

AI-m of the Game

Workshop #2: Minimax

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Recap

What happened last time?

AI-m of the Game Overview

Students will implement a variety of AI algorithms to beat simple 2-player classical competitive games

- Examples: Tic-Tac-Toe, Connect Four, checkers, chess, Battleships, etc.
- We'll discuss (not implement) more complex methods (eg. neural nets, Q-learning)

Workshop Structure

Workshops #1-4

- Discuss multiple game structures and basic AI techniques
- Take-away game, Tic-Tac-Toe, Sim, and Connect Four
- Walk you step-by-step

Workshops #5-10

- Choose, build, then “solve” your own game!
 - should be hard eg. checkers, Go, chess
- Do your own research
- Advisors will only give guidance and deadlines

Key Links for this Course

- Meeting Minutes: <https://tinyurl.com/y5e365wn>
 - workshop schedule and notes
 - links to resources for learning and reviewing
 - let me know if you can't access and edit it
- GitHub: <https://github.com/project-ignite-2021-ai>
 - upload/download all of the code
- CMU Canvas
 - upload/download all of the non-code file (eg. slides)

Introduction to AI Games

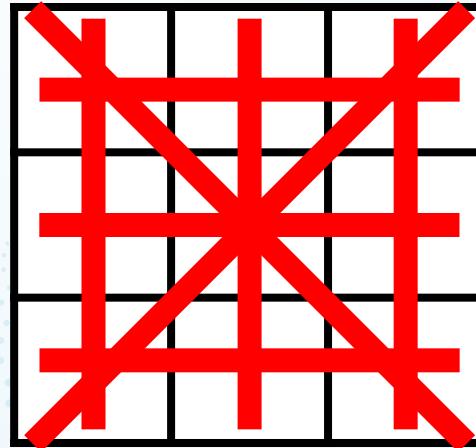
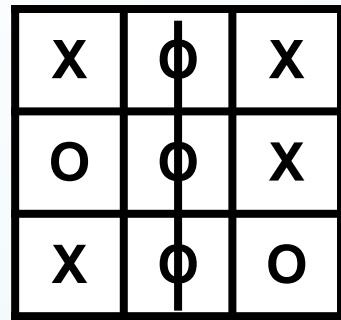
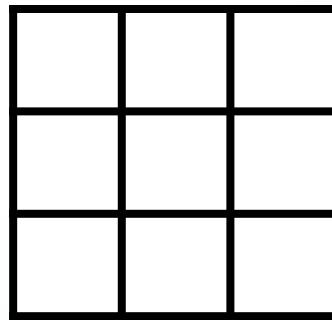
- What is AI?
- What are “games”? What games are there?
 - focus on 2-player sequential zero-sum
fully-observable discrete deterministic games
- What game strategies are there?
 - During-Each-...-Wins: say ‘2’ so that opponent says ‘1’
 - Takeaway Game: working backwards, put the
opponent in a losing state/position
 - Tic-Tac-Toe: ???

Tic-Tac-Toe

What are the basic rules and strategies?

Rules of Tic-Tac-Toe

- 3×3 grid of squares
- Turn player places their piece on an empty square
 - Player 1: O, Player 2: X
- The player who gets 3 of their pieces in a row, column, or diagonal wins!
- If all 9 squares of the board are filled, and no player has 3-in-a-row, it's a draw/tie



Tic-Tac-Toe Complexity

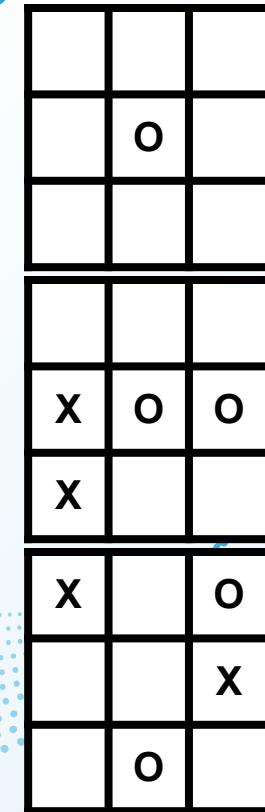
- Upper limit of games: $9! = 9*8*7*6*5*4*3*2*1 = 362,880$
 - actually has 255,168 games
- Upper limit of states: $3^9 = 19,683$
 - actually has 5,478 legal states
- Full game tree has 549,946 nodes
 - implies that on average, we visit each legal state over 100 times!

9	8	7
6	5	4
3	2	1

Typical Tic-Tac-Toe Strategy

1. start w/ piece in the center or corner
 - a. these move cover the most (4/8) and (3/8) winning positions!
2. try to automatically win
3. block any of opponent's attempts to 3-in-a-row
4. "trap" your opponent by having 2 ways to make a 3-in-a-row
 - a. opponent is helpless to stop it!

with perfect play, game is a **tie**

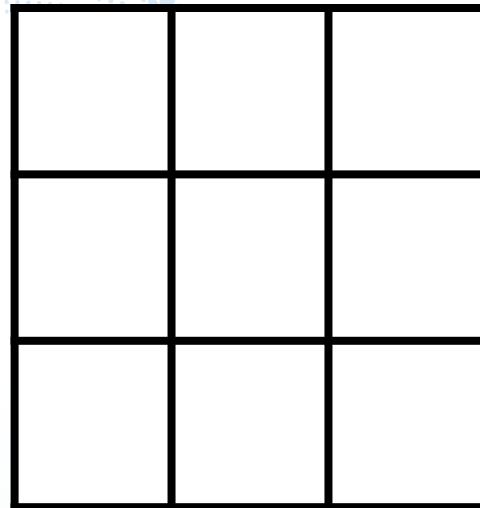


Other Tic-Tac-Toe Strategies

- Place randomly
- Place nearby your other pieces
- Place far from your other pieces
- Place randomly, but block opponent's 3-in-a-rows
- Block all of opponent's ways to get a 3-in-a-row
- Go for the fastest 3-in-a-row
- Go for "traps" (where you have multiple 3-in-a-rows after one move)

Optimal Strategy

But what is the best, most perfect, unbeatable strategy to win Tic-Tac-Toe?



