

Concurrency Theory

Winter 2025/26

Lecture 14: Interleaving Semantics of Petri Nets

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Programming Languages Group
University of Freiburg

<https://proglang.github.io/teaching/25ws/ct.html>

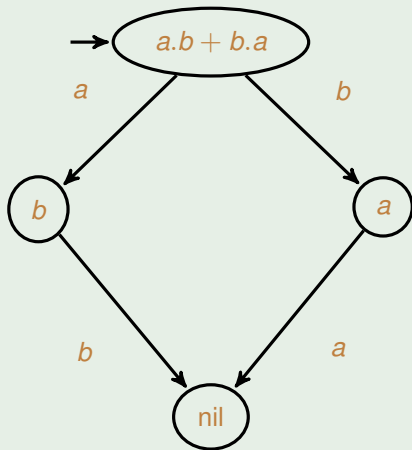
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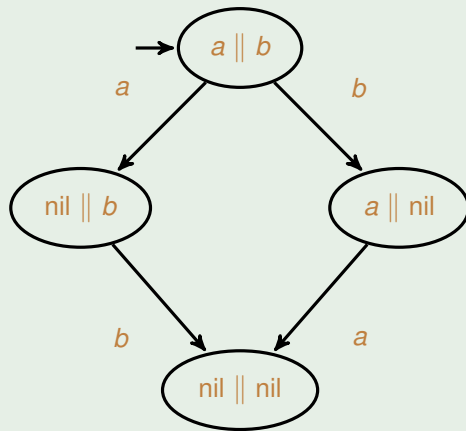
Outline of Lecture 14

- 1 Introduction
- 2 Basic Net Concepts
- 3 The Interleaving Semantics of Petri Nets
- 4 The Marking Graph
- 5 Summary

Example 14.1 (LTSs of CCS processes)

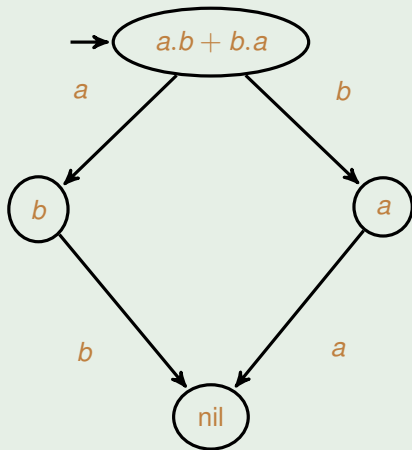


LTS of $a.b.nil + b.a.nil$

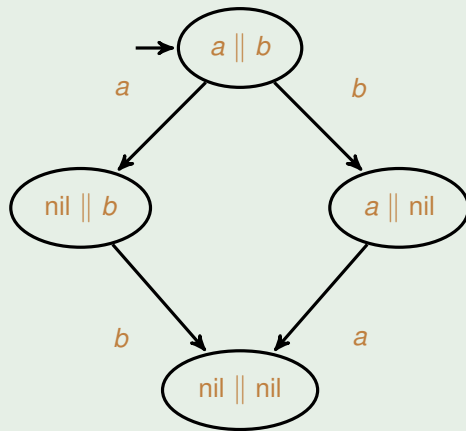


LTS of $a.nil || b.nil$

Example 14.1 (LTSs of CCS processes)



LTS of $a.b.nil + b.a.nil$



LTS of $a.nil || b.nil$

Carl Adam Petri (1926–2010)



Semantics: Executions and Traces

Models of computation in the 1960s: lambda-calculus, finite automata, Turing machines, ...

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States: current configurations of the machine

One or more initial states

Possibly some distinguished final states

Transitions: moves between configurations

Lambda calculus	$(\lambda x.xx)(\lambda y.y)$	\longrightarrow	$(\lambda y.y)(\lambda z.z)$
Turing machine	$0010q_1011$	\longrightarrow	$001q_201011$
Finite automaton	q_1	\xrightarrow{a}	q_2
Pushdown automaton	$(q_1, XYYZ)$	\xrightarrow{a}	$(q_2, XYXYYZ)$

Executions: alternating sequences of states and transitions



C.A. Petri points out a discrepancy between how **Theoretical Physics** and **Theoretical Computer Science** described systems in 1962:

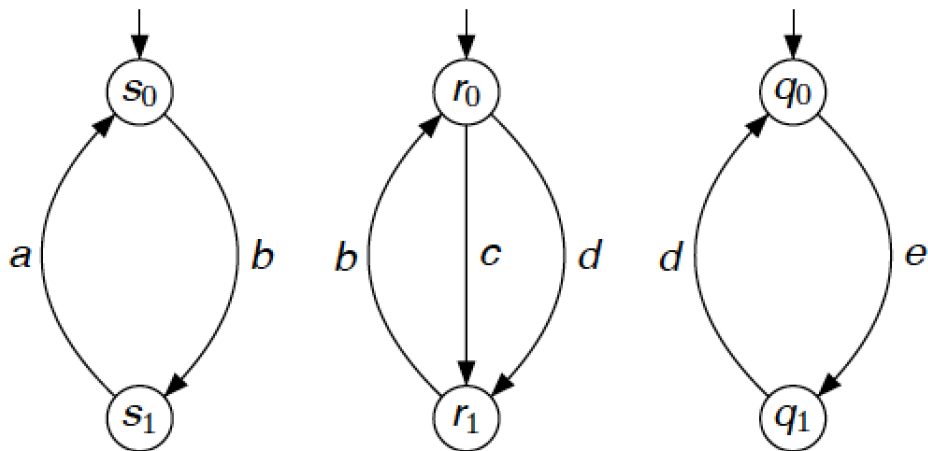
Theoretical Physics describes systems as a collection of interacting particles (subsystems), without a notion of global clock or simultaneity

