Data Structures

Lecture 15: Graph Traversals

การทราเวิสกราฟเข้าไปที่

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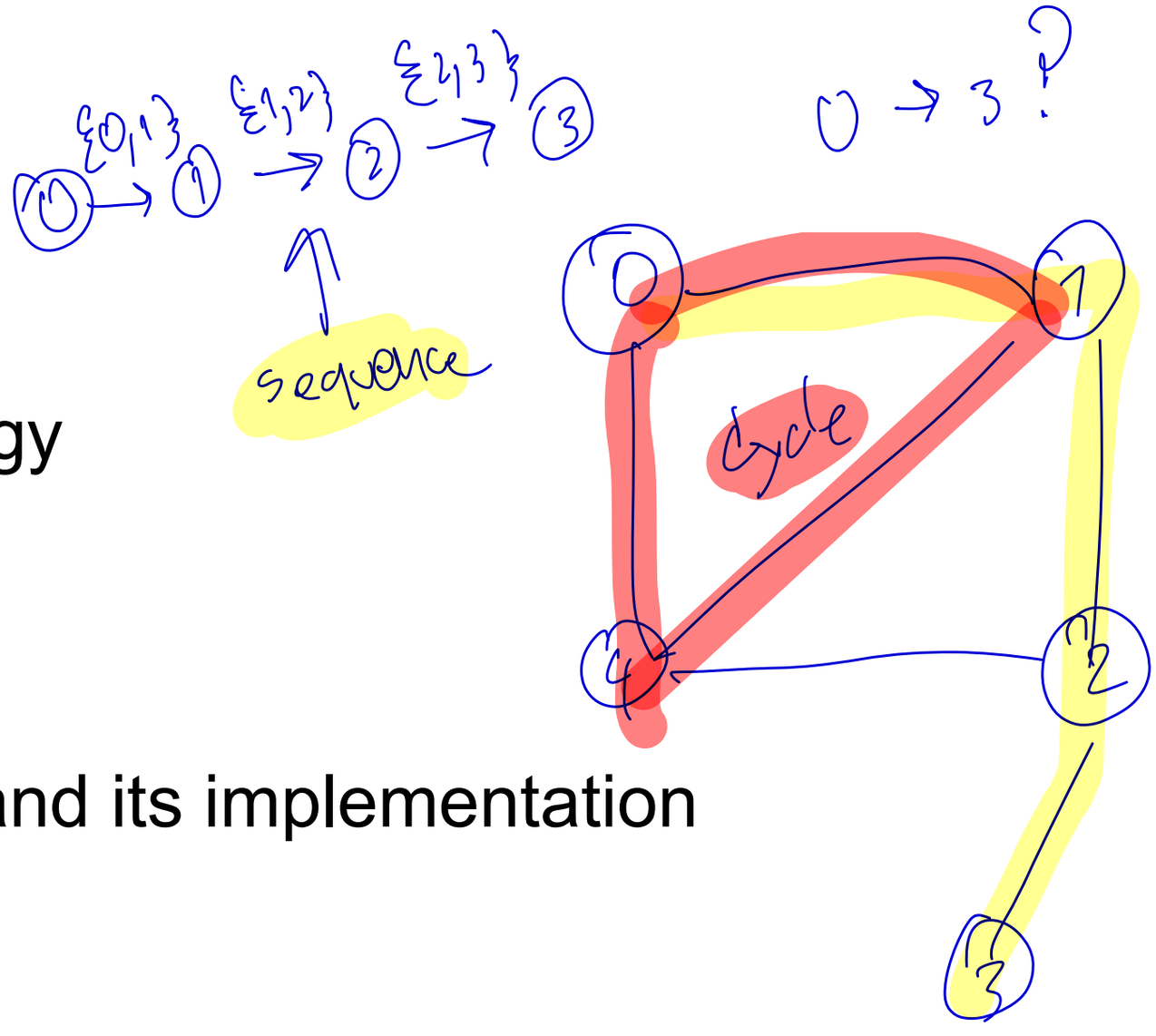
Outlines



- More graph terminology
- Graph traversals
  - Depth-first search and its implementation

