Data Structures and Algorithms

Lecture 16: Graph Traversals (cont.)

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Outlines

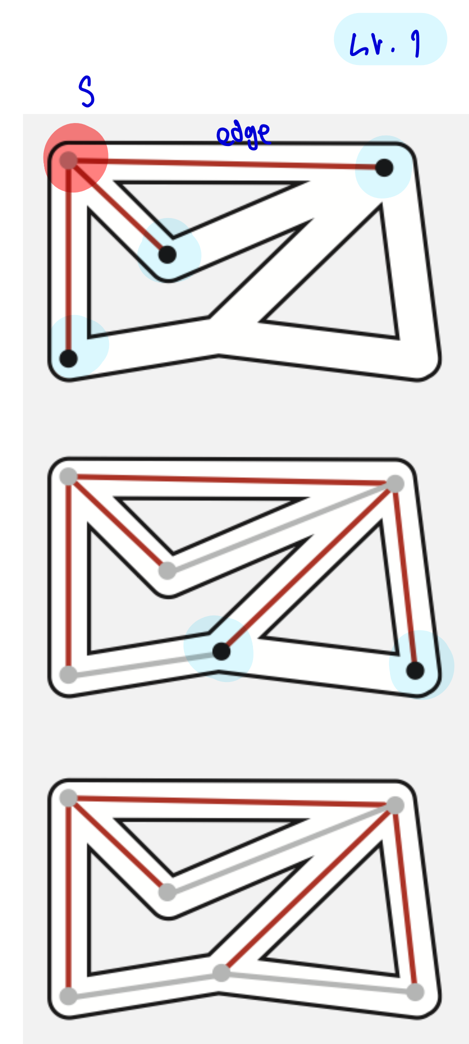
- Graph traversals (cont.)
    - Breath-first search and its implementation
    - Breath-first search properties

DFS < ลึกก่อน

Breath-First Search / Breath-First Traversal

แบ่งการทำงานเป็นรอบ

- ***Breath-First Search*** (**BFS**): Like DFS, but in BFS, we unroll the string in a more conservative manner ทุกจุดที่มาจาก s ได้ step เดียว
  - Start at vertex *s* which we assign it as level 0, define the anchor for the string
    - *First round*: let out the string the length of one edge, we visit all the vertices we can reach from *s.* These vertices are assigned as level 1
    - *Second round*: let out the string the length of two edges, we visit all the unvisited vertices we can reach from the level 1 vertices. These vertices are then assigned as level 2
    - ….
    - Terminates when every vertex has been visited เมื่อทุก Node ถูก Expo



BFS in Undirected Graph (1)

- BFS proceeds in rounds and subdivides the vertices into levels, we will need to memorize vertices at each level

start Node

```
BFS-visit(s, adjList, visited = {}):
    visited = visited U {s}          s ไปแล้ว → O(1)
    L_0 = {s}                        → O(1)
    i = 0                            → O(1)
    while L_i is not empty:          → O(1) per iteration
        L_{i+1} = {}
        for each u in L_i:           O(n) in total
            for each v in adjList[u]:
                if v is not in visited:
                    visited = visited U {v}      O(1) } O(1) × outdeg(u)
                    L_{i+1} = L_{i+1} U {v}
        i = i+1
```

total complexity

O(n) + O(1)

