HOMEWORK 2

MATH 2001

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ABSTRACT. This is the second homework assignment. The problems are from Hammack [Ham18, Ch. 1, §1.2]:

- **Chapter 1 Section 2**, Exercises: 2, 4, 8, 12, 18.

CONTENTS

CHAPTER 1 SECTION 1.2

Ch.1, §1.2, Exercise 2. Write out the indicated sets by listing their elements between braces. Suppose $A = \{\pi, e, 0\}$ and $B = \{0, 1\}$.

(a) $A \times B$

(b) $B \times A$

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(c) $A \times A$

(d) $B \times B$

(e) $\emptyset \times B$

(f) $(A \times B) \times B$

(g) $A \times (B \times B)$

(h) B^3

Solution to Ch.1, §1.2, Exercise 2. I worked with the entire class on this solution.

(a) $A \times B$

$$= (\pi, 0), (\pi, 1), (\mathrm{e}, 0), (\mathrm{e}, 1), (0, 0), (0, 1)$$

(b) $B \times A$

$$= (0, \pi), (0, \mathrm{e}), (0, 0), (1, \mathrm{pi}), (1, \mathrm{e}), (1, 0)$$

(c) $A \times A$

$$= (\pi, \pi), (\pi, e), (\pi, 0), (e, \pi),$$

$$(e, e), (e, 0), (0, \pi), (0, e), (0, 0)$$

(d) $B \times B$

$$= (0, 0), (0, 1), (1, 0), (1, 1)$$

(e) $A \times \emptyset$

$$= \emptyset$$

(f) $(A \times B) \times B$

$$= ((\pi, 0), 0), ((\pi, 0), 1), ((\pi, 1), 0), ((\pi, 1), 1), ((e, 0), 0), ((e, 0), 1),$$

$$((e,1),0),((e,1),1),((0,0),0),((0,0),1),((0,1),0),((0,1),1)$$

(g) $A \times (B \times B)$

$$= (\pi,(0,0)),(e,(0,0)),(0,(0,0)),(\pi,(0,1)),(e,(0,1)),(0,(0,1)),$$

$$(\pi,(1,0)),(e,(1,0)),(0,(1,0)),(\pi,(1,1)),(e,(1,1)),(0,(1,1))$$

(h) $A \times B \times B$

$$= (\pi,0,0),(\pi,0,1),(\pi,1,0),(\pi,1,1),(e,0,0),(e,0,1),$$

$$(e,1,0),(e,1,1),(0,0,0),(0,0,1),(0,1,0),(0,1,1)$$

□

Ch.1, §1.2, Exercise 4. Write the following set by listing its elements between braces: $\{n \in \mathbb{Z} : 2 < n < 5\} \times \{n \in \mathbb{Z} : |n| = 5\}$.

Solution to Ch.1, §1.2, Exercise 4.

$$A = \{n \in \mathbb{Z} : 2 < n < 5\}$$

$$A = \{3,4\}$$

$$B = \{n \in \mathbb{Z} : |n| = 5\}$$

$$B = \{5,-5\}$$

$$A \times B = (3,5),(3,-5),(4,5),(4,-5)$$

□

Ch.1, §1.2, Exercise 8. Write the following set by listing its elements between braces:

$$\{0,1\}^4$$

Solution to Ch.1, §1.2, Exercise 8.

$$\{0,1\}^4$$

$$= (0,0,0,0),(0,0,0,1),(0,0,1,0),(0,0,1,1),$$

$$(0,1,0,0),(0,1,0,1),(0,1,1,0),(0,1,1,1),$$

$$(1,0,0,0),(1,0,0,1),(1,0,1,0),(1,0,1,1),$$

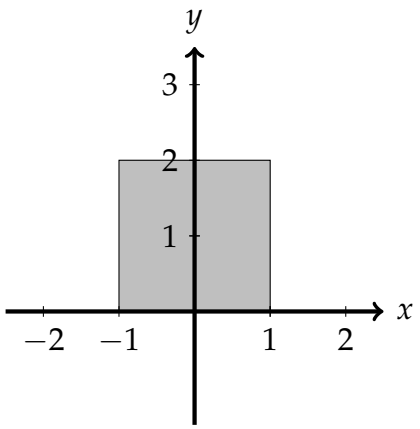
$$(1,1,0,0),(1,1,0,1),(1,1,1,0),(1,1,1,1)$$

□

Ch.1, §1.2, Exercise12. Sketch these Cartesian products on the $x-y$ plane $\mathbb{R}^2$:

$$[-1,1]\times[0,1]$$

Solution to Ch.1, §1.2, Exercise 12.

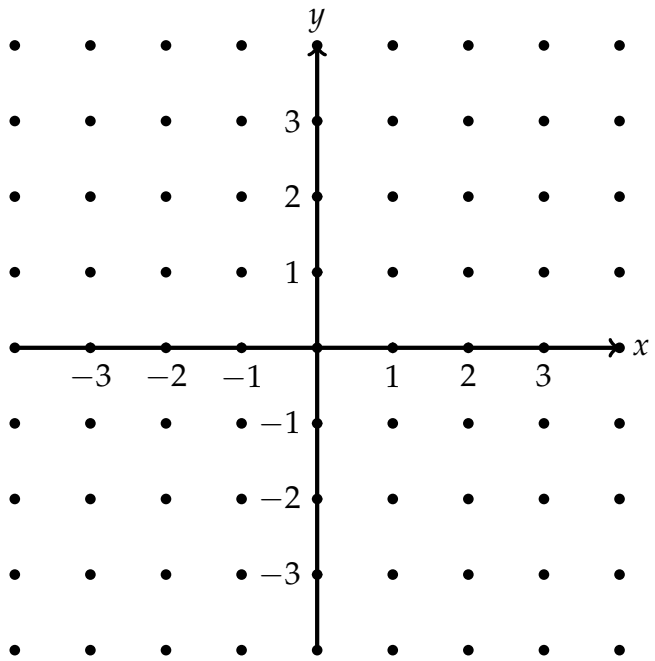


□

Ch.1, §1.2, Exercise 18. Sketch these Cartesian products on the $x - y$ plane $\mathbb{R}^2$:

$$\mathbb{Z} \times \mathbb{Z}$$

Solution to Ch.1, §1.2, Exercise 18.



□

REFERENCES

[Ham18] Richard Hammack, *Book of Proof*, 3 ed., Creative Commons, 2018.

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