

# Concurrency Theory

Winter 2025/26

## Lecture 14: Interleaving Semantics of Petri Nets

Thomas Noll, Peter Thiemann  
Programming Languages Group  
University of Freiburg

<https://proglang.github.io/teaching/25ws/ct.html>

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# Outline of Lecture 14

1 Introduction

2 Basic Net Concepts

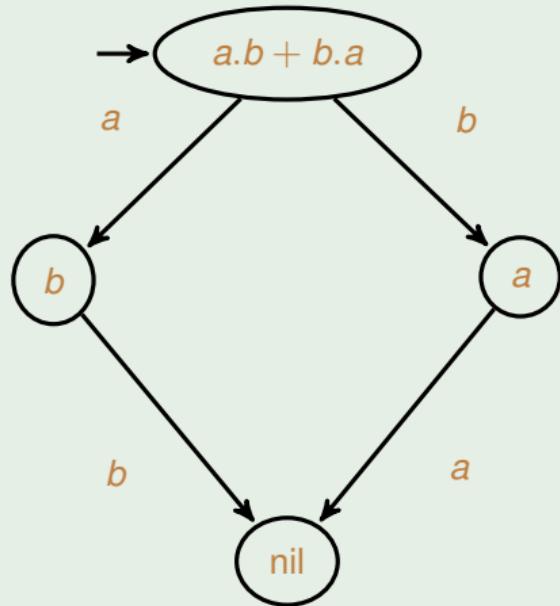
3 The Interleaving Semantics of Petri Nets

4 The Marking Graph

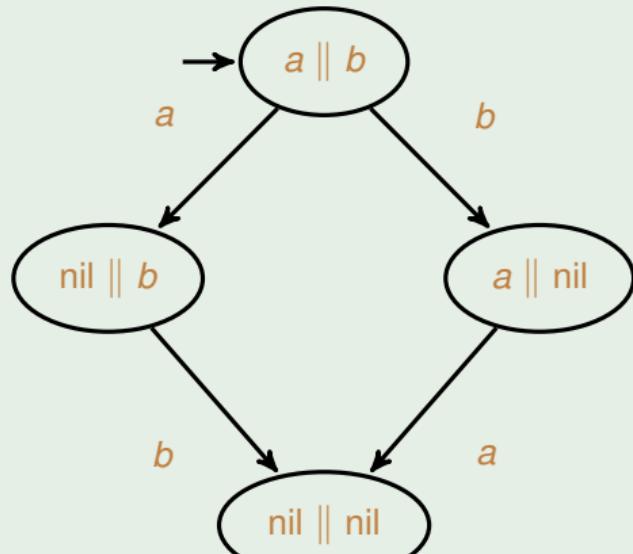
5 Summary

# Motivation

## Example 14.1 (LTSs of CCS processes)



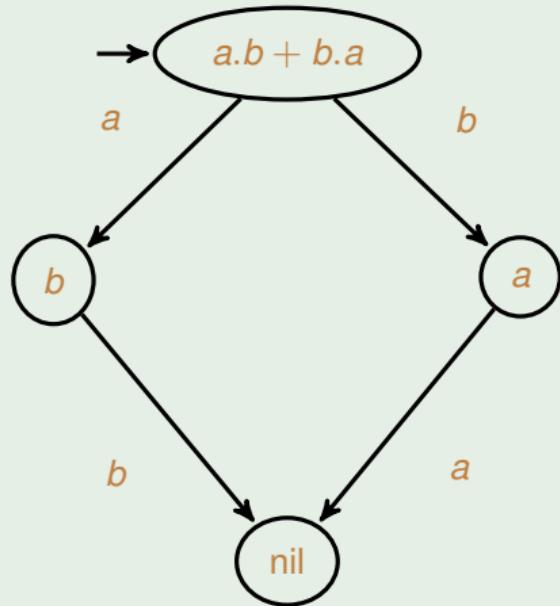
LTS of  $a.b.\text{nil} + b.a.\text{nil}$



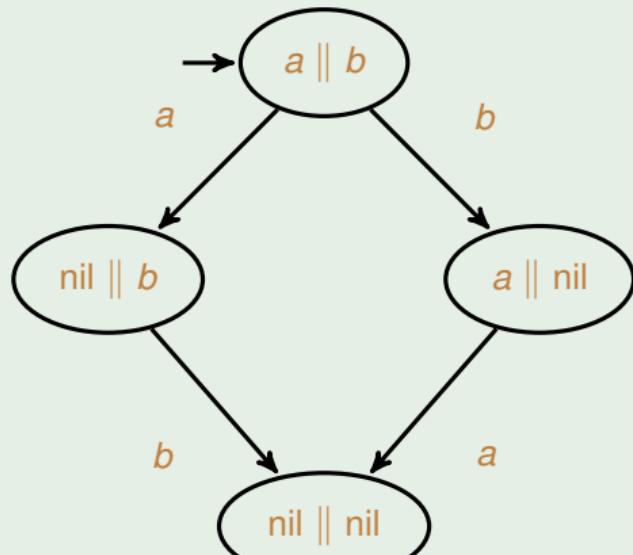
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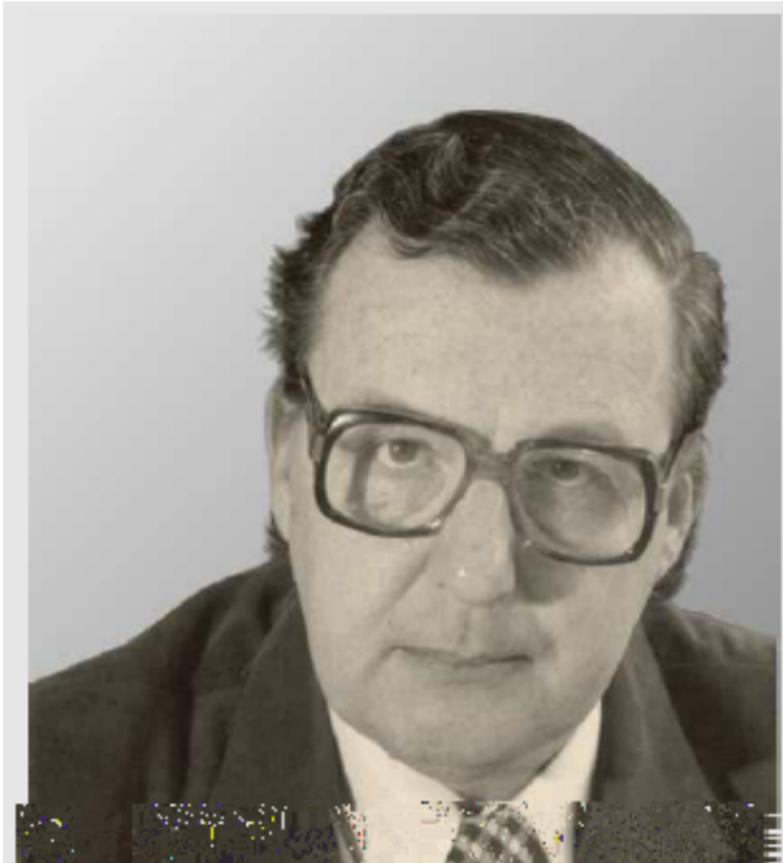


LTS of  $a.b.\text{nil} + b.a.\text{nil}$



LTS of  $a.\text{nil} \parallel b.\text{nil}$

# Carl Adam Petri (1926–2010)



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Models of computation in the 1960s: lambda-calculus, finite automata, Turing machines, ...

**States:** current configurations of the machine

One or more initial states

Possibly some distinguished final states

**Transitions:** moves between configurations

Lambda calculus	$(\lambda x.xx)(\lambda y.y)$	→	$(\lambda y.y)(\lambda z.z)$
Turing machine	0010 $q_1$ 011	→	001 $q_2$ 01011
Finite automaton	$q_1$	$\xrightarrow{a}$	$q_2$
Pushdown automaton	$(q_1, XYZ)$	$\xrightarrow{a}$	$(q_2, XYXYYZ)$

**Executions:** alternating sequences of states and transitions



C.A. Petri points out a discrepancy between how **Theoretical Physics** and **Theoretical Computer Science** described systems in 1962:

