E4.50: Sustainable Energy Systems

Part 2: Sustainable Energy Technologies

1 Wind Energy

Wind has always been an important source of energy for human use. It has powered sailing ships for thousands of years and some relatively complex wind-driven irrigation projects in the Middle East (especially in Persia) have been traced back to 250 B.C. Very early wind machines were of the vertical-axis type and they evolved into the horizontal-axis type in the Arab nations. In Europe, the windmill for grinding grain became well established in the 11th century AD, with the "Dutch mill" arrangement. Some ambitious efforts were made to build electric generator wind turbines in the late 1970s and early 1980s in response to the dramatic oil prices rises of the early 1970s but as prices returned to lower levels the efforts dwindled. Nevertheless, the USA emerged from that period with large numbers of turbines installed. Wind energy was reinvigorated in the 1990s as the emphasis on renewable energy grew. Denmark and Germany are notable for their efforts in this regard (and are home to some of the important manufacturers) but many nations are now actively supporting wind energy development, Figure 1, and China is installing very many large wind farms, including 43 GW of new plant in 2011.

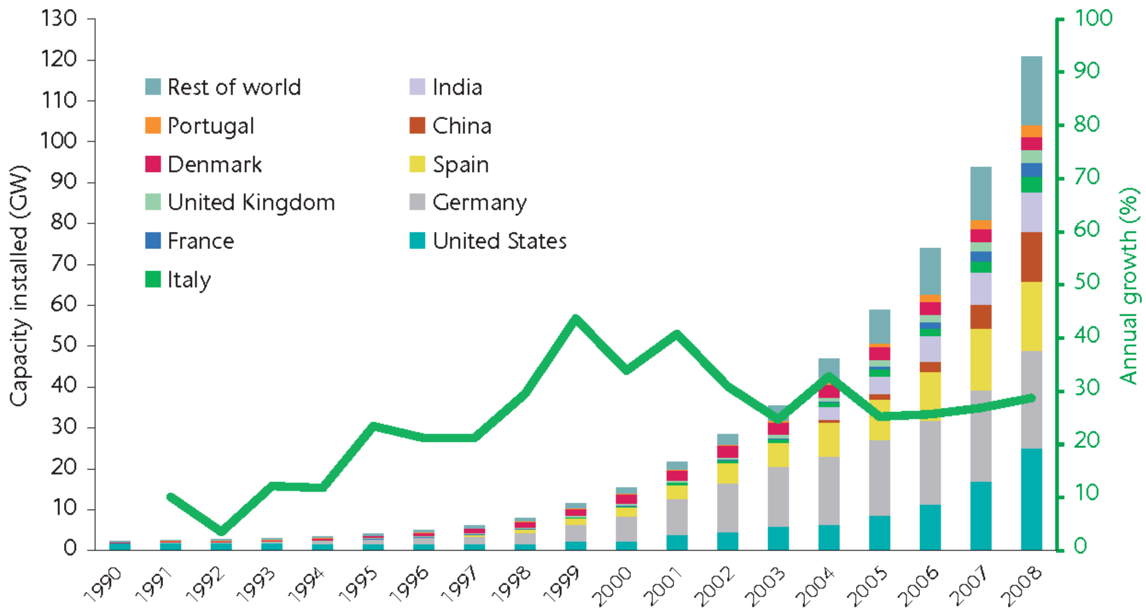


Figure 1 Install wind plant capacity by nation and annual global growth rate [Source: IEA Technology Roadmap: Wind Energy 2009]

The common form of wind turbine is illustrated in Figure 2. It is a horizontal axis machine with three blades (the typically number) rotating in a vertical plane. A strong but relatively slender tower supports a nacelle which contains the generator and various control elements, Figure 3. It is clearly important that the turbine faces into the wind. This requires the nacelle to rotate or "yaw" around the axis formed by the tower. The yaw is normally under active control using a set of motors. This video give a good view of key parts of a wind turbine http://youtu.be/LNXTm7aHvWc

Figure 2 A typical wind turbine [Source: Vesta Website]

The most obvious change in turbine design in recent years has been the drive toward larger diameter, high power turbines. Figure 4 shows the largest machine available over recent years. It has become clear that wind turbines scale up well and that wind farms are best composed of a relatively small number of large turbines. There have also been changes in the type of electrical generator used and we will return to this later.

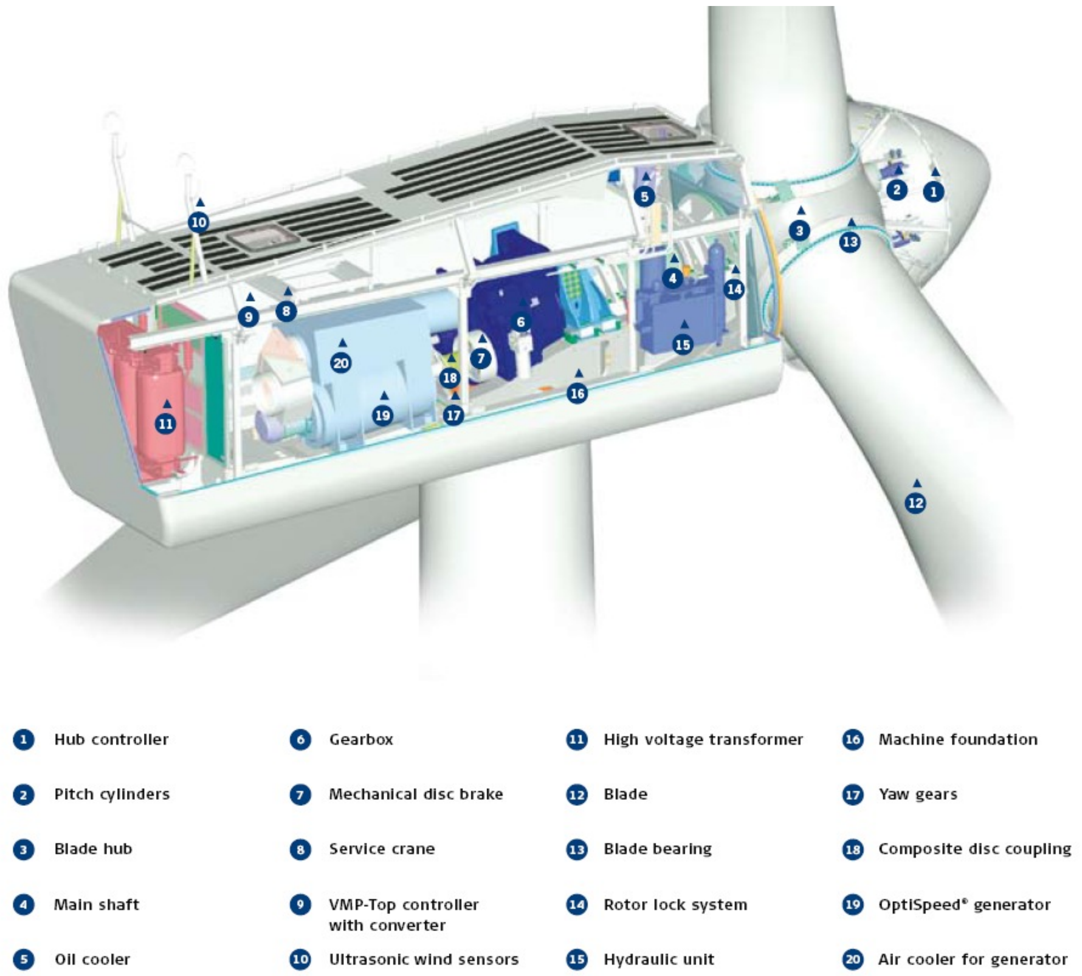


Figure 3 A cut-away diagram of a nacelle [Source: Vesta Website]

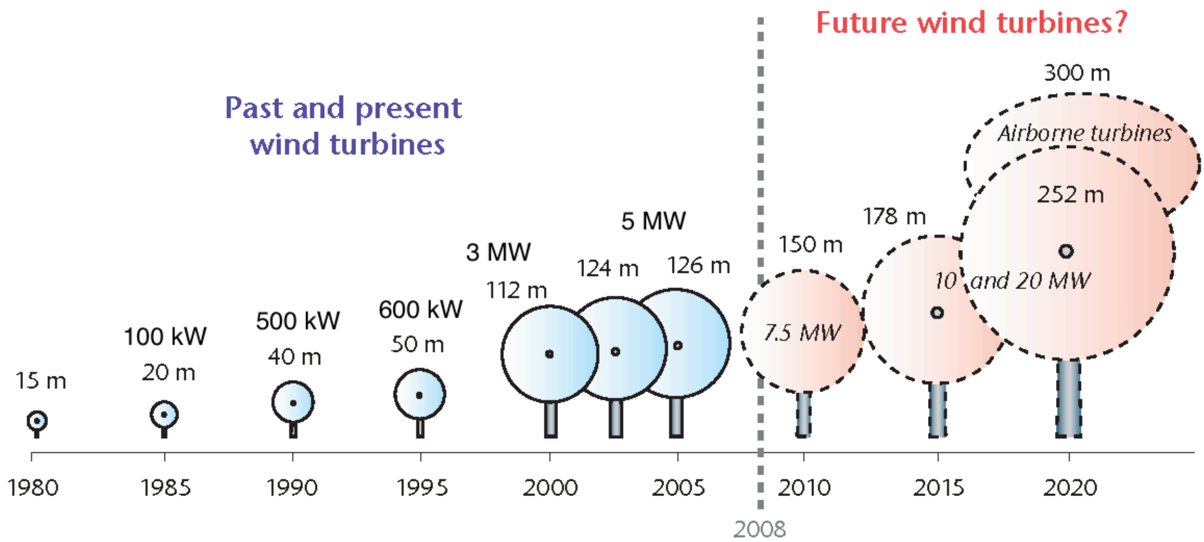


Figure 4 The growth of turbine size and power [IEA Technology Roadmap: Wind Energy 2009]

1.1 Extracting Kinetic Energy from a Moving Air Mass

A moving volume of air (composed of cross-sectional area, A [m^2] and a distance, x [m] in the direction of travel) has a kinetic energy of

$$E_k = \frac{1}{2} m V^2 = \frac{1}{2} \rho A x V^2 \qquad (3.1)$$

Where:

E_k	= Kinetic energy [J]
m	= Mass [kg]
ρ	= Air density [kg/m^3]
V	= Speed of the moving mass [m/s]

Air density is subject to variation according to the local conditions of temperature, humidity and pressure. At "standard" temperature and pressure conditions (which means 293 K and 1 bar) density of dry air is ρ=1.201 kg/m^3. For the UK, the most commonly cited figure in wind studies is ρ=1.29 kg/m^3, and for the US ρ=1.275 kg/m^3. Both figures are for dry conditions at 273 K.

To extract this kinetic energy we need to decelerate the moving air. If we decelerated the air to standstill we could extract a power equal to the kinetic energy in the volume of air being processed per unit time.

$$P_w = \frac{d}{dt} E_k = \tfrac{1}{2} \rho V^2 \frac{d(Ax)}{dt} = \tfrac{1}{2} \rho A V^3 \qquad (3.2)$$

Extracting energy from a fluid flow is a process that can not approach 100% efficiency. A simple view of this is that if we were to extract all the energy with a device by bring the air to rest, there would be an accumulation of still air at the device which would prevent more air arriving.

Figure 5 shows a simple representation of a turbine in an air-flow. The turbine blades exert a force on the air to decelerate it and the velocity drops. Although the velocity drops we assume the mass flow rate doesn't reduce and so the "stream tube" increases in area so that the same mass flow rate is achieved at lower velocity. In the treatment here we will assume that the pressure down stream once the wake has stabilised is equal to the pressure upstream. However, in exerting a force on the air mass, the turbine causes an increase in pressure in front of the blades and a drop in pressure behind the blades.