Minimax

Minimize the maximum, maximize the minimum

Example of a Game

o		x
x		
x	o	o

Option 1

o		x
x	o	
x	o	o

Option 2

o	o	x
x		
x	o	o

Option 3

o		x
x		o
x	o	o

P1

o		x
x		
x	o	o

P2

o		x
x	o	
x	o	o

+1

o	o	x
x		
x	o	o

o		x
x		o
x	o	o

P1

o	o	x
x	x	
x	o	o

-1

o	o	x
x		x
x	o	o

-1

o	o	x
x	o	x
x	o	o

+1

o	x	x
x	o	o
x	o	o

+1

15

P1

o		x
x		
x	o	o

P2

o		x
x	o	
x	o	o

+1

o	o	x
x		
x	o	o

o		x
x		o
x	o	o

P1

o	o	x
x	x	
x	o	o

-1

o	o	x
x		x
x	o	o

+1

o		x
x	x	o
x	o	o

-1

o	x	x
x	o	o
x	o	o

+1

o	o	x
x	o	x
x	o	o

+1

o	x	x
x	o	o
x	o	o

+1

16

P1

o		x
x		
x	o	o

P2

o		x
x	o	
x	o	o

+1

o	o	x
x		
x	o	o

-1

o		x
x		o
x	o	o

-1

o	o	x
x	x	
x	o	o

-1

o	o	x
x		x
x	o	o

+1

o		x
x	x	o
x	o	o

-1

o	x	x
x	o	o
x	o	o

+1

o	o	x
x	o	x
x	o	o

+1

o	x	x
x	o	o
x	o	o

+1

P1

o		x
x		
x	o	o

+1

P2

o		x
x	o	
x	o	o

+1

o	o	x
x		
x	o	o

-1

o		x
x		o
x	o	o

-1

P1

o	o	x
x	x	
x	o	o

-1

o	o	x
x		x
x	o	o

+1

o		x
x	x	o
x	o	o

-1

o	x	x
x	o	o
x	o	o

+1

o	o	x
x	o	x
x	o	o

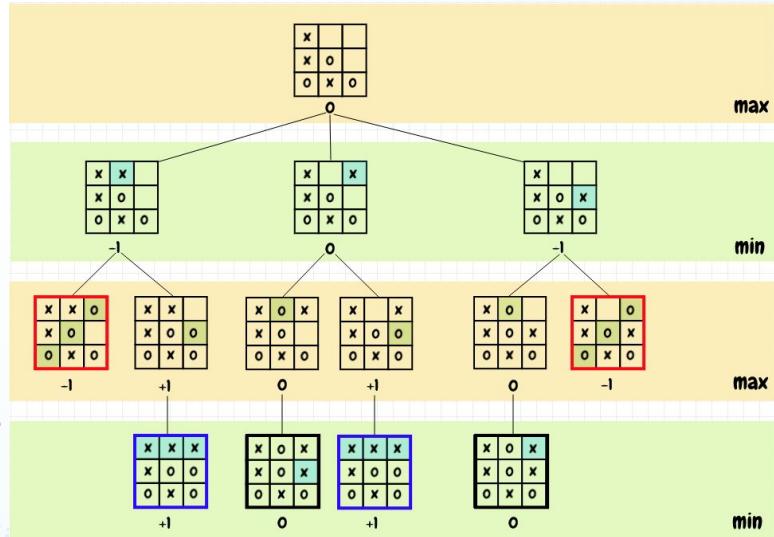
+1

o	o	x
x	o	o
x	o	o

+1

Minimax Algorithm

- you try to **maximize** your utility
- opponent tries to **minimize** your utility (ie. maximize his own)
- draw a full game tree, and evaluate every state based on how your opponent will minimize and how you'll maximize
 - for now, we only care about the **end states**



Minimax Issues

- we're still searching through all 255,168 games and 549,946 tree nodes
 - if we find the best move early on, why do we need to search through all of the worse moves too?
 - why should we have to play through all 9 moves just to make the first move?
 - many nodes are repeated states (ie. same board); it's a waste to have to evaluate them again!

