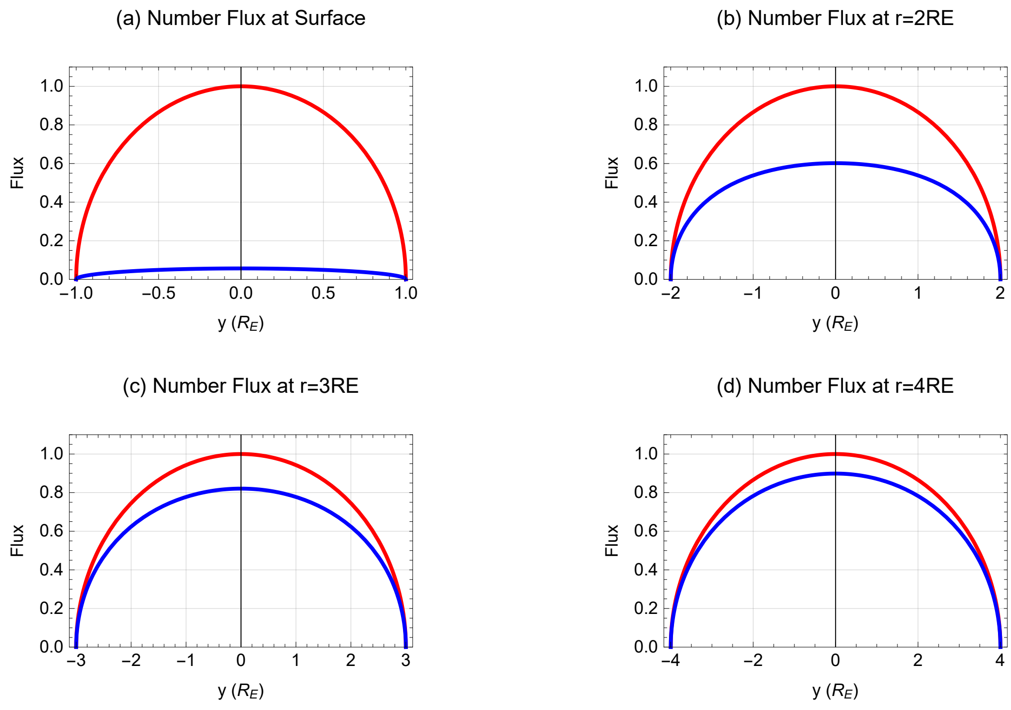

(c) Number Flux at r=3RE
Flux
1.0
0.8
0.6
0.4
0.2
0.0
−3
−2
−1
0
1
2
3
y (R_E)

Out[∘]=


(d) Number Flux at r=4RE
Flux
1.0
0.8
0.6
0.4
0.2
0.0
−4
−2
0
2
4
y (R_E)

In[ ]:=
```
GraphicsGrid[{{PlotA, PlotB}, {PlotC, PlotD}}]
```

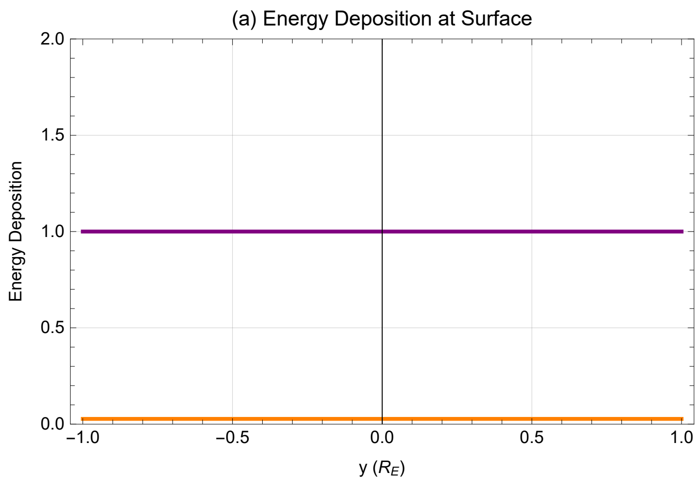
Out[ ]=

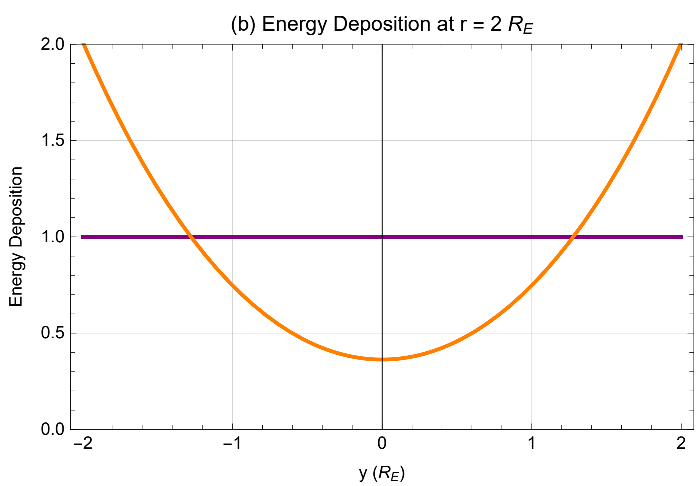



```
(*now let's do the same for the ENERGY DEPOSITION, NOT FLUX*)
```