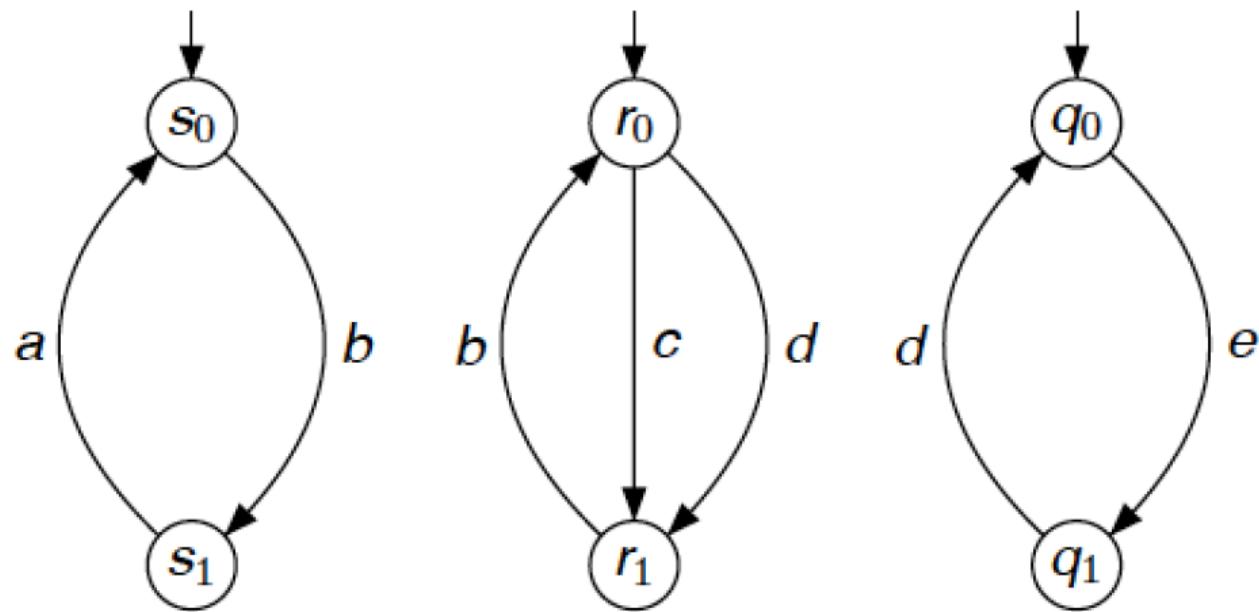
**Theoretical Physics** describes systems as a collection of interacting particles (subsystems), without a notion of global clock or simultaneity

**Theoretical Computer Science** describes systems as sequential virtual machines going through a temporally ordered sequence of global states

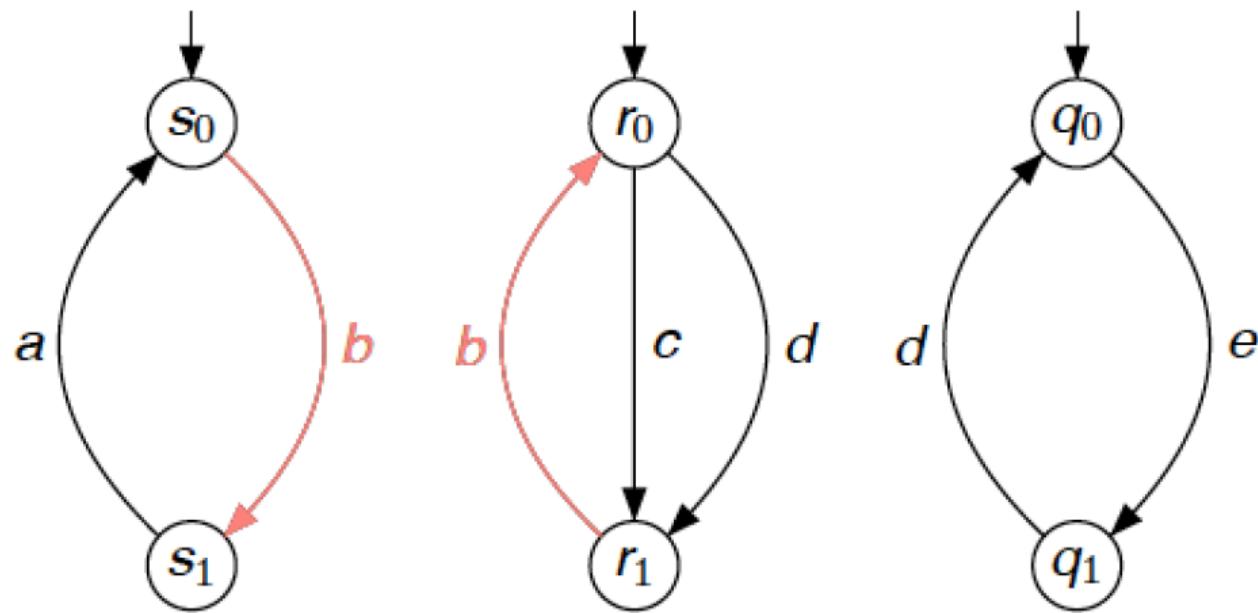
Petri's question:

Which kind of abstract machine should be used to describe the physical implementation of a Turing machine?

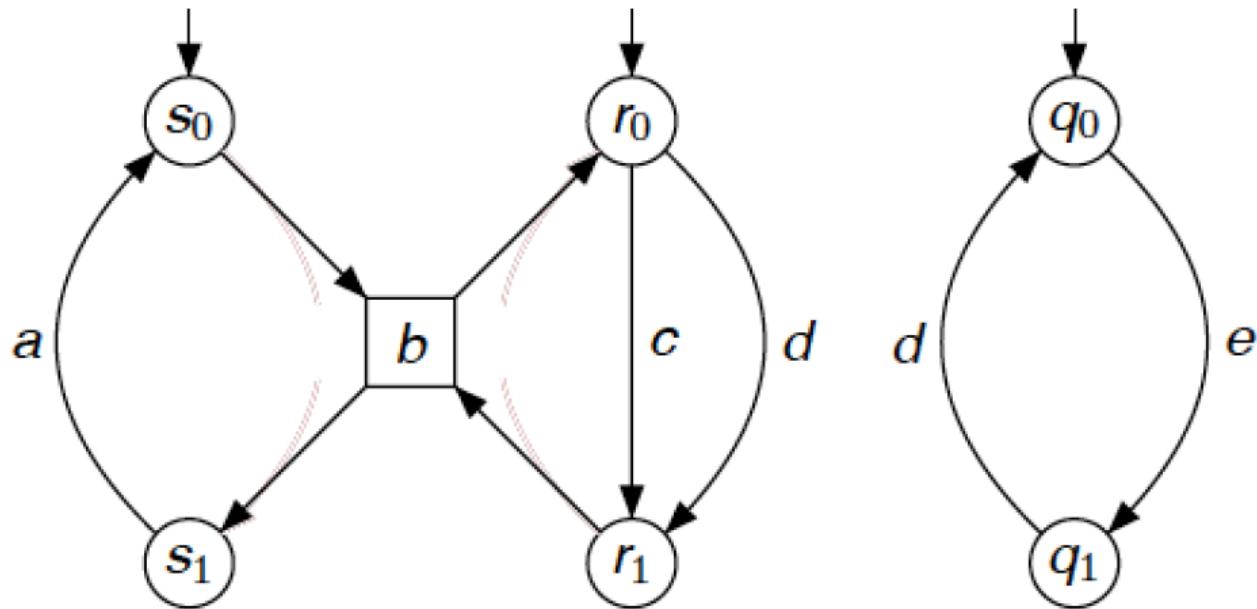
A graphical representation of interacting finite automata:



