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Modeling of a Variable Speed Wind Turbine with a Permanent Magnet Synchronous Generator

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Abstract.- **The aim of this work is to analyze a typical configuration of a Wind Turbine Generator System (WTGS) equipped with a Variable Speed Generator. Nowadays, doubly-fed induction generators are being widely used on WTGS, although synchronous generators are being extensively utilized too. There are different types of synchronous generators, but the multi-pole Permanent Magnet Synchronous Generator (PMSG) is chosen in order to obtain its model. It offers better performance due to higher efficiency and less maintenance since it does not have rotor current and can be used without a gearbox, which also implies a reduction of the weight of the nacelle and a reduction of costs. Apart from the generator, the analyzed WTGS consists of another three parts: wind speed, wind turbine and drive train. These elements have been modeled and the equations that explain their behavior have been introduced. What is more, the whole WTGS has been implemented in MATLAB/Simulink interface. Moreover, the concept of the Maximum Power Point Tracking (MPPT) has been presented in terms of the adjustment of the generator rotor speed according to instantaneous wind speed.**

I. INTRODUCTION

The utilization of wind energy has a very long tradition. Some historians suggest that wind turbines (windmills) were known over 3000 years ago [1]. Until the early twentieth century wind power was used to provide mechanical power to pump water or to grind grain.

The first wind turbines appeared at the beginning of the last century and technology was improved step by step from the early 1970s. By the end of the 1990s, wind energy has re-emerged as one of the most important sustainable energy resources, partly because of the increasing price of the oil, security concerns of nuclear power and its environmental issues. Moreover, as wind energy is abundant and it has an inexhaustible potential, it is one of the best technologies today to provide a sustainable electrical energy supply to the world development.

Actually, during the last decade of the twentieth century, worldwide wind capacity doubled approximately every three years. Currently, five countries (Germany, USA, Denmark, India and Spain) concentrate more than 83% of worldwide wind energy capacity in their countries [2]. Studies have shown that by the end of 2003, the total installed capacity of the wind turbines reached 39.234 GW and will exceed 110 GW by the year of 2012 [3].

The need for increased power production from the wind and economic reasons, when the rated power of today's wind turbines is still relatively small (2MW units are now typical), makes it necessary to group wind turbines into so-called wind farms.

Wind farms are built on land, but in recent years there has been (and will probably be in the future) a strong trend towards locating them offshore. The lack of suitable wind turbine sites on land (it is particularly the case of densely populated countries) and the highest wind speeds located near the sea (and consequently higher energy can be extracted from the wind) are the two main reasons for locating wind farms offshore. Horns Rev in Denmark [4] is an example of a current offshore wind farm, which is capable of producing 160 MW.

Both induction and synchronous generators can be used for wind turbine systems [6]. Mainly, three types of induction generators are used in wind power conversion systems: cage rotor, wound rotor with slip control and doubly fed induction rotors. The last one is the most utilized in wind speed generation because it provides a wide range of speed variation. However, the variable-speed directly-driven multi-pole permanent magnet synchronous generator (PMSG) wind architecture is chosen for this purpose and it is going to be modeled: it offers better performance due to higher efficiency and less maintenance because it does not have rotor current. What is more, PMSG can be used without a gearbox, which implies a reduction of the weight of the nacelle and reduction of costs.

The present research article analyzes the model of a variable speed wind turbine equipped with 2MW PMSG. It must be noted that the present research focuses neither on the converter (grid side and rotor side) nor on their controls.

II. SYSTEM DESCRIPTION

The system analyzed is a variable speed wind turbine based on a multi-pole PMSG. Due to the low generator speed, the rotor shaft is coupled directly to the generator, which means that no gearbox is needed. The generator is connected to the grid via an AC/DC/AC converter, which consists of an uncontrolled diode rectifier, an internal DC-Link modeled as a capacitor and a PWM voltage-source inverter.

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A transformer is located between the inverter and the Point of Common Connection (PCC) in order to raise the voltage by avoiding losses in the transport of the current. The layout of the electrical part is depicted in Fig. 1.