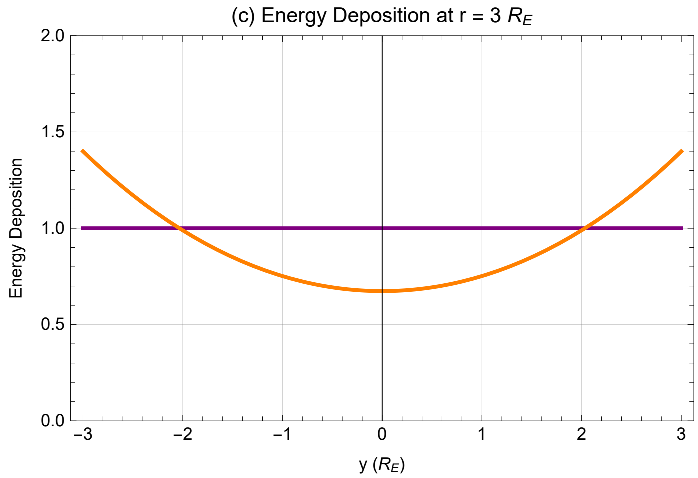
In[ ]:=
```
Edep[x_, y_] = Abs[n0 * m / 2 * (uxnorth[x, y] * uxnorth[x, y] +
      uynorth[x, y] * uynorth[x, y] + uznorth[x, y] * uznorth[x, y])];
(*deposition along the equator*)
Eqdep[y_] = Edep[√(RE^2 - y^2), y];
(*deposition along a circle of radius lam(bda) RE*)
Eqdepabove[y_, lam_] = Edep[√((lam * RE)^2 - y^2), y];
(*standard "bullseye deposition *)
Ebullseyedep[x_, y_] = Abs[n0 * m * u0 * u0 / 2];
(*bullseye deposition along equator*)
Eqbullseyedep[y_] = Ebullseyedep[√(RE^2 - y^2), y];
(*bullseye deposition along a circle of radius lam(bda) RE*)
Eqbullseyedepabove[y_, lam_] = Ebullseyedep[√((lam * RE)^2 - y^2), y];
```

In[ ]:=
```
PlotE =
 Plot[{Eeqbullseyedep[y * RE] / (n0 * m * u0 * u0 / 2), Eeqdep[y * RE] / (n0 * m * u0 * u0 / 2)},
  {y, -1, 1}, FrameLabel → {"y (RE)", "Energy Deposition"}, GridLines → Automatic,
  Frame → True, PlotLabel → "(a) Energy Deposition at Surface",
  PlotStyle → {{Purple, Thick}, {Orange, Thick}}, PlotRange -> {0, 2}]
PlotF = Plot[{Eeqbullseyedepabove[y * RE, 2] / (n0 * m * u0 * u0 / 2),
   Eeqdepabove[y * RE, 2] / (n0 * m * u0 * u0 / 2)}, {y, -2, 2},
  FrameLabel → {"y (RE)", "Energy Deposition"}, GridLines → Automatic,
  Frame → True, PlotLabel → "(b) Energy Deposition at r = 2 RE",
  PlotStyle → {{Purple, Thick}, {Orange, Thick}}, PlotRange -> {0, 2}]
PlotG = Plot[{Eeqbullseyedepabove[y * RE, 3] / (n0 * m * u0 * u0 / 2),
   Eeqdepabove[y * RE, 3] / (n0 * m * u0 * u0 / 2)}, {y, -3, 3},
  FrameLabel → {"y (RE)", "Energy Deposition"}, GridLines → Automatic,
  Frame → True, PlotLabel → "(c) Energy Deposition at r = 3 RE",
  PlotStyle → {{Purple, Thick}, {Orange, Thick}}, PlotRange -> {0, 2}]
PlotH = Plot[{Eeqbullseyedepabove[y * RE, 4] / (n0 * m * u0 * u0 / 2),
   Eeqdepabove[y * RE, 4] / (n0 * m * u0 * u0 / 2)}, {y, -4, 4},
  FrameLabel → {"y (RE)", "Energy Deposition"}, GridLines → Automatic,
  Frame → True, PlotLabel → "(d) Energy Deposition at r = 4 RE",
  PlotStyle → {{Purple, Thick}, {Orange, Thick}}, PlotRange -> {0, 2}]
```

