Introduction to Homotopy Theory
(Notes based on lectures by Sergiy Maksymenko)

Abstract

Homotopy theory studies spaces up to a homotopy, which is a continuous deformation of one continuous function to another. This documents is a work in progress done during a course audit. These notes are taken purposefully in English to strenghten intuition and simplify lookup of concepts in related literature.

Contents

1 Notes on Point-set Topology

Definition 1.1 (Topological space): A topological space is a pair $\langle X, \tau \rangle$, where X is a set and τ, a topology on X, is a collection of subsets ($\tau \subseteq \mathcal{P}(X)$) called open sets, such that:

- $\varnothing \in \tau$.
- $X \in \tau$.
- τ is closed under arbitrary finite intersections.
- τ is closed under abbitrary unions. **Maybe find some cute notation for this.**

Definition 1.2 (Trivial Topology): A topological space is called trivial, when the topology on X consists only of $\varnothing$ and X.

Definition 1.3 (Discrete Topology): A topological space is called discrete, when $\tau = \mathcal{P}(X)$.

Definition 1.4 (Continuous map): Let $\langle X, \tau \rangle$ and $\langle Y, \sigma \rangle$ be topological spaces. A map $f : X \to Y$ is **continuous** if $\forall s \in \sigma, f^{-1}(s) \in \tau$. In plain English, a map is continuous when a preimage of an open set in Y is an open set in X.

Base of topology and methods of inducing topologies on sets were discussed during Lecture 2.

Bonus. Compare topology to a field of sets to a σ-algebra to a Borel σ-algebra. Discussed during Lecture 5.

2 Homotopy

2.1 Definition

2.2 Examples

2.3 Homotopy and equivalence

Theorem 2.1: Homotopy equivalence between spaces is an equivalence relation.

Proof. ... □

Theorem 2.2: A homotopy between continuous maps is an equivalence relation.

Proof. ... □

2.4 Contractible Spaces

2.4.1 Convex Sets

2.4.2 Star-convex Sets

2.4.3 Trees

3 Quotient Spaces and Maps

See lectures 2 and 3.

Brouwer's Fixed Point Theorem was mentioned around here. Bring up the context?.

3.1 Quotients and Groups

3.2 Cones of Topological Spaces

4 Retractions and Deformation Retractions

5 Classes of Homotopy Maps

See lectures 4 and 5.

5.1 Mappings of S^1 to Itself

6 Resources

Course page: https://sites.google.com/site/kafedramatematikikau/products-services/homotopy-theory