

循环神经网络

Recurrent Neural Network

- 起源：

1982 John Joseph Hopfield提出带有反馈结构的Hopfield网络

Neural networks and physical systems with emergent collective computational abilities

- 发展：

1986 Michael Jordan提出循环结构的

Serial order: A parallel distributed processing approach

1990 Jeffrey Elman提出现代意义上的Recurrent Neural Network

Finding structure in time

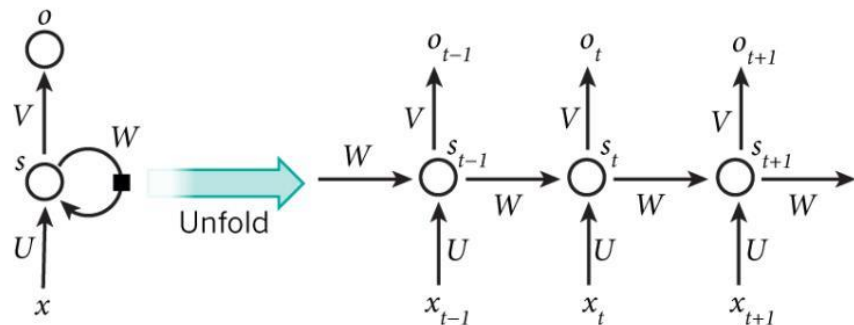
- 壮大：

1997 Hochreiter & Schmidhuber提出LSTM

LSTM: long short-term memory for vanishing gradients problem



Recurrent Neural Network



$$o_t = g(Vs_t) \quad (1)$$

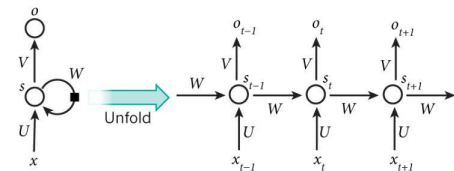
$$s_t = f(Ux_t + Ws_{t-1}) \quad (2)$$

$$\begin{aligned} \longrightarrow o_t &= g(Vs_t) \\ &= g(Vf(Ux_t + Ws_{t-1})) \\ &= g(Vf(Ux_t + Wf(Ux_{t-1} + Ws_{t-2}))) \\ &= g(Vf(Ux_t + Wf(Ux_{t-1} + Wf(Ux_{t-2} + Ws_{t-3})))) \\ &= g(Vf(Ux_t + Wf(Ux_{t-1} + Wf(Ux_{t-2} + Wf(Ux_{t-3} + \dots))))) \end{aligned} \quad (3)$$

$$\begin{aligned} x &\in \mathbb{R}(m) \\ U &\in \mathbb{R}(n, m) \\ s_{Ux} &= U \cdot x + b_s \\ \rightarrow s_{Ux} &\in \mathbb{R}(n) \end{aligned}$$

$$\begin{aligned} W &\in \mathbb{R}(n, n) \\ s_{Ws} &= W \cdot s_{t-1} + b_s \\ \rightarrow s_{Ws} &\in \mathbb{R}(n) \end{aligned}$$

Recurrent Neural Network



$$o_t = g(Vs_t) \quad (1)$$

$$s_t = f(Ux_t + Ws_{t-1}) \quad (2)$$

加入偏置



$$o_t = g(Vs_t + b_o) \quad (1)$$

$$s_t = f(Ux_t + Ws_{t-1} + b_s) \quad (2)$$

$$\begin{aligned} \longrightarrow o_t &= g(Vs_t) \\ &= g(Vf(Ux_t + Ws_{t-1})) \\ &= g(Vf(Ux_t + Wf(Ux_{t-1} + Ws_{t-2}))) \\ &= g(Vf(Ux_t + Wf(Ux_{t-1} + Wf(Ux_{t-2} + Ws_{t-3})))) \\ &= g(Vf(Ux_t + Wf(Ux_{t-1} + Wf(Ux_{t-2} + Wf(Ux_{t-3} + \dots)))) \end{aligned} \quad (3)$$

$$f(z) = \tanh(z)$$

$$g(z) = \text{softmax}(z)$$

$$U = \begin{bmatrix} 0.10 & 0.20 & 0.30 \\ 0.40 & 0.50 & 0.60 \end{bmatrix}, \quad W = \begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \end{bmatrix}, \quad V = \begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \\ 0.50 & 0.60 \end{bmatrix}, \quad b_s = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}, \quad b_o = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 1.00 \\ 2.00 \\ 3.00 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 2.00 \\ 3.00 \\ 4.00 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 3.00 \\ 4.00 \\ 5.00 \end{bmatrix} \quad 1,2,3,4,5$$

$$GroundTruth_1 = \begin{bmatrix} 1.00 \\ 0.00 \\ 0.00 \end{bmatrix}, \quad GroundTruth_2 = \begin{bmatrix} 0.00 \\ 0.00 \\ 1.00 \end{bmatrix}, \quad GroundTruth_3 = \begin{bmatrix} 0.00 \\ 1.00 \\ 0.00 \end{bmatrix}$$



Recurrent Neural Network

$$s_1 = \tanh(U \cdot x_1 + W \cdot s_0 + b_s)$$

$$= \tanh\left(\begin{bmatrix} 0.10 & 0.20 & 0.30 \\ 0.40 & 0.50 & 0.60 \end{bmatrix} \cdot \begin{bmatrix} 1.00 \\ 2.00 \\ 3.00 \end{bmatrix} + \begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \end{bmatrix} \cdot \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} + \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}\right)$$

$$= \tanh\left(\begin{bmatrix} 1.40 \\ 3.20 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0.88535 \\ 0.99668 \end{bmatrix}$$

$$s_2 = \tanh(U \cdot x_2 + W \cdot s_1 + b_s)$$

$$= \tanh\left(\begin{bmatrix} 0.10 & 0.20 & 0.30 \\ 0.40 & 0.50 & 0.60 \end{bmatrix} \cdot \begin{bmatrix} 2.00 \\ 3.00 \\ 4.00 \end{bmatrix} + \begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \end{bmatrix} \cdot \begin{bmatrix} 0.89 \\ 1.00 \end{bmatrix} + \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}\right)$$

$$= \tanh\left(\begin{bmatrix} 2.29 \\ 5.36 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0.97961 \\ 0.99996 \end{bmatrix}$$

$$s_3 = \tanh(U \cdot x_3 + W \cdot s_2 + b_s)$$

$$= \tanh\left(\begin{bmatrix} 0.10 & 0.20 & 0.30 \\ 0.40 & 0.50 & 0.60 \end{bmatrix} \cdot \begin{bmatrix} 3.00 \\ 4.00 \\ 5.00 \end{bmatrix} + \begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \end{bmatrix} \cdot \begin{bmatrix} 0.98 \\ 1.00 \end{bmatrix} + \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}\right)$$