ว่า 1 Node อยู่ได้ Lv เดียว

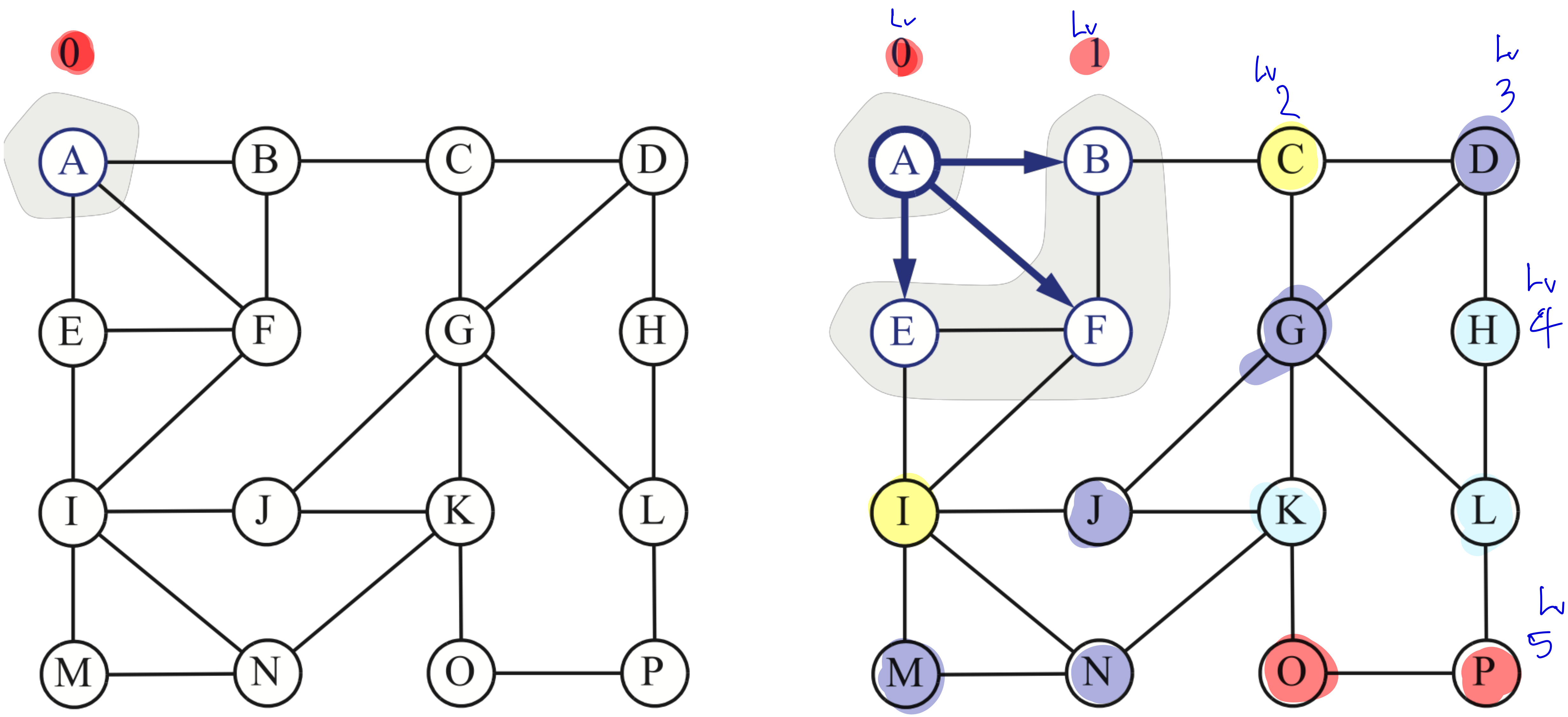
- *Remarks:* BFS-visit(s, adjList, {}) only sees stuff reachable from vertex s, so the search only explores the connected component that contains s. If the graph is connected, it will explore the entire graph.

BFS in Undirected Graph (2)

- To make sure that the search explores the entire graph, we need to apply BFS at each unvisited vertex

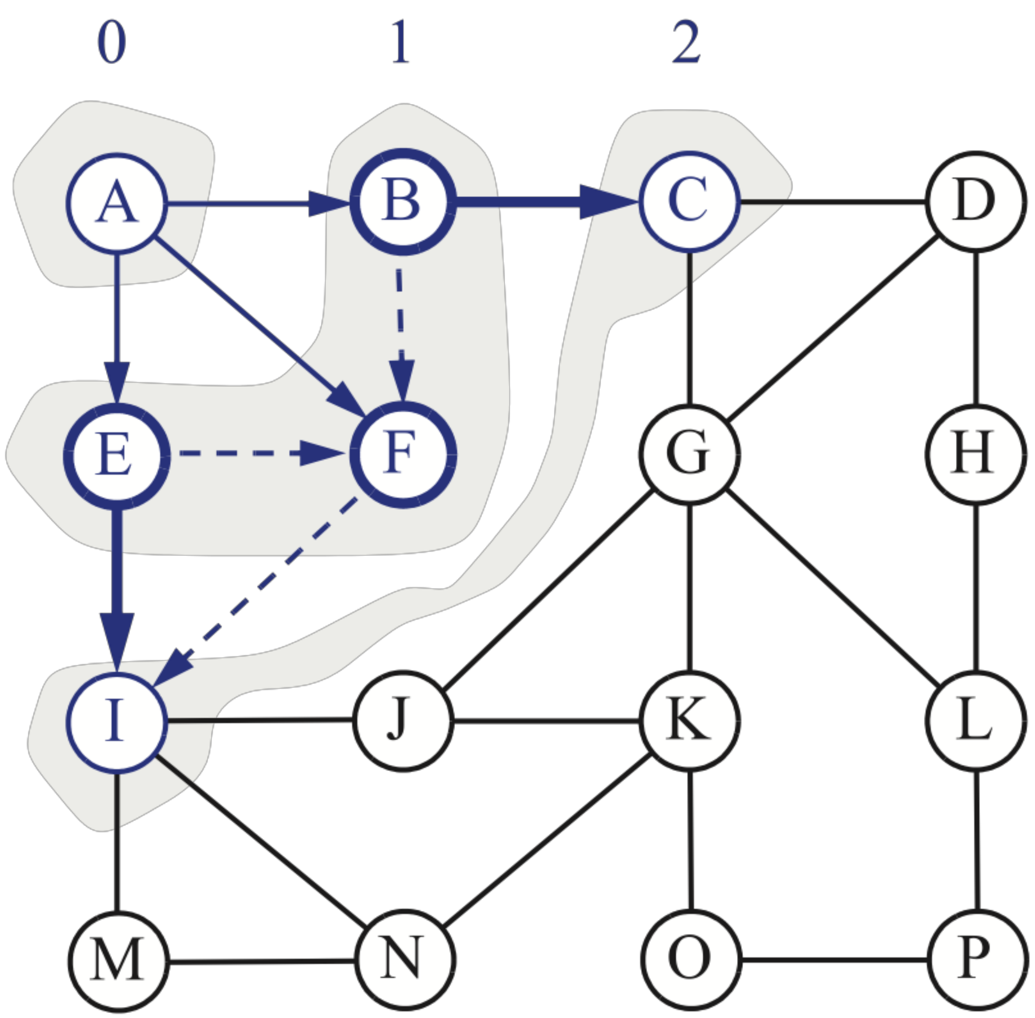
```
BFS-explore(V, adjList):
    visited = {}
    for each s in V:
        if s not in visited:
            BFS-visit(s, adjList, visited)
```

