

Matrix Multiplication - GeeksforGeeks

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Measures of Central Tendency in Statistics Set Theory Practice NCERT Solutions for Class 8 to 12 RD Sharma Class 8 Solutions for Maths: Chapter Wise PDF RD Sharma Class 9 Solutions RD Sharma Class 10 Solutions RD Sharma Class 11 Solutions for Maths RD Sharma Class 12 Solutions for Maths Three 90 Challenge 90% Refund Matrix Multiplication Last Updated : 22 Dec, 2025 Matrix multiplication is a binary operation that produces a new matrix from two given matrices. For the multiplication to be defined, the number of columns in the first matrix must equal the number of rows in the second matrix. The resulting matrix, called the matrix product, has the same number of rows as the first matrix and the same number of columns as the second matrix. For example , if matrices A and B satisfy this condition, their product results in a new matrix whose order is determined by the rows of A and the columns of B. If A is $(m \times p)$ and the order of B is $(p \times n)$, then the order of the multiplied matrix is $(m \times n)$. Matrix Multiplication How to Multiply Matrices Let's take two matrices A and B of order 2×2 Rules and Conditions for Matrix Multiplication If " $A = [a_{ij}]_{m \times n}$ " and " $B = [b_{ij}]_{n \times o}$ " are two matrices, then the product of A and B is denoted as AB , whose order is " $m \times o$ ". This condition is represented as given in the image. Matrix multiplication is not commutative, i.e., $AB \neq BA$, or, in simple words, the product of A and B matrices is AB , and AB is not equal to BA ; it is even possible that AB exists but BA does not exist. Compatibility Conditions for Matrix Multiplication : We can multiply two matrices if the number of columns in the 1st matrix is equal to the number of rows in the 2nd matrix, otherwise, the given matrices cannot be multiplied. For example: A 2×3 matrix can be multiplied by a 3×2 matrix, resulting in a 2×2 matrix. A 3×3 matrix cannot be multiplied by a 4×2 matrix because their dimensions are incompatible. Special Properties of Matrix Multiplication Apart from compatibility, here are some special properties of matrix multiplication: Both AB and BA matrix multiplication are defined if both A and B are square matrices. One of the matrices doesn't need to be a zero matrix if the product of two matrices A and B is zero. Note: French mathematician Jacques Philippe Marie Binet was the first to perform matrix multiplication in 1812. Step-by-Step Process for Matrix Multiplication To multiply two matrices, A and B, ensure that the number of columns in A equals the number of rows in B. Calculate Elements: Multiply each element in a row of the first matrix by the corresponding element in a column of the second matrix, and sum these products to compute each element of the product matrix. Repeat this process for all rows and columns. Form the Product Matrix: Place the computed elements in their respective positions to construct the resulting matrix.' For example , let's take a matrix of order (2×3) and another of order (3×2) . To multiply these matrices, follow these steps: Formula for Matrix Multiplication Let's take two matrices A and B of order 2×2 such that $A = [a_{ij}]$ and $B = [b_{ij}]$. Then the multiplication of A and B is obtained in the image such that, The resultant multiplication matrix X is represented as, $X = \left[\begin{array}{cc} X_{11} & X_{12} \\ X_{21} & X_{22} \end{array} \right]$ Where $X_{xy} = A_{x1}B_{y1} + A_{x2}B_{y2}$ Matrix Multiplication (Scalar) A matrix can be multiplied by a scalar value, which is called scalar multiplication. When a matrix "A = [a ij]" is multiplied by a scalar value "k," every element of the given matrix is multiplied by the scalar value. The resultant matrix is expressed as kA , where $kA = k[a_{ij}] = [ka_{ij}]$, for all the values of i and j.] A = $\left[\begin{array}{cc} a & b \\ c & d \end{array} \right]$ $kA = k \times \left[\begin{array}{cc} a & b \\ c & d \end{array} \right] = \left[\begin{array}{cc} ka & kb \\ kc & kd \end{array} \right]$

