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MASTER THESIS

Differential cryptanalysis with SAT solvers

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AFFIDAVIT

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ABSTRACT

Hash functions are ubiquitous in the modern information age. They provide preimage, second preimage and collision resistance which are needed in a wide range of applications.

In August 2006, Wang et al. showed efficient attacks against several hash function designs including MD4, MD5, HAVAL-128 and RIPEMD. With these results differential cryptanalysis has been shown useful to break collision resistance in hash functions. Over the years advanced attacks based on those differential approaches have been developed.

To find collisions like Wang et al., a cryptanalyst needs to specify a differential characteristic. Looking at the differential behavior of the underlying operations of the hash algorithm shows how differential values propagate in the algorithm. The goal is to find a differential characteristic whose differences cancel out in the output. Once such a differential characteristic was discovered, in a second step the actual values for those differences are defined yielding an actual hash collision.

Finding a differential characteristic can be a cumbersome and tedious task. Whereas propagation can be automated using dedicated tools, finding an initial differential characteristic is a difficult task as it can be specified with arbitrary levels of granularity.

SAT solvers inherently implement both tasks. They consecutively propagate values which narrow the search space. The probability to find a satisfiable assignment increases if the narrowed search space has many satisfiable assignments. And finally the assignment reveals initial values. On the other hand, SAT solvers have no notion of differential values and therefore problem encoding becomes an important topic.

In this thesis we look at differential characteristics and encode them as SAT problem. A SAT solver tells us whether a differential characteristic can represent a hash collision or not. We implemented a framework which allows us to verify differential behavior for integer operations. We then looked at the encoded problems in details and tried to change the encoding to improve the runtime of the SAT solver. We also provide a small CNF analysis library to compare an encoded problem with others.

Keywords: hash function, differential cryptanalysis, differential characteristic, MD4, SHA-256, collision resistance, satisfiability, SAT solver

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Thank you.

どもありがとうございました。

All source code is available at lukas-prokop.at/proj/megosat and published under terms and conditions of Free/Libre Open Source Software. This document was printed with Lua^AT_EX and Linux Libertine Font.

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Chapter 1

Introduction

1.1 Overview

Hash functions are used as cryptographic primitives in many applications and protocols. They take an arbitrary input message and provide a hash value. Input message and hash value are considered as byte strings in a particular encoding. The hash value is of fixed length and satisfies several properties which make it useful in a variety of applications.

In this thesis we will consider the hash algorithms MD4 and SHA-256. They use basic arithmetic functions like addition and bit-level functions such as XOR to transform an input to a hash value. We use a bit vector as input to this implementation and all operations applied to this bit vector will be represented as clauses of a SAT problem. Additionally we represent differential characteristics of hash collisions as SAT problem. If and only if satisfiability is given, the particular differential state is achievable using two different inputs leading to the same output. As far as SAT solvers return an actual model satisfying that state, we get an actual hash collision which can be verified and visualized. If the internal state of the hash algorithm is too large, the attack can be computationally simplified by modelling only a subset of steps of the hash algorithm or changing the modelled differential path.

Based on experience with these kind of problems with previous non-SAT-based tools we aim to apply best practices to a satisfiability setting. We will discuss which SAT techniques lead to best performance characteristics for our MD4 and SHA-256 testcases.

1.2 Thesis Outline

This thesis is organized as follows:

In Chapter 1 we briefly introduce basic subjects of this thesis. We explain our high-level goal involving hash functions and SAT solvers.

In Chapter 2 we introduce the MD4 and SHA-256 hash functions. Certain design decisions imply certain properties which can be used in differential cryptanalysis. We discuss those decisions in this chapter after a formal definition of the function itself. Beginning with this chapter we develop a theoretical notion of our tools.

In Chapter 3 we discuss approaches of differential cryptanalysis. We start off with work done by Wang, et al. and followingly introduce differential notation to simplify representation of differential states. This way we can easily dump hash collisions.

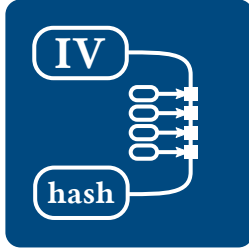
In Chapter 4 we discuss SAT solving. We give a brief overview over used SAT solvers and discuss how we can speed up SAT solvers for cryptographic problems.

In Chapter 5 we define SAT features which help us to classify SAT problems. This is a small subproject we did to look at properties of resulting DIMACS CNF files.

In Chapter 6 we discuss how we represent a problem (i.e. the hash function and a differential characteristic) as SAT problem. This ultimately allows us to solve the problem using a SAT solver.

In Chapter 7 we show data as result of our work. Runtimes are the main part of this chapter, but also results of the SAT features project are presented.

In Chapter 8 we suggest future work based on our results.



Chapter 2

Hash algorithms

In this chapter we will define hash functions and their desired security properties. Followingly we look at SHA256 and MD4 as established hash functions. We will represent them with Boolean algebra (in chapter 6) to make it possible to reason about states in those hash functions using SAT solvers.

2.1 Preliminaries Redux

Definition 2.1 (*Hash function*)

A *hash function* is a mapping $h : X \rightarrow Y$ with $X = \{0, 1\}^*$ and $Y = \{0, 1\}^n$ for some fixed $n \in \mathbb{N}_{\geq 1}$.

- Let $x \in X$, then $h(x)$ is called *hash value of x* .
- Let $h(x) = y \in Y$, then x is called *preimage of y* .

Hash functions are considered as cryptographic primitives used as building blocks in cryptographic protocols. A hash function has to satisfy the following security requirements:

Definition 2.2 (*Preimage resistance*)

Given $y \in Y$, a hash function h is *preimage resistant* iff it is computationally infeasible to find $x \in X$ such that $h(x) = y$.

Definition 2.3 (Second-preimage resistance)

Given $x \in X$, a hash function h is *second-preimage resistant* iff it is computationally infeasible to find $x_2 \in X$ with $x \neq x_2$ such that $h(x) = h(x_2)$. x_2 is called *second preimage*.

Definition 2.4 (Collision resistance)

A hash function h is *collision resistant* iff it is computationally infeasible to find any two $x \in X$ and $x_2 \in X$ with $x \neq x_2$ such that $h(x) = h(x_2)$. Tuple (x, x_2) is called *collision*.

As far as hash functions accept input strings of arbitrary length, but return a fixed size output string, existence of collisions is unavoidable [20]. However, good hash functions make it very difficult to find collisions or preimages.

Any digital data can be hashed (i.e. used as input to a hash function) by considering it in binary representation. The format or encoding is not part of the hash function's specification.

2.1.1 Merkle-Damgård designs

The Merkle-Damgård design is a particular design of hash functions providing the following security guarantee:

Definition 2.5 (Collision resistance inheritance)

Let F_0 be a collision resistant compression function. A hash function in Merkle-Damgård design is collision resistant if F_0 is collision resistant.

This motivates thorough research of collisions in compression functions. The design was found independently by Ralph C. Merkle and Ivan B. Damgård. It was described by Merkle in his PhD thesis [13, p. 13–15] and followingly used in popular hash functions such as MD4, MD5 and the SHA2 hash function family. The single-pipe design works as follows:

1. Split the input into blocks of uniform block size. If necessary, apply padding to the last block to achieve full block size.
2. Compression function F_0 is applied iteratively using the output y_{i-1} of the previous iteration and the next input block x_i , denoted $y_i = F_0(y_{i-1}, x_i)$.
3. An optional postprocessing function is applied.

2.1.2 Padding and length extension attacks

Hash functions of single-piped Merkle-Damgård design inherently suffer from length extension attacks. MD4 and SHA256 apply padding to their input to nor-

malize their input size to a multiple of its block size. The compression function is applied afterwards. This design is vulnerable to length extensions.

Consider some collision (x_0, x_1) with $F_0(x_0) = y = F_0(x_1)$ where x_0 and x_1 have a size of one block. Let p be a suffix with size of one block. Then also $(x_0 \parallel p, x_1 \parallel p)$ (where \parallel denotes concatenation) represents a collision in single-piped Merkle-Damgård designs, because it holds that:

$$F_0(F_0(x_0), p) = F_0(F_0(x_1), p) \iff F_0(y, p) = F_0(y, p)$$

Hence $(x_0 \parallel p, x_1 \parallel p)$ is a collision as well. As far as F_0 is applied recursively to every block, p can be of arbitrary size and (x_0, x_1) can be of arbitrary uniform size.

