

It makes the most sense to me to start from scratch with Euler's mass and momentum conservation equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (1)$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \rho \mathbf{g} + \mathbf{f}_{\text{rad}} \quad (2)$$

where ρ = wind density, \mathbf{v} = wind velocity, p = gas pressure, $\mathbf{g} = -\frac{GM_{\text{gal}}}{r^2}$, and \mathbf{f}_{rad} is the force per unit volume of wind material due to radiation. Murray (2005, ApJ, 618, 569) ignores gas pressure, so we set $p = 0$. With the galaxy having luminosity L and the gas having opacity κ , $\mathbf{f}_{\text{rad}} = \rho \frac{\kappa L}{4\pi cr^2}$.

If we assume a steady state for the flow, we can rewrite the above equations:

$$\nabla \cdot (\rho \mathbf{v}) = 0 \quad (3)$$

$$\rho (\mathbf{v} \cdot \nabla) \mathbf{v} = \rho \mathbf{g} + \mathbf{f}_{\text{rad}}; \quad (4)$$

Then, assuming spherical symmetry, we can rewrite them again in spherical coordinates:

$$\frac{d}{dr}(r^2 \rho v) = 0 \quad (5)$$

$$\rho v \frac{dv}{dr} = -\frac{\rho GM_{\text{gal}}}{r^2} + \rho \frac{\kappa L}{4\pi cr^2}. \quad (6)$$

Equation 5 of course implies $r^2 \rho v = \text{const}$, or in other words, $\frac{dM_{\text{wind}}}{dt} = \text{const}$. Dividing Equation 6 by ρ , and substituting $M_{\text{gal}} = 2\sigma^2 r/G$ (i.e., the galaxy is an isothermal sphere with velocity dispersion σ), we get

$$v \frac{dv}{dr} = \frac{\kappa L}{4\pi cr^2} - \frac{2\sigma^2}{r}. \quad (7)$$

Then it's easy to solve this equation, setting $v = 0$ at R_0 :

$$v(r) = 2\sigma \sqrt{R_g \left(\frac{1}{R_0} - \frac{1}{r} \right) + \ln(R_0/r)} \quad (8)$$

with $R_g = \frac{\kappa L}{8\pi c\sigma^2}$. These are Murray's equations 26 and 27, for an optically thin wind. If you then choose whatever mass outflow rate you want ($\frac{dM_{\text{wind}}}{dt}$), we can write $r^2 \rho v = \text{const} = \frac{dM_{\text{wind}}}{dt}$, and solve for ρ :

$$\rho(r) = \frac{dM_{\text{wind}}/dt}{r^2 v(r)}, \quad (9)$$

and you have some nice inputs for the radiative transfer code (I hope).

The requirement for the wind to be launched is that

$$\frac{\kappa L}{4\pi r^2 c} > \frac{2\sigma^2}{r}, \quad (10)$$

or

$$L > \frac{8\pi\sigma^2 R_0 c}{\kappa} = L_{Edd}. \quad (11)$$

Plugging in the definition for R_g (above), this implies $R_g = (L/L_{Edd})R_0$, and that R_g must be larger than R_0 .

And now a comment. I had originally thought that Murray’s “optically thick wind”, discussed in section 2.3 of his paper, would be a good input, as his velocity equation for this is very simple:

$$v(r) = 2\sigma\sqrt{\left(\frac{L}{L_M} - 1\right)\ln(r/R_0)}. \quad (12)$$

However, this equation no longer makes sense to me. To obtain this equation, he first sets up the forces on an “optically thick” spherical shell (his Equation 14):

$$\frac{dP}{dt} = M_g(r)\frac{dv}{dt} = -\frac{GM(r)M_g(r)}{r^2} + \frac{L(t)}{c}, \quad (13)$$

where $M_g(r)$ is the mass profile of this shell. The force due to radiation here has no dependence on r , I guess because the shell completely surrounds the source and absorbs every photon no matter what r it has. He uses this equation to find the minimum luminosity required for the galaxy to blow out all its gas, L_M , and derives the acceleration equation:

$$\frac{dv}{dt} = \frac{GM(r)}{r^2}\left(\frac{L}{L_M} - 1\right). \quad (14)$$

If you just think of this as the equation of motion of a single shell, then I think you can replace $\frac{dv}{dt}$ with $\frac{dv}{dr}\frac{dr}{dt}$ and integrate to obtain $v(r)$ (Equation 10) for this shell. But, he uses this equation as the momentum equation for a “time-independent, optically thick wind (not a shell)” – now he’s referring to an extended distribution of gas. And I think he’s saying that each shell that makes up this distribution, at each radius, feels an acceleration given by Equation 12. This just doesn’t seem right – how can all the luminosity from the galaxy get through the innermost shell to push on the next shell out? Maybe I’m missing something here. But my main point is, I like Equations 8 and 9 best for including in your models!