Lab 07 Regression

RE 519 Data Analytics and Visualization | Autumn 2025

In Lab 7, we will go back to the **King County sales** data from maintained by Andy Krause at Zillow. You can find the datasets and Readme file in this repository.

In Part A, we will use classical regression, which considers more on the model specification, assumptions, and parameter interpretation. In Part B, we will still use regression but under the context of machine learning. We care about the prediction accuracy on unseen test dataset. A seminal paper discusses the **two cultures in statistical modeling**: Statistical Modeling: The Two Cultures. Leo Breiman. 2001.

The due day for each lab can be found on the course wesbite. The submission should include Rmd, html, and any other files required to rerun the codes.

From Lab 4, the use of any generative AI tool (ChatGPT, Copilot, etc.) is **allowed** for coding assignments. But, I still encourage you to write codes by yourself and use AI tools as a way to debug and explore new knowledge. More information about Academic Integrity and the Use of AI.

Lab 07-A: Regression in Statistics

```
#install.packages("modelsummary") # you only need to run the installation once
#install.packages("devtools") # if you are using a Windows computer
#devtools::install_github('andykrause/kingCoData') # you only need to run the installation once
library('tidyverse')
library(modelsummary) # the package for formatting the regression result table
library('kingCoData') # load the data package
```

```
data(kingco_sales) # load the sale data
# only select the sales in 2023
sales <- kingco_sales %>%
  filter(sale_date >= as.Date("2023-01-01") & sale_date <= as.Date("2023-12-31"))
```

A Simple Linear Regression

Checking Correlation

Correlation (r) measures the strength and direction of a linear relationship between two variables. The most common one for continuous numeric data is **Pearson correlation**, which ranges from -1 to 1.

- +1 Perfect positive correlation
- 0 No linear relationship
- –1 Perfect negative correlation

Note: Correlation is a useful starting point for selecting variables. We often use $|r| > 0.3$ as a rough criterion for inclusion. But we still need to incorporate domain knowledge and some diagnostic testing. For example, correlation between sale price and year built is only ~0.1, which suggests a weak linear relationship, but it may still be theoretically relevant.

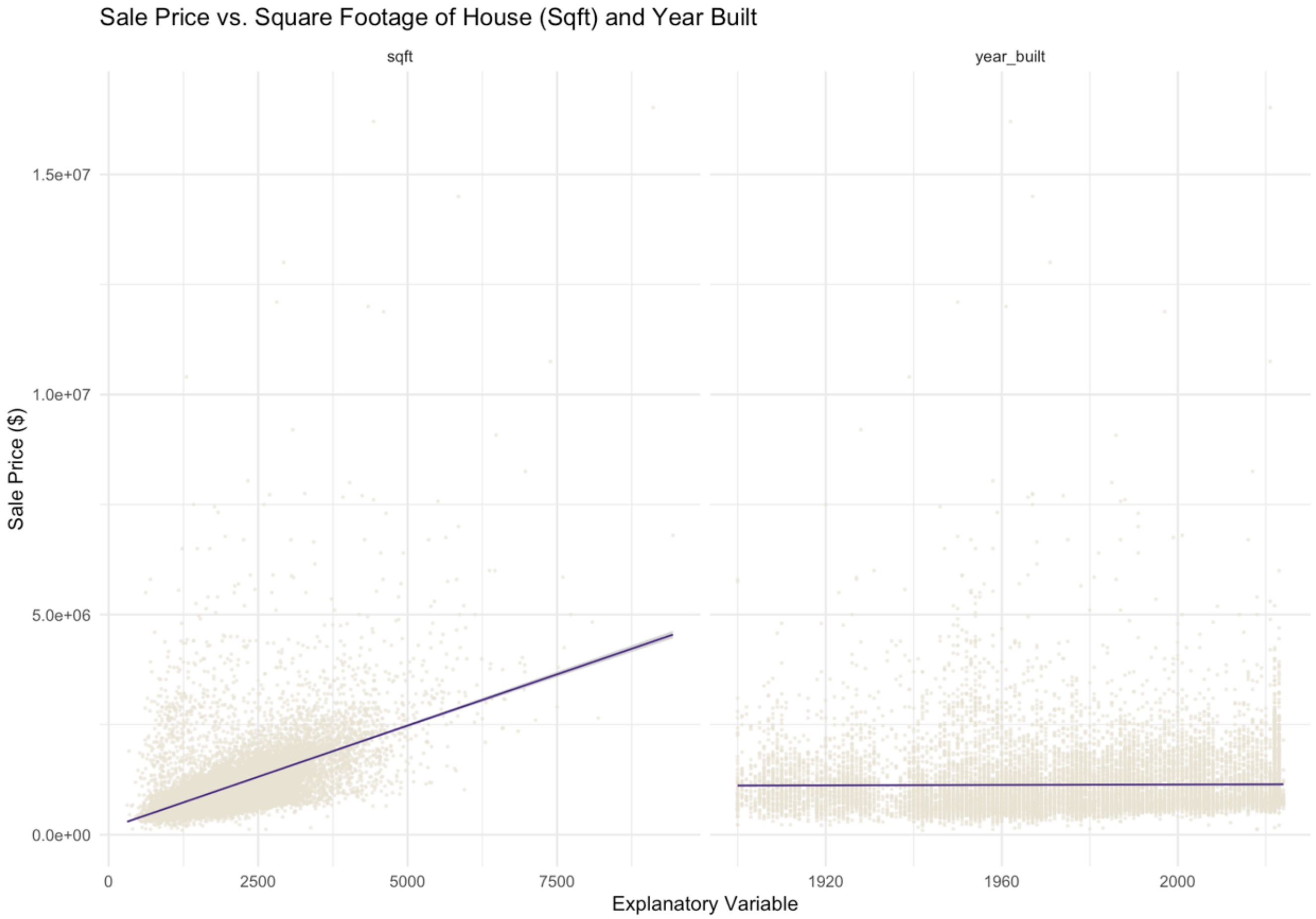
```
cor(sales[, c("sale_price", "sqft", "year_built", "condition")])
```

```
##             sale_price        sqft   year_built   condition
## sale_price 1.000000000  0.53952741  0.009373296  0.01784696
## sqft       0.539527410  1.00000000  0.297844019 -0.05792704
## year_built 0.009373296  0.29784402  1.000000000 -0.35992275
## condition  0.017846960 -0.05792704 -0.359922752  1.00000000
```

Scatterplots

```
sales_long <- sales |>
  pivot_longer(
    cols = c(sqft, year_built),
    names_to = "variable",
    values_to = "xvalue"
  )

ggplot(sales_long, aes(x = xvalue, y = sale_price)) +
  geom_point(size = 0.3, alpha = 0.5, color = "#e8e3d3") +
  geom_smooth(linewidth = 0.5, method = "lm", formula = "y~x", color = "#4b2e83") +
  facet_wrap(~ variable, scales = "free_x") +
  labs(
    x = "Explanatory Variable",
    y = "Sale Price ($)",
    title = "Sale Price vs. Square Footage of House (Sqft) and Year Built"
  ) +
  theme_minimal()
```



Fitting the Model

Starting to fit a regression, you define a family of models that express a precise, but generic, pattern that you want to capture. For example, the pattern might be a straight line, or a quadratic curve. You will express the model family as an equation like $y = \beta_0 + \beta_1 x$ or $y = \beta_0 + \beta_1 x^2$. Here, x and y are known variables from your data, and β_0 and β_1 are parameters that can vary to capture different patterns.

```r
# lm means linear model
# we don't need to add intercept, R will add it for us
simple_lm <- lm(data = sales, sale_price ~ sqft + year_built)
```

We can check the results using `summary()` (result table) and `confint()` (confidence intervals).

```r
summary(simple_lm)
```

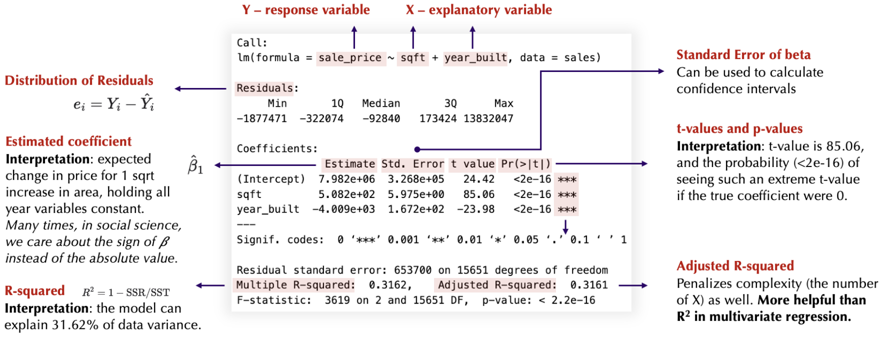
```
##
```

```
## Call:
## lm(formula = sale_price ~ sqft + year_built, data = sales)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1877471  -322074   -92840   173424 13832047
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.982e+06  3.268e+05   24.42   <2e-16 ***
## sqft         5.082e+02  5.975e+00   85.06   <2e-16 ***
## year_built  -4.009e+03  1.672e+02  -23.98   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 653700 on 15651 degrees of freedom
## Multiple R-squared:  0.3162, Adjusted R-squared:  0.3161
## F-statistic:  3619 on 2 and 15651 DF,  p-value: < 2.2e-16
```

```
confint(simple_lm)
```

```
##                    2.5 %       97.5 %
## (Intercept) 7341424.5882 8622695.8739
## sqft            496.5235     519.9459
## year_built    -4336.6278   -3681.2999
```

How to read the tables?



```
Call:
lm(formula = sale_price ~ sqft + year_built, data = sales)

Residuals:
     Min       1Q   Median       3Q      Max
-1877471  -322074   -92840   173424 13832047

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  7.982e+06  3.268e+05   24.42   <2e-16 ***
sqft         5.082e+02  5.975e+00   85.06   <2e-16 ***
year_built  -4.009e+03  1.672e+02  -23.98   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 653700 on 15651 degrees of freedom
Multiple R-squared:  0.3162,    Adjusted R-squared:  0.3161
F-statistic:  3619 on 2 and 15651 DF,  p-value: < 2.2e-16
```

Y – response variable

X – explanatory variable

Distribution of Residuals

$e_i = Y_i - \hat{Y}_i$

Estimated coefficient $\hat{\beta}_1$

Interpretation: expected change in price for 1 sqrt increase in area, holding all year variables constant.
Many times, in social science, we care about the sign of β instead of the absolute value.

