HOMEWORK 10

MATH 2001

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ABSTRACT. This is the first homework assignment. The problems are from Hammack [**?**, Ch. 1, §1.1]:

- **Chapter 12 Section 12.1**, Exercises: 4, 6. **Section 12.2**, Exercises: 5, 10. **Section 12.3**, Exercises: 1, 2.

CONTENTS

CHAPTER 12 SECTION 12.1

Ch.12, §12.1, Exercise 4. There are eight different functions $f : \{a, b, c\} \to \{0, 1\}$. List them all. Diagrams will suffice.

Date: April 12, 2020.

Solution to Ch.1, §1.1, Exercise 2.

$$f = \{(a,0),(b,0),(c,0)\}$$

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□

Ch.12, §12.1, Exercise 6. Suppose $f : \mathbb{Z} \to \mathbb{Z}$ is defined as $f = \{(x, 4x+5) : x \in \mathbb{Z}\}$. State the domain, codomain and range of f. Find $f(10)$.

Solution to Ch.12, §12.1, Exercise 6.

The domain of the function is $\{x : x \in \mathbb{Z}\}$.

The codomain of the function is $\{x : x \in \mathbb{Z}\}$.

The range of the function is $\{x : x = 4n, n \in \mathbb{Z}\}$.

□

Ch.12, §12.2, Exercise 5. A function $f : \mathbb{Z} \to \mathbb{Z}$ is defined as $f(n) = 2n + 1$. Verify whether this function is injective and whether it is surjective.

Solution to Ch.12, §12.2, Exercise 5.

Proposition. The function is injective but not surjective.

Step 1. Prove the function is injective. *Proof.* Assume $f(x) = f(y)$. We have $2x + 1 = 2y + 1$, thus $x = y$.

Step 2. Prove the function is not surjective.

Proof. There exist element 2 which $f(x) = 2x + 1 \neq 2$ for every $x \in \mathbb{Z}$. The function is injective but not surjective. □

Ch.12, §12.2, Exercise 10. Prove that the function $f : \mathbb{R} - 1 \to \mathbb{R} - 1$ defined by $f(x) = \left(\frac{x+1}{x-1}\right)^3$ is bijective.

Solution to Ch.12, §12.2, Exercise 10.

Proposition. The $f(x)$ is bijective.

Step 1. Prove $f(x)$ is injective.

Proof. Assume $f(x) = f(y)$, we have $\left(\frac{x+1}{x-1}\right)^3 = \left(\frac{y+1}{y-1}\right)^3$, taking the cubic root on both side get $\left(\frac{x+1}{x-1}\right) = \left(\frac{y+1}{y-1}\right)$. Therefor we have:

$$(x + 1) * (y - 1) = (y + 1) * (x - 1)$$

$$xy + y - x = xy + x - y$$

$$y - x = x - y$$

$$2y = 2x$$

$$x = y$$

Step 2. Prove $f(x)$ is sujective.

Proof. Suppose $f(x) = c$, we have $f(x) = \left(\frac{x+1}{x-1}\right)^3 = c$. Sove for x, we get $x = \frac{1+\sqrt[3]{c}}{1-\sqrt[3]{c}}, x \in \mathbb{R}$. It follows that $f(x)$ is surjective. The $f(x)$ is surjective and injective, so it is bijective. □

Ch.1, §1.1, Exercise 38.

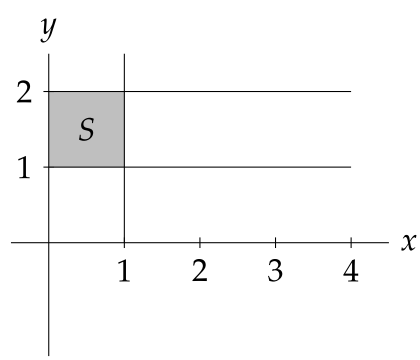
Solution to Ch.1, §1.1, Exercise 38. □

Ch.1, §1.1, Exercise 40. Sketch the following set of points in the x, y-plane:

$$S = \{(x, y) : x \in [0, 1],\ y \in 1, 2]\}$$

Solution to Ch.1, §1.1, Exercise 40. For this problem I first sketched my own solution by hand. However, to implement my solution in LaTeX, I modifed the tikz code from the webpage:

https://tex.stackexchange.com/questions/140312/tikz-shading-region-bounded-by-s



□

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