

# Равенство в Теории Типов

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# Intentional Type Theory

Per Martin-Löf “An Intuitionistic Theory of Types: Predicative Part”, 1975

Definitional equality  $=_{\text{def}}$

$$\text{Definition, Equivalence, } \alpha\beta, \quad \frac{a =_{\text{def}} b}{x[a] =_{\text{def}} x[b]},$$
$$\frac{a : A \quad A =_{\text{def}} B}{a : B}.$$

Propositional equality (a.k.a. Identity)  $I(A, a, b)$

$$r(a) : I(A, a, a)$$

$$\frac{\displaystyle \frac{x : A \quad y : A \quad z : I(A, x, y)}{C[x, y, z]} \quad c[x] : C[x, x, r(x)]}{f(x, x, r(x)) =_{\text{def}} c[x]}$$

# Extentional Type Theory

Per Martin-Löf “Intuitionstic Type Theory”, 1980

## Definitional equality

Definition, Equivalence, Substituiting equals for equals,  $\alpha$ . Checking that in  $\frac{A \quad B}{A \& B}$   $A, B$  on top are the same (i.e. definitionally equal) to the ones on the bottom.

## Judgemental equality $A = B, a = b : A$

Defined for every type,  $\beta\eta$ ,  
 $[x : A]$

$$\xi \frac{f(x) = g(x) : B(x)}{\lambda x. f(x) = \lambda x. g(x) : \Pi(x : A)B(x)},$$
$$\frac{a \in A \quad A = B}{a \in B}, \quad \frac{a = b \in A \quad A = B}{a = b \in B}.$$

## Propositional equality

$$\text{ER} \frac{c : I(A, a, b)}{a = b : A}, \quad \text{UIP} \frac{c : I(A, a, b)}{c = r : I(A, a, b)}.$$

# Propositions as Types

Proposition	$\leftrightarrow$	Type
Proof	$\leftrightarrow$	Term
Proof simplification	$\leftrightarrow$	Computation

# Propositions as Types

$$A \leftrightarrow A$$

$$A \wedge B \leftrightarrow A \times B$$

$$A \supset B \leftrightarrow A \rightarrow B$$

$$\forall x B(x) \leftrightarrow \Pi(x : A) B$$

$$x = y \leftrightarrow ???$$

Logic:

- ▶ Reflexivity  $x = x$
- ▶ Congruence  $\frac{x = y}{f(x) = f(y)}$
- ▶ Substiutivity  $\frac{x = y \quad A_x}{A_y}$

Type theory:

- ▶  $I_{xx}$
- ▶  $\frac{I_{xy}}{I(fx)(fy)}$
- ▶  $\frac{I_{xy} \quad A_x}{A_y}$

# Intentional Type Theory

aka Martin-Löf theory of types

Definitional  $f \equiv g$ .

- ▶ Intentional
- ▶ Up to  $\alpha$
- ▶ Substitutive

Checking that in  $\frac{A \quad B}{A \& B}$   $A, B$  on top are the same (i.e. definitionally equal) to the ones on the bottom.

# Intentional Type Theory

aka Martin-Löf theory of types

Judgemental  $A = B$ ,  $a = b \in A$

► Holds by definition for canonical elements

► Up to  $\beta$

► Substitutive

► 
$$\frac{a \in A \quad A = B}{a \in B}$$

► 
$$\frac{a = b \in A \quad A = B}{a = b \in B}$$



# Intentional Type Theory

aka Martin-Löf theory of types

Propositional  $I(a, b, A)$

- ▶ A type
- ▶ Canonical element  $r(a) : I(a, a, A)$
- ▶ Eliminator  $J :.$
- ▶ 
$$\frac{a = b \in A}{r(a) : I(a, b, A)}$$

# Extensional Type Theory

$$\frac{x \in I(a, b, A)}{a = b \in A}$$





# HoTT

# OTT



# ZOMBIE