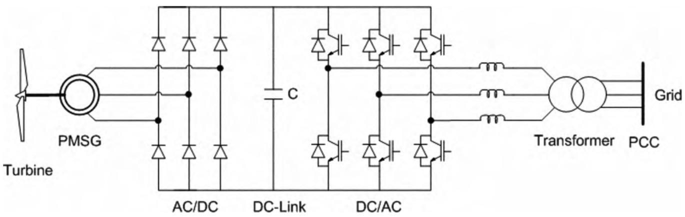

Fig. 1. Electrical scheme of a variable speed wind turbine equipped with a direct-drive PMSG.

It must be noted that this study is dedicated to analyze and implement the model from the wind turbine to the PMSG. For this reason, transformer, grid, rectifier and inverter models (and their controls) will not be considered.

III. Subsystem Models

A. Wind Speed Model

A model is required that can properly simulate the spatial effect of wind behavior, including gusting, rapid (ramp) changes, and background noise. The wind speed is modeled as the sum of the four components listed above [7]

$$v_w(t) = v_b(t) + v_r(t) + v_g(t) + v_n(t) \tag{1}$$

where v_b is the base (constant) wind component, v_r is the ramp wind component, v_g is the gust wind component and v_n is the base noise wind component, all of them in m/s.

Fig. 2. shows the graphics of the non-constant wind speed components: ramp, gust and noise components.

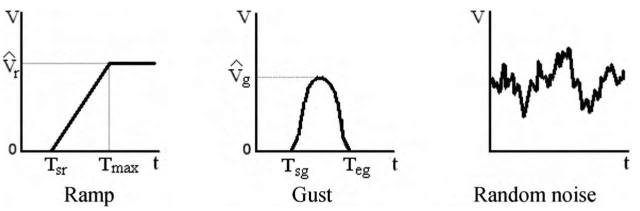

Fig. 2. Non-constant wind speed components.

The present work considers a constant wind speed equal to 12 m/s. Consequently, the model implementation of the wind speed in Simulink implies the consideration of the base wind speed component, as shown in Fig. 3.

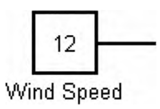

Fig. 3. Wind Speed model with Simulink.

B. Wind Turbine Model

The rotor aerodynamics are presented by the well-known static relations [1], [2], [8], [9]

$$P_w = c_P \frac{1}{2} \rho A v_w^3 \tag{2}$$

where P_w is the power extracted from the wind [W], ρ is the air density, which is equal to 1.225 kg/m^3 at sea level at temperature T = 288 K, c_P is the power coefficient, v_w is the wind speed upstream of the rotor [m/s] and A is the area swept by the rotor [m^2] ($A=\pi R^2$, being R the radius of the blade [m]).

The amount of aerodynamic torque (τ_w) in N·m is given by the ratio between the power extracted from the wind (P_w), in W, and the turbine rotor speed (ω_w), in rad/s, as follows

$$\tau_w = \frac{P_w}{\omega_w} \tag{3}$$

It should be noted that the mechanical torque transmitted to the generator (τ_{w_g}) is the same as the aerodynamic torque, since there is no gearbox. It implies that the gearbox ratio is $n_g = 1$. Therefore $\tau_w = \tau_{w\_g}$.

The power coefficient c_P reaches a maximum value equal to c_P = 0.593 [9] which means that the power extracted from the wind is always less than 59.3% (Betz's limit), because various aerodynamic losses depend on the rotor construction (number and shape of blades, weight, stiffness, etc.). This is the well-known low efficiency to produce electricity from the wind.

The power coefficient can be utilized in the form of look-up tables or in form of a function. The second approach is presented below, where the general function defining the power coefficient as a function of the tip-speed ratio and the blade pitch angle is defined as [8]

$$c_P(\lambda, \vartheta) = c_1\left(c_2 \frac{1}{\beta} - c_3\vartheta - c_4\vartheta^x - c_5\right) e^{-c_6\frac{1}{\beta}} \tag{4}$$

Since this function depends on the wind turbine rotor type, the coefficients c_1-c_6 and x can be different for various turbines. The proposed coefficients [1] are equal to: c_1 = 0.5, c_2 = 116, c_3 = 0.4, c_4 = 0, c_5 = 5, c_6 = 21 (x is not used because c_4 = 0). Additionally, the parameter β is also defined in different ways [1], [2], [8]. For example, the parameter $1/\beta$ in [1] is defined as

$$\frac{1}{\beta} = \frac{1}{\lambda + 0.08\vartheta} - \frac{0.035}{1 + \vartheta^3} \tag{5}$$

where ϑ is the pitch angle [°], which is the angle between the plane of rotation and the blade cross-section chord (Fig. 4), and the tip-speed ratio λ is defined as

$$\lambda = \frac{\omega_w R}{v_w} \tag{6}$$

where ω_w is the angular velocity of rotor [rad/s], R is the rotor radius [m] and v_w is the wind speed upstream of the rotor [m/s].