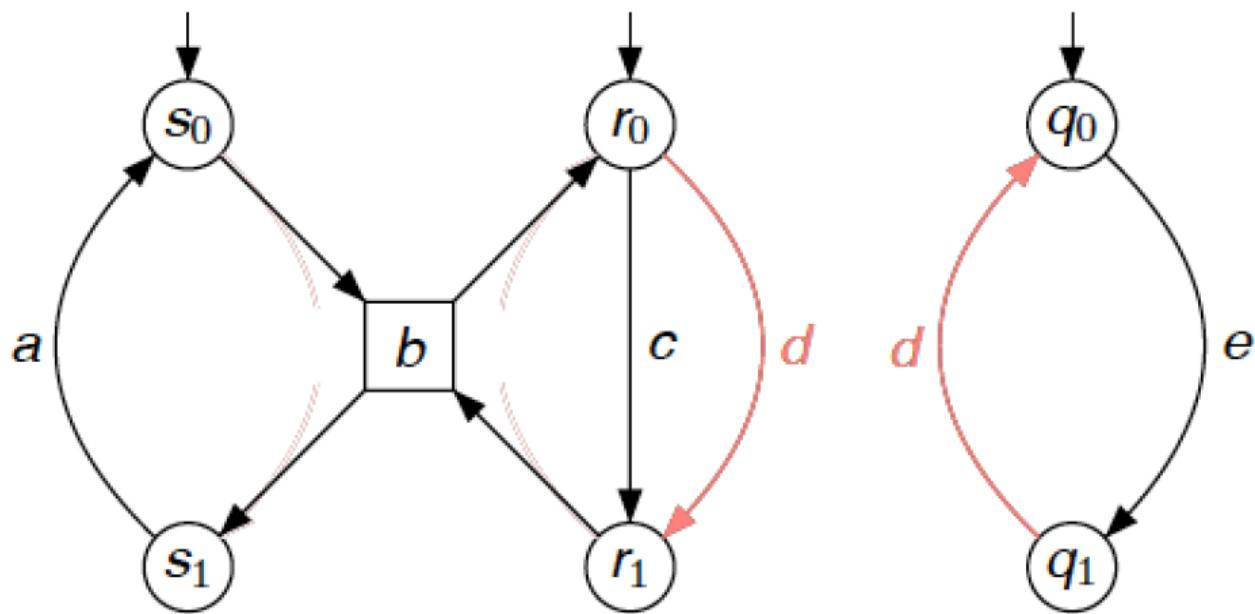
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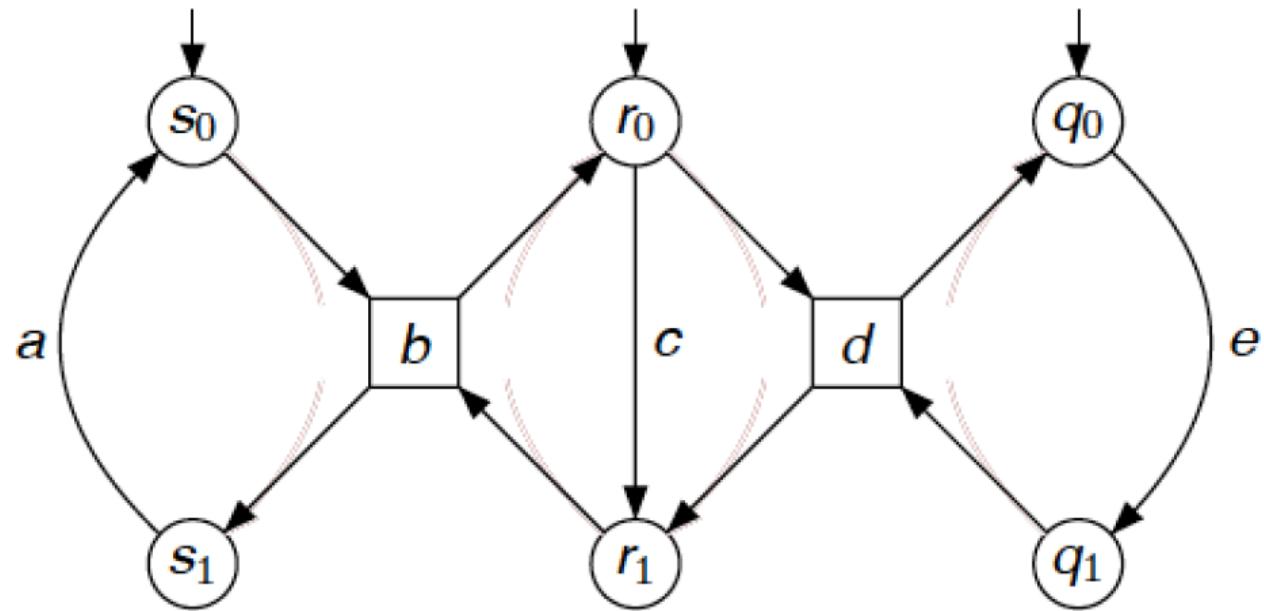
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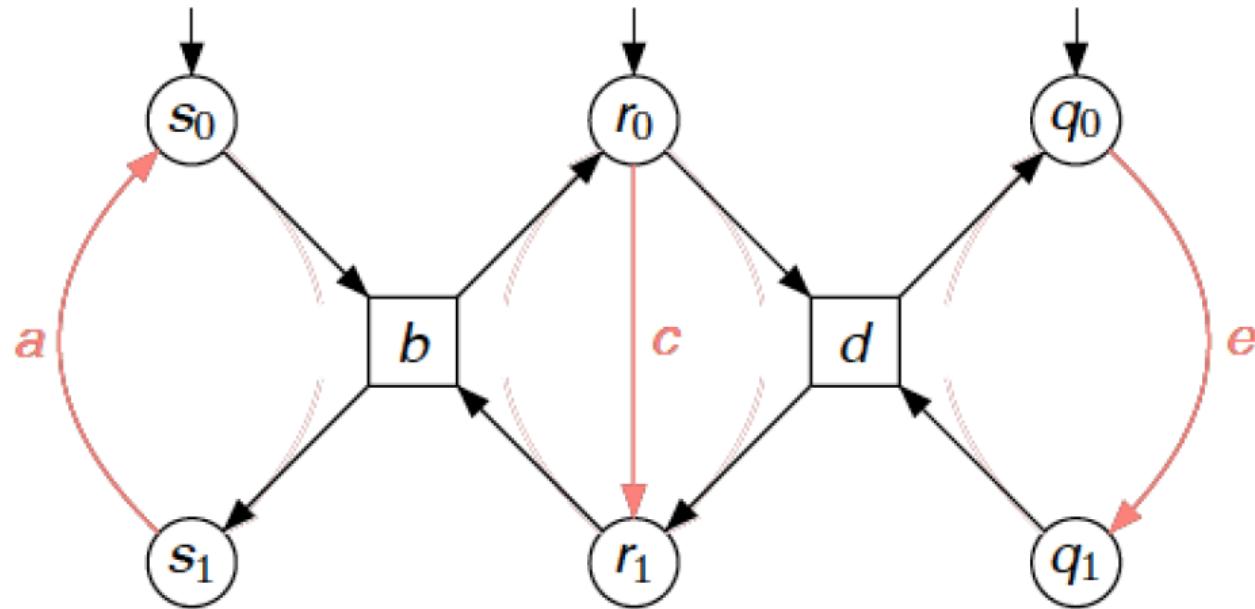
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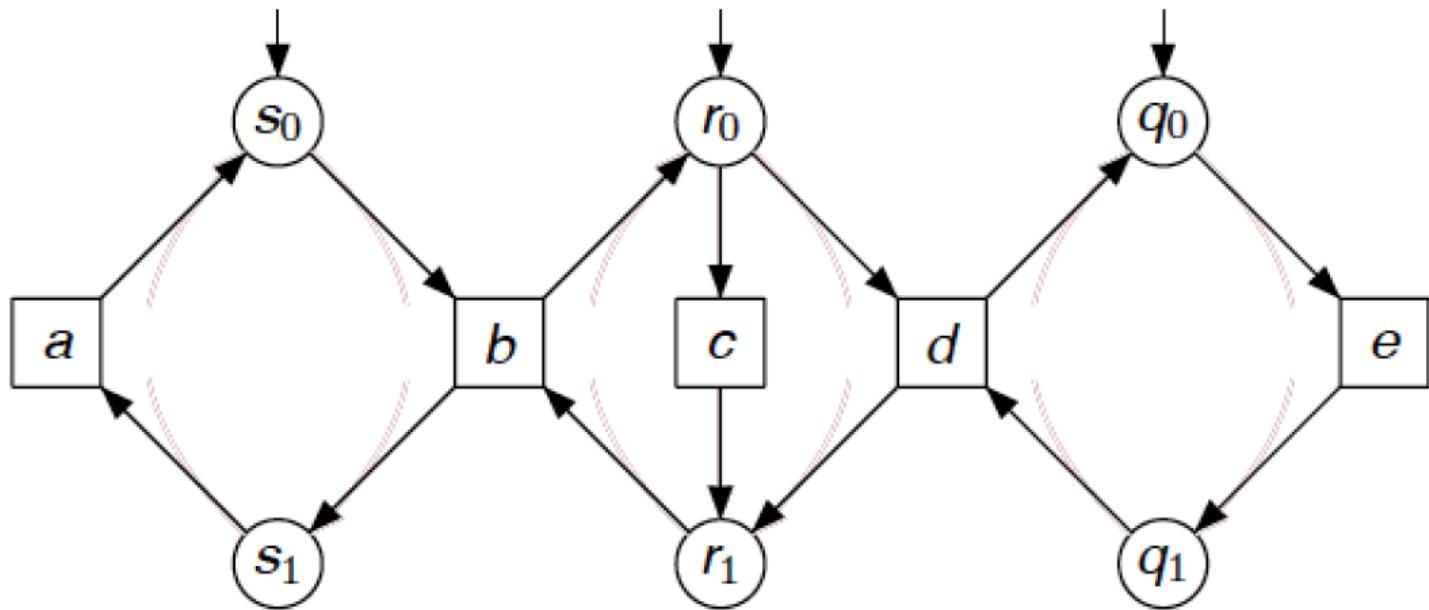
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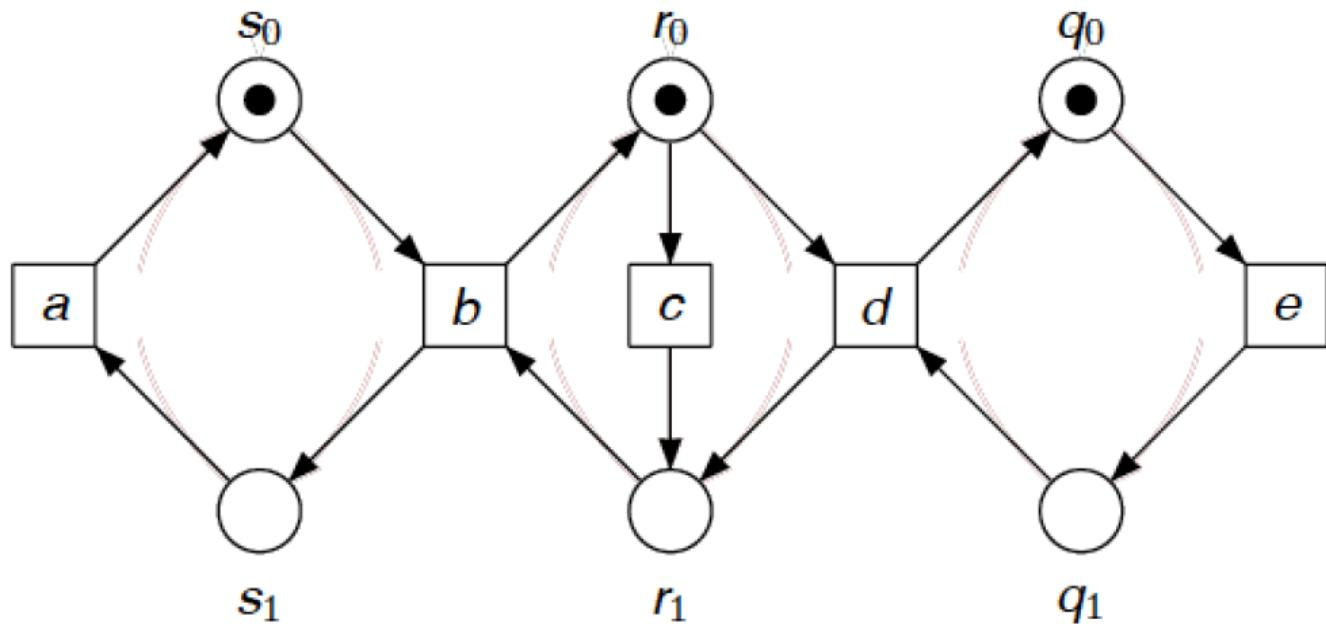
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2 Basic Net Concepts

