# **Concurrency Theory**

Winter 2025/26

Lecture 2: Calculus of Communicating Systems (CCS)

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https://proglang.github.io/teaching/25ws/ct.html

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Winter 2025/26



## Outline of Lecture 2

- The Approach
- Syntax of CCS
- 3 CCS Examples
- Formal Semantics of CCS
- Infinite State Spaces
- The CAAL Too
- Epilogue



# The Calculus of Communicating Systems

## History

First development:

Robin Milner: A Calculus of Communicating Systems, LNCS 92, Springer, 1980

Elaboration and larger case studies:

Robin Milner: Communication and Concurrency, Prentice-Hall, 1989

Extension to mobile systems:

Robin Milner: Communicating and Mobile Systems: the  $\pi$ -calculus, Cambridge University

Press, 1999

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## Approach

Description of concurrency on a simple and abstract level, using only a few basic primitives

- no explicit storage (variables)
- no explicit representation of values (numbers, Booleans, ...) or data structures
- concurrent system reduced to communication potential

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# Definition 2.1 (Syntax of CCS)

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- Let Pid be a set of process identifiers.
- The set *Prc* of process expressions is defined by the following grammarn:

```
P,Q ::= nil (inaction)

| \alpha.P  (prefixing)

| P+Q  (choice)

| P \parallel Q  (parallel composition)

| P \setminus L  (restriction)

| P[f]  (relabelling)

| C  (process call)
```

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- $\bullet$  *P*  $\parallel$  *Q* denotes the parallel execution of *P* and *Q*, involving interleaving or communication.

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- P + Q represents the nondeterministic choice between P and Q.
- ullet  $P \parallel Q$  denotes the parallel execution of P and Q, involving interleaving or communication.
- The restriction  $P \setminus L$  declares each  $a \in L$  as a local name which is only known within P.

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- The relabelling P[f] allows to adapt the naming of actions.
- The behaviour of a process call C is given by the right-hand side of the corresponding equation.

## Definition 2.1 (continued)

A (recursive) process definition is an equation system of the form

$$(C_i = P_i \mid 1 \leq i \leq k)$$

where  $k \ge 1$ ,  $C_i \in Pid$  (pairwise distinct), and  $P_i \in Prc$  (with identifiers from  $\{C_1, \ldots, C_k\}$ ).

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#### **Notational Conventions:**

- a means a
- $\sum_{i=1}^{n} P_i$   $(n \in \mathbb{N})$  means  $P_1 + \ldots + P_n$  (where  $\sum_{i=1}^{0} P_i := \text{nil}$ )
- $P \setminus a$  abbreviates  $P \setminus \{a\}$
- $[a_1 \mapsto b_1, \dots, a_n \mapsto b_n]$  stands for  $f : Act \to Act$  with

$$f(a_i) = b_i$$
 for  $i \in [n]$  and  $f(\alpha) = \alpha$  otherwise

• restriction and relabelling bind stronger than prefixing, prefixing stronger than parallel composition, parallel composition stronger than choice:

$$P \setminus a + b \cdot Q \parallel R$$
 means  $(P \setminus a) + ((b \cdot Q) \parallel R)$ 

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- The Approach
- Syntax of CCS
- 3 CCS Examples
- Formal Semantics of CCS
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# CCS Examples

# Example 2.2 (Bounded buffers)

(1) One-place buffer:

$$B = in.\overline{out}.B$$

# **CCS** Examples

## Example 2.2 (Bounded buffers)

(1) One-place buffer:

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(2) Two-place buffer:

$$B_0 = in.B_1$$

$$B_1 = \overline{out}.B_0 + in.B_2$$

$$B_2 = \overline{out}.B_1$$

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$$B_2 = \overline{out}.B_1$$

(3) Parallel two-place buffer:

$$B_{\parallel} = (B[out \mapsto com] \parallel B[in \mapsto com]) \setminus com$$
  
 $B = in.\overline{out}.B$ 

"Interaction diagram":



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# **Labelled Transition Systems**

Goal: represent system behaviour by (infinite) graph

- nodes = system states
- edges = transitions between states

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### Definition 2.3 (Labelled transition system)

An (Act-)labelled transition system (LTS) is a triple  $(S, Act, \longrightarrow)$  consisting of

- a set S of states
- a set Act of (action) labels
- a transition relation  $\longrightarrow \subseteq S \times Act \times S$

For  $(s, \alpha, s') \in \longrightarrow$  we write  $s \stackrel{\alpha}{\longrightarrow} s'$ . An LTS is called finite if S is so.