$$= \tanh\left(\begin{bmatrix} 2.90 \\ 6.89 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0.99394 \\ 1.00000 \end{bmatrix}$$

$$o_1 = \text{softmax}(V \cdot s_1 + b_o)$$

$$= \text{softmax}\left(\begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \\ 0.50 & 0.60 \end{bmatrix} \cdot \begin{bmatrix} 0.88535 \\ 0.99668 \end{bmatrix} + \begin{bmatrix} 0.00 \\ 0.00 \\ 0.00 \end{bmatrix}\right)$$

$$= \text{softmax}\left(\begin{bmatrix} 0.29 \\ 0.66 \\ 1.04 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0.22 \\ 0.32 \\ 0.46 \end{bmatrix}$$

$$o_2 = \text{softmax}(V \cdot s_2 + b_o)$$

$$= \text{softmax}\left(\begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \\ 0.50 & 0.60 \end{bmatrix} \cdot \begin{bmatrix} 0.97961 \\ 0.99996 \end{bmatrix} + \begin{bmatrix} 0.00 \\ 0.00 \\ 0.00 \end{bmatrix}\right)$$

$$= \text{softmax}\left(\begin{bmatrix} 0.30 \\ 0.69 \\ 1.09 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0.21 \\ 0.32 \\ 0.47 \end{bmatrix}$$

$$o_3 = \text{softmax}(V \cdot s_3 + b_o)$$

$$= \text{softmax}\left(\begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \\ 0.50 & 0.60 \end{bmatrix} \cdot \begin{bmatrix} 0.99394 \\ 1.00000 \end{bmatrix} + \begin{bmatrix} 0.00 \\ 0.00 \\ 0.00 \end{bmatrix}\right)$$

$$= \text{softmax}\left(\begin{bmatrix} 0.30 \\ 0.70 \\ 1.10 \end{bmatrix}\right)$$

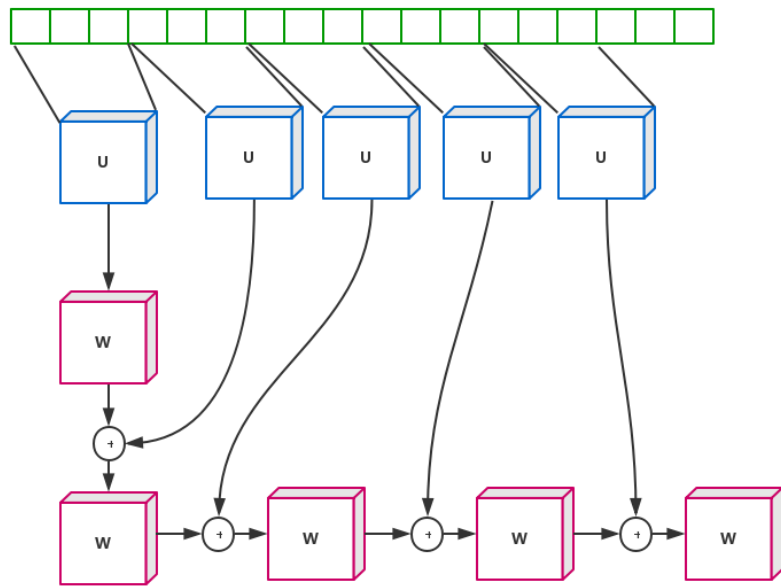
$$= \begin{bmatrix} 0.21 \\ 0.32 \\ 0.47 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.10 & 0.20 & 0.30 \\ 0.40 & 0.50 & 0.60 \end{bmatrix}, W = \begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \end{bmatrix}, V = \begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \\ 0.50 & 0.60 \end{bmatrix}, b_s = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}, b_o = \begin{bmatrix} 0.00 \\ 0.00 \\ 0.00 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 1.00 \\ 2.00 \\ 3.00 \end{bmatrix}, x_2 = \begin{bmatrix} 2.00 \\ 3.00 \\ 4.00 \end{bmatrix}, x_3 = \begin{bmatrix} 3.00 \\ 4.00 \\ 5.00 \end{bmatrix}$$

$$\text{GroundTruth}_1 = \begin{bmatrix} 1.00 \\ 0.00 \\ 0.00 \end{bmatrix}, \text{GroundTruth}_2 = \begin{bmatrix} 0.00 \\ 0.00 \\ 1.00 \end{bmatrix}, \text{GroundTruth}_3 = \begin{bmatrix} 0.00 \\ 1.00 \\ 0.00 \end{bmatrix}$$

- 在时间维度上：
 - 在输入上滑动
 - 对state进行深度计算



- 沿时间反向传播 (Back Propagation Through Time, BPTT) :

- Loss为各个时间点的Loss之和
- 权重的梯度同样为各个时间点的梯度之和
- 前馈传播中的权重包括了:

U: 链接 x_t (输入) 和 s_t (状态)

W: 链接 s_{t-1} 和 s_t

V: 链接 s_t 和 o_t (输出)

所以我们针对这三个权重来考虑

- $\delta_t^{(L)}$ 仍然定义为梯度参数, 即 $\frac{\partial Cost}{\partial logits_t^{(L)}}$

- 回忆一下普通神经网络时代的定义:

$$\delta^{(L)} = \nabla_y Cost \odot \sigma'(logit^{(L)}) \quad (BP1) \quad , \quad \text{且} \quad o_t = g(Vs_t + b_o) \quad (1)$$

$$\delta^{(l)} = ((w^{(l+1)})^T \delta^{(l+1)}) \odot \sigma'(logit^{(l)}) \quad (BP2) \quad , \quad s_t = f(Ux_t + Ws_{t-1} + b_s) \quad (2)$$

循环神经网络的反向传播

- 反向传播：

- Loss为各个时间点的Loss之和
- 权重的梯度同样为各个时间点的梯度之和
- 前馈传播中的权重包括了：

U：链接 x_t （输入）和 s_t （状态）

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V：链接 s_t 和 o_t （输出）

所以我们针对这三个权重来考虑

- $\delta_t^{(L)}$ 仍然定义为梯度参数，即 $\frac{\partial Cost}{\partial \text{logits}_t^{(L)}}$

- 回忆一下普通神经网络时代的定义：

$$\delta_t^{(L)} = \nabla_y Cost \odot \sigma'(\text{logit}_t^{(L)}) \quad (BP1) \quad o_t = g(Vs_t + b_o) \quad (1)$$

