Planck'd 2025 Hackathon Report

Quantum Algorithms Track

Team Name: placeholder

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Abstract

This report details my solutions for the "Reaching a Goal (Qu)bit by (Qu)bit" challenge. We explore the transition from classical random walks to quantum walks, demonstrating the effects of superposition and quantum interference on RMS displacement. We successfully simulated a classical diffusive walk (RMS Displacement $\propto \sqrt{t}$) and a discrete-time quantum walk (RMS Displacement $\propto t$), confirming a quadratic increase in the RMS Displacement, i.e, in the spreading characteristic.

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1 Problem 0: The Classical Random Walk

1.1 Theory and Approach

The classical random walk is a fundamental stochastic process. In this problem, we modeled "Bob" moving on a 1D integer line starting at $x = 0$. At each time step t, Bob moves one unit forward or backward with equal probability ($P = 0.5$).

To analyze the spreading behavior statistically, we had to simulate a larger number of walks instead of just one to cancel the noise. The Root-Mean-Square (RMS) displacement at time t is calculated as:

$$RMS(t) = \sqrt{\langle x(t)^2 \rangle} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} x_i(t)^2} \tag{1}$$

1.2 Implementation

We implemented the simulation using Python and NumPy. We utilized the NumPy library to simulate all 10,000 walks simultaneously for 100 time steps. A cumulative sum (‘np.cumsum‘) along the time axis was used to efficiently track the position history of every walk.

1.3 Results and Analysis

The resulting RMS displacement is plotted below against the number of steps t.

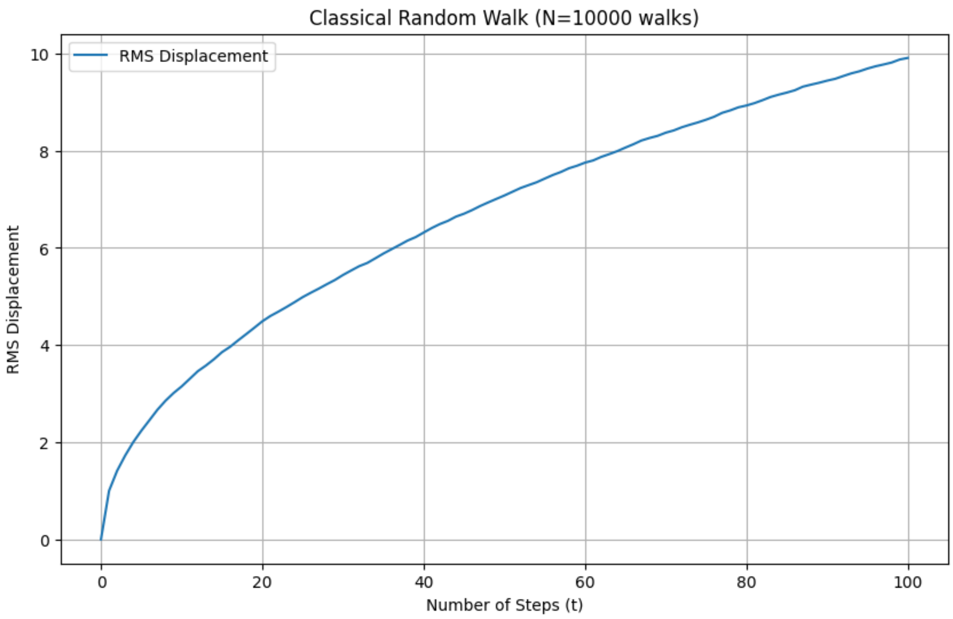

Figure 1: RMS displacement of 10,000 classical walkers over 100 steps. The curve clearly follows a $\sqrt{t}$ trend.

As seen in Figure 1, the RMS displacement grows non-linearly. The curve closely matches the theoretical prediction of $\sigma \propto \sqrt{t}$, characteristic of a classical diffusive walk. This is due to the fact that random steps often cancel each other out, limiting the rate of spread.

2 Problem 1: A Quantum Coin Flip

2.1 Theory and Approach

In this problem, Bob replaces his classical coin with a qubit, using a Pauli-X gate as the coin operation in place of the classical coin flip.

$$X(|0\rangle) = |1\rangle\,, \quad X(|1\rangle) = |0\rangle \tag{2}$$

Bob starts in the state $|x = 0\rangle \otimes |0\rangle$. He applies the X gate, then moves left if the coin is $|0\rangle$ and right if the coin is $|1\rangle$.

2.2 Results

Simulating this process for 5 steps yields the following deterministic path:

Step 0: $x = 0$, Coin = $|0\rangle$
Step 1: $x = 1$, Coin = $|1\rangle$
Step 2: $x = 0$, Coin = $|0\rangle$
Step 3: $x = 1$, Coin = $|1\rangle$
Step 4: $x = 0$, Coin = $|0\rangle$
Step 5: $x = 1$, Coin = $|1\rangle$

2.3 Analysis

This path is a simple, deterministic oscillation between positions x = 0 and x = 1. While it uses a quantum gate (Pauli-X), it does not exhibit any "true" quantum behavior because no superposition is created. The walker is always at a single, definite position. This serves as a baseline to contrast with the true quantum walk in Problem 2.