Theoretical Computer Science describes systems as sequential virtual machines going through a temporally ordered sequence of global states

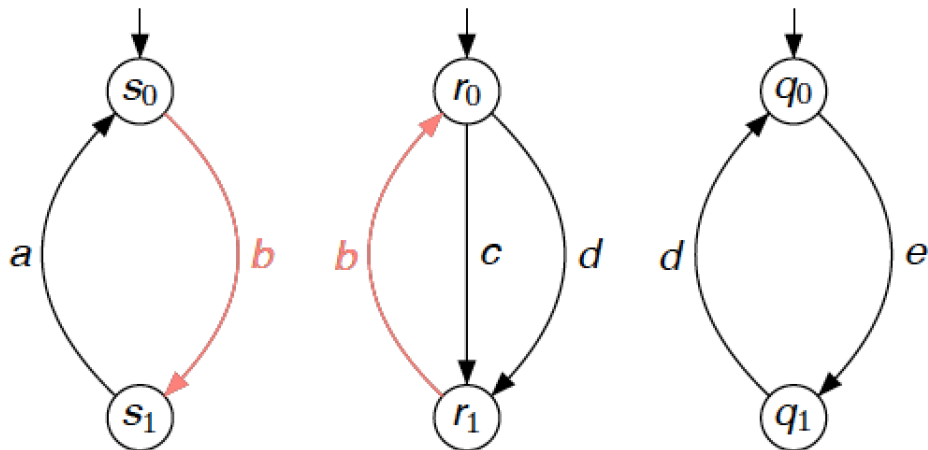
Petri's question:

Which kind of abstract machine should be used to describe the **physical implementation** of a Turing machine?

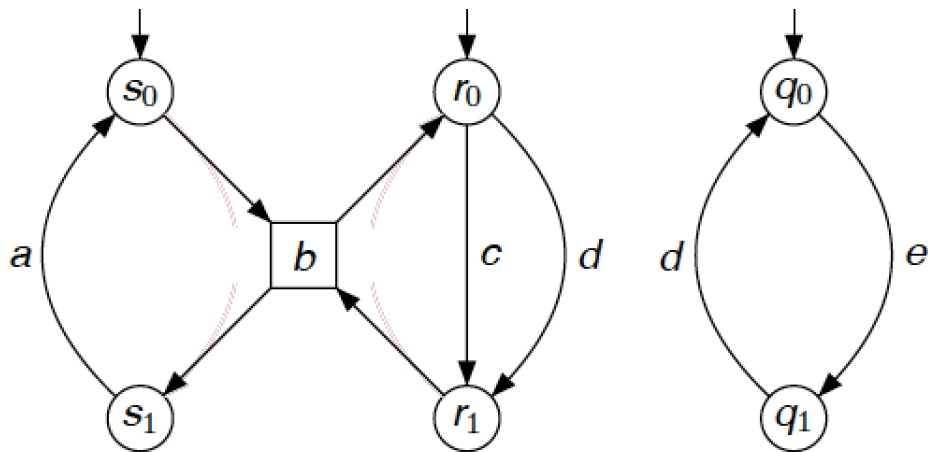
A graphical representation of interacting finite automata:



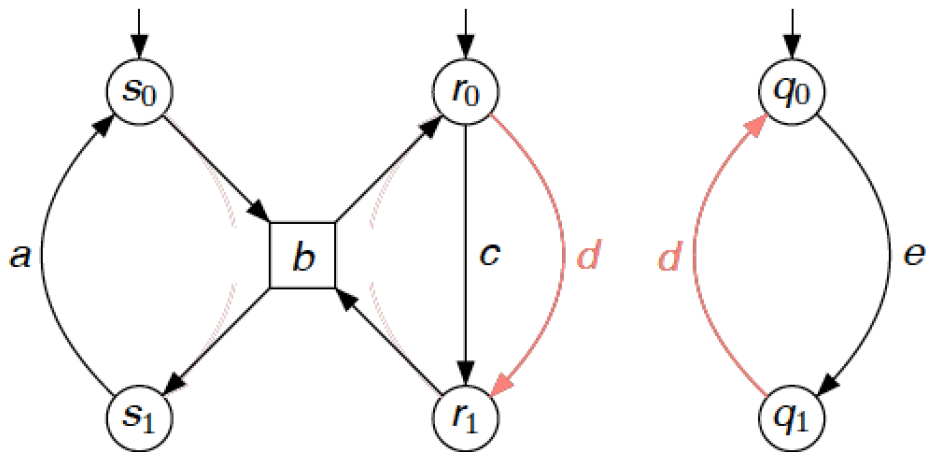
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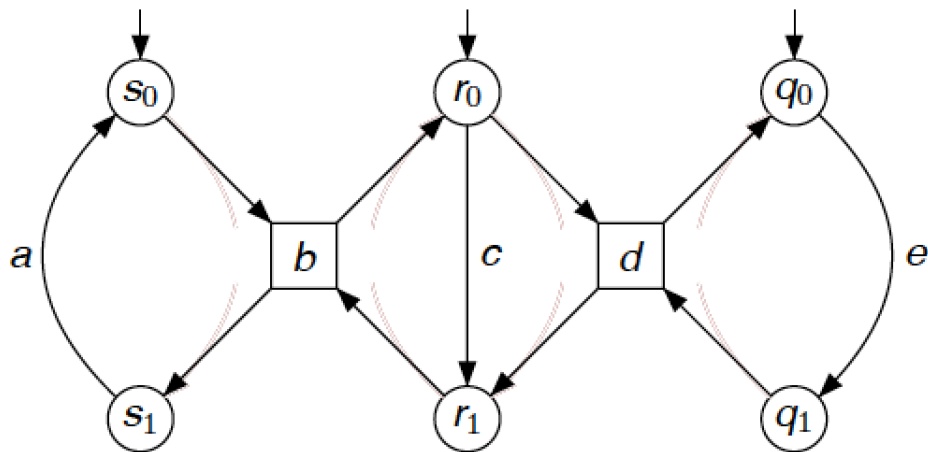
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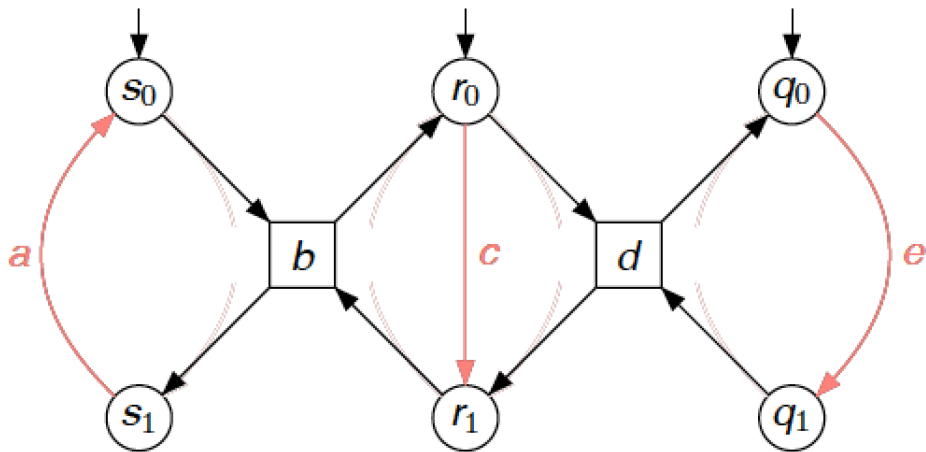
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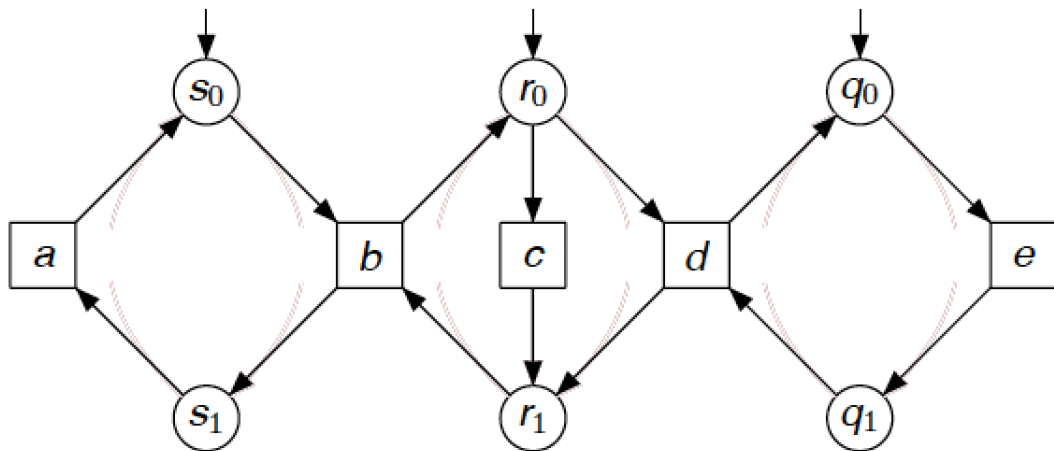
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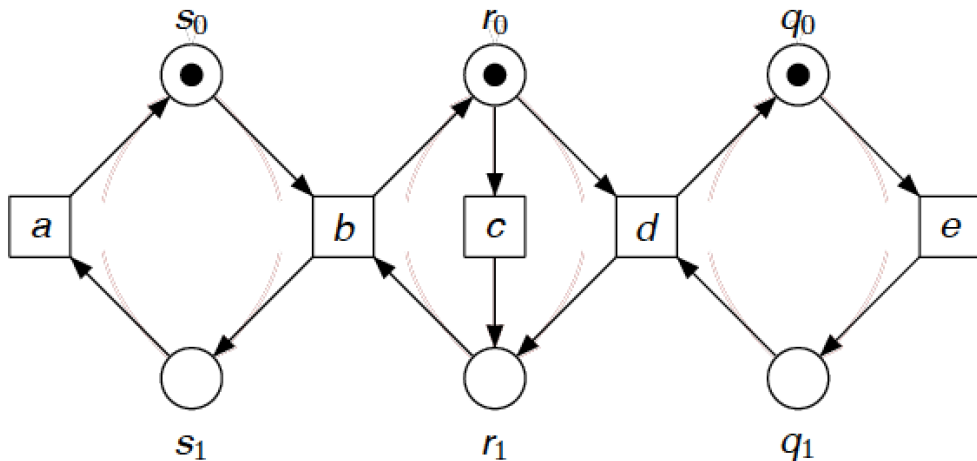
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Outline of Lecture 14

- 1 Introduction
- 2 **Basic Net Concepts**
- 3 The Interleaving Semantics of Petri Nets
- 4 The Marking Graph
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A **Petri net** is a structure with two kinds of elements: **places** and **transitions**.
These are connected by **arcs**.