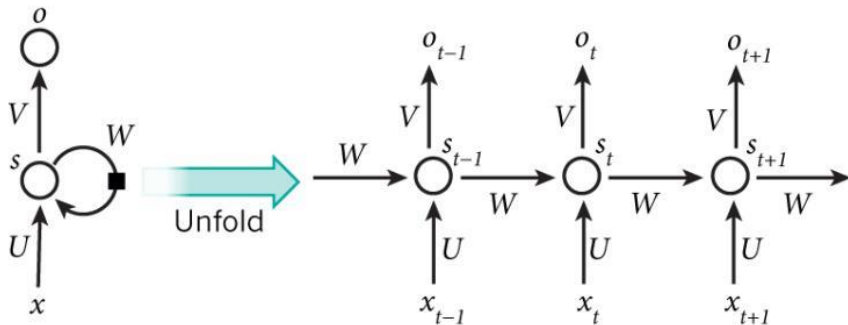
$$\delta_t^{(l)} = ((w^{(l+1)})^T \delta_t^{(l+1)}) \odot \sigma'(\text{logit}_t^{(l)}) \quad (BP2)' \quad \text{且} \quad s_t = f(Ux_t + Ws_{t-1} + b_s) \quad (2)$$

$\delta_t^{(L)}$ 可以由各个时刻的 o_t 与ground truth计算得出

$\delta_t^{(L-1)}$ 可以由（1）计算得出，仅与V有关，和普通神经网络一样

我们将（2）分解为： $\text{logit}_t = Ux_t + Ws_{t-1}$ 或写成 $z_t = Ux_t + Ws_{t-1}$
 $s_{t-1} = f(\text{logit}_{t-1})$ $s_{t-1} = f(z_{t-1})$

$\delta_{t-1}^{(L-1)}$ 可以由 $\delta_t^{(L-1)}$ 沿时间方向向前传播得出



循环神经网络的反向传播

$$\delta_{(T)}^{(L)} = \nabla_y Cost \odot \sigma'(\text{logit}_{(T)}^{(L)}) \quad (BP1)$$

$$\delta_{(T)}^{(l)} = ((V^{(l+1)})^T \delta_{(T)}^{(l+1)}) \odot \sigma'(\text{logit}_{(T)}^{(l)}) \quad (BP2)$$

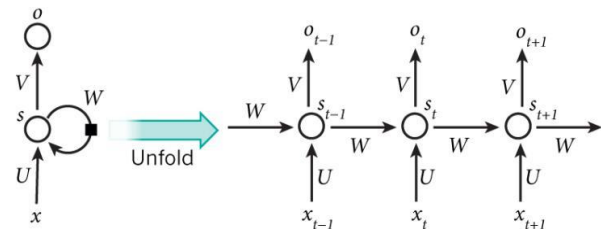
$$\delta_{(t)}^{(l)} = ((W^{(l)})^T \delta_{(t+1)}^{(l)} + (V^{(l+1)})^T \delta_{(t)}^{(l+1)}) \odot \sigma'(s_{(t)}^{(l)}) \quad (BP2T)$$

$$\frac{\partial Cost}{\partial bias^{(l)}} = \sum_{t=0}^T \delta_{(t)}^{(l)} \quad (BP3)$$

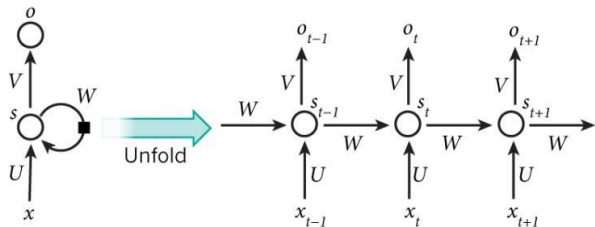
$$\frac{\partial Cost}{\partial V^{(l)}} = \sum_{t=0}^T \delta_{(t)}^{(l)} \cdot (s_{(t)}^{(l-1)})^T \quad (BP4V)$$

$$\frac{\partial Cost}{\partial W^{(l)}} = \sum_{t=1}^T \delta_{(t)}^{(l)} \cdot (s_{(t-1)}^{(l)})^T \quad (BP4W)$$

$$\frac{\partial Cost}{\partial U^{(l)}} = \sum_{t=0}^T \delta_{(t)}^{(l)} \cdot (x_{(t)})^T \quad (BP4U)$$



循环神经网络的反向传播



$$o_t = g(Vs_t + b_o) \quad (1)$$

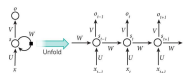
$$s_t = f(Ux_t + Ws_{t-1} + b_s) \quad (2)$$

$$U = \begin{bmatrix} 0.10 & 0.20 & 0.30 \\ 0.40 & 0.50 & 0.60 \end{bmatrix}, \quad W = \begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \end{bmatrix}, \quad V = \begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \\ 0.50 & 0.60 \end{bmatrix}, \quad b_s = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}, \quad b_o = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

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循环神经网络的反向传播



$$o_t = g(Vs_t + b_o) \quad (1)$$

$$s_t = f(Ux_t + Ws_{t-1} + b_s) \quad (2)$$

$$U = \begin{bmatrix} 0.10 & 0.20 & 0.30 \\ 0.40 & 0.50 & 0.60 \end{bmatrix}, W = \begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \end{bmatrix}, V = \begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \\ 0.50 & 0.60 \end{bmatrix}, b_s = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}, b_o = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 1.00 \\ 2.00 \\ 3.00 \end{bmatrix}, x_2 = \begin{bmatrix} 2.00 \\ 3.00 \\ 4.00 \end{bmatrix}, x_3 = \begin{bmatrix} 3.00 \\ 4.00 \\ 5.00 \end{bmatrix}$$

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$$\delta_3^3 = \nabla_y Cost \odot \sigma'(logit_{(T)}^{(L)}) = \begin{bmatrix} 0.21231 \\ -0.68366 \\ 0.47135 \end{bmatrix}$$

$$\begin{aligned} \nabla V_3 &= \delta_3^3 \cdot (s_3)^T \\ &= \begin{bmatrix} 0.21231 \\ -0.68366 \\ 0.47135 \end{bmatrix} \cdot \begin{bmatrix} 0.99394 & 1.00000 \end{bmatrix} \\ &= \begin{bmatrix} 0.21 & 0.21 \\ -0.68 & -0.68 \\ 0.47 & 0.47 \end{bmatrix} \end{aligned}$$

$$\delta_2^3 = \nabla_y Cost \odot \sigma'(logit_{(T)}^{(L)}) = \begin{bmatrix} 0.21308 \\ 0.31658 \\ -0.52965 \end{bmatrix}$$

$$\begin{aligned} \nabla V_2 &= \delta_2^3 \cdot (s_2)^T \\ &= \begin{bmatrix} 0.21308 \\ 0.31658 \\ -0.52965 \end{bmatrix} \cdot \begin{bmatrix} 0.97961 & 0.99996 \end{bmatrix} \\ &= \begin{bmatrix} 0.21 & 0.21 \\ 0.31 & 0.32 \\ -0.52 & -0.53 \end{bmatrix} \end{aligned}$$

$$\delta_1^3 = \nabla_y Cost \odot \sigma'(logit_{(T)}^{(L)}) = \begin{bmatrix} -0.78166 \\ 0.31813 \\ 0.46353 \end{bmatrix}$$

