Modeling the Maximum Throughput of Bluetooth Low Energy in an Error-Prone Link

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***Abstract*—We present an analytical model for the maximum throughput of Bluetooth Low Energy (BLE), considering the presence of uncorrelated bit errors and the impact of a key BLE parameter that defines the time between the start of two consecutive connection events. The derived analysis models the generic application of master-to-slave unidirectional data transmission, which also forms an upper bound for bidirectional data transmission throughput. Simulation results show that our model accurately predicts the maximum BLE throughput for all bit error rates and BLE parameter settings evaluated.**

***Index Terms*—Bluetooth Low Energy, maximum throughput, performance evaluation.**

I. INTRODUCTION

BLUETOOTH Low Energy (BLE) is an emerging wireless, low-power technology designed for control and monitoring applications. BLE is the main feature of the Bluetooth 4.0 specification [1], [2]. It is expected that BLE will be present in a large number of devices in the next few years. In fact, the IETF 6LoWPAN Working Group has identified BLE as a key technology for the Internet of Things and is currently writing a specification for the transmission of IPv6 packets on top of BLE [3]. Although the data transfer rate of BLE is defined to be 1 Mbit/s, the actual throughput values depend on the link quality and the communication layer overheads, specifically the overheads resulting from the new Physical Layer and Link Layer defined in BLE. This letter presents an analytical model of BLE throughput as a function of Bit Error Rate (BER) and a key BLE parameter called $connInterval$, which defines the time between the start of two consecutive connection events.

II. BLE OVERVIEW

The BLE protocol architecture comprises the Physical Layer, the Link Layer, the Logical Link Control and Adaptation Protocol (L2CAP), the Attribute Protocol (ATT), the Generic Attribute Profile (GATT) and the Generic Access Profile (GAP). In addition, the Security Manager Protocol (SMP) provides security functionality between devices.

At the Physical Layer, data are transmitted at 1 Mbit/s. BLE Link Layer provides a Time Division Multiple Access (TDMA) scheme by which a master device controls the communication with a number of slave devices. Data communication between a master and a slave requires the establishment of a connection. Within a connection, time is divided in non-overlapping periods of time called connection events. The master determines the connection event start time and the value of $connInterval$. The slave is required to listen to the channel at the beginning of a connection event, every *connSlaveLatency + 1* connection events. Each connection event starts with the transmission of a packet from the master. If the slave receives the packet, the slave must respond with a packet to the master. As shown in Fig. 1, a time interval called Inter Frame Space (IFS) is used between the complete reception of a packet and the start of the transmission of the next one, which is defined to be 150 μs. The connection event is considered to be open while master and slave continue to alternate in sending packets. If none of the devices has more data to transmit, the connection event will be closed and the slave will not be required to listen until the beginning of the next connection event determined by the *connSlaveLatency* parameter. A connection event may also be closed due to the reception of two consecutive packets with bit errors from a device, or due to corruption of the Access Address field of any packet. The Access Address identifies the Link Layer connection between a master and a slave. Bit error detection is carried out by including a 24-bit Cyclic Redundancy Check (CRC) code in each packet.

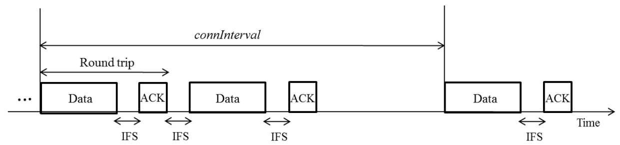

Fig. 1. Example of a connection event that contains two round trips (i.e., Data-ACK transactions). The time between the start of two consecutive connection events is equal to the $connInterval$ value.