Solving these Issues

- if we find the best move early on, why do we need to search through all of the worse moves too?
 - prune the trees! (we'll talk about this right now!)
- why should we have to play through all 9 moves just to make the first move?
 - stop the minimax code at a certain depth
- many nodes are repeated states (ie. same board); it's a waste to have to evaluate them again!
 - store the states in a hashtable; we'll skip this for now

Alpha-Beta Pruning

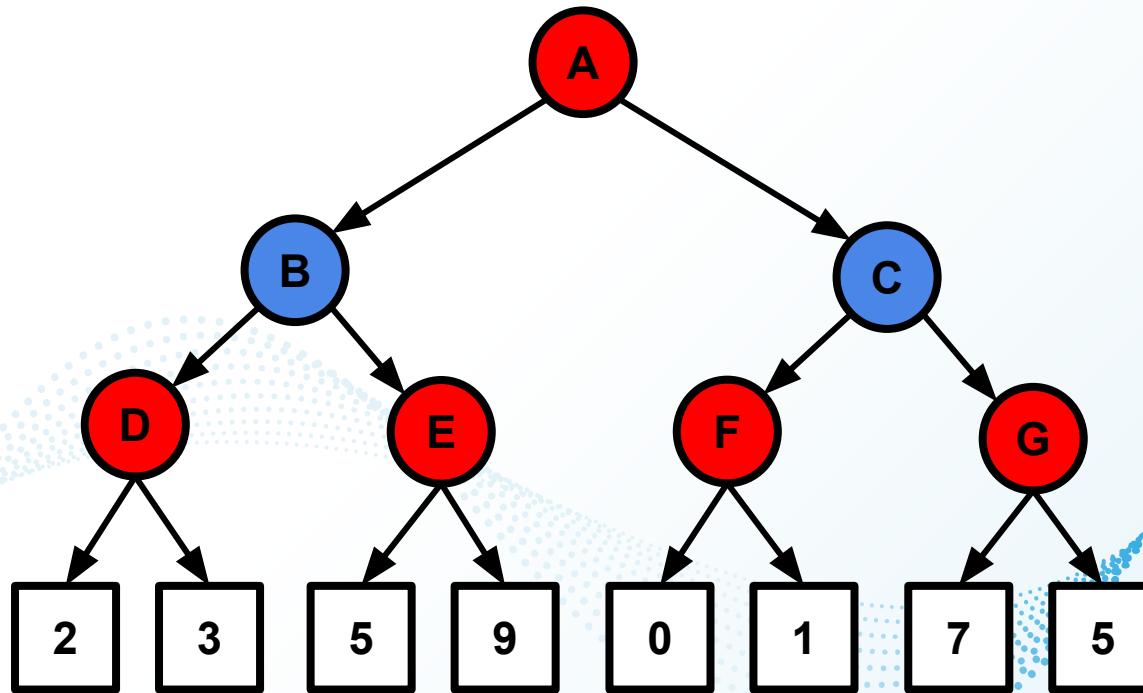
Cutting down the minimax search

Example of Game Tree

Player 1; MAX

Player 2; MIN

Player 1; MAX

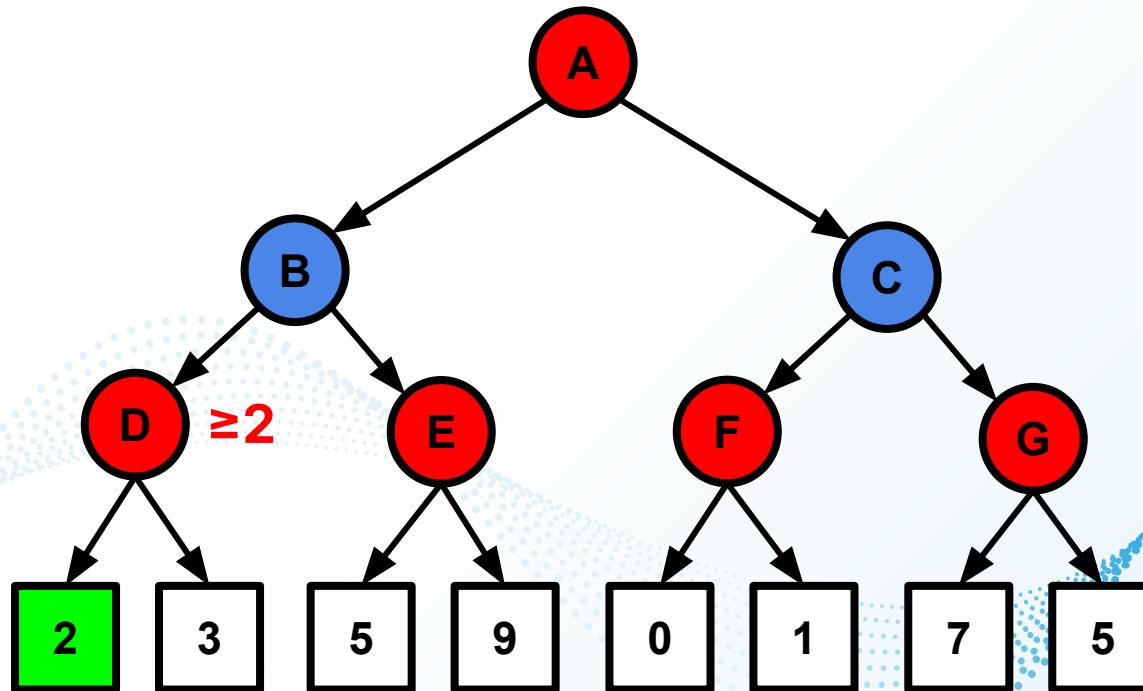


Example of Game Tree

Player 1; MAX

Player 2; MIN

Player 1; MAX