$$\begin{aligned} \nabla V_1 &= \delta_1^3 \cdot (s_1)^T \\ &= \begin{bmatrix} -0.78166 \\ 0.31813 \\ 0.46353 \end{bmatrix} \cdot \begin{bmatrix} 0.88535 & 0.99668 \end{bmatrix} \\ &= \begin{bmatrix} -0.69 & -0.78 \\ 0.28 & 0.32 \\ 0.41 & 0.46 \end{bmatrix} \end{aligned}$$

$$\delta_3^2 = (W^T \cdot \delta_4^2 + V^T \cdot \delta_3^3) \odot \sigma'(s_3^2)$$

$$\begin{aligned} &= ((\begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \end{bmatrix})^T \cdot \begin{bmatrix} 0.00000 \\ 0.00000 \end{bmatrix} + (\begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \\ 0.50 & 0.60 \end{bmatrix})^T \cdot \begin{bmatrix} 0.21231 \\ -0.68366 \\ 0.47135 \end{bmatrix}) \odot \begin{bmatrix} 0.01209 \\ 0.00000 \end{bmatrix} \\ &= \begin{bmatrix} 0.00063 \\ 0.00000 \end{bmatrix} \end{aligned}$$

$$\delta_2^2 = (W^T \cdot \delta_3^2 + V^T \cdot \delta_2^3) \odot \sigma'(s_2^2)$$

$$\begin{aligned} &= ((\begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \end{bmatrix})^T \cdot \begin{bmatrix} 0.00063 \\ 0.00000 \end{bmatrix} + (\begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \\ 0.50 & 0.60 \end{bmatrix})^T \cdot \begin{bmatrix} 0.21308 \\ 0.31658 \\ -0.52965 \end{bmatrix}) \odot \begin{bmatrix} 0.04036 \\ 0.00009 \end{bmatrix} \\ &= \begin{bmatrix} -0.00599 \\ -0.00001 \end{bmatrix} \end{aligned}$$

$$\delta_1^2 = (W^T \cdot \delta_2^2 + V^T \cdot \delta_1^3) \odot \sigma'(s_1^2)$$

$$\begin{aligned} &= ((\begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \end{bmatrix})^T \cdot \begin{bmatrix} -0.00599 \\ -0.00001 \end{bmatrix} + (\begin{bmatrix} 0.10 & 0.20 \\ 0.30 & 0.40 \\ 0.50 & 0.60 \end{bmatrix})^T \cdot \begin{bmatrix} -0.78166 \\ 0.31813 \\ 0.46353 \end{bmatrix}) \odot \begin{bmatrix} 0.21615 \\ 0.00662 \end{bmatrix} \\ &= \begin{bmatrix} 0.05370 \\ 0.00164 \end{bmatrix} \end{aligned}$$

$$\delta_{(T)}^{(L)} = \nabla_y Cost \odot \sigma'(logit_{(T)}^{(L)}) \quad (BP1)$$

$$\delta_{(T)}^{(l)} = ((V^{(l+1)})^T \delta_{(T)}^{(l+1)}) \odot \sigma'(logit_{(T)}^{(l)}) \quad (BP2)$$

$$\delta_{(t)}^{(l)} = ((W^{(l)})^T \delta_{(t+1)}^{(l)} + (V^{(l+1)})^T \delta_{(t)}^{(l+1)}) \odot \sigma'(s_{(t)}^{(l)}) \quad (BP2T)$$

$$\frac{\partial Cost}{\partial bias^{(l)}} = \sum_{t=0}^T \delta_{(t)}^{(l)} \quad (BP3)$$

$$\frac{\partial Cost}{\partial V^{(l)}} = \sum_{t=1}^T \delta_{(t)}^{(l)} \cdot (s_{(t-1)}^{(l-1)})^T \quad (BP4V)$$

$$\frac{\partial Cost}{\partial W^{(l)}} = \sum_{t=1}^T \delta_{(t)}^{(l)} \cdot (s_{(t-1)}^{(l-1)})^T \quad (BP4W)$$

$$\frac{\partial Cost}{\partial U^{(l)}} = \sum_{t=0}^T \delta_{(t)}^{(l)} \cdot (x_{(t)})^T \quad (BP4U)$$

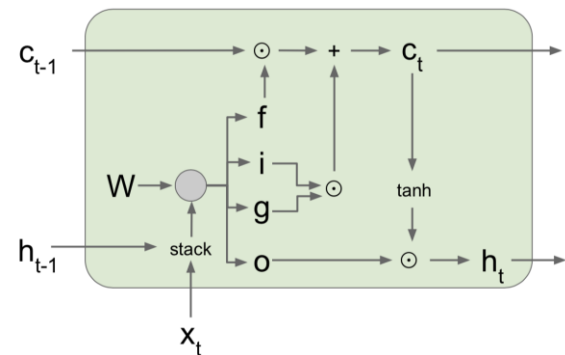
$$\begin{aligned} \nabla W &= \sum_{t=1}^3 \delta_{(t)}^{(2)} \cdot (s_{(t-1)}^{(2)})^T \\ &= \begin{bmatrix} 0.00061 & 0.00063 \\ 0.00000 & 0.00000 \end{bmatrix} + \begin{bmatrix} -0.00531 & -0.00597 \\ -0.00001 & -0.00001 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} -0.00469 & -0.00535 \\ -0.00001 & -0.00001 \end{bmatrix}$$

$$\begin{aligned} \nabla V &= \sum_{t=0}^3 \delta_{(t)}^{(3)} \cdot (s_{(t)}^{(2)})^T \\ &= \begin{bmatrix} -0.27229 & -0.35369 \\ -0.08774 & -0.05002 \\ 0.36003 & 0.40372 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \nabla U &= \sum_{t=0}^3 \delta_{(t)}^{(1)} \cdot (x_{(t)})^T \\ &= \begin{bmatrix} -0.27229 & -0.35369 \\ -0.08774 & -0.05002 \\ 0.36003 & 0.40372 \end{bmatrix} \end{aligned}$$

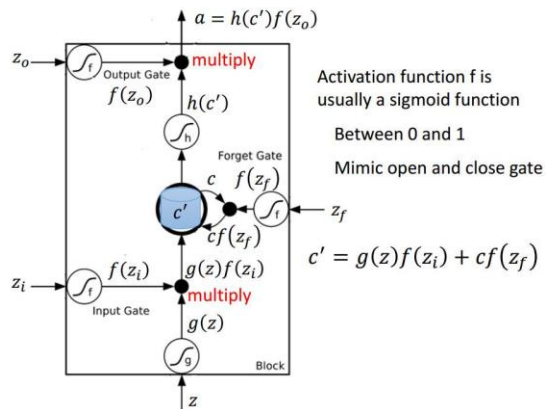
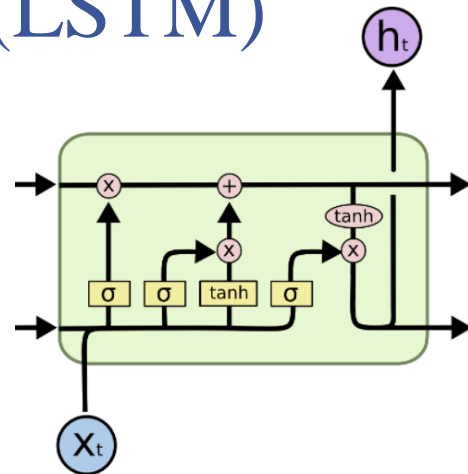
Long Short-Term Memory(LSTM)



