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Master Thesis

Differential cryptanalysis with SAT solvers

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AFFIDAVIT

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Abstract

Hash functions are ubiquitous in the modern information age. They provide preimage, second preimage and collision resistance which are needed in a wide range of applications.

In August 2006, Wang et al. showed efficient attacks against several hash function designs including MD4, MD5, HAVAL-128 and RIPEMD. With these results differential cryptanalysis has been shown useful to break collision resistance in hash functions. Over the years advanced attacks based on those differential approaches have been developed.

To find collisions like Wang et al., a cryptanalyst needs to specify a differential characteristic. Looking at the differential behavior of the underlying operations of the hash algorithm shows how differential values propagate in the algorithm. The goal is to find a differential characteristic whose differences cancel out in the output. Once such a differential characteristic was discovered, in a second step the actual values for those differences are defined yielding an actual hash collision.

Finding a differential characteristic can be a cumbersome and tedious task. Whereas propagations can be automated using dedicated tools, finding an initial differential characteristic is a difficult task as they can be specified with arbitrary levels of granularity.

SAT research at same time faces similar problems. SAT solvers implement heuristics to find satisfying assignments for Boolean functions. They also propagate values once knowledge about the problem gets derived.

In this thesis we look at differential characteristics and encode them as SAT problem. A SAT solver tells us whether a differential characteristic can represent a hash collision or not. We implemented a framework which allows us to verify differential behavior for integer operations. We then looked at the encoded problems in details and tried to change the encoding to improve the runtime of the SAT solver. We also provide a small CNF analysis library to compare an encoded problem with others.

Keywords: hash function, differential cryptanalysis, differential characteristic, MD4, SHA-256, collision resistance, satisfiability, SAT solver

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And finally I am grateful for the support by Martina, who also supported me during good and bad days with this thesis, and my parents which provided a prosperous environment to me to be able to stand where I am today.

Thank you.
どもありがとうございました。

All source code is available at lukas-prokop.at/proj/megosat and published under terms and conditions of Free/Libre Open Source Software. This document was printed with LuaLaTeX and Linux Libertine Font.

Contents

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Chapter 1

Introduction

Hash functions are used as cryptographic primitives in many applications and protocols. They take an arbitrary input message and provide a hash value. Input message and hash value are considered as byte strings in a particular encoding. The hash value is of fixed length and satisfies several properties which make it useful in a variety of applications.

In this thesis we will consider the hash algorithms MD4 and SHA-256 and represent differential characteristics of hash collisions as SAT problem. If and only if satisfiability is given, the particular differential state is achievable using two different inputs leading to the same output. As far as SAT solvers return an actual model satisfying that state, we get an actual hash collision which can be verified and visualized. If the internal state of the hash algorithm is too large, the attack can be computationally simplified by modelling only a subset of steps of the hash algorithm or changing the modelled differential path.

Based on experience with these kind of problems with previous non-SAT-based tools we aim to apply best practices to a satisfiability setting. We will discuss which SAT techniques lead to best performance characteristics for our MD4 and SHA-256 testcases.

1.1 Cryptanalysis preliminaries

Definition 1.1 (*Hash function*)

A *hash function* is a mapping $h : X \to Y$ with $X = \{0,1\}^*$ and $Y = \{0,1\}^n$ for some fixed $n \in {}_{\geqslant 1}$.

- Let $x \in X$, then $h(x)$ is called *hash value of x*.
- Let $h(x) = y \in Y$, then x is called *preimage of y*.

One example showing the use of hash functions as primitives are JSON Web Tokens (JWT) specified in RFC 7519 [7]. Section 8 defines implementation requirements and refers to RFC 7518 [5], which specifies cryptographic algorithms to be implemented. "HMAC SHA-256" (besides "none") is the only signature and MAC algorithm required to be implemented. SHA-256 as hash algorithm is used as cryptographic primitive in this configuration.

A hash function has to satisfy the following security requirements:

Definition 1.2 (*Preimage resistance*)

Given $y \in Y$, a hash function h is *preimage resistant* iff it is computationally infeasible to find $x \in X$ such that $h(x) = y$.

