Combinatorics - GeeksforGeeks

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Courses Tutorials Practice Jobs Number System Algebra Set Theory Geometry Linear Algebra Trigonometry Logarithms Statistics Probability Calculus Discrete Mathematics Engineering Math Practice Problems Ace GATE 2027 Linear Algebra Matrices Row Echelon Form Eigenvalues and Eigenvectors System of Linear Equations Matrix Diagonalization LU Decomposition Finding Inverse of a Square Matrix using Cayley Hamilton Theorem in MATLAB Sequence & Series Mathematics | Sequence, Series and Summations Binomial Theorem Finding nth term of any Polynomial Sequence Calculus Limits, Continuity and Differentiability Cauchy's Mean Value Theorem Taylor Series Inverse functions and composition of functions Definite Integral Application of Derivative - Maxima and Minima Probability & Statistics Mean, Variance and Standard Deviation Conditional Probability Bayes' Theorem Probability Distribution - Function, Formula, Table Covariance and Correlation Practice Questions Last Minute Notes - Engineering Mathematics Engineering Mathematics - GATE CSE Previous Year Questions GATE Preparation Explore Combinatorics Last Updated : 23 Jul, 2025 Combinatorics is a branch of mathematics that focuses on studying the selection, arrangement, and operation of countable discrete structures. This is essential in computer science because it can be used to solve problems regarding statistics and probability. Moreover, combinatorics also plays a big role in the optimization of various applications. Combination and Permutation are two terms that are often used to solve problems in combinatorics. Table of Content Combinatorics Definition Permutation and Combination under Combinatorics Combination Permutation Properties of Combinatorics Addition Principle Multiplication Principle Uses of Combinatorics Combinatorics Definition Combinatorics is a branch of mathematics focused on counting, arranging, and analyzing finite sets. It deals with the study of combinations, permutations, and the structures that arise from these arrangements. Combinatorics Permutation and Combination under Combinatorics Combinatorics is a branch of mathematics that deals with counting, arranging, and analyzing sets of elements. Within combinatorics, permutations and combinations are two fundamental concepts used to calculate the number of ways to arrange or select items. Combination Combination is selections of objects where the order does not matter. The formula for number of combinations of 'n' objects taken 'r' at a time is: n C r = C(n, r) = n!/r!(n - r)! Permutation Permutation is arrangements of objects where the order matters. The formula for number of permutations of 'n' objects taken 'r' at a time is: n P r = P(n, r) = n!/(n - r)! Example of Permutation and Combination: Suppose we have 3 variables named x, y, and z. Find the combination and the permutation of two variables from the given variables. Solution: Combination Combination: xy, yz, and zx Permutation: xy, yx, yz, zy, zx, and xz From the example above, we can see that in the permutation both the xy and yx are written because the order of arrangement matters. However, in the combination, only one of them is written because both are the same things since the order of arrangement does not matter. Properties of Combinatorics Combinatorics is the branch of mathematics that deals with counting, arrangement, and combination of objects. Here are some fundamental properties and principles of combinatorics: Addition Principle If there are n ways to perform task A and m ways to perform task B, and these tasks cannot be done simultaneously, then there are n + m ways to choose one of these tasks. Example: If you have 3 shirts and 4 pants, there are 3+4 = 7 ways to choose either a shirt or a pant. Multiplication Principle If there are n ways to perform task A and m ways to perform task B, and these tasks are independent (i.e., performing one does not affect the other), then there are n×m ways to perform both tasks. Example: If you have 3 shirts and 4 pants, there are 3×4 = 12 ways to choose a shirt and a pant. Uses of Combinatorics Combinatorics, the branch of mathematics dealing with counting, arrangement, and combination of objects, has numerous applications across various fields. Here are some key areas where combinatorics is used: Probability Theory: Combinatorics helps in calculating probabilities by counting the number of favorable outcomes over the total possible outcomes. Graph Theory: Used to study graphs, which are structures made up of nodes connected by edges. Design and Analysis of Experiments: Combinatorial designs help in planning experiments to ensure that the data collected is statistically valid and can be analyzed effectively. Algorithms and Data Structures: Combinatorial algorithms are used in sorting, searching, and optimization problems. Articles Related to Combinatorics: Permutation and Combination Binomial Theorem Pegionhole Principle Fundamental Principle of Counting Combinatorics Solved Examples Example 1: How Many Words with