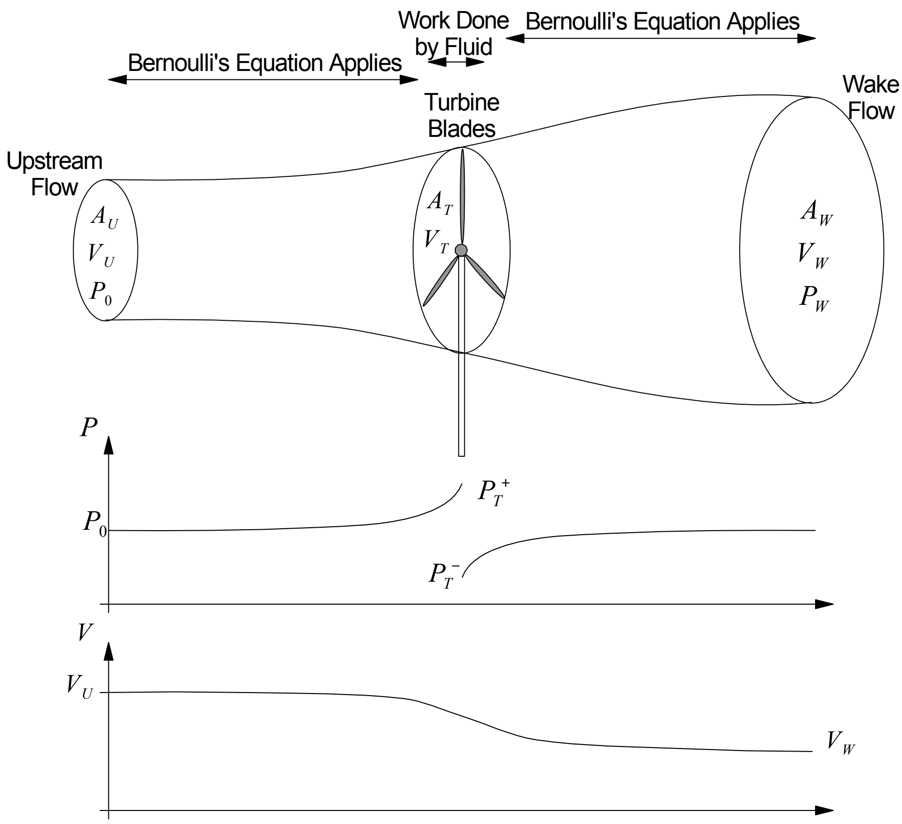


Figure 5 The stream-tube joining the upstream and wake of the flow over a turbine and the associated pressures and velocity of the flow.

We can determine the power extracted as air passes through the turbine as the product of the force exerted on the air and the velocity of the air.

$$P = F\,V \tag{3.3}$$

The wind velocity at the turbine will be lower than the upstream wind because the force is causing the air to decelerate. Here we can see that a small force will allow the air to pass over the blades at high velocity but will not extract much power. A higher force would be better but if the force is too high, the velocity drops too far and the power extracted falls. There will be a force that maximises the power extracted but we will not expect extraction of all of the available power.

Albert Betz undertook analysis of the problem in a general form in 1919 and the theoretical maximum energy extraction of a turbine is known as the Betz limit. The analysis proceeds using conservation of mass flow rate, momentum theory and Bernoulli's equation. Mass flow rate is the mass in the stream tube passing an observation point. The mass of air in a volume is simply the volume multiplied by the air density, ρ. The volume in question is the volume swept by the area, A of the stream tube in unit time, t and the distance swept, x is given by the velocity of the airflow.

$$\begin{aligned} m &= \rho A x \\ x &= Vt \end{aligned} \tag{3.4}$$

So the mass flow rate is given by mass per unit time:

$$\dot{m} = \rho A V \tag{3.5}$$

The stream tube drawn in Figure 5 is defined by the path taken by the mass of air that passes over the turbine (in other words mass flow rate is preserved within the stream tube).

$$\dot{m} = \rho A_U V_U = \rho A_T V_T = \rho A_W V_W \tag{3.6}$$

For an incompressible fluid, the density of the fluid is a constant. This is true of liquids in most cases and also true for gasses in simple flows like this provided that the velocity does not approach the speed of sound (taken to mean a speed of less than Mach 0.3) and so we can use a common value of ρ throughout our system.

A change in momentum comes about when a force is exerted on a mass for a period of time. The deceleration of the airflow between the upstream and wake represents a change in momentum.

$$\Delta(mV) = \rho A_U (V_U t) V_U - \rho A_W (V_W t) V_W \tag{3.7}$$

The conservation of mass in the volume of air moved (in unit time) means we can take that as a common term also.

$$\Delta(mV) = \rho A_U (V_U t)(V_U - V_W) \tag{3.8}$$

The rate-of-change of momentum indicates the force being exerted on the airflow. In our case, this is the force exerted by the turbine.

$$F = \frac{d}{dt}(mV) = \frac{\Delta(mV)}{t} = \rho A_U V_U (V_U - V_W) \tag{3.9}$$

By applying the principle of conservation of mass flow rate we can express the force in terms of the turbine area and velocity at the turbine.

$$F = \rho A_T V_T (V_U - V_W) \tag{3.10}$$

We also need to define something called the flow induction factor, *a*. It is a relative change in velocity. It is illustrated here for the velocity of the air at the turbine compared to the upstream velocity.

$$\begin{aligned} a &= \frac{V_U - V_T}{V_U} \\ V_T &= (1-a)V_U \end{aligned} \tag{3.11}$$

Finally we need to note that Bernoulli's equation states that the total energy per unit volume in a fluid is constant. This applies to flow of incompressible fluids along streamlines in steady conditions provided no work is done on the fluid.

$$\tfrac{1}{2}\rho V^2 + P + \rho g z = const \tag{3.12}$$

where *z* is the height of the streamline

1.1.1 Power Production as a Function of Flow Induction

The force exerted by the turbine on the air in the stream tube is determined by the cross-sectional area and the pressure drop. The velocity of the flow at turbine depends on the induction factor. Together these determine the power.

$$\begin{aligned} P_T &= F_T V_T \\ &= A_T\left(P_T^+ - P_T^-\right) \times V_U(1-a) \end{aligned} \tag{3.13}$$

The pressure change across turbine can be found from Bernoulli's equation. We will assume that there is no change in height of the stream tube so that the potential energy term can be ignored. We also recall that the pressure in the far wake is equal to the upstream pressure. We must apply Bernoulli's equation to the upstream and downstream flows separately because these are the regions in which no work is done on the air. Work is done as the air passes over the turbine.

On the upstream side:

$$\tfrac{1}{2}\rho V_U^2 + P_O = \tfrac{1}{2}\rho V_T^2 + P_T^+ \tag{3.14}$$

On the downstream side:

$$\tfrac{1}{2}\rho V_W^2 + P_O = \tfrac{1}{2}\rho V_T^2 + P_T^- \tag{3.15}$$

The pressure change is found from these two equations

$$P_T^+ - P_T^- = \tfrac{1}{2}\rho V_U^2 - \tfrac{1}{2}\rho V_W^2 \tag{3.16}$$

We also know that this pressure difference over the area of the turbine gives the force exerted on the airflow which in turn dictates the rate of change of momentum.

$$\begin{aligned} A_T\left(P_T^+ - P_T^-\right) &= \frac{d}{dt}(mV) = \rho A_T V_T \left(V_U - V_W\right) \\ &= \rho A_T V_U (1-a)\left(V_U - V_W\right) \end{aligned} \tag{3.17}$$

$$P_T^+ - P_T^- = \rho V_U (1-a)\left(V_U - V_W\right)$$

Combining these two equations gives a relation between the wake and upstream velocities.

$$\begin{aligned} P_T^+ - P_T^- = \rho V_U (1-a)\left(V_U - V_W\right) &= \tfrac{1}{2}\rho\left(V_U^2 - V_W^2\right) \\ \rho V_U (1-a)\left(V_U - V_W\right) &= \tfrac{1}{2}\rho\left(V_U - V_W\right)\left(V_U + V_W\right) \\ V_W &= V_U(1-2a) \end{aligned} \tag{3.18}$$

Note that the solution $V_W = V_U(1-2a)$ is only valid for a up to $a = 0.5$ (beyond which the assumptions used in the derivation no longer apply and the implication is that the wake flow has become negative). This result is interesting because it says that half the velocity reduction occurs upstream of the turbine and half downstream. In other words:

$$V_T = \tfrac{1}{2}\left(V_U + V_W\right) \tag{3.19}$$

Although this is sometimes stated as an assumption, it is not since we have been able to derive it. Putting this result back into the equation for pressure difference we have:

$$\begin{aligned} P_T^+ - P_T^- &= \left(V_U - V_U\left(1-2a\right)\right)\rho V_U\left(1-a\right) \\ &= 2\rho V_U^2\, a\left(1-a\right) \end{aligned} \tag{3.20}$$

So, returning to the power equation we now have power as a function of the flow induction factor:

$$\begin{aligned} P_T &= F_T\, V_T \\ &= 2\rho A_T V_U^3 a\left(1-a\right)^2 \end{aligned} \tag{3.21}$$

1.1.2 Power Coefficient, C_P

We define the power coefficient of a turbine as the ratio of the power extracted to power represented by the transfer of kinetic energy in the unimpeded flow:

$$\begin{aligned} P_T &= C_P P_K \\ &= \tfrac{1}{2} C_P \rho\, A_T V_U^3 \end{aligned} \tag{3.22}$$

From which we obtain

$$\begin{aligned} C_P &= \frac{P_T}{P_K} = {2\rho A_T V_U^3 a\left(1-a\right)^2} \Big/ {\tfrac{1}{2}\rho A_T V_U^3} \\ &= 4a\left(1-a\right)^2 \end{aligned} \tag{3.23}$$

1.1.3 Maximum Power Limit (the Betz Limit)

If we take the power coefficient and differentiate with respect to a we can find the maximum power point.

$$\frac{d}{da}C_P = \frac{d}{da}4\left(a - 2a^2 + a^3\right) = 4\left(1 - 4a + 3a^2\right) = 0 \tag{3.24}$$

We obtain a=1/3 for the maximum point and

$$C_P^{Betz} = 4 \times \tfrac{1}{3}\left(1 - \tfrac{1}{3}\right)^2 = \tfrac{16}{27} \approx 59.3\% \tag{3.25}$$

The Betz limit can be approached in practice but not exceeded regardless of the design of the turbine. Practical values of the power coefficient depend on the construction of the turbine and are influenced by:

- blade aerodynamics
- blade pitch angle
- non-ideal fluid phenomena (such as swirling)
- friction losses at the turbine rotor
- frame/tower obstruction.

These factors are mostly determined by the design of the turbine and are then unchangeable but the blade pitch is something which can be changed during operation in a way that allows the power extracted to be controlled.

1.2 Blade Speed and Number of Blades

It is noticeable that wind turbines have very few blades (normally three) compared to a gas turbine or steam turbine. Interestingly, wind-driven water pumps use more blades than wind turbines for electricity generation, Figure 6.

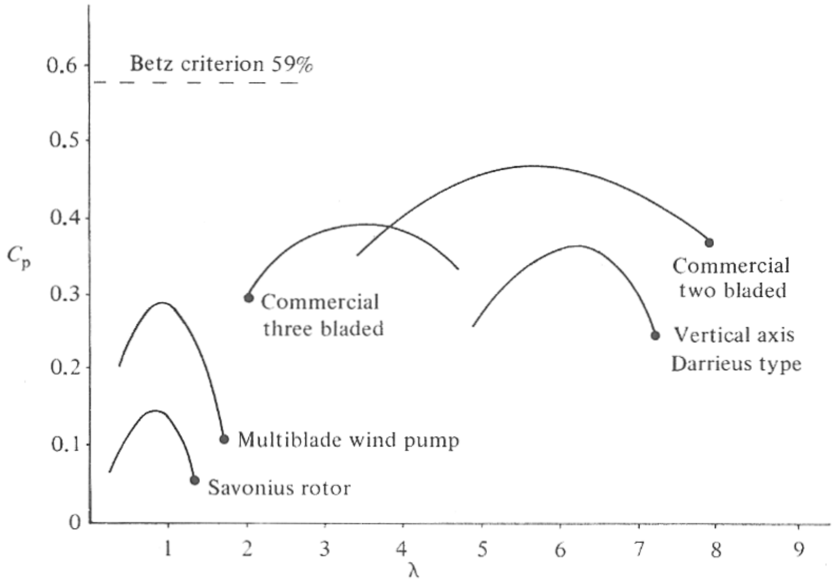
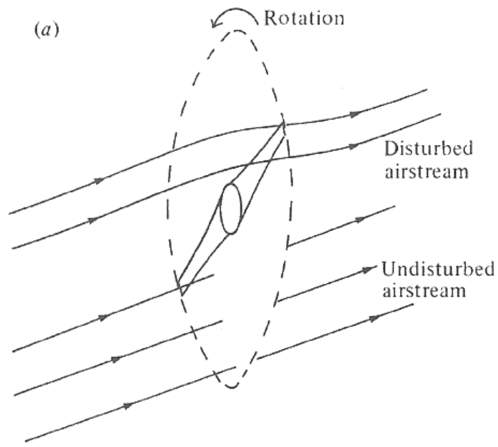


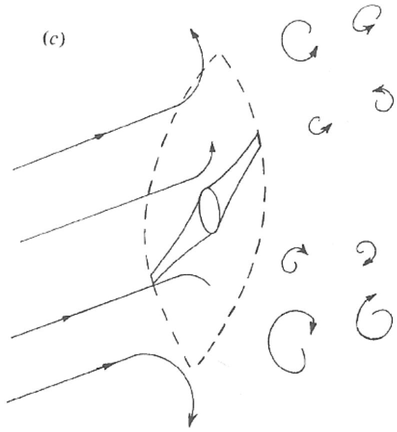
Figure 6. Power coefficient versus TSR for a variety of wind turbines

The reason is to do with the ratio of the flow rate to the turbine speed. The designer needs to find a compromise between two factors (and some ill-informed criticism of wind turbine design ignores this).