Definition 1.3 (*Second-preimage resistance*)

Given $x \in X$, a hash function h is *second-preimage resistant* iff it is computationally infeasible to find $x_2 \in X$ with $x \neq x_2$ such that $h(x) = h(x_2)$. x_2 is called *second preimage*.

Definition 1.4 (*Collision resistance*)

A hash function h is *collision resistant* iff it is computationally infeasible to find any two $x \in X$ and $x_2 \in X$ with $x \neq x_2$ such that $h(x) = h(x_2)$.

As far as hash functions accept input strings of arbitrary length, but return a fixed size output string, existence of collisions is unavoidable [14]. However, good hash functions make it very difficult to find collisions or preimages.

The considered hash functions apply padding to their input to normalize their input size to a multiple of its block size. The round function follows afterwards. In the following, we always consider input of block size instead of the original input message as bytestring. Padding is negligible, because once we have two colliding blocks, the collision will be reflected in the output in these single-pipe Merkle-Darmgård designs. This results in a length extension attack, making input padding negligible for cryptanalysis.

Message 1			
4d7a9c83	**d6cb927a**	**29d5a578**	57a7a5ee
de748a3c	dcc366b3	b683a020	3b2a5d9f
c69d71b3	f9e99198	d79f805e	a63bb2e8
45dc8e31	97e31fe5	2794bf08	b9e8c3e9
Message 2			
4d7a9c83	**56cb927a**	**b9d5a578**	57a7a5ee
de748a3c	dcc366b3	b683a020	3b2a5d9f
c69d71b3	f9e99198	d79f805e	a63bb2e8
45dd8e31	97e31fe5	2794bf08	b9e8c3e9
Hash value of Message 1 and Message 2			
5f5c1a0d	71b36046	1b5435da	9b0d807a

Table 1.1: One of two MD4 hash collisions provided in [17]. Values are given in hexadecimal, message words are enumerated from left to right, top to bottom. Differences are highlighted in bold for illustration purposes. For comparison the first bits of Message 1 are `11000001`… and the last bits are …`10011101`. A message represents one block of 512 bits.

1.2 Cryptanalysis of Hash Functions

In August 2004, Wang et al. published results at Crypto'04 [17] which revealed that MD4, MD5, HAVAL-128 and RIPEMD can be broken practically using differential cryptanalysis. Their work is based on preliminary work by Hans Dobbertin [4]. On an IBM P690 machine, an MD5 collision can be computed in about one hour using this approach. Collisions for HAVAL-128, MD4 and RIPEMD were found as well. Patrick Stach's `md4coll.c` program [15] implements Wang's approach and can find MD4 collisions in few seconds on my Thinkpad x220 setup specified in Appendix C.

Let n denote the digest size, i.e. the size of the hash value $h(x)$ in bits. Due to the birthday paradox, a collision attack has a generic complexity of $2^{n/2}$ whereas preimage and second preimage attacks have generic complexities of 2^n. In other words it is computationally easier to find any two colliding hash values than the preimage or second preimage for a given hash value.

Following results by Wang et al., differential cryptanalysis was shown as powerful tool for cryptanalysis of hash algorithms. This thesis applies those ideas to satisfiability approaches.

1.3 Differential cryptanalysis

Definition 1.5 (*Hash collision*)

Given a hash function h, a hash collision is a pair (x, x_2) with $x \neq x_2$ such that $h(x) = h(x_2)$.

Differential cryptanalysis is based on the idea to consider two execution states of hash algorithms for slightly different input messages. We trace those difference to learn about the propagation of message differences.

Hash algorithms consume input values as blocks of bits. As far as the length of the input must not conform to the block size, padding is applied. Now consider such a block of input values and another copy of it. We use those two blocks as inputs for two hash algorithm implementations, but provide slight modifications in few bits. MD4 has 48 round function applications in 3 rounds. Differential cryptanalysis considers the difference in the evaluation state between the two instances (compare with Figure 1.1).