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# Components of a Net

A Petri net is a structure with two kinds of elements: places and transitions.  
These are connected by arcs.

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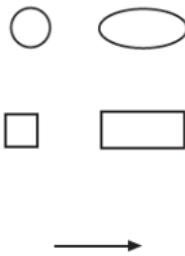
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Places and transitions are connected to each other by directed arcs.  
Graphically, an arc is represented by an arrow. An arc models an abstract, sometimes only notional relation between components.



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Places and transitions are connected to each other by directed **arcs**.

Graphically, an arc is represented by an arrow. An arc models an abstract, sometimes only notional relation between components. Arcs run from places to transitions or vice versa.



## Definition 14.2 (Petri net)

A Petri net  $N$  is a triple  $(P, T, F)$  where:

- $P$  is a finite set of places,
- $T$  is a finite set of transitions with  $P \cap T = \emptyset$ , and
- $F \subseteq (P \times T) \cup (T \times P)$  are the arcs.<sup>a</sup>

Places and transitions are generically called nodes.

---

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# Nets

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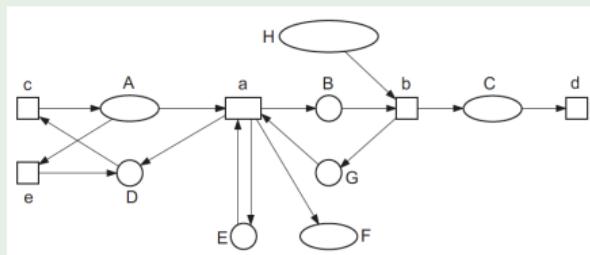
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## Example 14.3



$$\begin{aligned}P &= \{A, B, C, \dots\} \\T &= \{a, b, c, \dots\} \\F &= \{(A, a), (a, B), (B, b), \dots\}\end{aligned}$$

# Pre- and Post-Sets

## Definition 14.4 (Pre- and post-sets)

Let node  $x \in P \cup T$ .

- The **pre-set** of  $x$  is defined by  $\bullet x := \{y \mid (y, x) \in F\}$ .
- The **post-set** of  $x$  is defined by  $x^\bullet = \{y \mid (x, y) \in F\}$ .

Two nodes  $x, y \in P \cup T$  form a **loop** if  $x \in \bullet y$  and  $y \in \bullet x$ .

# Pre- and Post-Sets

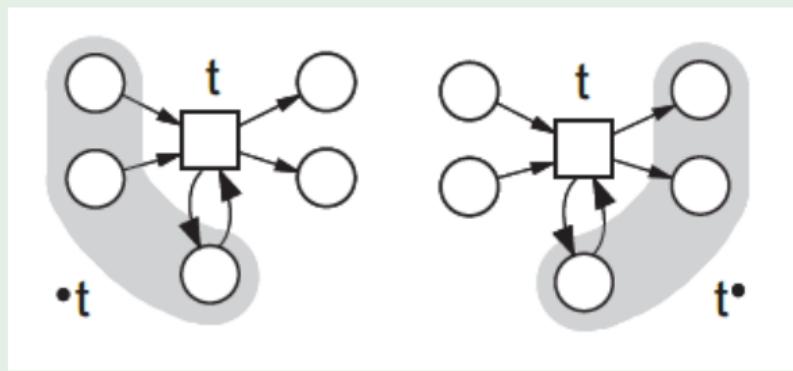
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## Example 14.5



## Definition 14.6 (Marking)

- A marking  $M$  of a net  $N = (P, T, F)$  is a mapping  $M : P \rightarrow \mathbb{N}$ .
- For net  $N = (P, T, F)$  and marking  $M_0$ , the quadruple  $(P, T, F, M_0)$  is called an **elementary system net** with **initial marking**  $M_0$ .

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### Intuition:

- A marking can be seen as a **multiset** of places.
- It defines a distribution of **tokens** across places.
- Tokens are depicted as black dots.



**Remark:** In **generic** (= non-elementary) system nets, several types (colours) of tokens can be distinguished.

## Definition 14.7 (Enabling and occurrence of a transition)

Let  $(P, T, F, M_0)$  be an elementary system net and  $M : P \rightarrow \mathbb{N}$ .

- Marking  $M$  enables a transition  $t \in T$  if  $M(p) \geq 1$  for each place  $p \in {}^{\bullet}t$ .
- Transition  $t \in T$  can occur in marking  $M$  if  $t$  is enabled in  $M$ .
- Its occurrence or firing leads to marking  $M'$ , denoted by the step relation  $M \xrightarrow{t} M'$  and defined for each place  $p \in P$  by

$$M'(p) := M(p) - F(p, t) + F(t, p)$$

where we represent relation  $F$  by its characteristic function.

# Transition Firing

## Definition 14.7 (Enabling and occurrence of a transition)

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**Intuition:** Transition  $t$  is enabled whenever every  $p \in {}^{\bullet}t$  holds at least one token.

On  $t$ 's occurrence, one token is removed from each place in  ${}^{\bullet}t$ , and one token is put in each place in  $t^{\bullet}$ :

$$M'(p) = \begin{cases} M(p) - 1 & \text{if } p \in {}^{\bullet}t \text{ and } p \notin t^{\bullet} \\ M(p) + 1 & \text{if } p \in t^{\bullet} \text{ and } p \notin {}^{\bullet}t \\ M(p) & \text{otherwise} \end{cases}$$

# Transition Occurrence

## Definition (Enabling and occurrence of a transition)

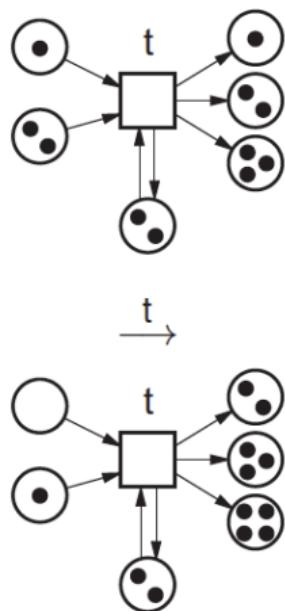
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## Example 14.8



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# The Interleaving Semantics of Petri Nets I

**Goal:** Establish an **execution semantics** by mapping a Petri net to a labelled transition system

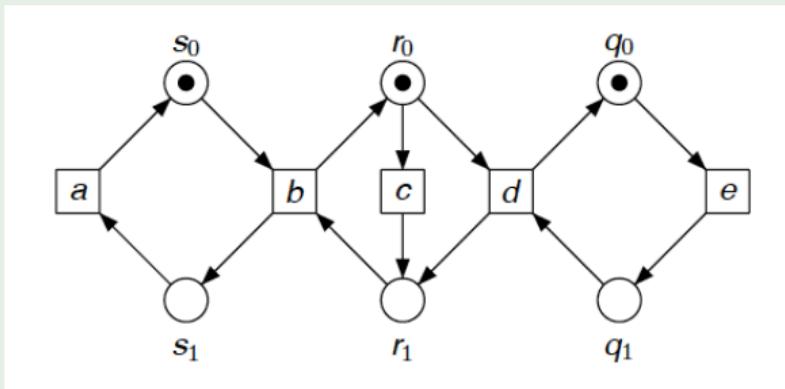
States: markings (i.e., distributions of tokens over the net)