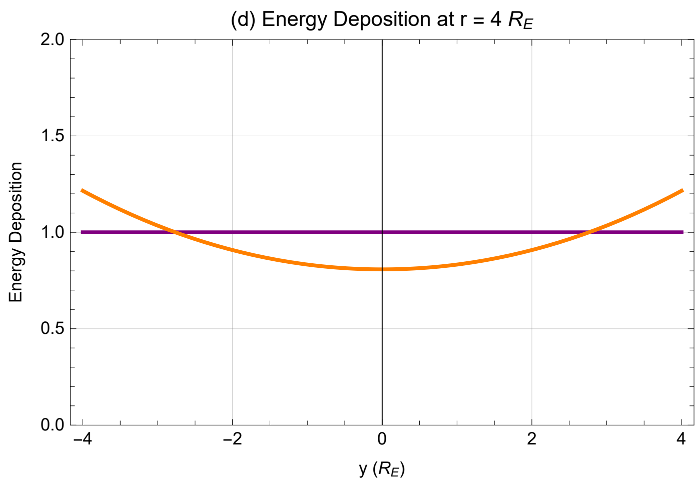
Out[ ]=


Out[ ]=


Out[ ]=

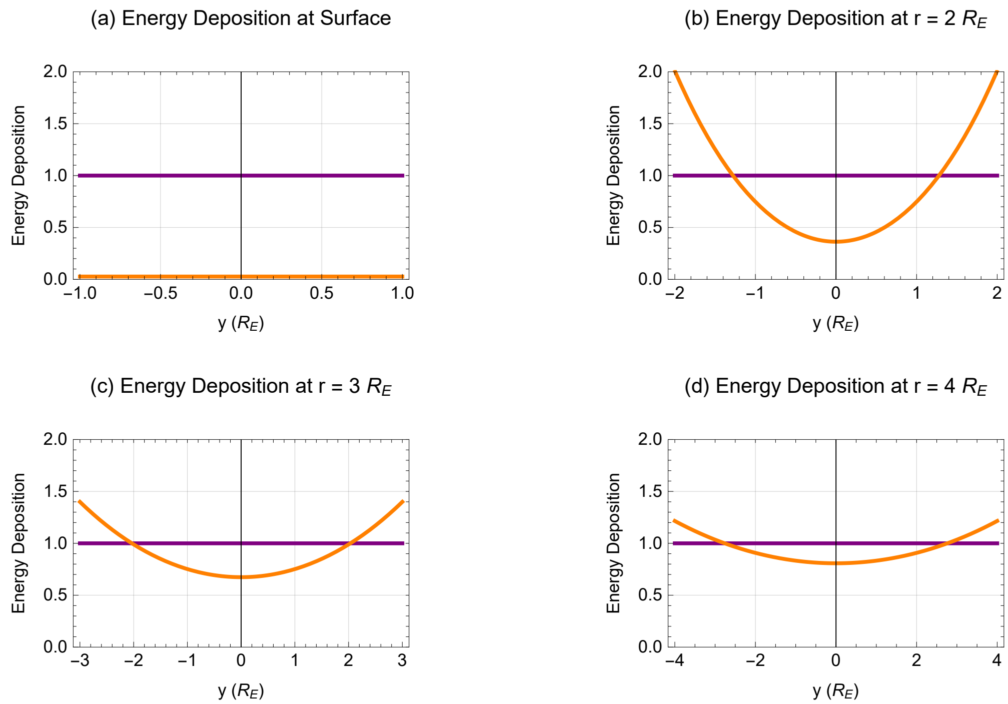

(c) Energy Deposition at r = 3 R_E
Energy Deposition
2.0
1.5
1.0
0.5
0.0
−3
−2
−1
0
1
2
3
y (R_E)

Out[ ]=


(d) Energy Deposition at r = 4 R_E
Energy Deposition
2.0
1.5
1.0
0.5
0.0
−4
−2
0
2
4
y (R_E)

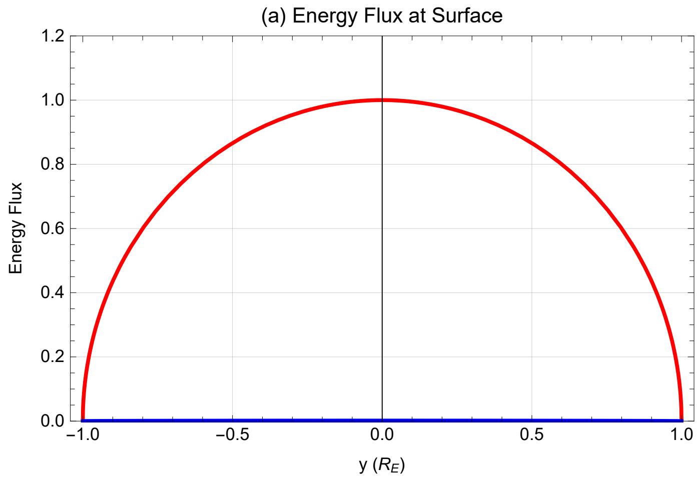
In[ ]:= `GraphicsGrid[{{PlotE, PlotF}, {PlotG, PlotH}}]`

