

```

In[ ]:= (*05/16/21*)
(*velocity u_z (constant) inside the northern Alfvén wing generated by a box-
like ionospheric obstacle*)
B0 = 5.1369 * 10-9; (*upstream magnetic field in Tesla*)
n0 = 0.11 * 106; (*upstream number density in 1/m3*)
mp = 1.6726231 * 10-27; (*proton mass*)
m = 7.5 * mp; (*upstream ion mass*)
mu0 = 4 * 3.14159265359 * 10-7; (*magnetic permeability of vacuum*)
vA =  $\frac{B0}{\sqrt{\mu0 * n0 * m}}$ ; (*Alfvén velocity*)
u0 = 43 * 103; (*magnitude of upstream flow velocity in m/s*)
MA = u0 / vA (*Alfvénic Mach number, Europa*)

uz[theta_, a_] =  $\frac{B0 * \text{Cos}[\text{theta}]}{\sqrt{\mu0 * n0 * m} (1 + MA^2 - 2 MA \text{Sin}[\text{theta}])} * \\ (-\sqrt{(1 + MA^2 - 2 MA \text{Sin}[\text{theta}] - MA^2 a^2 (1 - \text{Sin}[\text{theta}] * \text{Sin}[\text{theta}]))} + \\ MA * (a - 2) * \text{Sin}[\text{theta}] + MA^2 (1 - a) + 1);$ 
ux[theta_, a_] = u0 +  $\frac{B0}{\sqrt{\mu0 * n0 * m} (1 + MA^2 - 2 MA \text{Sin}[\text{theta}])} * ((\text{Sin}[\text{theta}] - MA) * \\ \sqrt{(1 + MA^2 - 2 MA \text{Sin}[\text{theta}] - MA^2 a^2 (1 - \text{Sin}[\text{theta}] * \text{Sin}[\text{theta}]))} + \\ MA * a * \text{Cos}[\text{theta}] * \text{Cos}[\text{theta}] - \text{Sin}[\text{theta}] * (1 + MA^2 - 2 MA \text{Sin}[\text{theta}])));$ 

Out[ ]:= 0.348577