Components of a Net

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A **place** is represented by a circle or ellipse. A place p always models a passive component: p can store, accumulate or show things.

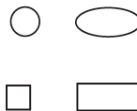


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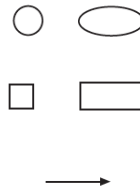
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Places and transitions are connected to each other by directed **arcs**. Graphically, an arc is represented by an arrow. An arc models an abstract, sometimes only notional relation between components.



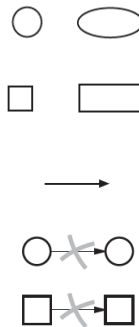
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Definition 14.2 (Petri net)

A **Petri net** N is a triple (P, T, F) where:

- P is a finite set of **places**,
- T is a finite set of **transitions** with $P \cap T = \emptyset$, and
- $F \subseteq (P \times T) \cup (T \times P)$ are the **arcs**.^a

Places and transitions are generically called **nodes**.

^a F is also called the **flow** relation.

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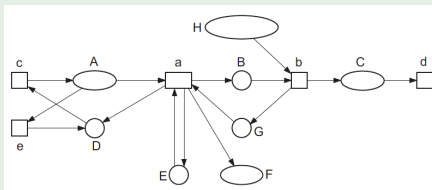
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Example 14.3



$$P = \{A, B, C, \dots\}$$

$$T = \{a, b, c, \dots\}$$

$$F = \{(A, a), (a, B), (B, b), \dots\}$$

Definition 14.4 (Pre- and post-sets)

Let node $x \in P \cup T$.

- The **pre-set** of x is defined by $\bullet x := \{y \mid (y, x) \in F\}$.
- The **post-set** of x is defined by $x^\bullet = \{y \mid (x, y) \in F\}$.

Two nodes $x, y \in P \cup T$ form a **loop** if $x \in \bullet y$ and $y \in \bullet x$.

Pre- and Post-Sets

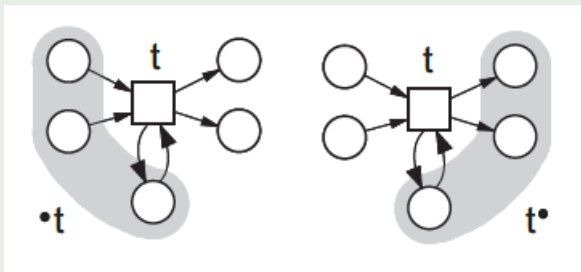
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Example 14.5



Definition 14.6 (Marking)

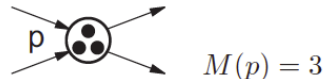
- A **marking** M of a net $N = (P, T, F)$ is a mapping $M : P \rightarrow \mathbb{N}$.
- For net $N = (P, T, F)$ and marking M_0 , the quadruple (P, T, F, M_0) is called an **elementary system net** with **initial marking** M_0 .

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Intuition:

- A marking can be seen as a **multiset** of places.
- It defines a distribution of **tokens** across places.
- Tokens are depicted as black dots.



Remark: In **generic** (= non-elementary) system nets, several types (colours) of tokens can be distinguished.

Definition 14.7 (Enabling and occurrence of a transition)

Let (P, T, F, M_0) be an elementary system net and $M : P \rightarrow \mathbb{N}$.

- Marking M **enables** a transition $t \in T$ if $M(p) \geq 1$ for each place $p \in {}^\bullet t$.
- Transition $t \in T$ can **occur** in marking M if t is enabled in M .
- Its **occurrence** or **firing** leads to marking M' , denoted by the **step** relation $M \xrightarrow{t} M'$ and defined for each place $p \in P$ by

$$M'(p) := M(p) - F(p, t) + F(t, p)$$

where we represent relation F by its characteristic function.

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Intuition: Transition t is enabled whenever every $p \in {}^\bullet t$ holds at least one token.

On t 's occurrence, one token is removed from each place in ${}^\bullet t$, and one token is put in each place in t^\bullet :

$$M'(p) = \begin{cases} M(p) - 1 & \text{if } p \in {}^\bullet t \text{ and } p \notin t^\bullet \\ M(p) + 1 & \text{if } p \in t^\bullet \text{ and } p \notin {}^\bullet t \\ M(p) & \text{otherwise} \end{cases}$$

Definition (Enabling and occurrence of a transition)

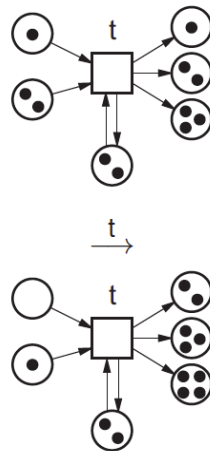
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Example 14.8



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The Interleaving Semantics of Petri Nets I

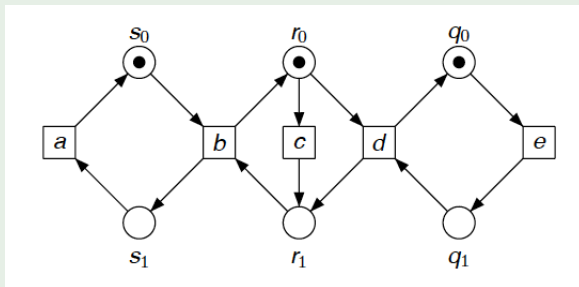
Goal: Establish an **execution semantics** by mapping a Petri net to a labelled transition system