Transitions:  $M \xrightarrow{t} M'$  (“steps”)

Sequential runs:  $M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} M_2 \xrightarrow{t_3} \dots$  (step sequences)

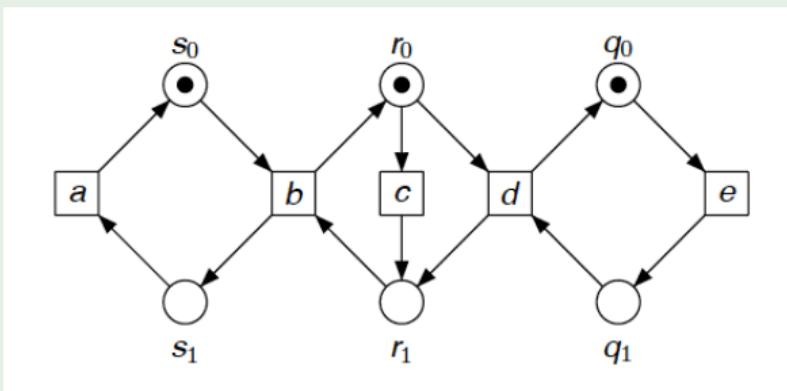
# The Interleaving Semantics of Petri Nets II

## Example 14.9



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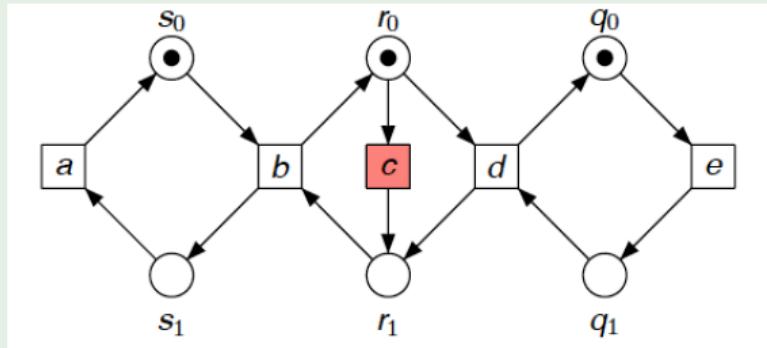


$$\begin{matrix} s_1 \\ r_1 \\ q_1 \end{matrix} \left[ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right]$$

(As the marking for *s*<sub>0</sub> is the complement of *s*<sub>1</sub>, the marking for *s*<sub>0</sub> is omitted.  
The same applies to the places *r*<sub>0</sub> and *q*<sub>0</sub>.)

# The Interleaving Semantics of Petri Nets III

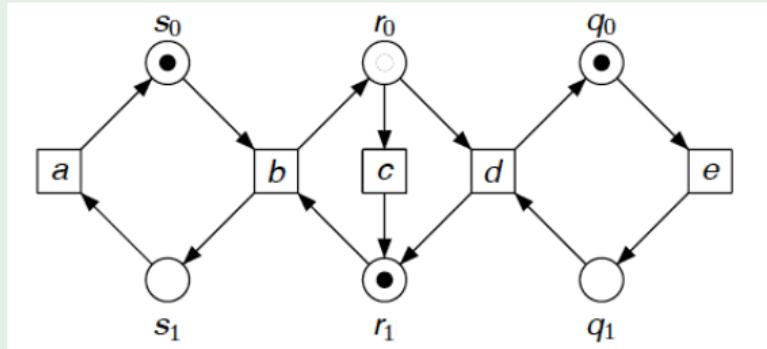
## Example 14.9 (continued)



$$\begin{matrix} s_1 \\ r_1 \\ q_1 \end{matrix} \xrightarrow{c} \left[ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right]$$

# The Interleaving Semantics of Petri Nets IV

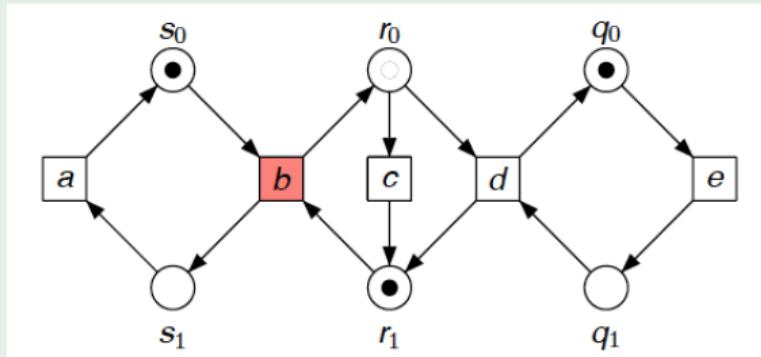
## Example 14.9 (continued)



$$\begin{matrix} s_1 \\ r_1 \\ q_1 \end{matrix} \xrightarrow{c} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{c} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

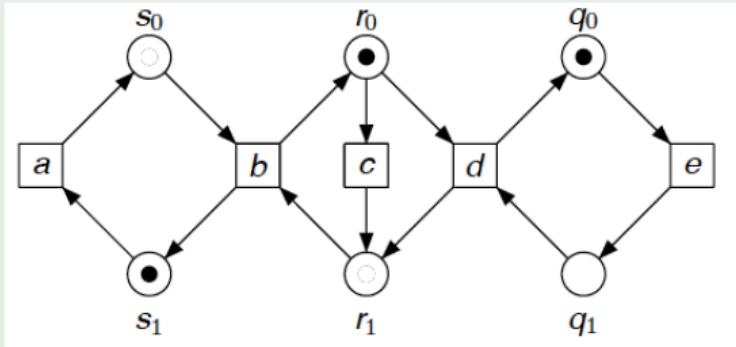
# The Interleaving Semantics of Petri Nets V

## Example 14.9 (continued)



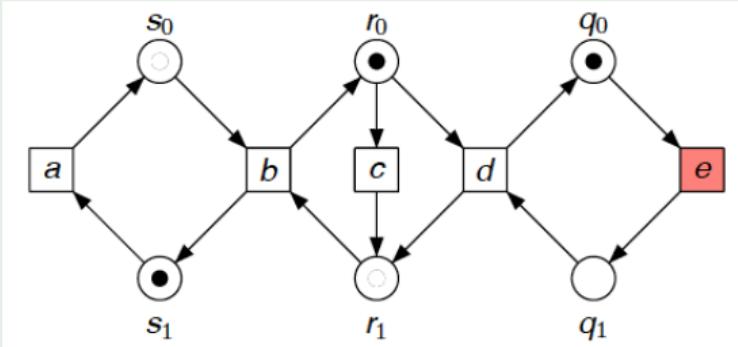
$$\begin{matrix} s_1 \\ r_1 \\ q_1 \end{matrix} \xrightarrow{c} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{b} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

## Example 14.9 (continued)



$$\begin{array}{l} s_1 \\ r_1 \\ q_1 \end{array} \xrightarrow{c} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{b} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \xrightarrow{b} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

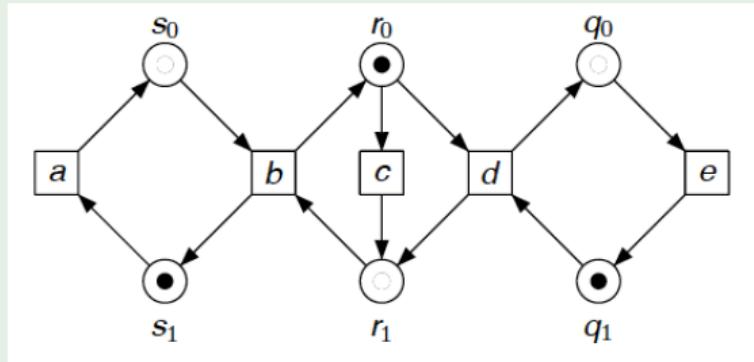
## Example 14.9 (continued)



$$\begin{array}{ll} s_1 & \left[ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right] \xrightarrow{c} \left[ \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right] \xrightarrow{b} \left[ \begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right] \xrightarrow{e} \\ r_1 & \\ q_1 & \end{array}$$

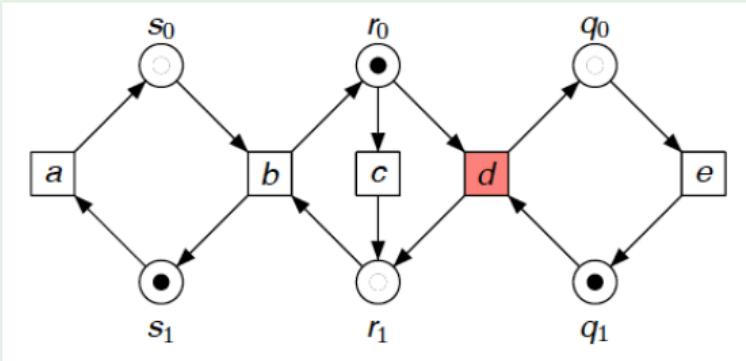
# The Interleaving Semantics of Petri Nets VIII

