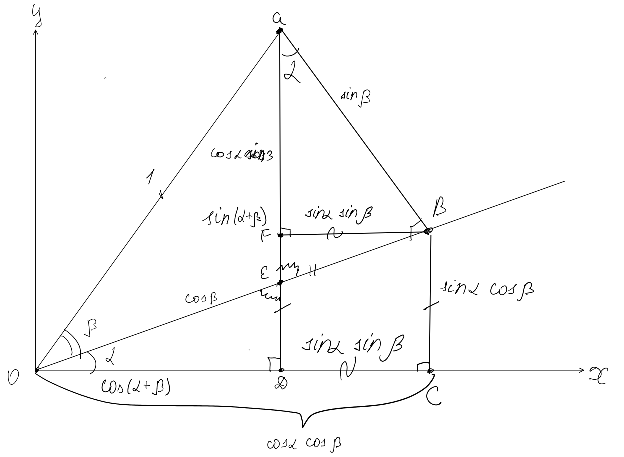
Deriving sum and sub sin and cos formula

13 February, 2023 16:12

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$



$\overline{Oa} = 1$

$\sin\beta = \frac{\overline{aB}}{\overline{Oa}} = \frac{\overline{aB}}{1} \Rightarrow \overline{aB} = \sin\beta$

$\sin\alpha = \frac{BC}{OB} = \frac{BC}{\cos\beta} \Rightarrow BC = \sin\alpha\cos\beta$ $\quad\quad$ $\cos\alpha = \frac{OC}{OB} = \frac{OC}{\cos\beta} \Rightarrow OC = \cos\alpha\cos\beta$

$\measuredangle OED = \measuredangle aEB$ — vertical $\rightarrow \triangle OED \sim \triangle aEB \Rightarrow \measuredangle EaB = \alpha$

$\cos\alpha = \frac{aF}{aB} = \frac{aF}{\sin\beta} \Rightarrow aF = \cos\alpha\sin\beta$

$\sin\alpha = \frac{FB}{\sin\beta} \Rightarrow FB = \sin\alpha\sin\beta$

$\cos(\alpha+\beta) = OD = OC - DC = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

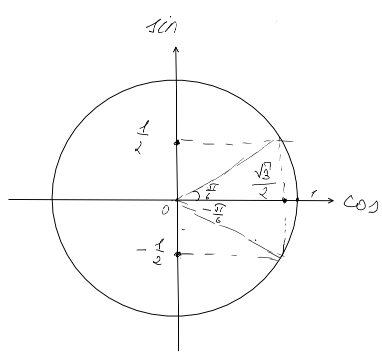
$FD = BC = \sin\alpha\cos\beta$

$\sin(\alpha+\beta) = aD = aF + FD = \cos\alpha\sin\beta + \sin\alpha\cos\beta$

$\cos(\alpha-\beta) = \cos(\alpha+(-\beta))$

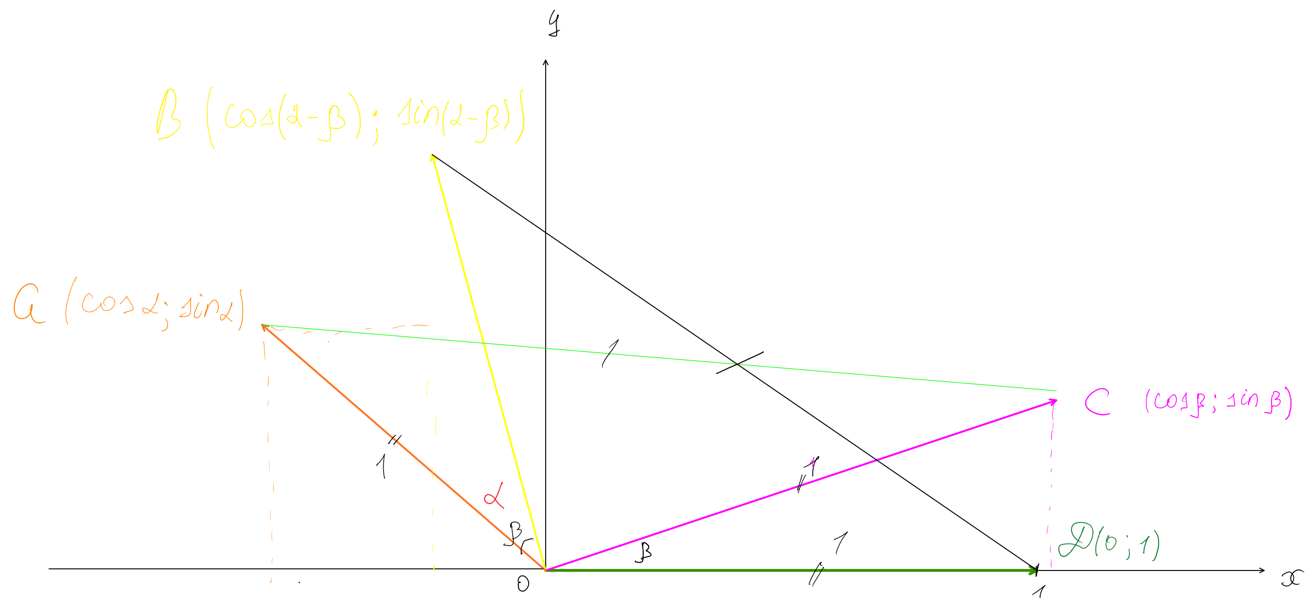
$\cos(\alpha-\beta) = \cos\alpha\cos(-\beta) - \sin\alpha\sin(-\beta) =$
$\Rightarrow \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(-\beta) = -\sin\beta$



