Storing messages with neural multipartite cliques

L

number of neurons per cluster l
number of clusters c
number of erased clusters c_e
number of messages : m
number of activities per cluster : a
density : d
edge : (i, j)
probability for i and j to be active in their respective clusters for one message : $\frac{a}{l}$
probability of the edge not being added in the network for one message : $1 - \left(\frac{a}{l}\right)^2$

$$d = 1 - \left(1 - \left(\frac{a}{l}\right)^2\right)^m$$

gap with density in simulations : under 1 percent
probability for a vertex v to achieve the maximum score, that is to mean $a(c - c_e)$: $d^{a(c-c_e)}$
probability for a vertex v not to achieve the maximum : $1 - d^{a(c-c_e)}$
probability for none of the vertices of one erased cluster, excepting the correct ones, to achieve the maximum : $\left(1 - d^{a(c-c_e)}\right)^{l-a}$
probability for none of the vertices in any erased cluster, excepting the correct ones, to achieve the maximum in their respective cluster : $\left(1 - d^{a(c-c_e)}\right)^{c_e(l-a)}$
Whence error rate is :

$$P_{err} = 1 - \left(1 - d^{a(c-c_e)}\right)^{c_e(l-a)}$$

The simulations agree with the analytical result.

For one iteration (for erasures, not for errors), winner takes all is the same as a-winners take all.

For multiple iterations a-winner take all is an improvement.

"a-Winners take all" better than "winner takes all" with errors instead of erasures.

For errors : only better than only 1 activity per cluster if multiple iterations of the a-winners take all rule. For one iteration, it is less efficient.

Spéculations en Français :

Peut-être un intérêt pour cluter based associative memories build from unreliable storage : si une arête porte moins d'info, on peut peut-être en supprimer plus (mais rajout ?!)

Marche un petit peu, mais pas forcément significatif : à tester plus en détail.

Marche mieux pour des bruits importants

$P(n_{v_c} = n_0) = \binom{ac_k}{n_0}(1-\psi)^{n_0}\psi^{ac_k - n_0}$

$P_+ = \psi(1-d) + (1-\psi)d$

$P(n_v = x) = \binom{ac_k}{x} P_+^x (1-P_+)^{ac_k - x}$

$$P(\text{no other vertex activated in this cluster}) = \sum_{n_0=1}^{ac_k} P(n_{v_c} \geqslant n_0)^a \left[\sum_{x=0}^{n_0-1} P(n_v = x)\right]^{l-a}$$

$$P(\text{no other vertex activated in any cluster}) = \left(\sum_{n_0=1}^{ac_k} P(n_{v_c} \geqslant n_0)^a \left[\sum_{x=0}^{n_0-1} P(n_v = x)\right]^{l-a}\right)^{c_e}$$

Sans tenir compte du choix au hasard si plus sont activés.

References

[LPGRG14] François Leduc-Primeau, Vincent Gripon, Michael Rabbat, and Warren Gross. Cluster-based associative memories built from unreliable storage. In *ICASSP*, May 2014. To appear.