# Storing messages with multipartite neural cliques

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## Abstract

We extend recently introduced associative memories based on cliques to multipartite cliques. We propose a variant of the classic retrieving rule. We study their relative performance for retrieving partially erased and corrupted messages. We provide both analytical and simulation results showing improvements of capacity over.

**Index terms**— associative memory, error correcting code, cliques, multipartite cliques, neural networks

# I Introduction

# II Networks of neural cliques

## 1 Learning messages

## 2 Retrieving messages

### a The "winner takes all" rule

### b The "$a$ winners take all" rule

less biologically plausible rule

same algorithmic complexity

# III Retrieval performance

## 1 Retrieving partially erased messages

### a Analytical result

number of neurons per cluster $l$

number of clusters $c$

number of erased clusters $c_e$

number of messages : $m$

number of activities per cluster : $a$

density : $d$

edge : $(i, j)$

probability for $i$ and $j$ to be active in their respective clusters for one message : $\frac{a}{l}$

probability of the edge not being added in the network for one message : $1 - \left(\frac{a}{l}\right)^2$

$$d = 1 - \left(1 - \left(\frac{a}{l}\right)^2\right)^m$$

gap with density in simulations : under 1 percent

probability for a vertex $v$ to achieve the maximum score, that is to mean $a(c - c_e)$ : $d^{a(c-c_e)}$

probability for a vertex $v$ not to achieve the maximum : $1 - d^{a(c-c_e)}$

probability for none of the vertices of one erased cluster, excepting the correct ones, to achieve the maximum : $\left(1-d^{a(c-c_e)}\right)^{l-a}$

probability for none of the vertices in any erased cluster, excepting the correct ones, to achieve the maximum in their respective cluster : $\left(1-d^{a(c-c_e)}\right)^{c_e(l-a)}$

Whence error rate is :

$$P_{err} = 1 - \left(1-d^{a(c-c_e)}\right)^{c_e(l-a)}$$

$\eta_m = (P_{retrieve} m) \frac{2\left(c\log_2\binom{l}{a} - \log_2(m)+1\right)}{c(c-1)l^2} \times \frac{c}{(c-c_e)\log_2\binom{l}{a}}$

$\eta_{max} = \max_x \eta_x$

## b Simulations

Improves the retrieval rate for multiple iterations and/or corrupted messages.

The simulations agree with the analytical results for density and error rate.

For one iteration (for erasures, not for errors), winner takes all is the same as $a$-winners take all.

For multiple iterations $a$-winners take all is a significant improvement.

"$a$-winners take all" better than "winner takes all" with errors instead of erasures, even for one iteration.

For errors : only better than only 1 activity per cluster if multiple iterations of the $a$-winners take all rule. For one iteration, it is less efficient.