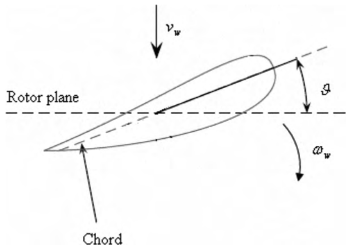

Fig. 4 . Blade pitch angle ϑ .

The $c_p = c_p(\lambda, \vartheta)$ characteristics computed taking into account (12) and (13), the above parameters c_1- c_6 and the wind turbine parameters (table I), for various blade pitch angles ϑ, are presented in Fig. 5.

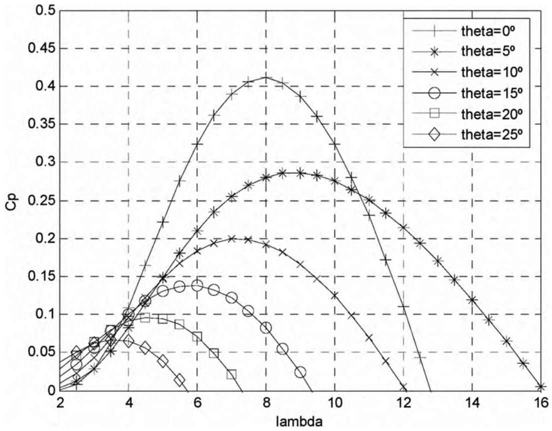

Fig. 5. Analytical approximation of $c_P(\lambda, \vartheta)$ characteristics.

The model of the wind turbine implemented in Simulink is shown in Fig. 6.

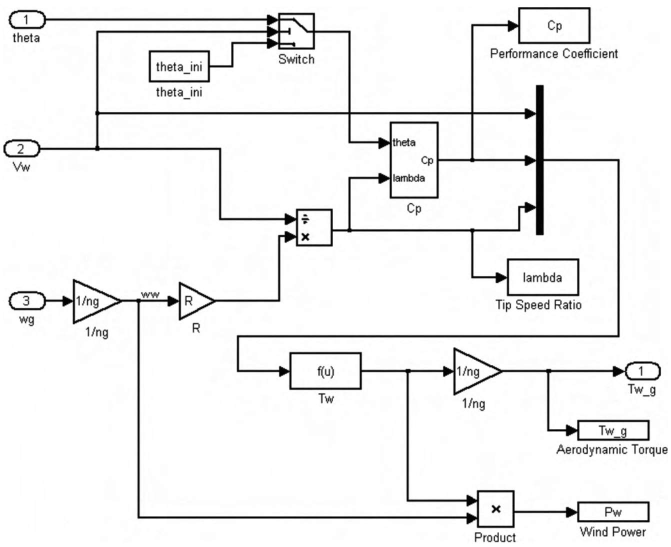

Fig. 6. Wind Turbine modeled with Simulink.

Table I shows the parameters of the analyzed wind turbine.

TABLE I
WIND TURBINE PARAMETERS

Parameter	Symbol	Value and Units
Air density	ρ	1.205 kg/m^3
Rotor radius	R	38 m
Rated wind speed	$V_{w\ rated}$	11.8 m/s
Maximum C_p	$C_{p\ max}$	0.4412

C. Drive Train Model

The drive train of a wind turbine generator system consists of the following elements: a blade-pitching mechanism with a spinner, a hub with blades, a rotor shaft and a gearbox with breaker and generator. It must be noted that gearbox is not considered because the analyzed system consists of a wind turbine equipped with a multi-pole PMSG.