R-squared $R^2 = 1 - \mathrm{SSR/SST}$

Interpretation: the model can explain 31.62% of data variance.

Standard Error of beta

Can be used to calculate confidence intervals

t-values and p-values

Interpretation: t-value is 85.06, and the probability (<2e-16) of seeing such an extreme t-value if the true coefficient were 0.

Adjusted R-squared

Penalizes complexity (the number of X) as well. **More helpful than R^2 in multivariate regression.**

Results from R and Interpretations

▶ **Click to view additional notes on reading the results**

Check Assumptions

For a normal linear model to be a good model, there are some conditions/assumptions that need to be fulfilled.

- **Linearity**: Y changes linearly with X. Y can be described by a linear combination of X (or transformation of X).
- **Independence**: the residuals are independent of each other. We are using time series and spatial data so we are likely violate this assumption.
- **Normality and homoscedasticity for error terms**: the distribution of the residuals is normal with equal variance.
- **No perfect multicollinearity among X**: X should not be a linear combination of each other. Example: using all three scores as X if total score = part A score + part B score.

If these assumptions are not met, the inference with the computed standard error is invalid. That is, if the assumptions are not met, the standard error should not be trusted, or should be computed using alternative methods.

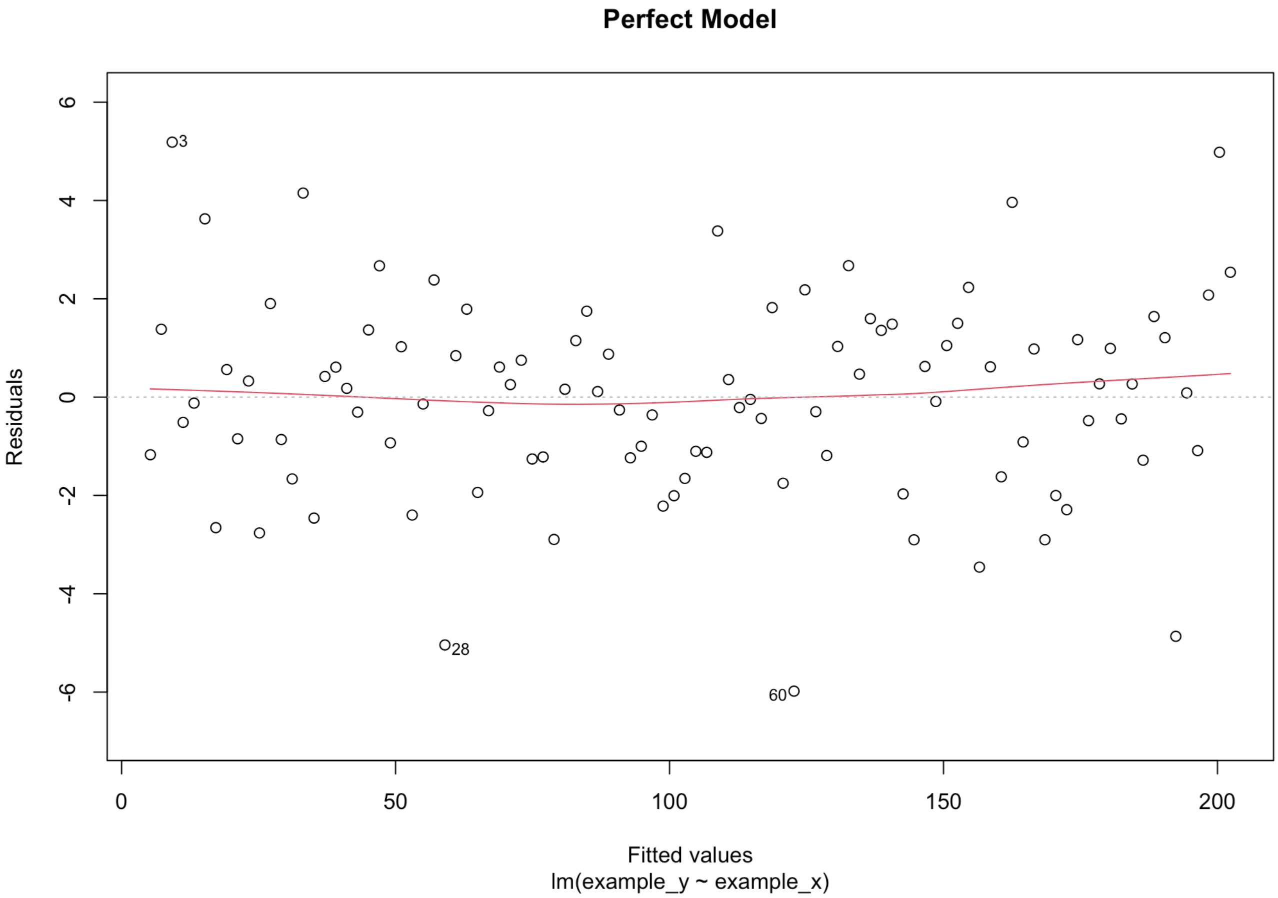
The quickest way to check many assumptions is to use `plot(model)` and it will generated several key plots for us. We use `which` to specify the plot but in practice, you can generate them at the same time.

We will setup a 'perfect' model as the example to compare:

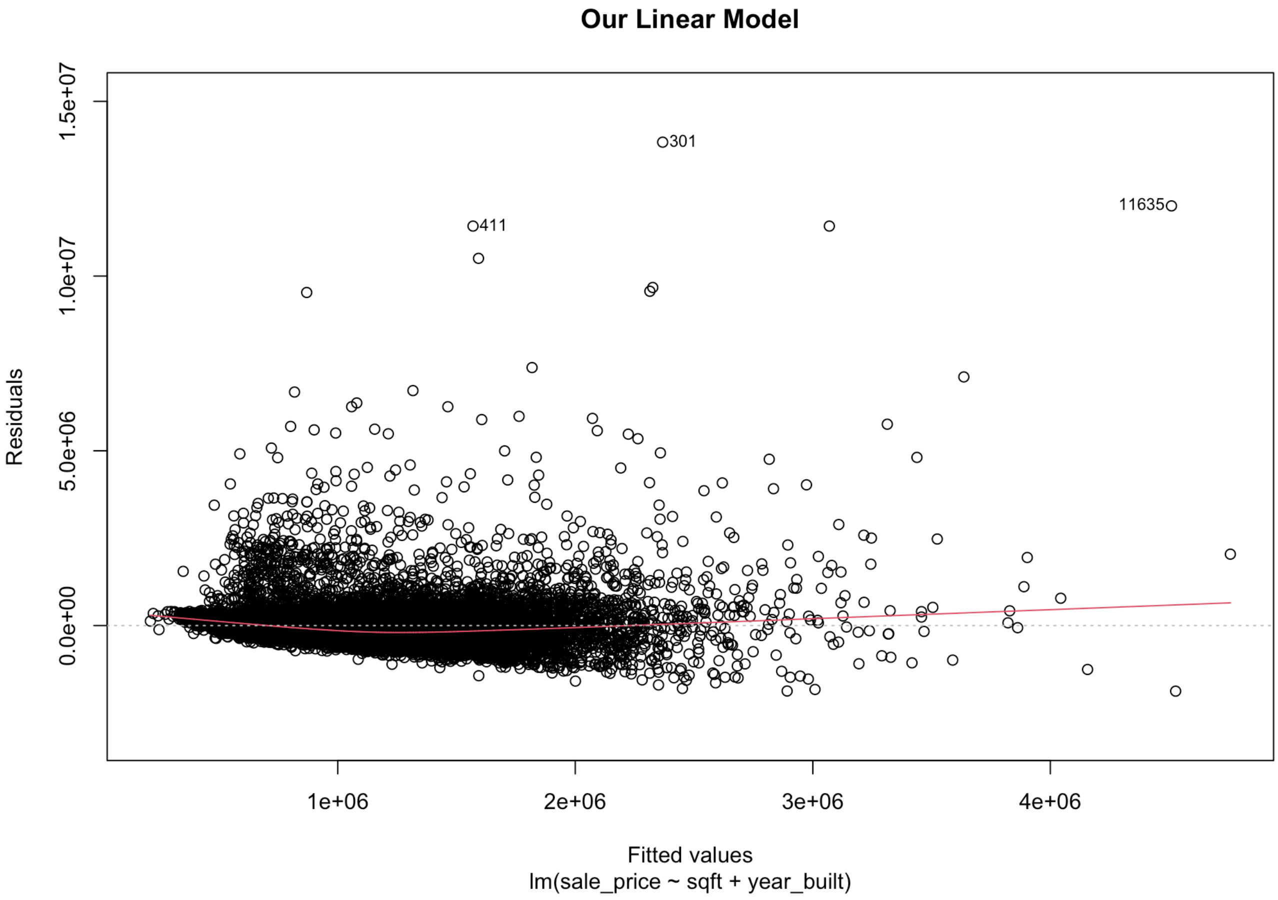
```
example_x <- seq(1, 100, by = 1)
example_y <- 3 + 2 * example_x + rnorm(100, mean = 0, sd = 2)
example_model <- lm(example_y ~ example_x)
```

Residuals vs Fitted

```
plot(example_model, which = 1, title("Perfect Model"))
```



```
plot(simple_lm, which = 1, title("Our Linear Model"))
```



Horizontal Axis: Fitted values (predicted values) from the regression model.

Vertical Axis: Residuals (the differences between the observed values and the predicted values).

Interpretation: This plot helps you check for linearity and homoscedasticity (equal variance of error terms). Ideally, you should see a horizontal red line (for linearity) and random scatter of points with no discernible pattern (for homoscedasticity). If there's a pattern (e.g., a curve or fan shape), it suggests non-linearity or heteroscedasticity, which might indicate a problem with the model.