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#### Remarks:

- Sometimes an initial state  $s_0 \in S$  is distinguished (" $LTS(s_0)$ ").
- (Finite) LTSs correspond to (finite) automata without final states.

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## Semantics of CCS I

We define the assignment

by induction over the syntactic structure of process expressions.

Here we employ derivation rules of the form

$$(rule name) \frac{premise(s)}{conclusion}$$

whose instances are composed to form derivation trees (where axioms, i.e., rules without premises, correspond to leaves).

12/23

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**Reminder:**  $P, Q ::= \text{nil} \mid \alpha.P \mid P + Q \mid P \parallel Q \mid P \setminus L \mid P[f] \mid C$ 

## Definition 2.4 (Semantics of CCS)

A process definition  $(C_i = P_i \mid 1 \le i \le k)$  determines the LTS  $(Prc, Act, \longrightarrow)$  whose transitions can be inferred from the following rules  $(P, P', Q, Q' \in Prc, \alpha \in Act, \lambda \in A \cup \overline{A}, L \subseteq A, f : Act \to Act)$ :

$$(Act)\frac{P\overset{\alpha}{\longrightarrow}P}{P+Q\overset{\alpha}{\longrightarrow}P'} \qquad (Sum_1)\frac{P\overset{\alpha}{\longrightarrow}P'}{P+Q\overset{\alpha}{\longrightarrow}P'} \qquad (Sum_2)\frac{Q\overset{\alpha}{\longrightarrow}Q'}{P+Q\overset{\alpha}{\longrightarrow}Q'}$$

$$(Par_1)\frac{P\overset{\alpha}{\longrightarrow}P'}{P\parallel Q\overset{\alpha}{\longrightarrow}P'\parallel Q} \qquad (Par_2)\frac{Q\overset{\alpha}{\longrightarrow}Q'}{P\parallel Q\overset{\alpha}{\longrightarrow}P\parallel Q'} \qquad (Com)\frac{P\overset{\lambda}{\longrightarrow}P'}{P\parallel Q\overset{\overline{\longrightarrow}}{\longrightarrow}P'\parallel Q'}$$

$$(Res)\frac{P\overset{\alpha}{\longrightarrow}P'}{P\setminus L\overset{\alpha}{\longrightarrow}P'\setminus L} \qquad (Rel)\frac{P\overset{\alpha}{\longrightarrow}P'}{P[f]\overset{f(\alpha)}{\longrightarrow}P'[f]} \qquad (Call)\frac{P\overset{\alpha}{\longrightarrow}P'}{C\overset{\alpha}{\longrightarrow}P'}$$

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## Example 2.5 (Bounded buffers; cf. Example 2.2)

- (1) One-place buffer:  $B = in.\overline{out}.B$ 
  - First step:

$$(Call) \frac{\overbrace{in.\overline{out}.B \stackrel{in}{\longrightarrow} \overline{out}.B}^{}}{B \stackrel{in}{\longrightarrow} \overline{out}.B}$$

## Example 2.5 (Bounded buffers; cf. Example 2.2)

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  - First step:

$$(Call) \frac{(Act) \quad \underline{\qquad \qquad } \\ \underline{\qquad \qquad in.\overline{out}.B \stackrel{in}{\longrightarrow} \overline{out}.B} \\ B \stackrel{in}{\longrightarrow} \overline{out}.B$$