Because of this vulnerability, cryptanalysts only need to find a collision in compression functions. In our tests will only consider input of one block and padding will be neglected.

2.1.3 Example usage

One example showing the use of hash functions as primitives are JSON Web Tokens (JWT) specified in RFC 7519 [11]. Its application allows web developers to represent claims to be transferred between two parties.

Section 8 defines implementation requirements and refers to RFC 7518 [7], which specifies cryptographic algorithms to be implemented. “HMAC SHA-256” (besides “none”) is the only signature and MAC algorithm required to be implemented. Hence SHA-256 as hash algorithm is used as primitive in this configuration.

2.2 MD4

MD4 is a cryptographic hash function originally described in RFC 1186 [17], updated in RFC 1320 [18] and declared obsolete by RFC 6150 [22]. It was invented by Ronald Rivest in 1990 with properties given in Table 2.1. In 1995 [4] successful full-round attacks have been found to break collision resistance. Followingly preimage and second-preimage resistance in MD4 have been broken as well. Some of those attacks are described in [19] and [14]. We derived a Python 3 implementation based on a Python 2 implementation and made it available on github [16].

MD4 uses three auxiliary Boolean functions:

Definition 2.6

The Boolean IF function is defined as follows: If the first argument is true, the second argument is returned. Otherwise the third argument is returned.

block size	512 bits	namely variable block in RFC 1320 [18]
digest size	128 bits	as per Section 3.5 in RFC 1320 [18]
internal state size	128 bits	namely variables A, B, C and D
word size	32 bits	as per Section 2 in RFC 1320 [18]

TABLE 2.1: MD4 hash algorithm properties

The Boolean MAJ function returns true if the number of Boolean values true in arguments is at least 2. The Boolean XOR function returns true if the number of Boolean values true in arguments is odd.

Using the logical operators \wedge (AND), \vee (OR) and \neg (NEG) we can define them as (see section 4.1 for a thorough discussion of these operators):

$$\text{IF}(X, Y, Z) = (X \wedge Y) \vee (\neg X \wedge Z) \quad (2.1)$$

$$\text{MAJ}(X, Y, Z) = (X \wedge Y) \vee (X \wedge Z) \vee (Y \wedge Z) \quad (2.2)$$

$$\begin{aligned} \text{XOR}(X, Y, Z) = & (X \wedge \neg Y \wedge \neg Z) \vee (\neg X \wedge Y \wedge \neg Z) \\ & \vee (\neg X \wedge \neg Y \wedge Z) \vee (X \wedge Y \wedge Z) \end{aligned} \quad (2.3)$$

In the following a brief overview over MD4's design is given.

Padding and length extension. First of all, padding is applied. A single bit 1 is appended to the input. As long as the input does not reach a length congruent 448 modulo 512, bit 0 is appended. Followingly, length appending takes place. Represent the length of the input (without the previous modifications) in binary and take its first 64 less significant bits. Append those 64 bits to the input.

Initialization. The message is split into 512-bit blocks (i.e. 16 32-bit words). Four state variables A_i with $-4 \leq i < 0$ are initialized with these hexadecimal values:

$$[A_{-4}] \ 01234567 \quad [A_{-1}] \ 89abcdef \quad [A_{-2}] \ fedcba98 \quad [A_{-3}] \ 76543210$$

Round function with state variable updates. We also need an auxiliary matrix $(i_{k,l})$ which stores indices. Let $i_{k,l}$ be the value in the k -th row and l -th column of matrix $(i_{k,l})$. Analogously $j_{k,l}$ is defined for matrix $(j_{k,l})$.

$$(i_{k,l}) = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 0 & 4 & 8 & 12 & 1 & 5 & 9 & 13 & 2 & 6 & 10 & 14 & 3 & 7 & 11 & 15 \\ 0 & 8 & 4 & 12 & 2 & 10 & 6 & 14 & 1 & 9 & 5 & 13 & 3 & 11 & 7 & 15 \end{pmatrix}$$

$$(j_{k,l}) = \begin{pmatrix} 3 & 7 & 9 & 11 \\ 3 & 5 & 9 & 13 \\ 3 & 9 & 11 & 15 \end{pmatrix}$$

Then the round function is applied to this block in three rounds with 16 iterations each. Let $1 \leq k \leq 3$ be the round counter and $1 \leq l \leq 16$ be the iteration counter. For every round, for every iteration apply the following function:

The values of state variable A_{-1} , A_{-2} and A_{-3} are taken as arguments for function F where F is IF in the first 16 iterations, MAJ in the following 16 iterations and finally XOR in the last 16 iterations. This return value is added to the value of state variable A_{-4} , the current message block M and $X_{i_{k,l}}$. This sum modulo 2^{32} is then left-rotated (see Definition 2.7) by $j_{k,l} \bmod 4$ bits and stored in value A_{-1} . State variables A_{-1} , A_{-2} and A_{-3} update variables A_{-2} , A_{-3} and A_{-4} respectively.

This round function design is visualized in Figure 2.1.

2.3 SHA-256

SHA-256 is a hash function from the SHA-2 family designed by the National Security Agency (NSA) and published in 2001 [6]. It uses a Merkle-Damgård construction with a Davies-Meyer compression function. The best known preimage attack was found in 2011 and breaks preimage resistance for 52 rounds [8]. The best known collision attack breaks collision resistance for 31 rounds of SHA-256 [12] and pseudo-collision resistance for 46 rounds [9].

block size	512 bits	as per Section 1 of the standard [6]
digest size	256 bits	mentioned as Message Digest size [6]
internal state size	256 bits	as per Section 1 of the standard [6]
word size	32 bits	as per Section 1 of the standard [6]

TABLE 2.2: SHA-256 hash algorithm properties

Definition 2.7 (Shifts, rotations and a notational remark)

Consider a 32-bit word X with 32 binary values b_i with $0 \leq i \leq 31$. b_0 refers to the least significant bit. Shifting (\ll and \gg) and rotation (\lll and \ggg) creates a

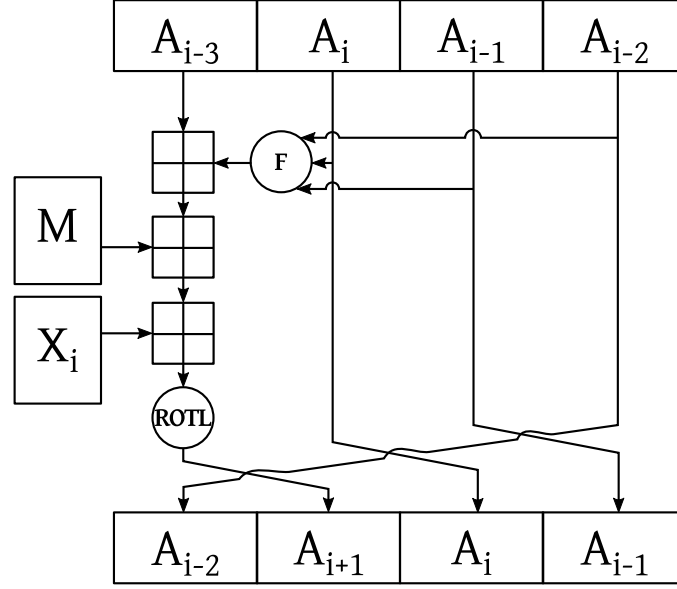


Figure 2.1: MD4 round function updating state variables A , B , C and D

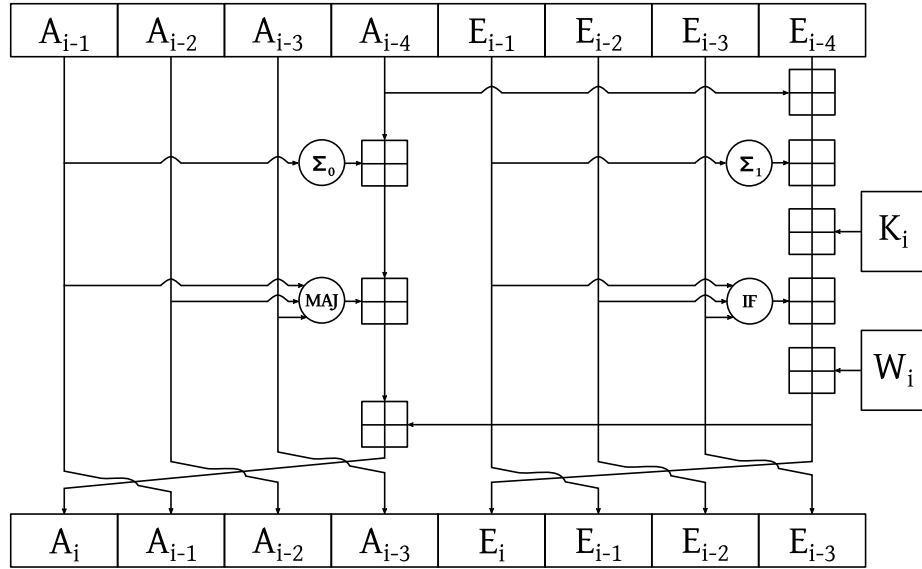


Figure 2.2: SHA-256 round function as characterized in [5]