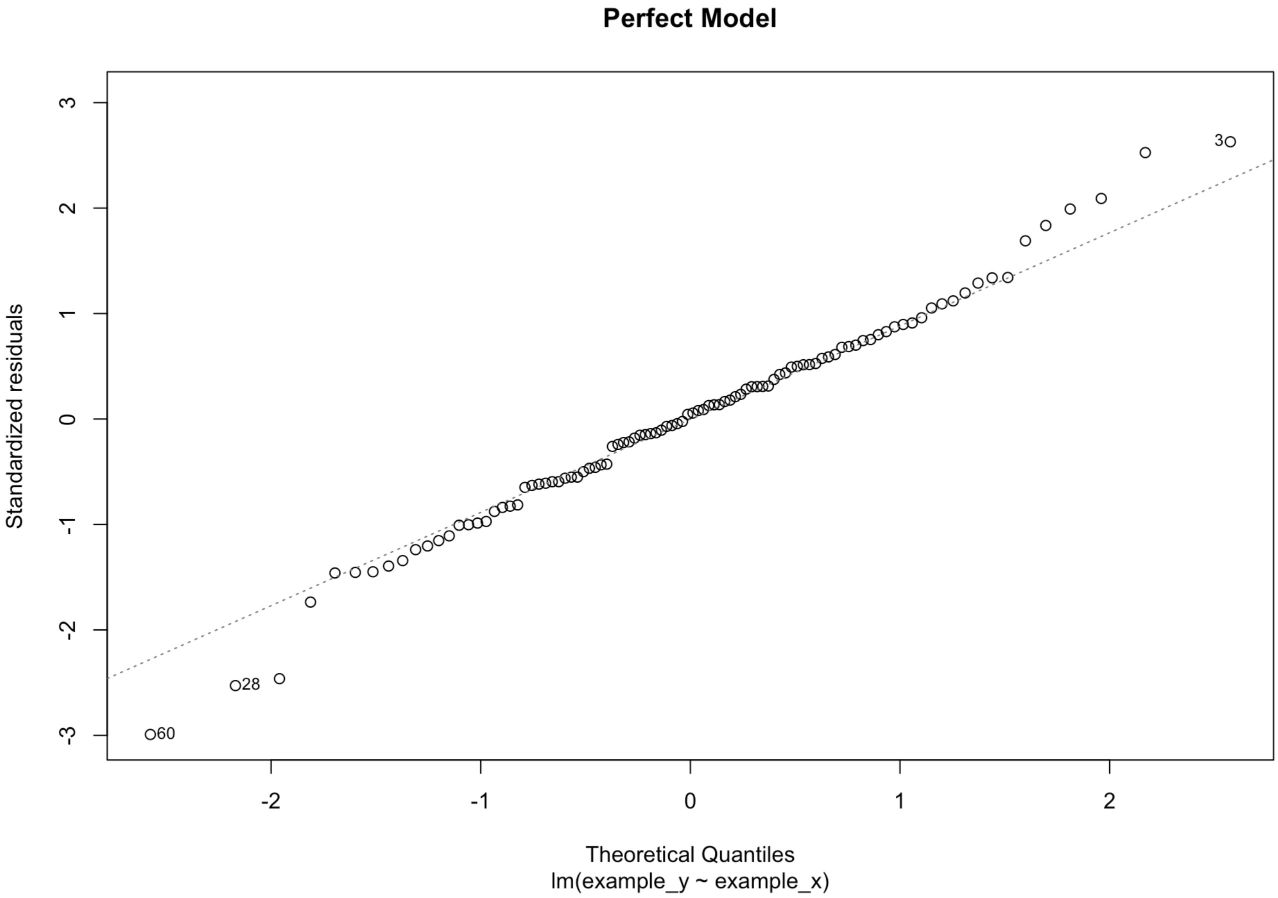
In our case: no much concern on the linearity (red line is around 0); some level of heteroscedasticity (fan shape - residuals variance increase with the increase of fitted values). That happens a lots in price data, which is often right-skewed/long-tailed and log transformation to Y is a common strategy.

Q–Q Plot

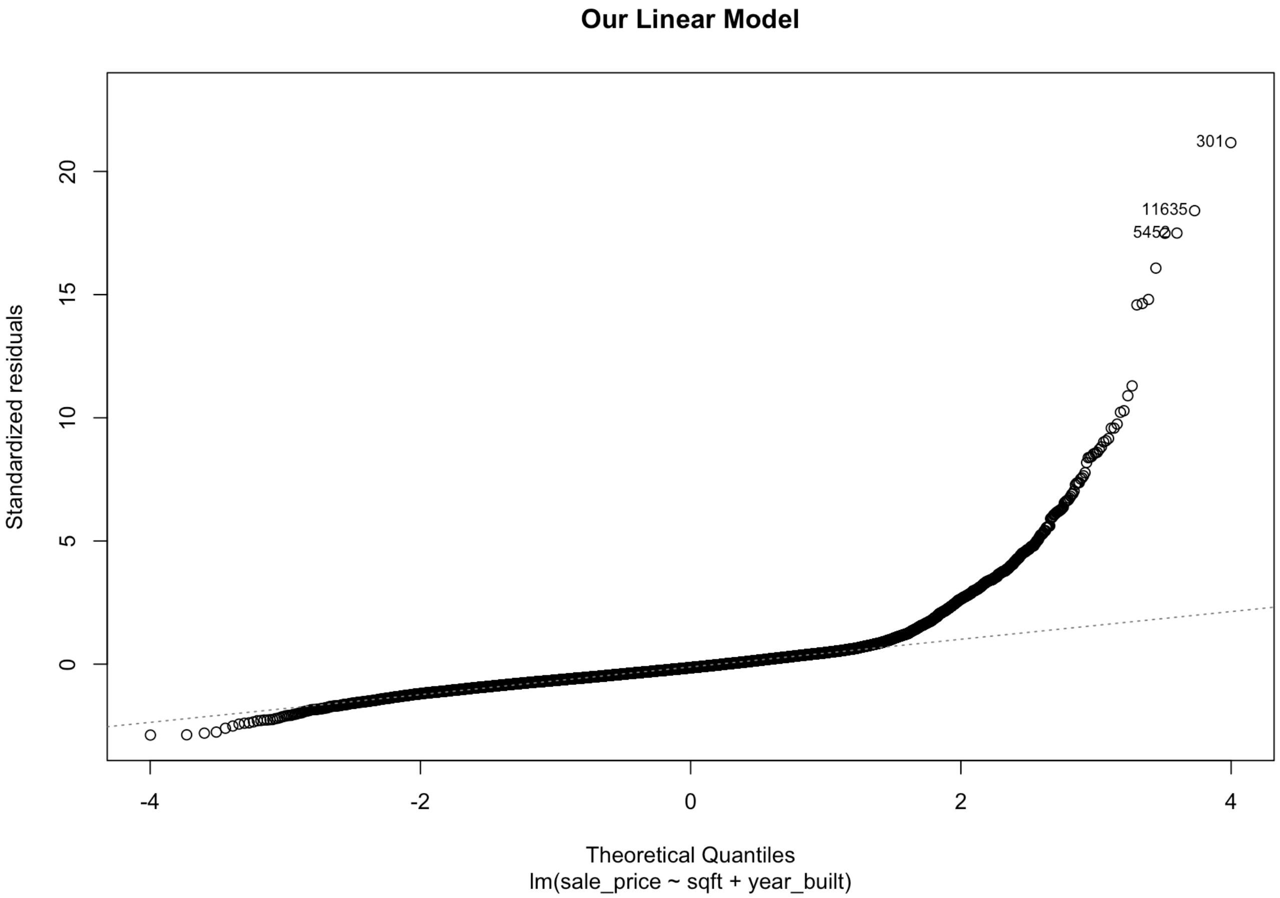
Q-Q plots helps us to compare a set of data with normal distribution. If the data is perfectly normally distributed, it should follow the dash line. In our case, we can see the residuals are not normally distributed. It reconfirms the price data is right-skewed/long-tailed.

- S-shaped curve: heavier tails or more peaked than the normal distribution (for example, a t-distribution).
- Curved upward or downward: skewness (the data are not symmetric).
- Clearly deviating from the straight line: Suggests the data are not normally distributed.

```
plot(example_model, which = 2, title("Perfect Model"))
```

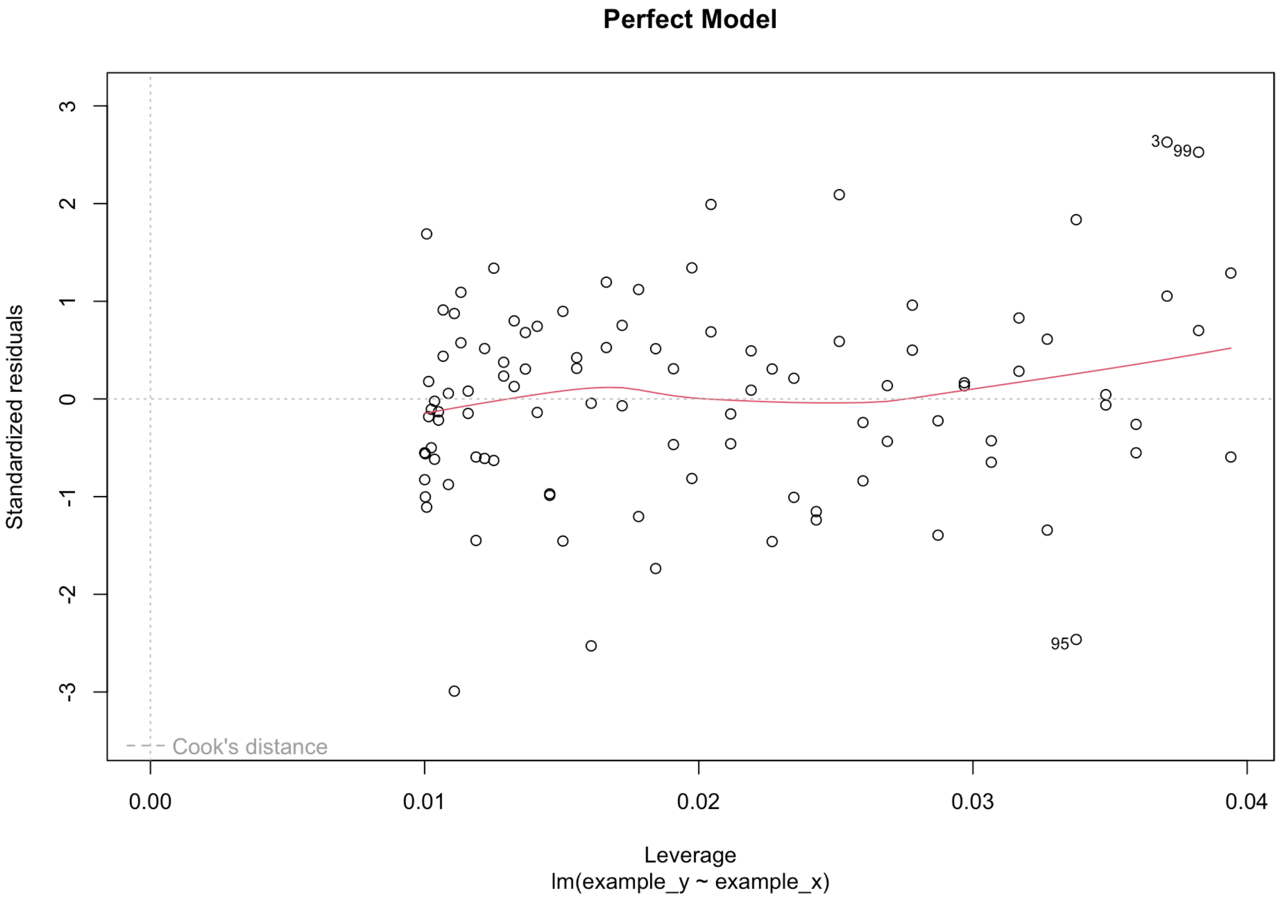


```
plot(simple_lm, which = 2, title("Our Linear Model"))
```