3 Problem 2: The Superposed Walker

3.1 Theory: Discrete-Time One-Dimensional Quantum Walk

By replacing the X gate with a Hadamard (H) gate, we introduce superposition. The coin state evolves as:

$$H(|0\rangle) = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad H(|1\rangle) = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \tag{3}$$

This causes the walker's state to "split" into multiple paths simultaneously. These paths can then meet at the same position and undergo quantum interference (constructive or destructive), drastically changing the spreading behavior. On expanding the state expression at each step, we notice that the end terms always propagate the walk to a new position.

3.2 Implementation

We implemented a hybrid simulation:

- Position Space: A classical NumPy matrix storing amplitudes for both coin states at every possible position x.
- Coin Operator: We used Qiskit's 'Operator' class to apply the Hadamard matrix to the amplitudes at each position.
- Shift Operator: Implemented via array indexing, moving amplitudes for $|0\rangle$ left ($i \to i-1$) and $|1\rangle$ right ($i \to i+1$).

3.3 Results: Position Amplitudes

The simulation tracked the exact complex amplitudes for the first three steps.

Step (t)	**Position (x)**	**Non-Zero Amplitudes**
1	-1	$\|0\rangle : 0.7071$
	+1	$\|1\rangle : 0.7071$
2	-2	$\|0\rangle : 0.5000$
	0	$\|0\rangle : 0.5000$, $\|1\rangle : 0.5000$
	+2	$\|1\rangle : -0.5000$
3	-3	$\|0\rangle : 0.3536$
	-1	$\|0\rangle : 0.7071$, $\|1\rangle : 0.3536$
	+1	$\|0\rangle : -0.3536$
	+3	$\|1\rangle : 0.3536$

Table 1: Evolution of position amplitudes for the first 3 steps. Note the destructive interference that would occur at $t = 3, x = +1$ for the $|1\rangle$ coin state.

3.4 Results: RMS Displacement

We calculated the RMS displacement for the quantum walk over 100 steps and plotted it.

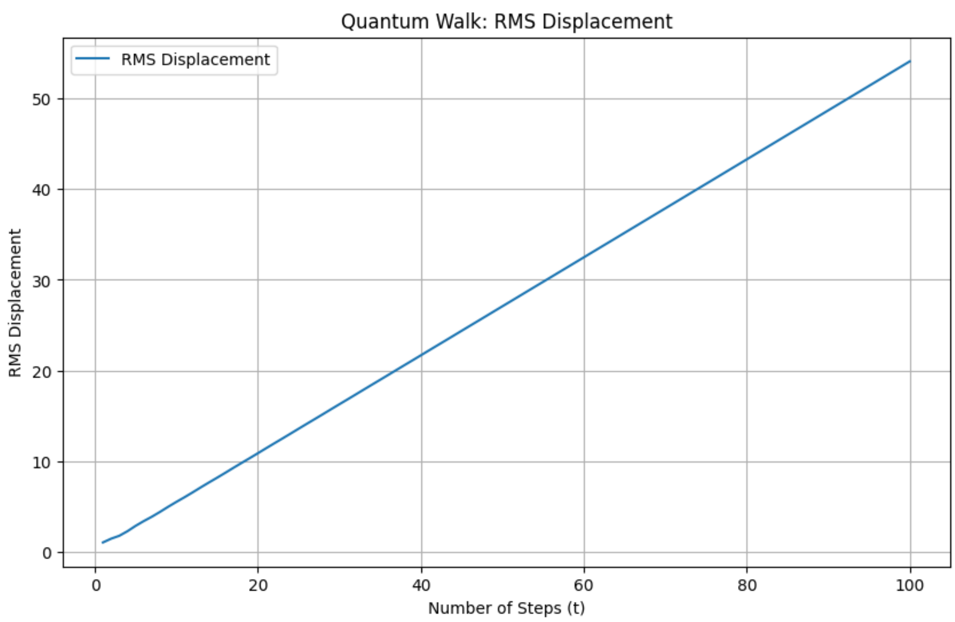

Figure 2: RMS Displacement of quantum walk over 100 steps following a linear trend.

3.5 Analysis

The plot in Figure 2 shows a dramatic difference. While the classical walk spreads diffusively ($\propto \sqrt{t}$), the quantum walk spreads much faster, with RMS displacement growing linearly ($\propto t$).

This increase is due to the combined effect of two principles: Superposition and Interference.

The Hadamard "coin" operator places the walker in a superposition of moving left and right simultaneously. This allows the "edges" of the state's wavefunction to *always* propagate outwards at the maximum possible speed (one step per time unit).

The Hadamard gate's

$$H(|1\rangle) = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \tag{4}$$

term introduces negative amplitudes. Near the origin, paths recombine with opposite phases and cancel each other out(destructive interference). This decreases the probability of the walker remaining near the center.

At the "edges" of the walk, paths recombine with the same phase, increasing the probability at these extreme positions(constructive interference).