new 32-bit word Y with 32 binary values a_i . We define the following notations:

$$Y := X \ll n \iff a_i := b_{i-n} \text{ if } 0 \leq i - n < 32 \text{ and } 0 \text{ otherwise}$$

$$Y := X \gg n \iff a_i := b_{i+n} \text{ if } 0 \leq i + n < 32 \text{ and } 0 \text{ otherwise}$$

$$Y := X \lll n \iff a_i := b_{i-n \bmod 32}$$

$$Y := X \ggg n \iff a_i := b_{i+n \bmod 32}$$

Furthermore $X \oplus Y$ denotes XOR with arguments X and Y .

Besides MD4's two auxiliary functions MAJ and IF, another four auxiliary functions are defined. Be aware that \oplus denotes the XOR functions whereas \boxplus denotes addition modulo 2^{32} .

$$\Sigma_0(X) := (X \ggg 2) \oplus (X \ggg 13) \oplus (X \ggg 22)$$

$$\Sigma_1(X) := (X \ggg 6) \oplus (X \ggg 11) \oplus (X \ggg 25)$$

$$\sigma_0(X) := (X \ggg 7) \oplus (X \ggg 18) \oplus (X \gg 3)$$

$$\sigma_1(X) := (X \ggg 17) \oplus (X \ggg 19) \oplus (X \gg 10)$$

Padding and length extension The padding and length extension scheme of MD4 is used also in SHA-256. Append bit 1 and followed by a sequence of bit 0 until the message reaches a length of 448 modulo 512 bits. Afterwards the first 64 bits of the binary representation of the original input are appended.

Initialization In a similar manner to MD4, initialization of internal state variables (called "working variables" in [6, Section 6.2.2]) takes place before running the round function. The eight state variables are initialized with the following hexadecimal values:

$$\begin{aligned} A_{-1} &= 6a09e667 & A_{-2} &= bb67ae85 & A_{-3} &= 3c6ef372 & A_{-4} &= a54ff53a \\ E_{-1} &= 510e527f & E_{-2} &= 9b05688c & E_{-3} &= 1f83d9ab & E_{-4} &= 5be0cd19 \end{aligned}$$

Furthermore SHA-256 uses 64 constant values in its round function. We initialize step constants K_i for $0 \leq i < 64$ with the following hexadecimal values (which must be read left to right and top to bottom):

428a2f98	71374491	b5c0fbcf	e9b5dba5	3956c25b	59f111f1
923f82a4	ab1c5ed5	d807aa98	12835b01	243185be	550c7dc3
72be5d74	80deb1fe	9bdc06a7	c19bf174	e49b69c1	efbe4786
0fc19dc6	240ca1cc	2de92c6f	4a7484aa	5cb0a9dc	76f988da
983e5152	a831c66d	b00327c8	bf597fc7	c6e00bf3	d5a79147
06ca6351	14292967	27b70a85	2e1b2138	4d2c6dfc	53380d13
650a7354	766a0abb	81c2c92e	92722c85	a2bfe8a1	a81a664b
c24b8b70	c76c51a3	d192e819	d6990624	f40e3585	106aa070
19a4c116	1e376c08	2748774c	34b0bcb5	391c0cb3	4ed8aa4a
5b9cca4f	682e6ff3	748f82ee	78a5636f	84c87814	8cc70208
90bfeffa	a4506ceb	bef9a3f7	c67178f2		

Precomputation of W Let W_i for $0 \leq i < 16$ be the sixteen 32-bit words of the padded input message. Then compute W_i for $16 \leq i < 64$ the following way:

$$W_i := \sigma_1(W_{i-2}) + W_{i-7} + \sigma_0(W_{i-15}) + W_{i-16}$$

Round function For every block of 512 bits, the round function is applied. The eight state variables are updated iteratively for i from 0 to 63.

$$\begin{aligned} E_i &:= A_{i-4} + E_{i-4} + \Sigma_1(E_{i-1}) + \text{IF}(E_{i-1}, E_{i-2}, E_{i-3}) + K_i + W_i \\ A_i &:= E_i - A_{i-4} + \Sigma_0(A_{i-1}) + \text{MAJ}(A_{i-1}, A_{i-2}, A_{i-3}) \end{aligned}$$

W_i and K_i refer to the previously initialized values.

Computation of intermediate hash values Intermediate hash values for the Davies-Meyer construction are initialized with the following values:

$$\begin{aligned} H_0^{(0)} &:= A_{-1} & H_1^{(0)} &:= A_{-2} & H_2^{(0)} &:= A_{-3} & H_3^{(0)} &:= A_{-4} \\ H_4^{(i)} &:= E_{-1} & H_5^{(i)} &:= E_{-2} & H_6^{(i)} &:= E_{-3} & H_7^{(i)} &:= E_{-4} \end{aligned}$$

Every block creates its own E_i and A_i values for $60 \leq i < 64$. These are used to compute the next intermediate values:

$$\begin{aligned} H_0^{(j)} &:= A_{63} + H_0^{(i-1)} & H_4^{(j)} &:= E_{63} + H_4^{(i-1)} \\ H_1^{(j)} &:= A_{62} + H_1^{(i-1)} & H_5^{(j)} &:= E_{62} + H_5^{(i-1)} \\ H_2^{(j)} &:= A_{61} + H_2^{(i-1)} & H_6^{(j)} &:= E_{61} + H_6^{(i-1)} \\ H_3^{(j)} &:= A_{60} + H_3^{(i-1)} & H_7^{(j)} &:= E_{60} + H_7^{(i-1)} \end{aligned}$$

Finalization The final hash digest of size 256 bits is provided as

$$H_0^{(N)} \parallel H_1^{(N)} \parallel H_2^{(N)} \parallel H_3^{(N)} \parallel H_4^{(N)} \parallel H_5^{(N)} \parallel H_6^{(N)} \parallel H_7^{(N)}$$

where N denotes the index of the last block and operator \parallel denotes concatenation. Hence $H_0^{(N)}$ are the four least significant bytes of the digest.



“JUST BECAUSE IT’S AUTOMATIC DOESN’T
MEAN IT WORKS.”
—Daniel J. Bernstein

Chapter 3

Differential cryptanalysis

In chapter 2 we defined two hash functions. In this chapter we consider such functions from a differential perspective. Differential considerations will turn out to yield successful collision attacks on hash functions. We introduce a notation to easily represent differential characteristics.

3.1 Motivation

In August 2004, Wang et al. published results at Crypto’04 [23] which revealed that MD4, MD5, HAVAL-128 and RIPEMD can be broken practically using differential cryptanalysis. Their work is based on preliminary work by Hans Dobbertin [4]. On an IBM P690 machine, an MD5 collision can be computed in about one hour using this approach. Collisions for HAVAL-128, MD4 and RIPEMD were found as well. Patrick Stach’s `md4coll.c` program [21] implements Wang’s approach and can find MD4 collisions in few seconds on my Thinkpad x220 setup specified in [Appendix B](#).

Let n denote the digest size, i.e. the size of the hash value $h(x)$ in bits. Due to the birthday paradox, a collision attack has a generic complexity of $2^{n/2}$ whereas preimage and second preimage attacks have generic complexities of 2^n . In other words it is computationally easier to find any two colliding hash values than the preimage or second preimage for a given hash value.

Following results by Wang et al., differential cryptanalysis was shown as powerful tool for cryptanalysis of hash algorithms. This thesis applies those ideas to satisfiability approaches.