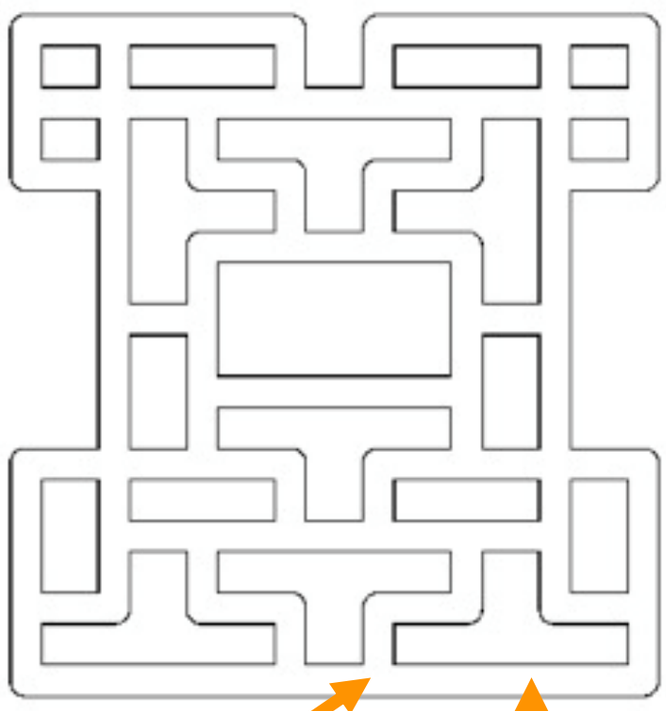
More Graph Terminology

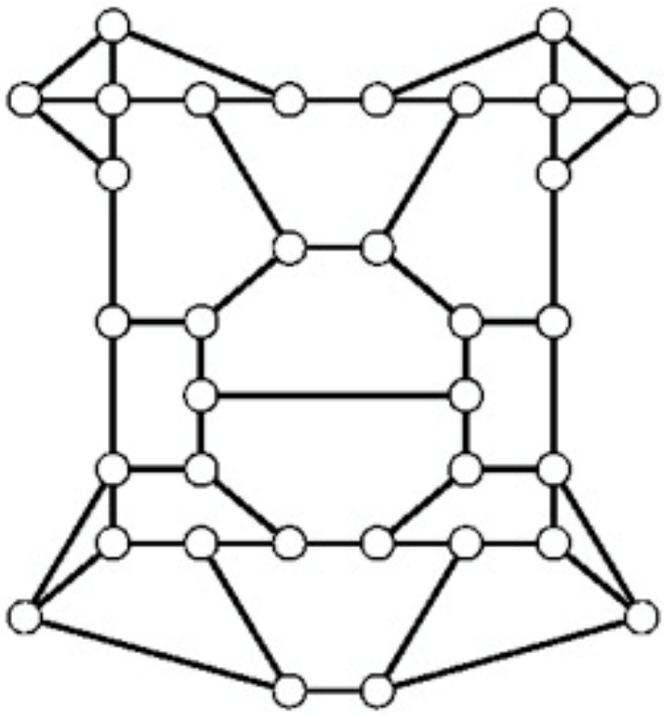
- ***Path*** := A sequence of alternating vertices and edges that starts at a vertex and ends at a vertex such that each edge is incident to its predecessor and successor vertex in the sequence
- ***Cycle*** := A path with more than one edge whose first and last vertices are the same
- ***Path's length*** := The number of vertices in the path minus one or the number of edges in the path
- ***Connected graph*** := A graph is *connected* if, for any two vertices, there is a path between them
- ***Connected components*** := If a graph is not connected, its maximal connected subgraphs are called the *connected components* of the graph

Graph Traversals (Graph Search)

- ***Traverse*** := Travel across the data structure to visit each element exactly once
  - *Example*: Traverse an array; traverse a linked list
  - In graph, the elements to visit are vertices and edges
- ***Graph traversal*** is a systematic procedure for exploring a graph by examining all its vertices and edges