1. If the blades are too far apart or rotating too slowly then some of the air passes through the cross-section of the turbine between the blades without interacting with the blades then some useful work is missed and the C_P value will be low.



2. If blades are too close together or rotating too rapidly then turbulent air created by one blade interferes with the operation of the following blade and again the C_P value will be low.



In the case of a wind turbine, we are dealing with a relatively slow fluid flow (in the region of 10 m/s) and yet trying to achieve a high rotational speed for the generator (because higher speeds mean smaller sized generators). For this reason, a low number of blades is best (and even so a gearbox may still be needed to get a high enough generator speed). Two-blade, and even one-blade, turbines have been tried but have various problems occur with interactions with "shadow" of the tower or with balancing dynamic forces. Three blades is the practical minimum.

(In a gas turbine, the fluid flow is very fast and the problem is to hold the rotation speed down so that the bearings don't fail. Rotational speeds are limited to about 100,000 rpm but even this very high speed requires a very large number of blades to be used.)

1.3 Blade Aerodynamics

The traditional wind mill utilised drag produced by flow over the blades to give a force to rotate the blades. Modern wind turbines have blades that are operated more like aircraft wings to give a lift force. The lift force produced depends on the aerofoil design and on the "angle of attack" between the chord-line of the aerofoil and the flow direction of the air. The lift force acts perpendicular to the flow and the drag force is in line with the flow.

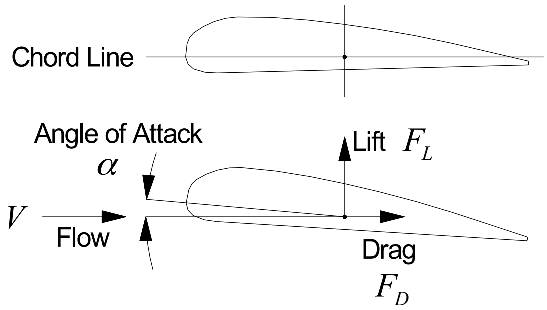


Figure 7 Lift and drag forces on an aerofoil

The airflow over blade is composed of two components: the airflow due to the wind and the flow because the blade is itself moving (rotating). In Figure 8 a blade is shown that is rotating but presently in the horizontal position such that its velocity, V_B, at this point is vertically upwards (note that V_B depends on the radial distance from the hub and so is not constant along the blade length being greatest at the tip). The flow over the blade, V', is thus the vector sum of the air flow past the turbine, V_T, (acting left to right) and a flow due to the movement of the blade, $-V_B$, (acting top to bottom) and acts at an angle of θ with respect to the plane of rotation. The blade is shown pitched into the flow by an angle β relative to the plane of the blades. The difference between the flow angle, θ, and the pitch angle, β, gives the angle of attack, α upon which the lift and drag forces depend. In recent designs of turbine, the blades can be rotated about their own axis (pitched) to control the angle of attack and control the lift and drag forces.

The lift and drag forces need to be resolved into components in the plane of rotation to give the force F_{Rot} that causes rotation and perpendicular to the plane of rotation to give force F_{Axial} which axis along the axis of rotation and has to be resisted by the tower of the turbine.

The angle of attack of the blade is key in determining the magnitudes of the lift and drag forces. The angle of attack depends on how the blade is pitched (β) but also the relative flows due to the wind and the blade rotation (V_T and V_B). In fact the performance of the turbine as whole can be characterised in terms of the ratio of the velocity of the blade tip to the velocity of the wind, known as the tip-speed ratio:

$$\lambda = \frac{\omega_T R}{V_U} \tag{3.26}$$

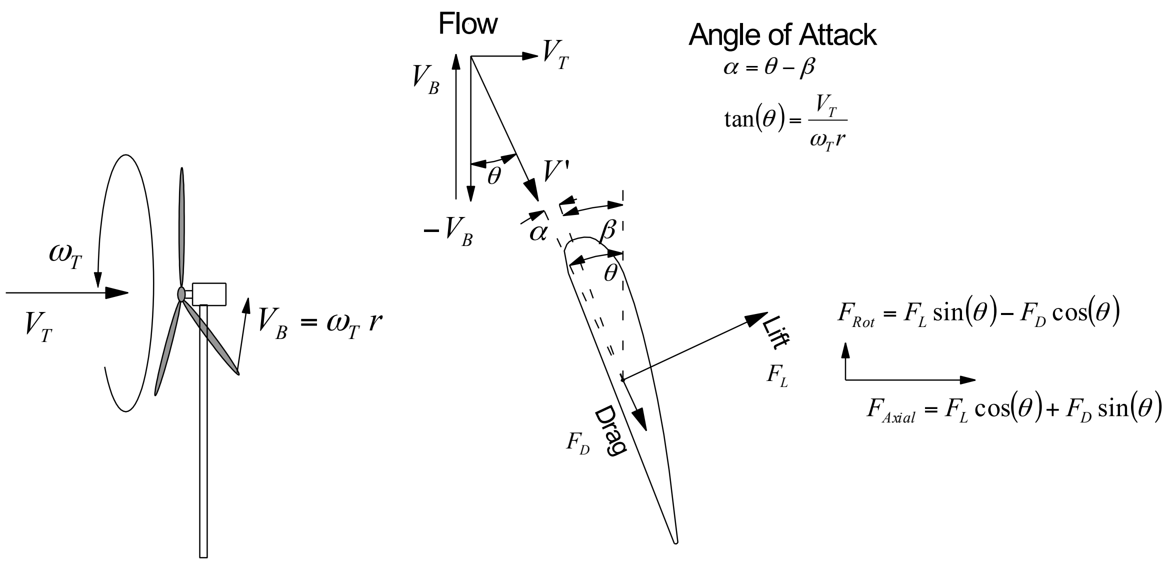


Figure 8 Lift and drag forces on a rotating blade

If *Cp* is plotted against the wind speed, *V*, there is a different curve for every value of the rotational speed of the blades, ω_T. Similarly, if *Cp* is plotted against ω_T, there is a different curve for every value of *V*. The number of curves involved makes it very difficult to visualise the characteristics of *Cp*. However, if the power coefficient is plotted against the tip-speed ratio, a single characteristic curve is obtained. This single curve incorporates *V* and ω, and it provides an easier way to evaluate the performance of a particular wind turbine. This is the reason why the tip speed ratio is so widely used by manufacturers, developers and researchers. A typical *Cp* versus tip speed ratio characteristic is shown in Figure 9.

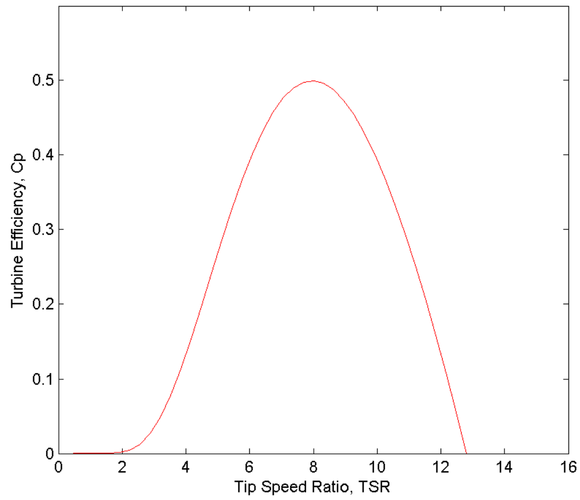


Figure 9. Power coefficient, C_P versus tip-speed ratio, λ

The Figure 9 above indicates that for a particular wind speed, there is a tip-speed ratio that maximises the power extraction from the wind. The common three-bladed turbine has an optimal λ of about 4 but individual designs vary around this value.

1.3.1 Power Output as Function of Wind and Rotational Speed

Recognising that the power coefficient is a function of the tip speed ratio, the power equation is

$$P_T = \tfrac{1}{2} C_P(\lambda) \rho A_T V_U^3$$

If we take some example wind speeds, the typical $C_P(\lambda)$ curve and an example blade radius (12m here), we can plot the power extraction. Figure 10 shows a series of curves for wind speeds between 3 m/s and 10 m/s. Clearly as the wind speed increases more power is obtained according to a cube law. It is also clear that for each wind speed there is an optimum rotational speed to be used and that this optimal speed increases with wind speed.

This last observation is important since it implies that in order to most effectively capture wind energy we need a generator that is able to vary its speed. In other words, the generator cannot be synchronous with the grid frequency if it is to be optimal in capturing wind.

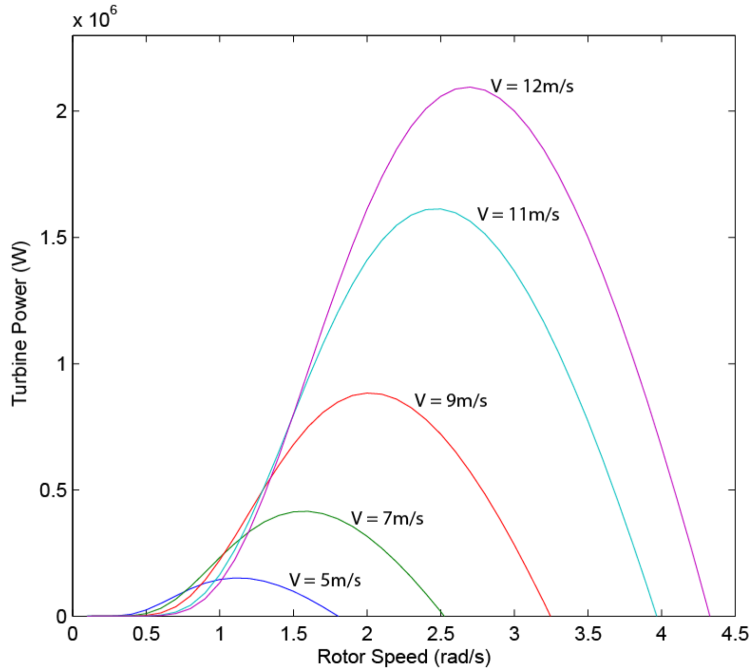


Figure 10 Example power extraction as a function of rotational speed and wind speed for various wind speeds

1.3.2 Pitch Control

The C_P curve shown in Figure 9 is for a turbine where the blades have not been pitched. If the blade pitch angle is changed then we would obtain a different curve for each angle such as shown in Figure 11. Increasing the pitch angle tends to reduce the angle of attack and reduce the power being produced. The control is quite sensitive and only about 20° of pitch is required to reduce the power extracted from maximum to less than 20%. One has to bear in mind that the blades of the turbine are large and heavy and pitching is typically at a rate-of-change of about 15° per second. This is fast enough for good regulation of the turbine but slow compared with some of the disturbances in the grid to which a turbine might have to react.