Link Layer connections use a stop-and-wait flow control mechanism based on cumulative acknowledgments (ACKs), which at the same time provides error recovery capabilities. Devices maintain two one-bit variables whose values are copied in the header of each packet sent. These variables are a sequence number, which is incremented by one when a device transmits a new packet, and the next expected sequence number, which is incremented by one when a device correctly receives a packet. A packet without payload only plays the role of an ACK. If a device receives a packet with an invalid CRC check, the device resends its last transmitted packet, which serves as a negative ACK.

On top of the Link Layer, many BLE implementations will use the Basic L2CAP mode, which mainly multiplexes upper layer protocols. One of them is GATT/ATT, which allows the manipulation of resources called attributes by supporting

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request/reply transactions and the transmission of one-way messages, such as notifications. In this letter, we specifically analyze the throughput for such one-way data communication from master to a slave device.

III. Analysis of the Maximum Throughput

The maximum throughput can be achieved under the circumstances that new data are always available from the application layer for transmission, and the parameter *connSlaveLatency* is assigned the value 0. Hence these circumstances are assumed in the analysis. The transmission of the data between master and slave leads to Link Layer round trips. We define a round trip as the transaction that includes a packet containing data and the ACK sent in response (see Fig. 1).

Let N be the number of data packets that are successfully delivered and acknowledged within a connection event. The maximum BLE throughput, denoted δ_{max}, can be obtained as

$$\delta_{max} = \frac{E\left[N\right] \times L}{connInterval} \tag{1}$$

where $E[N]$ is the expected value of N, and L is the amount of user data in a packet.

Let $P\{i|k\}$ denote the probability of i acknowledged data packet deliveries in a connection event that contains k round trips, where k does not include the eventual round trips that end a connection event due to bit errors. Then, $E[N]$ can be calculated as

$$E\left[N\right] = \sum_{k=0}^{N_{Max}} \sum_{i=0}^{k} iP\left\{i|k\right\}, \tag{2}$$

where N_{Max} denotes the maximum number of round trips that can take place within one connection event. N_{Max} can be expressed as

$$N_{Max} = \left\lfloor \frac{connInterval}{T_{Data} + T_{Ack} + 2IFS} \right\rfloor, \tag{3}$$

where T_{Data} and T_{Ack} denote the transmission time of a data and an ACK packet, respectively.

A round trip is called successful, if both data packet and its ACK are correctly received within that round trip. We approximate $P\{i|k\}$ by the probability that i out of the k round trips that take place in a connection event are successful[1], where k does not include the eventual round trips that end a connection event due to bit errors. Define Γ_{RT} to represent the probability of a round trip being successful, which can be formulated as,

$$\Gamma_{RT} = \Gamma_{Data}\Gamma_{Ack}, \tag{4}$$

where Γ_{Data} and Γ_{Ack} are the probabilities of successful transmission of a data packet and an ACK, respectively. And, let Γ_{AA} represent the probability that the Access Address field of a packet does not suffer bit errors. If Ω_{RT} denotes the probability of having an unsuccessful round trip that does not end connection event alone, i.e., where the data packet or the ACK suffer bit errors that do not affect their Access Address field, then

$$\Omega_{RT} = \Gamma_{Data}\Omega_{Ack} + \Omega_{Data}\Gamma_{Ack} + \Omega_{Data}\Omega_{Ack}, \tag{5}$$

where Ω_{Data} and Ω_{Ack} denote the probabilities of reception of a data packet and an ACK packet with an invalid CRC check, however with a correct Access Address field, respectively. Assuming that bit errors are not correlated,

$$\Gamma_{Data} = (1 - BER)^{L_{Data}}, \tag{6}$$
$$\Gamma_{Ack} = (1 - BER)^{L_{Ack}}, \tag{7}$$
$$\Gamma_{AA} = (1 - BER)^{L_{AA}}, \tag{8}$$
$$\Omega_{Data} = \Gamma_{AA}(1 - (1 - BER)^{L_{Data} - L_{AA}}), \tag{9}$$
$$\Omega_{Ack} = \Gamma_{AA}(1 - (1 - BER)^{L_{Ack} - L_{AA}}), \tag{10}$$

where L_{Data}, L_{Ack} and L_{AA} denote the length of a data packet, the length of an ACK packet and the length of the Access Address field, respectively.