States: markings (i.e., distributions of tokens over the net)

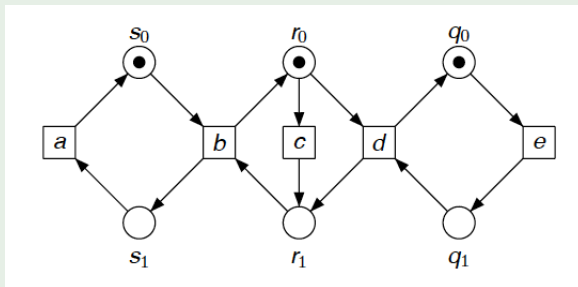
Transitions: $M \xrightarrow{t} M'$ (“steps”)

Sequential runs: $M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} M_2 \xrightarrow{t_3} \dots$ (step sequences)

Example 14.9



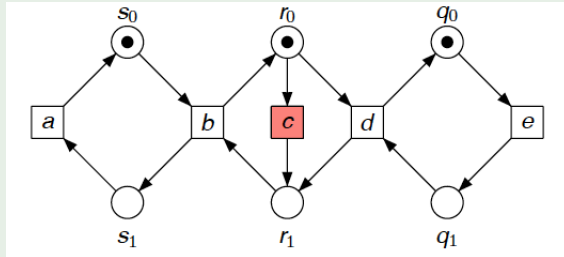
Example 14.9



$$\begin{matrix} s_1 \\ r_1 \\ q_1 \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

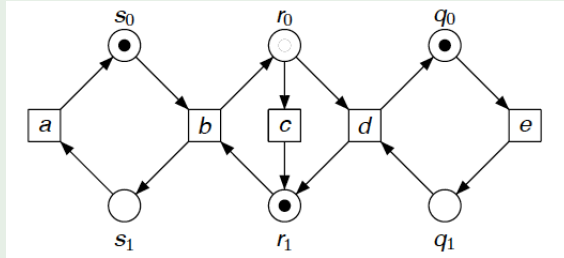
(As the marking for s_0 is the complement of s_1 , the marking for s_0 is omitted.
The same applies to the places r_0 and q_0 .)

Example 14.9 (continued)



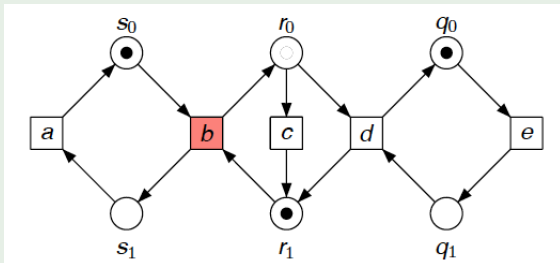
$$\begin{matrix} s_1 \\ r_1 \\ q_1 \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{c}$$

Example 14.9 (continued)



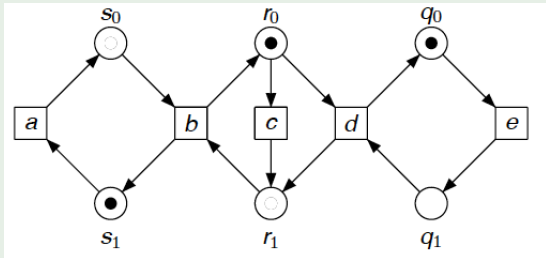
$$\begin{array}{c} s_1 \\ r_1 \\ q_1 \end{array} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{c} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Example 14.9 (continued)



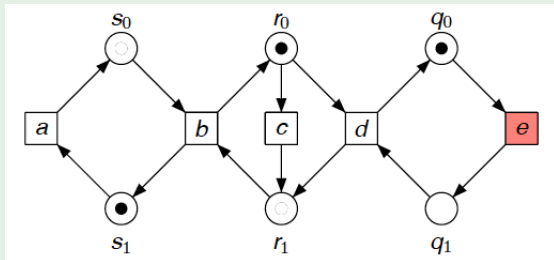
$$\begin{array}{l} s_1 \\ r_1 \\ q_1 \end{array} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{c} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \xrightarrow{b}$$

Example 14.9 (continued)



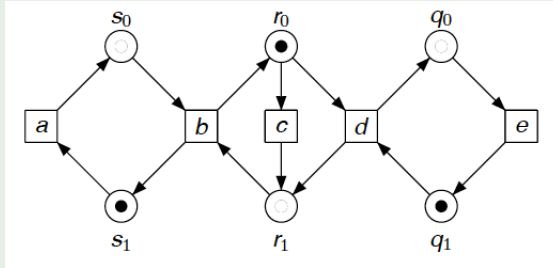
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Example 14.9 (continued)



