Равенство в Теории Типов

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Propositions as Types

Proposition	$\leftrightarrow$	Type
Proof	$\leftrightarrow$	Term
Proof simplification	$\leftrightarrow$	Computation

Propositions as Types

$$
\begin{array}{rcl}
A & \leftrightarrow & A \\
A \wedge B & \leftrightarrow & A \times B \\
A \supset B & \leftrightarrow & A \to B \\
\forall x B(x) & \leftrightarrow & \Pi(x : A)B \\
x = y & \leftrightarrow & ???
\end{array}
$$

$=$

Logic:

- Reflexivity $x = x$
- Congruence $\dfrac{x = y}{f(x) = f(y)}$
- Substiutivity $\dfrac{x = y \qquad A_x}{A_y}$

Type theory:

- $\mathcal{I}xx$
- $\dfrac{\mathcal{I}xy}{\mathcal{I}(fx)(fy)}$
- $\dfrac{\mathcal{I}xy \qquad A_x}{A_y}$

Intentional Type Theory

aka Martin-Löf theory of types

Definitional $f \equiv g$.

- Intentional
- Up to α
- Substiutive

Checking that in $\dfrac{A \qquad B}{A \& B}$ A, B on top are the same (i.e. definitionally equal) to the ones on the bottom.

Intentional Type Theory

aka Martin-Löf theory of types

Judgemental $A = B, \quad a = b \in A$

- Holds by definition for canonical elements
- Up to β
- Substiutive
- $\dfrac{a \in A \qquad A = B}{a \in B}$
- $\dfrac{a = b \in A \qquad A = B}{a = b \in B}$

Intentional Type Theory

aka Martin-Löf theory of types

Propositional $I(a, b, A)$

- A type
- Canonical element $r(a) : I(a, a, A)$
- Eliminator J :.
- $\dfrac{a = b \in A}{r(a) : I(a, b, A)}$

Extensional Type Theory

$$\frac{x \in I(a, b, A)}{a = b \in A}$$

CiC

UTT

HoTT

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NuPRL

ZOMBIE