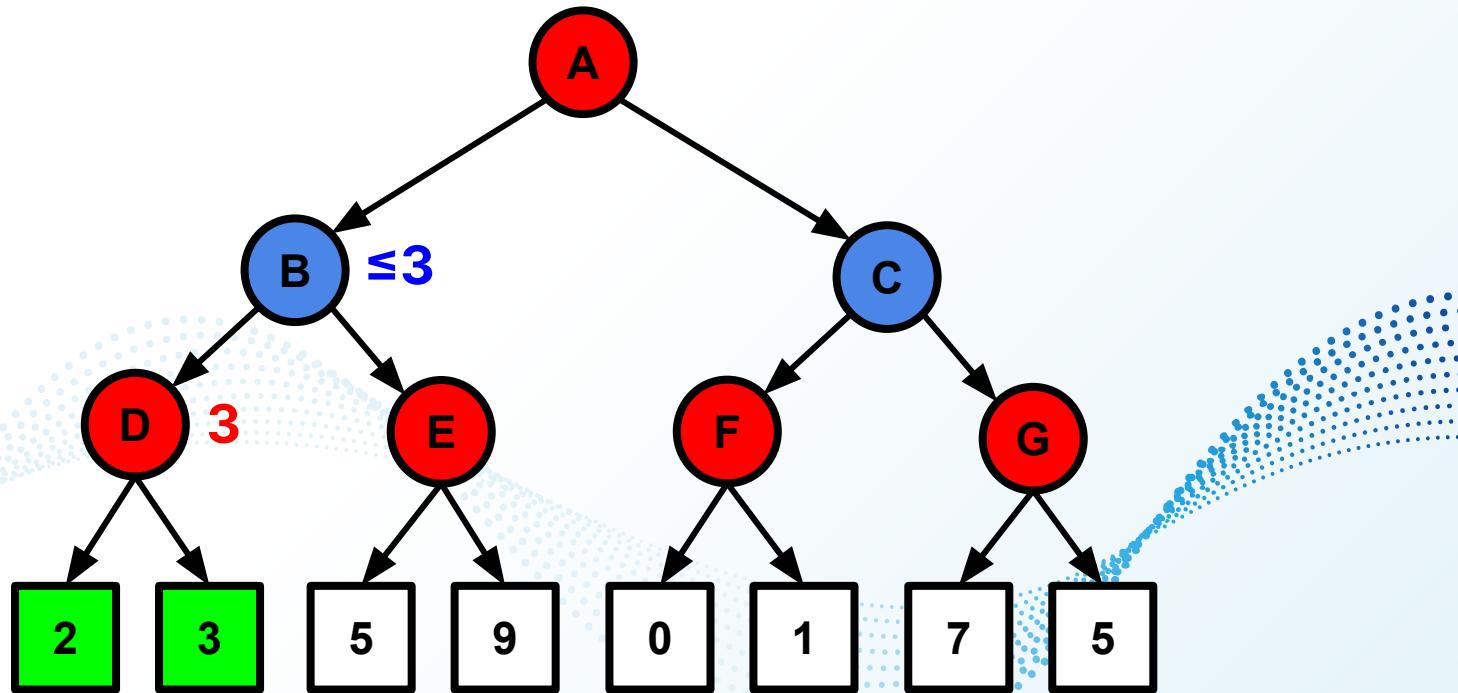


Example of Game Tree

Player 1; MAX

Player 2; MIN

Player 1; MAX

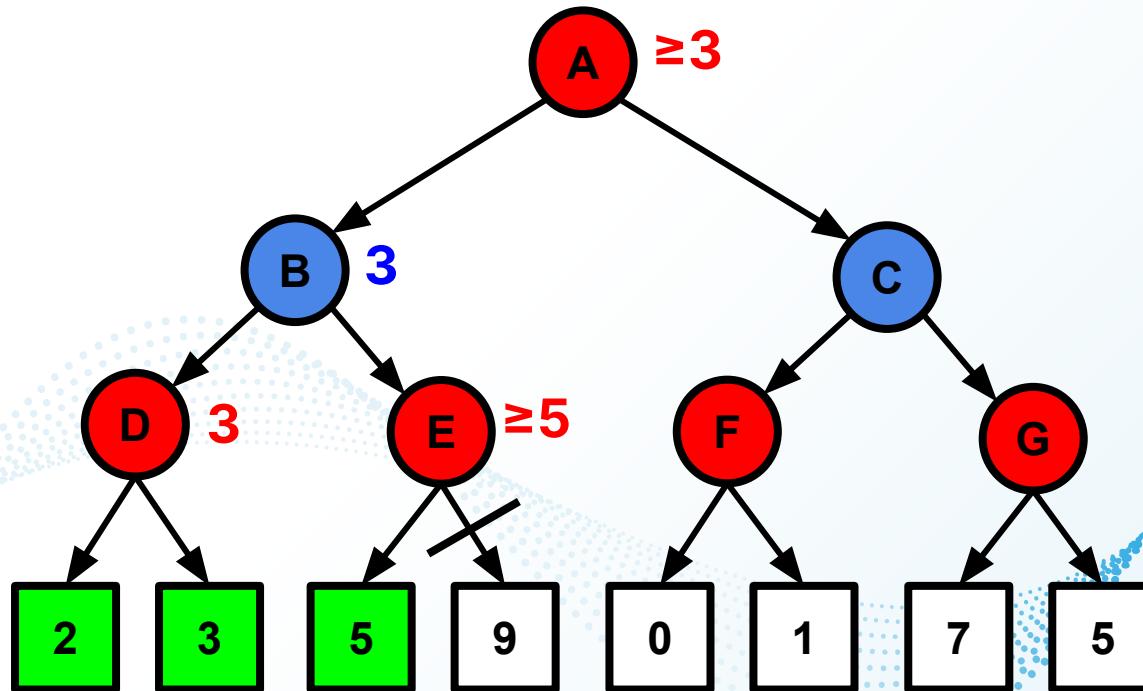


Example of Game Tree

Player 1; MAX

Player 2; MIN

Player 1; MAX

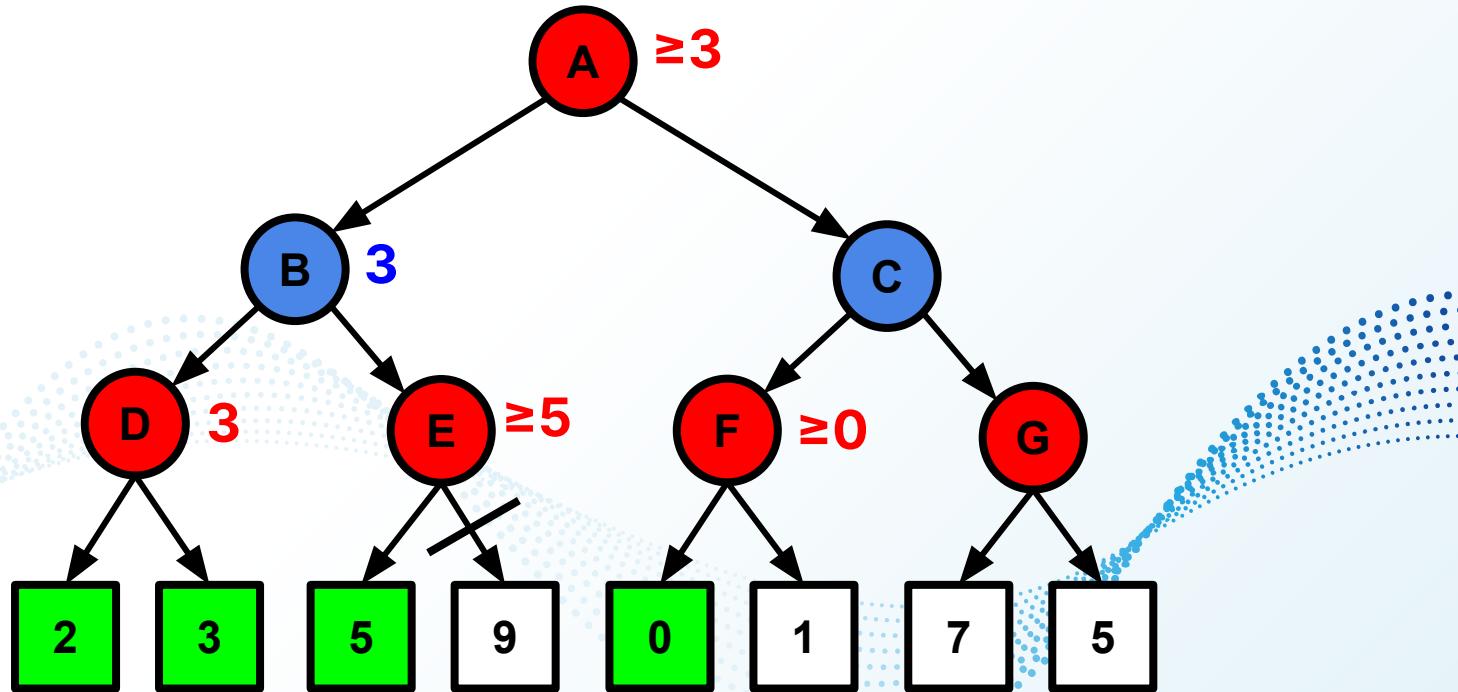


Example of Game Tree

Player 1; MAX

Player 2; MIN

Player 1; MAX

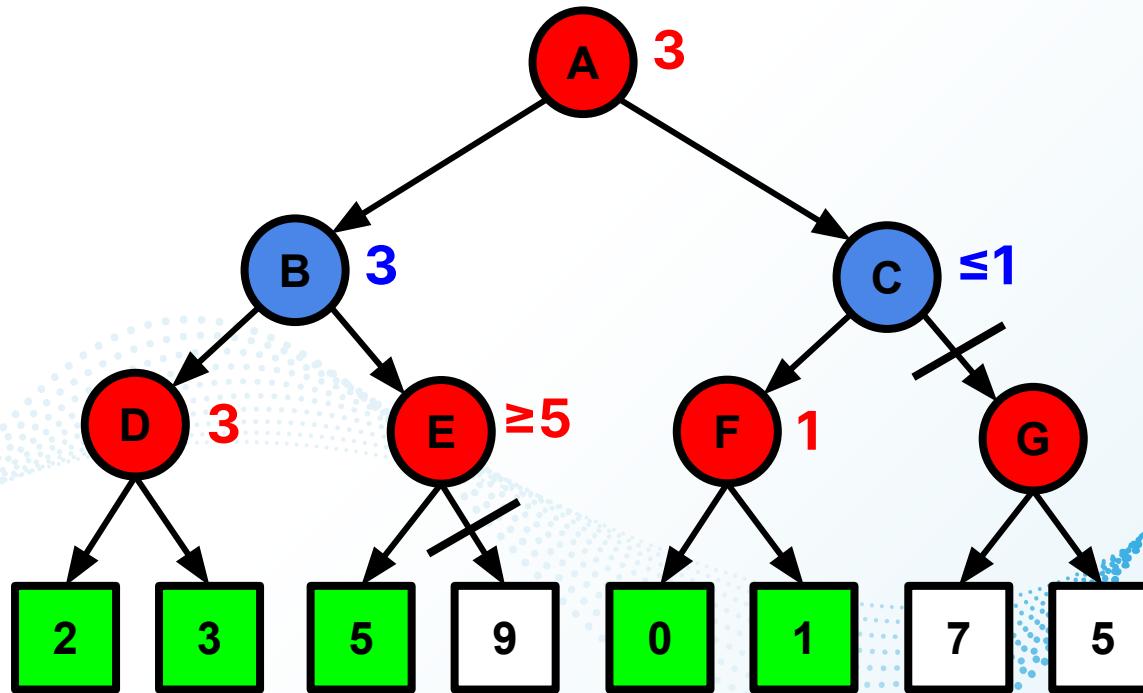