Out[ ]=

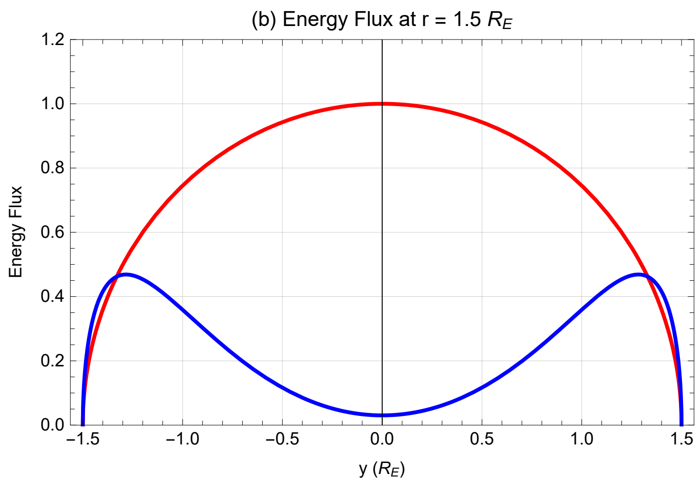


```
In[]:= (*12/11/2020: Now let's do the actual energy flux! EFlux= 1/2 n m u^2 vec{u}* e_r,
see plasma physics HW, sheet 4*)
Eflux[x_, y_] = Abs[n0 * m / 2 * (uxnorth[x, y] * uxnorth[x, y] +
       uynorth[x, y] * uynorth[x, y] + uznorth[x, y] * uznorth[x, y]) *
     (cosphi[x, y] * uxnorth[x, y] + sinphi[x, y] * uynorth[x, y])];
(*deposition along the equator*)
Eeqflux[y_] = Eflux[√(RE^2 - y^2), y];
(*deposition along a circle of radius lam(bda) RE*)
Eeqfluxabove[y_, lam_] = Eflux[√((lam * RE)^2 - y^2), y];
(*standard "bullseye deposition *)
Ebullseyeflux[x_, y_] = Abs[n0 * m * u0 * u0 * (cosphi[x, y] * u0) / 2];
(*bullseye deposition along equator*)
Eeqbullseyeflux[y_] = Ebullseyeflux[√(RE^2 - y^2), y];
(*bullseye deposition along a circle of radius lam(bda) RE*)
Eeqbullseyefluxabove[y_, lam_] = Ebullseyeflux[√((lam * RE)^2 - y^2), y];
PlotFluxE = Plot[
  {Eeqbullseyeflux[y * RE] / (n0 * m * u0 * u0 * u0 / 2), Eeqflux[y * RE] / (n0 * m * u0 * u0 * u0 / 2)},
  {y, -1, 1}, FrameLabel → {"y (R_E)", "Energy Flux"}, GridLines → Automatic,
  Frame → True, PlotLabel → "(a) Energy Flux at Surface",
  PlotStyle → {{Red, Thick}, {Blue, Thick}}, PlotRange -> {0, 1.2}]
PlotFluxF = Plot[{Eeqbullseyefluxabove[y * RE, 1.5] / (n0 * m * u0 * u0 * u0 / 2),
   Eeqfluxabove[y * RE, 1.5] / (n0 * m * u0 * u0 * u0 / 2)}, {y, -1.5, 1.5},
  FrameLabel → {"y (R_E)", "Energy Flux"}, GridLines → Automatic,
  Frame → True, PlotLabel → "(b) Energy Flux at r = 1.5 R_E",
  PlotStyle → {{Red, Thick}, {Blue, Thick}}, PlotRange -> {0, 1.2}]
PlotFluxG = Plot[{Eeqbullseyefluxabove[y * RE, 2] / (n0 * m * u0 * u0 * u0 / 2),
   Eeqfluxabove[y * RE, 2] / (n0 * m * u0 * u0 * u0 / 2)}, {y, -2, 2},
  FrameLabel → {"y (R_E)", "Energy Flux"}, GridLines → Automatic,
  Frame → True, PlotLabel → "(c) Energy Flux at r = 2 R_E",
  PlotStyle → {{Red, Thick}, {Blue, Thick}}, PlotRange -> {0, 1.2}]
PlotFluxH = Plot[{Eeqbullseyefluxabove[y * RE, 2.5] / (n0 * m * u0 * u0 * u0 / 2),
   Eeqfluxabove[y * RE, 2.5] / (n0 * m * u0 * u0 * u0 / 2)}, {y, -2.5, 2.5},
  FrameLabel → {"y (R_E)", "Energy Flux"}, GridLines → Automatic,
  Frame → True, PlotLabel → "(d) Energy Flux at r = 2.5 R_E",
  PlotStyle → {{Red, Thick}, {Blue, Thick}}, PlotRange -> {0, 1.2}]
```

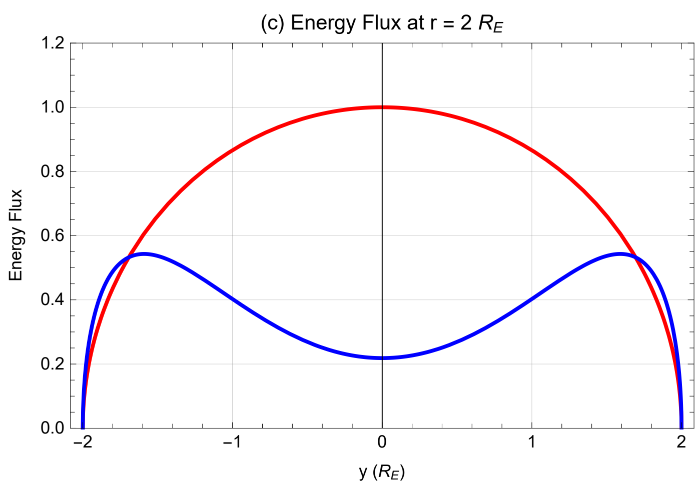
Out[ ]=


(a) Energy Flux at Surface
Energy Flux
y (R_E)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−1.0
−0.5
0.0
0.5
1.0

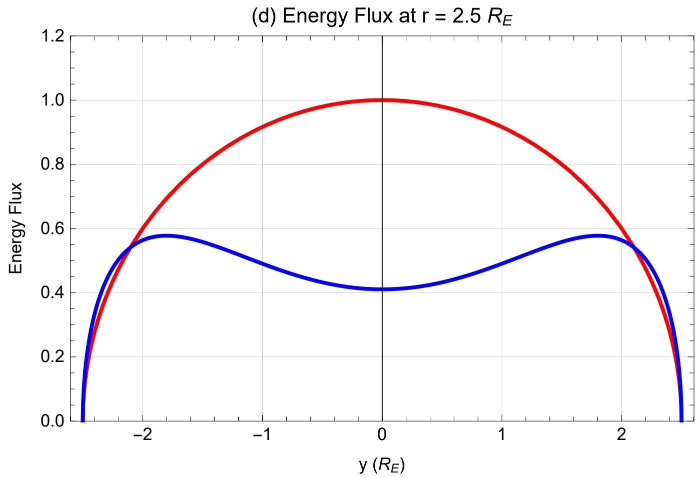
Out[ ]=


(b) Energy Flux at r = 1.5 R_E
Energy Flux
y (R_E)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

