Homework 1

Geophysical Fluid Dynamics I

Due 1/22/2024 (grace period until 1/24/2024)

You are encouraged to work with your peers on homework problems. I am also available to help with parts you are struggling with. You may not, however, consult graded homework assignments or homework solutions from previous years.

1. Consider the geoid, which is the surface of constant geopotential at sea level. In class we defined the geopotential (Φ) to satisfy

$$\vec{g} = -\nabla\Phi \tag{1}$$

with $\vec{g}$ being the combined effect of Newtonian gravity due to the mass of the earth and the centrifugal force associated with earth's rotation.

   (a) Derive expressions for Φ at the equator (Φ_e) and pole (Φ_p) as a function of distance from earth's center (r), as well as earth's mass (M) and rotation rate (Ω).

   (b) Using these expressions, estimate the size of the equatorial bulge of the geoid, i.e., how much larger the equatorial radius is than the polar radius. (Hint: You can simply the calculation by approximating that the size of the bulge is tiny compared to the radius of the earth and Taylor expanding. Google can help with numbers; for example, google calculates g when you enter `G*(mass of earth)/(radius of earth)^2`.)

   [Note that you should find that your answer noticeably underestimates the standard textbook value, which is 21 km (e.g., Cushman-Roisin & Beckers, p. 45). This is because for gravity we assumed that the earth's mass forms a sphere of uniform density, which the Shell Theorem tells us is equivalent to a point mass at the center. But this is inaccurate because the mass of the earth has a bulged shape, rather than being spherical, and this has a gravitational effect.]

   (c) In class we considered a rock on a flat ocean bottom. Calculate how much the rock deflects the ocean surface, i.e., how much higher the geoid is above the rock compared to a location far from the rock. (For this problem you can ignore the centrifugal term that is considered above.) (Hint: First calculate the geopotential far from the rock, where the rock has no influence. Then calculate the geopotential directly above the rock. Specify that the geoid is at a baseline height plus a deviation due to the rock, and then you can simplify your math by Taylor expanding for a small height deviation.)

2. In class we derived the equation for the Coriolis force when velocities are slow compared with earth's rotation. Here we will look at the full momentum equation, which also allows fast velocities. This vector equation is given in eq. (2.47a,b,c) of the Vallis textbook. (Note that in the pressure gradient force term, Vallis is using $r\,\delta\theta$ for meridional displacement and $r\cos\theta\,\delta\lambda$ for zonal displacement, rather than y and x as we have used in class.) Consider the acceleration of spatially-uniform horizontal motion with no pressure gradient or friction.

   (a) Use Matlab (or similar) to plot the meridional acceleration for purely zonal velocity (Dv/Dt vs u), allowing the velocity to vary over a wide range of positive and negative values. Compare this full equation with the approximate equation that we derived in class (i.e., include both curves in your plot).

(over)

(b) Explain physically why Dv/Dt has an extremum at one value of u and is zero at two values of u, as well as why the motion is deflected to the *left* in the Northern Hemisphere for certain velocities. (Hint: expand the range of plotted velocities if you don't see these features in your plot; you may perhaps find it useful to normalize u by $R\Omega$ and to find a relevant way to nondimensionalize t as well.)

3. Look at the paper Engelhardt and Engelhardt (2017) [here]. You don't need to worry about the details of the derivations in the paper, but try to get a sense of the big picture. Based on this paper and the concepts that were discussed in class for why the earth's geoid is not spherical, briefly describe why an iceberg that is initially at rest floating in the ocean would start drifting equatorward if the ocean current velocity, sea surface tilt, and atmospheric wind velocity were all zero.