Message 1			
4d7a9c83	d6cb927a	29d5a578	57a7a5ee
de748a3c	dcc366b3	b683a020	3b2a5d9f
c69d71b3	f9e99198	d79f805e	a63bb2e8
45dc8e31	97e31fe5	2794bf08	b9e8c3e9
Message 2			
4d7a9c83	56cb927a	b9d5a578	57a7a5ee
de748a3c	dcc366b3	b683a020	3b2a5d9f
c69d71b3	f9e99198	d79f805e	a63bb2e8
45dd8e31	97e31fe5	2794bf08	b9e8c3e9
Hash value of Message 1 and Message 2			
5f5c1a0d	71b36046	1b5435da	9bod807a

TABLE 3.1: One of two MD4 hash collisions provided in [23]. Values are given in hexadecimal, message words are enumerated from left to right, top to bottom. Differences are highlighted in bold for illustration purposes. For comparison the first bits of Message 1 are 11000001... and the last bits are ...10011101. A message represents one block of 512 bits.

3.2 Fundamentals

Definition 3.1 (Hash collision)

Given a hash function h , a hash collision is a pair (x_1, x_2) with $x_1 \neq x_2$ such that $h(x_1) = h(x_2)$.

Differential cryptanalysis is based on the idea to consider two execution states of hash algorithms for slightly different input messages. We trace those difference to learn about the propagation of message differences.

TODO: explain pseudo collisions

Hash algorithms consume input values as blocks of bits. As far as the length of the input must not conform to the block size, padding is applied. Now consider such a block of input values and another copy of it. We use those two blocks as inputs for two hash algorithm implementations, but provide slight modifications in few bits. MD4 has 48 round function applications in 3 rounds. Differential cryptanalysis considers the difference in the evaluation state between the two instances (compare with Figure 3.1).

Visualizing those differences helps the cryptanalyst to find modifications yielding a small number of differences in the evaluation state. The cryptanalyst consecutively modifies the input values to eventually receive a collision in the output value. If the number of differences in the evaluation state is small, this trail is expected to result in a hash collision with higher probability.

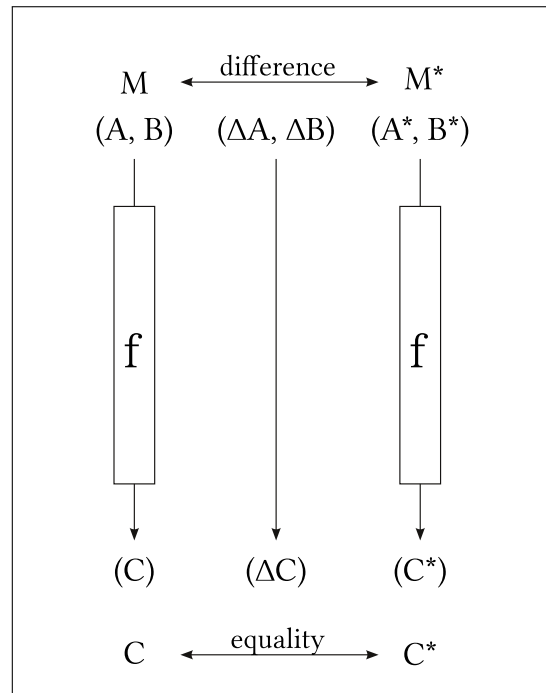


Figure 3.1: Typical attack setting for a collision attack: Hash function f is applied to two inputs M and M^* which differ by some predefined bits. M describes the difference between these values. A hash collision is given if and only if output values C and C^* show the same value. In differential cryptanalysis we observe the differences between two instances applying function f to inputs M and M^* .

bit	binary	hexadecimal representation / differential notation
x_0	d6cb927a	11010110110010111001001001111010
x_1	29d5a578	0010100111010101010010101111000
x_2	45dc8e31	01000101110111001000111000110001
x_0^*	56cb927a	01010110110010111001001001111010
x_1^*	b9d5a578	1011100111010101010010101111000
x_2^*	45dd8e31	01000101110111011000111000110001
Δx		u1010110110010111001001001111010 n01n100111010101010010101111000 010001011101110n1000111000110001

TABLE 3.2: The three words different between Message 1 and Message 2 of the original MD4 hash collision. The last three lines show how differences can be written down using bit conditions. As far as 4 symbols are not from the set $\{0, 1\}$ it holds that the messages differ by 4 bits.

3.3 Differential notation

Differential notation helps us to visualize differential characteristics by defining so-called *generalized bit conditions*. It was introduced by Christian Rechberger and Christophe de Cannière in 2006 [2, Section 3.2], inspired by *signed differences* by Wang et al. and is shown in Table 3.3.

Consider two hash algorithm implementations. Let x_i be some bit from the first implementation and let x_i^* be the corresponding bit from the second implementation. Differences are computed using a XOR and commonly denoted as $\Delta x = x_i \oplus x_i^*$. Bit conditions allow us to encode possible relations between bits x_i and x_i^* .

For example, let us take a look at the original Wang et al. hash collision in MD4 provided in Table 3.1. We extract all values with differences and represent them using differential notation. This gives us Table 3.2.

The following properties hold for bit conditions:

- If $x_i = x_i^*$ holds and some value is known, $\{0, 1\}$ contains its bit condition.
- If $x_i \neq x_i^*$ holds and some value is known, $\{u, n\}$ contains its bit condition.
- If $x_i = x_i^*$ holds and the values are unknown, its bit condition is $-$.
- If $x_i \neq x_i^*$ holds and the values are unknown, its bit condition is x .

Applying this notation to hash collisions means that arbitrary bit conditions (except for $\#$) can be specified for the input values. In one of the intermediate iterations, we enforce a difference using one of the bit conditions $\{u, n, x\}$. This excludes trivial solutions with no differences from the set of possible solutions. And the final values need to lack differences thus are represented using $-$.

3.4 A simple addition example

TODO:

- illustrate how differences propagate by an addition example illustrated in differential notation
- reference to Magnus Daum's thesis (\neq signed bit diff desc)

3.5 Differential path

TODO:

- refer to some testcase which shows a differential path with many unresolved differences.
- Then show the corresponding testcase where ? became - and x.
- Illustrate how MD4 and SHA-256 descriptions maps to matrix representation.

(x_i, x_i^*)	(0, 0)	(1, 0)	(0, 1)	(1, 1)	(x_i, x_i^*)	(0, 0)	(1, 0)	(0, 1)	(1, 1)
?	✓	✓	✓	✓	3	✓	✓		
-	✓			✓	5	✓		✓	
x		✓	✓		7	✓	✓	✓	
0	✓				A		✓		✓
u		✓			B	✓	✓		✓
n			✓		C			✓	✓
1				✓	D	✓		✓	✓
#					E		✓	✓	✓

TABLE 3.3: Differential notation as introduced in [2]. The left-most column specifies a symbol called “bit condition” and right-side columns indicate which bit configurations are possible for two given bits x_i and x_i^* .

Δx	conjunctive normal form	Δx	conjunctive normal form
#	$(x) \wedge (\neg x)$	1	$(x) \wedge (x^*)$
0	$(\neg x) \wedge (\neg x^*)$	-	$\neg(x \oplus x^*)$
u	$(x) \wedge (\neg x^*)$	A	(x)
3	$(\neg x^*)$	B	$(x \vee \neg x^*)$
n	$(\neg x) \wedge (x^*)$	C	(x^*)
5	$(\neg x)$	D	$(\neg x \vee x^*)$
x	$(x \oplus x^*)$	E	$(x \vee x^*)$
7	$(\neg x \vee \neg x^*)$?	

TABLE 3.4: All bit conditions represented as CNF using two boolean variables x and x^* to represent two bits.



“WHAT IDIOT CALLED THEM LOGIC
ERRORS RATHER THAN BOOL SHIT?”
—Unknown

Chapter 4

Satisfiability

Boolean algebra allows us to describe functions over two-valued variables. Satisfiability is the question for an assignment such that a function evaluates to true. Satisfiability problems are solved by SAT solvers. We discuss the basic theory behind satisfiability. We will learn that any computation can be represented as satisfiability problem. In Chapter 6 we will represent a differential cryptanalysis problem such that it is solvable iff the corresponding SAT problem is satisfiable.

4.1 Basic notation and definitions

Definition 4.1 (Boolean function)

A *Boolean function* is a mapping $h : X \rightarrow Y$ with $X = \{0, 1\}^n$ for $n \in \mathbb{N}_{\geq 1}$ and $Y = \{0, 1\}$.

