1.14.

$$f(x) = \frac{(3x^4 + x^3 - 2x^2 - 1)}{2x^2 - 3}$$

$$g(x) = x^2$$

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \lim_{x \to \infty} \frac{3x^4 + x^3 - 2x^2 - 1}{(2x^2 - 3)\, x^2} = \lim_{x \to \infty} \frac{3 + \frac{x^3}{x^4} - 2\frac{x^2}{x^4} - \frac{1}{x^4}}{2 - 3\frac{x^2}{x^4}} =$$

$$= \lim_{x \to \infty} \frac{3}{2} \Rightarrow f(x) = \Theta(g(x))$$

1.9.

$$f(x) = x + e^{2x}$$

$$g(x) = 5x^2$$

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \lim_{x \to \infty} \frac{x + e^{2x}}{5x^2} = \left[\frac{\infty}{\infty}\right] = \lim_{x \to \infty} \frac{\overset{1}{\cancel{2}} e^{2x}}{\underset{5}{\cancel{10}} x} = \left[\frac{\infty}{\infty}\right] =$$

$$= \lim_{x \to \infty} \frac{2e^{2x}}{5} = \infty$$

$$\Downarrow$$

$$f(x) = \Omega(g(x))$$

2.1.

$$T(n) = 2T\left(\frac{n}{3}\right) + n^3$$

$$\left(\frac{n}{3^h}\right)^3 = 1$$

$$\frac{n}{3^h} = 1$$

$$3^h = n$$

$$h = \log_3 n$$

$$n^3$$

$$\left(\frac{n}{3}\right)^3 \quad \left(\frac{n}{3}\right)^3$$

$$\left(\frac{n}{9}\right)^3 \quad \left(\frac{n}{9}\right)^3 \quad \left(\frac{n}{9}\right)^3 \quad \left(\frac{n}{9}\right)^3$$

$$\vdots$$

$$\left(\frac{n}{3^h}\right)^3$$

Sumos:

$$n^3 = 2^0\left(\frac{n}{3^0}\right)^3$$

$$2\left(\frac{n}{3}\right)^3 = 2^1\left(\frac{n}{3^1}\right)^3$$

$$4\left(\frac{n}{3}\right)^3 = 2^2\left(\frac{n}{3^2}\right)^3$$

$$\vdots$$

$$2^h\left(\frac{n}{3^h}\right)^3$$

$$T(n) = \sum_{i=1}^{h} 2^i\left(\frac{n}{3^i}\right)^3 = n^3 \sum_{i=1}^{\log_3 n}\left(\frac{2}{27}\right)^i =$$

$$= n^3\left(\frac{1-\left(\frac{2}{27}\right)^{\log_3 n}}{1-\frac{2}{27}}\right) = n^3\left(\frac{1-\frac{n^{\log_3 2}}{n^3}}{\frac{25}{27}}\right) = \frac{n^3 - n^{\log_3 2}}{\frac{25}{27}} = O(n^3)$$

Tikriname, ar tenkina sprendinį $T(n) = O(n \log_2 n)$

$$O(n \log_2 n) \leq cn \log_2 n$$

$$T\left(\frac{n}{3}\right) \leq c\frac{n}{3}\log_2\frac{n}{3}$$

$$T(n) \leq \frac{2}{3}cn\log_2\frac{n}{3} + n^3 \leq cn\log_2 n$$

$$\frac{2}{3}c\log_2\frac{n}{3} + n^2 \leq c\log_2 n$$

$$\frac{2}{3}c\log_2\frac{n}{3} - c\log_2 n \leq -n^2$$

$$c\,\frac{2}{3}\log_2\frac{1}{3} \leq -n^2$$

$$c \leq \frac{-n^2}{\frac{2}{3}\log_2\frac{1}{3}}$$

bet kokiam n, c – neigiamas, todėl netenkina.

3.8

$T(n) = 2T(\frac{n}{4}) + n^{0,51}$

$a = 2 \quad b = 4 \quad f(n) = n^{0,51}$

$n^{\log_4 2} = n^{0,5} < n^{0,51}$ III atv.

$f(n) = \Omega(n^{0,5+\varepsilon})$

$\frac{f(n)}{\Omega(n^{0,5+\varepsilon})} = \frac{n^{0,51}}{n^{0,5+\varepsilon}} = n^{-0,49-\varepsilon}$

$-0,49 - \varepsilon = 0$

$\varepsilon = -0,49$ $\quad \varepsilon \not> 0$, todėl netinka

3.14

$T(n) = 4T(\frac{n}{2}) + cn$

$a = 4 \quad b = 2 \quad f(n) = cn$

$n^{\log_2 4} = n^2 > cn$ I atv.

$f(n) = O(n^{\log_2 4 - \varepsilon})$

$\frac{f(n)}{O(n^{2-\varepsilon})} = \frac{cn}{n^{2-\varepsilon}} = cn^{1-2+\varepsilon}$

$1 - 2 + \varepsilon = 0$

$\varepsilon = 1$

$T(n) = \Theta(n^2)$

4.3 6 metrų rąstas

l	1	2	3	4	5	6	7
c	3	6	7	9	11	13	15

Kai $l = 2$

$\max(\ldots) = \begin{matrix} 3+3 \\ 6 \end{matrix} = 6$

Kai $l = 3$

$\max(\ldots) = \begin{matrix} 9 \\ 9 \\ 7 \end{matrix} = 9$

Kai $l = 4$

$\max(\ldots) = \begin{matrix} 12 \\ 12 \\ 12 \\ 9 \end{matrix} = 12$

Kai $l = 5$

$\max(\ldots) = \begin{matrix} 15 \\ 15 \\ 15 \\ 15 \\ 11 \end{matrix} = 15$

Kai $l = 6$

$\max(\ldots) = \begin{matrix} 18 \\ 18 \\ 18 \\ 18 \\ 18 \\ 13 \end{matrix} = 18$

Pelnas bus didžiausias pjaustant kas 1 metrą arba kas 2 metrus.