Sparse associative memories

L

number of neurons per cluster l
number of clusters c
number of erased clusters c_e
number of messages : m
number of activities per cluster : a
density : d
edge : (i, j)
probability for i and j to be active in their respective clusters for one message : $\frac{a}{l}$
probability of the edge not being added in the network for one message : $1 - \left(\frac{a}{l}\right)^2$

$$d = 1 - \left(1 - \left(\frac{a}{l}\right)^2\right)^m$$

probability for a vertex v to achieve the maximum score, that is to mean $a(c - c_e)$: $d^{a(c-c_e)}$
probability for a vertex v not to achieve the maximum : $1 - d^{a(c-c_e)}$
probability for none of the vertices of one erased cluster, excepting the correct ones, to achieve the maximum : $\left(1 - d^{a(c-c_e)}\right)^{l-a}$
probabilities for none of the vertices in any erased cluster, excepting the correct ones, to achieve the maximum in their respective cluster : $\left(1 - d^{a(c-c_e)}\right)^{c_e(l-a)}$
Whence error rate is :

$$P_{err} = 1 - \left(1 - d^{a(c-c_e)}\right)^{c_e(l-a)}$$

The analytical result agree with the simulations.
For one iteration, winner takes all is the same as a-winners take all.
For multiple iterations a-winner take all is an improvement.
"a-Winners take all" better than "winner takes all" with errors instead of erasures.