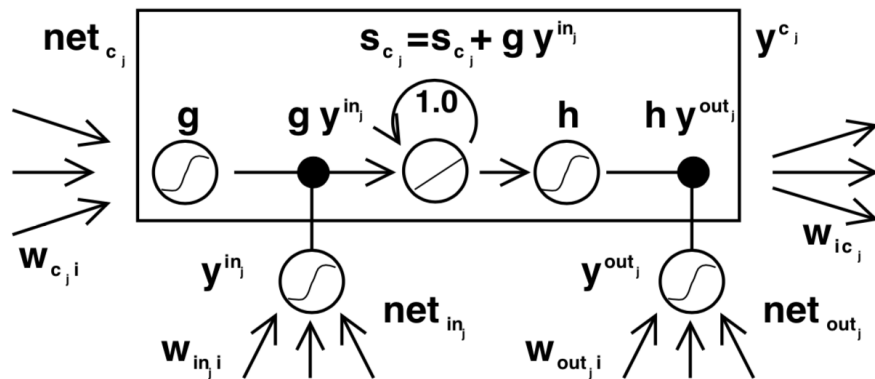
$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

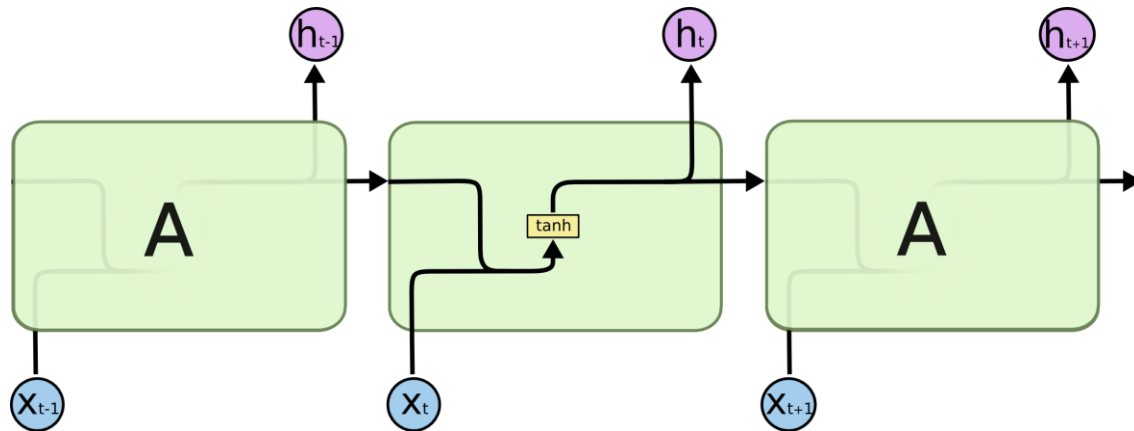


Activation function f is usually a sigmoid function
Between 0 and 1
Mimic open and close gate

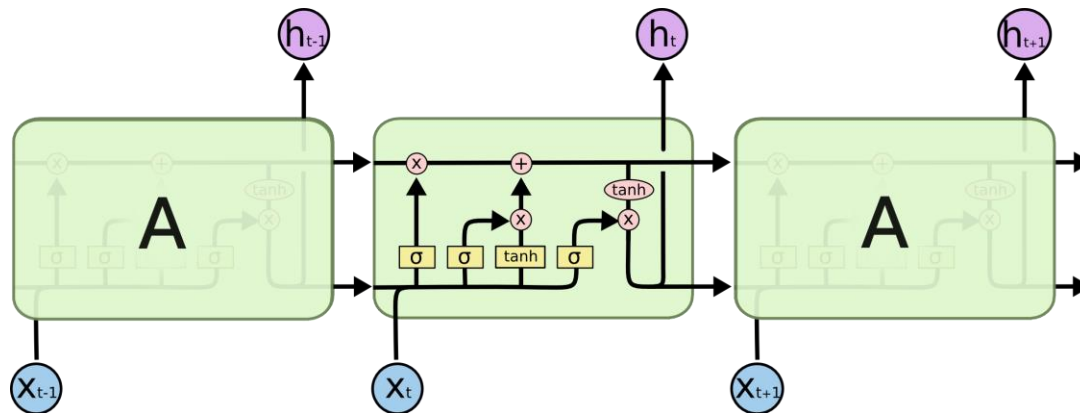


Long Short-Term Memory(LSTM)

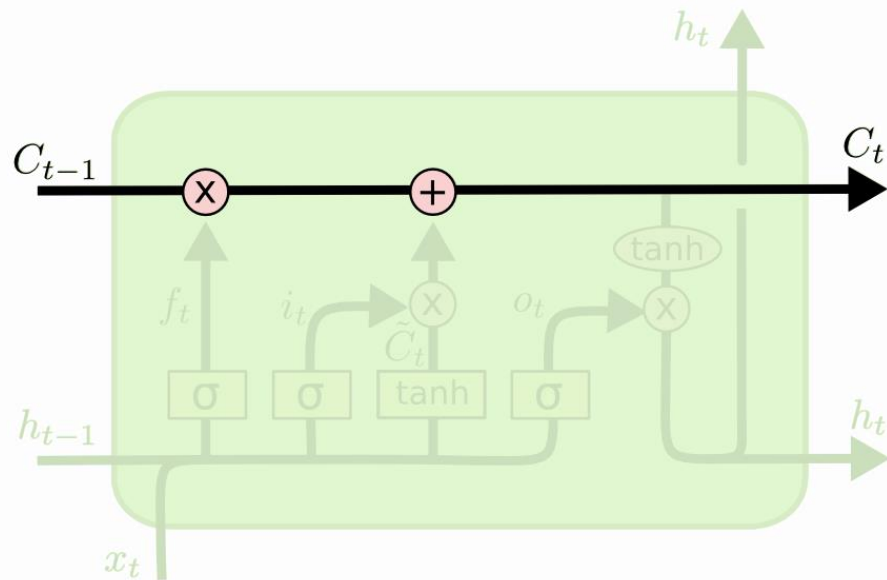
RNN:



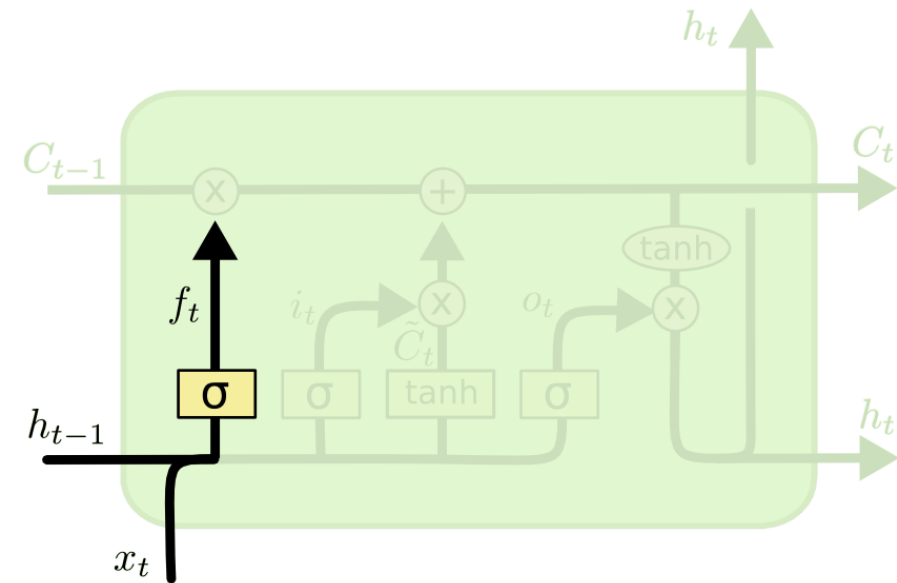
LSTM:



Long Short-Term Memory(LSTM)

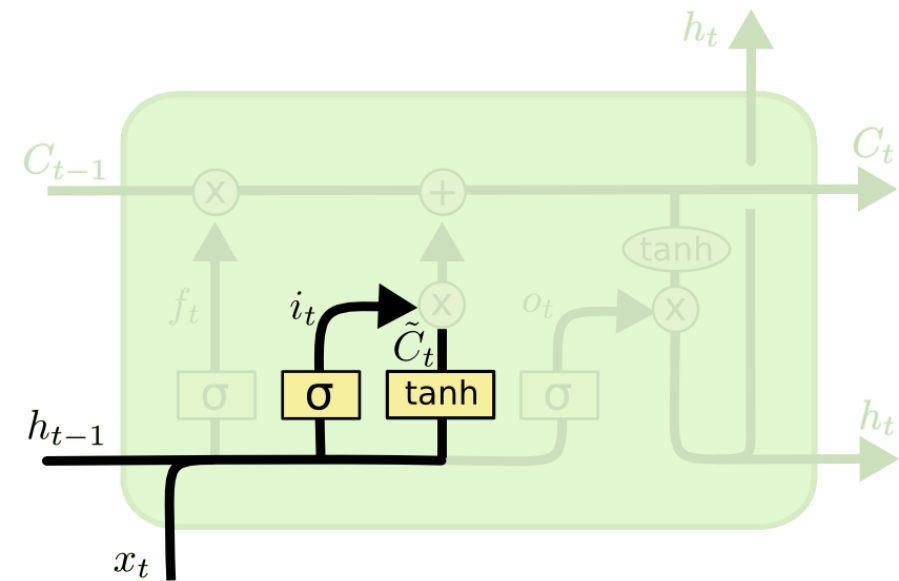


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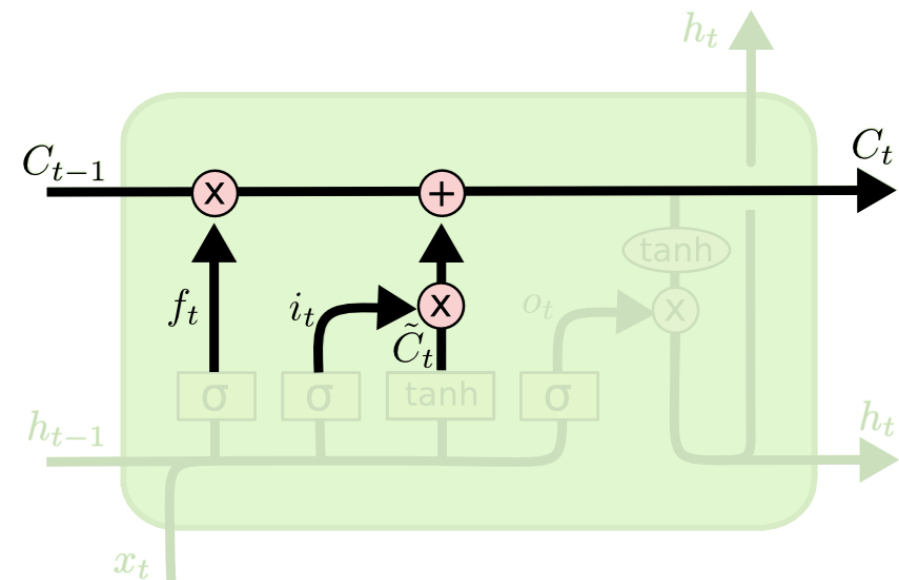
$$f_t = \sigma (W_f \cdot [h_{t-1}, x_t] + b_f)$$

Long Short-Term Memory(LSTM)



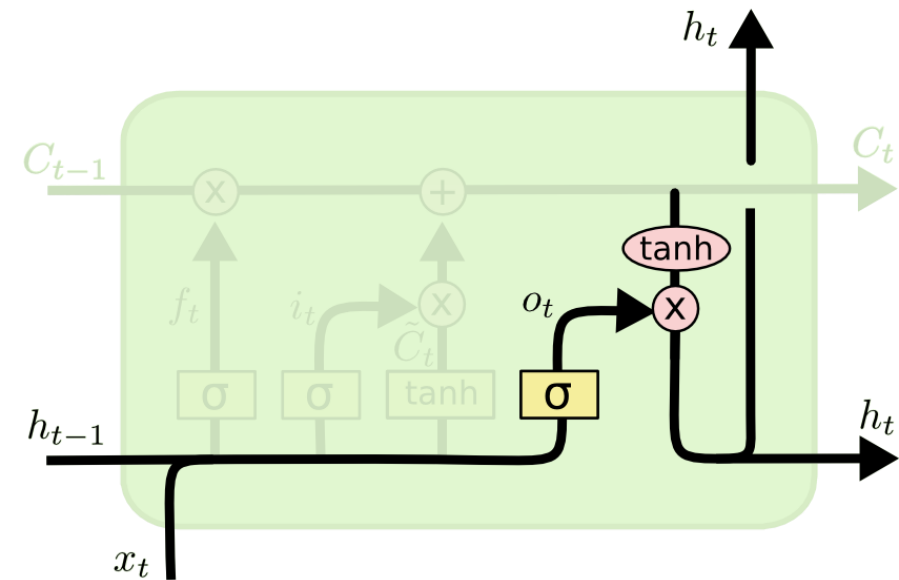
$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