$\left[\begin{array}{cc} ka & kb \\ kc & kd \end{array} \right]$ Matrix Multiplication (2x2) Let us consider two matrices A and B of order "2 × 2". Then its multiplication is achieved using the formula. $A = \left[\begin{array}{cc} a & b \\ c & d \end{array} \right]$ and $B = \left[\begin{array}{cc} p & q \\ r & s \end{array} \right]$ $AB = \left[\begin{array}{cc} a & b \\ c & d \end{array} \right] \times \left[\begin{array}{cc} p & q \\ r & s \end{array} \right]$ $AB = \left[\begin{array}{cc} (ap+br) & (aq+bs) \\ (cp+dr) & (cq+ds) \end{array} \right]$ Read in detail: How to Multiply 2 × 2 Matrices. Matrix Multiplication (3x3) Let us consider two matrices, P and a "3 × 3" matrix. Now, the matrix multiplication formula of "3 × 3" matrices is, $X = \left[\begin{array}{ccc} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{array} \right]$ $Y = \left[\begin{array}{ccc} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{array} \right]$ $XY = \left[\begin{array}{ccc} (x_{11}y_{11}+x_{12}y_{21}+x_{13}y_{31}) & (x_{11}y_{12}+x_{12}y_{22}+x_{13}y_{32}) & (x_{11}y_{13}+x_{12}y_{23}+x_{13}y_{33}) \\ (x_{21}y_{11}+x_{22}y_{21}+x_{23}y_{31}) & (x_{21}y_{12}+x_{22}y_{22}+x_{23}y_{32}) & (x_{21}y_{13}+x_{22}y_{23}+x_{23}y_{33}) \\ (x_{31}y_{11}+x_{32}y_{21}+x_{33}y_{31}) & (x_{31}y_{12}+x_{32}y_{22}+x_{33}y_{32}) & (x_{31}y_{13}+x_{32}y_{23}+x_{33}y_{33}) \end{array} \right]$ Related Articles How to Multiply 3x3 Matrices How to Multiply a 3 × 3 Matrix with a 1 × 3 Matrix Properties of Matrix Multiplication The following are some important properties of matrix multiplication: Commutative Property The matrix multiplication is usually not commutative, i.e., the multiplication of the first matrix with the second matrix is not similar to the multiplication of the second matrix with the first. If A and B are two matrices, then $AB \neq BA$. Associative Property The matrix multiplication is associative in nature. If A, B, and C are three matrices, then $A(BC) = (AB)C$ This property holds true if the products A(BC) and (AB)C are defined. Distributive Property The Distributive property also holds true for matrix multiplication. If A, B, and C are three matrices, then by applying the distributive property, we get. $A(B + C) = AB + AC$ $(B + C)A = BA + CA$ Note: This property is only true if and only if A, B, and C are compatible. Multiplicative Identity Property Matrix multiplication has an identity property that states that, if we multiply a matrix A by an Identity matrix of the same order, then it results in the same matrix. $A \cdot I = I \cdot A = A$ Transpose Property The transpose property also holds true for matrix multiplication. If A and B are two matrices, then by applying the transpose property. $(AB)^T = B^T A^T$ Note: This property is only true if the product AB is defined. Multiplicative Property of Zero Matrix multiplication has the property of zero, which states that if a matrix is multiplied by a zero matrix, then the resultant matrix is a zero matrix(O). $A \cdot O = O \cdot A = O$ Also, the product of any two non-zero matrices may result in a zero matrix, i.e., $AB = O$ Then that doesn't mean that $A = O$ or $B = O$. Product with a Scalar If A and B are two matrices and AB is defined, then the product of the matrix with the scalar (k) is defined as, $k(AB) = (kA)B = A(kB)$ Articles related to Matrix Multiplication: Square Matrix Adjoint of a Matrix Minors and Cofactors of Determinants Algorithm for Matrix Multiplication Various matrix multiplication algorithms are widely used for finding matrix multiplication, and some of the most common matrix multiplication algorithms are, Iterative Algorithm Divide and Conquer Algorithm Sub-Cubic Algorithms Parallel and Distributed Algorithms These algorithms are widely used in computer programming to find the multiplication of two matrices, such that the results are efficient and take less memory and time. They are used to find the 2×2, 3×3, and 4×4 multiplication of matrices. We use these matrix multiplication algorithms for a variety of purposes, and the method to multiply matrices is similar for any order of matrix for a particular algorithm. Solved Question on Matrix Multiplication Example 1. Let $A = \begin{bmatrix} 1 & 8 & 3 \\ 9 & 4 & 5 \\ 6 & 2 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 7 & 4 \\ 1 & 3 & 2 \\ 5 & 9 & 8 \end{bmatrix}$ Find AxB ? Solution: $A \times B = \begin{bmatrix} 1 & 8 & 3 \\ 9 & 4 & 5 \\ 6 & 2 & 7 \end{bmatrix} \times \begin{bmatrix} 6 & 7 & 4 \\ 1 & 3 & 2 \\ 5 & 9 & 8 \end{bmatrix} = \begin{bmatrix} (1 \times 6 + 8 \times 1 + 3 \times 5) & (1 \times 7 + 8 \times 