Definition 4.2 (Assignment)

A k -*assignment* is an element of $\{0, 1\}^k$.
Let f be some k -ary Boolean function. An *assignment for function f* is any k -assignment.

Definition 4.3 (Truth table)

Let f be some k -ary Boolean function. The *truth table of Boolean function f* assigns truth value 0 or 1 to any assignment of f .

Boolean functions are characterized by their corresponding truth table.

x_1	x_2	$f(x_1, x_2)$	x_1	x_2	$f(x_1, x_2)$	v	$f(v)$
1	1	1	1	1	1	1	0
1	0	0	1	0	1	0	1
0	1	0	0	1	1	(c) NOT	
0	0	0	0	0	0		

(A) AND

(B) OR

TABLE 4.1: Truth tables for AND, OR and NOT

Table 4.1 shows example truth tables for the Boolean AND, OR and NOT functions. A different definition of the three functions is given the following way:

Definition 4.4

Let AND, OR and NOT be three Boolean functions.

- AND maps $X = \{0, 1\}^2$ to 1 if all values of X are 1.
- OR maps $X = \{0, 1\}^2$ to 1 if any value of X is 1.
- NOT maps $X = \{0, 1\}^1$ to 1 if the single value of X is 0.

All functions return 0 in the other case.

Those functions are denoted $a_0 \wedge a_1$, $a_0 \vee a_1$ and $\neg a_0$ respectively, for input parameters a_0 and a_1 .

It is interesting to observe, that any Boolean function can be represented using only these three operators. This can be proven by complete induction over the number of arguments k of the function.

Let $k = 1$. Then we consider any possible 2-assignment for one input variable x_1 and one value of $f(x_1)$. Then four truth tables are possible listed in Table 4.2. The description shows the corresponding definition of f using AND, OR and NOT only.

Now let g be some k -ary function. Let (a_0, a_1, \dots, a_k) be the k input arguments to g and $x_1 := g(a_0, a_1, \dots, a_k)$. Then we can again look at Table 4.2 to discover that 4 cases are possible: 2 cases where the return value of our new $(k + 1)$ -ary function depends on value x_1 and 2 cases where the return value is constant.

This completes our proof.

x_1	$f(x_1)$	x_1	$f(x_1)$	x_1	$f(x_1)$	x_1	$f(x_1)$
1	1	1	1	1	0	1	0
0	1	0	0	0	1	0	0
(A) $f : x \mapsto 1$		(B) $f : x \mapsto x$		(C) $f : x \mapsto \neg x$		(D) $f : x \mapsto 0$	

TABLE 4.2: Unary f and its four possible cases

Boolean functions have an important property which is described in the following definition:

Definition 4.5

A Boolean function f is *satisfiable* iff there exists at least one input $x \in X$ such that $f(x) = 1$. Every input $x \in X$ satisfying this property is called *model*.

The corresponding tool to determine satisfiability is defined as follows:

Definition 4.6

A *SAT solver* is a tool to determine satisfiability (SAT or UNSAT) of a Boolean function. If satisfiability is given, it returns some model.

4.1.1 Computational considerations

The generic complexity of SAT determination is given by 2^n for n Boolean variables.

Let n be the number of variables of a Boolean function. No known algorithm exists to determine satisfiability in polynomial runtime. This means no algorithm solves the SAT problem with runtime behavior which depends polynomially on the growth of n .

This is known as the famous $\mathcal{P} \stackrel{?}{\neq} \mathcal{NP}$ problem.

However, SAT solver can take advantage of the problem's description. For example consider function f in Display 4.1.

$$f(x_0, x_1, x_2) = x_0 \wedge (\neg x_1 \vee x_2) \quad (4.1)$$

Instead of trying all possible 8 cases for 3 Boolean variables, we can immediately see that x_0 is required to be 1. So we don't need to test $x_0 = 0$ and can skip 4 cases. This particular strategy is called *unit propagation*.

4.1.2 SAT competitions

SAT research is heavily concerned with finding good heuristics to find some model for a given SAT problem as fast as possible. Biyearly [SAT competitions](#) take place to challenge SAT solvers in a set of benchmarks. The committee evaluates the most successful SAT solvers solving the most problems within a given time frame.

SAT 2016 is currently ongoing, but in 2014 lingeling by Armin Biere has won first prize in the Application benchmarks track and second prize in the Hard Combinatorial benchmarks track for SAT and UNSAT instances respectively. Its parallelized sibling plingeling and Cube & Conquer sibling treengeling have won prizes in parallel settings.

In chapter 7 we will look at runtime results shown by (but not limited to) those SAT solvers.

4.2 The DIMACS de-facto standard

Definition 4.7

A *conjunction* is a sequence of Boolean functions combined using a logical OR. A *disjunction* is a sequence of Boolean functions combined using a logical AND. A *literal* is a Boolean variable (*positive*) or its negation (*negative*).

A SAT problem is given in *Conjunctive Normal Form* (CNF) if the problem is defined as conjunction of disjunctions of literals.

A simple example for a SAT problem in CNF is the exclusive OR (XOR). It takes two Boolean values a and b as arguments and returns true if and only if the two arguments differ.

$$(a \vee b) \wedge (\neg a \vee \neg b) \quad (4.2)$$

Display 4.2 shows one conjunction (denoted \wedge) of two disjunctions (denoted \vee) of literals (denoted a and b where prefix \neg represents negation). This structure constitutes a CNF.

Analogously we define a *Disjunctive Normal Form* (DNF) as disjunction of conjunctions of literals. The negation of a CNF is in DNF, because literals are negated and conjunctions become disjunctions, vice versa.

Theorem 4.1

Every Boolean function can be represented as CNF.

Theorem 4.1 is easy to prove. Consider the truth table of an arbitrary Boolean function f with k input arguments and j rows of output value false. We represent f as CNF.

Consider Boolean variables $b_{i,l}$ with $0 \leq i \leq j$ and $0 \leq l \leq k$. For every row i of the truth table with assignment (r_i) , add one disjunction to the CNF. This disjunction contains $b_{i,l}$ if $r_{i,l}$ is false. The disjunction contains $\neg b_{i,l}$ if $r_{i,l}$ is true.

As far as f is an arbitrary k -ary Boolean function, we have proven that any function can be represented as CNF.

SAT problems are usually represented in the DIMACS de-facto standard. Consider a SAT problem in CNF with $nbclauses$ clauses and enumerate all variables from 1 to $nbvars$. A DIMACS file is an ASCII text file. Lines starting with “c” are skipped (comment lines). The first remaining line has to begin with “p cnf” followed by $nbclauses$ and $nbvars$ separated by spaces (header line). All following non-comment lines are space-separated indices of Boolean variables optionally

prefixed by a minus symbol. Then one line represents one clause and must be terminated with a zero symbol after a space. All lines are conjuncted to form a CNF.

Variations of the DIMACS de-facto standard also allow multiline clauses (the zero symbol constitutes the end of a clause) or arbitrary whitespace instead of spaces. The syntactical details are individually published on a per competition basis.

LISTING 4.1: Display 4.2 represented in DIMACS format

```
p cnf 2 2
a b
-a -b
```

4.3 Terminology

Given a conjunctive structure of disjunctions, we can define terms related to this structure:

Definition 4.8

A *clause* is a disjunction of literals. A *k-clause* is a clause consisting of exactly *k* literals. A *unit clause* is a 1-clause. A *Horn clause* is a clause with at most one positive literal. A *definite clause* is a clause with exactly one positive literal. A *goal clause* is a clause with no positive literal.

Definition 4.9

Given a literal, its *negated literal* is the literal with its sign negated. A literal is *positive*, if its sign is positive. A literal is *negative* if its sign is negative. An *existential literal* is a literal which occurs exactly once and its negation does not occur. A *used variable* is a variable which occurs at least once in the CNF.

The *literal frequency* is the number of occurrences of a literal in the CNF divided by the number of clauses declared. Equivalently *variable frequency* defines the number of variable occurrences divided by the number of clauses declared.

Definition 4.10

The *clause length* of a clause is the number of literals contained. A clause is called *tautological* if a literal and its negated literal occurs in it.

4.4 Basic SAT solving techniques

Definition 4.11

Given two CNFs A and B , they are called *equisatisfiable* if and only if A is satisfiable if and only if B .

