1. Krets

1.1. Elements

1.1.1. Resistor

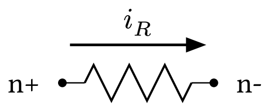

Figure 1: A resistor.

The current through a resistor is given by Ohm's law:

$$i_R = \frac{v^+ - v^-}{R} = G(v^+ - v^-) \tag{1}$$

To get the element stamps for the resistor in the conductance matrix, we use Kirchhoff's Current Law (KCL) at nodes n+ and n-.

At node n+:

$$i_{n+} = i_R = G(v^+ - v^-) \tag{2}$$

At node n-:

$$i_{n-} = -i_R = -G(v^+ - v^-) \tag{3}$$

This leads to the following conductance matrix stamps:

Table 1: Element stamps for a resistor in the conductance matrix in group 1.

	v^+	v^-	RHS
v^+	$+G$	$-G$	
v^-	$-G$	$+G$	

To get the stamps in group 2, we introduce a current variable i_R for the resistor:

$$v^+ - v^- = Ri_R \Rightarrow v^+ - v^- - Ri_R = 0 \tag{4}$$

Table 2: Element stamps for a resistor in the conductance matrix in group 2.

	v^+	v^-	i_R	RHS
v^+			$+1$	
v^-			-1	
i_R	$+1$	-1	$-R$	

1.1.2. BJT

1.1.3. Capacitor

Table 3: Element stamps for a capacitor in the conductance matrix in group 1.

	v^+	v^-	RHS
v^+	$+C$	$-C$	
v^-	$-C$	$+C$	

Table 4: Element stamps for a capacitor in the conductance matrix in group 2.

	v^+	v^-	i_C	RHS
v^+				
v^-				
i_C	$-C$	$+C$	$+1$	

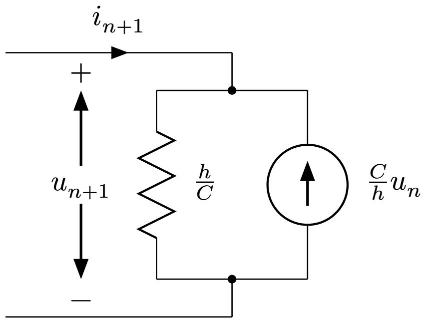

Figure 2: Capacitor Companion model for Backwards Euler.

The dynamic element equation

$$i(t_{n+1}) = C(u(t_{n+1}))u'(t_{n+1}) \tag{5}$$

using $i(t_{n+1}) \approx i_{n+1}$ and $u(t_{n+1}) \approx u_{n+1}$

$$i_{n+1} = C(u_{n+1})u'(t_{n+1}) \approx C(u_{n+1})\left(\frac{u_{n+1} - u_n}{h}\right) \tag{6}$$

$$u_{n+1} = \frac{h}{C}i_{n+1} + u_n \tag{7}$$

so

$$G_{n+1} = \frac{h}{C} \text{ and } u_n = u_{n+1} - G_{n+1}i_{n+1} \tag{8}$$

1.1.4. Current Source

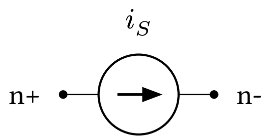

Figure 3: A current source.

The current through an independent current source is given by:

$$i_S = I \tag{9}$$

To get the element stamps for the independent current source in the conductance matrix, we use Kirchhoff's Current Law (KCL) at nodes n+ and n-.

At node n+:

$$i_{n+} = i_S = I \tag{10}$$

At node n-:

$$i_{n-} = -i_S = -I \tag{11}$$

This leads to the following conductance matrix stamps:

Table 5: Element stamp for an independent current source in group 1

	v^+	v^-	RHS
v^+			$+I$
v^-			$-I$

Table 6: Element stamp for an independent current source in group 2

	v^+	v^-	i	RHS
v^+			$+1$	
v^-			-1	
i			$+1$	I

1.1.5. Diode

The diode is modeled as a nonlinear element with a current-voltage relationship defined by the Shockley diode equation:

$$I_D = I_S\left(e^{\frac{V_D}{nV_T}} - 1\right) \tag{12}$$

Where I_D is the diode current, I_S is the reverse saturation current, V_D is the voltage across the diode, V_T is the thermal voltage, and, n is the ideality factor, also known as the quality factor, emission coefficient, or the material constant.