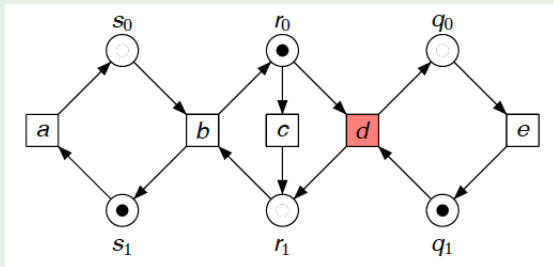
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Example 14.9 (continued)



$$\begin{array}{l}
 s_1 \\
 r_1 \\
 q_1
 \end{array}
 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
 \xrightarrow{c}
 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}
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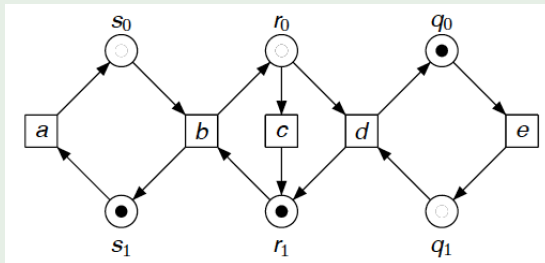
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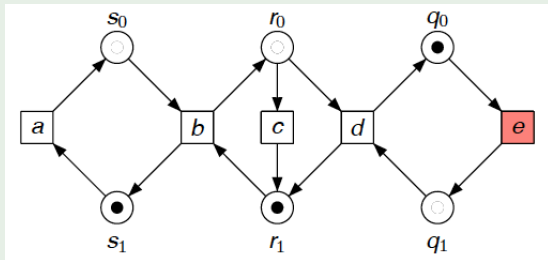
The Interleaving Semantics of Petri Nets X

Example 14.9 (continued)



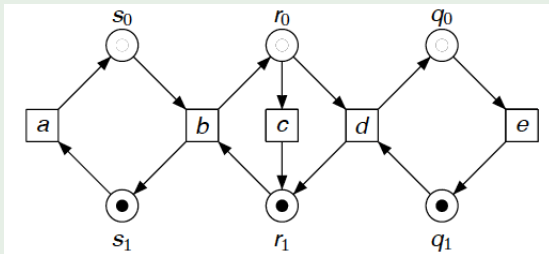
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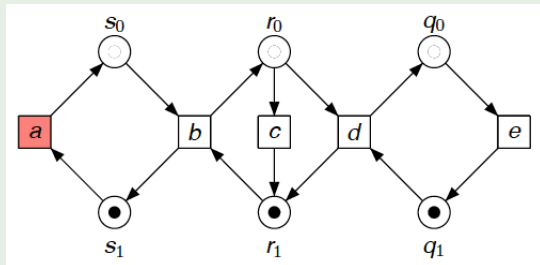
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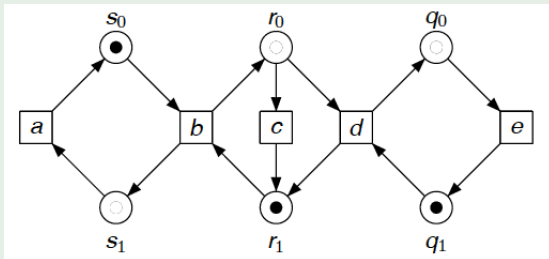
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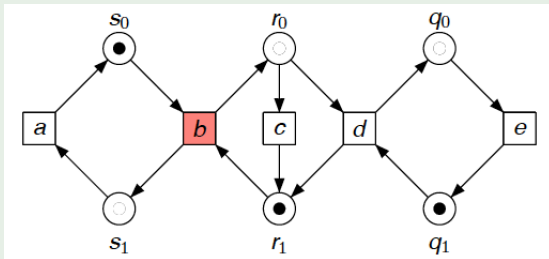
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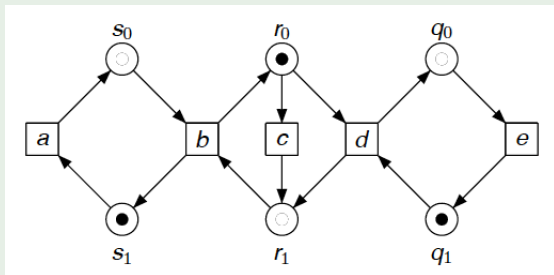
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Definition 14.10 (Step sequence)

Let (P, T, F, M_0) be an elementary system net.

- A sequence of transitions $\sigma = t_1 t_2 \dots t_n \in T^*$ is a **step sequence** if there exist markings M_1, \dots, M_n such that

$$M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} \dots \xrightarrow{t_{n-1}} M_{n-1} \xrightarrow{t_n} M_n.$$

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- Marking M_n is then **reached** by the occurrence of σ , denoted $M_0 \xrightarrow{\sigma} M_n$.
- M is a **reachable marking** if there exists a step sequence σ such that $M_0 \xrightarrow{\sigma} M$.

Reachable Markings

Definition 14.10 (Step sequence)

Let (P, T, F, M_0) be an elementary system net.

- A sequence of transitions $\sigma = t_1 t_2 \dots t_n \in T^*$ is a **step sequence** if there exist markings M_1, \dots, M_n such that

$$M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} \dots \xrightarrow{t_{n-1}} M_{n-1} \xrightarrow{t_n} M_n.$$

- Marking M_n is then **reached** by the occurrence of σ , denoted $M_0 \xrightarrow{\sigma} M_n$.
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Example 14.11