Long Short-Term Memory(LSTM)



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

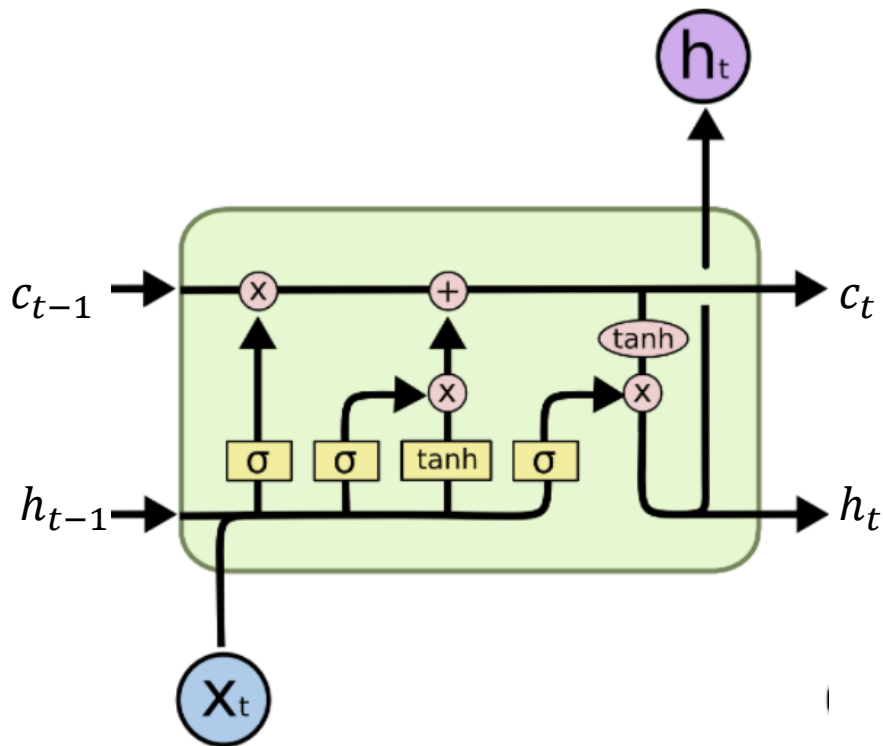
Long Short-Term Memory(LSTM)



$$o_t = \sigma (W_o [h_{t-1}, x_t] + b_o)$$

$$h_t = o_t * \tanh (C_t)$$

Long Short-Term Memory(LSTM)



$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f) \quad (1)$$

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i) \quad (2)$$

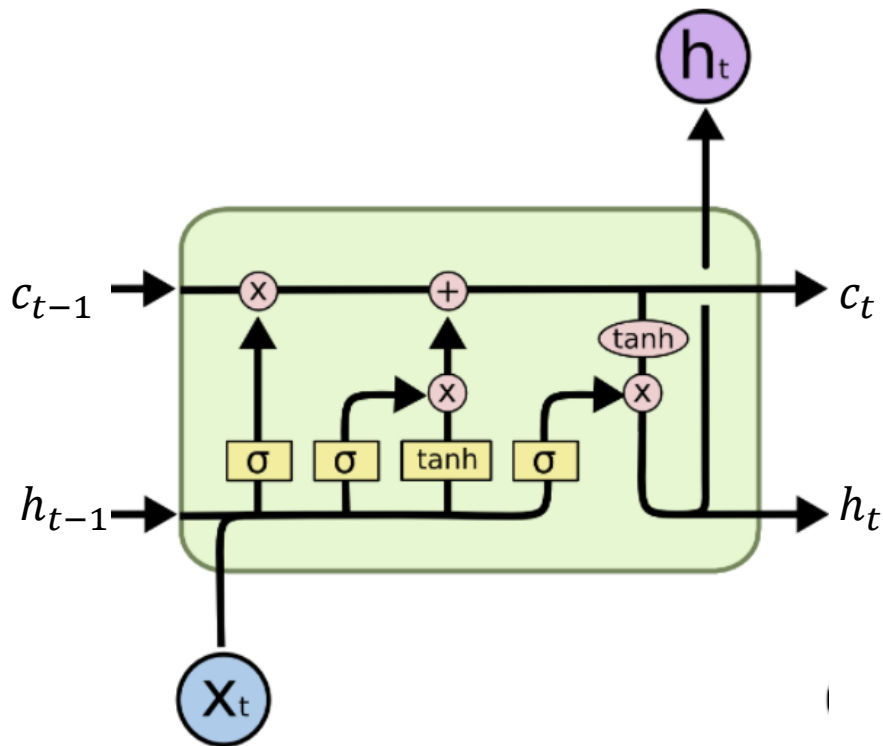
$$\tilde{c}_t = \tanh(W_c \cdot [h_{t-1}, x_t] + b_c) \quad (3)$$

$$c_t = f_t \odot c_{t-1} + i_t \odot \tilde{c}_t \quad (4)$$

$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o) \quad (5)$$

$$h_t = o_t \odot \tanh(c_t) \quad (6)$$

Long Short-Term Memory(LSTM)



$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f) \quad (1)$$

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i) \quad (2)$$

$$\tilde{c}_t = \tanh(W_c \cdot [h_{t-1}, x_t] + b_c) \quad (3)$$

$$c_t = f_t \odot c_{t-1} + i_t \odot \tilde{c}_t \quad (4)$$

$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o) \quad (5)$$

$$h_t = o_t \odot \tanh(c_t) \quad (6)$$

$$\delta_t^{(L)} = \frac{\partial Cost}{\partial logits_t^{(L)}}$$

- 词嵌入表示
 - Efficient Estimation of Word Representation in Vector Space (2013)
- 什么是词嵌入？

