HOMEWORK 5

MATH 2001

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ABSTRACT. This is the first homework assignment. The problems are from Hammack [Ham18, Ch. 2, §2.5]:

- **Chapter 2 Section 2.5**, Exercises: 4, 6, 8. **Section 2.6**, Exercises: 4, 6. **Section 2.7**, Exercises: 2, 4, 8.

CONTENTS

CHAPTER 1 SECTION 1.1

Ch.2, §2.5, Exercise 4, 6, 8. Write a truth table for the logical statements.

4. $\neg(\boldsymbol{P} \vee \boldsymbol{Q}) \vee (\neg \boldsymbol{P})$
6. $(\boldsymbol{P} \wedge \neg \boldsymbol{P}) \wedge \boldsymbol{Q}$
8. $\boldsymbol{P} \vee (\boldsymbol{Q} \wedge \neg \boldsymbol{R})$

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Solution to Ch.2, §2.5, Exercise 4, 6, 8.

4. $\neg(\boldsymbol{P} \vee \boldsymbol{Q}) \vee (\neg \boldsymbol{P})$

P	Q	$\neg P$	$P \vee Q$	$\neg(P \vee Q)$	$\neg(P \vee Q) \vee (\neg P)$
T	T	F	T	F	F
T	F	F	T	F	F
F	T	T	T	F	T
F	F	T	F	T	T

6. $(\boldsymbol{P} \wedge \neg \boldsymbol{P}) \wedge \boldsymbol{Q}$

P	Q	$\neg P$	$P \wedge \neg P$	$(P \wedge \neg P) \wedge Q$
T	T	F	F	F
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

8. $\boldsymbol{P} \vee (\boldsymbol{Q} \wedge \neg \boldsymbol{R})$

P	Q	R	$\neg R$	$Q \wedge \neg R$	$P \vee (Q \wedge \neg R)$
T	T	T	F	F	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	F

$\square$

Ch.2, §2.6, Exercise 4, 6. Use truth tables to show that the following statements are logically equivalent.

4. $\neg(\boldsymbol{P} \vee \boldsymbol{Q}) = (\neg \boldsymbol{P}) \wedge (\neg \boldsymbol{Q})$
6. $\neg(\boldsymbol{P} \wedge \boldsymbol{Q} \wedge \boldsymbol{R}) = (\neg \boldsymbol{P}) \vee (\neg \boldsymbol{Q}) \vee (\neg \boldsymbol{R})$

Solution to Ch.2, §2.6, Exercise 4, 6.

4. $\neg(\boldsymbol{P} \vee \boldsymbol{Q}) = (\neg \boldsymbol{P}) \wedge (\neg \boldsymbol{Q})$

P	Q	$\neg P$	$\neg Q$	$(P \vee Q)$	$\neg(P \vee Q)$	$(\neg P) \wedge (\neg Q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

6. $\neg(\boldsymbol{P} \wedge \boldsymbol{Q} \wedge \boldsymbol{R}) = (\neg \boldsymbol{P}) \vee (\neg \boldsymbol{Q}) \vee (\neg \boldsymbol{R})$

P	Q	R	$\neg P$	$\neg Q$	$\neg R$	$P \wedge Q \wedge R$	$\neg(P \wedge Q \wedge R)$	$(\neg P) \vee (\neg Q) \vee (\neg R)$
T	T	T	F	F	F	T	F	F
T	T	F	F	F	T	F	T	T
T	F	T	F	T	F	F	T	T
T	F	F	F	T	T	F	T	T
F	T	T	T	F	F	F	T	T
F	T	F	T	F	T	F	T	T
F	F	T	T	T	F	F	T	T
F	F	F	T	T	T	F	T	T

$\square$

Ch.2, §2.7, Exercise 2, 4, 8. Write the following as English sentences. Say weather they are true or false.

2. $\forall x \in \mathbb{R}, \exists n \in \mathbb{N}, x^n \geq 0$
4. $\forall \boldsymbol{X} \in \wp(\mathbb{N}), \boldsymbol{X} \subseteq \mathbb{R}$
8. $\forall n \in \mathbb{Z}, \exists \boldsymbol{X} \subseteq \mathbb{N}, |\boldsymbol{X}| = n$

Solution to Ch.2, §2.7, Exercise 18.

2. $\forall x \in \mathbb{R}, \exists n \in \mathbb{N}, x^n \geq 0$

   For all x in the $\mathbb{R}$, there exists n in $\mathbb{N}$ such that x^n is greater than and equal to 0. **TRUE**

4. $\forall \boldsymbol{X} \in \wp(\mathbb{N}), \boldsymbol{X} \subseteq \mathbb{R}$

   All X in $\wp(\mathbb{N})$ is subsets of $\mathbb{R}$. **FALSE**

8. $\forall n \in \mathbb{Z}, \exists \boldsymbol{X} \subseteq \mathbb{N}, |\boldsymbol{X}| = n$

   For all n in the $\mathbb{Z}$, there exists $\boldsymbol{X}$ in $\mathbb{N}$ such that $|\boldsymbol{X}| = n$. **FALSE**

$\square$

REFERENCES

[Ham18] Richard Hammack, *Book of Proof*, 3 ed., Creative Commons, 2018.

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