Example of Game Tree

Player 1; MAX

Player 2; MIN

Player 1; MAX



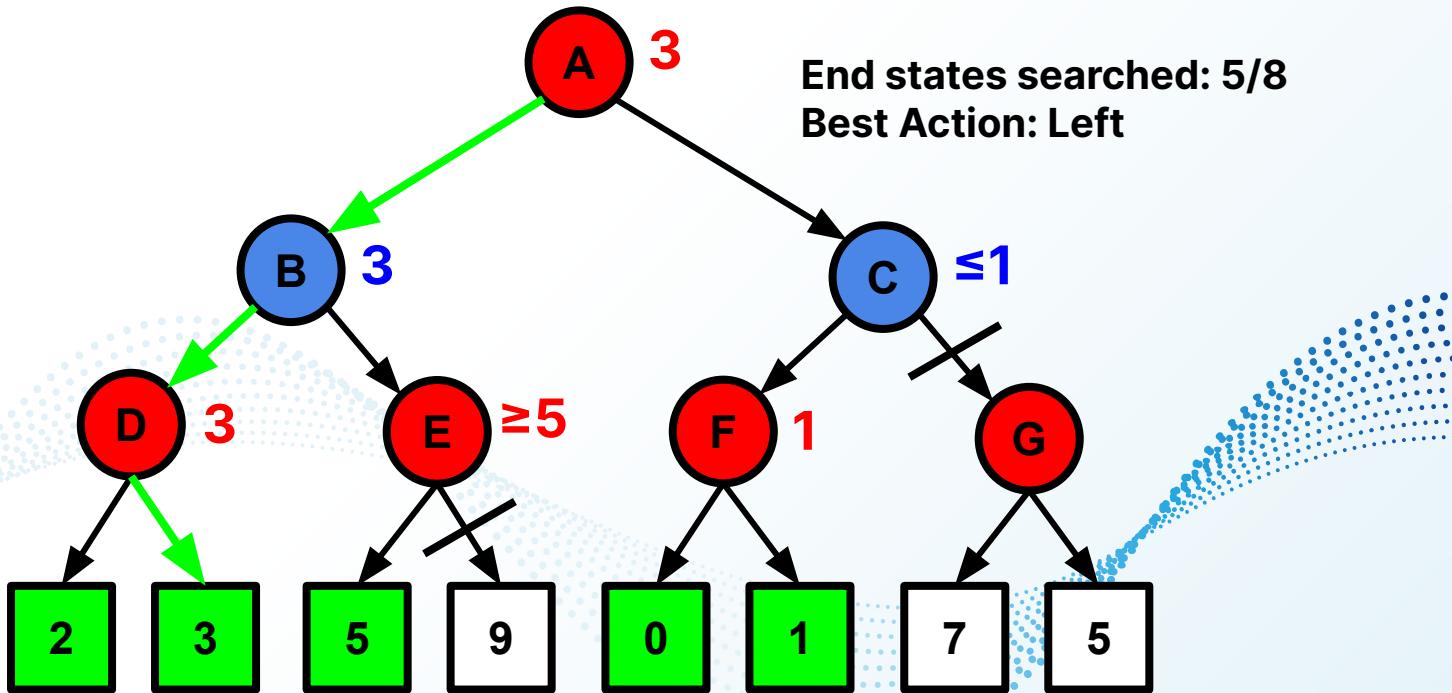
Example of Game Tree

Player 1; MAX

Player 2; MIN

Player 1; MAX

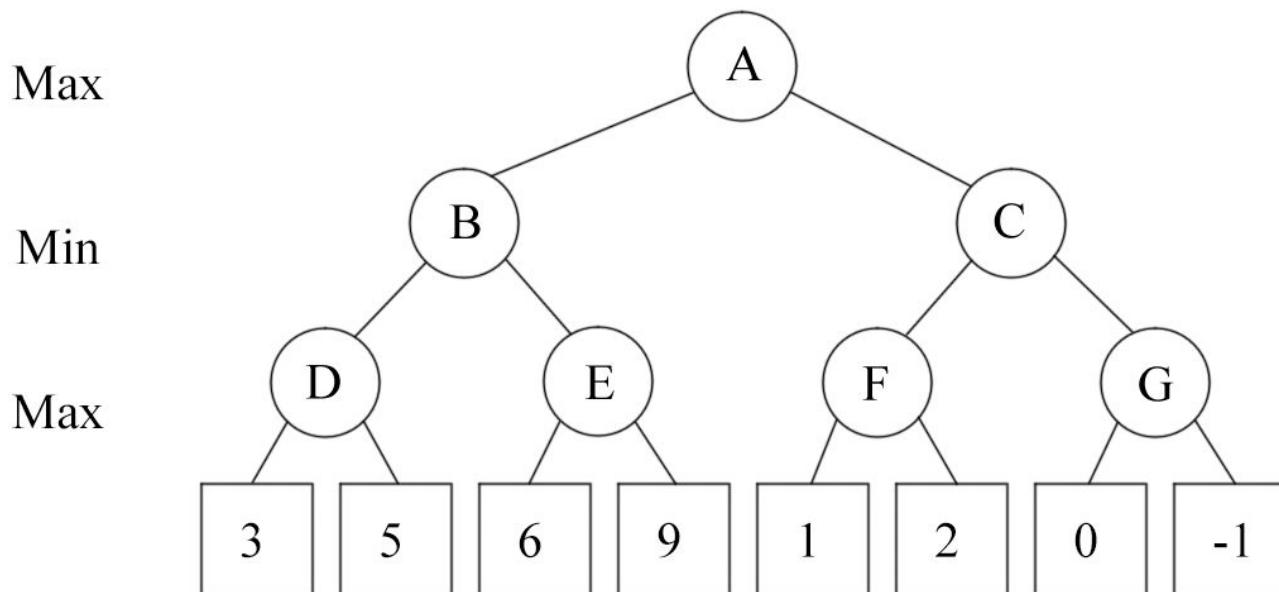
End states searched: 5/8
Best Action: Left



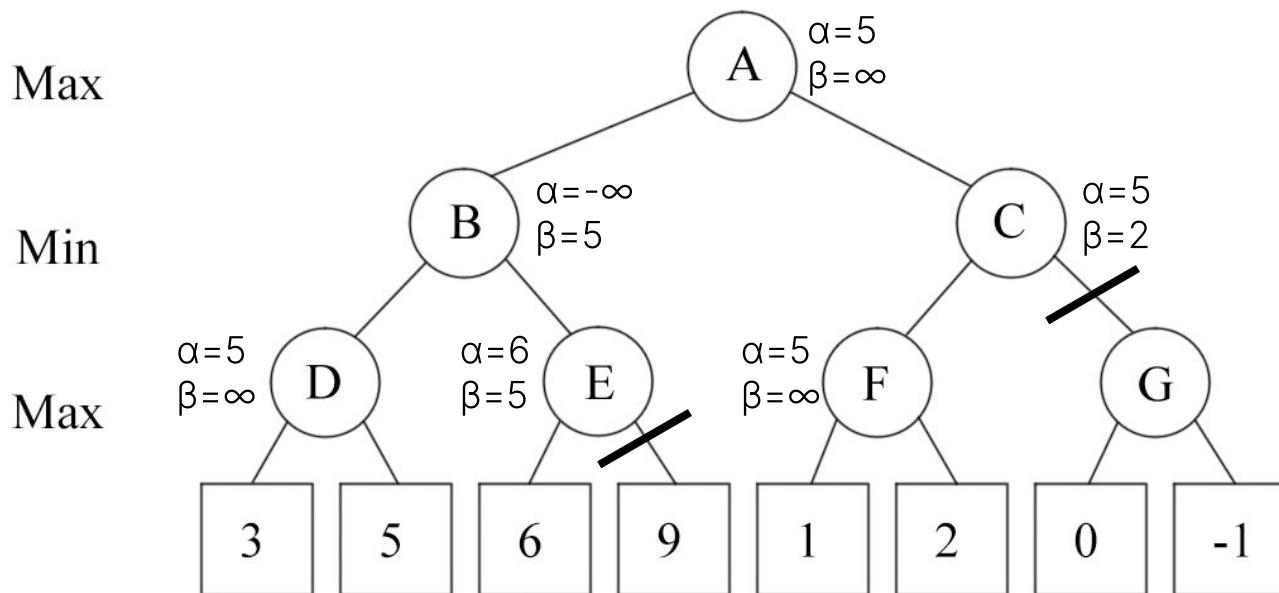
Alpha-Beta Pruning

- alpha (α) = best value for Player 1 (maximizer)
- beta (β) = best value for Player 2 (minimizer)
- alpha pruning: stop searching a MIN node if the new β value becomes \leq the old α value
- beta pruning: stop searching a MAX node if the new α value becomes \geq the old β value
- to see this exact alpha-beta pruning example again with α and β , go to [this website](#)