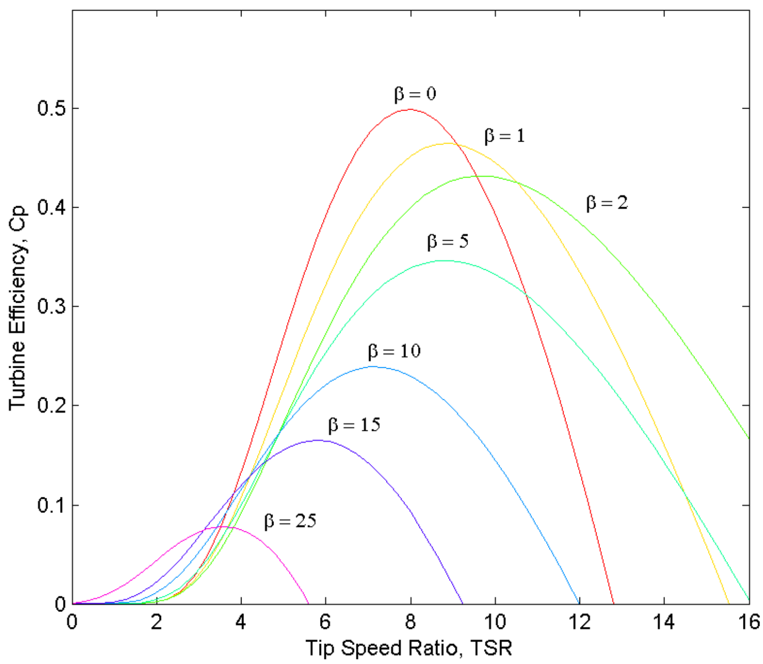


Figure 11 Power Coefficient as a function of tip-speed ratio and blade pitch

1.4 Control Regime

1.4.1 Physical Limits on Speed and Torque

Winds speeds vary across a wide range in stochastic manner. Strom force winds occur occasionally and the turbine must be strong enough to at least withstand these wind speeds if not actually generate power under those conditions. All the elements of the drive train of a turbine (the blades, the hub/shaft, the gearbox, the generator, the electrical path) will have some physical limitations. The blades must support the centripetal forces and the stress limitation of the material will place a limit on the maximum speed of rotation. The gearbox and generator will have been designed to have a corresponding maximum speed rating. The gearbox (and probably other elements too) will have been designed for some maximum torque capability and the generator will have been designed for a corresponding maximum current rating. In section 1.5 we will discuss how a designer might choose what maximum torque and maximum speed to design for given some knowledge of wind speeds. For now we just need to note that the control regime must respect these physical limitations of the plant and ensure operation remains in a safe range.

We also need to note that the product of speed and torque is power and so the physical limits on speed and torque give rise of a maximum power capability of the turbine. Also note that for a turbine, the torque and power are related to speed (for a constant angle of attack). For these reasons we normally just talk about trying to observe a power limit for the turbine but underlying this are individual speed and torque limits.

An example of failure to observe a speed limit can be seen here: http://youtu.be/CqEccgR0q-o

This turbine was an early design: a stall regulated fixed speed generator which was designed to stall in storm force winds and then have a parking brake applied. The parking brake had failed to operate and the run-away in turbine speed has dramatic consequences. The reason that news crews were there to record it is that the turbine controller had reported the fault via a modem link to a control centre and the area had been cordoned off to avoid injury.

1.4.2 Control Regions: Maximum Power and Constant Power

There are two basic regions of control. The first region is for relatively low wind speeds where the power being produced is less than the turbine rated (i.e. maximum) value. Here we wish to optimise the power by using the optimum pitch angel and the optimum tip-speed ratio. Thus, the pitch angle is held constant at the value that gives highest C_P curve (typically β=0) and the turbine rotational speed should then be set to achieve the optimal tip-speed ratio for the prevailing wind speed so that C_{Popt} is achieved. Turbine speed is itself controlled by small adjustments to the generator reaction torque. The generator reaction torque is determined by the current drawn from the generator. This optimum power region is the left hand side of Figure 12 where the operating curve (red) is a cube-law curve passing through the peaks of the family of power aginast rotor speed. The figure is actually plotted against generator is speed; the generator speed is often higher than the turbine speed because a gearbox is used).

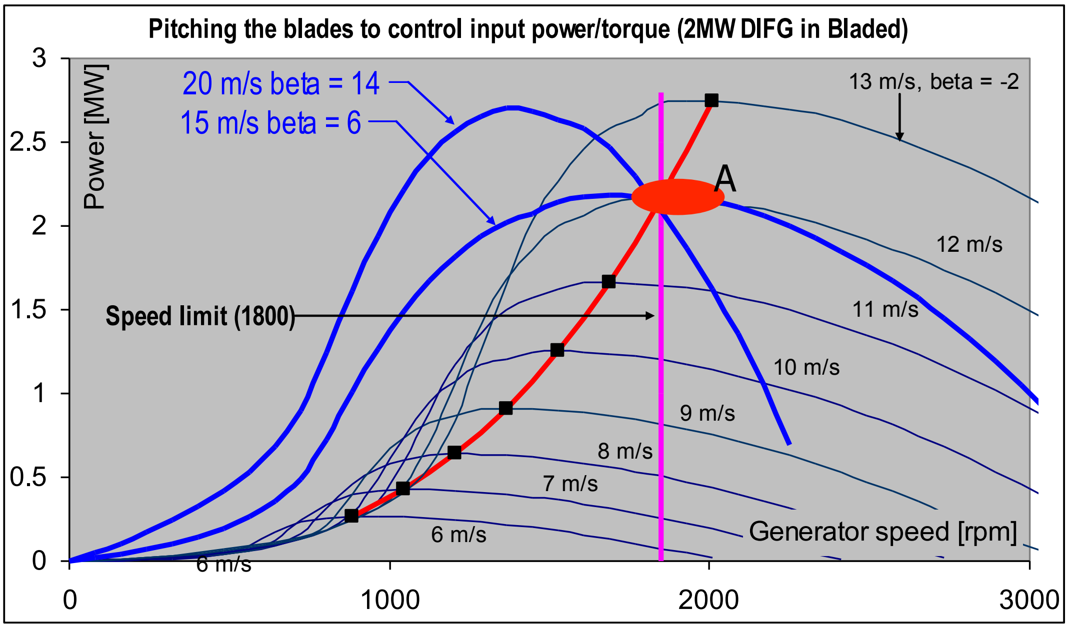


Figure 12 Use of Pitch Control to Limit Turbine Speed and Power (Source: N Jenkins, Cardiff University]

The second region is actually the small region mark A in Figure 12 that lies at a generator speed of 1800 rpm and a power of 2MW which represents the maximum operating point of the generator. Clearly, if the turbine speed is being held constant at its maximum value while the wind speed rises, the turbine is not being operated at $\lambda = \lambda_{opt}$ and this is not required since we are also seeking to limit the generator power at its maximum rather than take every thing these very high wind speeds could provide. To hold the turbine at this speed, the blades are pitched to reduce magnitude and change the shape of the of the C_P curve as represented by the blue curves. The pitch angle for these higher wind speeds has been chosen so that they pass through point A. The electrical controls of the generator are set to extract exactly 2MW and the turbine speed is made the subject of a speed control loop in which the blade pitch angle is changed in response to speed error. In effect, the blade pitch is continuously adjusted to keep the power developed by the turbine equal to the power being taken from the generator.

Figure 13 is another view of the "optimal power" and the "constant power" regions. In this figure, various quantities have been plotted against wind speed (not turbine or generator speed). To the left operation is at $\beta = \beta_{opt}$ and $\lambda = \lambda_{opt}$ so that $C_P = C_{Pmax}$ and the power increases as the cube of wind speed. To the right, the power and turbine speed are held constant by pitching the blades and also reducing the tip-speed ratio.

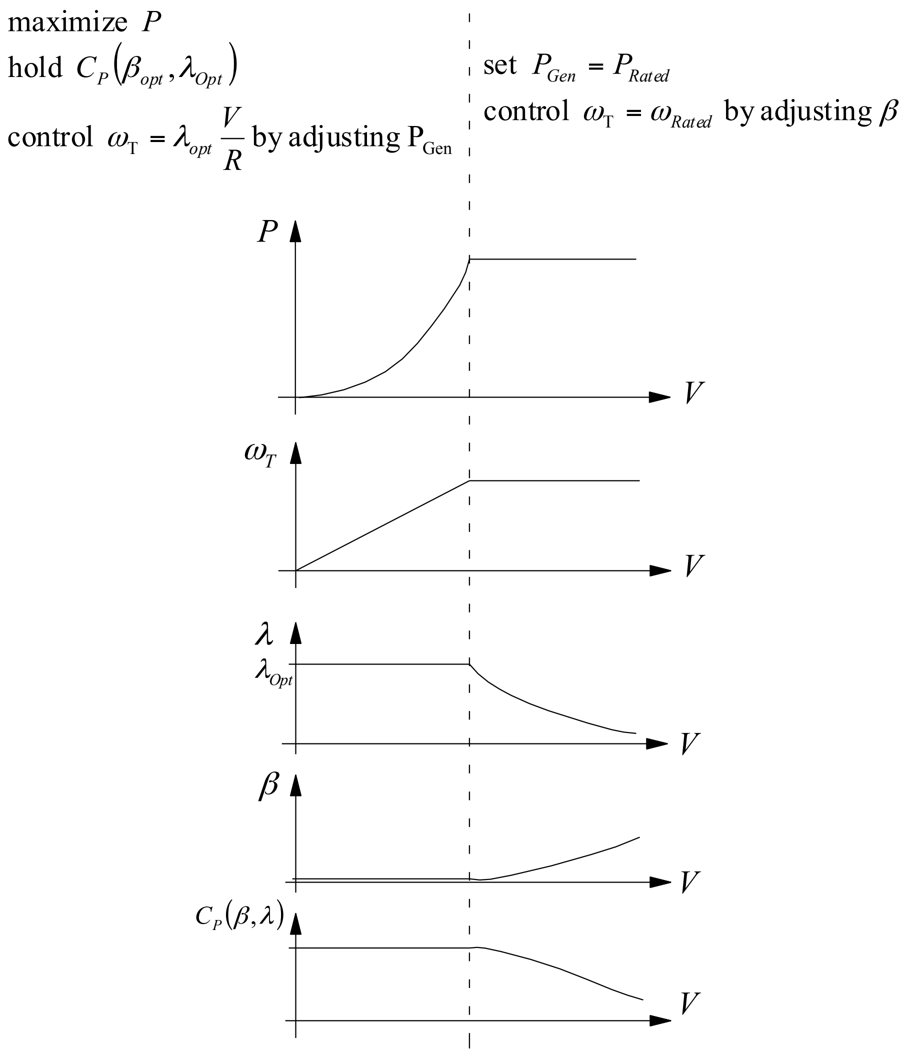


Figure 13 Turbine operating regions

Some further explanation is needed of how the turbine is made to operate at the maximum of the C_P curve in the "optimal power" region in the face of variation of wind speed. The main task is to identify the optimal rotational speed for the turbine. The obvious approach is to measure the wind speed and directly determine the optimal rotational speed from knowledge of the optimal tip-speed ratio of the turbine blades. The generator torque could then be used to manipulate the speed in a closed-loop speed regulator. The difficulty is in obtaining a good measurement of the undisturbed upstream wind speed.

In many implementations, a different approach is taken which relies on some inherent negative feedback that forms a stable equilibrium (a stable speed) if the generator torque is set according to the expected optimal torque. From the power we can find the torque expected at the maximum power point and we can express that power as a function of rotational speed assuming that the optimal tip-speed ration applies.

$$\begin{aligned} P_T &= \tfrac{1}{2}\rho A_T C_P(\lambda,\beta) V_U^3 \\ &= \tfrac{1}{2}\rho A_T C_P(\lambda,\beta)\frac{\omega^3 R^3}{\lambda^3} \end{aligned} \tag{3.27}$$

Assume that $\beta = \beta_{opt}$ and $\lambda = \lambda_{opt}$ so that $C_P = C_{Pmax}$

$$P_{Max} = \tfrac{1}{2}\rho A_T C_{Pmax} \frac{\omega_{opt}^3 R^3}{\lambda_{opt}^3}$$
$$T_{MPP} = \frac{P_{Max}}{\omega_{opt}} = \tfrac{1}{2}\rho A_T C_{Pmax} \frac{\omega_{opt}^2 R^3}{\lambda_{opt}^3} \tag{3.28}$$

This equation for the expected torque at the maximum power point is then applied as the set point for generator torque using the measured rotational speed of the turbine.

$$T_G = \tfrac{1}{2}\rho A_T C_{Pmax} \frac{\omega_T^2 R^3}{\lambda_{opt}^3} \tag{3.29}$$

Operation is illustrated in Figure 14. The solid red line is the optimal generator torque and the dashed lines are the turbine torque for three different wind speeds (obtained by dividing power by speed). Assume that the wind speed is V_a , for which the turbine torque and optimal generator torque characteristics cross at point A at slightly above 1.5 rad/s. At point A, there is no net torque on the shaft and so the speed is constant. If the speed was perturbed up a little, the turbine torque would fall and the control system would apply a higher generator torque. Both affects would tend to reduce the turbine speed back to point A which can be described as a stable equilibrium. If the wind speed increased to V_b, and in the first instance the turbine speed is unchanged then the turbine torque would have increased to that at B'. The generator torque unchanged because the turbine speed has not yet changed also (it would still be at point A) and so a torque difference appears on the shaft which is an accelerating torque which would move the turbine along the dashed line from B' to B. As that happens, the control system would increase the generator torque according to equation 3-29 which is the solid line from A to B. At B a new stable operating point would be achieved.