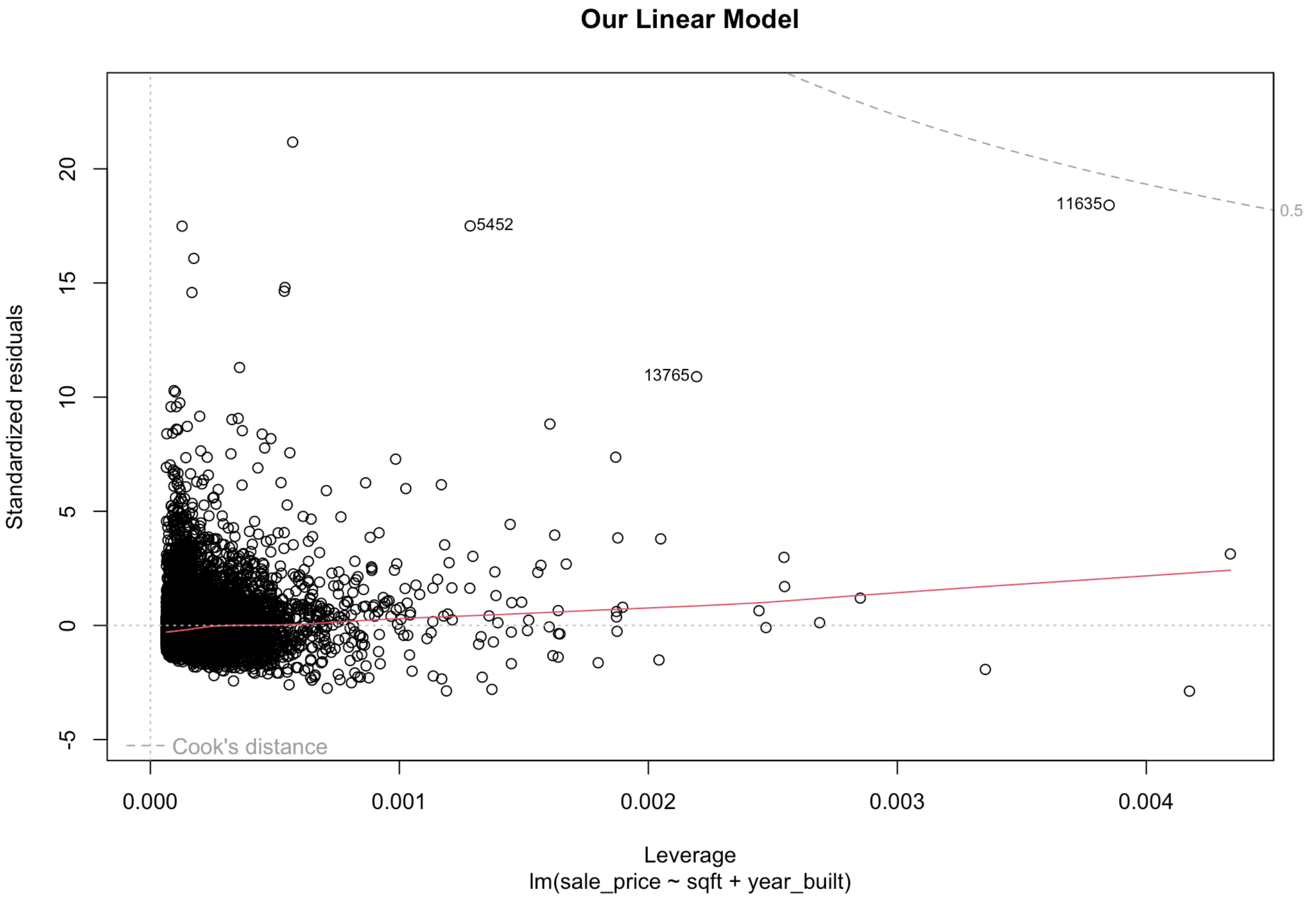


Residuals vs Leverage

```
plot(example_model, which = 5, title("Perfect Model"))
```



```
plot(simple_lm, which = 5, title("Our Linear Model"))
```



Horizontal Axis: Leverage values, which indicate how much the corresponding observation influences the fit.

Vertical Axis: Standardized residuals (residuals divided by their standard deviation).

Interpretation: This plot helps you identify *influential data points (outliers)* and *influential cases (observations with high leverage)*. Points that are far from the horizontal line at zero indicate observations with high standardized residuals. Points that are far from the vertical line at 1/n (where n is the number of observations) have high leverage. The gray dash line is the Cook's Distance; points above the line have will have noticeable change to regression line if removed.

In our case: There are a few outliers (5452, 13765, 11635) but it is not a super serious problem (within Cook's distance = 0.5). We can check whether the original data is correctly collected. After that, we can decide whether to remove them.

Multicollinearity

The Variance Inflation Factor (VIF) measures how much the variance of a regression coefficient is inflated due to collinearity with other predictors. If there is perfect multicollinearity, R will automatically drop one of the variables. If there is imperfect multicollinearity. R won't drop and the model estimates will be biased.

- VIF = 1: no correlation with other predictors.
- VIF > 5: moderate multicollinearity concern, potential for removal.
- VIF > 10: serious multicollinearity, must remove the variable

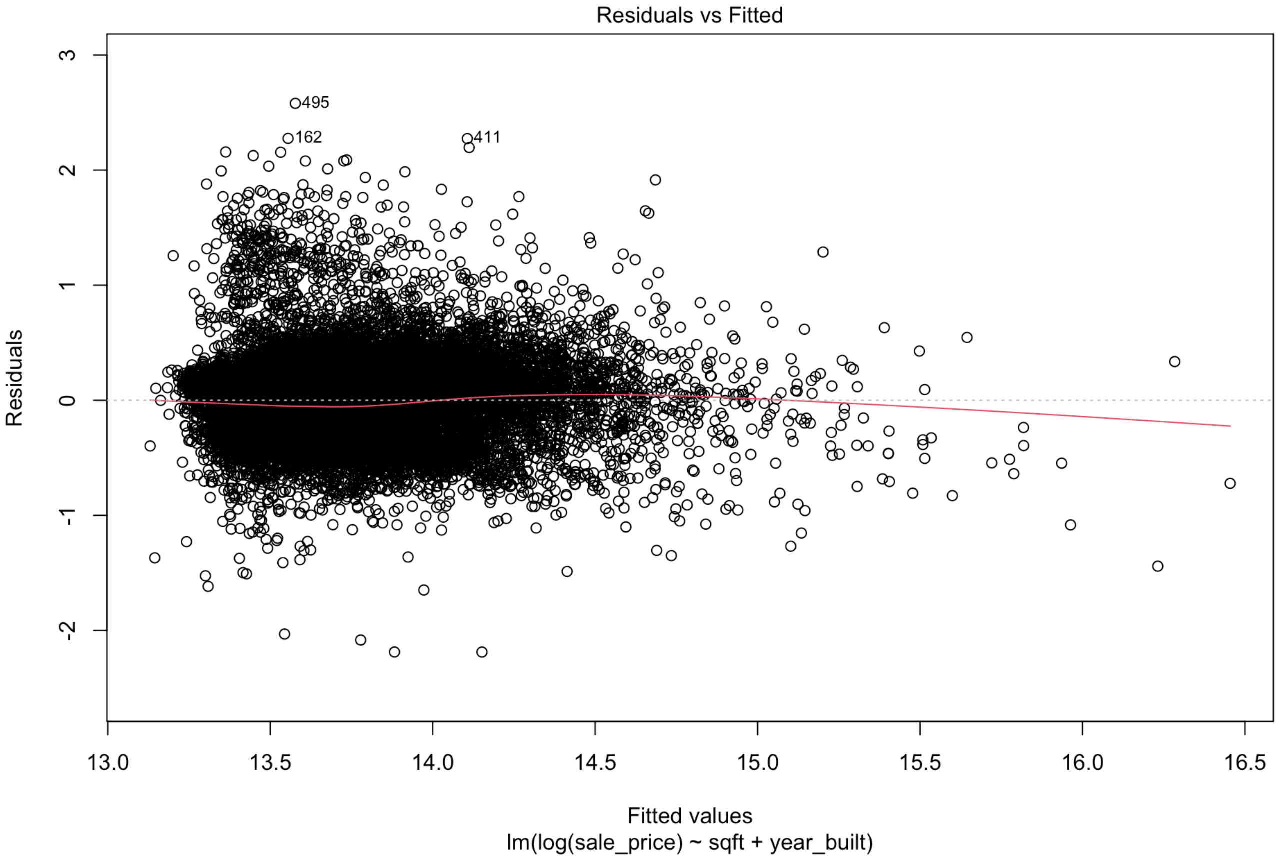
```
#install.packages('car')
library(car)
vif(simple_lm)
```