BFS Example (1)

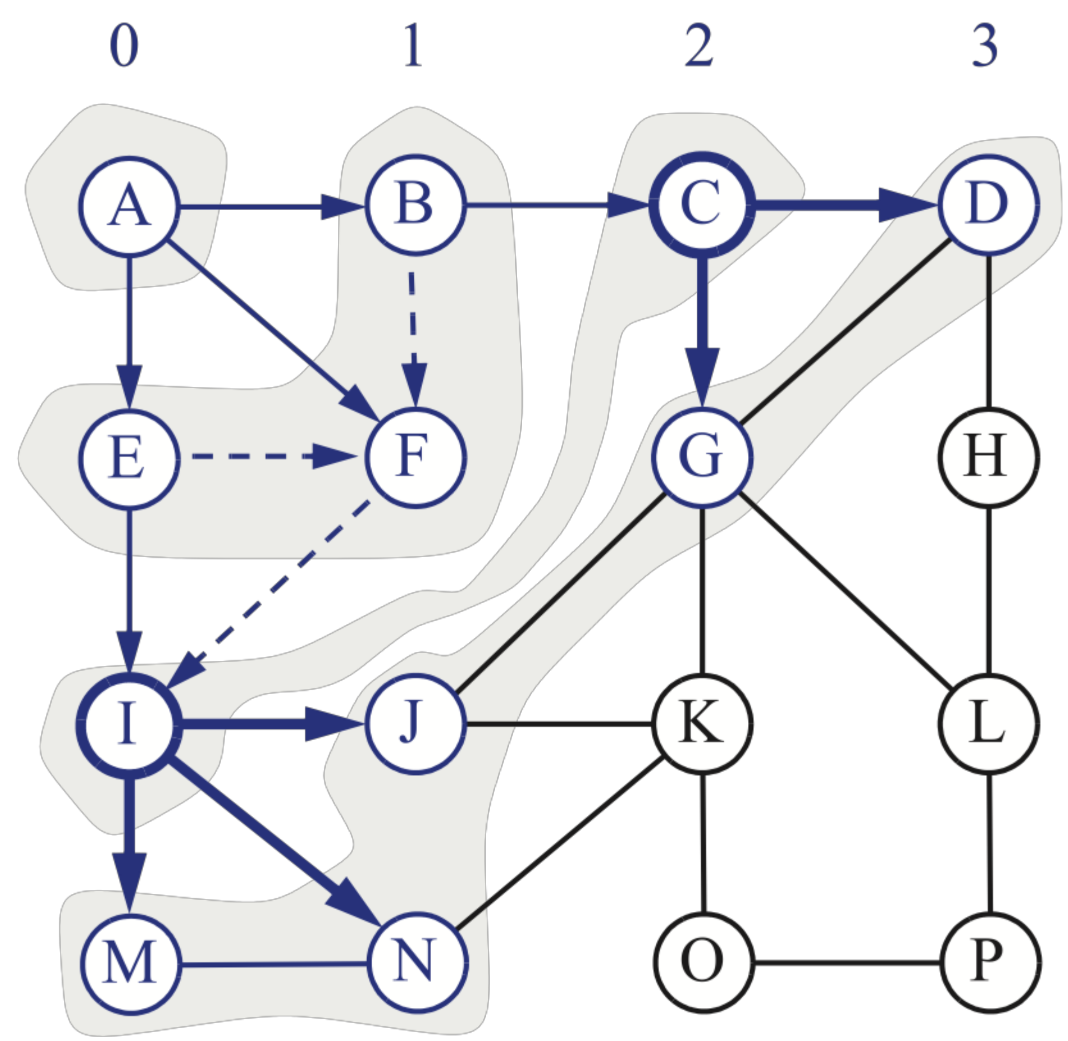


(a) (b)

BFS Example (2)