Each of the operating points identified in Figure 14 will yield the maximum power for the prevailing wind speed (provided the characterisation of the turbine in equation 3.29 has been accurate). This is illustrated in Figure 15 where one should note that the if the torque / speed curves on the upper axes is multiplied by speed one obtained the power / speed curves on the lower axes.

Alternative Approaches

Instead of decreasing the angle of attack in high wind speeds one can increase it. This will begin to give a turbulent flow on the top surface of the blade meaning the lift reduces. This is described as stall. Some turbines use this affect to limit the power in high wind speeds. This is known as active stall. Controlled pitch (pitching towards the "feather" position) and active stall (pitching toward the stall position) can both achieve power control. The relative merits are:

Active stall

- Small (slow) pitch actuators
- The total span of the pitch angle is relatively small
- The size of the brake required is smaller than in passive stall

Controlled pitch

- May also be deployed at speeds lower than the rated speed
- The total span of the pitch angle is relatively large
- Requires larger (faster) pitch actuators
- The size of the brake is smaller than in active stall

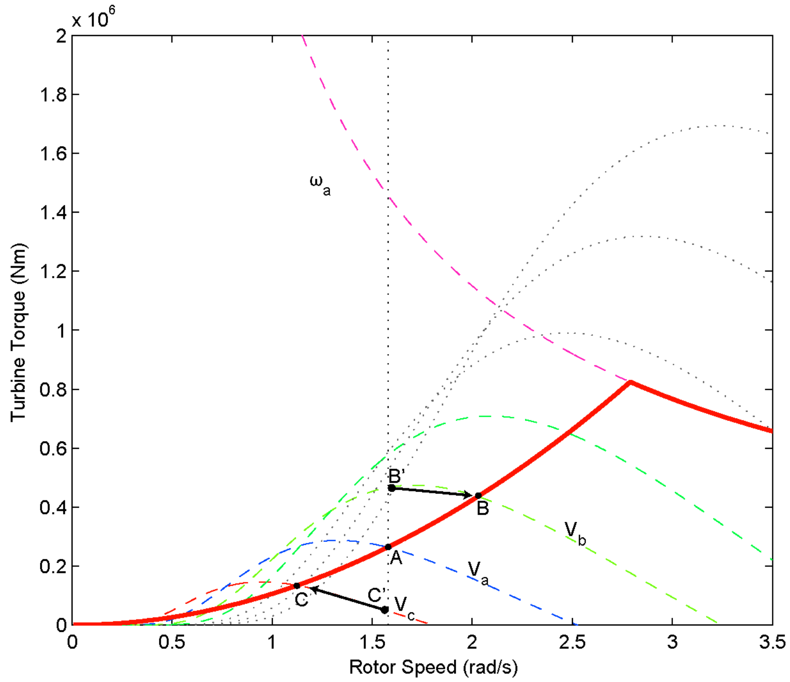


Figure 14 Illustration of equilibrium around optimal torque characteristic (source, P. Clemow, PhD thesis, Imperial College)

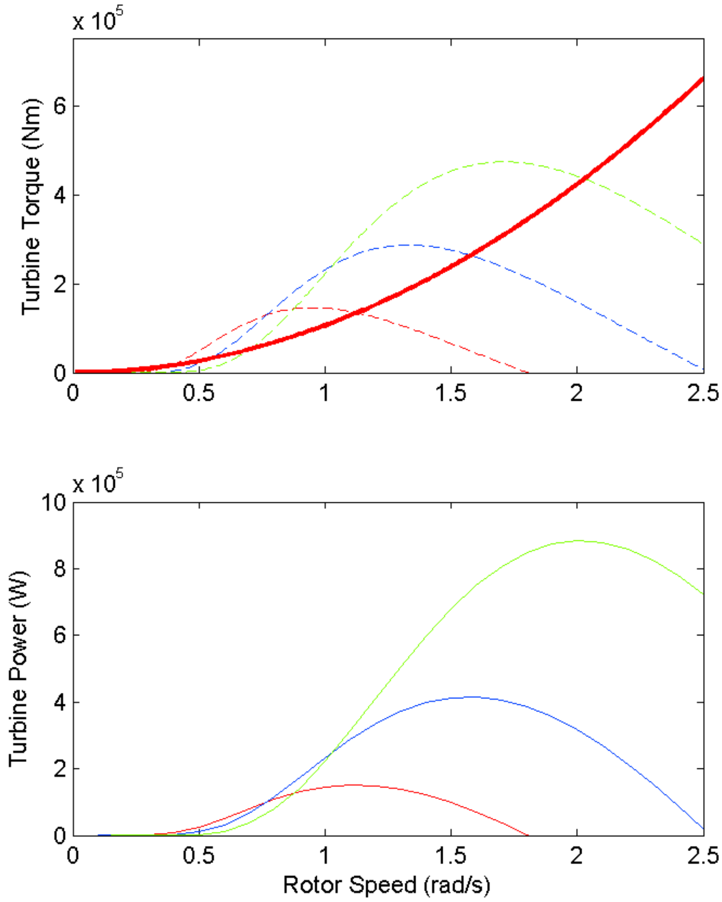
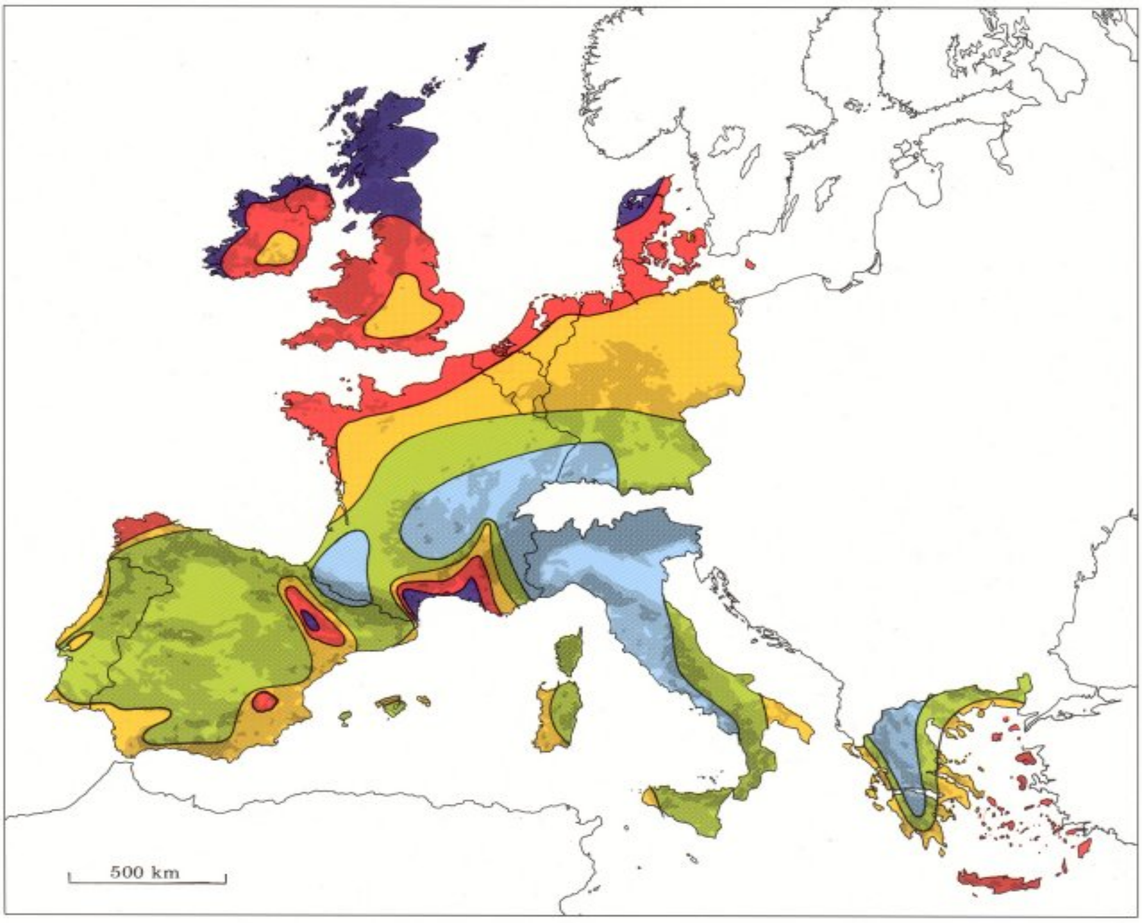


Figure 15 Comparison of Torque/speed and Power Speed/speed curves showing location of maximum power point (MPP)

1.5 Influence of Wind Speed Statistics on Turbine Design

So far we have not discussed what wind speed is expected. It is common to see the wind pattern across a country described in terms of the average wind speed that can be expected, for example Figure 16. Although this is a useful shorthand for describing good and bad sites for wind turbines, it does not provide nearly enough data for a proper assessment.

Figure 16 also reveals two other important factors. It states that the wind speed is for a height of 50 m above ground. The interaction of the airflow with the ground gives a drag which means that wind speeds are lower close to the ground dropping to zero at the surface (a boundary layer affect). Further, the extent of this affect depends on the roughness of the surface and so the figure quotes different wind speeds for different types of terrain.



Wind resources[1] at 50 metres above ground level for five different topographic conditions										
	Sheltered terrain[2]		Open plain[3]		At a sea coast[4]		Open sea[5]		Hills and ridges[6]	
	$m\,s^{-1}$	Wm^{-2}	$m\,s^{-1}$	Wm^{-2}	$m\,s^{-1}$	Wm^{-2}	$m\,s^{-1}$	Wm^{-2}	$m\,s^{-1}$	Wm^{-2}
	> 6.0	> 250	> 7.5	> 500	> 8.5	> 700	> 9.0	> 800	> 11.5	> 1800
	5.0-6.0	150-250	6.5-7.5	300-500	7.0-8.5	400-700	8.0-9.0	600-800	10.0-11.5	1200-1800
	4.5-5.0	100-150	5.5-6.5	200-300	6.0-7.0	250-400	7.0-8.0	400-600	8.5-10.0	700-1200
	3.5-4.5	50-100	4.5-5.5	100-200	5.0-6.0	150-250	5.5-7.0	200-400	7.0- 8.5	400- 700
	< 3.5	< 50	< 4.5	< 100	< 5.0	< 150	< 5.5	< 200	< 7.0	< 400

Figure 16 Average Wind Speeds in Western Europe

There is much historic data on wind speeds and it allows a statistical analysis of the properties of wind. Wind shows variations on several timescales, as seen in Figure 17 and Figure 18. In the very short term (seconds and minutes) "gusting" variation of the wind speed is apparent. In the timescale of seasons there are changes in the average wind speed. In the time frame of days we see significant variation due to changing weather patterns. Over period of an hour or a few hours, wind speed is reasonably predictable based on the recent history or a weather forecast.