Out[ ]=

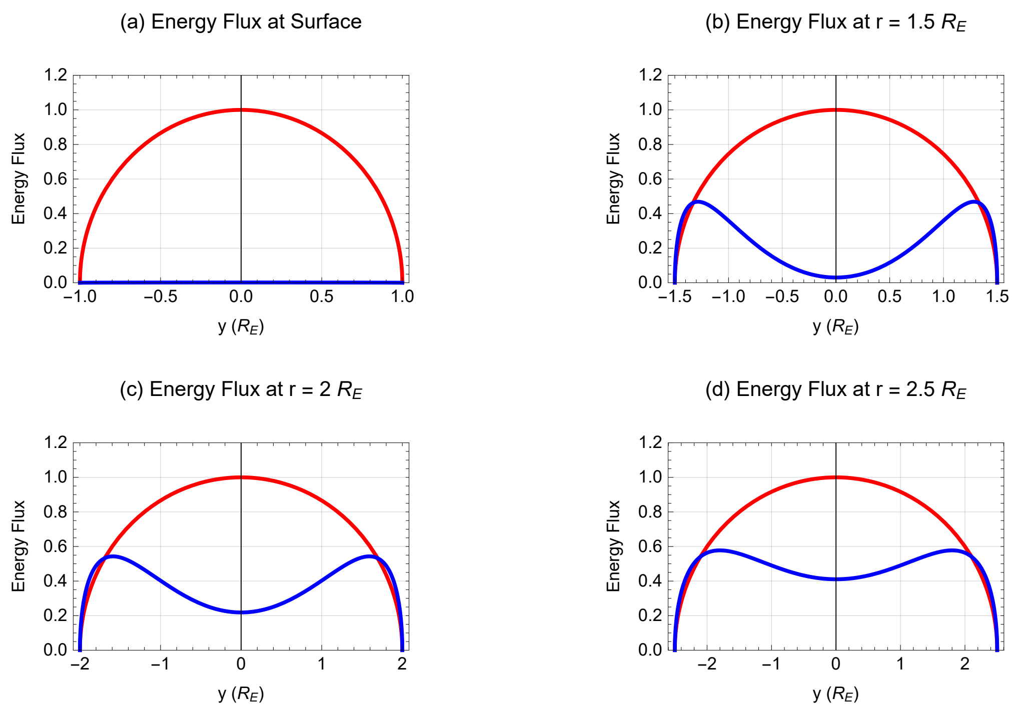

(c) Energy Flux at r = 2 R_E
Energy Flux
y (R_E)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−2
−1
0
1
2

Out[ ]=



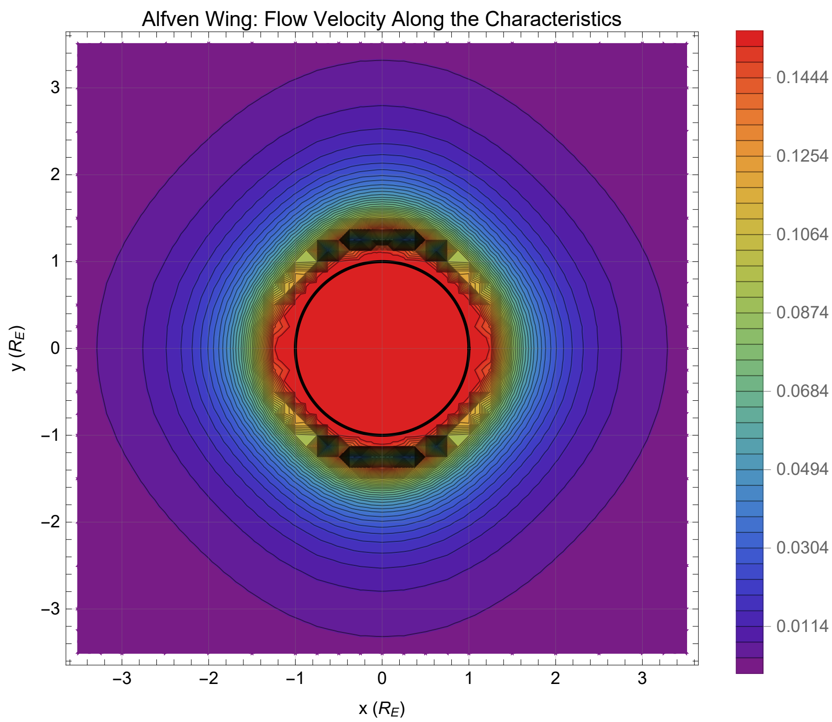
In[ ]:=
```
GraphicsGrid[{{PlotFluxE, PlotFluxF}, {PlotFluxG, PlotFluxH}}]
```