Visualizing those differences helps the cryptanalyst to find modifications yielding a small number of differences in the evaluation state. The cryptanalyst consecutively modifies the input values to eventually receive a collision in the output value. If the number of differences in the evaluation state is small, this trail is expected to result in a hash collision with higher probability.

1.4 Satisfiability

Definition 1.6

A *Boolean function* is a mapping $h : X \to Y$ with $X = \{0, 1\}^n$ for $n \in {}_{\geqslant 1}$ and $Y = \{0, 1\}$.

The following definition gives three basic Boolean functions:

Definition 1.7

Let `AND`, `OR` and `NOT` be three Boolean functions.

- `AND` maps $X = \{0, 1\}^2$ to 1 if all values of X are 1.
- `OR` maps $X = \{0, 1\}^2$ to 1 if any value of X is 1.
- `NOT` maps $X = \{0, 1\}^1$ to 1 if the single value of X is 0.

All functions return 0 in the other case.

Definition 1.8

A *truth table* unambiguously defines a Boolean function by enlisting the evaluated truth value for all possible sets of inputs.

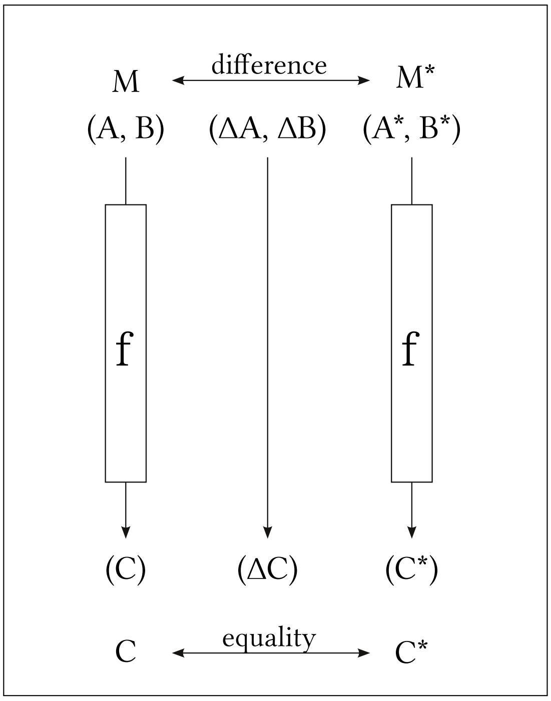


Figure 1.1: Typical attack setting for a collision attack: Hash function f is applied to two inputs M and M^* which differ by some predefined bits. M describes the difference between these values. A hash collision is given if and only if output values C and C^* show the same value. In differential cryptanalysis we observe the differences between two instances applying function f to inputs M and M^*.

v_1	v_2	$f(v_1, v_2)$
1	1	1
1	0	0
0	1	0
0	0	0

(A) AND

v_1	v_2	$f(v_1, v_2)$
1	1	1
1	0	1
0	1	1
0	0	0

(B) OR

v	$f(v)$
1	0
0	1

(C) NOT

TABLE 1.2: Truth tables for AND, OR and NOT

Table 1.2 shows truth tables for AND, OR and NOT.

Boolean functions have an important property which is characterized in the following definition:

Definition 1.9

A Boolean function f is *satisfiable* iff there exists at least one input $x \in X$ such that $f(x) = 1$. Every input $x \in X$ satisfying this property is called *model.* Every element of X is called *assignment.*

The generic complexity of SAT determination is given by 2^n for n Boolean variables. The corresponding tool to determine satisfiability is defined as follows:

Definition 1.10

A *SAT solver* is a tool to determine satisfiability of a Boolean function. If satisfiability is given, it returns some model.

SAT research is heavily concerned with finding good heuristics to find some model for a given SAT problem as fast as possible. Biyearly SAT competitions take place to challenge SAT solvers in a set of benchmarks. The committee evaluates the most successful SAT solvers solving the most problems within a given time frame.