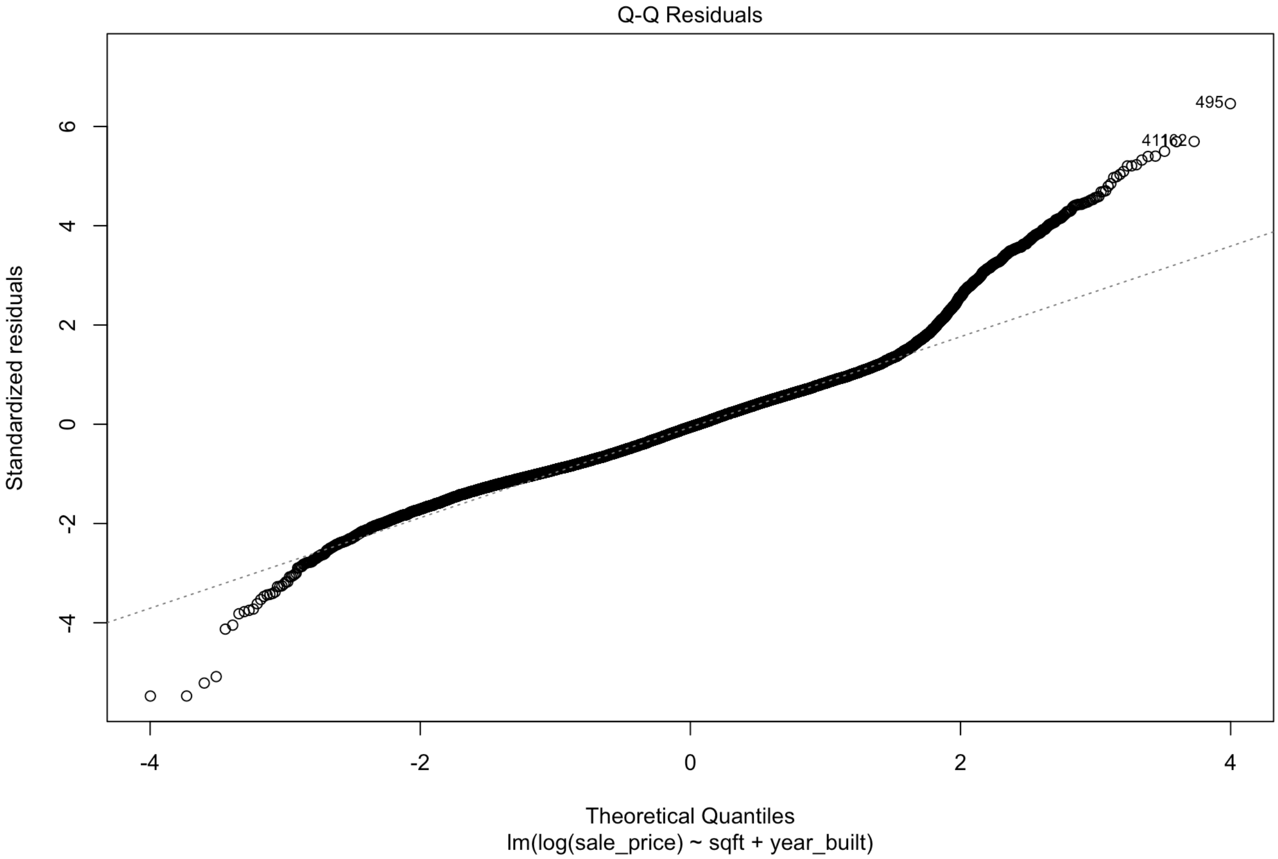
```
##       sqft year_built 
##   1.097347   1.097347
```

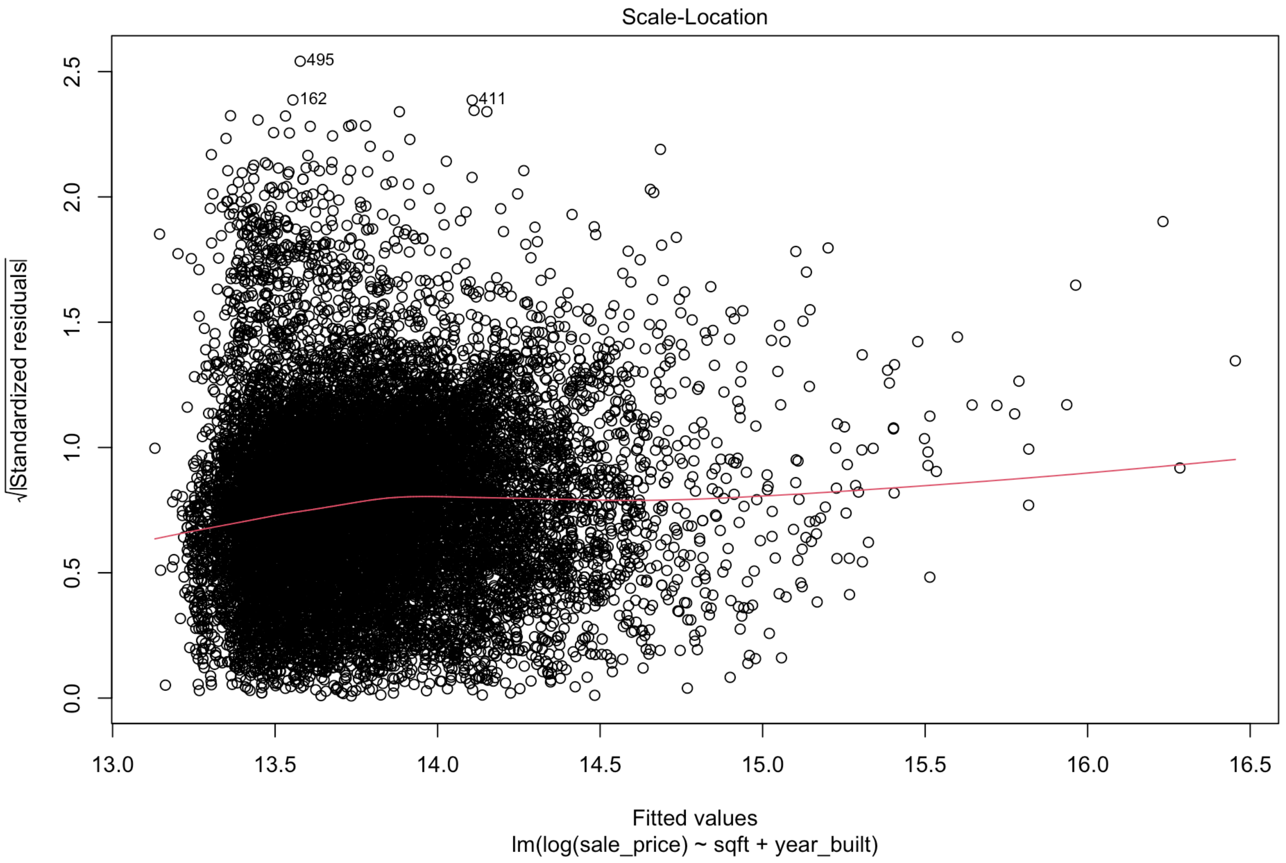
Adjustment to the Regression

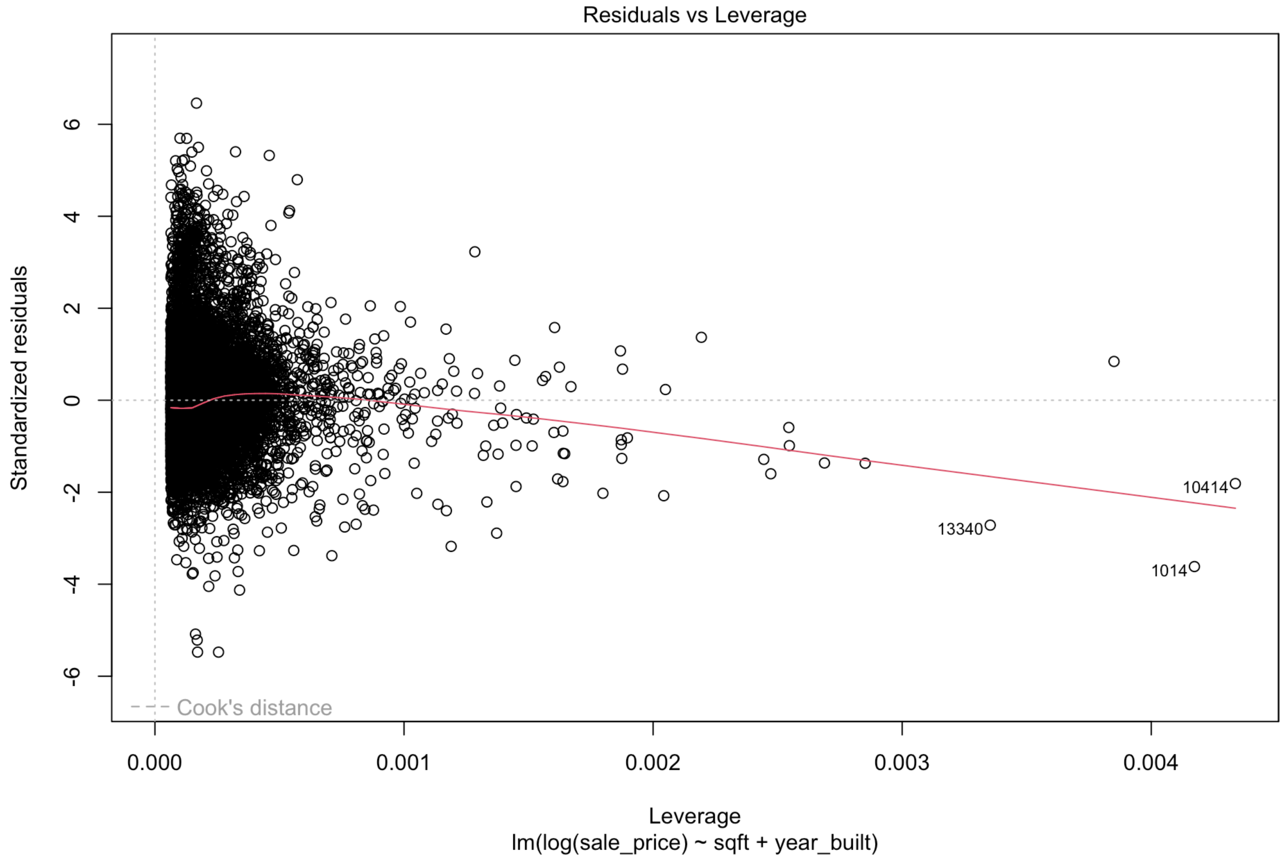
Because the original model showed heteroscedasticity (the "fan shape" pattern in residual plots), we apply a log transformation to the Y `sale_price`. Almost everything is better after the transformation.

```
log_simple_lm <- lm(data = sales, log(sale_price) ~ sqft + year_built)
plot(log_simple_lm)
```