Out[ ]=



In[ ]:=
```
(*have a closer look at uz*)
Plot11 = ParametricPlot[{Cos[t], Sin[t]}, {t, 0, 2 * Pi}, AspectRatio -> Automatic,
   AxesLabel → {"x (R_E)", "y (R_E)"}, PlotStyle → {Black, thick, solid}];
Plot12 = ContourPlot[uznorth[x * RE, y * RE] / (u0), {x, -3.5, 3.5}, {y, -3.5, 3.5},
   Frame → True, GridLines → Automatic, FrameLabel → {"x (R_E)", "y (R_E)"},
   Contours → 40, ColorFunction → "Rainbow", PlotLegends → Automatic];
Plot13 = Show[Plot12, Plot11, PlotLabel ->
   "Alfven Wing: Flow Velocity Along the Characteristics"]
```

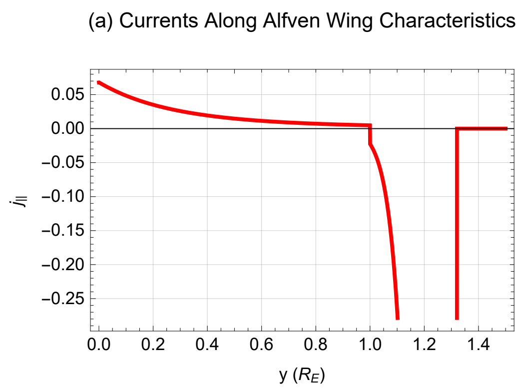
Out[ ]=

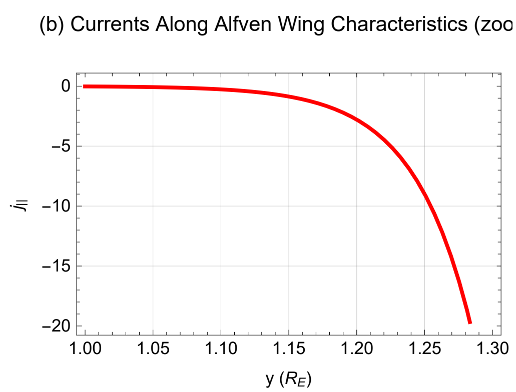


In[ ]:=
```
(*12/07/20: Currents along the Alfven wing characteristics, j = - sigmaA laplace psi*)
jpar1[x_, y_] = SigmaA * beta * sinphi[x, y] * K1 *
   (1 - Exp[-beta * r[x, y]] * (beta * r[x, y] + 1)) / (r[x, y] * r[x, y]); (*region 1: r ≤ R1*)
jpar2[x_, y_] = -SigmaA * delta * sinphi[x, y] *
   (K3 + K4 * Exp[delta * r[x, y]] * (delta * r[x, y] - 1)) / (r[x, y] * r[x, y]); (*region 2: R1 ≤r ≤ R2*)
jpar3[x_, y_] = 0; (*region 3: R2 ≤ r*)
jpar[x_, y_] =
  Piecewise[{{SigmaA * beta * K1 * beta^2 / 2, r[x, y] == 0}, {jpar1[x, y], 0 < r[x, y] ≤ R1},
    {jpar2[x, y], R1 < r[x, y] ≤ R2}, {jpar3[x, y], R2 < r[x, y] }}];
```

In[•]:=
```
(*Plotting*)
jparalongy[y_] = jpar[0, y];
Plotcurr1 = Plot[jparalongy[y * RE] / (SigmaA E0 / RE), {y, 0, 1.5},
   FrameLabel → {"y (R_E)", "j_||"}, GridLines → Automatic, Frame → True, PlotLabel →
    "(a) Currents Along Alfven Wing Characteristics", PlotStyle → {Red, Thick}] ;
Plotcurr2 = Plot[jparalongy[y * RE] / (SigmaA E0 / RE), {y, 1., 1.3},
   FrameLabel → {"y (R_E)", "j_||"}, GridLines → Automatic, Frame → True, PlotLabel →
    "(b) Currents Along Alfven Wing Characteristics (zoom)", PlotStyle → {Red, Thick}] ;
GraphicsGrid[{{Plotcurr1}, {Plotcurr2}}]
```