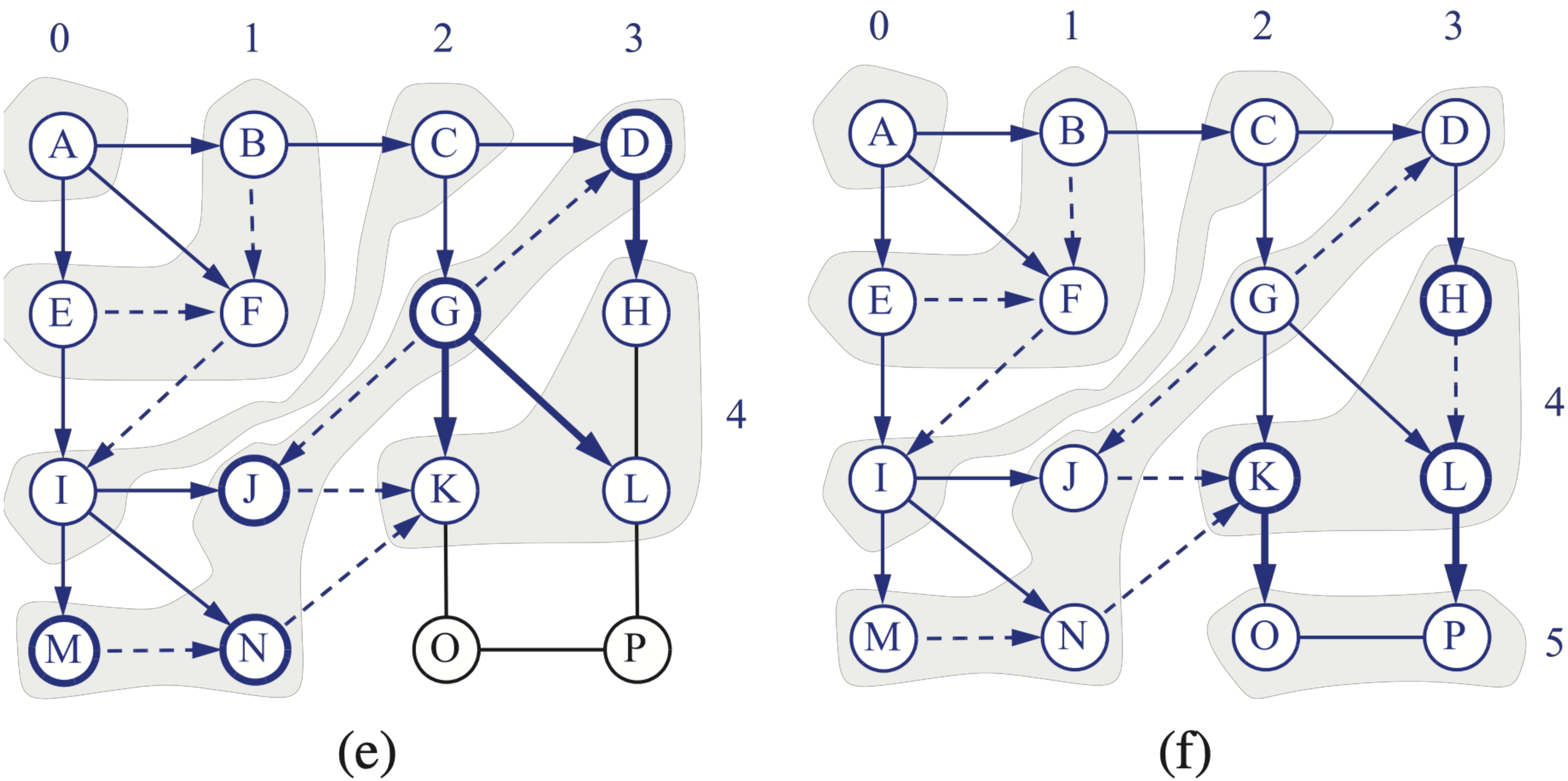


(c)



(d)

BFS Example (3)



BFS's Complexity

- With adjacency-list representation, like DFS, BFS takes time $O(n+m)$ to traverse a graph with n vertices and m edges

- *Analysis*: In BFS-visit, we only need to visit the neighbors of u, for each u in V

$$\sum_{u \in V} deg(u) = O(m)$$

- The time taken by BFS-explore just adds $O(n)$

BFS Implementation in C++ (1)

```
// C++ program to print BFS traversal from a given vertex in a  given graph
#include<iostream>
#include<list>
using namespace std;

// Graph class represents a undirected graph using adjacency list
// representation
class Graph
{
   int V;    // No. of vertices
   list<int> *adj; // Pointer to an array containing adjacency lists

public:
   Graph(int V);   // Constructor
   void addEdge(int v, int w); // Function to add an edge to graph
   void BFSVisit(int s); // BFS traversal of the vertices reachable from s
};

Graph::Graph(int V)
{
   this->V = V;
   adj = new list<int>[V];
}

void Graph::addEdge(int v, int w)
{
   adj[v].push_back(w); // Add w to v’s list
   adj[w].push_back(v); // Add v to w’s list
}
```

BFS Implementation in C++ (2)

assume $s = 4$

จากนั้น Dequeue ออกไป Process

Lv1 Lv2

Q(~~4~~ 1, 0, 6, 2, 1 –)

Lv0 จะถูกใส่เข้าไปก่อน

```
// BFS traversal of the vertices reachable from s
void Graph::BFSVisit(int s)
{
   // Mark all the vertices as not visited
   bool *visited = new bool[V];
   for (int i = 0; i < V; i++)
      visited[i] = false;
   // Create a queue for BFS
   list<int> queue;  ทำ Queue ที่ โครงสร้างเป็น list
   // Mark the current node as visited and enqueue it
   visited[s] = true;
   queue.push_back(s);
   // 'i' will be used to get all adjacent vertices of a vertex u
   list<int>::iterator i;
   int u;

   while(!queue.empty()){
      // Dequeue a vertex from queue and print it
      u = queue.front();
      cout << u << endl;
      queue.pop_front();
      // Get all adjacent vertices of the dequeued vertex u. If a adjacent has not been visited,
      // then mark it visited and enqueue it
      for (i = adj[u].begin(); i != adj[u].end(); ++i) {
         if (!visited[*i]) {
            visited[*i] = true;
            queue.push_back(*i);
         }
      }
   }
}
```