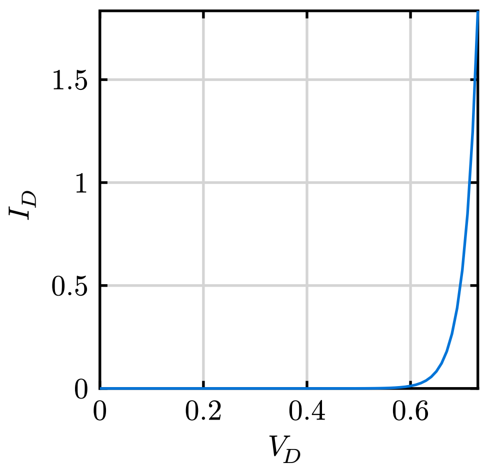

Figure 4: Diode IV Curve

The conductance of the diode is G_D and is given by the derivative of the Shockley diode equation with respect to the voltage:

$$G_D = \frac{dI_D}{dV_D} = \frac{I_S}{nV_T} e^{\frac{V_D}{nV_T}} \tag{13}$$

The companion model for the diode can be represented as a current source in parallel with a conductance.

$$I_{\text{eq}} = I_D - G_{\text{eq}} V_D \tag{14}$$

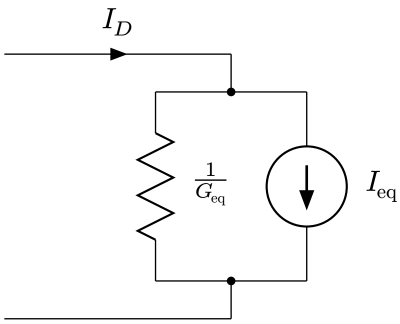

Figure 5: Diode Companion model

The element stamps for the diode in the conductance matrix are given by:

Table 7: Element stamps for a diode in group 1.

	v^+	v^-	RHS
v^+	$+G_{\text{eq}}$	$-G_{\text{eq}}$	$-I_{\text{eq}}$
v^-	$-G_{\text{eq}}$	$+G_{\text{eq}}$	$+I_{\text{eq}}$

1.1.6. Inductor

The dynamic element equation

$$u(t_{n+1}) = L(i(t_{n+1}))i'(t_{n+1}) \tag{15}$$

using $u(t_{n+1}) \approx u_{n+1}$ and $i(t_{n+1}) \approx i_{n+1}$

$$u_{n+1} = L(i_{n+1})i'(t_{n+1}) \approx L(i_{n+1})\left(\frac{i_{n+1} - i_n}{h}\right) \tag{16}$$

$$i_{n+1} = \frac{h}{L}u_{\text{n+1}} + i_n \tag{17}$$

$$G_{n+1} = \frac{h}{L} \text{ and } i_n = i_{n+1} - G_{n+1}u_{n+1} \tag{18}$$

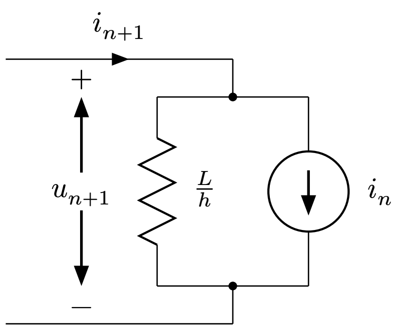

Figure 6: Inductor companion model for Backwards Euler.

1.1.7. Mosfet

1.1.8. Voltage Source

In the conductance matrix the stamps for a voltage source are given by:

If the positive terminal is connect to node `i` and the node is not grounded, the stamp is: 1

1.2. Analyses

$$|x_{\text{new}} - x_{\text{old}}| \leq \text{relative\_tolerance} * \max(|x_{\text{new}}|, |x_{\text{old}}|) + \begin{cases} \text{current\_absolute\_tolerance} \\ \text{voltage\_absolute\_tolerance} \end{cases} \tag{19}$$

1.2.1. DC

During DC analysis, the circuit is analyzed under steady-state conditions with all capacitors treated as open circuits and all inductors treated as short circuits.

1.2.1.1. Diode IV Curve

$\begin{cases} V_1 = 1 \\ R_1 = 1000 \end{cases}$

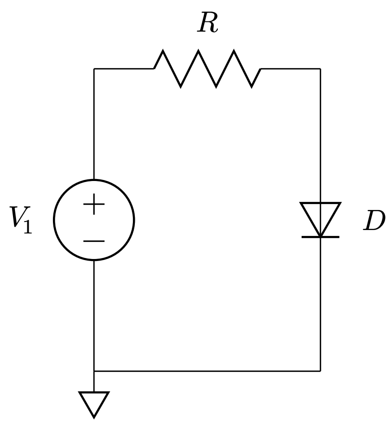

Figure 7: Diode IV Curve

Lets build the conductance matrix for this circuit.

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & \frac{1}{R} & -\frac{1}{R} \\ 0 & -\frac{1}{R} & \frac{1}{R} + G_D \end{pmatrix} \begin{vmatrix} I(V_1) \\ V_{\text{in}} \\ V_{\text{out}} \end{vmatrix} = \begin{pmatrix} 1 \\ 0 \\ I_D \end{pmatrix} \tag{20}$$

A Appendix

A.1 Constants

The following physical constants are used throughout this document:

$k_B = 1.380649 \cdot 10^{-23}$ (Boltzmann constant)

$q = 1.602176634 \cdot 10^{-19}$ (Elementary charge)

$T = 300$ (Standard temperature)

$V_T = \frac{k_B T}{q} \approx 0.02585$ (Thermal voltage at 300K)

$I_S = 1 \cdot 10^{-}12$ (reverse saturation current)

0.000000000001