Residuals vs Fitted
495
162
411
3
2
1
0
-1
-2
Residuals
13.0
13.5
14.0
14.5
15.0
15.5
16.0
16.5
Fitted values
lm(log(sale_price) ~ sqft + year_built)


Q-Q Residuals
495
41162
6
4
2
0
-2
-4
Standardized residuals
-4
-2
0
2
4
Theoretical Quantiles
lm(log(sale_price) ~ sqft + year_built)


Scale-Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
2.0
2.5
495
162
411
13.0
13.5
14.0
14.5
15.0
15.5
16.0
16.5
Fitted values
lm(log(sale_price) ~ sqft + year_built)


Residuals vs Leverage
Standardized residuals
-6
-4
-2
0
2
4
6
10414
13340
1014
Cook's distance
0.000
0.001
0.002
0.003
0.004
Leverage
lm(log(sale_price) ~ sqft + year_built)

```r
summary(log_simple_lm)
```

```
## 
## Call:
## lm(formula = log(sale_price) ~ sqft + year_built, data = sales)
## 
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -2.18762 -0.26969 -0.02224  0.22164  2.58006 
## 
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)    
## (Intercept)  1.774e+01  1.997e-01   88.80   <2e-16 ***
## sqft         3.717e-04  3.651e-06  101.80   <2e-16 ***
## year_built  -2.395e-03  1.022e-04  -23.44   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 0.3995 on 15651 degrees of freedom
## Multiple R-squared:  0.3995, Adjusted R-squared:  0.3995 
## F-statistic:  5207 on 2 and 15651 DF,  p-value: < 2.2e-16
```

You may notice that the β in log-transformed model is quite small. Because of the transformation, our interpretation of β could be different! Holding other variables constant, we change X_1 for one unit, we can write as:

$$\begin{aligned}
\log(Y_{changed}) &= \beta_0 + \beta_1(X_1 + 1) + \beta_2 X_2 + \varepsilon \\
&= \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon + \beta_1 \\
&= \log(Y_{original}) + \beta_1 \\
\beta_1 &= \log(Y_{changed}) - \log(Y_{original}) \\
&= \log(\frac{Y_{changed}}{Y_{original}}) \\
\exp(\beta_1) &= \frac{Y_{changed}}{Y_{original}}
\end{aligned}$$

We use `coef()` to get accurate β:

```r
coef(log_simple_lm)
```

```
##   (Intercept)          sqft    year_built 
## 17.7377723721  0.0003717023 -0.0023949085
```

```r
exp(0.0003717023)
```

```
## [1] 1.000372
```

So, we can explain the relationship between sale price and square footage of house as: holding all year variables constant, the expected **percentage change** in price for 1 sqft increase is 0.3%.

Note: if we want to know expected percentage change in price for 100 sqft, it should be 1.000372^{100} (+3.79%) instead of 0.000372×100 (+3.72%).

Interactions and Non-linear Variables

An interaction means that the **effect** of one variable depends on the level of another variable. For example, we may think of the effect of house size on price changes depending on the house's condition. We can add interaction terms by using `*`. Note: R will add `condition` itself as an explanatory variable in the following model:

```
log_no_interaction_lm <- lm(data = sales, log(sale_price) ~ year_built + sqft + condition)
log_interaction_lm <- lm(data = sales, log(sale_price) ~ year_built + sqft + sqft * condition)
```

You could also have variables in you specification that are squared, cubed etc. such as:

```
model7 <- lm(sale_price ~  sqft + I(sqft^2) + cos(sqft), data = sales)
```

The `I()` is a base R command that forces R to evaluate anything inside the `I()` before anything else. Sometimes it is needed when conducting regressions.

Compare Models

When compare with several models, we can use `modelsummary` package to display and compare regression results in one formatted table. It helps us quickly see how coefficients, significance levels, and model fit change.

```
modelsummary(
  list(
    "Without Interaction Model" = log_no_interaction_lm,
    "Interaction Model" = log_interaction_lm
  ),
  stars = TRUE,
  fmt = 5,
  title = 'Comparsion Table between Models with/without Interaction',
  output = "html")
```

Comparsion Table between Models with/without Interaction

	Without Interaction Model	Interaction Model
(Intercept)	17.71652***	17.81729***
	(0.22059)	(0.22081)
year_built	−0.00239***	−0.00232***
	(0.00011)	(0.00011)
sqft	0.00037***	0.00026***
	(0.00000)	(0.00002)
condition	0.00104	−0.06512***
	(0.00458)	(0.01093)
sqft × condition		0.00003***
		(0.00000)
Num.Obs.	15654	15654
R2	0.400	0.401
R2 Adj.	0.399	0.401
AIC	447439.8	447397.5
BIC	447478.1	447443.4
Log.Lik.	−7846.290	−7824.117
F	3471.285	2621.781
RMSE	0.40	0.40

+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001

In an interaction model, the β of individual variables (those also appearing in the interaction term) represent their effects only when the other interacting variable equals zero. So, it is a **baseline effect** and does not have much direct meanings. We must always interpret them together with the interaction term. In our interaction model:

- 0.00026: effect of sqft when condition = 0 (condition cannot be 0 in the data)
- -0.06512: effect of condition when sqft = 0 (square footage of house is not likely to be 0)

For the interaction term, we can explain as: with a one-unit increase in condition, holding other variables constant, the coefficient (slope) of sqft increases by 0.0003.

We see a significant positive interaction: the positive effect of square footage on price is stronger for houses in better condition. In other words, larger houses benefit more from good condition than smaller ones.

Performance Metrics

R-squared and Adjusted R-squared [larger - better]: Mentioned in the lecture, R-squared shows how much of the variation in the response variable is explained by the model. Adjusted R-squared corrects for the number of predictors, so it is better for comparing models with different numbers of variables.

Root Mean Squared Error (RMSE) [smaller - better]: Root of MSE, measures the average distance between the observed and predicted values. Smaller RMSE means better model fit.

Akaike's Information Criterion (AIC) [smaller - better]: AIC is good for unknown data generating process(The AIC tries to select the model that most adequately describes an unknown, high dimensional reality. Akaike's Information Criteria is good for making asymptotically equivalent to cross-validation. You can check AIC using `AIC(model)`.

Bayesian Information Criterion (BIC) [smaller - better]: BIC tries to find the TRUE model among the set of candidates. A lot of times, BIC and AIC will agrees on each other, but BIC penalize models that have a lot of parameters. Danger of underfit. Bayesian Information Criteria is good for consistent estimation. You can check BIC using `BIC(model)`.

In general, our interaction model perform better in terms of all four metrics. And we will use this model to conduct prediction. Note: we may still need same assumption check for the new model.

Prediction

We can use `predict` function to generate predicted values from a regression model.

```
# we add a new column in sales dataframe
sales$predicted <- predict(log_interaction_lm, data = sales)
# check the prediction
sales$predicted[1]
```

```
##        1 
## 13.56881
```

Check the prediction, we find the value is extremely small (13.57 << real sale price). We used log transformation to response variable! We need to transform the predicted values back by taking the exponential (exp) of each prediction to return them to the original sale price scale.

```
sales$predicted_exp <- exp(sales$predicted)
sales$predicted_exp[1]
```

```
##        1 
## 781375.6
```

📚 TODO: Explain the Results of Logistic Regression

10 points

You may discuss in small group, but you must prepare the submission by yourself. Generative AI is NOT allowed for write-up questions. You ask conceptual questions but you need to cite properly. How to cite?

Your submission should be HTML or PDF. Handwritten PDFs are acceptable as long as they are scanned and easy to read. Part B will not require any coding as well.

In this section, we will read a recent research paper published by Runstad Department faculty Rebecca J. Walter, Arthur Acolin, Ruoniu (Vince) Wang, Gregg Colburn, along with people from other institutions: **Exploring the association between household compositional change and mobility of subsidized householders in the United States**. You are able to find the PDF version within the lab folder.

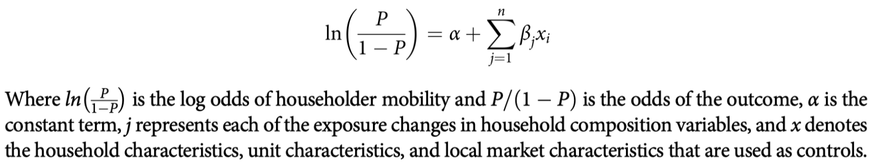
This study examines the relationship between household compositional change (such as a partner, child, or adult entering or leaving the household) and residential mobility of subsidized householders in the US. They use the Annual Longitudinal Files 2005–2018 from

U.S. Department of Housing and Urban Development, which is restricted-use data required researchers to apply to access. Northwest Federal Statistical Research Data Center is the center within UW partnered with U.S. Census Bureau to provide such data access.

Please skim the paper first and use the following to guide your further reading, especially about the **Data and methods** and **Results** parts.