## Example 14.9 (continued)



$$\begin{matrix} s_1 \\ r_1 \\ q_1 \end{matrix} \xrightarrow{c} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{b} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \xrightarrow{d} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{e} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

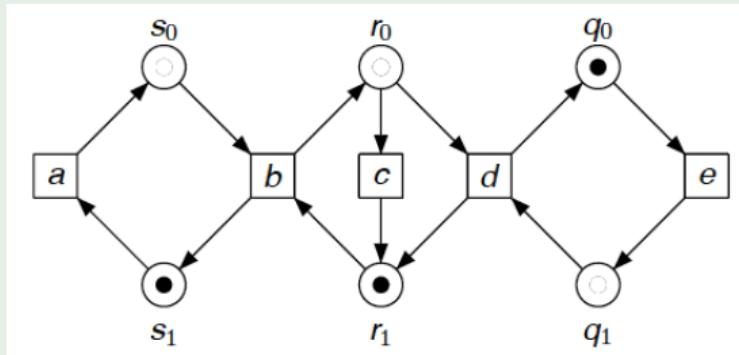
## Example 14.9 (continued)



$$\begin{array}{l} s_1 \\ r_1 \\ q_1 \end{array} \xrightarrow{c} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \xrightarrow{b} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{e} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \xrightarrow{d} \quad$$

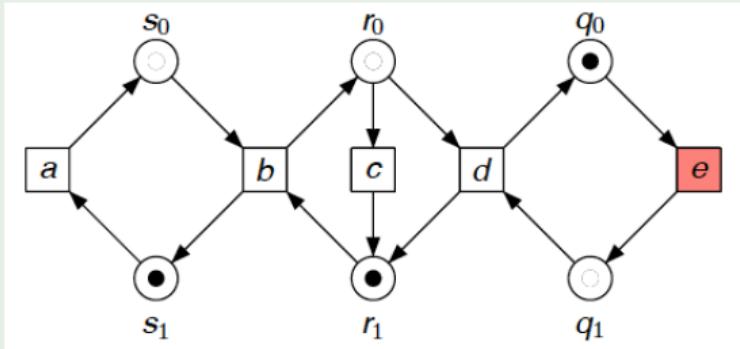
# The Interleaving Semantics of Petri Nets X

## Example 14.9 (continued)



$$\begin{array}{ll} s_1 & \left[ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right] \\ r_1 & \xrightarrow{c} \left[ \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right] \\ q_1 & \xrightarrow{b} \left[ \begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right] \\ & \xrightarrow{e} \left[ \begin{array}{c} 1 \\ 0 \\ 1 \end{array} \right] \\ & \xrightarrow{d} \left[ \begin{array}{c} 1 \\ 1 \\ 0 \end{array} \right] \end{array}$$

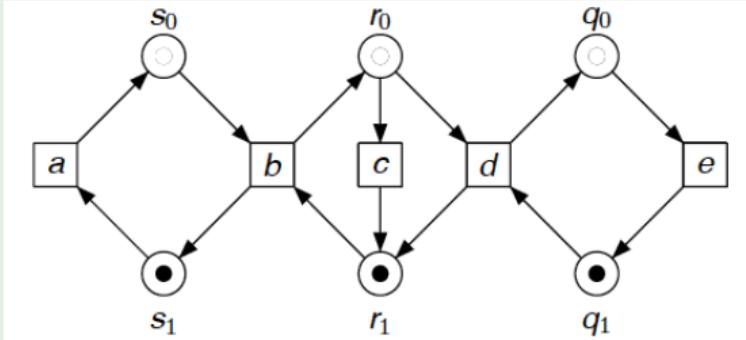
## Example 14.9 (continued)



$$\begin{array}{l} s_1 \\ r_1 \\ q_1 \end{array} \left[ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right] \xrightarrow{c} \left[ \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right] \xrightarrow{b} \left[ \begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right] \xrightarrow{e} \left[ \begin{array}{c} 1 \\ 0 \\ 1 \end{array} \right] \xrightarrow{d} \left[ \begin{array}{c} 1 \\ 1 \\ 0 \end{array} \right] \xrightarrow{e} \left[ \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right]$$

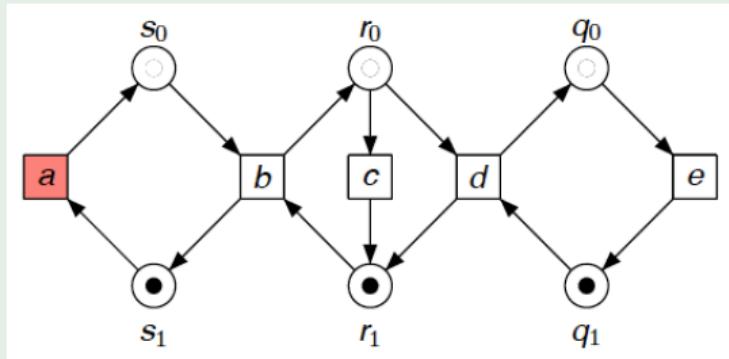
# The Interleaving Semantics of Petri Nets XII

## Example 14.9 (continued)



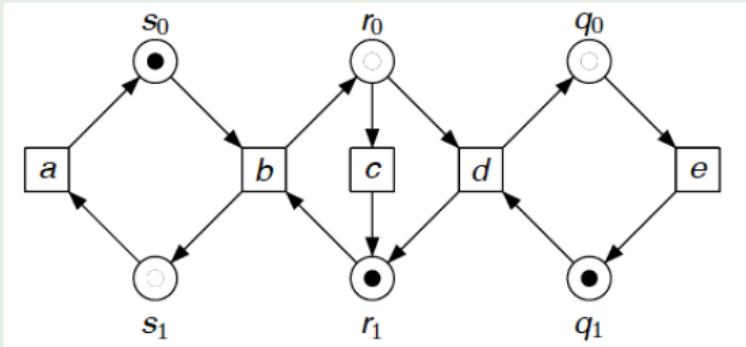
$$\begin{array}{ll} s_1 & \left[ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right] \xrightarrow{c} \left[ \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right] \xrightarrow{b} \left[ \begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right] \xrightarrow{e} \left[ \begin{array}{c} 1 \\ 0 \\ 1 \end{array} \right] \xrightarrow{d} \left[ \begin{array}{c} 1 \\ 1 \\ 0 \end{array} \right] \xrightarrow{e} \left[ \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right] \\ r_1 & \\ q_1 & \end{array}$$

## Example 14.9 (continued)



$$\begin{array}{l} \begin{matrix} s_1 & r_1 & q_1 \end{matrix} \\ \xrightarrow{c} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \xrightarrow{b} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{e} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \xrightarrow{d} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \xrightarrow{e} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \xrightarrow{a} \end{array}$$

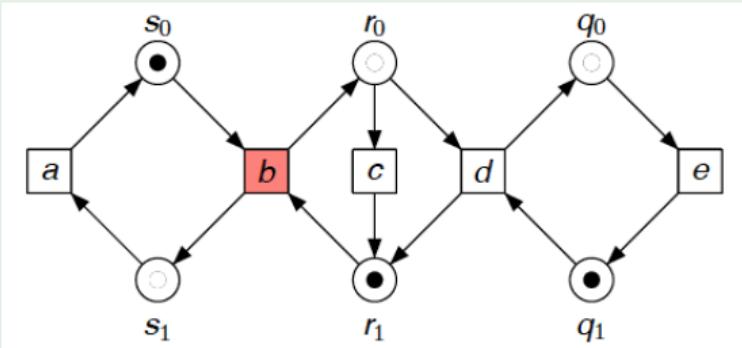
## Example 14.9 (continued)



$$\begin{array}{ll} s_1 & \left[ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right] \xrightarrow{c} \left[ \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right] \xrightarrow{b} \left[ \begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right] \xrightarrow{e} \left[ \begin{array}{c} 1 \\ 0 \\ 1 \end{array} \right] \xrightarrow{d} \left[ \begin{array}{c} 1 \\ 1 \\ 0 \end{array} \right] \xrightarrow{e} \left[ \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right] \xrightarrow{a} \left[ \begin{array}{c} 0 \\ 1 \\ 1 \end{array} \right] \\ r_1 & \\ q_1 & \end{array}$$

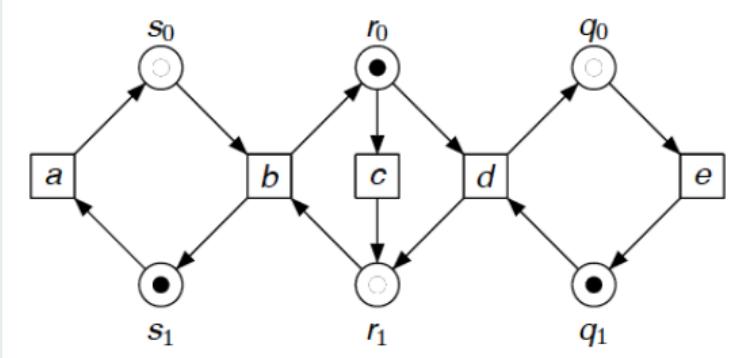
# The Interleaving Semantics of Petri Nets XV

## Example 14.9 (continued)



$$\begin{array}{l} s_1 \\ r_1 \\ q_1 \end{array} \left[ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right] \xrightarrow{c} \left[ \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right] \xrightarrow{b} \left[ \begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right] \dots \xrightarrow{e} \left[ \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right] \xrightarrow{a} \left[ \begin{array}{c} 0 \\ 1 \\ 1 \end{array} \right] \xrightarrow{\textcolor{red}{b}} \left[ \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right]$$

## Example 14.9 (continued)



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# Reachable Markings

## Definition 14.10 (Step sequence)

Let  $(P, T, F, M_0)$  be an elementary system net.