BFS Implementation in C++ (3)

```
int main()
{
   Graph g(4);
   g.addEdge(0, 1);
   g.addEdge(0, 2);
   g.addEdge(1, 2);
   g.addEdge(2, 0);
   g.addEdge(2, 3);

   cout << "Following is Breath-First Traversal (starting from vertex 0) \n";
   g.BFSVisit(0);
   return 0;
}
```

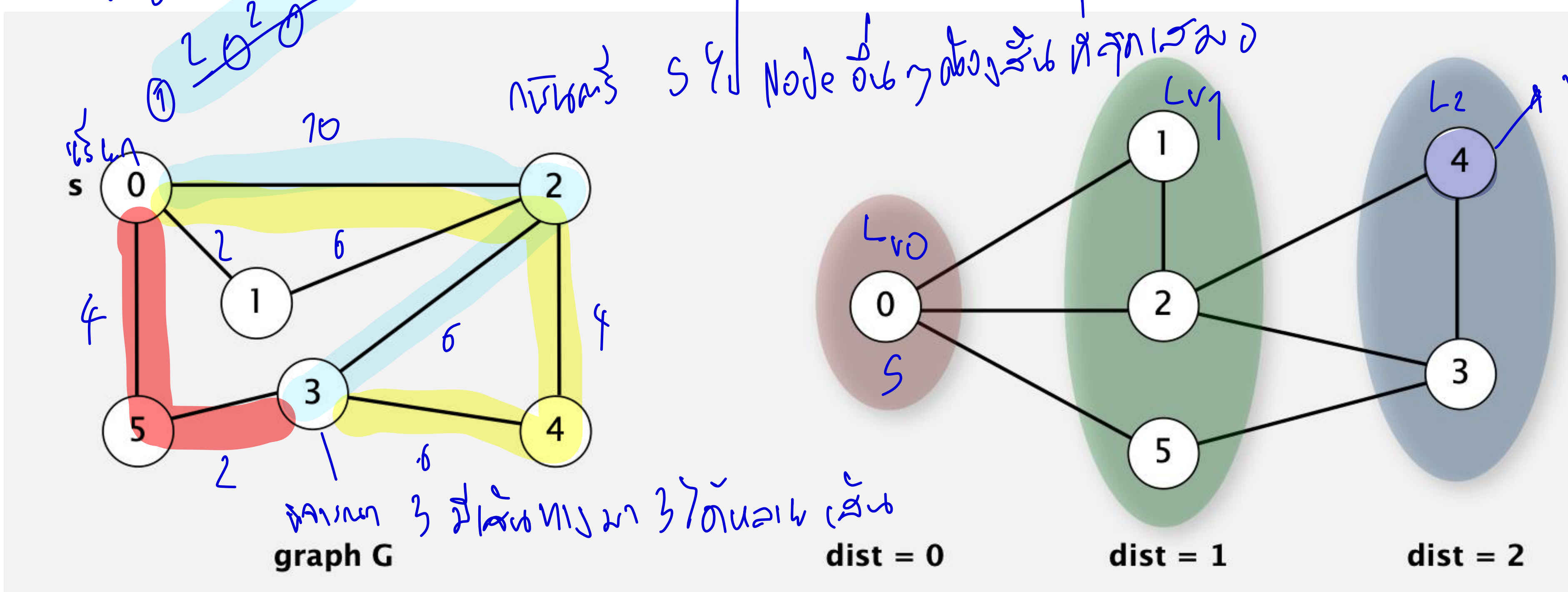
```
Output:
Following is Breath-First Traversal (starting from vertex 0)
0
1
2
3
```

DFS & BFS การเช็คว่า connected แต่ DFS ไม่การันตีว่า shortest

BFS's Properties

Node v อยู่ Lv I แสดงว่าห่างจาก Node s I เส้นเชื่อม

- **Proposition**: If G is a connected graph, BFS computes a *shortest path* (a path of shortest length) from *s* to all other vertices.