Define $P_{cl}(k)$ to denote the probability that a connection event is closed after k round trips, for $0 \leq k \leq N_{Max}$, where k does not include the eventual round trips that end a connection event due to bit errors. Taking into account the different reasons that may lead to the end of a connection event, $P_{cl}(k)$ can be expressed as

$$P_{cl}(k) = \begin{cases} 2\Omega_{AA} + \Omega_{Data}^2 + \Omega_{Ack}^2, & 0 \leq k \leq N_{Max} - 2, \\ 2\Omega_{AA}, & k = N_{Max} - 1, \\ 1, & k = N_{Max}, \end{cases} \tag{11}$$

where Ω_{AA} denotes the probability that the Access Address field of a transmitted packet becomes corrupted, and thus,

$$\Omega_{AA} = 1 - \Gamma_{AA}. \tag{12}$$

Then, the approximation of $P\{i|k\}$ becomes

$$P\{i|k\} \cong \Gamma_{RT}^{i}\, \Omega_{RT}^{k-i}\, \Phi(i,k)\, P_{cl}(k), \tag{13}$$

where $\Phi(i,k)$ denotes the number of possible combinations whereby a connection event is closed after k round trips, when i out of k round trips are successful, for $i \leq k$, and k does not include the eventual round trips that end a connection event due to bit errors. We approximate $\Phi(i,k)$ as

$$\Phi(i,k) = \Phi_0(i,k) + \Phi_1(i,k)P_1, \tag{14}$$

where $\Phi_n(i,k)$ denotes the number of possible sequences of k round trips that have i successful round trips, and the event of exactly two consecutive unsuccessful round trips is present n times in a sequence, for $i \leq k$. In (14), P_1 denotes the probability that two consecutive unsuccessful round trips do not cause the end of a connection. P_1 can be obtained as

$$P_1 = \frac{2\Omega_{Data}\Omega_{Ack}(1 - \Omega_{Ack})(1 - \Omega_{Data})}{(\Omega_{Ack} + \Omega_{Data})^2}. \tag{15}$$

We next calculate $\Phi_0(i,k)$. This process can be viewed as uniformly random selection of $k - i$ distinct numbers from $\{1, \ldots, k\}$ to represent the unsuccessful round trip numbers, where the remaining i numbers represent the successful round trip numbers. Let $A = \{a_1, a_2, \ldots, a_{k-i}\}$ represent the set of selected unsuccessful round trip numbers. If A contains no consecutive integers, then A is called a nonconsecutive

[1] For the sake of brevity, we do not consider the low probability case, where two consecutive unsuccessful round trips do not result in end of connection event, but result in acknowledged data, e.g., with a transmission sequence of correct DATA, erroneous ACK, erroneous DATA and correct ACK.

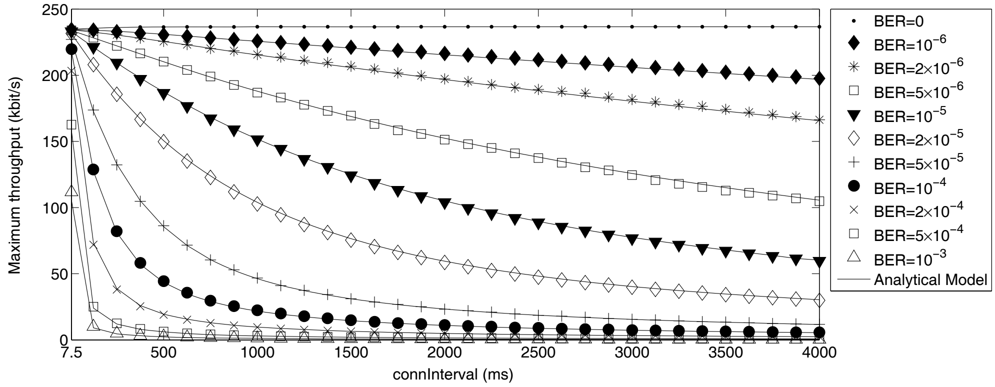

Fig. 2. Maximum throughput of a Bluetooth Low Energy link for various $connInterval$ and BER values: simulation (symbols) vs. analysis (lines).