- A sequence of transitions  $\sigma = t_1 \ t_2 \dots t_n \in T^*$  is a **step sequence** if there exist markings  $M_1, \dots, M_n$  such that

$$M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} \dots \xrightarrow{t_{n-1}} M_{n-1} \xrightarrow{t_n} M_n.$$

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## Example 14.11

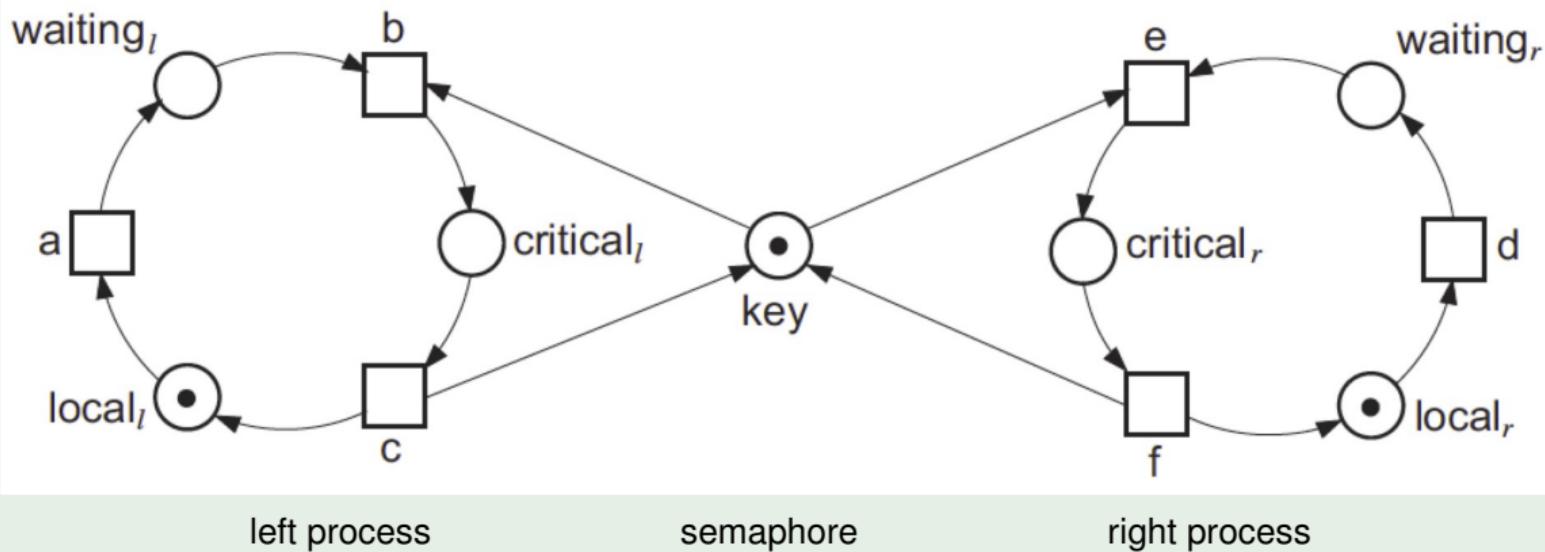
In the previous example,

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{cbedeab} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

# Mutual Exclusion I

## Example 14.12

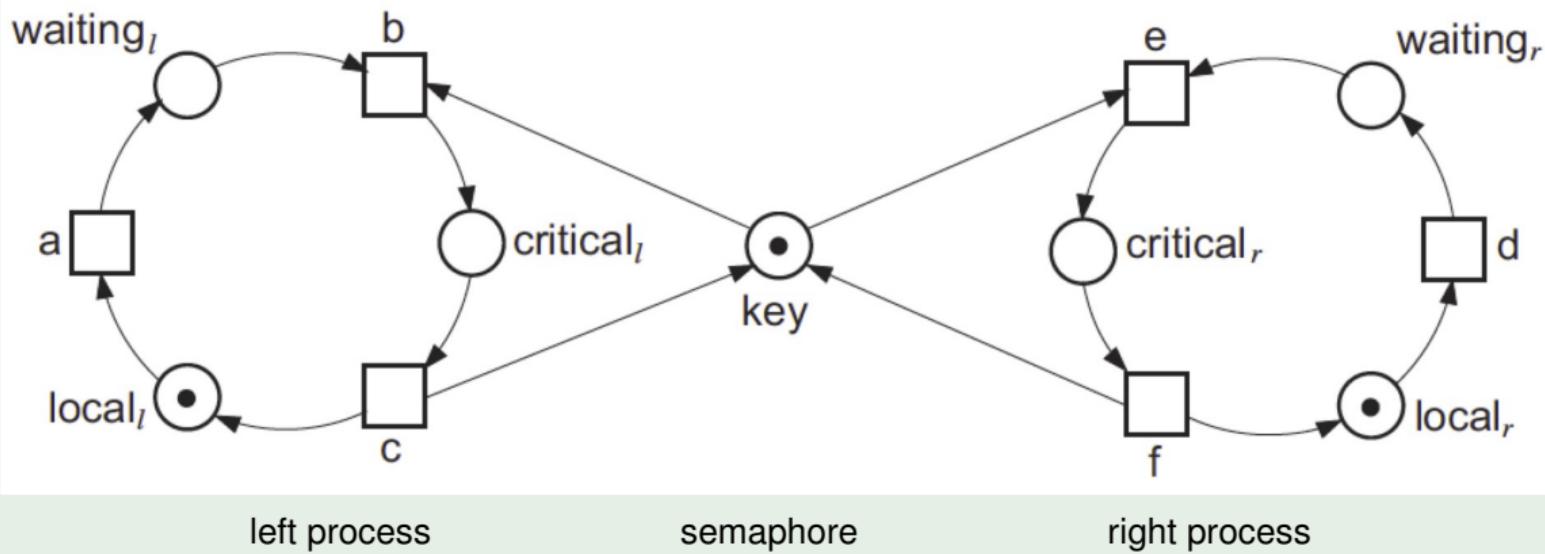
Two processes cycling through the states *local*, *waiting* and *critical*:



# Mutual Exclusion I

## Example 14.12

Two processes cycling through the states *local*, *waiting* and *critical*:



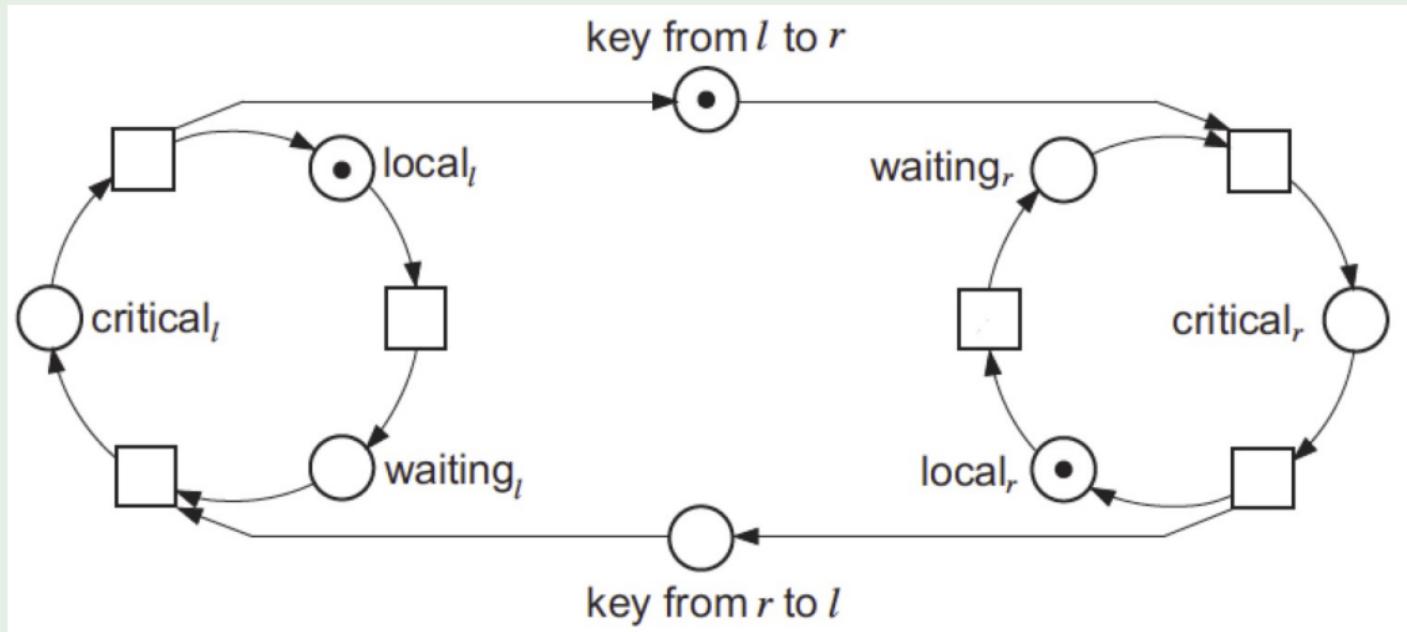
Between transitions *b* and *e*, a conflict can arise infinitely often.