2 Different Vowels and 2 Different Consonants can be Formed from Alphabet? Solution: To solve this problem, we have to know that the alphabet contains 26 letters including 5 vowels and 21 consonants. Step 1: Ways of selecting 2 different vowels out of 5 vowels C(5, 2) = 5! / (2! × (5 – 2)!) = 5! / (2! × 3!) = 10 Step 2: Ways of selecting 2 different consonants out of 21 consonants C(21, 3) = 21! / (2! × (21 – 2)!) = 21! / (3! × 19!) = 210 Hence, combinations of 2 different vowels and 3 different consonants is 10 × 210 = 2100 Step 3: Ways arranging or forming 2 vowels and 2 consonants out of 4 letters P(4, 4) = 4! / (4 – 4)! = 4! / 0! = 4! = 24 Therefore, the total number of ways is 2100 × 24 = 54000 Answer: 54000 words Example 2: How many distinct ways are there to arrange 10 people in a row if exactly 3 people must be together in one block? Solution: To solve this problem, we need to treat the block of 3 people who must be together as a single unit. The 3 people within this block can be arranged among themselves in 3! ways. [3! = 6] Since the 3 people are treated as one unit, we now have 8 units to arrange (the block plus the other 7 individual people). The 8 units can be arranged in (8!) ways. [8! = 40320] To find the total number of distinct ways to arrange the 10 people with the block treated as a single unit, multiply the number of ways to arrange the units by the number of ways to arrange the people within the block. [8! × 3! = 40320 × 6 = 241920] Therefore, there are (241,920) distinct ways to arrange 10 people in a row if exactly 3 of them must be together in one block. Problem 3: How many 5-digit numbers can be formed such that the digit 3 appears exactly twice? Solution: To determine how many 5-digit numbers can be formed such that the digit 3 appears exactly twice, we can break down the problem as follows: Choose the positions for the two 3's: We need to choose 2 positions out of 5 for the digit 3. The number of ways to choose 2 positions out of 5 is given by the combination formula 5 C 2 = 5!/2!(5-2)! = 10 The remaining 3 positions can be filled with any digits from 0 to 9 except for 3. Since we are forming 5-digit numbers, the first digit cannot be 0. The first digit (one of the remaining three positions) can be any digit from 1 to 9 except 3. That gives us 8 choices for the first digit. The other two positions can be any digit from 0 to 9 except 3. That gives us 9 choices for each of these two positions. For the first position (not 0 and not 3), we have 8 choices. For the second position (can be 0 but not 3), we have 9 choices. For the third position (can be 0 but not 3), we have 9 choices. Therefore, the number of ways to fill these remaining three positions is: 8 × 9 × 9 = 648 We now combine the number of ways to choose the positions for the 3's and the number of ways to fill the remaining positions. 10 × 648 = 6480 Therefore, there are (6,480) different 5-digit numbers where the digit 3 appears exactly twice. Problem 4: How many ways can you arrange the letters in the word "MATHEMATICS" such that the two 'M's are next to each other? Solution: Treat two 'M's as a single unit: This reduces the problem to arranging the units: "MM", A, T, H, E, A, T, I, C, S (10 units). Count permutations of these units: Frequencies: A: 2 T: 2 Other letters appear once. Use the formula: 10!/2!×2! Calculate: 3628800/4=907200 Problem 5: Problem: I n how many ways can you choose 4 books from a shelf of 12 books? Solution: 4 Books from the shelf of 12 books can be choose in 4 C 12 number of ways. Use the combination formula 12 C 4 = 12!//4!(12–4)! Calculate = (12×11×10×9)/(4×3×2×1) = 495 There are 495 ways to choose 4 books from 12. Practice Problems Combinatorics Problem 1: How many distinct ways can you arrange 7 people around a circular table? Problem 2: How many ways can you distribute 5 identical candies into 3 distinct boxes? Problem 3: In how many ways can you arrange 4 items so that none of them is in its original position? Problem 4: How many ways can you arrange 6 people around a circular table? Problem 5: How many ways can you distribute 5 identical candies to 3 children? Problem 6: How many 4-digit numbers can be formed using the digits 1, 2, and 3, with repetition allowed? Problem 7: How many subsets can be formed from a set of 8 elements? Problem 8: Find the number of ways to select 4 students from a class of 20 where at least 1 student must be from a specific group of 5 students. Problem 9: How many different paths are there from the bottom-left corner to the top-right corner of a 3x3 grid, moving only right or up? Problem 10: How many 5-digit numbers can be formed using the digits 1, 2, 3, 4, 5, if no digit is repeated?