Settings

1. [1 pt] What is the unit of analysis of this study? What is the sample size N? Is this study cross-sectional or longitudinal (lecture 2 data)?
2. [0.5 pt] Why do you think U.S. Census Bureau requires us apply for the data usage instead of providing public access like American Community Survey (ACS) data?
3. [0.5 pt] In a research paper, we tend to include two parts of explanatory variables: (1) the **key explanatory variables (or variables of interest)**, which directly address the research question; and (2) the **control variables**, which account for other factors that might also influence the response variable. In this paper, what are the key explanatory variables and response variable?
4. [1 pt] What is the purpose of including local market characteristics as part of X? Even though we do not have access to the data, which variables could be multicollinear with local market characteristics?
5. [0.5 pt] Can we use normal linear regression for this study?
6. [0.5 pt] The follow picture shows model specification in page 8 of this paper. Do you find any problem with this equation and explanation, especially for the explanatory variable part?



$$\ln\left(\frac{P}{1-P}\right) = \alpha + \sum_{j=1}^{n} \beta_j x_i$$

Where $ln(\frac{P}{1-P})$ is the log odds of householder mobility and $P/(1-P)$ is the odds of the outcome, α is the constant term, j represents each of the exposure changes in household composition variables, and x denotes the household characteristics, unit characteristics, and local market characteristics that are used as controls.

Page 8 - Model Specification

Model Configurations and Results

This study use logistic regression, which is similar to linear regression but for binary response variable (1/0). We change Y with $\log(P/(1-P))$, where P is the probability of $Y = 1$. This post from IBM would be helpful to understand logistic regression. Think about this model:

$$\log(\frac{P}{1-P}) = \beta_0 + \beta_1 X_1 + \beta_2 X_2$$

1. [1 pt] How should we interpret β_1?
2. [0.5 pt] In the result tables of this paper, they use **Odds Ratio** rather than the coefficient of X. How could you calculate the coefficient β for Household Member Enters or Exits (with odd ratio of 1.671)? The table below shows part of the results.
3. [0.5 pt] How much more likely are households with a compositional change (a member entering or exiting) to move, compared to those without, controlling for other factors?
4. [1 pt] What are the null and alternative hypotheses being tested by the t-value (398.7) for "Household Member Enters or Exits" in Model 1? What is the meaning of '***'?
5. [1 pt] They conducted at least three logistic regression models. Why did they do that?
6. [1 pt] The authors only use tables to present results. What is one way to visualize the adds ratio from this logistic model to show uncertainty of the results? You can use a plot example to illustrate.
7. [1 pt] The title of this paper uses *association* rather than *effect*, *impact*, or *influence*. Authors are also very cautious term usages in main text. What is the reason?

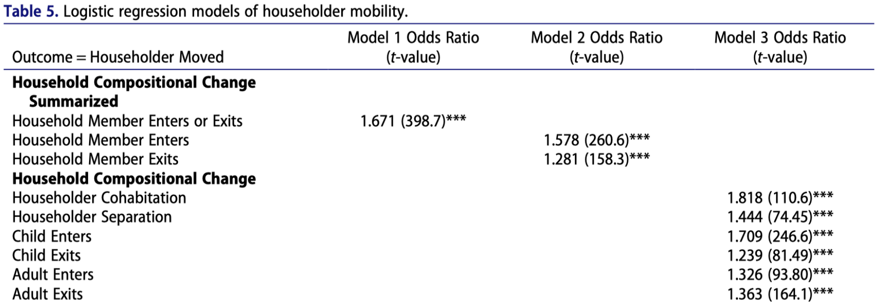


Table 5. Logistic regression models of householder mobility.

Outcome = Householder Moved	Model 1 Odds Ratio (*t*-value)	Model 2 Odds Ratio (*t*-value)	Model 3 Odds Ratio (*t*-value)
Household Compositional Change Summarized			
Household Member Enters or Exits	1.671 (398.7)***		
Household Member Enters		1.578 (260.6)***	
Household Member Exits		1.281 (158.3)***	
Household Compositional Change			
Householder Cohabitation			1.818 (110.6)***
Householder Separation			1.444 (74.45)***
Child Enters			1.709 (246.6)***
Child Exits			1.239 (81.49)***
Adult Enters			1.326 (93.80)***
Adult Exits			1.363 (164.1)***

Table 5 - Logistic Regression Results

Lab 07-B: Regression in Machine Learning

We will use `tidymodels`, which is a R package for modeling and machine learning in R using tidyverse principles. It provides us a framework. It is helpful to know that machine learning and deep learning are normally run with Python. For the consistence of this class, we will continue use R. But it is not difficult to transfer to Python! Some resources:

- An Introduction to Python for R Users
- Conversion R and Python

```
#install.packages("tidymodels") # you only need to install once
library(tidymodels)
```

We will use the same dataset as section A but let's forget the most assumptions (normality, linearity, etc.) and focus purely on predictive performance! We will train and evaluate linear regression, ridge regression, and lasso regression using cross-validation to see which approach yields the best testing predictions.

Data Preparation

There are too many variables can be added to the model. Some are not suitable for a prediction task, such as longitude and latitude. We start from remove some data columns and merge some of them. After that, do some cleaning based on inspection.

In a more rigorous machine learning process, we may conduct feature engineering and selection before training. For example, we could use Principal Component Analysis (PCA) to reduce dimension or select most important features based on correlation results. A more comprehensive book on this: Feature Engineering and Selection: A Practical Approach for Predictive Models.

```
sales_pred <- sales %>%
  mutate(view_mountains = view_rainier + view_olympics + view_cascades + view_territorial,
         view_water = view_sound + view_lakewash + view_lakesamm + view_otherwater)
# select some features (subjective here)
sales_pred <- sales_pred %>%
  dplyr::select(sale_price, year_built, sqft_lot, sqft, beds, view_skyline, view_mountains, view_water)
```

Before train the model, we need to split the dataset into a training set and a testing set. The training data will be used to train the models, while the testing data will be reserved to evaluate how well the model generalizes to unseen observations.

```
set.seed(123)
split <- initial_split(sales_pred, prop = 0.8) # 80% training data
train_data <- training(split)
test_data  <- testing(split)
```

We are going to build a recipe for data preprocessing. A recipe is the steps to do data preparation under `tidymodels` framework. For Lasso and ridge, we need to normalize features.

```
rec <- recipe(sale_price ~ ., data = train_data) %>% # model and dataset
  step_poly(year_built, degree = 5) %>% # add polynomial expansion to x, x^2, x^3, x^4, x^5
  step_poly(sqft, degree = 5) %>%
  step_poly(sqft_lot, degree = 5) %>%
  step_poly(beds, degree = 5) %>%
  step_zv(all_numeric()) %>% # remove variables that contain only a single value
  step_normalize(all_predictors()) # normalize (center and scale) the numeric variables
```

Please note that The object `rec` is a recipe that has **not been trained** on data yet (for example, does not normalize).

Model Specification

We start from specify the lasso regression. In `tidymodels`, the model type (`linear_reg`) and the computational engine (`glmnet`) are separated.

```
lasso_spec <- linear_reg(
  penalty = tune(), # penalty hyperparameter: tuneable and selected by cross-validation
  mixture = 1
  # mixture = 1 is lasso and mixture = 0 is ridge
  # mixture in (0,1) is elastic net (combination of lasso and ridge)
) %>%
  set_engine("glmnet") # specify which package or system will be used to fit the model
  # `glmnet` is the engine that implements penalized linear models.
```

A workflow is a container that bundles together a recipe (data preprocessing steps) and a model specification.

```
# use `workflow()` to put receipt and models together. We have not train yet!
lasso_workflow <- workflow() %>%
  add_recipe(rec) %>%
  add_model(lasso_spec)
```

Tuning Grid for Penalty

We can define the search space for λ. `penalty(range = c(a, b))` specifies the log10 scale from 10^a - 10^b.

```
lambda_grid <- grid_regular(
  penalty(range = c(2, 5)), # 100 - 10000; determined by yourself after attempts
  levels = 50 # try 50 values between the previous range
)
```

Cross-validation Setup

We make 5 folds to separate the `train_data` into five equally sized subsets for cross-validation.

```
folds <- vfold_cv(train_data, v = 5)
```

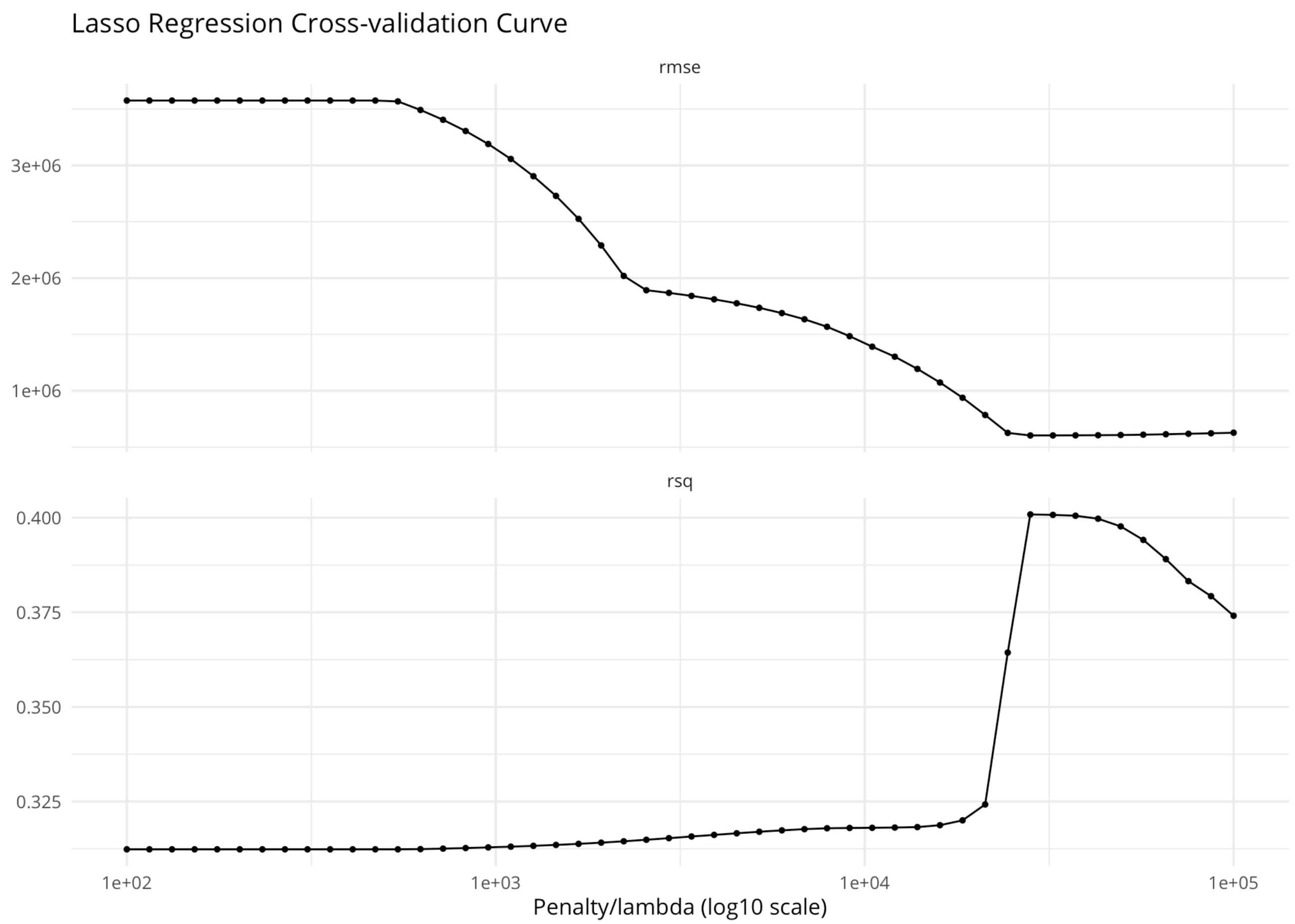
Model Training via Grid Search

```
tune_results <- tune_grid(
  lasso_workflow,
  resamples = folds,
  grid = lambda_grid,
  metrics = metric_set(rmse, rsq) # we evaluate two metrics: RMSE (error) and R² (fit)
)
```