The acceptable (and common) way to model the drive train is to treat the system as a number of discrete masses connected together by springs defined by damping and stiffness coefficients (Fig. 7). Therefore, the equation of ith mass motion can be described as follows [1], [11]:

$$\begin{aligned}\frac{d^2\theta_i}{dt^2} &= \frac{v_i c_i}{J_i}\frac{d\theta_{i-1}}{dt} - \frac{v_{i+1}^2 c_{i+1} + c_i}{J_i}\frac{d\theta_i}{dt} + \frac{v_{i+1}c_{i+1}}{J_i}\frac{d\theta_{i+1}}{dt} + \\ &+ \frac{v_i k_i}{J_i}\theta_{i-1} - \frac{v_{i+1}^2 k_{i+1} + k_i}{J_i}\theta_i + \frac{v_{i+1}k_{i+1}}{J_i}\theta_{i+1} \\ &+ \frac{\tau_i}{J_i} - D_i\frac{d\theta_i}{dt}\end{aligned} \tag{7}$$

where v_i is the transmission rate between i and i-1masses, c_i is the shaft viscosity [kg/(m·s)], k_i is the shaft elastic constant [N/m], J is the moment of inertia of the ith mass [kg·m^2] , τ_i is the external torque [N·m] applied to the ith mass and D_i is the damping coefficient [N·m/s], which represents various damping effects. For the purposes of the present research, neither viscosity nor damping effects have been considered.

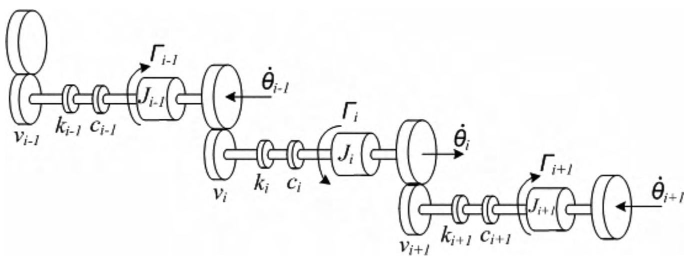

Fig. 7. Transmission model of N masses connected together.

When the complexity of the study varies, the complexity of the drive train differs. For example, when the problems such as torsional fatigue are studied, dynamics from all parts have to be considered. For these purposes, two-lumped mass or more sophisticated models are required. However, when the study focuses on the interaction between wind farms and AC grids, the drive train can be treated as one-lumped mass model for the sake of time efficiency and acceptable precision [5], [11]. The last approximation has been considered in the present study and it is defined by the following equation

$$\frac{d\omega_g}{dt} = \frac{\tau_e - \tau_{w\_g}}{J_{eq}} - \frac{B_m}{J_{eq}} \cdot \omega_g \tag{8}$$

where the sub-index g represents the parameters of the generator side, ω_g is the mechanical angular speed [rad/s] of the generator, B_m is the damping coefficient [N·m/s], τ_e is the electromechanical torque [N·m], τ_{w_g} is the aerodynamic torque that has been transferred to the generator side (3), which is equal to the torque produced in the rotor side because there is no gearbox, and J_{eq} is the equivalent rotational inertia of the generator [kg·m^2], which is derived from [5]

$$J_{eq} = J_g + \frac{J_w}{n_g^2} \tag{9}$$

where J_g and J_w are the generator and the rotor rotational inertias [kg·m^2] respectively, n_g is the gear ratio, which is equal to 1, because no gearbox is utilized.

The model of the one mass drive train implemented in Simulink is depicted in Fig. 8.

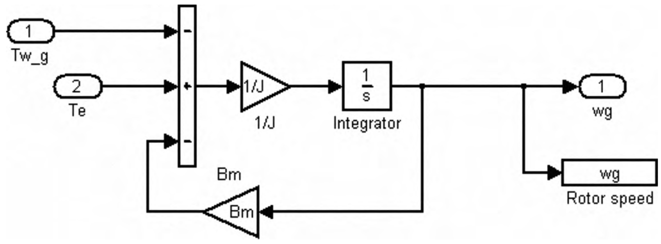

Fig. 8. Drive Train modeled with Simulink.

Table II shows the parameters of the drive train that has been considered.

TABLE II
DRIVE TRAIN PARAMETERS

Parameter	Symbol	Value and Units
Gear ratio	n_g	1
Rotational damping coefficient	B_m	0
Equivalent inertia (turbine+generator)	J_{eq}	0.3 kg·m^2

D. Generator Model

The PMSG has been considered as a system which makes possible to produce electricity from the mechanical energy obtained from the wind.

