HOMEWORK 1

MATH 2001

SEBASTIAN CASALAINA

ABSTRACT. This is the first homework assignment. The problems are from Hammack [Ham18, Ch. 1, §1.1]:

- **Chapter 2 Section 2.9**, Exercises: 2, 3, 4, 5. **Section 2.10**, Exercises: 2, 4, 10. **Section 4**, Exercises: 1, 2, 5, 7, 9.

CONTENTS

CHAPTER 2

Ch.2, §2.9, Exercise 2, 3, 4, 5. Translate each of the following sentences into symbolic logic.

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2. The number x is positive but the number y is not positive.
3. If x is prime, then $\sqrt{x}$ is not a rational number.
4. For every prime number p there is another prime number q with q ¿ p.
5. For every positive number ε, there is a positive number δ for which $|x-a| < \delta$ implies $|f(x) - f(a)| < \varepsilon$.

Solution to Ch.2, §2.9, Exercise 2, 3, 4, 5.

2. (x is positive) $\wedge$ (y is not positive)
3. (x is prime) $\implies$ ($\sqrt{x}$ is not prime)
4. $\exists q \in \textbf{primes}, (\forall p \in \textbf{primes}, q > p)$
5. $\exists \delta, (\delta > 0) \wedge [\forall \varepsilon, ((\varepsilon > 0) \wedge (|x - \alpha| < \delta)) \implies (f(x) - f(a)| < \varepsilon]$

$\square$

Ch.2, §2.10, Exercise 2, 4, 10. Negate the following sentences.

2. If x is prime, then $\sqrt{x}$ is not a rational number.
4. For every positive number ε, there is a positive number δ such that $|x-a| < \delta$ implies $|f(x) - f(a)| < \varepsilon$.
10. If f is a polynomial and its degree is greeter than 2, then f' is not constant.

Solution to Ch.2, §2.10, Exercise 2, 4, 10.

2. x is prime, $\sqrt{x}$ is a rational number.

4. There is a positive number ε, such that for all positive number δ, $(|x-a|<\delta) \wedge (|f(x)-f(a)| \geq \varepsilon)$.
10. f is a polynomial and it s degree is greeter than 2, f' is constant.

$\square$

Ch.4, §4, Exercise 1. Use the method of direct proof to prove the following statements: If x is an even integer, then x^2 is even.

Solution to Ch.4, §4, Exercise 1.

Proposition If x is an even integer, then x^2 is even.

Proof. Suppose x is even. Then $x = 2a$ for some $a \in \mathbb{Z}$, by definition of an even number. Thus $x^2 = 4a^2 = 2(2a^2)$, so $x^2 = 2b$ where $b = 2a^2 \in \mathbb{Z}$. Therefore x^2 is even, by definition of an even number.

$\square$

Ch.4, §4, Exercise 3. Use the method of direct proof to prove the following statements: If a is an odd integer, then $a^2 + 3a + 5$ is odd.

Solution to Ch.1, §1.1, Exercise 30.

Proposition If a is an odd integer, then $a^2 + 3a + 5$ is odd.

Proof. Suppose a is odd. Then $a = 2b + 1$ for some $b \in \mathbb{Z}$, by definition of an odd number. Thus $a^2 + 3a + 5 = 4b^2 + 10b + 9 = 2(2b^2 + 5b + 4) + 1$, so $a^2 + 3a + 5 = 2b + 1$ where $b = (2b^2 + 5b + 4) \in \mathbb{Z}$. Therefor $a^2 + 3a + 5$ is odd, by definition of an odd number. $\square$

Ch.4, §4, Exercise 5. Use the method of direct proof to prove the following statements: Suppose $x, y \in \mathbb{Z}$. If x is even, then xy is even.

Solution to Ch.1, §1.1, Exercise 38.

Proposition Suppose $x, y \in \mathbb{Z}$. If x is even, then xy is even.

Proof. x is even. Then $x = 2a$ for some $a \in \mathbb{Z}$, by definition of even numbers.

Case 1. Suppose y is odd. Then $y = 2b + 1$ for some $b \in \mathbb{Z}$, by definition of odd numbers. Thus $x * y = 2a * (2b + 1) = 4ab + 2a = 2(2ab + a)$, so $x * y = 2c$ where $c = (2ab + a) \in \mathbb{Z}$. Therefor $x * y$ is even, by definition of an even number.

Case 2. Suppose y is even. Then $y = 2d$ for some $d \in \mathbb{Z}$, by definition of an even number. Thus $x * y = 2a * 2d = 4ad = 2(2ad)$, so $x * y = 2e$ where $e = 2ad \in \mathbb{Z}$. Therefor $x * y$ is even, by definition of an even number.

Because $y \in \mathbb{Z}$, y is either even or odd. In both cases, $x * y$ is even.

□

Ch.4, §4, Exercise 7. Use the method of direct proof to prove the following statements: Suppose $a, b \in \mathbb{Z}$. If $a|b$, then $a^2|b^2$.

Solution to Ch.1, §1.1, Exercise 40.

Proposition Suppose $a, b \in \mathbb{Z}$. If $a|b$, then $a^2|b^2$.

Proof Suppose $a|b$ where $a, b \in \mathbb{Z}$. Then $a = b * c$ for some $c \in \mathbb{Z}$, by definition. Thus $a^2 = (b * c)^2 = b^2 * c^2$, so $a^2 = b^2 * d$ where $d = c^2 \in \mathbb{Z}$. There for $a^2|b^2$, by definition.

□

References

[Ham18] Richard Hammack, *Book of Proof*, 3 ed., Creative Commons, 2018.

University of Colorado, Department of Mathematics, Campus Box 395, Boulder, CO 80309-0395

Email address: casa@math.colorado.edu