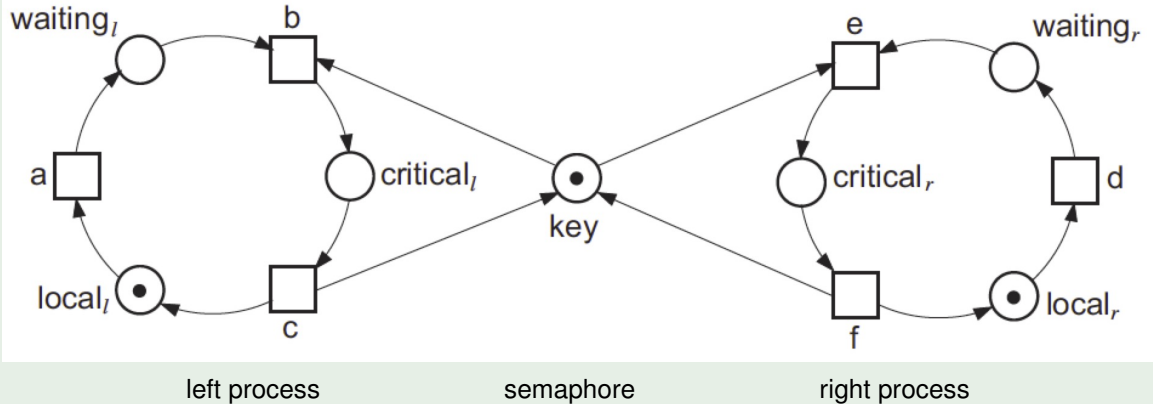
In the previous example,

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{cbdeab} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Mutual Exclusion I

Example 14.12

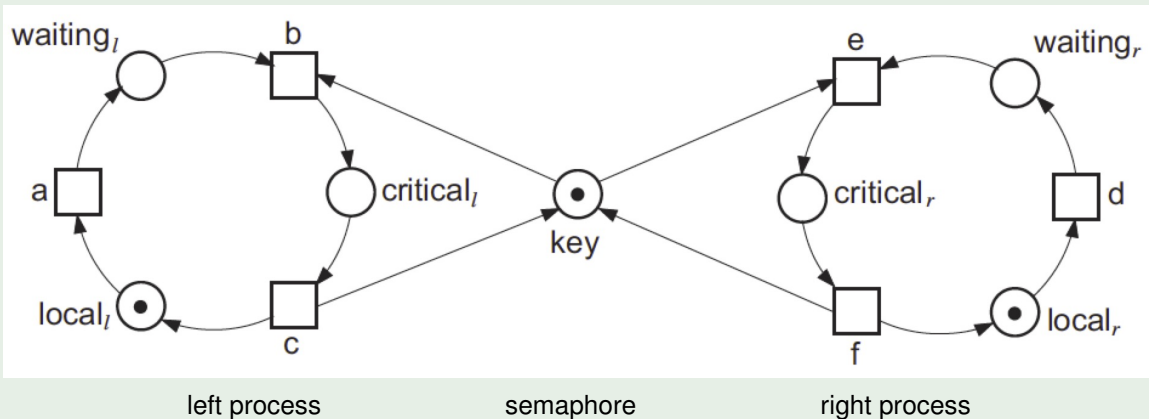
Two processes cycling through the states *local*, *waiting* and *critical*:



Mutual Exclusion I

Example 14.12

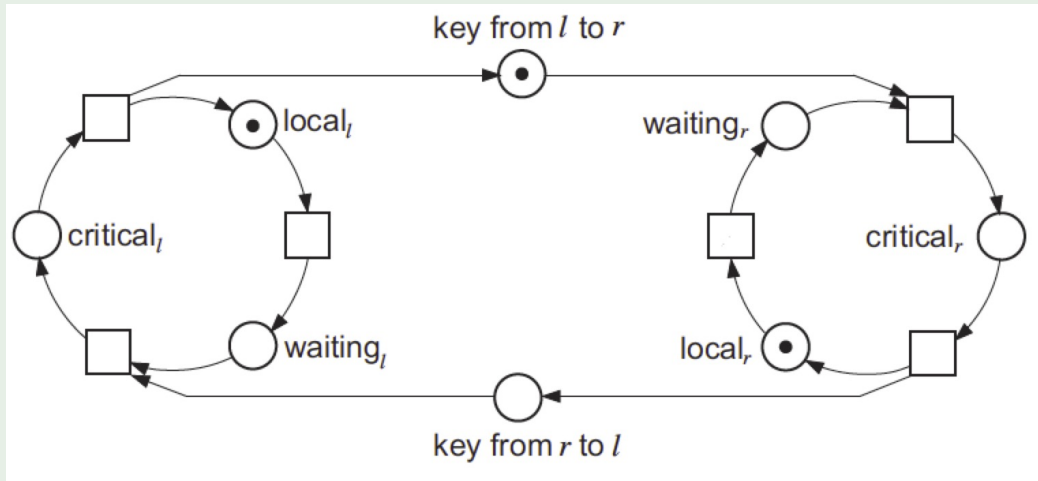
Two processes cycling through the states *local*, *waiting* and *critical*:



Between transitions *b* and *e*, a conflict can arise infinitely often.
No strategy has been modelled to solve this conflict.

Example 14.13

A strategy where processes are acquiring access in an **alternating** fashion:



One-Bounded Elementary System Nets

Definition 14.14 (One-boundedness)

An elementary system net $N = (P, T, F, M_0)$ is called **one-bounded** if for each reachable marking M and place $p \in P$,

$$M(p) \leq 1.$$

One-Bounded Elementary System Nets

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Remark: Markings of one-bounded elementary system nets can be described as a **set** of places.