1.5 Satisfiability of hash algorithm states

We discussed Boolean functions and satisfiability. At the same time we looked at basic properties of hash algorithms. But the question remains how we can link those areas together? This section is dedicated to this question.

Definition 1.11

An *algorithm* is a step-wise set of instructions to solve a problem. An *I/O algorithm* transforms given input values to output values.

Hash algorithms are one example of I/O algorithms.

$$\begin{array}{lccc} \text{1st arg:} & & a_1 & a_0 \\ \text{2nd arg:} & + & b_1 & b_0 \\ \hline \text{carry:} & & & c_0 \\ \text{sum:} & = & s_1 & s_0 \end{array} \quad \rightsquigarrow \quad \begin{array}{l} s_0 = \texttt{XOR}(a_0, b_0) \\ c_0 = a_0 \land b_0 \\ s_1 = \texttt{XOR}(a_0, b_0, c_0) \end{array}$$

Figure 1.2: Modelling 2bit addition (left) as Boolean function (right)

I/O algorithms can be implemented as a sequence of instructions for computers. At the same time I/O algorithms can be represented as combination of Boolean functions. This claim is backed in more detail in Section 3.1 with Theorem 3.1. It follows immediately that we can represent I/O algorithms such as hash algorithms entirely as Boolean function.

Theorem 1.1

Every algorithm can be represented as Boolean function.

We consider 2bit addition as small example. Let a_i be the first argument where i denotes the binary position. If $i = 0$, the *least significant bit* (LSB) is considered. If $i = 1$, the *most significant bit* (MSB) is considered.

Let b_i be the second argument and s_i be the output value. Furthermore c_i is the carry bit, where c_1 is left out, because it is not used in 2bit addition. This model of 2bit addition as Boolean function can be seen in Figure 1.2.

1.6 Thesis Outline

This thesis is organized as follows:

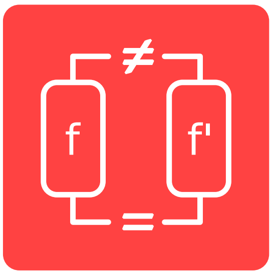
In Chapter 1 we discussed the basic properties and fundamentals of the tools in discussion including hash functions and SAT solvers.

In Chapter 2 we introduce the MD4 and SHA-256 hash functions and discuss possible approaches in differential cryptanalysis.

In Chapter 3 we discuss SAT solving and potential approaches to speed up SAT solvers for cryptographic problems.

In Chapter 4 we show results of our work and discuss its implications.

In Chapter 5 we suggest future work based on our results.



> “Just because it’s automatic
> doesn’t mean it works.”
> —Daniel J. Bernstein

Chapter 2

Differential cryptanalysis

2.1 MD4

MD4 is a cryptographic hash function originally described in RFC 1186 [11], updated in RFC 1320 [12] and declared obsolete by RFC 6150 [16]. It was invented by Ronald Rivest in 1990 with properties given in Table 2.1. Since 1995 [4] successful attacks have been found to break collisions, preimage and second-preimage resistance in MD4; including but not limited to [13] and [8]. A Python 3 implementation derived from a previous Python version is available at github [10].

block size	512 bits	namely variable `block` in RFC 1320 [12]
digest size	128 bits	as per Section 3.5 in RFC 1320 [12]
internal state size	128 bits	namely variables A, B, C and D
word size	32 bits	as per Section 2 in RFC 1320 [12]

Table 2.1: MD4 hash algorithm properties

MD4 uses three auxiliary Boolean functions:

$$\texttt{IF}\,(X, Y, Z) = (X \land Y) \lor (\neg X \land Z) \tag{2.1}$$

$$\texttt{MAJ}\,(X, Y, Z) = (X \land Y) \lor (X \land Z) \lor (Y \land Z) \tag{2.2}$$

$$\begin{aligned}\texttt{XOR}\,(X, Y, Z) = {} & (X \land \neg Y \land \neg Z) \lor (\neg X \land Y \land \neg Z) \\ & \lor (\neg X \land \neg Y \land Z) \lor (X \land Y \land Z)\end{aligned} \tag{2.3}$$

Definition 2.1

The Boolean IF function behaves the following way: If the first argument is true, the second argument is returned. If the first argument is false, the third argument is returned.