Second step:

$$(Act) \xrightarrow{\overline{out}.B \xrightarrow{\overline{out}} B}$$

## Example 2.5 (Bounded buffers; cf. Example 2.2)

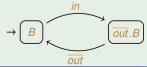
- (1) One-place buffer:  $B = in.\overline{out}.B$ 
  - First step:

$$(Call) \frac{(Act) \frac{}{in.\overline{out}.B \xrightarrow{in} \overline{out}.B}}{B \xrightarrow{in} \overline{out}.B}$$

Second step:

$$(Act) \xrightarrow{\overline{out}.B \xrightarrow{\overline{out}} B}$$

→ Complete LTS:



# Semantics of CCS IV

# Example 2.5 (continued)

(2) Sequential two-place buffer: 
$$B_0 = in.B_1$$
  
 $B_1 = \overline{out}.B_0 + in.B_2$   
 $B_2 = \overline{out}.B_1$ 

First step:

$$(Call) \xrightarrow[B_0 \xrightarrow{in} B_1]{in.B_1 \xrightarrow{in} B_1}$$

## Semantics of CCS IV

## Example 2.5 (continued)

(2) Sequential two-place buffer: 
$$B_0 = in.B_1$$
  
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First step:

(Call) 
$$\xrightarrow[B_0 \xrightarrow{in} B_1]{in.B_1 \xrightarrow{in} B_1}$$

Second step:

$$(Call) \xrightarrow{\text{(Act)}} \frac{\frac{}{\overline{out}.B_0} \xrightarrow{\overline{out}} B_0}{\overline{out}.B_0 + in.B_2} \xrightarrow{\overline{out}} B_0}{B_1 \xrightarrow{\overline{out}} B_0}$$

## Semantics of CCS IV

## Example 2.5 (continued)

(2) Sequential two-place buffer: 
$$B_0 = in.B_1$$
  
 $B_1 = \overline{out}.B_0 + in.B_2$   
 $B_2 = \overline{out}.B_1$ 

• First step:

$$(Call) \xrightarrow[B_0 \xrightarrow{in} B_1]{in.B_1 \xrightarrow{in} B_1}$$

Second step:

$$(Call) \xrightarrow{\text{(Act)}} \frac{\frac{}{\overline{out}.B_0} \xrightarrow{\overline{out}} B_0}{\overline{out}.B_0 + in.B_2 \xrightarrow{\overline{out}} B_0}$$

$$B_1 \xrightarrow{\overline{out}} B_0$$

• Like first step:  $B_2 \xrightarrow{\overline{out}} B_1$ 

#### Semantics of CCS IV

#### Example 2.5 (continued)

(2) Sequential two-place buffer: 
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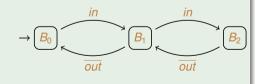
• First step:

$$(Call) \xrightarrow[B_0 \xrightarrow{in} B_1]{in.B_1 \xrightarrow{in} B_1}$$

Second step:

$$(Call) \xrightarrow{\begin{array}{c} (Act) & \xrightarrow{\overline{out}} B_0 \xrightarrow{\overline{out}} B_0 \\ \hline \overline{out}.B_0 + in.B_2 \xrightarrow{\overline{out}} B_0 \\ \hline B_1 \xrightarrow{\overline{out}} B_0 \end{array}}$$

- Like first step:  $B_2 \xrightarrow{\overline{out}} B_1$
- Complete LTS:



## Semantics of CCS V

#### Example 2.5 (continued)

(3) Parallel two-place buffer ( $f := [out \mapsto com], g := [in \mapsto com]$ ):

$$B_{\parallel} = (B[\underline{f}] \parallel B[g]) \setminus com$$
  
 $B = in.\overline{out}.B$ 

#### First step:

$$(Call) \frac{(Act) \frac{}{\underbrace{in.\overline{out}.B \xrightarrow{in} \overline{out}.B}}}{\underbrace{B \xrightarrow{in} \overline{out}.B}}}{\underbrace{(Par_1) \frac{}{B[f] \parallel B[g] \xrightarrow{in} (\overline{out}.B)[f]}}}{\underbrace{B[f] \parallel B[g] \xrightarrow{in} (\overline{out}.B)[f] \parallel B[g]}}{\underbrace{(B[f] \parallel B[g]) \setminus com \xrightarrow{in} ((\overline{out}.B)[f] \parallel B[g]) \setminus com}}}$$