3 + 3 \times 9) & (1 \times 4 + 8 \times 2 + 3 \times 8) \\ (9 \times 6 + 4 \times 1 + 5 \times 5) & (9 \times 7 + 4 \times 3 + 5 \times 9) & (9 \times 4 + 4 \times 2 + 5 \times 8) \\ (6 \times 6 + 2 \times 1 + 7 \times 5) & (6 \times 7 + 2 \times 3 + 7 \times 9) & (6 \times 4 + 2 \times 2 + 7 \times 8) \end{bmatrix} = \begin{bmatrix} 29 & 58 & 44 \\ 83 & 120 & 84 \\ 73 & 111 & 84 \end{bmatrix}$ Example 2. Let $A = \begin{bmatrix} 1 & 5 & 4 \\ 9 & 3 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 7 \\ 1 & 3 \\ 5 & 9 \end{bmatrix}$. Find AxB ? Solution: $A \times B = \begin{bmatrix} (1 \times 6 + 5 \times 1 + 4 \times 5) & (1 \times 7 + 5 \times 3 + 4 \times 9) \\ (9 \times 6 + 3 \times 1 + 8 \times 5) & (9 \times 7 + 3 \times 3 + 8 \times 9) \end{bmatrix} = \begin{bmatrix} 31 & 58 \\ 97 & 144 \end{bmatrix}$ Example 3. Let $A = \begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 4 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}$. Find $(AB + AC)$? Solution: $A \times B = \begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ -1 & 0 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} (2 \times 3 + 0 \times (-1) + (-3) \times 4) & (2 \times 1 + 0 \times 0 + (-3) \times 2) \\ (1 \times 3 + 4 \times (-1) + 5 \times 4) & (1 \times 1 + 4 \times 0 + 5 \times 2) \end{bmatrix} = \begin{bmatrix} -6 & -4 \\ 19 & 11 \end{bmatrix}$ $A \times C = \begin{bmatrix} 2 & 0 & -3 \\ 1 & 4 & 5 \end{bmatrix} \times \begin{bmatrix} 4 & 7 \\ 2 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} (2 \times 4 + 0 \times 2 + (-3) \times 1) & (2 \times 7 + 0 \times 1 + (-3) \times (-1)) \\ (1 \times 4 + 4 \times 2 + 5 \times 1) & (1 \times 7 + 4 \times 1 + 5 \times (-1)) \end{bmatrix} = \begin{bmatrix} 1 & 10 \\ 11 & 1 \end{bmatrix}$

$$\begin{aligned} -1\end{bmatrix} &= \begin{bmatrix} 2x4 + 0x2 + (-3)x1 & (2x7 + 0x1 + (-3)x(-1)) \\ (1x4 + 4x2 + 5x1) & (1x7 + 4x1 + 5x(-1)) \end{bmatrix} = \begin{bmatrix} 5 & 17 \\ 17 & 6 \end{bmatrix} \text{ Now calculate } (AB + AC) = \begin{bmatrix} -6 & -4 \\ 19 & 11 \end{bmatrix} + \begin{bmatrix} 5 & 17 \\ 17 & 6 \end{bmatrix} = \begin{bmatrix} -1 & 13 \\ 36 & 17 \end{bmatrix} \text{ Example 4. Let } A = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}, A^2 = pA, \text{ then find the value of } p? \text{ Solution: Calculating, } A^2 = A \times A = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} (2x2 + (-2)x(-2)) & ((2x(-2) + (-2)x2) \\ ((-2)x2 + 2x(-2)) & ((-2)x(-2) + 2x2) \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} \text{ Given, } A^2 = pA \text{ Taking } A^2 \text{ in the equation, } \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = p \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2p & -2p \\ -2p & 2p \end{bmatrix} \text{ Now, } 8 = 2p, -8 = -2p \Rightarrow p = 4 \text{ Thus, the value of } p \text{ is 4} \text{ Example 5: Find the value of } 3P \text{ if } P = \left[\begin{array}{ccc} 2 & -3 & 4 \\ 1 & 0 & 5 \\ 7 & -4 & 6 \end{array} \right]. \text{ Solution: } 3P = 3 \times \left[\begin{array}{ccc} 2 & -3 & 4 \\ 1 & 0 & 5 \\ 7 & -4 & 6 \end{array} \right] = \left[\begin{array}{ccc} 3 \times 2 & 3 \times -3 & 3 \times 4 \\ 3 \times 1 & 3 \times 0 & 3 \times 5 \\ 3 \times 7 & 3 \times -4 & 3 \times 6 \end{array} \right] = \left[\begin{array}{ccc} 6 & -9 & 12 \\ 3 & 0 & 15 \\ 21 & -12 & 18 \end{array} \right] \text{ Practice Question on Matrix Multiplication Question 1: Find } 9P \text{ if } P = \left[\begin{array}{ccc} 2 & -3 & 4 \\ 1 & 0 & 5 \\ 7 & -4 & 6 \end{array} \right]. \text{ Question 2: Multiply, } P = \left[\begin{array}{ccc} 2 & -3 & 4 \\ 1 & 0 & 5 \\ 7 & -4 & 6 \end{array} \right] \text{ and } Q = \left[\begin{array}{ccc} 3 & -6 & 9 \\ 7 & 0 & 8 \\ 1 & -4 & 1 \end{array} \right] \text{ Question 3: Find the product of } AB \text{ where } A = \left[\begin{array}{ccc} 1 & -2 & 3 \\ 9 & 0 & 6 \\ 8 & -4 & 1 \end{array} \right] \text{ and } B = \left[\begin{array}{ccc} 0 & -12 & 7 \\ 0 & 0 & 3 \\ -21 & -6 & 8 \end{array} \right] \text{ Question 4: Matrix Multiplication with Identity Matrix } A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ 0 & 4 \\ 1 & 1 \end{bmatrix} \text{ What is } AI \text{ and } IA? \text{ More Questions on Matrix Multiplication - [Check here !] Comment Article Tags: Article Tags: Technical Scripter Mathematics School Learning Class 12 Technical Scripter 2020 Maths-Class-12 + 2 More}$$