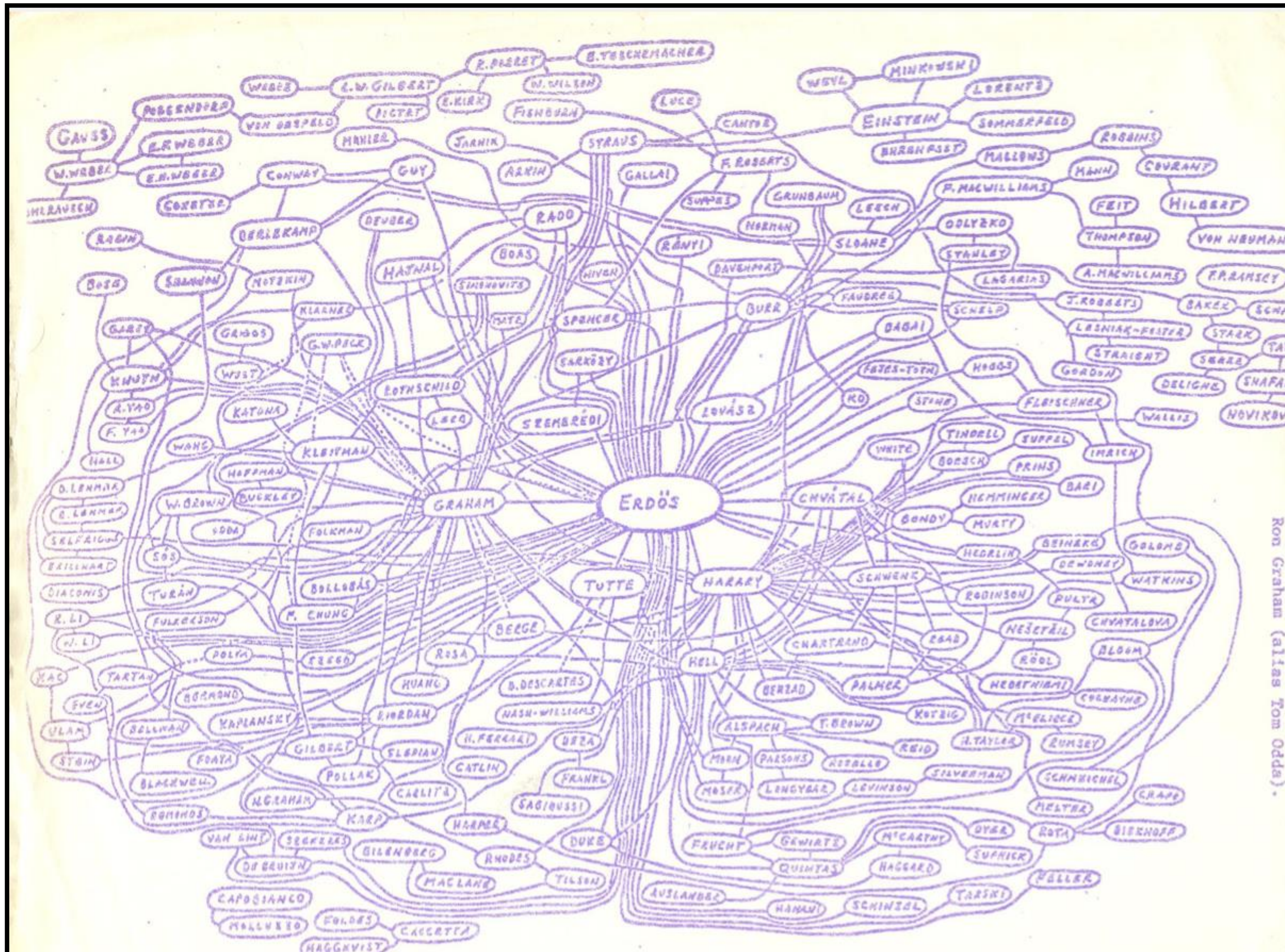


- *Remark*: BFS examines the vertices in increasing distance order from *s.* So, if *v* is at distance *k* from *s*, i.e., the length of shortest paths from *u* to *v* is *k,* then *k* will appear as one of the level k vertices during the search