4.4.1 Boolean constraint propagation (BCP)

One of the most basic techniques to SAT solving is boolean constraint propagation. It is so fundamental that SATzilla, introduced in section 5.2, applies it even when looking at SAT features.

Let l be the literal of a unit clause in a CNF. Remove any clause containing l and replace any occurrences of $\neg l$ from the CNF. It is easy to see, that the resulting CNF is equisatisfiable, because due to the unit clause l must be true. So any clause containing l is satisfied and $\neg l$ yields false, where $A \vee \perp$ is equivalent to A for any boolean function A .

4.4.2 Watched literals

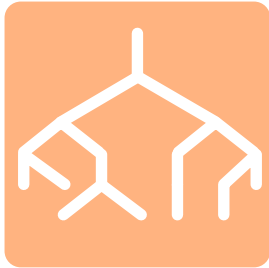
Watched Literals are another fundamental concept in SAT solving. It is very expensive to check satisfiability of all clauses for every value of a literal. Watched Literals is a neat technique to reduce the number of checks.

TODO: finish

4.5 SAT solvers in use

TODO: discuss minisat, treengeling, lingeling, plingeling, cmsat 4.5, cmsat 5, glucose syrup static, glucose static, lingeling ats101, ats102, ats104

TODO: discuss how Armin modified lingeling



“WHAT IDIOT CALLED THEM LOGIC
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—Unknown

Chapter 5

SAT features

At the very beginning I was very intrigued by the question “What is an ‘average’ SAT problem?”. Answers to this question can help to optimize SAT solver memory layouts. Specifically for this thesis I wanted to find out whether our problems distinguish from “average” problems in any way such that we can use this distinction for runtime optimization.

I came up with 8 questions related to basic properties of SAT problems we will discuss in depth in this section:

1. Given an arbitrary literal. What is the percentage it is positive?
2. What is the variables / clauses ratio?
3. How many literals occur only either positive or negative?
4. What is the average and longest clause length among CNF benchmarks?
5. How many Horn clauses exist in a CNF?
6. Are there any tautological clauses?
7. Are there any CNF files with more than one connected variable component?
8. How many variables of a CNF are covered by unit clauses?

We will now define the terms used in those questions.

5.1 SAT features and CNF analysis

Definition 5.1 (*SAT feature*)

A *SAT feature* is a statistical value (named *feature value*) retrievable from some given SAT problem.

The most basic example of a SAT feature is the number of variables and clauses of a given SAT problem. This SAT feature is stored in the CNF header of a SAT problem encoded in the DIMACS format.

The general goal is to write a tool which evaluates several SAT features at the same time and retrieve them for comparison with other problems. Therefore it should be computationally easy to evaluate SAT features of a given SAT problem. A suggested computational limit is given with polynomial complexity in terms of number of variables and number of clauses for memory as well as runtime for evaluation algorithms. Sticking to this convention implies that evaluation of satisfiability must not be necessary to evaluate a SAT feature under the assumption that $\mathcal{P} \neq \mathcal{NP}$. Hence the number of valid models cannot be a SAT feature as far as satisfiability needs to be determined. But no actual hard computational limit is defined.

5.2 Related work

The most similar resource I found looking at SAT features was the SATzilla project [15, 24] in 2012. The authors systematically defined 138 SAT features categorized in 12 groups. The features themselves are not defined formally, but an implementation is provided bundled with example data. The following list provides an excerpt of the features:

nvarsOrig number of variables defined in the CNF header

nvars number of active variables

reducedVars nvarsOrig - nvars, divided by nvars

vars-clauses-ratio nvars divided by number of active clauses

POSNEG-RATIO-CLAUSE-mean mean of $2 \cdot \left\| 0.5 - \frac{\text{pos}_c}{\text{length}_c} \right\|$ where pos_c is the number of positive literals of clause c and length_c clause length of c

POSNEG-RATIO-CLAUSE-entropy like POSNEG-RATIO-CLAUSE-mean but entropy

TRINARY+ number of clauses with clause length 1, 2 or 3 divided by number of active clauses

HORNY-VAR-min minimum number of times a variable occurs in a horn clause

cluster-coeff-mean let neighbors of a clause be all clauses containing any literal negated and let clauses c_1 and c_2 be conflicting if c_1 contains literal l and c_2 contains $-l$, then return the mean of 2 times the number of conflicting neighbors of a clause c divided by the number of unordered pairs of neighbors, returned iff computable within 20 seconds for all clauses

Please recognize that active clauses are the unsatisfied clauses after BCP has been applied. Equivalently active variables are remaining variables after application of BCP.

Many SAT solvers collect feature values to improve algorithm selection, restart strategies and estimate problem sizes. Recent trends to apply Machine Learning to SAT solving imply feature evaluation. SAT features and the resulting satisfiability runtime are used as training data for Machine Learning. One example using SAT features for algorithm selection is ASlib [1].

Previous work has shown that expensive algorithms can provide useful data in a small time frame if they are limited to a constant subproblem size.

5.3 Statistical features

For our SAT features we need to define some basic statistical terminology. Let x_1, x_2, \dots, x_n be a finite sequence of numbers ($n \in \mathbb{N}$).

Arithmetic mean (or short: mean) is defined as

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

Standard deviation (or short: sd) with mean \bar{x} is defined as

$$\sigma(x) = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

Median with $x_1 \leq x_2 \leq \dots \leq x_n$ (i.e. sorted ascendingly) is defined as

$$m = \begin{cases} x_{\text{mid}} & \text{if } n \text{ odd} \\ \frac{x_{\text{mid}} + x_{\text{mid}+1}}{2} & \text{if } n \text{ even} \end{cases} \quad \text{with } \text{mid} = \frac{n}{2}$$

and often considered more “robust” than the arithmetic mean.

Entropy is defined according to Claude Shannon's information theory:

$$H(x) = - \sum_{i=1}^n x_i \cdot \log_2(x_i)$$

where $0 \cdot \log_2(0) := 0$.

Furthermore *count* refers to the number of elements n , *largest* refers to the maximum element $\max_{1 \leq i \leq n}(x_i)$ and *smallest* refers to the minimum element $\min_{1 \leq i \leq n}(x_i)$.

5.4 Suggested SAT features

We wrote a tool called *cnf-analysis*. The evaluated features are partially inspired by SATzilla and *lingeling*. The latter prints basic statistics for every CNF it evaluates.

A summary of our suggested SAT features is given:

clause_variables_sd_mean

mean of sd of variables in a clause

clauses_length_(largest, smallest, mean, median, sd)

statistics related to the clause length

connected_(literal, variable)_components_count

two literals (variables) are connected if they occur in some clause together, count the number of connected components

definite_clauses_count

number of definite clauses in the CNF

existential_literals_count

number of existential literals in the CNF

existential_positive_literals_count

number of positive, existential literals in the CNF

(false, true)_trivial

is the CNF satisfied if all variables are claimed to be false (true)?

goal_clauses_count

number of goal clauses in the CNF

literals_count

number of literals in the CNF (i.e. sum of clause lengths)

literals_frequency_k_to_k + 5

let n_l be the literal frequency of literal l , count the number of n_l satisfying $\frac{k}{100} \leq n_l < \frac{k+5}{100}$ where k is a variable in $\{0, 5, 10, \dots, 90, 95\}$ and $k = 95$ counts $\frac{k}{100} \leq n_l \leq \frac{k+5}{100}$.

literals_frequency_(largest, smallest, mean, median, sd)_entropy

statistics related to literal frequencies

literals_occurence_one_count

number of literals with occurence 1

nbclauses, nbvars number of clauses (variables) as defined in the CNF header

negative_literals_in_clause_(smallest, largest, mean)

statistics related to number of negative literals in clauses

(positive, negative)_unit_clause_count

number of unit clauses with a positive (negative) literal

positive_literals_count

number of positive literals in CNF

positive_literals_in_clause_(largest, smallest, mean, median, sd)

statistics related to number of positive literals in clauses

positive_negative_literals_in_clause_ratio_(mean, entropy)

let r_c be the number of positive literals divided by clause length of clause c , mean and related of all r_c

positive_negative_literals_in_clause_ratio_mean

mean of all r_c

tautological_literals_count

number of clauses which contain a tautological literal

two_literals_clause_count

number of clauses with two literals

variables_frequency_k_to_k + 5

same as literals_frequency_k_to_k + 5 but for variables

variables_frequency_(largest, smallest, mean, median, sd, entropy)

same as literals_frequency but for variables

variables_used_count

number of variables with occurence greater o

In the following section we want to evaluate SAT features and compare test cases.