$\sin(\alpha-\beta) = \sin(\alpha+(-\beta)) =$
$= \sin\alpha\cos(-\beta) + \cos\alpha\sin(-\beta) =$
$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$

II



$\triangle aOC = \triangle BOD \Leftarrow aO = BO \;\&\; OC = OD \;\&\; \sphericalangle aOC = \sphericalangle BOD$

$\triangle aOC = \triangle BOD \Rightarrow aC = BD$

$aB\ (x_a; y_a)\ (x_b; y_b) \qquad |aB| = \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2}$

$$aC = \sqrt{(\cos\beta - \cos\alpha)^2 + (\sin\beta - \sin\alpha)^2}$$

$aC^2 = (\cos\beta - \cos\alpha)^2 + (\sin\beta - \sin\alpha)^2 \quad | \; (a-b)^2 = a^2 - 2ab$

$aC^2 = \cos^2\beta - 2\cos\beta\cos\alpha + \cos^2\alpha + \sin^2\beta - 2\sin\beta\sin\alpha + \sin^2\alpha \quad | \; \sin$

$aC^2 = 2 - 2\cos\beta\cos\alpha - 2\cos\beta\sin\alpha$

$BD^2 = (1 - \cos(\alpha-\beta))^2 + (0 - \sin(\alpha-\beta))^2 \quad | \; (a-b)^2 = a^2$

$+ b^2$

$\sin^2\alpha + \cos^2\alpha = 1$

$- 2ab + b^2$

$$BD^2 = 1 - 2\cos(\alpha-\beta) + \cos^2(\alpha-\beta) + \sin^2(\alpha-\beta)$$

$$BD^2 = 2 - 2\cos(\alpha-\beta)$$

$$AC = BD \Rightarrow AC^2 = BD^2$$

$$2 - 2\cos\beta\cos\alpha - 2\cos\beta\sin\alpha = 2 - 2\cos(\alpha-\beta) \quad \Big| -2$$

$$-2\cos\beta\cos\alpha - 2\cos\beta\sin\alpha = -2\cos(\alpha-\beta) \quad \Big| : -2$$

$$\cos\beta\cos\alpha + \cos\beta\sin\alpha = \cos(\alpha-\beta)$$

$$\boxed{\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

$$\cos(\alpha+\beta) = \cos(\alpha-(-\beta)) = \cos\alpha\cos(-\beta) + \sin\alpha\sin(-\beta) =$$

$$= \boxed{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

$$\cos\alpha = \sin\left(\frac{\pi}{2} - \alpha\right) \qquad \sin\alpha = \cos\left(\frac{\pi}{2} - \alpha\right)$$

$$\sin(\alpha-\beta) = \cos\left(\frac{\pi}{2} - (\alpha-\beta)\right) = \cos\left(\frac{\pi}{2} - \alpha + \beta\right) =$$

$$= \cos\left(\frac{\pi}{2} - \alpha\right)\cos\beta - \sin\left(\frac{\pi}{2} - \alpha\right)\sin\beta =$$

$$= \boxed{\sin\alpha\cos\beta - \cos\alpha\sin\beta}$$

$$\sin(\alpha+\beta) = \sin(\alpha-(-\beta)) =$$

$$= \sin\alpha\cos(-\beta) - \cos\alpha\sin(-\beta) =$$

$$= \boxed{\sin\alpha\cos\beta + \cos\alpha\sin\beta}$$