TABLE I
PACKET AND PACKET FIELD SIZES

Parameter	Definition	Size (bytes)
L_{AA}	Access Address size	4
L_{Ack}	ACK (i.e., empty packet) size	10
L_{Data}	Maximum data packet size	37
L	Max. application layer payload size	20

set. The term $\Phi_0(i,k)$, then, corresponds to the number of possible sequences that would result in a nonconsecutive set of A. Based on [4], $\Phi_0(i,k)$ can be written as

$$\Phi_0(i,k) = \binom{k-(k-i)+1}{k-i} = \binom{i+1}{k-i}. \tag{16}$$

The derivation of $\Phi_1(i,k)$ requires considering two separate cases. In the first one, the two consecutive unsuccessful round trips are the first two or the last two round trips of the connection event. Thus, the round trip after the first two or the one that precedes the last two, respectively, is a successful round trip. Hence, the number of combinations of k round trips where only the first two or the last two are consecutively unsuccessful, denoted by N_{1A}, is

$$N_{1A} = 2\Phi_0(i-1,k-3) = 2\binom{i}{k-i-2}. \tag{17}$$

In the second case, the two consecutive unsuccessful round trips are neither the first two, nor the last two round trips of the connection event. Hence, the round trip that precedes, and the round trip that follows the two consecutively unsuccessful ones are both successful. In consequence, the number of combinations of k round trips where there are exactly two consecutive unsuccessful round trips that are not the first two or the last two ones, denoted by N_{1B}, can be found as

$$N_{1B} = (k-3)\Phi_0(i-2,k-4) = (k-3)\binom{i-1}{k-i-2}. \tag{18}$$

Considering (17) and (18), $\Phi_1(i,k)$ can be expressed as $N_{1A}+N_{1B}$, that is,

$$\Phi_1(i,k) = 2\binom{i}{k-i-2} + (k-3)\binom{i-1}{k-i-2}. \tag{19}$$

Finally, plugging (2)-(16) and (19) into (1), the maximum achievable throughput of BLE can be found analytically.

IV. EVALUATION

To validate the analytical model derived, we performed simulations for various BER and $connInterval$ values. For each parameter set, 1,000,000 connection events are simulated. The BER value is ranged from 0 to 10^{-3}, i.e., up to the BER value for which the receiver sensitivity is defined in BLE standard [1], whereas the $connInterval$ is varied between 7.5 ms and 4000 ms, i.e., between the minimum and maximum values allowed for this parameter.

Table I shows other parameter settings of the evaluations. The values used for L_{AA} and L_{Ack} are the ones defined by the Bluetooth specification [1]. In order to evaluate the maximum BLE throughput, we set the maximum payload size of notification messages allowed in the specification by default, i.e. 20 bytes. The corresponding data packet size is 37 bytes, which is also the maximum data packet size allowed.

Fig. 2 illustrates the results obtained by the analytical model and by the simulation runs. As shown in Fig. 2, the analytical model is accurate in predicting the maximum BLE throughput. In absence of bit errors, the maximum achievable BLE throughput is 236.7 kbit/s. For high BER values, BLE throughput decreases rapidly with $connInterval$, because the connection events end prematurely and a large amount of time cannot be exploited for data transmission.

V. CONCLUSION

We have presented an analytical model of the maximum BLE throughput as a function of BER and $connInterval$. Simulation results validate the analytical model derived.

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