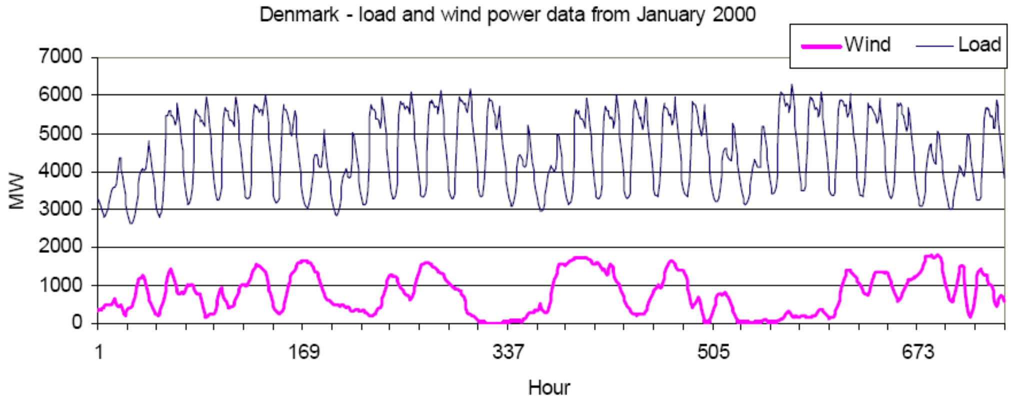


Figure 17 Wind Power Variation over 4 Weeks (in comparison to load variation)

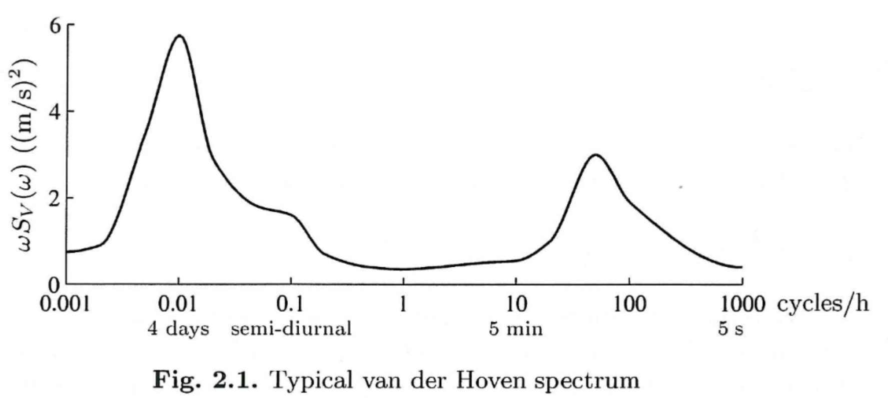


Figure 18 Typical "van der Hoven" spectrum of wind (F.D. Bianchi *et al.*, "Wind Turbine Control Systems")

Over a long period, the probability, $p(V)$ of a particular wind speed, V occurring is found to correspond closely to a statistical distribution known as a Wiebull distribution.

$$p(V) = \frac{k}{C}\left(\frac{V}{C}\right)^{k-1} e^{-\left(\frac{V}{C}\right)^k} \tag{3.30}$$

This distribution has two scaling parameters, k and C, that can be used to fit this distribution to a particular case. For assessing a wind turbine we multiply the probability by 8760 to obtain the number of hours per year for which a given wind speed would prevail. As an example, with k=1.8 and C=7.5, we obtain the times shown in Figure 19 for wind speed quantised into 1 m/s intervals. This graph shows that the Wiebull distribution is not symmetric and in particular there is a large "tail" on the high wind speed side meaning that very high wind speeds are possible but unlikely. In this case, the most likely wind speed (the mode) is 5 m/s but the average wind speed is 6.6 m/s.

However, wind speed is not really the issue, wind power is. We know that wind power is a cube law function of wind speed. Figure 20 shows this for a turbine with a swept area of 6,000 m^2 (a blade of about 90 m diameter). The power present in the range 1-3 m/s is very small, relative to the turbine size, and the turbine would not be operated in this range because it would generate less power than it itself consumes. This is described as having a cut-in speed (which is taken to be 4 m/s for this example).

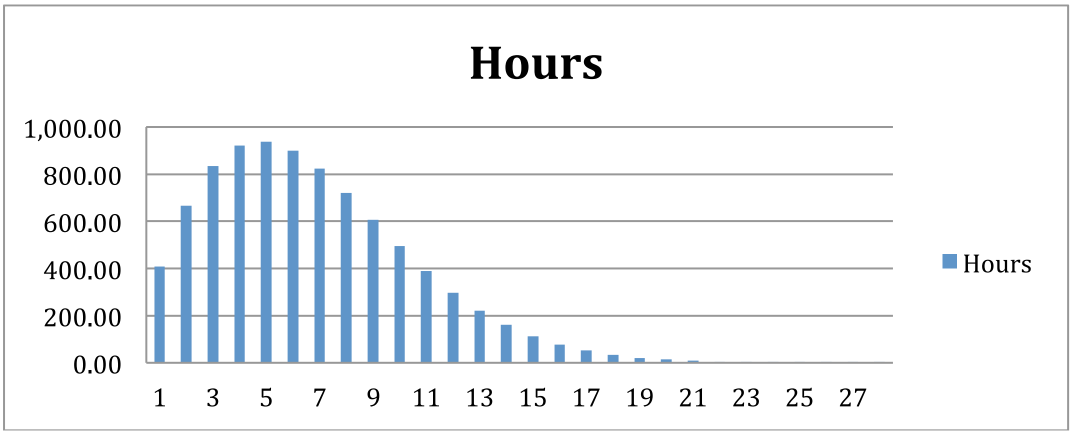


Figure 19 Time (in hours) for which each wind speed occurs

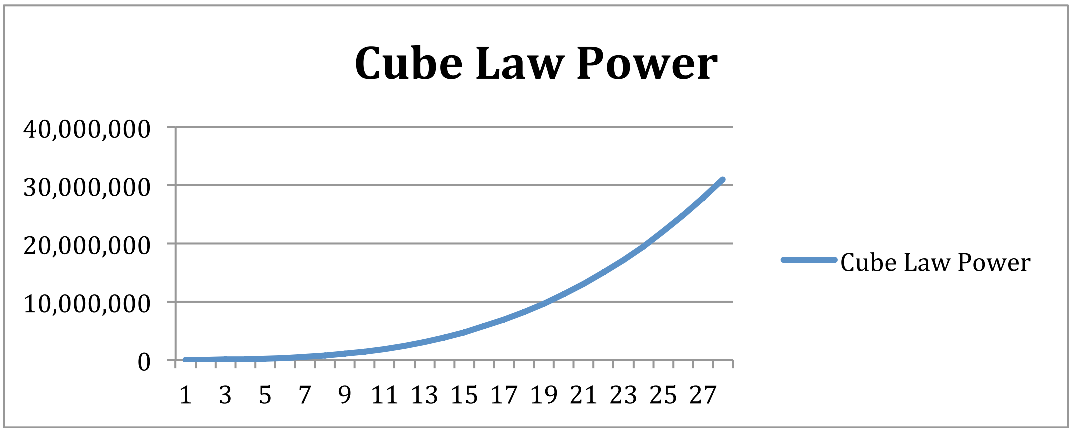


Figure 20 Cube Law Wind Power

So, the product of the distribution of wind speed (in hours) and the power law will give the energy yield (in watt hours, Wh) as shown in Figure 21. This graph reveals a lot about how turbine design and operation is approached. Wind speeds above 25 m/s are uncommon and so would yield very little energy. The forces exerted on the structure of the turbine (the tower) and on the drive train of the generator at such high wind speeds are very high and would require a very expensive structure. It is simply not economic to attempt to extract this energy. Instead, the wind turbine will be "cut-out" at 25 m/s so that it does not generate. This will be achieved by some combination of pitching the blades fully into the wind so they produce no lift and by applying a break so that the turbine stalls. This reduces the forces on the turbine structure but considerable forces remain.

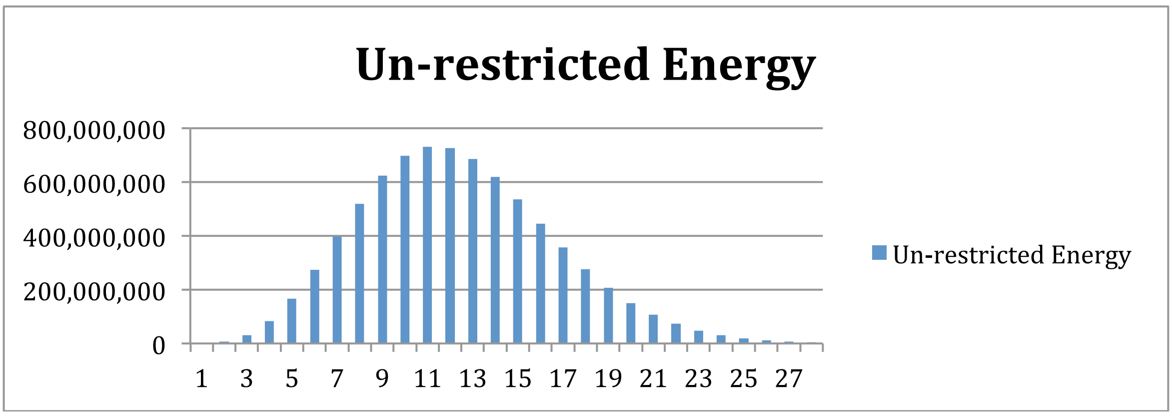


Figure 21 Ideal Energy Yield

The economic argument of balancing the cost of equipment against the energy yield extends further. From about 12 m/s upwards the energy yield is dropping but the size and cost of electrical generator needed to capture it needs to rise. In this range, it is best to limit the power generated so that a smaller generator can be used. The turbine will reach its power rating at a rated speed of (typically) 12 m/s and run in constant power mode above this speed. Below this speed, it will be optimised to run at the tip-speed ratio that yields the maximum power available. To limit the power in constant power mode, the blades are pitched into the wind as the wind speed rises. If pitch control is not included in the design, the blades will have been design to begin to stall at about the rated speed (known as stall control which can be active or passive)

With the cut-in, cut-out and constant power regions included, the power against wind speed graph is given by Figure 22. The corresponding energy yield is shown in Figure 23 in comparison to what could be obtained with an unrestricted turbine. It can be send that the energy not captured is small and the economic case is in favour of turbines designed this way.

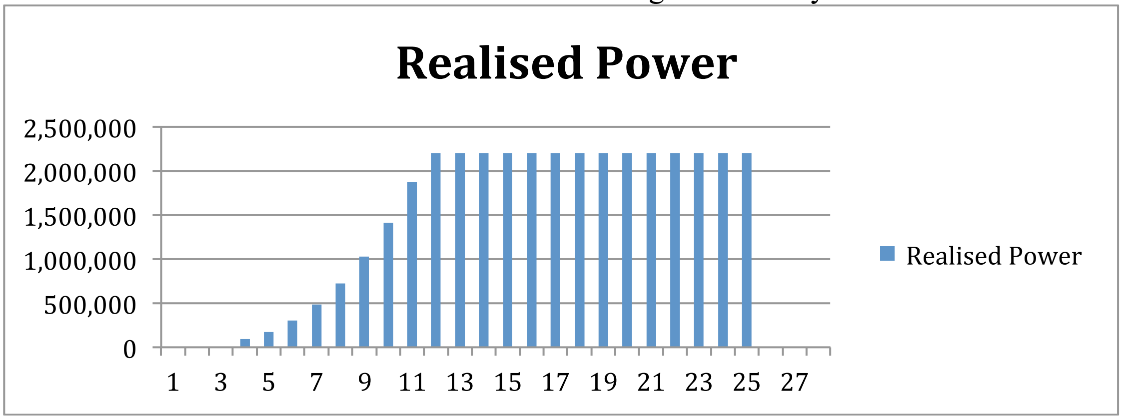


Figure 22 Power Yield of a 2.2 MW Turbine with a Rated Speed of 12 m/s

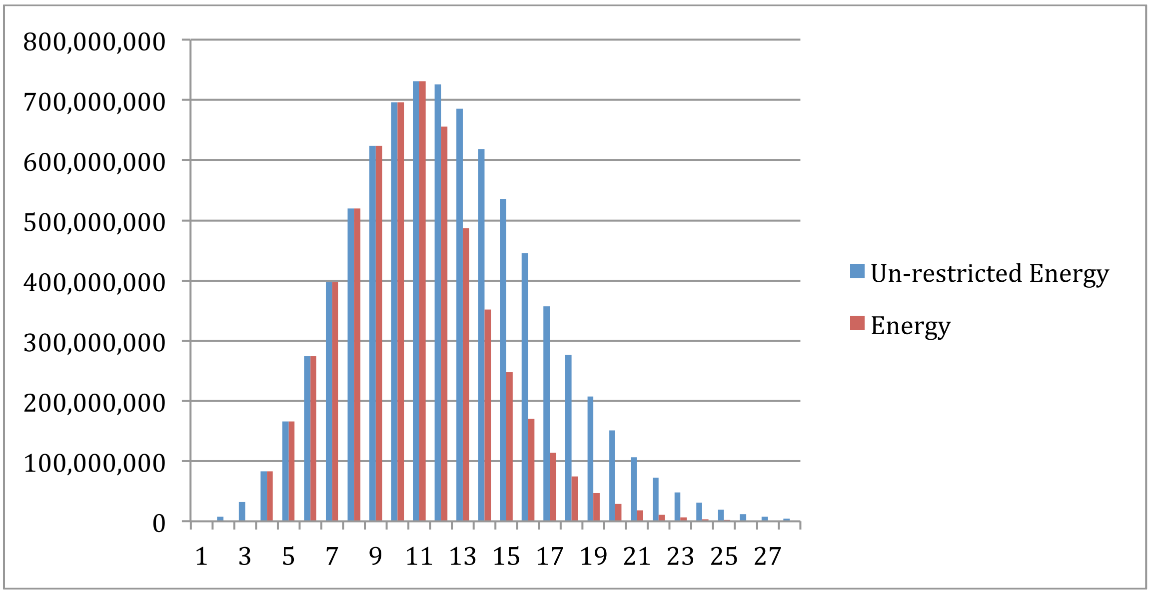


Figure 23 Turbine Energy Yield with and without Practical Limits

1.6 Vertical-Axis Wind Turbines

Although commercial large-scale turbines are almost exclusively of the horizontal-axis design, vertical-axis designs have been tried at many scales are much discussed for small-scale roof top installations. An example design from a company called Quiet Revolution is shown in Figure 24