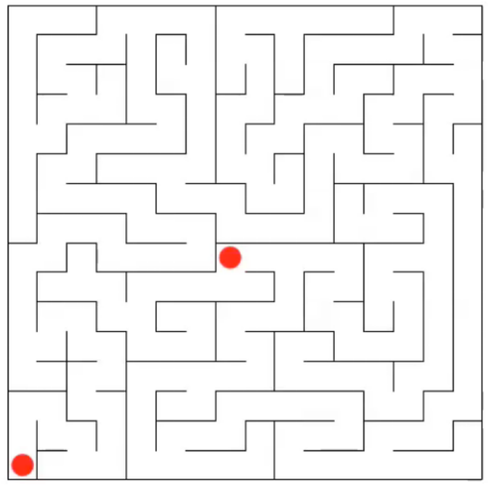
Maze Exploration: Motivation



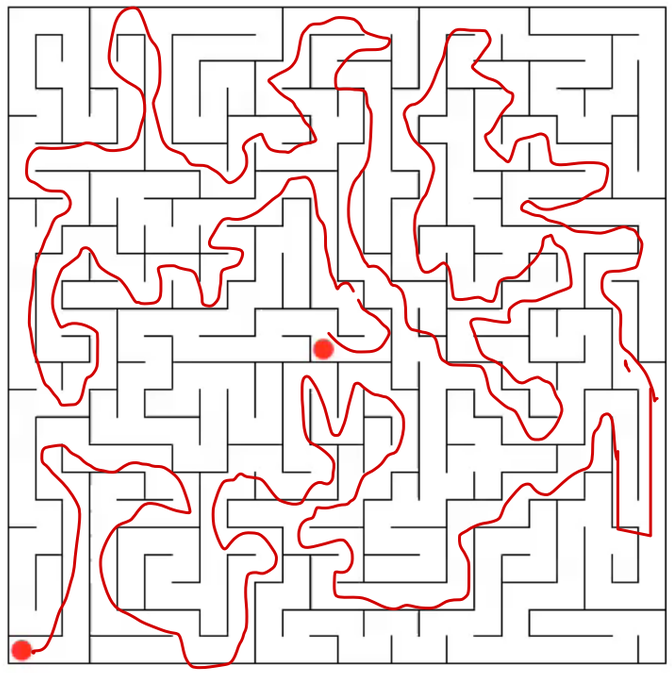


- Maze graph
  - Vertex = intersection
  - Edge = passage between intersections

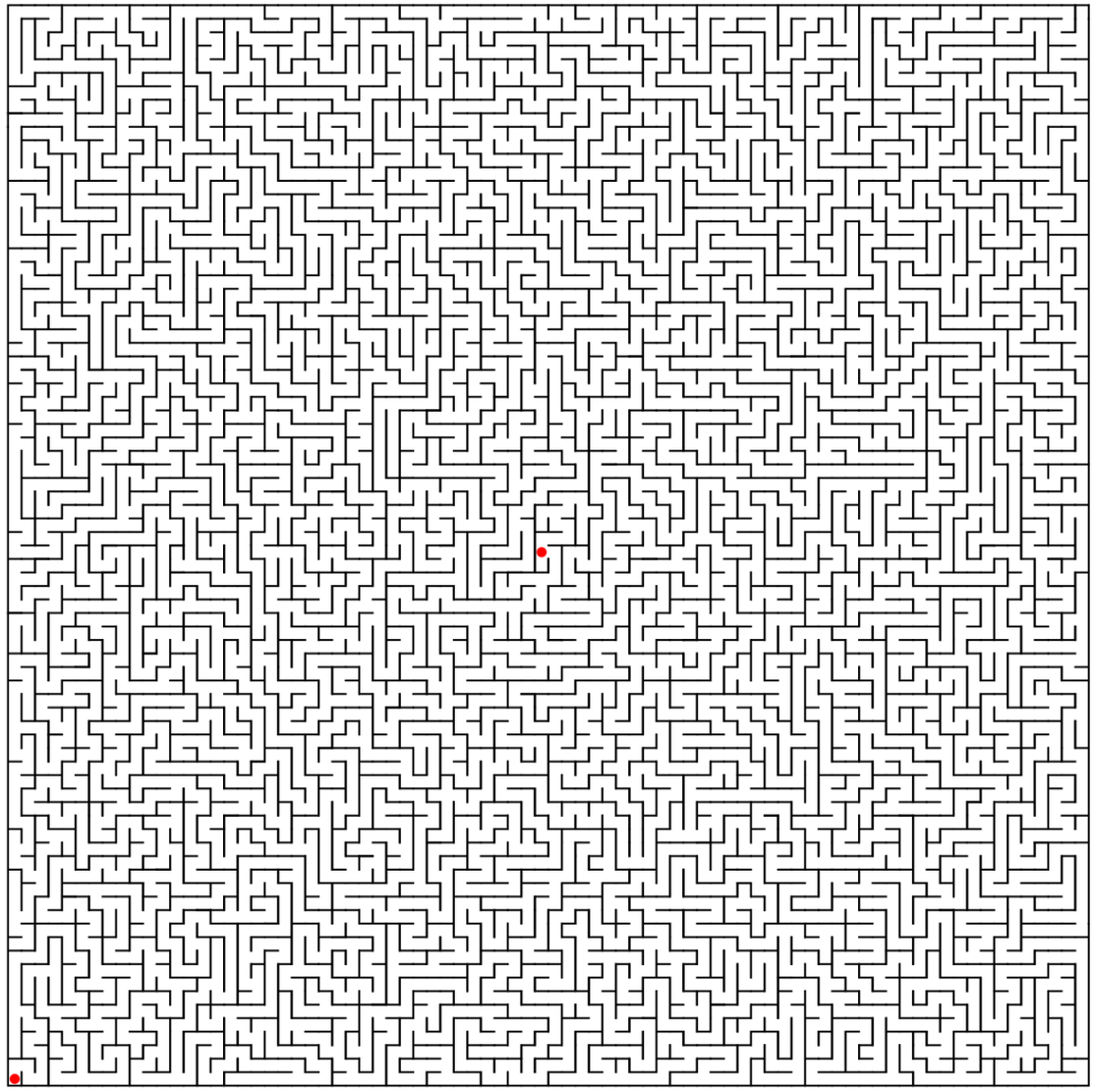
Maze Exploration Problem



Easy



Medium



Challenged

Trémaux's Algorithm

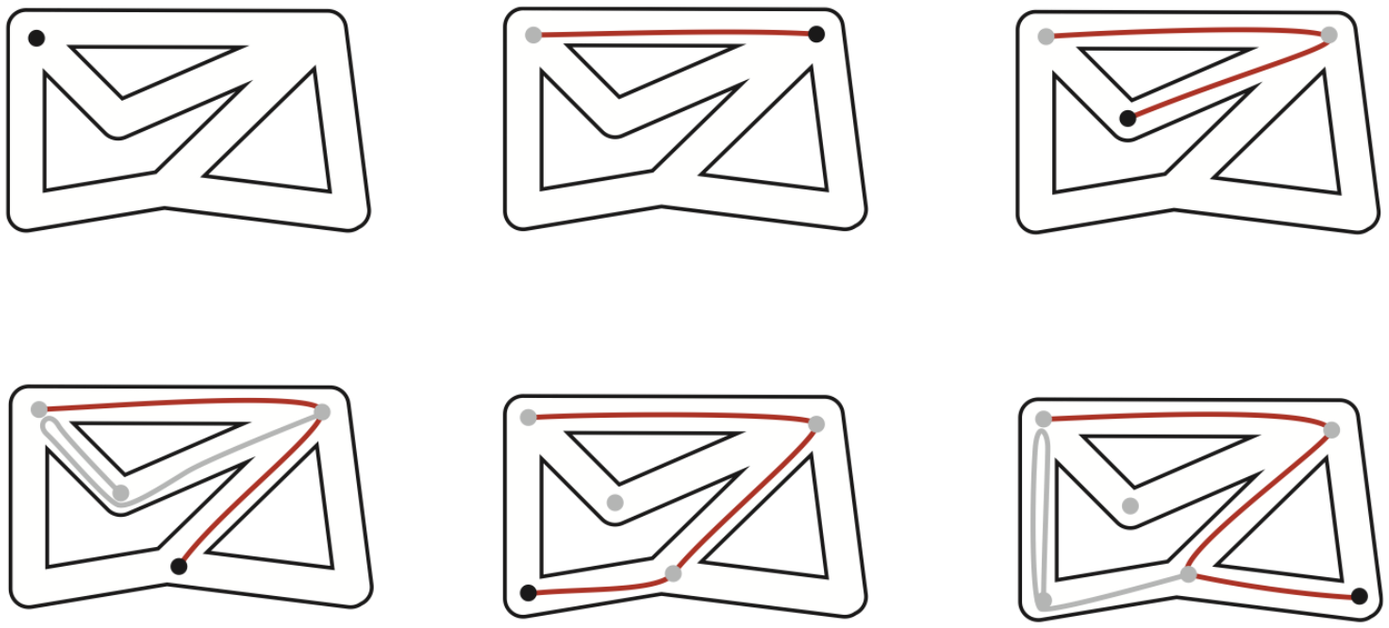
Key คือ กลับไปกลับมา

Mark ทุก แยก และ ไรเหี้ยก

- Unroll a ball of string behind you
- Mark each visited intersection and each visited passage

ถ้าไปมาแล้ว ไม่มี option ไปต้องกลับมา

- Backtrack when there is no unvisited options (hits a dead end)



Depth-First Search / Depth-First Traversal

- ***Depth-First Search*** (**DFS**): The idea is like exploring a maze
  - Follow the current path until you get stuck (hit a dead end)
  - Backtrack along breadcrumbs (~the string) until reach unvisited neighbor
  - Recursively explore
  - Careful not to repeat a vertex
- *Applications*: DFS can be used for testing several properties of graphs ใช้ทดสอบคุณสมบัติของ Graph
  - Test whether there is a path from one vertex to another
  - Test whether a graph is connected
  - Test whether a graph has a cycle

DFS in Undirected Graph (1)

- Implements breadcrumb by using a map or a list for collecting the visited vertices during the search

```
DFS-init(s, adjList):
    visited = {}
    DFS-visit(s, adjList, visited)
```

- Use recursion when visiting neighbors of the current (starting) vertex s

```
DFS-visit(s, adjList, visited):
    visited = visited U {s}
    for v in adjList[s]:
        if v not in visited:
            DFS-visit(v, adjList, visited)
```