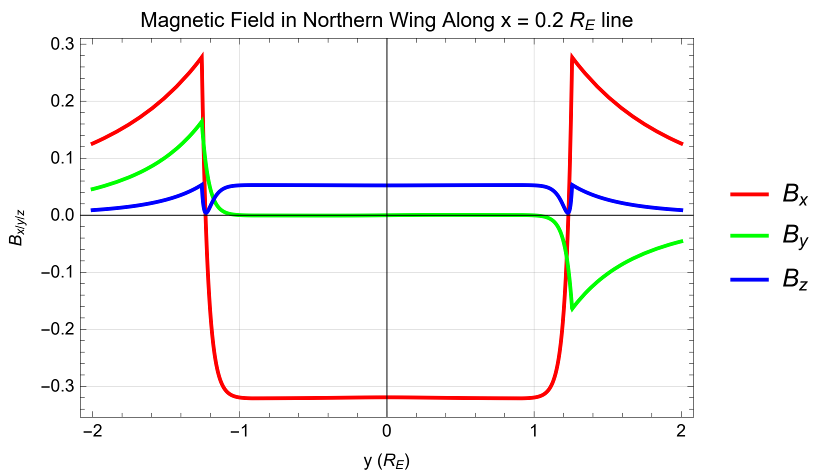
Out[•]=

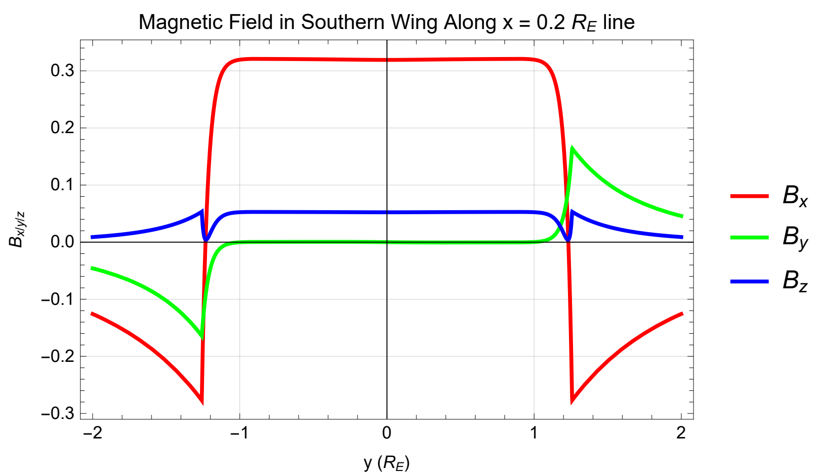





```
(*How does this jump of Jparallel manifest in the
 magnetic field? NOTE: Bz has been detrended for the plot*)
```

In[ ]:=
```
Plotmagnorth = Plot[{Bxnorth[0.4 RE, y * RE] / (B0),
    Bynorth[0.4 RE, y * RE] / (B0), Bznorth[0.4 RE, y * RE] / (B0) + B0 / B0}, {y, -2.0, 2.0},
  FrameLabel → {"y (RE)", "Bx/y/z"}, GridLines → Automatic, Frame → True,
  PlotLabel → "Magnetic Field in Northern Wing Along x = 0.2 RE line",
  PlotStyle → {{Red, Thick}, {Green, Thick}, {Blue, Thick}},
  PlotLegends -> {"Bx", "By", "Bz"}]
Plotmagsouth = Plot[{Bxsouth[0.4 RE, y * RE] / (B0), Bysouth[0.4 RE, y * RE] / (B0),
    Bzsouth[0.4 RE, y * RE] / (B0) + B0 / B0}, {y, -2.0, 2.0},
  FrameLabel → {"y (RE)", "Bx/y/z"}, GridLines → Automatic, Frame → True,
  PlotLabel → "Magnetic Field in Southern Wing Along x = 0.2 RE line",
  PlotStyle → {{Red, Thick}, {Green, Thick}, {Blue, Thick}},
  PlotLegends -> {"Bx", "By", "Bz"}]
(*GraphicsGrid[{{Plotmagnorth},{Plotmagsouth}}]*)
```

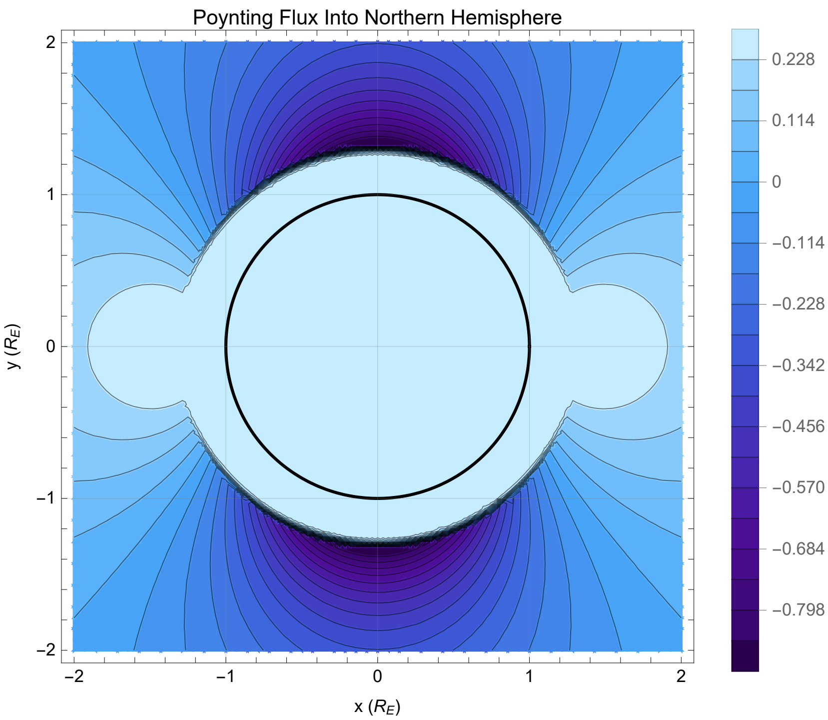
Out[ ]=


Out[ ]=


In[ ]:=
```
(*Poynting flux Sz along Alfven wing characteristics,
wlog into northern hemisphere. Normalized to background flux u0 B0^2/mu0*)
Sz[x_, y_] = (dxpsi[x, y] * Bynorth[x, y] - (dypsi[x, y] - u0 * Bznorth[x, y]) * Bxnorth[x, y]) /
   (u0 * B0^2);
```