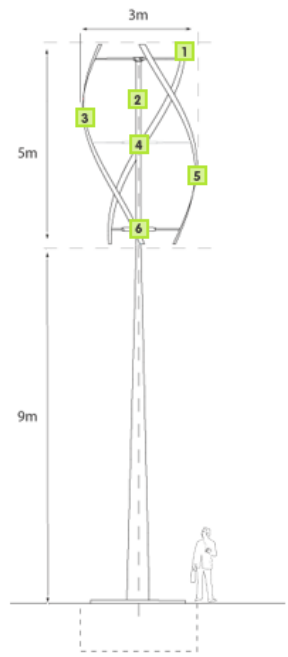


Figure 24 A 6 kW Vertical Axis Wind Turbine (VAWT) by Quiet Revolution

The reasons VAWT are being considered for roof-top urban sites is that they perform better than HAWT in wind that frequently changes direction, which is common for the disturbed wind patterns in urban settings. Some argue that they are also less visually intrusive. The drawbacks of VAWT are that they generally under perform in comparison to HAWT in winds of steady direction (because of the drag of the blade that is moving the "wrong way" in the airflow); they produce pulsating power (as blades move backwards and forwards through wind) and are prone to fatigue failures as a result of the pulsating forces on the blades.

1.7 Turbine Electrical Systems

In its early days of wind turbine development, a wide variety of generator and control scheme were tried. With time and experience, the industry has converged on small number of popular arrangements. The most important dividing point in a design is between fixed speed and variable speed operation.

1.7.1 Fixed Speed Turbines

With fixed speed operation, the rotational speed does not track up and down as the wind speed changes and therefore the turbine will not be operating at the peak of the $C_P(\lambda)$ curve expect for one special wind speed. There is obviously a penalty to be paid in the energy yield. The advantage lies in a much less sophisticated, and therefore relatively cheap, electrical system. Although described as fixed-speed, the common designs use an induction machine as the generator and so the machine actually operates over a 0-3% speed variation around synchronous speed (which is essentially a fixed speed compared to the variation in wind speed). Figure 25 shows the power against speed characteristic of an induction generator. Its power output close to synchronous speed would have almost the same shape. With no wind the machine will rotate at synchronous speed. As the wind speed increases and exerts more torque on the turbine, the speed will rise slightly until the reaction torque of the generator rises to balance the turbine torque.

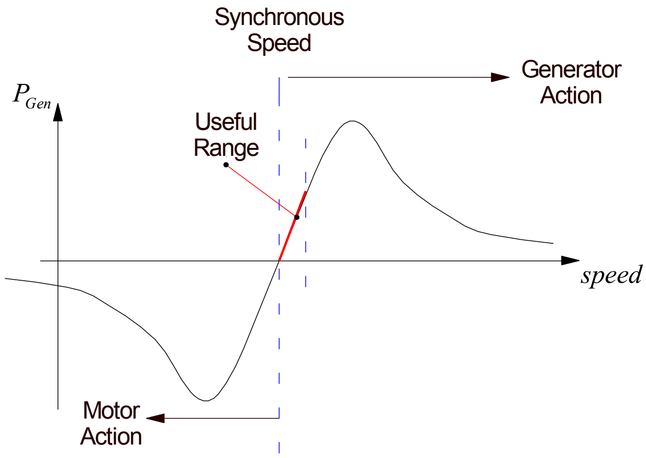


Figure 25 Power characteristic of an Induction Generator

In order to limit the power when the rated power of the generator is reached, the generator will be paired with a turbine that begins to stall at the appropriate speed and holds the turbine power approximately constant at higher wind speeds. The paring of a fixed-speed generator and a passive stall turbine was a cost effective and relatively simple design. It was popularised by Danish manufactures such as Vestas and was sometimes called the Danish system.

Figure 26 shows the market share of various forms of turbine system over the years 1995 to 2004. In this figure, type A is the fixed-speed design and it is clear that its market share has dramatically reduced. As turbines have become larger and the cost and sophistication of the civil engineering, the blades, the gearboxes and the control systems have risen, it has become more worthwhile to try to optimise the turbine energy yield through variable speed operation.

1.7.2 Variable Speed Turbines

There are some simple developments of the fixed speed turbine which involve the introduction of resistors in the rotor of an induction generator to alter its speed. This is type B in Figure 26. This relatively crude method is no longer in favour.

The straight forward way of achieving variable speed is to allow the generator to produce AC at whatever frequency is dictated by its rotational speed and then perform a frequency conversion in order to connect to the grid. The frequency conversion would normally be rectification to DC followed by inversion to 50 or 60 Hz as shown in Figure 27. This is known as a full-converter design because all of the generated power is passed through power electronic power converters and is type D in Figure 26. The magnetic field on the rotor would be produced by a permanent magnet in low power designs and a wound field in high power designs. Such a system would normally be paired with a variable-pitch turbine and a control scheme that actively controls both the pitch angle and turbine speed to follow the maximum power or the power limit as appropriate. The full-converter design places no restriction on the range of rotational speeds that can be accommodated which means the optimal power point can be tracked across a wide range. These are sometimes known as wide speed-range wind turbines (WSR) or full-converter wind turbines.

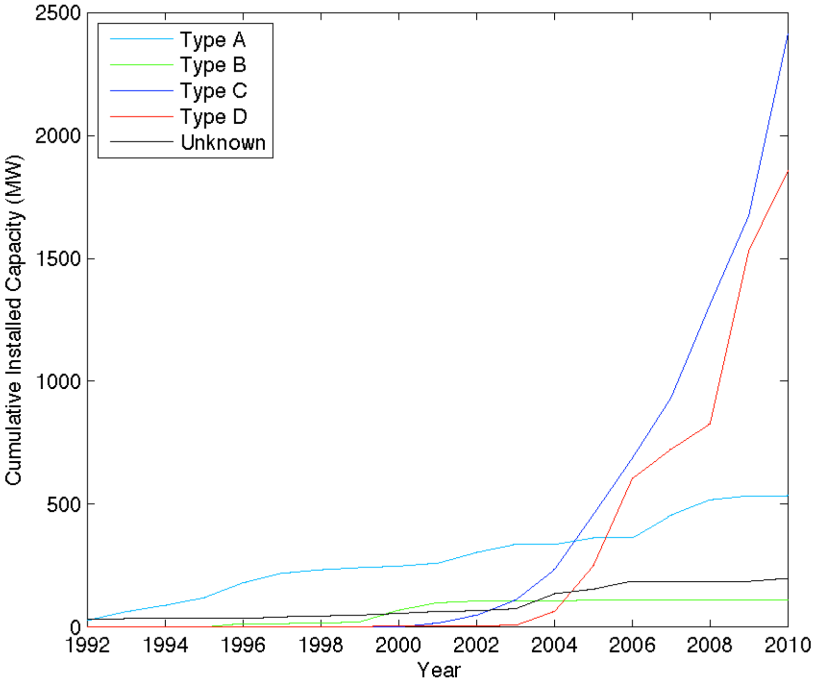


Figure 26 Market Share of Four Turbine Types {Data Source bwea.com]

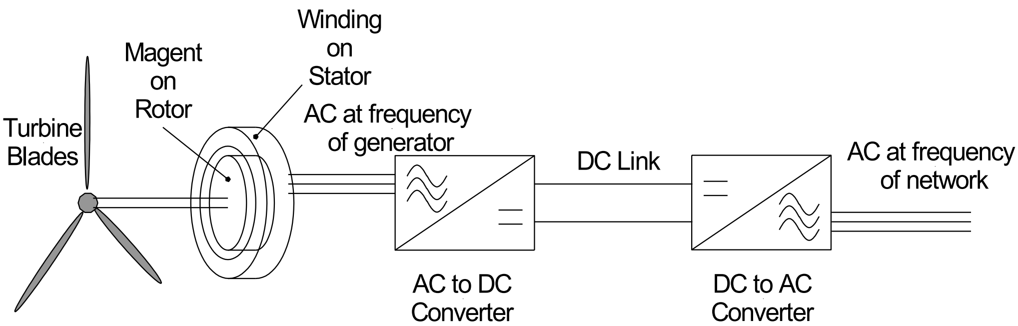


Figure 27 A Full-Converter Wind Turbine

This design is favoured by Enercon, Figure 28. Enercon takes this one stage further and does not provide a gearbox between the turbine and the generator. This means that the generator is a high-torque, low speed design. There is a clear advantage in obviating the gearbox because a gearbox at a power rating of several megawatt capable of handling sharply fluctuating torque is large, expensive and, in some cases, not sufficiently reliable. The disadvantage is that high torque generators are proportionately larger. Enercon uses a very large diameter generator which takes the form of an annulus to reduce weight.

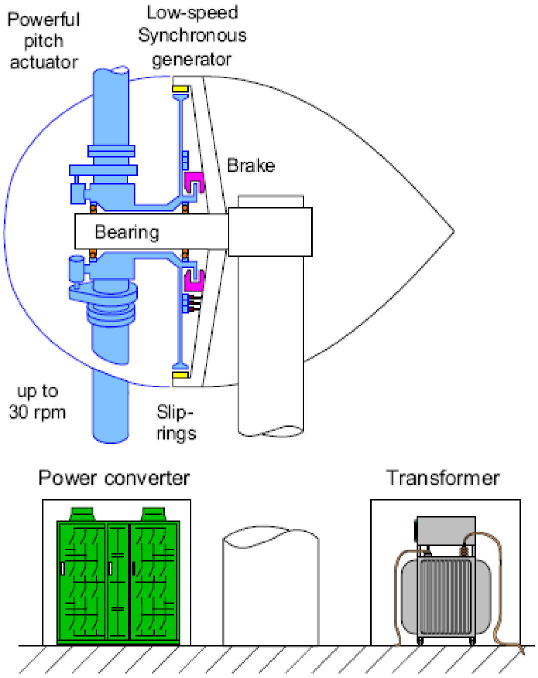


Figure 28 A direct-drive full-converter turbine system (after Enercon)

It has been considered that the power electronics of a full-converter design is too costly. It is also noted that wide speed-range is not really necessary. Because of the cube law power equation, a 3 to 1 speed range yields a 27 to 1 power range. This opens up the possibility of a compromise in which we settle for a limited speed range in exchange for using lower rated power converters. This is achieved using a doubly-fed induction generator, DFIG which is type C in Figure 26. The DFIG differs from the standard induction generator by having slip-rings which allows the rotor currents to be processed through an external circuit, in this case a power converter, Figure 29. Put simply the allowable slip range can be widened (in comparison to a standard induction machine) by exercising control over the rotor currents and indirectly the stator currents.