TODO: classify SAT features

5.5 CNF dataset

TODO: list data set explicitly

TODO: Illustrate basic properties about those CNF files

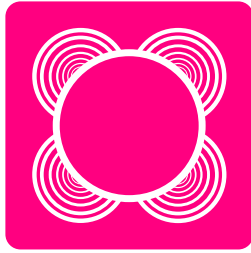
5.6 The average SAT problem

Proposition 5.1

The set of public benchmarks in SAT competitions between 2008 and 2015 represent average SAT problems

TODO: what is an average SAT problem?

Given a set of clauses, return a subset of clauses satisfying given criterion	
clauses_allLitsNeg	all literals are negative
clauses_oneLitNeg	exactly one literal is negative
clauses_geqOneLitNeg	more than one literal is negative
clauses_allLitsPos	all literals are positive
clauses_oneLitPos	exactly one literal is positive
clauses_geqOneLitPos	more than one literal is positive
clauses_length1	clause contains exactly one literal (“unit clause”)
clauses_length2	clause contains exactly two literals
clauses_unique	clause did not yet occur
clauses_tautological	clause contains some literal and its negation
Given a set of literals/variables, return Boolean property	
literals_existential	literal does not occur negated
literals_unit	literal occurs in clause of length 1
literals_contradiction	literal occurs with its negation on one clause
literals_1occ	literal occurs only in one clause once
literals_2occs	literal occurs two times in clauses
literals_3occs	literal occurs three times in clauses
variables_unit	variable occurs in clause of length 1
Given a set of clauses, return real number based on this clause	
clauses_mapLength	number of literals in clause
clauses_mapRatioPosNeg	number of positive literals divided by total number of literal
clauses_mapNumPos	number of positive literals in clause
Given one clause, return Boolean property	
clauselits_someEx	any is literal existential
clauselits_allEx	all literals are existential
clauselits_someUnit	contains unit variable
clauselits_someContra	contains contradiction variable
clauselits_all1occ	all variables occur only once in all clauses
clauselits_all12occ	all variables occur only once or twice in all clauses
Given all clauses, return the following property	
concomp_variable	number of connected components where two variables are in the same component iff they occur in at least one clause together
concomp_literal	number of connected components where two literals are in the same component iff they occur in at least one clause together
xor2_count	Number of clause pairs $(a \vee b, \neg a \vee b)$ for two variables a and b



“THERE IS CONSENSUS THAT ENCODING
TECHNIQUES USUALLY HAVE A DRAMATIC
IMPACT ON THE EFFICIENCY OF THE SAT
SOLVER”

—Magnus Björk

Chapter 6

Problem encoding

We already discussed how SAT solvers work and which input they take. We also sketched how hash algorithm properties got broken using differential cryptanalysis. In this section we combine those subjects and describe how we designed an attack setting.

6.1 STP approach

TODO: describe approach

6.2 Two instances and its difference

TODO: we wrote a tool called `algotocnf` for our differential needs

6.3 Approach with a differential description

TODO: approach with differential desc

6.4 Influencing evaluation order



Chapter 7

Results

7.1 SAT features results

TODO: do cryptoproblems distinguish from other problems?

TODO: do our benchmark distinguish from other problems?

TODO: answer the 8 questions posed previously

7.2 Attack results

TODO: is simplification worth it?

TODO: discuss runtime and development, tuning by doing difference variables first, diff desc makes a difference

Appendix ?? provide a more exhaustive list of runtimes retrieved.

7.3 Related work

7.4 Conclusion

TODO: we attacked MD₄ and SHA₂, but can see the problems with

7.5 Contributions

To strengthen Reproducible Research, the source code and data resulting from this thesis is available online. It allows the reader to run the experiments again and verify our claims. At the following website, any results part of this project are collected:

<http://lukas-prokop.at/proj/megosat/>

Several subprojects are part of this master thesis:

algotocnf

A python library implementing the encoding described in chapter 6.

Python3 library and program: <https://github.com/prokls/algotocnf>

cnf-hash

A standardized way to produce a unique hash for CNF files

Go implementation: <https://github.com/prokls/cnf-hash-go>

Python3 implementation: <https://github.com/prokls/cnf-hash-py>

Testsuite: <https://github.com/prokls/cnf-hash-tests2>

cnf-analysis

Evaluate SAT features for a given CNF file.

Go implementation: <https://github.com/prokls/cnf-analysis-go>

Python3 implementation: <https://github.com/prokls/cnf-analysis-py>

Testsuite: <https://github.com/prokls/cnf-analysis-tests>

Chapter 8

Summary and Future Work

8.1 Summary of results

8.2 Future work

Appendices

Appendix A

Illustration

i		$VS_{i,0}$	$VS_{i,1}$	$VS_{i,2}$
-4	A:	01100111010001010010001100000001		
-3	A:	00010000001100100101010001110110		
-2	A:	10011000101110101101110011111110		
-1	A:	1110111110011011010101110001001		
0	A:	01101011110101001110010000010010	W:	01001101011110101001110010000011
1	A:	011011001001111111011100u110001	W:	u1010110110010111001001001111010
2	A:	101010110100000001110u01n1110010	W:	n01n100111010101101001010111000
3	A:	101011u1001111010101001001010001	W:	0101011110100111101001011110110
4	A:	00101100011000110101010111110010	W:	1101110011101001000101000111100
5	A:	000110100110001010u1101000000001	W:	1101100110000110110011010110011
6	A:	0001101100unuu110001000001111010	W:	1011011010000011010000000100000
7	A:	00101011100000010unnn011001010000	W:	00111011001010100101110110011111
8	A:	011100110010001u1111111110110000	W:	11000110100111010111000110110011
9	A:	101011n01unnnu0001111100110011111	W:	11111001111010011001000110011000
10	A:	10n00100100001010100000010101110	W:	11010111100111111000000001011110
11	A:	u1000110101101100100101011111111	W:	10100110001110111011001011101000
12	A:	001011u00u101011111110001111011	W:	010001011101110n1000111000110001
13	A:	10un1n01001100010100000111100101	W:	10010111111000110001111111100101
14	A:	00001010010100011000100011010110	W:	00100111100101001011111100001000
15	A:	0001111010101u010110011011010100	W:	10111001111010001100001111101001
16	A:	n00n0un0110100101001101101011111		
17	A:	00011111001110100001001000011110		
18	A:	01010111000011010000000010010100		
19	A:	u1n10000000101111001101011000100		
20	A:	n1un1001111111011101000000110100		
21	A:	1111001110110000010111111010100		
22	A:	01011101110011010011001100111010		
23	A:	01010000111011101100011110001111		
24	A:	00000010000100100011011100011010		
25	A:	10110000100101100001010011101010		
26	A:	00001010100010010111011101000001		
27	A:	00000110111011101011010110011		
28	A:	10110110010111010110110000100101		
29	A:	10100010000011010100100001101001		
30	A:	00101001110101111100011101100011		
31	A:	1111110010010010101011110110110		
32	A:	0100111110100100110100000101111		
33	A:	00111000001111010110111011100100		
34	A:	00100000011101011110100000010101		
35	A:	n0100000001100110000010001110010		
36	A:	n0000111111010111101111001011001		
37	A:	11001000000110100100001100001100		
38	A:	10110000011001111110100110101100		
39	A:	00010010000010100001101100011100		
40	A:	1100000010010000111000110000101		
41	A:	00000110100001101111010100100110		
42	A:	01001110110111011111111010000110		
43	A:	01010000011000111101000001101101		
44	A:	11111000000101101111011100001100		
45	A:	10001010110110110010110000000100		
46	A:	10000010100110010101100011011100		
47	A:	10000001111001011011010010111101		

TABLE A.1: One of the original MD₄ collision given by Wang, et al.

Appendix B

Hardware setup

In the following we introduce two hardware setups which were used to run our testcases. The first setup is referred to as “Thinkpad x220” throughout the document whereas the second setup is referred to as “Cluster”.