The dynamic model of the PMSG is derived from the two-phase synchronous reference frame, which the q-axis is 90° ahead of the d-axis with respect to the direction of rotation. The synchronization between the d-q rotating reference frame and the abc-three phase frame is maintained by utilizing a phase locked loop (PLL) [10]. Fig. 9 shows the d-q reference frame used in a salient-pole synchronous machine (which is the same reference as the one used in a PMSG), where θ is the mechanical angle, which is the angle between the rotor d-axis and the stator axis.

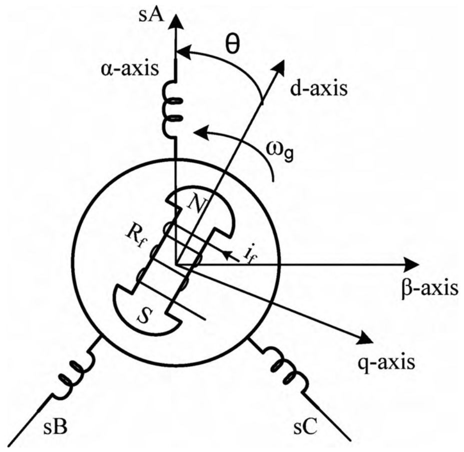

Fig. 9. d-q and α-β axis of a typical salient-pole synchronous machine.

The mathematical model of the PMSG for power system and converter system analysis is usually based on the following assumptions [9], [10]: the stator windings are positioned sinusoidal along the air-gap as far as the mutual effect with the rotor is concerned; the stator slots cause no appreciable variations of the rotor inductances with rotor position; magnetic hysteresis and saturation effects are negligible; the stator winding is symmetrical; damping windings are not considered; the capacitance of all the windings can be neglected and the resistances are constant (this means that power losses are considered constant).

The mathematical model of the PMSG in the synchronous reference frame (in the state equation form) is given by [10], [11]

$$\begin{aligned}\frac{di_d}{dt} &= \frac{1}{L_{ds} + L_{ls}}(-R_s i_d + \omega_e (L_{qs} + L_{ls}) i_q + u_d) \\ \frac{di_q}{dt} &= \frac{1}{L_{qs} + L_{ls}}(-R_s i_q - \omega_e [(L_{ds} + L_{ls}) i_d + \psi_f] + u_q)\end{aligned} \tag{10}$$

where subscripts d and q refer to the physical quantities that have been transformed into the d-q synchronous rotating reference frame, R_s is the stator resistance [Ω], L_d and L_q are the inductances [H] of the generator on the d and q axis, L_{ld} and L_{lq} are the leakage inductances [H] of the generator on the d and q axis, respectively, Ψ_f is the permanent magnetic flux [Wb] and ω_e is the electrical rotating speed [rad/s] of the generator, defined by

$$\omega_e = p\omega_g \tag{11}$$

where p is the number of pole pairs of the generator.

In order to complete the mathematical model of the PMSG the mechanical equation is needed, and it is described by the following electromagnetic torque equation [10]

$$\tau_e = 1.5p((L_{ds} - L_{ls}) i_d i_q + i_q \psi_f) \tag{12}$$

Fig. 10 shows the equivalent circuit of the PMSG in de *d-q* synchronous rotating reference frame.

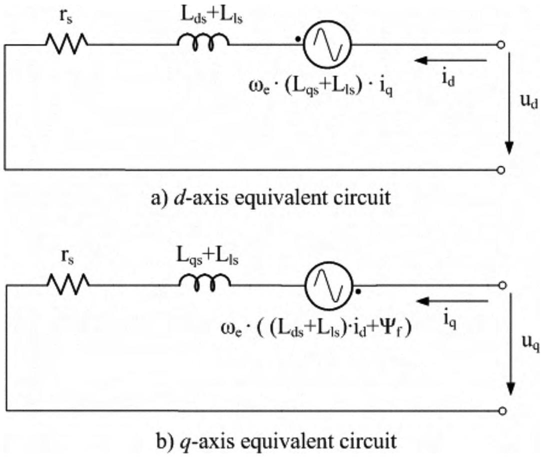

Fig. 10. Equivalent circuit of the PMSG in the synchronous frame.

The model of the PMSG implemented in Simulink is depicted in Fig. 11.

