Равенство в Теории Типов

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Intentional Type Theory

Per Martin-Löf "An Intuitionstic Theory of Types: Predicative Part", 1975

Definitional equality $=_{\text{def}}$

Reflexivity, Transitivity, Symmetry, αβ,

$$\frac{a =_{\text{def}} b}{x[a] =_{\text{def}} x[b]}\ ,\quad \frac{a : A \qquad A =_{\text{def}} B}{a : B}\ .$$

Propositional equality (a.k.a. Identity) $I(A, a, b)$

$$r(a) : I(A, a, a)$$

$$\frac{\dfrac{x : A \qquad y : A \qquad z : I(A, x, y)}{C[x, y, z]} \qquad c[x] : C[x, x, r(x)]}{f(x, x, r(x)) =_{\text{def}} c[x]}$$

Propositions as Types

Proposition	$\leftrightarrow$	Type
Proof	$\leftrightarrow$	Term
Proof simplification	$\leftrightarrow$	Computation

Propositions as Types

$$\begin{array}{rcl} A & \leftrightarrow & A \\ A \wedge B & \leftrightarrow & A \times B \\ A \supset B & \leftrightarrow & A \rightarrow B \\ \forall x B(x) & \leftrightarrow & \Pi(x : A)B \\ x = y & \leftrightarrow & ??? \end{array}$$

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Logic:

- Reflexivity $x = x$
- Congruence $\dfrac{x = y}{f(x) = f(y)}$
- Substiutivity $\dfrac{x = y \qquad A_x}{A_y}$

Type theory:

- $\mathcal{I}xx$
- $\dfrac{\mathcal{I}xy}{\mathcal{I}(fx)(fy)}$
- $\dfrac{\mathcal{I}xy \qquad A_x}{A_y}$

Intentional Type Theory

aka Martin-Löf theory of types

Definitional $f \equiv g$.

- Intentional
- Up to α
- Substiutive

Checking that in $\frac{A \qquad B}{A \& B}$ A, B on top are the same (i.e. definitionally equal) to the ones on the bottom.

Intentional Type Theory

aka Martin-Löf theory of types

Judgemental $A = B, \quad a = b \in A$

- Holds by definition for canonical elements
- Up to β
- Substiutive
- $\dfrac{a \in A \qquad A = B}{a \in B}$
- $\dfrac{a = b \in A \qquad A = B}{a = b \in B}$

Intentional Type Theory

aka Martin-Löf theory of types

Propositional $I(a, b, A)$

- A type
- Canonical element $r(a) : I(a, a, A)$
- Eliminator J :.
- $\dfrac{a = b \in A}{r(a) : I(a, b, A)}$

Extensional Type Theory

$$\frac{x \in I(a, b, A)}{a = b \in A}$$

CiC

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