Problem Set #2

OCEA/EART 172/272: Geophysical Fluid Dynamics

Due: Tuesday, April 21, 2020

1. Cushman-Roisin (2nd edition) Analytical Problem 7-1 (1st edition problem 4.1)

2. Consider a broad, weak, geostrophic ocean current of uniform amplitude 5 cm/s and width 200 km directed from west-to-east at a latitude of 60S. Which side of this current has a higher sea surface elevation and by how much?

3. Cushman-Roisin (2nd edition) Analytical Problem 7-2. (1st edition problem 4.2) (Hint: First, come up with the necessary relationship between the rotation vector and each component of velocity. Second, prove this vector relationship for each velocity component from the equations that include f and $f_\star$ terms.)

4. The shallow water model discussed in class is given by

$$\begin{aligned} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv &= -g\frac{\partial \eta}{\partial x} \\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu &= -g\frac{\partial \eta}{\partial y} \\ \frac{\partial h}{\partial t} + \frac{\partial uh}{\partial x} + \frac{\partial vh}{\partial y} &= 0. \end{aligned}$$

   Take the $\hat{k}\cdot\nabla\times$ of the horizontal momentum equations (i.e., take the curl of the horizontal components) and use the height equation to derive the potential vorticity equation

$$\frac{Dq}{Dt} = 0$$

   where $q = \frac{f+\zeta}{h}$ for the shallow-water model. Note that this problem can be done exactly (i.e., no need to drop terms that are small), and for general Coriolis parameter (i.e., not $f =$const).

5. Cushman-Roisin (2nd edition) Analytical Problem 7-6. (1st edition problem 4.6)

6. Extra Credit: A very conceptual **thought** experiment: Consider that the atmosphere is an inviscid, single layer of fluid. A westerly wind (blowing from west to east) must cross a mountain range oriented meridionally (i.e., north-south). Approximate this range as a single mountain whose elevations are unchanged in the north-south direction. Assume that the upper boundary (tropopause) does not vary appreciably, but that the height variation of the mountain range is significant and requires changes to latitude and/or relative vorticity to compensate. (Hints: The goal of this thought experiment is to work out plausible trajectories for zonal flow crossing a meridionally-oriented barrier while conserving PV. You will need to consider changes in relative vorticity, height and Coriolis parameter. Assume no changes in the flow in the meridional direction so $\frac{\partial u}{\partial y}$ is zero. The answers for eastward and westward motion are different.)

   (a) Use conservation of potential vorticity to schematically draw the trajectory of the westerly wind (i.e., wind from the west) before, over, and east of the mountain range.

      i. First do this problem assuming that latitudinal excursions over the mountain are sufficiently small that the Coriolis parameter does not change appreciably.

      ii. Now expand your picture to imagine that over large distances the Coriolis parameter does change, but that latitudinal excursions over the mountain range itself are not large enough for a meaningful change in f. Does this impact your trajectories much over the mountain? How about east of the mountain?

iii. Finally, consider the case where latitudinal excursions over the mountain itself are large enough that the Coriolis parameter changes meaningfully over the mountain. The case where changes in f are small is similar to ii, so for this answer, allow sufficiently large changes in f to have a qualitative change in the trajectory over the mountain.

(b) Now consider an easterly wind for case iii above. What is a plausible new trajectory? (you might need to go through i and ii on your own first.)