

## Concurrency Theory

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Sheet 9

**Due: Monday, 2026-01-19**

### Exercise 9.1 (Petri Nets)

Consider the alternating mutual exclusion system described by the Petri net  $N = (P, T, F)$ , where:

$$P = \{\text{local}_L, \text{waiting}_L, \text{critical}_L, \text{local}_R, \text{waiting}_R, \text{critical}_R, \text{key}_{LR}, \text{key}_{RL}\}$$

$$T = \{\text{lw}_L, \text{wc}_L, \text{cl}_L, \text{lw}_R, \text{wc}_R, \text{cl}_R\}$$

$$F = \begin{cases} (\text{local}_i, \text{lw}_i), (\text{lw}_i, \text{waiting}_i), (\text{waiting}_i, \text{wc}_i), (\text{wc}_i, \text{critical}_i), (\text{critical}_i, \text{cl}_i), (\text{cl}_i, \text{local}_i) & i \in \{L, R\} \\ (\text{cl}_L, \text{key}_{LR}), (\text{key}_{LR}, \text{wc}_R), (\text{cl}_R, \text{key}_{RL}), (\text{key}_{RL}, \text{wc}_L) \end{cases}$$

- (a) Draw the Petri net.
- (b) Define the pre-set  $\bullet \text{wc}_L$  and the post-set  $\text{wc}_L^\bullet$ .
- (c) Consider the initial marking:

$$M_0 = \{(p, 1) \mid p \in \{\text{local}_L, \text{key}_{LR}, \text{local}_R\}\} \cup \{(p, 0) \mid p \in P \setminus \{\text{local}_L, \text{key}_{LR}, \text{local}_R\}\}$$

Is this marking one bounded?

- (d) Draw the initial marking  $M_0$  onto the Petri net.
- (e) Which transitions are enabled at  $M_0$ ? Fire a sequence until process  $L$  is in the critical section. List the step sequence and resulting markings.
- (f) Can this system deadlock?
- (g) Draw the complete marking graph (interleaving semantics). Is this graph unique up to isomorphism with respect to other reachable markings?
- (h) List all actions of the net.
- (i) List all pairs of actions that are causally related ( $A < B$ ) or independent ( $A \perp\!\!\!\perp B$ ).
- (j) Provide the true concurrency semantics for a full cycle as a causal net.
- (k) Attach depth information to all nodes in your causal net.