```

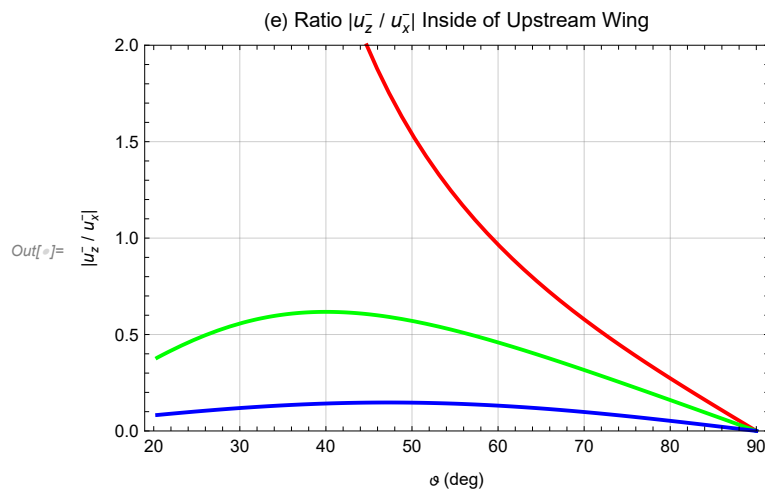
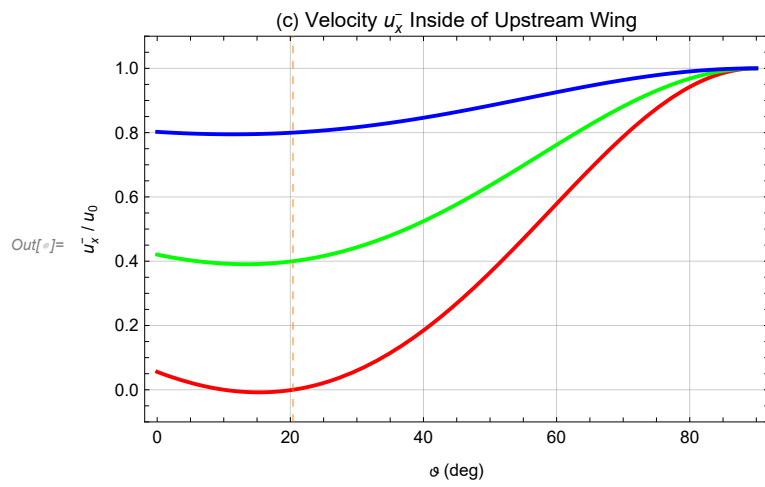
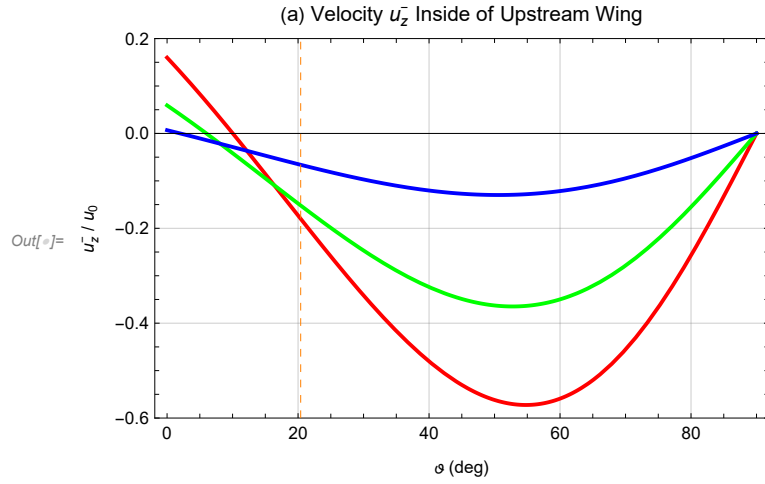
```

In[ ]:= Plot1 = Plot[
  {uz[theta *  $\frac{2\pi}{360}$ , 0.0] / (u0), uz[theta *  $\frac{2\pi}{360}$ , 0.4] / (u0), uz[theta *  $\frac{2\pi}{360}$ , 0.8] / (u0)},
  {theta, 0, 90}, FrameLabel -> {" $\theta$  (deg)", " $u_z^- / u_0$ "}, GridLines -> Automatic,
  Frame -> True, PlotLabel -> "(a) Velocity  $u_z^-$  Inside of Upstream Wing",
  PlotStyle -> {{Red, Thick}, {Green, Thick}, {Blue, Thick}},
  (*PlotLegends -> {" $\lambda^- = 1.0$ ", " $\lambda^- = 0.6$ ", " $\lambda^- = 0.2$ "}, *)
  PlotRange -> {-0.6, 0.2}, Epilog -> {Directive[Dashed, Orange],
    InfiniteLine[{ArcSin[MA] *  $\frac{360}{2\pi}$ , -0.6}, {ArcSin[MA] *  $\frac{360}{2\pi}$ , 0.2}]}]}]

Plot2 = Plot[{ux[theta *  $\frac{2\pi}{360}$ , 0.0] / (u0), ux[theta *  $\frac{2\pi}{360}$ , 0.4] / (u0),
  ux[theta *  $\frac{2\pi}{360}$ , 0.8] / (u0)}, {theta, 0, 90},
  FrameLabel -> {" $\theta$  (deg)", " $u_x^- / u_0$ "}, GridLines -> Automatic, Frame -> True,
  PlotLabel -> "(c) Velocity  $u_x^-$  Inside of Upstream Wing",
  PlotStyle -> {{Red, Thick}, {Green, Thick}, {Blue, Thick}},
  (*PlotLegends -> {" $\lambda^- = 1.0$ ", " $\lambda^- = 0.6$ ", " $\lambda^- = 0.2$ "}, *)
  PlotRange -> {-0.1, 1.1}, Epilog -> {Directive[Dashed, Orange],
    InfiniteLine[{ArcSin[MA] *  $\frac{360}{2\pi}$ , -0.1}, {ArcSin[MA] *  $\frac{360}{2\pi}$ , 1.1}]}]}]

Plot3 = Plot[{Abs[uz[theta *  $\frac{2\pi}{360}$ , 0.0] / ux[theta *  $\frac{2\pi}{360}$ , 0.0]],
  Abs[uz[theta *  $\frac{2\pi}{360}$ , 0.4] / ux[theta *  $\frac{2\pi}{360}$ , 0.4]],
  Abs[uz[theta *  $\frac{2\pi}{360}$ , 0.8] / ux[theta *  $\frac{2\pi}{360}$ , 0.8]]}, {theta, ArcSin[MA] *  $\frac{360}{2\pi}$ , 90},
  FrameLabel -> {" $\theta$  (deg)", " $|u_z^- / u_x^-|$ "}, GridLines -> Automatic, Frame -> True,
  PlotLabel -> "(e) Ratio  $|u_z^- / u_x^-|$  Inside of Upstream Wing",
  PlotStyle -> {{Red, Thick}, {Green, Thick}, {Blue, Thick}}, (*,
  PlotLegends -> {" $\lambda^+ = 1.0$ ", " $\lambda^+ = 0.6$ ", " $\lambda^+ = 0.2$ "}, *) PlotRange -> {0, 2}]

```



```
In[ ]:= Export["figure3a_new.pdf", Plot1]
       Export["figure3c_new.pdf", Plot2]
       Export["figure3e_new.pdf", Plot3]
```

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Out[ ]:= figure3a_new.pdf
```

```
Out[ ]:= figure3c_new.pdf
```

```
Out[ ]:= figure3e_new.pdf
```


```
In[ ]:= uxroots[theta_] = u0 + 
$$\frac{B0}{\sqrt{\mu0 * n0 * m (1 + MA^2 - 2 MA \sin[\theta])}}$$
 * ((Sin[theta] - MA) *
      
$$\sqrt{(1 + MA^2 - 2 MA \sin[\theta] - MA^2 a^2 (1 - \sin[\theta] * \sin[\theta]))} +$$

      MA * a * Cos[theta] * Cos[theta] - Sin[theta] * ((1 + MA^2 - 2 MA Sin[theta])));
```


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In[ ]:= a = 0.1
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```
Out[ ]:= 0.1
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```
In[ ]:= Solve[uxroots[theta] == 0, theta]
       Solve[uz[theta, 0.1] == 0, theta]
```

 **Solve:** Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

```
Out[ ]:= {{theta -> 0.270824 - 0.317337 I}, {theta -> 0.270824 + 0.317337 I},
          {theta -> 2.87077 - 0.317337 I}, {theta -> 2.87077 + 0.317337 I}}
```

 **Solve:** Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

```
Out[ ]:= {{theta -> -1.5708}, {theta -> 0.15751}, {theta -> 1.5708}}
```