一直以来，我们都将数据以one-hot向量的形式输入

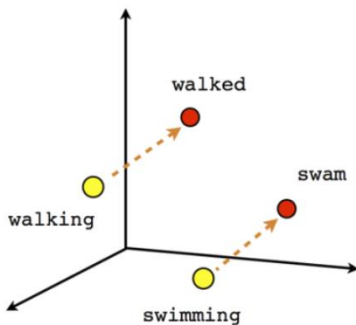
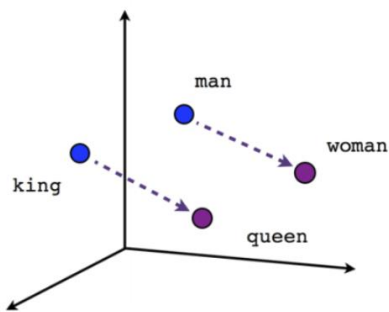
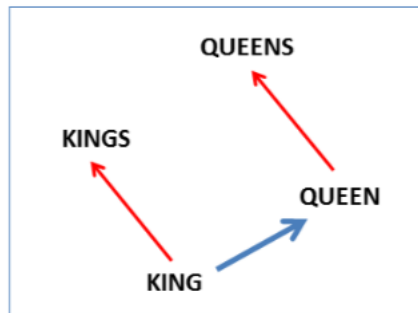
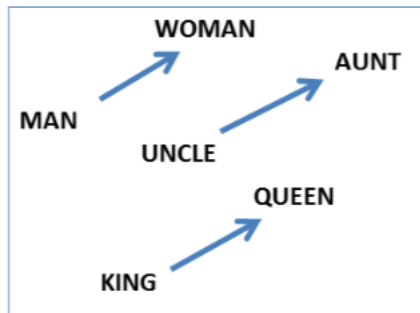
$$\begin{aligned} \text{我} &= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \text{爱} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \text{学} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \text{习} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} & \text{深度} = \begin{bmatrix} 0 \\ \cdots \\ 1 \\ \cdots \\ 0 \end{bmatrix}, \text{学习} = \begin{bmatrix} 0 \\ \cdots \\ 1 \\ \cdots \\ 0 \end{bmatrix} \end{aligned}$$

- 计算量大
- 词之间的关联无法体现

$$\text{深度} = \begin{bmatrix} 0.92 \\ 0.14 \\ 0.37 \\ 0.88 \\ -0.03 \end{bmatrix}, \text{学习} = \begin{bmatrix} 0.11 \\ 0.52 \\ -0.82 \\ 0.004 \\ 0.02 \end{bmatrix}$$

Word Embedding

- 词向量在语义上的优势



- Word2Vec训练
 - Skip-gram
 - CBOW
- 使用最简单的神经网络
- 看起来好像是在预测和输入词关联的词
- 但是我们需要的其实只是权重矩阵本身
 - 例如：输入数据：“我爱学习爱劳动，是个好青年”
→ 我爱学习爱劳动是个好青年
→ (我,爱), (我,学), (爱,我), (爱,学), (爱,习), (学,我), ...

$$\text{我} = \begin{bmatrix} 1 \\ 0 \\ \dots \\ 0 \end{bmatrix}^T \cdot \begin{bmatrix} 0.75 & 0.82 \\ 0.43 & -0.81 \\ \dots & \dots \\ -0.2 & 0.9 \end{bmatrix} = [0.75 \quad 0.82]$$



Attention Model

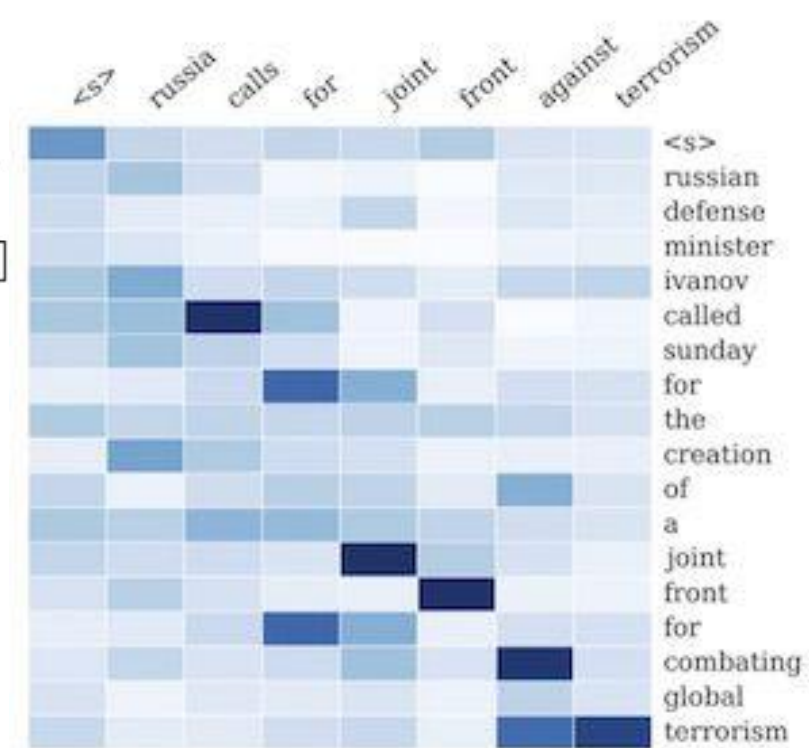
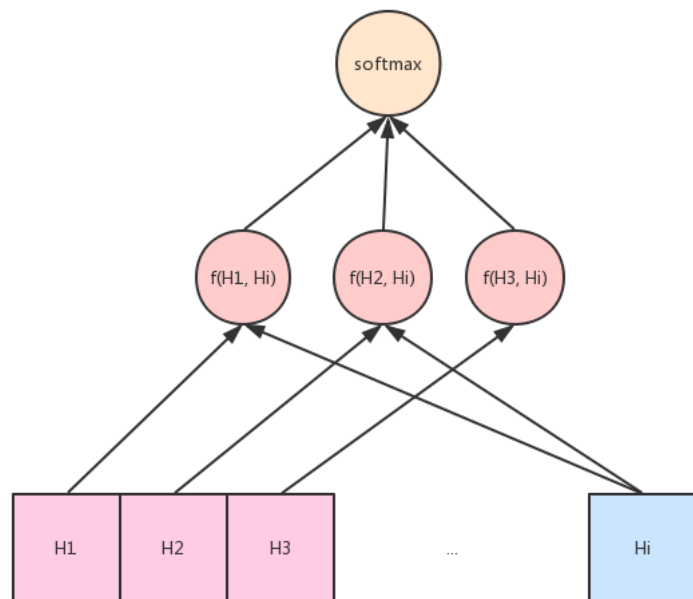
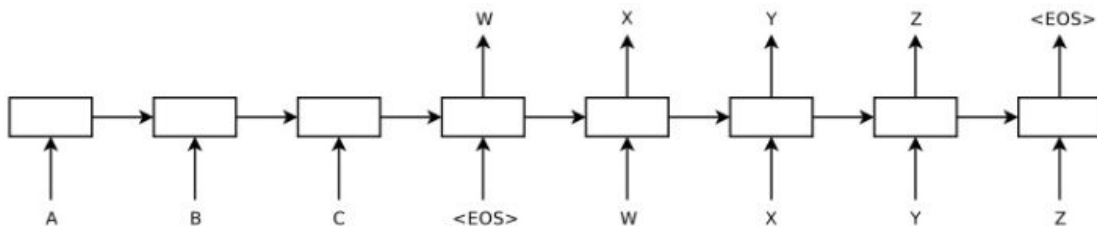


Figure 1: Example output of the attention-based summarization (ABS) system. The heatmap represents a soft alignment between the input (right) and the generated summary (top). The columns represent the distribution over the input after generating each word.

THANK YOU



AI100