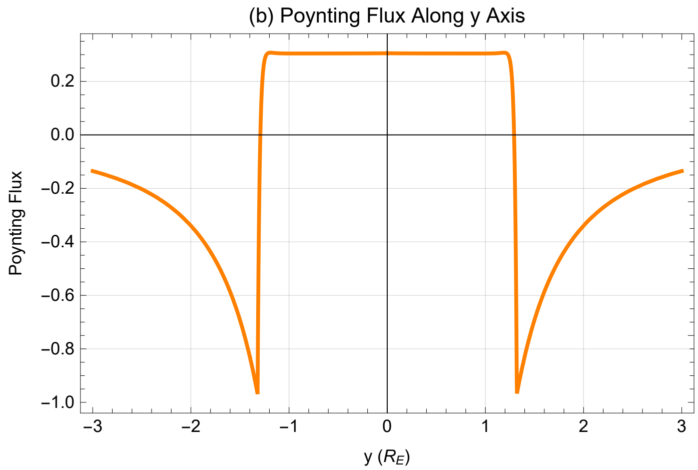
In[ ]:=
```
PlotS1 = ParametricPlot[{Cos[t], Sin[t]}, {t, 0, 2 * Pi}, AspectRatio -> Automatic,
    AxesLabel → {"x (R_E)", "y (R_E)"}, PlotStyle → {Black, thick, solid}];
PlotS2 = ContourPlot[Sz[x * RE, y * RE], {x, -2.0, 2.0}, {y, -2.0, 2.0},
    Frame → True, GridLines → Automatic, FrameLabel → {"x (R_E)", "y (R_E)"},
    Contours → 20, ColorFunction → "DeepSeaColors", PlotLegends → Automatic];
PlotS3 = Show[PlotS2, PlotS1, PlotLabel -> "Poynting Flux Into Northern Hemisphere"]
```

Out[ ]=

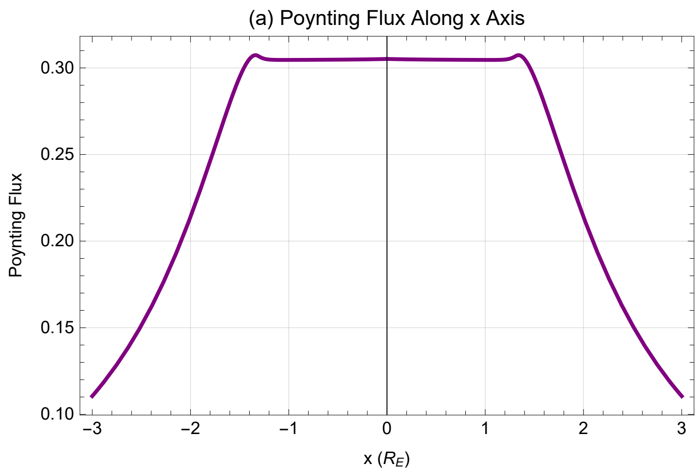


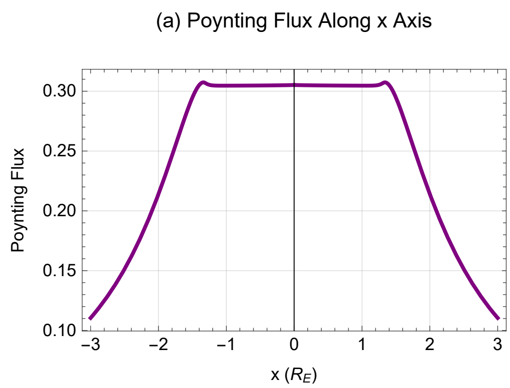
In[ ]:=
```
(*now: cut along y axis*)
PlotSalongx = Plot[Sz[0, y * RE], {y, -3, 3},
   FrameLabel → {"y (R_E)", "Poynting Flux"}, GridLines → Automatic, Frame → True,
   PlotLabel → "(b) Poynting Flux Along y Axis", PlotStyle → {Orange, Thick}]
PlotSalongy = Plot[Sz[x * RE, 0], {x, -3, 3}, FrameLabel → {"x (R_E)", "Poynting Flux"},
   GridLines → Automatic, Frame → True,
   PlotLabel → "(a) Poynting Flux Along x Axis", PlotStyle → {Purple, Thick}]
GraphicsGrid[{{PlotSalongy}, {PlotSalongx}}]
```

Out[ ]=

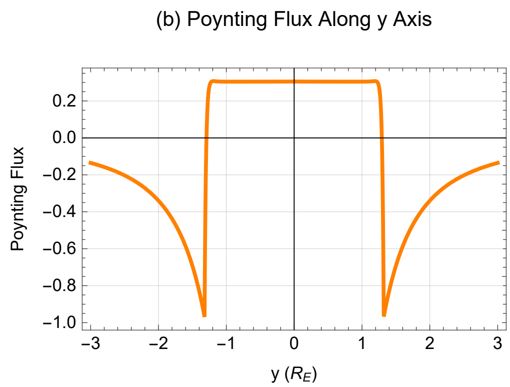

(b) Poynting Flux Along y Axis
Poynting Flux
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−3
−2
−1
0
1
2
3
y (R_E)

Out[ ]=


(a) Poynting Flux Along x Axis
Poynting Flux
0.30
0.25
0.20
0.15
0.10
−3
−2
−1
0
1
2
3
x (R_E)


(a) Poynting Flux Along x Axis
Poynting Flux
0.30
0.25
0.20
0.15
0.10
−3
−2
−1
0
1
2
3
x (R_E)

Out[∘]=


(b) Poynting Flux Along y Axis
Poynting Flux
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−3
−2
−1
0
1
2
3
y (R_E)