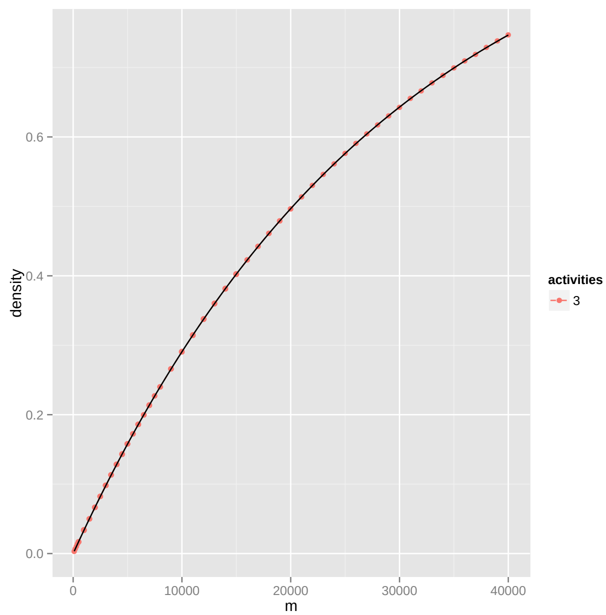


Figure 1: Theoretical and empirical densities

a

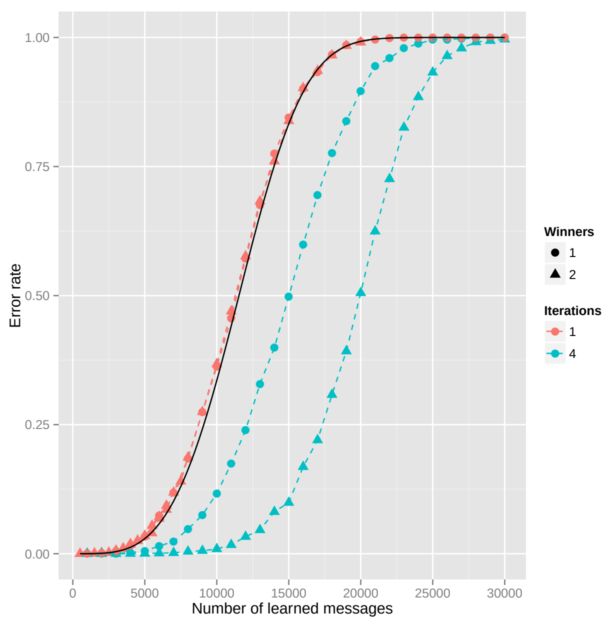


Figure 2: 2 activities per cluster, 4 clusters, 512 neurons per cluster, two erasures, each point is the mean of 5 networks with 1000 sampled messages, analytical result in continuous black

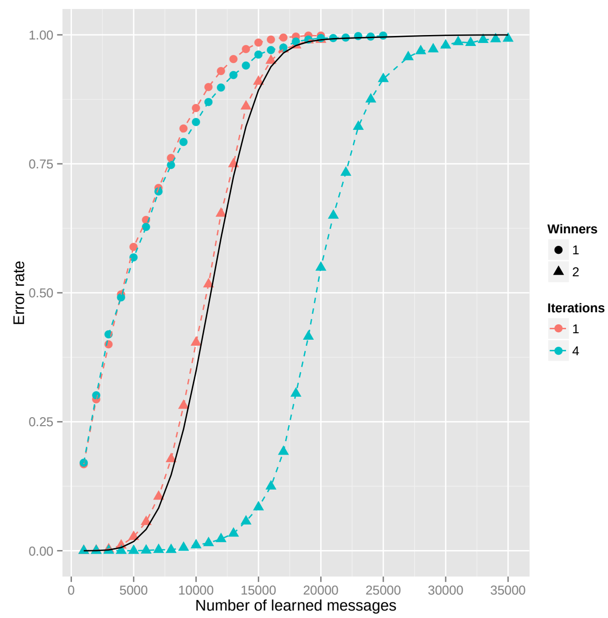


Figure 3: 2 activities per cluster, 4 clusters, 512 neurons per cluster, one erroneous cluster, each point is the mean of 5 networks with 1000 sampled messages, analytical result in continuous black

## 2 Retrieving corrupted messages

Spéculations en Français :

Peut-être un intérêt pour cluter based associative memories build from unreliable storage : si une arête porte moins d'info, on peut peut-être en supprimer plus (mais rajout ?!)

Marche mieux pour des bruits importants

$P(n_{v_c} = n_0) = \binom{ac_k}{n_0}(1-\psi)^{n_0}\psi^{ac_k - n_0}$

$P_+ = \psi(1-d) + (1-\psi)d$

$P(n_v = x) = \binom{ac_k}{x}P_+^x(1-P_+)^{ac_k - x}$

Sans tenir compte du choix au hasard si plus sont activés.

a priori for noise on edges :

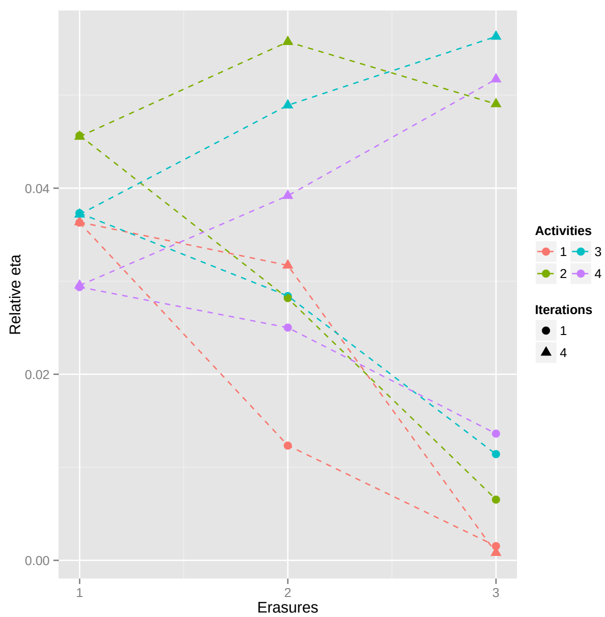


Figure 4: Comparison between different number of activities per cluster : 4 clusters, 512 neurons per cluster, 1000 sampled messages