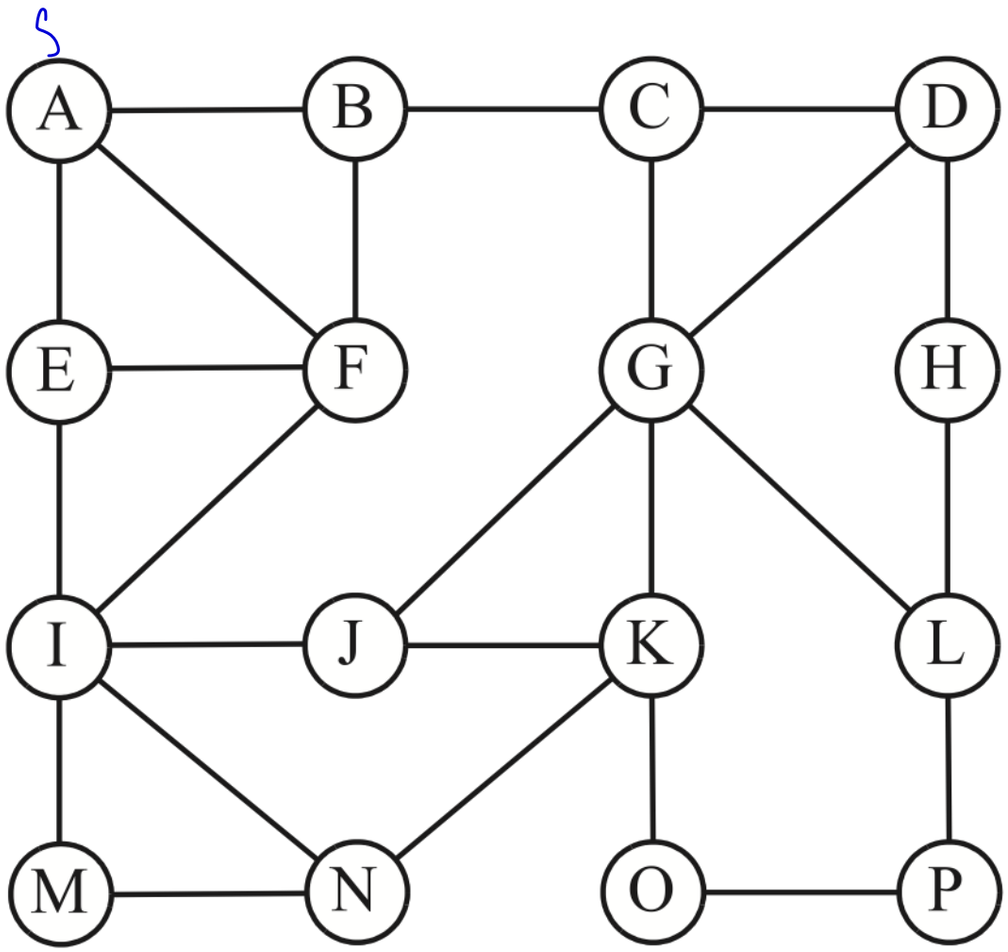
- *Remarks:* DFS-visit(s, adjList, visited) only sees stuff reachable from vertex s, so the search only explores the connected component that contains s. If the graph is connected, then it will explore the entire graph.

DFS in Undirected Graph (2)

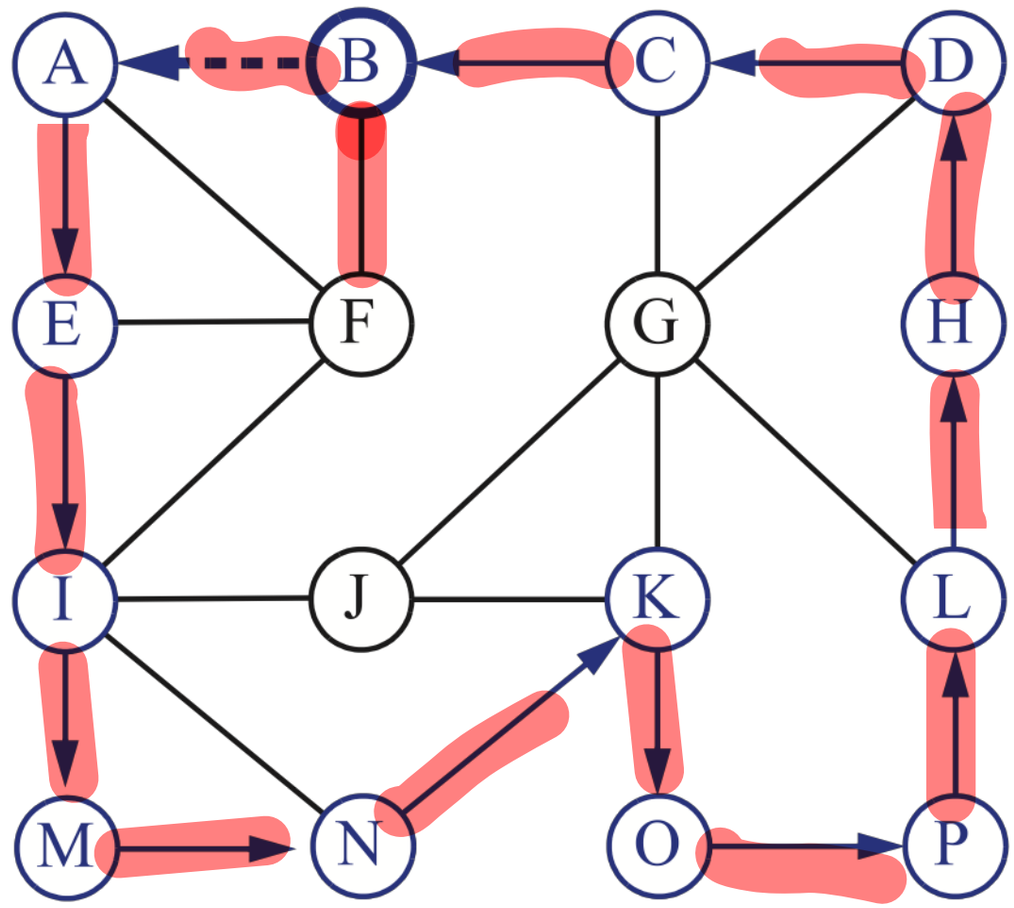
- To make sure that the search explores the entire graph, we need to apply DFS at each unvisited vertices

```
DFS-explore(V, adjList):
    visited = {}
    for s in V:
        if s not in visited:
            DFS-visit(s, adjList, visited)
```