One-Bounded Elementary System Nets

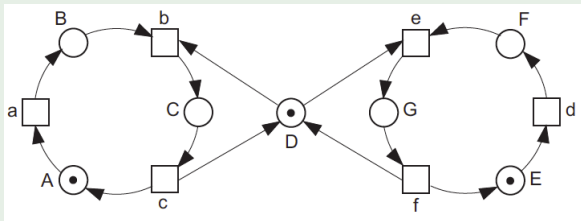
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Remark: Markings of one-bounded elementary system nets can be described as a **set** of places.

Example 14.15



Two steps beginning in marking ADE : $ADE \xrightarrow{a} BDE$ and $ADE \xrightarrow{d} ADF$.

Outline of Lecture 14

- 1 Introduction
- 2 Basic Net Concepts
- 3 The Interleaving Semantics of Petri Nets
- 4 The Marking Graph**
- 5 Summary

Definition 14.16 (Sequential run)

Let $N = (P, T, F, M_0)$ be an elementary system net.

- A **sequential run** of N is a sequence $M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} \dots$ of steps of N starting with the initial marking M_0 .
- A run can be finite or infinite.
- A finite run $M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} \dots \xrightarrow{t_n} M_n$ is **complete** if M_n does not enable any transition.

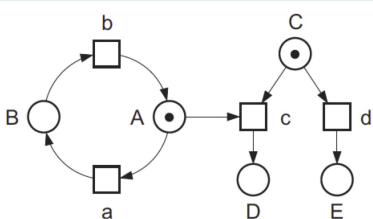
Sequential Runs

Definition 14.16 (Sequential run)

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Example 14.17



A sample complete run:

$$AC \xrightarrow{a} BC \xrightarrow{b} AC \xrightarrow{c} D$$

A sample incomplete run:

$$AC \xrightarrow{d} AE \xrightarrow{a} BE$$

Marking Graph

Definition 14.18 (Marking graph)

The **marking graph** of a net N has as nodes the reachable markings of N and as edges the corresponding steps of N .^a

^aSince firing an (enabled) transition in a marking yields a unique successor marking, marking graphs are a **deterministic** LTS.

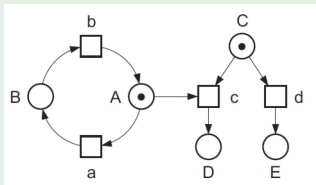
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Example 14.19



A sample elementary system net

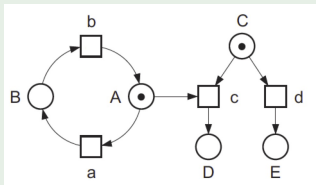
Marking Graph

Definition 14.18 (Marking graph)

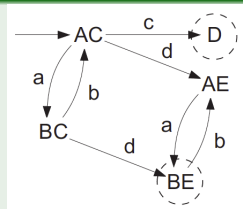
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Example 14.19



A sample elementary system net



... and its marking graph

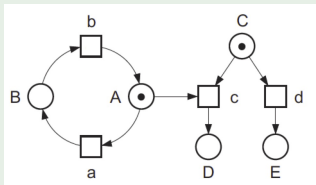
Marking Graph

Definition 14.18 (Marking graph)

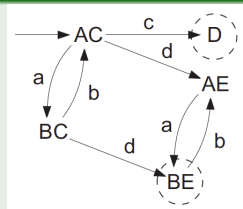
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Example 14.19



A sample elementary system net



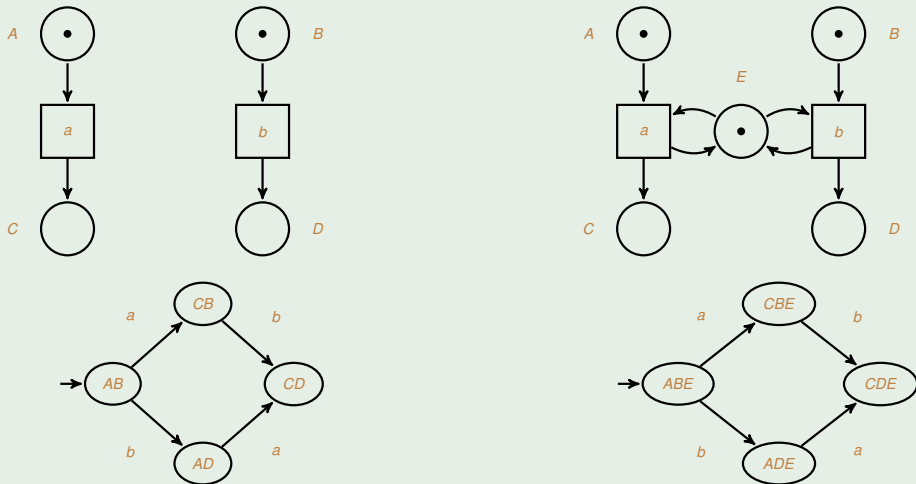
... and its marking graph

Interleaving semantics

The marking graph represents the **interleaving semantics** of a Petri net.

Interleaving vs. True Concurrency

Example 14.20 (Petri nets and their marking graphs)



Thus: Marking graphs are isomorphic even though the nets behave differently (a and b can occur simultaneously on the left, but not on the right).

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- A **Petri net** consists of places, transitions and arcs.
- An **elementary system net** is a Petri net plus a marking.
- Firing a single transition in a marking is a **step**.
- A **sequential run** is a sequence of steps starting in the initial marking.
- The **marking graph** has as nodes the reachable markings of the net and as edges its reachable steps.
- The marking graph represents the **interleaving semantics** of a net.