Difference Logic

Satisfiability Checking Seminar

Alex Ryndin
Supervision: Gereon Kremer

WS 2016/2017

Abstract

Short abstract. Motivation is important: where the Difference Logic (DL) can be used? (circuits checking, scheduling problems etc.)

1 Introduction

Introduction into the topic (1-2 pages).

1.1 Difference Logic

Definition of difference logic + examples of valid and invalid formulas.

Definition 1.1 (Difference Logic) *Let $\mathcal{P} = \{p_1, p_2, \ldots, p_n\}$ be ... (take definition from the main article [1])*

A valid formula:

$$\phi_{\text{valid}} = (x_1 - x_2 < 5) \wedge (x_1 - x_3 \leq 7) \wedge (x_5 = -15) \wedge (x_1 - x_5 < 10 \vee x_5 - x_3 < -3)$$

and examples of invalid one is:

$$\phi_{\text{invalid 1}} = (x_1 - x_2 < 5) \wedge (x_1 - x_3 \leq \pi)$$

Because $\pi \notin \mathbb{Z}$. Though, in practice (i.e. for the real life applications) it is usually possible to round π to e.g. 3.14 and then multiply both part of the equation by 100 and then introduce new variables:

$$\begin{aligned} x_1 - x_3 &\leq 3.14 \\ 100x_1 - 100x_3 &\leq 314 \\ x_1' - x_3' &\leq 314 \\ \text{where } x_1' = 100x_1 &\text{ and } x_3' = 100x_3 \end{aligned}$$

Another one:

$$\phi_{\text{invalid 2}} = x_1 + x_2 < 5$$

Because here we have addition. Although, the reader can spot that it is possible to introduce a new variable $x_2' = -x_2$ and turn $\phi_{\text{invalid 2}}$ into a valid formula. Yet another one:

$$\phi_{\text{invalid 3}} = x_1 x_2 < 25$$

Because multiplication, division, exponentiation etc. are not covered by the Difference Logic. Only difference of two variables is valid. Difference of more than two variable is not covered either:

$$\phi_{\text{invalid } 4} = x_1 - x_2 - x_3 < 250$$

In such cases one should try to transform the "invalid" formula into a valid "canonical" one by introducing new variables (in the above's case it can be e.g. $x_2' = x_2 + x_3$), multiplying left and right side of the inequality by some value etc.

1.2 General Approach to SAT Solving

Describe here the general approach to SAT solving presented in the main paper [1] (procedure Solve, DPLL algorithm, generic scheme of tandem of SAT Solver + Theory Solver). It is a good idea to include some picture/diagram describing the DPLL approach (e.g. a block scheme of the algorithm/procedure). Maybe it is also a good idea to give a simple example. Give definitions: implication graph, unique implication point.

1.3 Approaches to Solve SAT of Difference Logic

Shortly describe possible approaches (their core ideas) to solve DL SAT problem. According to the main article [1], they are:

- The lazy approach
- The preprocessing approach
- Incremental approaches

2 Preliminaries

Theoretical stuff, on which the solver from [1] is based on (1-3 pages).

2.1 Constraint Graph (CG)

Give definition of Constraint Graph and an example. Tell also about Difference Bound Matrix (DBM). Allude to the fact that SAT problem can be described in terms of a testing a CG on a negative cycle.

2.2 Bellman-Ford Algorithm

Describe Bellman-Ford algorithm and how is it applied to the DL SAT problem: "We detect cycles using a depth-first variant of the Bellman-Ford-Moore algorithm [GR93] which has much better average case complexity in practice" [1]

3 Topic

Describe how the proposed solver [1] works (3-5 pages).

3.1 Negative Cycles Detection

Show how the satifiability of a DL formula is related to the negative cycle detection.

3.2 Implementation Details

Some implementation details (Numeric Conflict Analysis, Reducing Feasibility Checks).

3.3 Experimental Results

Tell a reader about some experimental results.

4 Conclusion

Conclusion on the topic ($\frac{1}{2}$ of a page).

References

[1] S. Cotton, E. Asarin, O. Maler, and P. Niebert. Some progress in satisfiability checking for difference logic. In *Formal Techniques, Modelling and Analysis of Timed and Fault-Tolerant Systems*, pages 263–276. Springer, 2004.