$P(\text{no other vertex activated in this cluster}) =$ a
$\sum_{n=1}^{a(c-c_e)} \left[\sum_{k=1}^{a} \binom{a}{k} P(n_{v_c}=n)^k P(n_{v_c}>n)^{a-k}\right] \left[\sum_{x=0}^{n-1} P(n_v=x)\right]^{l-a}$

$P(\text{no other vertex activated in any cluster}) =$
$\left(\sum_{n=1}^{a(c-c_e)} \left[\sum_{k=1}^{a} \binom{a}{k} P(n_{v_c}=n)^k P(n_{v_c}>n)^{a-k}\right] \left[\sum_{x=0}^{n-1} P(n_v=x)\right]^{l-a}\right)^{c_e}$

Formula for errors instead of erasures :

gc : good (à remplacer par right) neuron in correct cluster

ge : good neuron in erroneous cluster

abe : bad activated neuron in erroneous cluster

be : others bad (à remplacer par wrong ou incorrect etc.) neurons in erroneous cluster

bc : bad neuron in correct cluster

$P(n_{gc} = (c-c_e-1)+\gamma+x) = \binom{c_e}{x} d^x (1-d)^{ce-x}$

$P(n_{ge} = (c-c_e)+x) = \binom{c_e-1}{x} d^x (1-d)^{c_e-1-x}$

$P(n_{abe} = \gamma+x) = \binom{c-1}{x} d^x (1-d)^{c-1-x}$

$P(n_{be} = x) = P(n_{bc} = x) = \binom{c-1}{x} d^x (1-d)^{c-1-x}$

$$P_{retrieve} = \left[\sum_{n=1}^{c-1+\gamma} P(n_{gc} = n) \left[\sum_{x=0}^{n-1} P(n_{bc} = x)\right]^{l-1}\right]^{c-c_e}$$
$$\times \left[\sum_{n=1}^{c-1+\gamma} P(n_{ge} = n) \left[\sum_{x=0}^{n-1} P(n_{bc} = x)\right]^{l-2} \left[\sum_{x=0}^{n-1} P(n_{abc} = x)\right]\right]^{c_e}$$

generalisation to multiple activities per cluster :

$P(n_{gc} = a(c-c_e-1)+\gamma+x) = \binom{ac_e}{x} d^x (1-d)^{ace-x}$

$P(n_{ge} = a(c-c_e)+x) = \binom{a(c_e-1)}{x} d^x (1-d)^{a(c_e-1)-x}$

$P(n_{abe} = \gamma+x) = \binom{a(c-1)}{x} d^x (1-d)^{a(c-1)-x}$

$P(n_{be} = x) = P(n_{bc} = x) = \binom{a(c-1)}{x} d^x (1-d)^{a(c-1)-x}$

$P_{retrieve} = \left[\sum_{n=1}^{a(c-1)+\gamma} \left[\sum_{k=1}^{a} \binom{a}{k} P(n_{gc} = n)^k P(n_{gc} > n)^{a-k}\right] P(n_{bc} < n)^{l-a}\right]^{c-c_e}$
$\times \left[\sum_{n=1}^{a(c-1)+\gamma} \left[\sum_{k=1}^{a} \binom{a}{k} P(n_{ge} = n)^k P(n_{ge} > n)^{a-k}\right] P(n_{bc} < n)^{l-2a} P(n_{abc} < n)^a\right]^{c_e}$

$P_{err} = 1 - P_{retrieve}$

# IV Conclusion

## References

[LPGRG14] François Leduc-Primeau, Vincent Gripon, Michael Rabbat, and Warren Gross. Cluster-based associative memories built from unreliable storage. In *ICASSP*, May 2014. To appear.