Cross-validation Results

We show the model performance changes as λ increases via a curve.

```
autoplot(tune_results)+
  theme_minimal(base_family = "opensans", base_size = 12) +
  labs(title = "Lasso Regression Cross-validation Curve",
       x = "Penalty/lambda (log10 scale)")
```



Fit the Final Model using the Best λ

```
best_lasso <- select_best(tune_results, metric = "rmse")

# another workflow
final_lasso <- workflow() %>%
  add_model(finalize_model(lasso_spec, best_lasso)) %>% # finalize_model() inserts the chosen lambda ir
  add_recipe(rec) %>% # the same recipe
  fit(train_data) # train the final Lasso model using the full training set
```

We can check the final coefficients using `tidy(workflow)`. variables with estimate = 0 have been removed by lasso regression.

```
tidy(final_lasso) %>% filter(estimate != 0)
```

```
## # A tibble: 8 × 3
##   term                estimate penalty
##   <chr>                  <dbl>   <dbl>
## 1 (Intercept)         1129450.  28118.
## 2 view_skyline        32961.  28118.
## 3 view_water         152808.  28118.
## 4 year_built_poly_1  -56966.  28118.
## 5 year_built_poly_3    6192.  28118.
## 6 sqft_poly_1        389353.  28118.
## 7 sqft_poly_2         99505.  28118.
## 8 sqft_poly_5          1415.  28118.
```

Testing

We evaluate the performance of the final Lasso model on the held-out test set. The `predict()` function generates predicted sale prices for the test data, and `metrics()` computes evaluation measures such as RMSE and R².

```
pred_lasso <- predict(final_lasso, test_data) %>%
  bind_cols(test_data)
metrics(pred_lasso, truth = sale_price, estimate = .pred)
```

```
## # A tibble: 3 × 3
##   .metric .estimator .estimate
##   <chr>   <chr>          <dbl>
## 1 rmse    standard   663413.  
## 2 rsq     standard        0.387
## 3 mae     standard   357505.  
```

📚 TODO: Review the Code for Regression

2 points

Carefully review and run all code chunks. You are encouraged to modify the model and explore ridge regression. Reply “yes” once you have reviewed them.

📚 TODO: Machine Learning to Predict the Rental Value

4 points

In this section, we will read a recent research paper that applies several machine learning regression techniques—including Lasso, Ridge, and tree-based models—to predict housing rental values in three African countries. The paper, Predicting the rental value of houses using hedonic and machine learning regression models examines how different modeling approaches perform when estimating rental values based on a wide range of housing characteristics, infrastructure indicators, and household attributes. You can find the PDF version of the article in the lab folder. Please skim the paper first and response to following questions. You can skip the parts beyond regression.

1. [1 pt] Why do we need regularization in Lasso and Ridge regression? What effect does regularization have on variance and bias, respectively?
2. [1 pt] In the model, the variable "roof" equals 1 if the house has a mud roof and 0 otherwise. If the Lasso regression sets the coefficient of this variable to zero in 2010, can we conclude that roof type is not associated with rental value? Why or why not?
3. [1 pt] Why do the authors use cross-validation in their analysis? And how cross-validation is used to choose the regularization parameter (λ) in Lasso and Ridge regression?
4. [1 pt] In the prediction performance table, OLS performs best in the in-sample evaluation, while Lasso and Ridge outperform OLS in the out-of-sample evaluation. Could you guess the reason for this pattern?

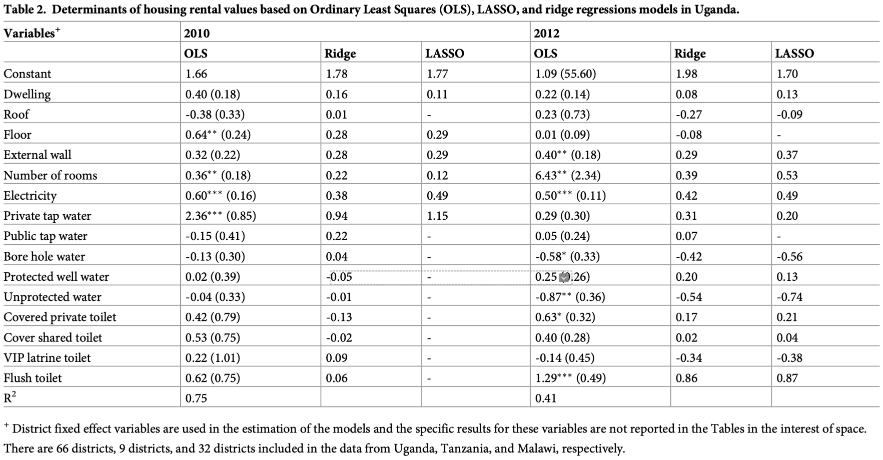


Table 2. Determinants of housing rental values based on Ordinary Least Squares (OLS), LASSO, and ridge regressions models in Uganda.

Variables$^+$	2010			2012		
	OLS	Ridge	LASSO	OLS	Ridge	LASSO
Constant	1.66	1.78	1.77	1.09 (55.60)	1.98	1.70
Dwelling	0.40 (0.18)	0.16	0.11	0.22 (0.14)	0.08	0.13
Roof	-0.38 (0.33)	0.01	-	0.23 (0.73)	-0.27	-0.09
Floor	0.64** (0.24)	0.28	0.29	0.01 (0.09)	-0.08	-
External wall	0.32 (0.22)	0.28	0.29	0.40** (0.18)	0.29	0.37
Number of rooms	0.36** (0.18)	0.22	0.12	6.43** (2.34)	0.39	0.53
Electricity	0.60*** (0.16)	0.38	0.49	0.50*** (0.11)	0.42	0.49
Private tap water	2.36*** (0.85)	0.94	1.15	0.29 (0.30)	0.31	0.20
Public tap water	-0.15 (0.41)	0.22	-	0.05 (0.24)	0.07	-
Bore hole water	-0.13 (0.30)	0.04	-	-0.58* (0.33)	-0.42	-0.56
Protected well water	0.02 (0.39)	-0.05	-	0.25 (0.26)	0.20	0.13
Unprotected water	-0.04 (0.33)	-0.01	-	-0.87** (0.36)	-0.54	-0.74
Covered private toilet	0.42 (0.79)	-0.13	-	0.63* (0.32)	0.17	0.21
Cover shared toilet	0.53 (0.75)	-0.02	-	0.40 (0.28)	0.02	0.04
VIP latrine toilet	0.22 (1.01)	0.09	-	-0.14 (0.45)	-0.34	-0.38
Flush toilet	0.62 (0.75)	0.06	-	1.29*** (0.49)	0.86	0.87
R^2	0.75			0.41		

$^+$ District fixed effect variables are used in the estimation of the models and the specific results for these variables are not reported in the Tables in the interest of space. There are 66 districts, 9 districts, and 32 districts included in the data from Uganda, Tanzania, and Malawi, respectively.

Determinants (~Coefficients) of Housing Rental Values

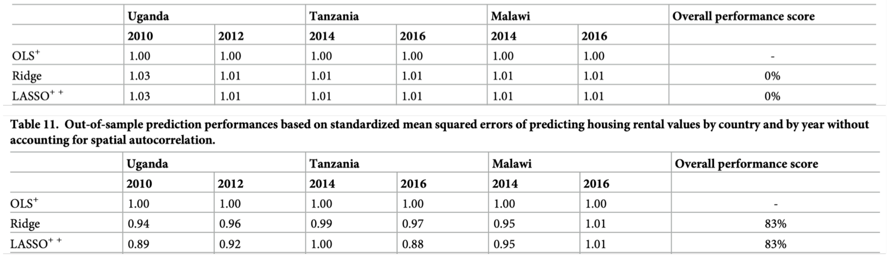


Table 8. In-sample prediction performances based on standardized mean squared errors of predicting housing rental values by country and years of analysis.

	Uganda		Tanzania		Malawi		Overall performance score
	2010	2012	2014	2016	2014	2016	
OLS$^+$	1.00	1.00	1.00	1.00	1.00	1.00	-
Ridge	1.03	1.01	1.01	1.01	1.01	1.01	0%
LASSO$^{+\,+}$	1.03	1.01	1.01	1.01	1.01	1.01	0%

Table 11. Out-of-sample prediction performances based on standardized mean squared errors of predicting housing rental values by country and by year without accounting for spatial autocorrelation.

	Uganda		Tanzania		Malawi		Overall performance score
	2010	2012	2014	2016	2014	2016	
OLS$^+$	1.00	1.00	1.00	1.00	1.00	1.00	-
Ridge	0.94	0.96	0.99	0.97	0.95	1.01	83%
LASSO$^{+\,+}$	0.89	0.92	1.00	0.88	0.95	1.01	83%

In-sample and Out-of-sample Prediction Performances

Acknowledgement

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