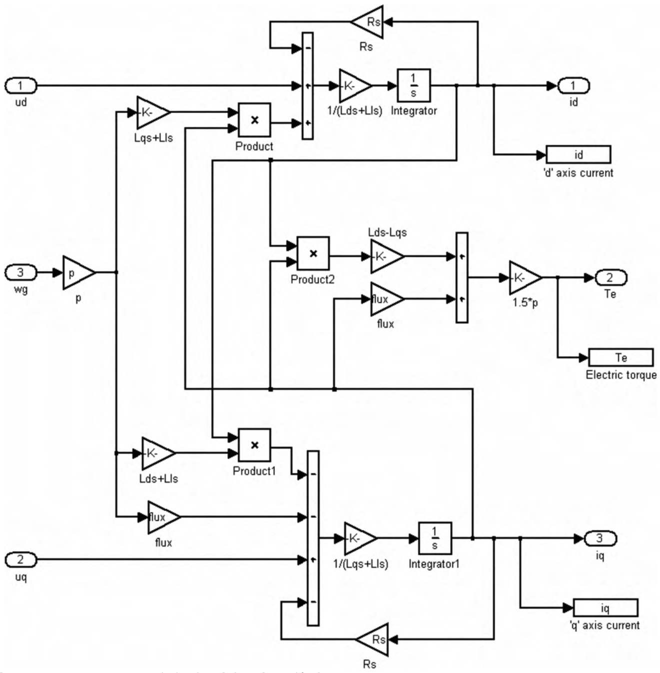

Fig. 11. PMSG modeled with Simulink.

By analyzing the power produced by the wind turbine at various wind and rotor speeds, as depicted in Fig. 12, it can be appreciated that an optimum power coefficient constant $K_{p\ opt}$ exists. This coefficient show the generated power associated with the corresponding optimum rotor speed [1], [2], [8]. $K_{p\ opt}$ is calculated from individual wind turbine characteristics. By measuring generated power, the corresponding optimum rotor speed can be calculated and set as the reference speed according to [1]

$$\omega_{r\ opt} = \sqrt[3]{\frac{P_{gen}}{K_{p\ opt}}} \tag{13}$$

where $\omega_{r\ opt}$ is the optimum rotor speed [rad/s] and P_{gen} is the measured generated power [W].

This is the base of the well-known Maximum Power Point Tracking (MPPT) [12], [13]: from the prior treatment of the wind turbine model it can be appreciated that in order to extract the maximum amount of power from the incident wind, C_p should be maintained at a maximum. In order to achieve this objective, it can be appreciated from Fig. 5 that the speed of the generator rotor must be optimized according to instantaneous wind speed (this optimization is achieved by using (13)).

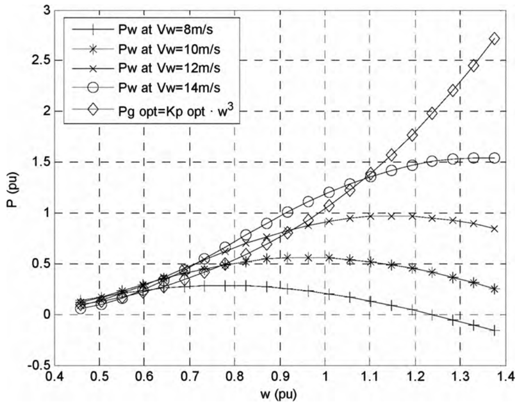

Fig. 12. Power vs. speed curves for different wind speeds and optimum power generated as a function of generator speed and wind speed.

The Basic parameters of the PMSG are given in Table III.

TABLE III
GENERATOR PARAMETERS

Parameter	Symbol	Value and Units
Rated generated power	$P_{gen\ rated}$	2 MW
Rated mechanical speed	$\omega_{g\ rated}$	2.18 rad/s
Stator resistance	R_s	0.08 Ω
Stator *d*-axis inductance	L_{ds}	0.334 H
Stator *q*-axis inductance	L_{qs}	0.217 H
Stator leakage inductance	L_{ls}	0.0334 H
Permanent magnet flux	Ψ_f	0.4832 Wb
Pole pairs	p	3

IV. CONCLUSION

The modeling of a variable speed wind turbine with a permanent magnet synchronous generator has been treated.

The model has been implemented in MATLAB/Simulink in order to validate it. C_P curves and power-speed characteristics have been obtained.

The generator has been modeled in the *d-q* synchronous rotating reference frame, taking into account different simplifications. Moreover, the concept of the maximum power point tracking has been presented in terms of the adjustment of the generator rotor speed according to instantaneous wind speed.

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