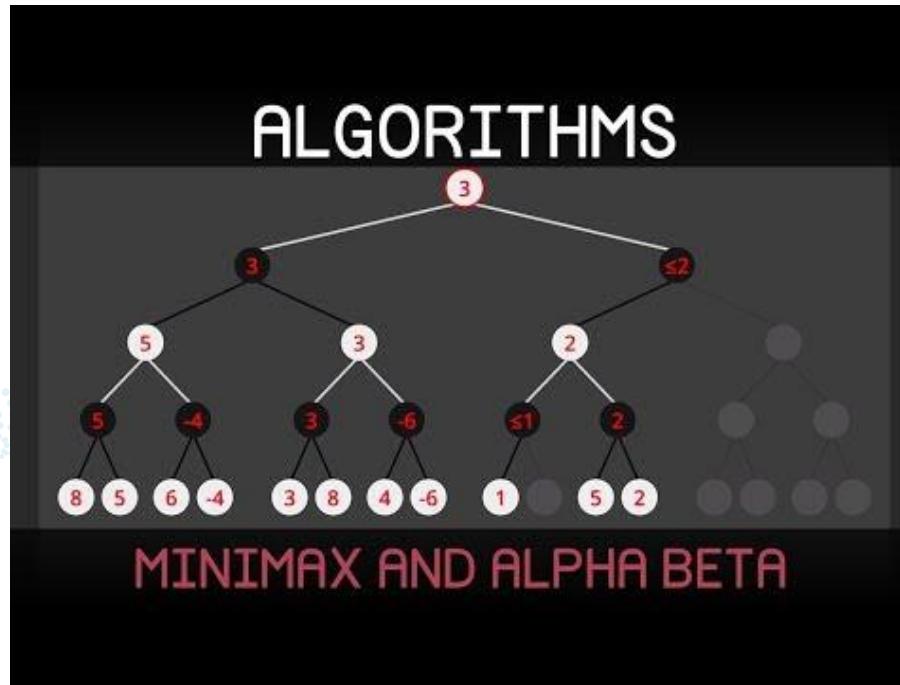
One More Example



One More Example



Minimax + Alpha-Beta Pruning



Tic-Tac-Toe in Python

Let's use minimax and pruning on Tic-Tac-Toe!

Solve Tic-Tac-Toe

Download tictactoe.py from GitHub

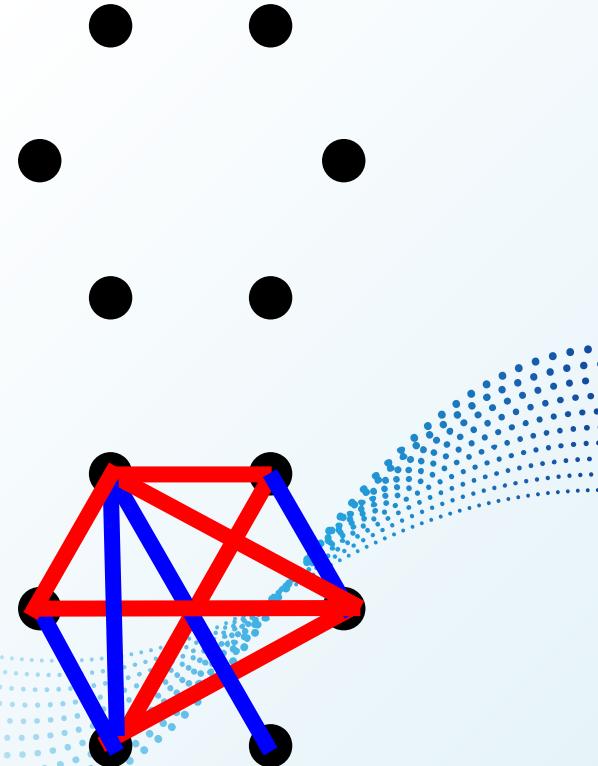
- Scan through the file to understand the structure
- Try playing it for yourself by running the code
- Complete the minimax() function
 - Use the YouTube video or the web as reference
- Complete the mmpruning() function
 - Use the YouTube video or the web as reference
- Test them with Player1Strategy and/or Player2Strategy

Sim (pencil game)

How can 6 dots be so complicated?

Rules of Sim

- 6 nonlinear dots (ie. a hexagon)
- Turn player draws a new line b/w any 2 points
 - Player 1: red, Player 2: blue
- The player who creates any triangle from 3 of their edges **loses!**
- Mathematically (by Ramsey theory), there are no draws/ties

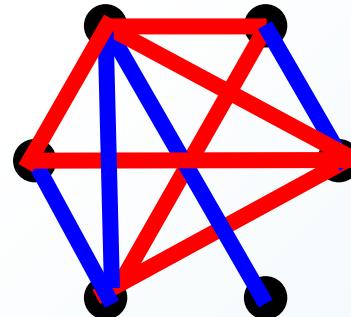


Let's Play Sim!



Sim vs Tic-Tac-Toe

- Game ends with 3-in-a-row
- In Sim, 3-in-a-row = you lose!
- In Sim, there are no ties
- Upper limit of Sim games: $15! = 1.31E12$ (over 1 trillion!)
 - more moves, higher depth
- Similar to a larger Tic-Tac-Toe game where the goal is to lose



		o	x
o	x	x	o
		o	
	o	x	

Solve Sim

Download sim.py from GitHub

- Scan through the file to understand the structure
- Try playing it for yourself by running the code
- Complete the mmpruning() function
 - Reference tictactoe.py or the rest of the sim.py
 - See negamax(), which is logically the same as the mmpruning() (but less code)
- Test them with Player1Strategy and/or Player2Strategy

To-do

Things to do by the next workshop:

- Complete tictactoe.py, or look at our sol'n on GitHub
- Complete sim.py, or look at our sol'n
- Get comfortable with minimax and alpha-beta pruning;
they're the basis of all AI games!

Thanks for coming!