DFS Example (1)

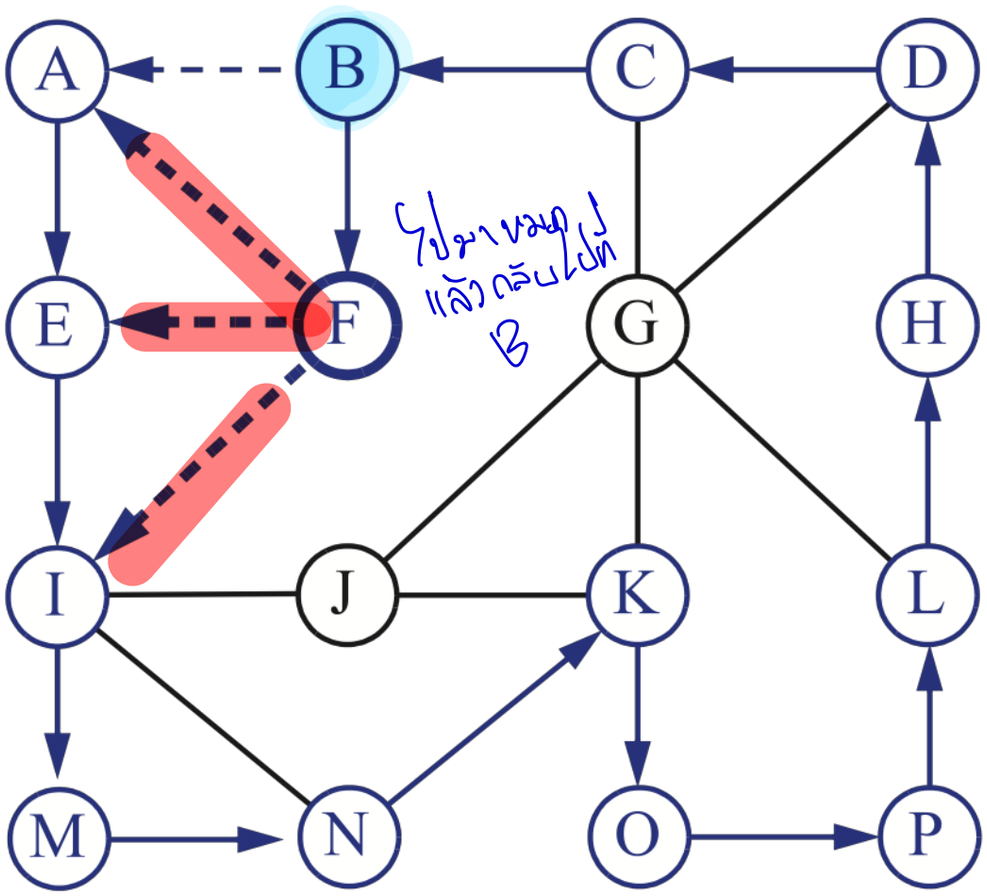


(a)

