1 Intro

I need to prove that haskell types and terms that I expose wouldn't break the system. It means two things:

1. Types preserve the same set of invariants
2. Terms have the same interface: any combination of `APPLY` that can be used(ignoring types) to original term must be usable with generated one; and primitives(numbers, strings, ... and their ops) are the same.

2 My transformations

2.1 Kinds

$$KT[\![Kind]\!] = HaskellKind$$

$$\begin{aligned} KT[\![Set_0]\!] &= * \\ KT[\![Kind_1 \to Kind_2]\!] &= KT[\![Kind_1]\!] \to KT[\![Kind_2]\!] \end{aligned}$$

2.2 Type declarations

$DTMA$ gives MAlonzo generated type name.
$DT,\ DTD$ are defined when {`-# EXPORT` $AgdaTypeName\ HaskellTypeName$ `#-`} is specified.

$$\begin{aligned} DTMA[\![AgdaTypeName]\!] &= HaskellTypeName \\ DT[\![AgdaTypeName]\!] &= HaskellTypeName \\ DTD[\![AgdaTypeName]\!] &\doteq HaskellTypeDeclaration \end{aligned}$$

Considering declaration:

data $AgdaDataType\ (A_1 : Kind_1) \cdots (A_n : Kind_n) : Kind_{n+1} \to \cdots \to Kind_m \to Set$ **where** $\ldots$

$$\begin{aligned} &DTD[\![AgdaDataType]\!] \doteq \\ &\quad \textbf{newtype}\ DT[\![AgdaDataType]\!]\ (a_0 :: KT[\![Kind_1]\!]) \cdots (a_m :: KT[\![Kind_m]\!]) \\ &\qquad = DT[\![AgdaRecordType]\!]\ (\forall b_0 \cdots b_k.\ DTMA[\![AgdaDataType]\!]\ b_0\ \cdots\ b_k) \end{aligned}$$

k is an arity of type constructor generated by MAlonzo.
It also works for **record**s.

2.3 Types

$$\begin{aligned} TT[\![AgdaType]\!](Context) &= HaskellType \\ Context &= \{AgdaTypeVarName \mapsto HaskellTypeVarName\} \end{aligned}$$

$$\begin{aligned} TT[\![A\ args\ldots]\!](\Gamma) &= a\ TT[\![args\ldots]\!](\Gamma), \quad (A \mapsto a) \in \Gamma \\ TT[\![CT\ args\ldots]\!](\Gamma) &= CT\ TT[\![args\ldots]\!](\Gamma), \quad CT \text{ is a } \texttt{COMPILED\_TYPE} \text{ or a primitive postulate} \\ TT[\![ET\ args\ldots]\!](\Gamma) &= DT[\![ET]\!]\ TT[\![args\ldots]\!](\Gamma) \\ TT[\![(A : Kind) \to T]\!](\Gamma) &= \forall (a :: KT[\![Kind]\!]).\ TT[\![T]\!](\Gamma \cup \{A \mapsto a\}) \\ TT[\![(x : T_1) \to T_2]\!](\Gamma) &= TT[\![T_1]\!](\Gamma) \to TT[\![T_2]\!](\Gamma), \quad x \notin freevars(T_2) \\ TT[\![(x : T_1,\ T_2)]\!](\Gamma) &= (TT[\![T_1]\!](\Gamma),\ TT[\![T_2]\!](\Gamma)), \quad x \notin freevars(T_2) \end{aligned}$$

2.4 Terms

$Wrap$ is defined only when $TT[\![AgdaType]\!](\varnothing)$ is defined.

$$Wrap^{2k}[\![AgdaType]\!](MAlonzoTerm) = MyTerm$$
$$Wrap^{2k+1}[\![AgdaType]\!](MyTerm) = MAlonzoTerm$$

$$\begin{aligned}
Wrap^{k}[\![A\ args\ldots]\!](term) &= \texttt{unsafeCoerce}\ term \\
Wrap^{2k}[\![(A : Kind) \to T]\!](term) &= Wrap^{2k}[\![T]\!](term\ ()) \\
Wrap^{2k+1}[\![(A : Kind) \to T]\!](term) &= Wrap^{2k+1}[\![T]\!](\lambda_.\ term) \\
Wrap^{k}[\![(x : T_1) \to T_2]\!](term) &= \lambda x.\ Wrap^{k}[\![T_2]\!](term\ Wrap^{k+1}[\![T_1]\!](x)) \\
Wrap^{k}[\![(x : T_1,\ T_2)]\!]((term_1,\ term_2)) &= (Wrap^{k}[\![T_1]\!](term_1),\ Wrap^{k}[\![T_2]\!](term_2))
\end{aligned}$$

2.5 Value declarations

$VTMA$ gives MAlonzo generated value name
VT, VTD are defined when {`-# EXPORT` $AgdaName\ HaskellName$ `#-`} is specified.

$$VTMA[\![AgdaName]\!] = HaskellName$$
$$VT[\![AgdaName]\!] = HaskellName$$
$$VTD[\![AgdaName]\!] \doteq HaskellDeclaration$$

Considering declaration:

$$AgdaName : AgdaType$$
$$AgdaName = \ldots$$

$$\begin{aligned}
&VTD[\![AgdaName]\!] \doteq \\
&\quad VT[\![AgdaName]\!] :: TT[\![AgdaType]\!](\varnothing) \\
&\quad VT[\![AgdaName]\!] = Wrap^{0}[\![AgdaType]\!](VTMA[\![AgdaName]\!])
\end{aligned}$$

It works in the same way for constructors. It also works seamlessly with parametrized modules and, consequently, with record functions.

3 Preserving type invariants

Two things to watch for:

- `newtype` wrappers preserve internal invariants of underlying Agda datatype.
- Transformation from Church polymorphism to Curry polymorphism.

In every other case type is exactly the same.

4 Preserving term interface

$Wrap$ clearly deals with the issue of passing and skipping type parameters with MAlonzo-generated code.
A thing to watch for is `unsafeCoerce`. It is legal because it's either:

- The same term(when its type is a type variable)
- A newtype around MAlonzo generated type
- A primitive