

# Sparse associative memories

L

number of neurons per cluster  $l$   
 number of clusters  $c$   
 number of erased clusters  $c_e$   
 number of messages :  $m$   
 number of activities per cluster :  $a$   
 density :  $d$   
 edge :  $(i, j)$   
 probability for  $i$  and  $j$  to be active in their respective clusters for one message :  $\frac{a}{l}$   
 probability of the edge not being added in the network for one message :  $1 - \left(\frac{a}{l}\right)^2$

$$d = 1 - \left(1 - \left(\frac{a}{l}\right)^2\right)^m$$

gap with density in simulations : under 1 percent  
 probability for a vertex  $v$  to achieve the maximum score, that is to mean  $a(c - c_e)$  :  $d^{a(c-c_e)}$   
 probability for a vertex  $v$  not to achieve the maximum :  $1 - d^{a(c-c_e)}$   
 probability for none of the vertices of one erased cluster, excepting the correct ones, to achieve the maximum :  $(1 - d^{a(c-c_e)})^{l-a}$   
 probabilities for none of the vertices in any erased cluster, excepting the correct ones, to achieve the maximum in their respective cluster :  $(1 - d^{a(c-c_e)})^{c_e(l-a)}$   
 Whence error rate is :

$$P_{err} = 1 - \left(1 - d^{a(c-c_e)}\right)^{c_e(l-a)}$$

The simulations agree with the analytical result.

For one iteration (for erasures, not for errors), winner takes all is the same as  $a$ -winners take all.

For multiple iterations  $a$ -winner take all is an improvement.

" $a$ -Winners take all" better than "winner takes all" with errors instead of erasures.

For errors : only better than only 1 activity per cluster if multiple iterations of the  $a$ -winners take all rule.

Spéculations en Français :

Peut-être un intérêt pour cluter based associative memories build from unreliable storage : si une arête porte moins d'info, on peut peut-être en supprimer plus (mais rajout ?!)

Marche un petit peu, mais pas forcément significatif : à tester plus en détail