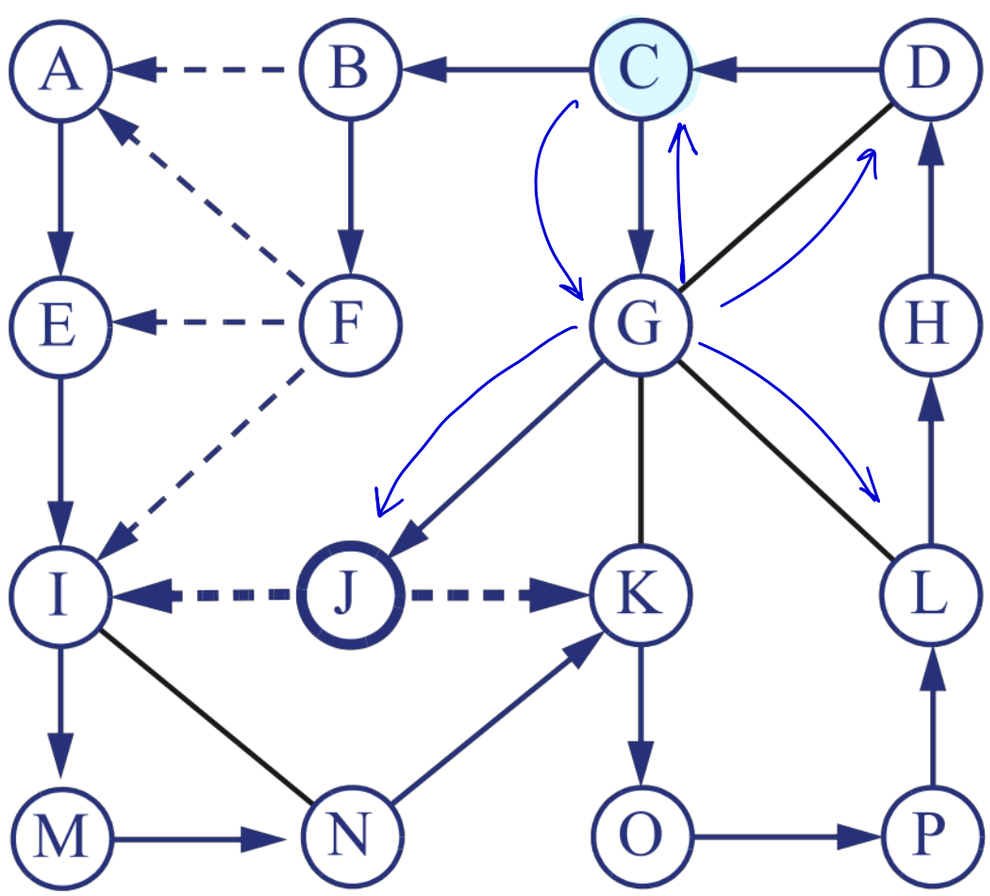


(b)

DFS Example (2)

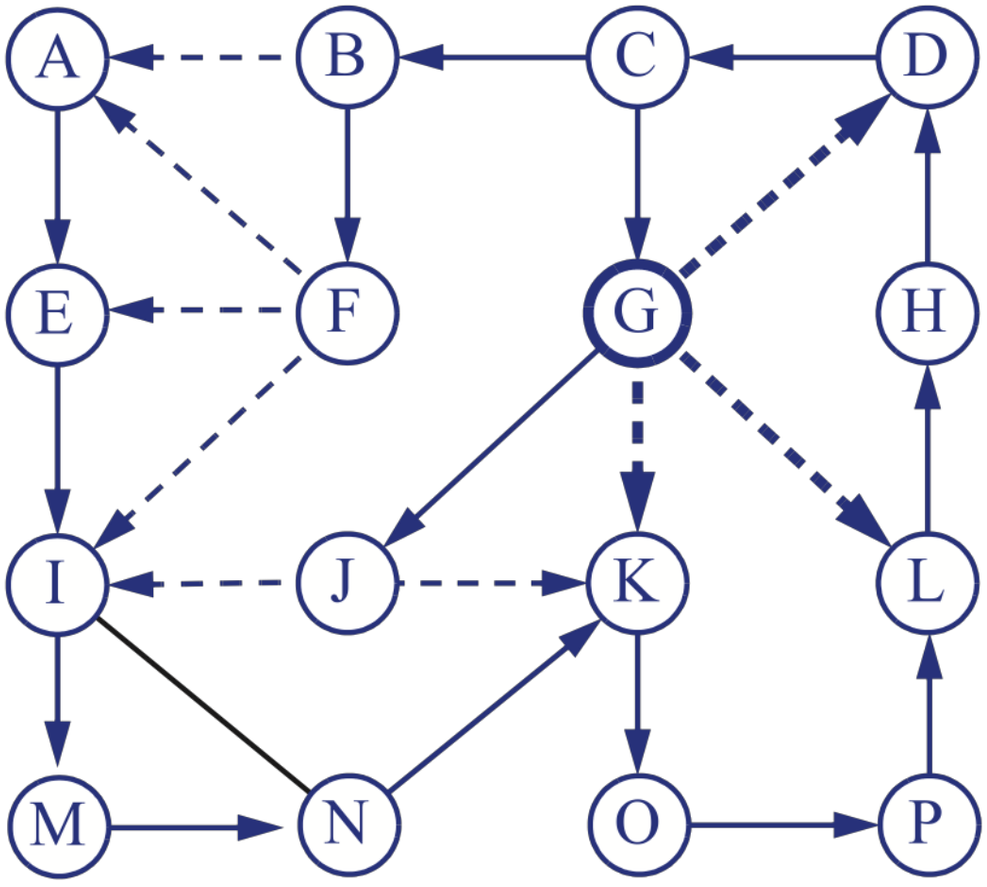


(c)