BFS Applications

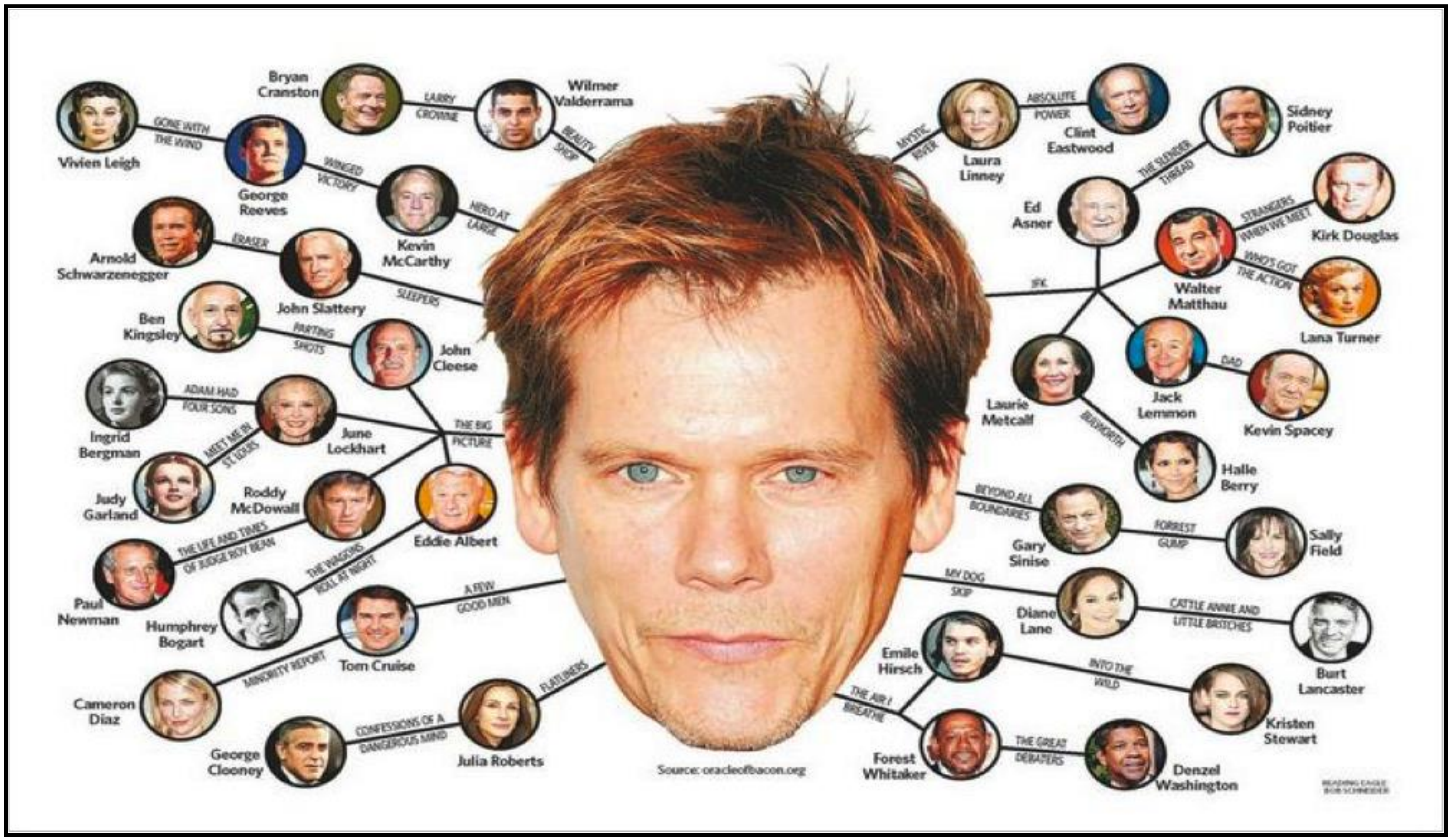
- *Applications*: Like DFS, BFS can be used to testing a number of properties of graphs
  - Test whether there is a path from one vertex to another (why?)
  - Test whether a graph is connected (how?)
  - Test whether a graph has a cycle (how?)