#### Example 2.5 (continued)

(3) Parallel two-place buffer ( $f := [out \mapsto com], g := [in \mapsto com]$ ):

$$B_{\parallel} = (B[\underline{f}] \parallel B[g]) \setminus com$$
  
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#### First step:

# $(\mathsf{Call}) \xrightarrow{(\mathsf{Act}) \frac{\mathsf{(Act)} \frac{(Act)} \frac{\mathsf{(Act)} \frac{\mathsf{(Act)} \frac{\mathsf{(Act)} \frac{(Act)} \frac{\mathsf{(Act)} \frac{\mathsf{(Act)} \frac{(Act)} \frac{\mathsf{(Act)} \frac{(Act)} \frac{Act} \frac{\mathsf{(Act)} \frac{(Act)} \frac{(Act)} \frac{Act} {(Act)} \frac{Act} \frac{Act} {(Act)} \frac{Act} \frac{Act} {(Act)}$

#### A failing attempt:

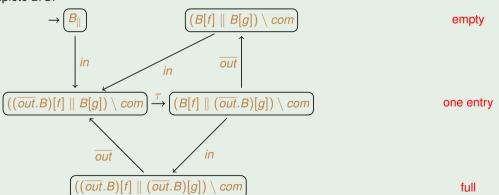
$$(\operatorname{Call}) \xrightarrow{(\operatorname{Act}) \frac{(\operatorname{Act}) \frac{(\operatorname{Act})$$

#### Semantics of CCS VI

## Example 2.5 (continued)

(3) Parallel two-place buffer:  $B_{\parallel} = (B[f] \parallel B[g]) \setminus com \quad (f := [out \mapsto com], g := [in \mapsto com])$ B = in.out.B

#### Complete LTS:



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So far: only finite state spaces

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#### Example 2.6 (Counter)

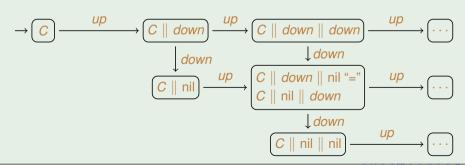
$$C = up.(C \parallel down.nil)$$

So far: only finite state spaces

#### Example 2.6 (Counter)

$$C = up.(C \parallel down.nil)$$

gives rise to infinite LTS (abbreviating down := down.nil):

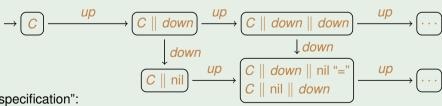


So far: only finite state spaces

#### Example 2.6 (Counter)

$$C = up.(C \parallel down.nil)$$

gives rise to infinite LTS (abbreviating down := down.nil):



Sequential "specification":

$$C_0 = up.C_1$$

$$C_n = up.C_{n+1} + down.C_{n-1} \qquad (n > 0)$$

$$C \parallel \text{nil} \parallel \text{nil}$$

$$Up$$

$$C \parallel \text{nil} \parallel \text{nil}$$

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#### The CAAL Tool



# CAAL (Concurrency Workbench, Aalborg Edition; https://caal.cs.aau.dk/)

- Smart editor
- Visualisation of generated LTS
- Equivalence checking w.r.t. several bisimulation, simulation and trace equivalences
- Model checking of (recursive) HML formulae
- Generation of distinguishing formulae for non-equivalent processes
- (Bi)simulation and model checking games

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# Summary: CCS

#### Summary

- Process behaviour defined by (synchronising) actions
- Syntax given by recursive definitions of processes
  - inaction nil
  - prefixing  $\alpha.P$
  - choice P + Q
  - parallel composition P || Q
  - restriction P \ L
  - relabelling *P*[*f*]
- Semantics given by (finite or infinite) labelled transition system
- Implemented by CAAL Tool

23/23

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