If a relatively small amount of power is extracted from the rotor, it allows the machine to generate at higher speeds and if a relatively small amount of power is injected into the rotor it allows generation down to lower speeds.

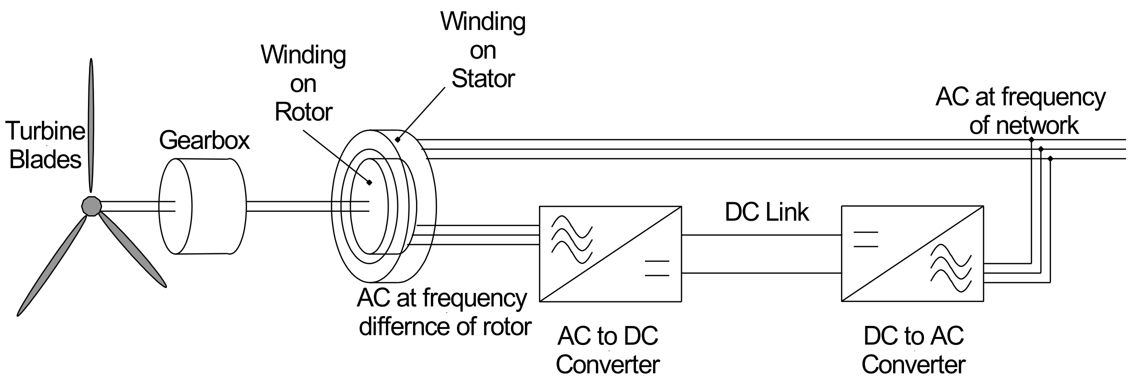


Figure 29 A Doubly-Fed induction Generator with a AC/DC/AC converter on the Rotor

As with any induction machine, the frequency of currents and voltages in the rotor is given by the difference between the rotational speed of the magnetic field (the synchronous speed imposed by the stator frequency) and the rotational speed of the rotor. The rotor-side AC/DC converter must be synchronised to this frequency.

$$\omega_{slip} = \omega_S - \omega_R$$
$$\omega_S = \frac{\omega_E}{P} \qquad (3.31)$$
$$s = \frac{\omega_S - \omega_R}{\omega_S}$$

$$P_{Mech} = P_S + P_R \qquad (3.32)$$

The voltages on the rotor are scaled by the turns ratio and the slip; the currents are scaled only by the turns ratio. This means that the power in the rotor is related to the power on the stator by the slip (ignoring loss in winding resistances).

$$P_R = sP_S \qquad (3.33)$$

When operating below synchronous speed, the slip is positive and the rotor power is in the opposite sense to the stator power. When acting as a generator, power is introduced in mechanical form and exported from the stator. Below synchronous speed, power needs to be injected into the rotor. Above synchronous speed, power is exported from the rotor. The power exchange between the rotor and the rotor-side AC/DC converter is passed through the DC-link and grid-side AC/DC converter. The comparisons of the power flows between sub- and super-synchronous operating modes are shown in Figure 30. (Note that losses in the generator and power converters have been ignored and that the same magnitude of mechanical power has been used in the illustration even though this would be dependent on speed in a wind turbine.)

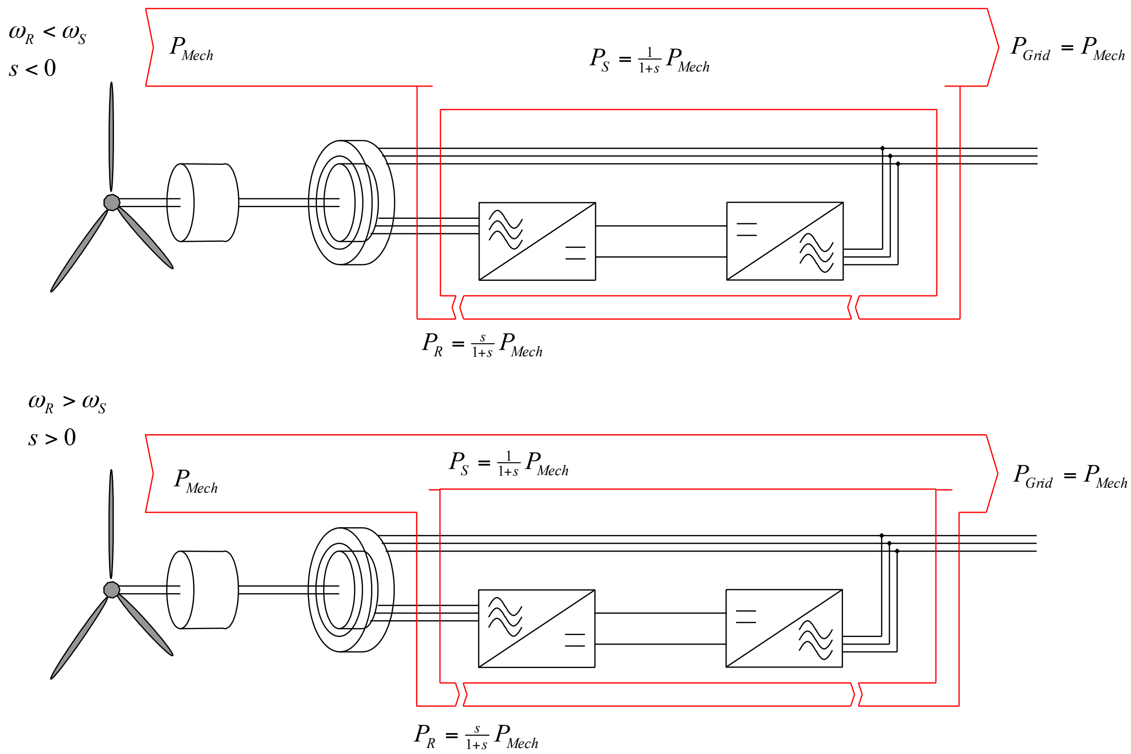


Figure 30 Power Flow in Sub- and Super-synchronous Operation

The key design parameter to be chosen is the range of *s* to be used. A typical figure is *s*=±0.4 (0.03 is typical of a cage-rotor machine). This means that the power converters are rated at 40% of the stator power and 0.4/1.4=28% of the total power of the generator. This is significant saving compared to the full converter (at the expense of having slip-rigs in the generator). A slip of 0.4

gives a speed range from 0.6 to 1.4 times the synchronous speed which is a 2.3 to 1 range and enables a reasonable range of wind speeds to be captured effectively.

Vestas is one of the main proponents of this design and in their design it can be paired with either a pitch-controlled turbine or an active-stall turbine. Because the bulk of the power flow is via the stator winding connected directly to the 50 or 60 Hz grid, it is necessary to have a generator with a synchronous speed of 1,500 rpm for a 2 pole-pair machine, 1,000 rpm for a 3 pole-pair machine, etc. To achieve these speeds (and avoid very high pole numbers) will require a gearbox between the turbine and the generator.

1.8 Turbine Drive Train Efficiency

The power flow for a three-stage generation system is shown in Figure 31.

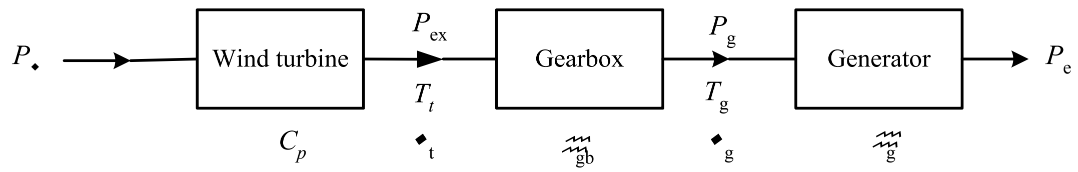


Figure 31 Power flow in a wind-generator system

The total efficiency of this system can be estimated by multiplying the power ratios of the several stages:

$$\eta = \frac{\textit{Electrical output power}}{\textit{Power available from the wind}} = \frac{P_e}{P_w} = \frac{P_{ex}}{P_w}\frac{P_g}{P_{ex}}\frac{P_e}{P_g} \qquad (3.\ 34)$$

The efficiency of the wind turbine (which is effectively the power coefficient) is given by:

$$0 \le \mathrm{Cp} \le \mathrm{C_p^{max}}$$

Where:

$$C_p^{max} \approx \begin{cases} 40-50\% & \text{for large machines (MW)} \\ 30-40\% & \text{for small machines (kW)} \end{cases}$$

The efficiency of the mechanical drive train depends largely on the gearbox although some designs avoid using a gear box. Gearbox efficiency may be estimated as:

$$\eta_{gb} = \frac{P_g}{P_{ex}} \approx \begin{cases} 80-95\% & \text{for large machines} \\ 70-90\% & \text{for small machines} \end{cases}$$

The efficiency of the electrical system is dominated by the generator and any power converters. (Inclusion of power converters will lead to efficiencies of this stage being lowered.) The expect range is:

$$\eta_g = \frac{P_e}{P_g} \approx \begin{cases} 85-97\% & \text{for large machines} \\ 70-92\% & \text{for small machines} \end{cases}$$

The electrical power available at the terminals of the generators is then estimated as follows:

$$P_e = P_w \times C_p \times \eta_{gb} \times \eta_g$$

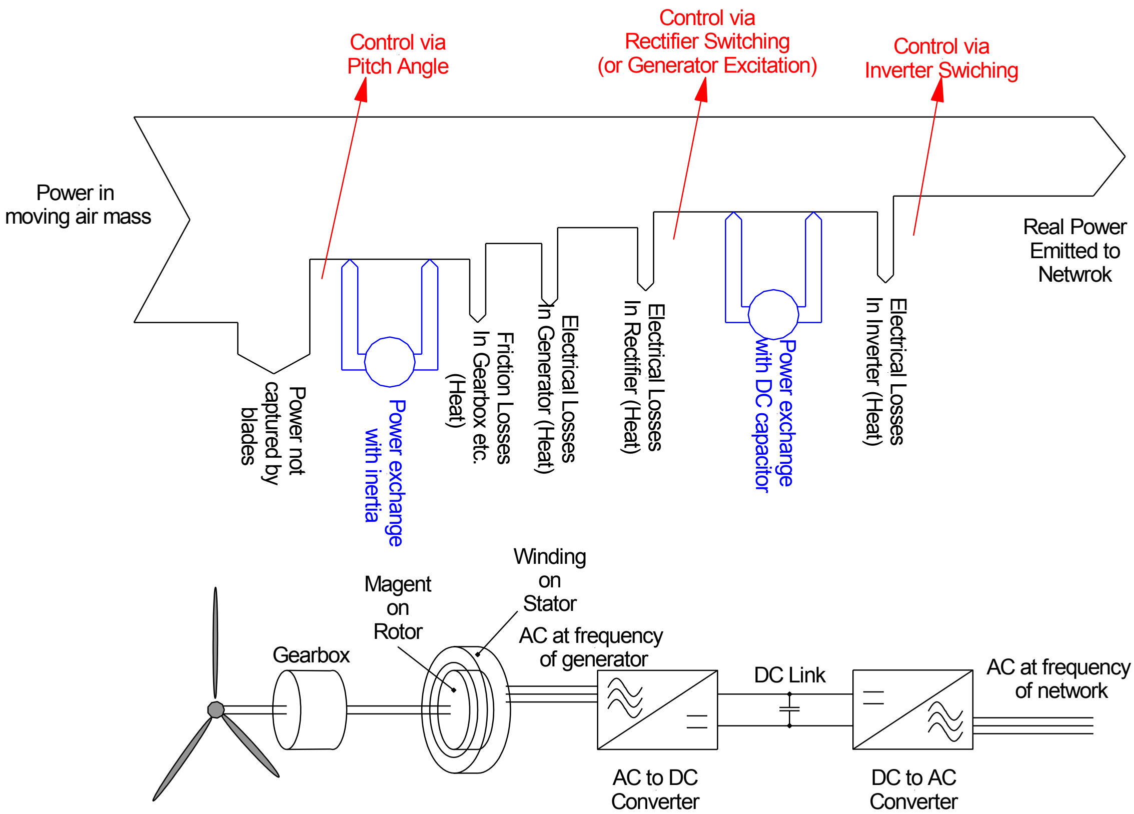


Figure 32 Power Flow Diagram for a Full Converter Wind Turbine