Example of BFS Applications: Erdös Number

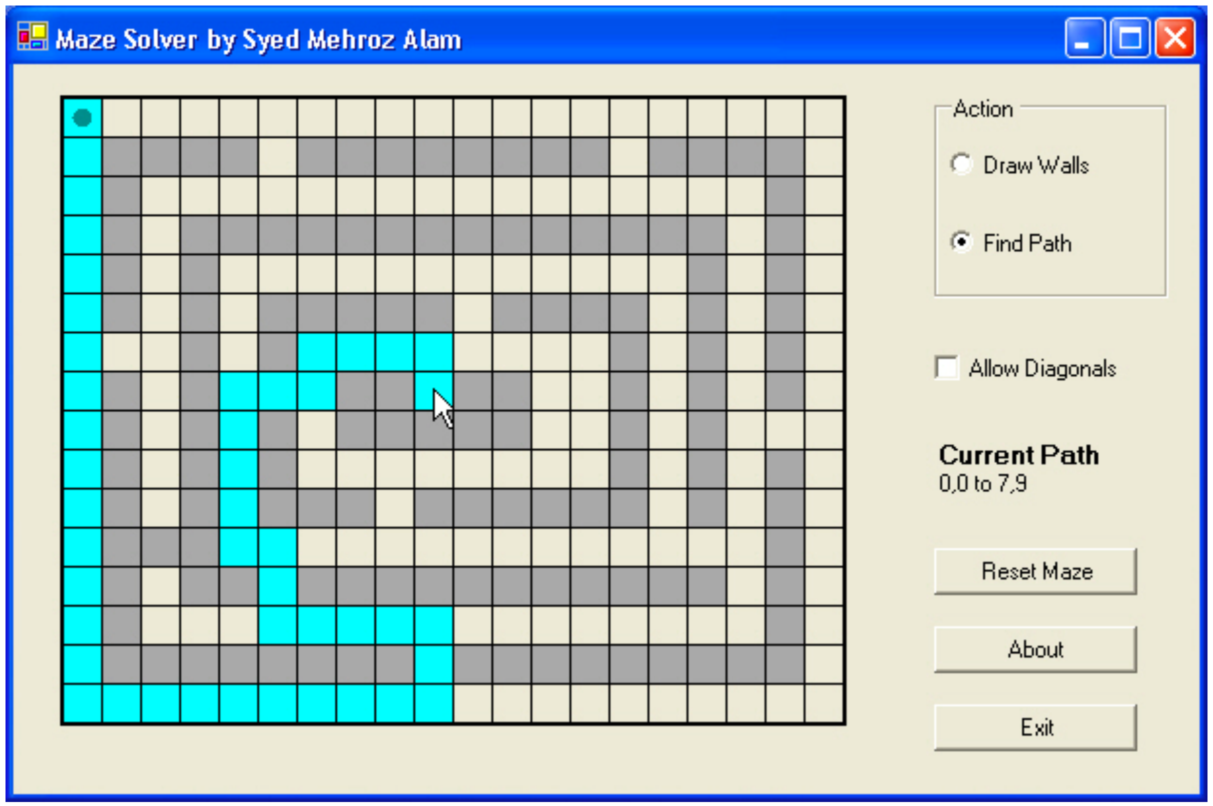


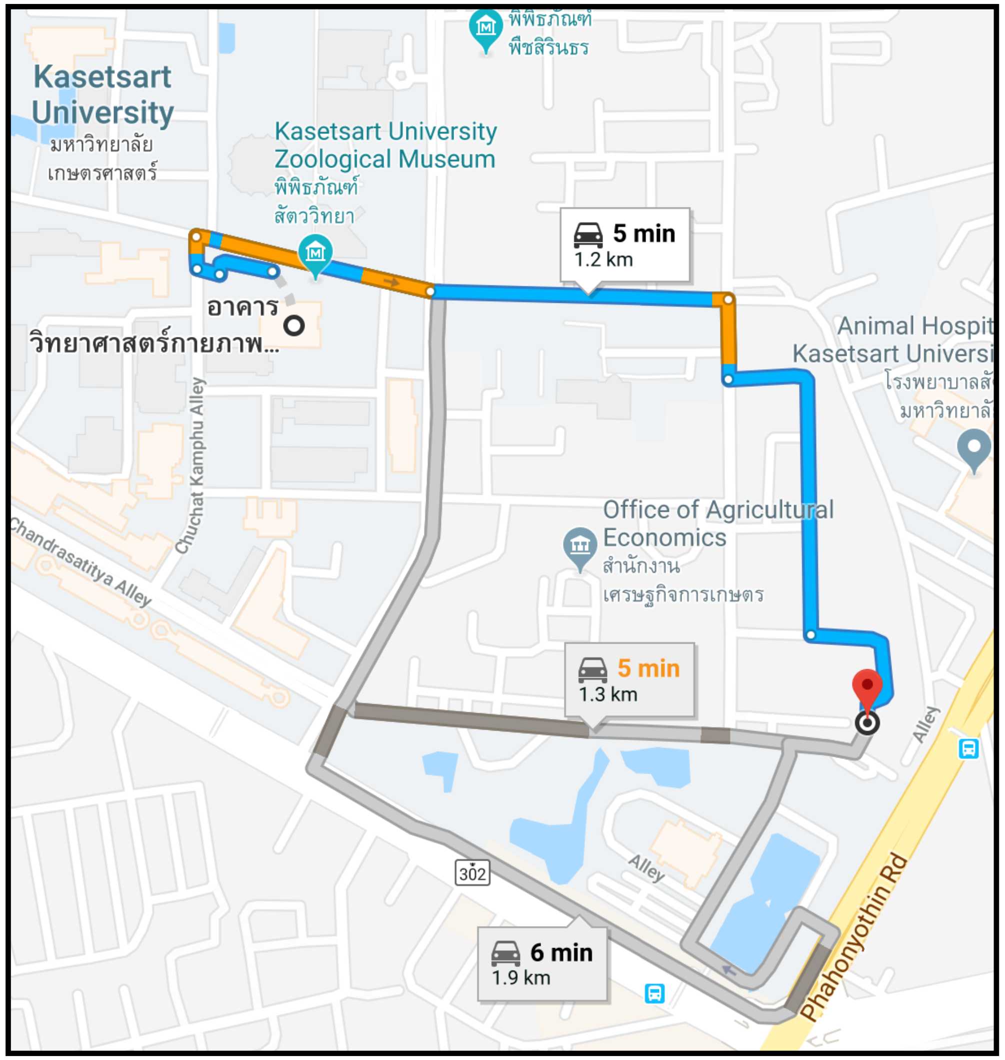
hand-drawing of part of the Erdös graph by Ron Graham

Example of BFS Applications: Oracle of Kevin Bacon

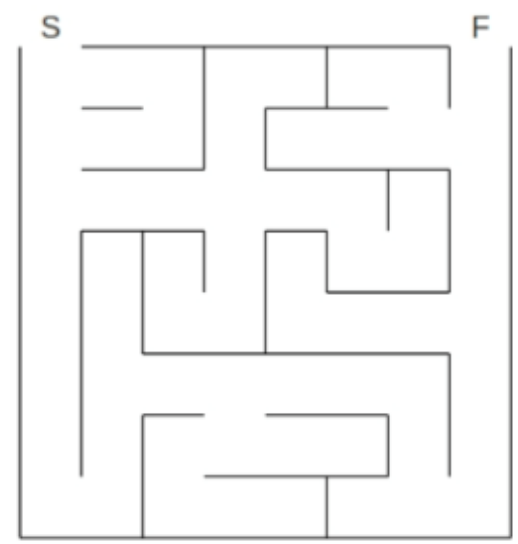


Example of BFS Applications: Path Finder





Programming Excerise



You are to create a C/C++ program that does the followings:

1. Create the graph that can represents the above maze.

2. Traverse the graph using BFS to check whether there is a path from S to F (answer “YES” if there is path, and answer “NO” otherwise)