<i>Type model</i>	Thinkpad Lenovo x220 tablet, 4299-2P6
<i>Processor</i>	Intel i5-2520M, 2.50 GHz, dual-core, Hyperthreaded
<i>RAM</i>	16 GB (extension to common retail setup)
<i>Memory</i>	160 GB SSD
<i>L3 cache size</i>	3072 KB

TABLE B.1: Thinkpad x220 Tablet specification [10]

<i>Processor</i>	Intel Xeon X5690, 3.47 GHz, 6 cores, Hyperthreaded
<i>RAM</i>	192 GB
<i>L3 cache size</i>	12288 KB

TABLE B.2: Cluster node nehalem192go specification [3]

Appendix C

Testcases

Figures C.1, C.2, C.3 and C.4 show testcases used to test performance measures.

i		$\nabla S_{i,0}$	$\nabla S_{i,1}$	$\nabla S_{i,2}$
-4	A:	01100111010001010010001100000001		
-3	A:	00010000001100100101010001110110		
-2	A:	1001100010111010101110011111110		
-1	A:	11101111110011011010101110001001		
0	A:	x-----	W:	--x-----
1	A:	-----	W:	-----
2	A:	-----x-----	W:	x-----
3	A:	xxx-----	W:	-----
4	A:	-----xx	W:	x-----
5	A:	-----xxxxxxxxxxxxx-x-----	W:	-----
6	A:	x-----x-----x-x-xxxx--x	W:	-----
7	A:	-----x-x-x-----	W:	-----
8	A:	-----x-x-x-x-----	W:	x-----
9	A:	-----x-x-x-----	W:	-----
10	A:	-----x-x-xxx-xxx-----	W:	-----
11	A:	x-----xxx-x-----	W:	-----
12	A:	--x-x-----	W:	x-----
13	A:	-----	W:	-----
14	A:	-x-----	W:	-----
15	A:	x-x-----x-----	W:	-----
16	A:	-xxx-----		
17	A:	-----		
18	A:	-----		
19	A:	x-----		
20	A:	x-----		
21	A:	-----		
22	A:	-----		
23	A:	-----		
24	A:	-----		
25	A:	-----		
26	A:	-----		
27	A:	-----		
28	A:	-----		
29	A:	-----		
30	A:	-----		
31	A:	-----		
32	A:	x-----		
33	A:	-----		
34	A:	-----		
35	A:	-----		
36	A:	-----		
37	A:	-----		
38	A:	-----		
39	A:	-----		
40	A:	-----		
41	A:	-----		
42	A:	-----		
43	A:	-----		
44	A:	-----		
45	A:	-----		
46	A:	-----		
47	A:	-----		

TABLE C.1: TODO description

i		$\nabla S_{i,0}$	$\nabla S_{i,1}$	$\nabla S_{i,2}$
-4	A:	01100111010001010010001100000001		
-3	A:	0001000000110010010101010001110110		
-2	A:	10011000101110101101110011111110		
-1	A:	1110111110011011010101110001001		
0	A:	????????????????????????????????	W:	--x-----
1	A:	????????????????????????????????	W:	-----
2	A:	????????????????????????????????	W:	x-----
3	A:	????????????????????????????????	W:	-----
4	A:	????????????????????????????????	W:	x-----
5	A:	????????????????????????????????	W:	-----
6	A:	????????????????????????????????	W:	-----
7	A:	????????????????????????????????	W:	-----
8	A:	????????????????????????????????	W:	x-----
9	A:	????????????????????????????????	W:	-----
10	A:	????????????????????????????????	W:	-----
11	A:	????????????????????????????????	W:	-----
12	A:	????????????????-----	W:	x-----
13	A:	????????????????-----	W:	-----
14	A:	????????????????-----	W:	-----
15	A:	????????????????-----	W:	-----
16	A:	???x-----		
17	A:	?-----		
18	A:	?-----		
19	A:	?-----		
20	A:	x-----		
21	A:	-----		
22	A:	-----		
23	A:	-----		
24	A:	-----		
25	A:	-----		
26	A:	-----		
27	A:	-----		
28	A:	-----		
29	A:	-----		
30	A:	-----		
31	A:	-----		
32	A:	x-----		
33	A:	-----		
34	A:	-----		
35	A:	-----		
36	A:	-----		
37	A:	-----		
38	A:	-----		
39	A:	-----		
40	A:	-----		
41	A:	-----		
42	A:	-----		
43	A:	-----		
44	A:	-----		
45	A:	-----		
46	A:	-----		
47	A:	-----		

TABLE C.2: TODO description

i		$\nabla S_{i,0}$	$\nabla S_{i,1}$	$\nabla S_{i,2}$
-4	A:	01100111010001010010001100000001		
-3	A:	00010000001100100101010001110110		
-2	A:	10011000101110101101110011111110		
-1	A:	1110111110011011010101110001001		
0	A:	????????????????????????????????	W:	--x-----
1	A:	????????????????????????????????	W:	-----
2	A:	????????????????????????????????	W:	x-----
3	A:	????????????????????????????????	W:	-----
4	A:	????????????????????????????????	W:	x-----
5	A:	????????????????????????????????	W:	-----
6	A:	????????????????????????????????	W:	-----
7	A:	????????????????????????????????	W:	-----
8	A:	????????????????????????????????	W:	x-----
9	A:	????????????????????????????????	W:	-----
10	A:	????????????????????????????????	W:	-----
11	A:	????????????????????????????????	W:	-----
12	A:	????????????????????????????????	W:	x-----
13	A:	????????????????????????????????	W:	-----
14	A:	????????????????????????????????	W:	-----
15	A:	????????????????????????????????	W:	-----
16	A:	????????????????????????????????		
17	A:	????????????????????????????????		
18	A:	????????????????????????????????		
19	A:	????????????????????????????????		
20	A:	????????????????????????????????		
21	A:	-----		
22	A:	-----		
23	A:	-----		
24	A:	-----		
25	A:	-----		
26	A:	-----		
27	A:	-----		
28	A:	-----		
29	A:	-----		
30	A:	-----		
31	A:	-----		
32	A:	x-----		
33	A:	-----		
34	A:	-----		
35	A:	-----		
36	A:	-----		
37	A:	-----		
38	A:	-----		
39	A:	-----		
40	A:	-----		
41	A:	-----		
42	A:	-----		
43	A:	-----		
44	A:	-----		
45	A:	-----		
46	A:	-----		
47	A:	-----		

TABLE C.3: TODO description

i		$VS_{i,0}$	$VS_{i,1}$	$VS_{i,2}$
-4	A:	01100111010001010010001100000001		
-3	A:	00010000001100100101010001110110		
-2	A:	100110001011101010110011111110		
-1	A:	1110111110011011010101110001001		
0	A:	????????????????????????????????	W:	????????????????????????????????
1	A:	????????????????????????????????	W:	????????????????????????????????
2	A:	????????????????????????????????	W:	????????????????????????????????
3	A:	????????????????????????????????	W:	????????????????????????????????
4	A:	????????????????????????????????	W:	????????????????????????????????
5	A:	????????????????????????????????	W:	????????????????????????????????
6	A:	????????????????????????????????	W:	????????????????????????????????
7	A:	????????????????????????????????	W:	????????????????????????????????
8	A:	????????????????????????????????	W:	????????????????????????????????
9	A:	????????????????????????????????	W:	????????????????????????????????
10	A:	????????????????????????????????	W:	????????????????????????????????
11	A:	????????????????????????????????	W:	????????????????????????????????
12	A:	????????????????????????????????	W:	????????????????????????????????
13	A:	????????????????????????????????	W:	????????????????????????????????
14	A:	????????????????????????????????	W:	????????????????????????????????
15	A:	????????????????????????????????	W:	????????????????????????????????
16	A:	????????????????????????????????		
17	A:	????????????????????????????????		
18	A:	????????????????????????????????		
19	A:	????????????????????????????????		
20	A:	????????????????????????????????		
21	A:	-----		
22	A:	-----		
23	A:	-----		
24	A:	-----		
25	A:	-----		
26	A:	-----		
27	A:	-----		
28	A:	-----		
29	A:	-----		
30	A:	-----		
31	A:	-----		
32	A:	x????????????????????????????????		
33	A:	-----		
34	A:	-----		
35	A:	-----		
36	A:	-----		
37	A:	-----		
38	A:	-----		
39	A:	-----		
40	A:	-----		
41	A:	-----		
42	A:	-----		
43	A:	-----		
44	A:	-----		
45	A:	-----		
46	A:	-----		
47	A:	-----		

TABLE C.4: TODO description

Appendix D

Runtimes retrieved

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