No strategy has been modelled to solve this conflict.

## Mutual Exclusion II

### Example 14.13

A strategy where processes are acquiring access in an **alternating** fashion:



# One-Bounded Elementary System Nets

## Definition 14.14 (One-boundedness)

An elementary system net  $N = (P, T, F, M_0)$  is called **one-bounded** if for each reachable marking  $M$  and place  $p \in P$ ,

$$M(p) \leq 1.$$

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**Remark:** Markings of one-bounded elementary system nets can be described as a **set** of places.

# One-Bounded Elementary System Nets

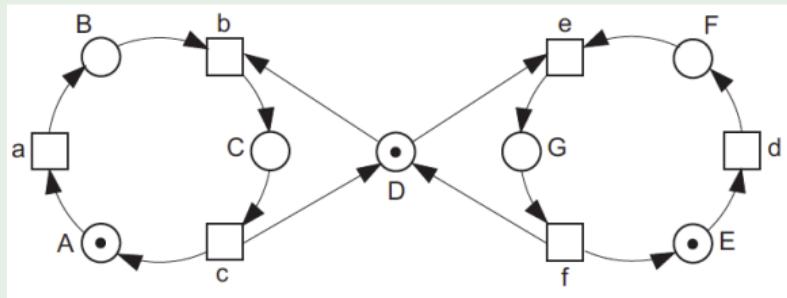
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**Remark:** Markings of one-bounded elementary system nets can be described as a **set** of places.

## Example 14.15



Two steps beginning in marking  $ADE$ :  $ADE \xrightarrow{a} BDE$  and  $ADE \xrightarrow{d} ADF$ .

# Outline of Lecture 14

1 Introduction

2 Basic Net Concepts

3 The Interleaving Semantics of Petri Nets

4 The Marking Graph

5 Summary

## Definition 14.16 (Sequential run)

Let  $N = (P, T, F, M_0)$  be an elementary system net.

- A **sequential run** of  $N$  is a sequence  $M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} \dots$  of steps of  $N$  starting with the initial marking  $M_0$ .
- A run can be finite or infinite.
- A finite run  $M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} \dots \xrightarrow{t_n} M_n$  is **complete** if  $M_n$  does not enable any transition.

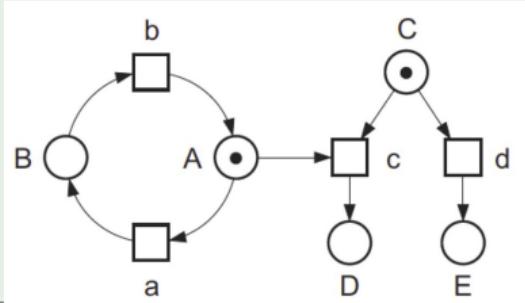
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## Example 14.17



A sample complete run:

$$AC \xrightarrow{a} BC \xrightarrow{b} AC \xrightarrow{c} D$$

A sample incomplete run:

$$AC \xrightarrow{d} AE \xrightarrow{a} BE$$

# Marking Graph

## Definition 14.18 (Marking graph)

The **marking graph** of a net  $N$  has as nodes the reachable markings of  $N$  and as edges the corresponding steps of  $N$ .<sup>a</sup>

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<sup>a</sup>Since firing an (enabled) transition in a marking yields a unique successor marking, marking graphs are a **deterministic LTS**.

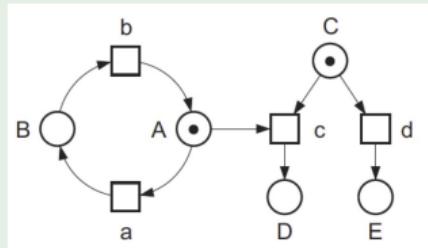
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## Example 14.19



A sample elementary system net

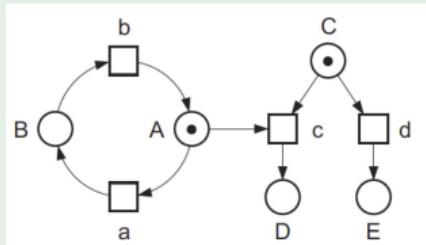
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## Definition 14.18 (Marking graph)

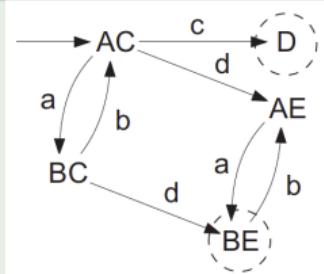
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... and its marking graph

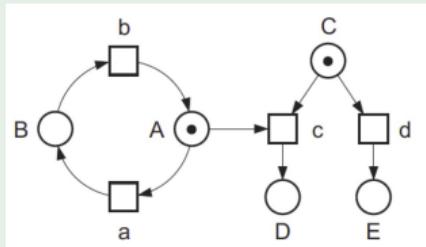
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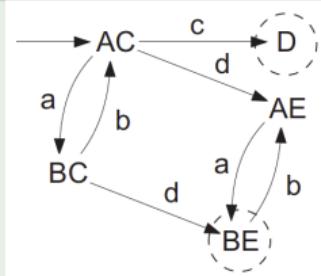
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## Example 14.19



A sample elementary system net



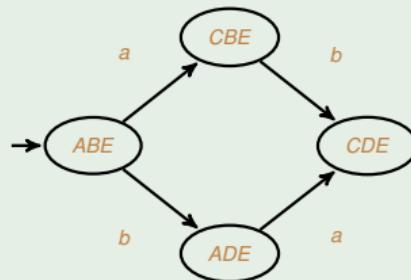
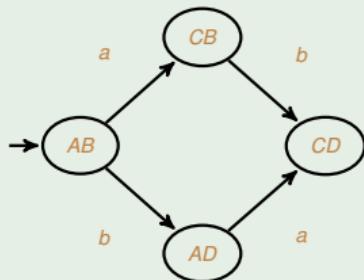
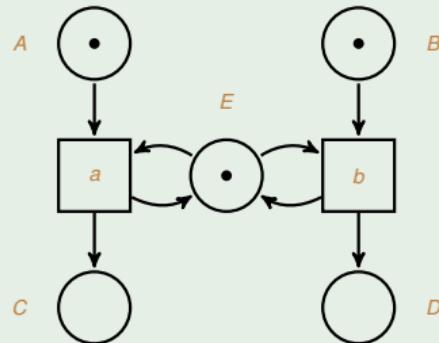
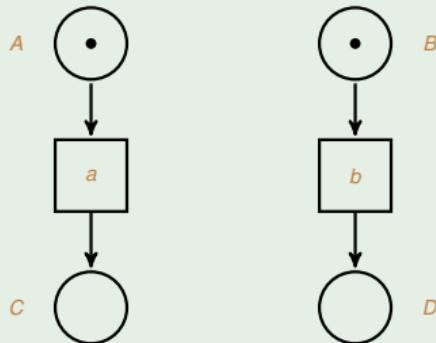
... and its marking graph

## Interleaving semantics

The marking graph represents the **interleaving semantics** of a Petri net.

# Interleaving vs. True Concurrency

Example 14.20 (Petri nets and their marking graphs)



**Thus:** Marking graphs are isomorphic even though the nets behave differently  
(*a* and *b* can occur simultaneously on the left, but not on the right).

# Outline of Lecture 14

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- A Petri net consists of places, transitions and arcs.
- An elementary system net is a Petri net plus a marking.
- Firing a single transition in a marking is a step.
- A sequential run is a sequence of steps starting in the initial marking.
- The marking graph has as nodes the reachable markings of the net and as edges its reachable steps.
- The marking graph represents the interleaving semantics of a net.