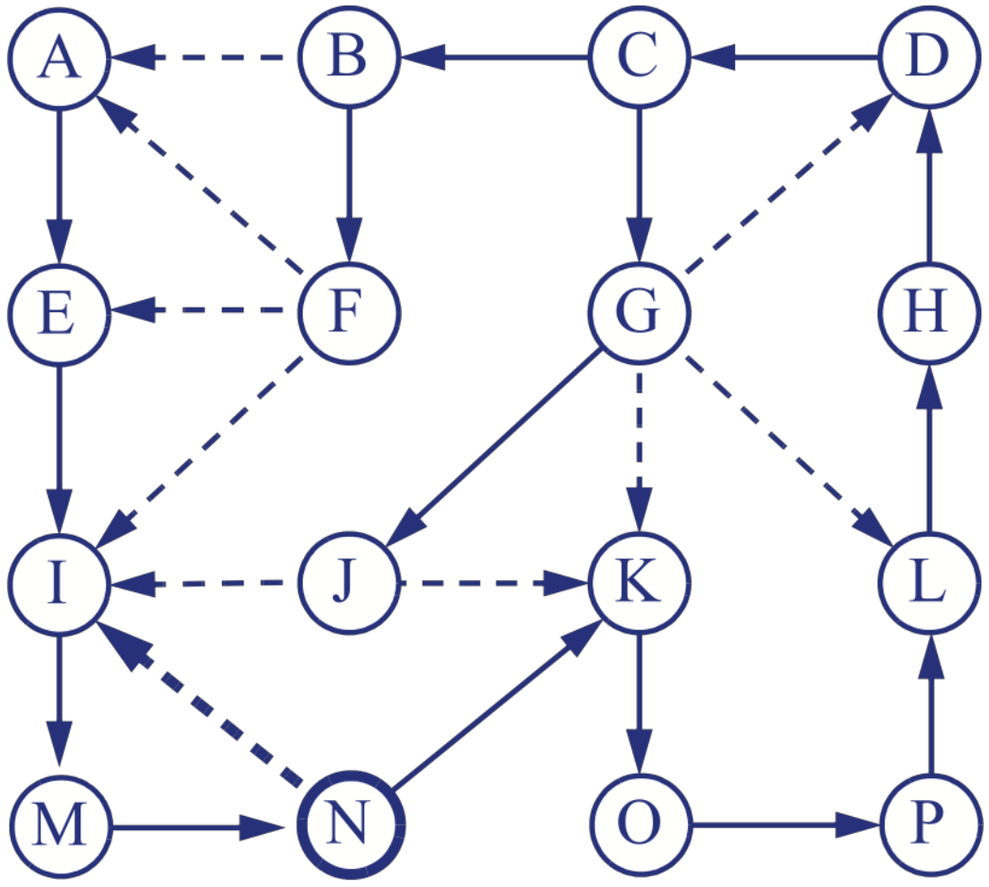


(d)

DFS Example (3)



(e)



(f)

DFS's Complexity

- With adjacency-list representation, DFS takes time $O(n+m)$ to traverse a graph with n vertices and m edges
- *Analysis*: DFS-visit gets called with a vertex s only once (before s is added to visited)
- The time taken by DFS-visit in the worst case can be

$$\sum_{s \in V} deg(s) = O(m)$$

- The time taken by DFS-explore just adds $O(n)$

DFS Implementation in C++ (1)

```
// C++ program to print DFS traversal from a given vertex in a  given graph
#include<iostream>
#include<list>
using namespace std;

// Graph class represents a undirected graph using adjacency list representation
class Graph
{
    int V;    // No. of vertices
    list<int> *adj; // Pointer to an array containing adjacency lists

    void DFSVisit(int s, bool visited[]);  // A recursive function used by DFS

public:
    Graph(int V);   // Constructor
    void addEdge(int v, int w); // Function to add an edge to graph
    void DFSInit(int s); // DFS traversal of the vertices reachable from s
};
```

DFS Implementation in C++ (2)

```
Graph::Graph(int V)
{
   this->V = V;
   adj = new list<int>[V];
}

void Graph::addEdge(int v, int w)
{
   adj[v].push_back(w); // Add w to v's list
   adj[w].push_back(v); // Add v to w's list
}

void Graph::DFSVisit(int s, bool visited[])
{
   // Mark the current node as visited and print it
   visited[s] = true;
   cout << s << " ";

   // Recurse for all the vertices adjacent to this vertex
   list<int>::iterator i;
   for (i = adj[s].begin(); i != adj[s].end(); ++i)
      if (!visited[*i])
         DFSVisit(*i, visited);
}
```

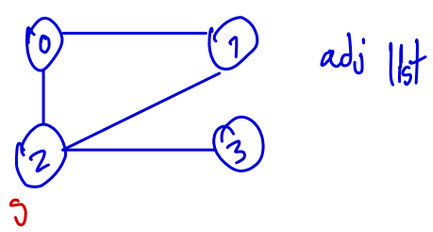
DFS Implementation in C++ (3)

```
// DFS traversal of the vertices reachable from v
void Graph::DFSInit(int s)
{
   // Mark all the vertices as not visited
   bool *visited = new bool[V];
   for (int i = 0; i < V; i++)
      visited[i] = false;

   // Call the recursive helper function to print DFS traversal
   DFSVisit(s, visited);
}

int main()
{
   Graph g(4);
   g.addEdge(0, 1);
   g.addEdge(0, 2);
   g.addEdge(1, 2);
   g.addEdge(2, 0);
   g.addEdge(2, 3);

   cout << "Following is Depth First Traversal (starting from vertex 2) \n";
   g.DFSInit(2);
   return 0;
}
```



Output:
Following is Depth First Traversal (starting from vertex 2)
2 0 1 3

Exercise

- As we have seen, the C++ implementation of DFS uses the technique of recursion.

- *Questions*:

- a) How can we implement DFS in iterative version?

- b) How can we modify DFS to check whether a graph is connected?

- c) How can we modify DFS to check whether a graph has a cycle?