The Boolean MAJ function returns true if the number of Boolean values true in arguments is at least 2. The Boolean XOR function returns true if the number of Boolean values true in arguments is odd.

In the following a quick overview over MD4's design is given.

Padding and length extension First of all, padding is applied. A single bit 1 is appended to the input. As long as the input does not reach a length congruent 448 modulo 512, bit 0 is appended. Followingly, length appending takes place. Represent the length of the input (without the previous modifications) in binary and take its first 64 less significant bits. Append those 64 bits to the input.

Initialization The message is split into 512-bit blocks (i.e. 16 32-bit words). Four state variables A, B, C and D are initialized with these hexadecimal values:

[A] 01234567 [B] 89abcdef [C] fedcba98 [D] 76543210

Round function with state variable updates We also need an auxiliary matrix $(i_{k,l})$ which stores indices. Let $i_{k,l}$ be the value in the k-th row and l-th column of matrix $(i_{k,l})$. Analogously $j_{k,l}$ is defined for matrix $(j_{k,l})$.

$$(i_{k,l}) = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 0 & 4 & 8 & 12 & 1 & 5 & 9 & 13 & 2 & 6 & 10 & 14 & 3 & 7 & 11 & 15 \\ 0 & 8 & 4 & 12 & 2 & 10 & 6 & 14 & 1 & 9 & 5 & 13 & 3 & 11 & 7 & 15 \end{pmatrix}$$

$$(j_{k,l}) = \begin{pmatrix} 3 & 7 & 9 & 11 \\ 3 & 5 & 9 & 13 \\ 3 & 9 & 11 & 15 \end{pmatrix}$$

Then the round function is applied to this block in three rounds with 16 iterations each. Let $1 \leqslant k \leqslant 3$ be the round counter and $1 \leqslant l \leqslant 16$ be the iteration counter. For every round, for every iteration apply the following function:

The values of state variable B, C and D are taken as arguments for function F where F is IF in the first 16 iterations, MAJ in the following 16 iterations and finally XOR in the last 16 iterations. This return value is added to the value of state variable A, the current message block M_i and $X[i_{k,l}]$. This sum is then left-rotated by $j_{k,l \ mod \ 4}$ bits and stored in value B. State variables B, C and D update variables C, D and A respectively.

This round function design is visualized in Figure 2.1.

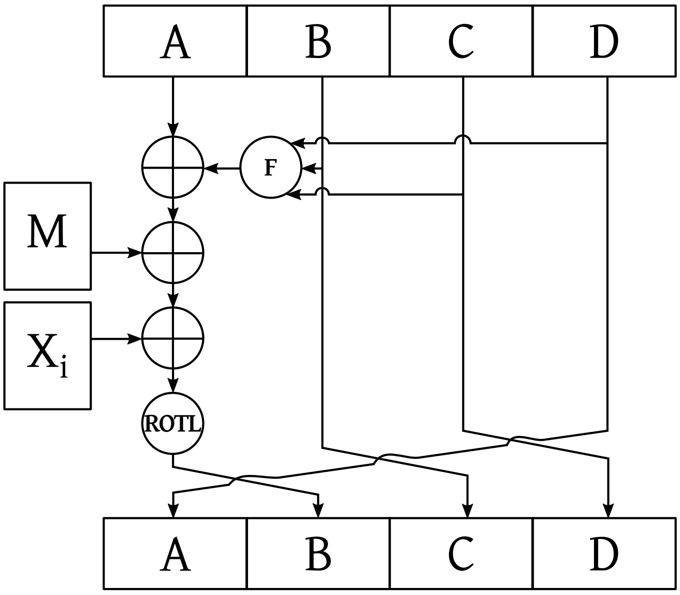


Figure 2.1: MD4 round function updating state variables *A*, *B*, *C* and *D*

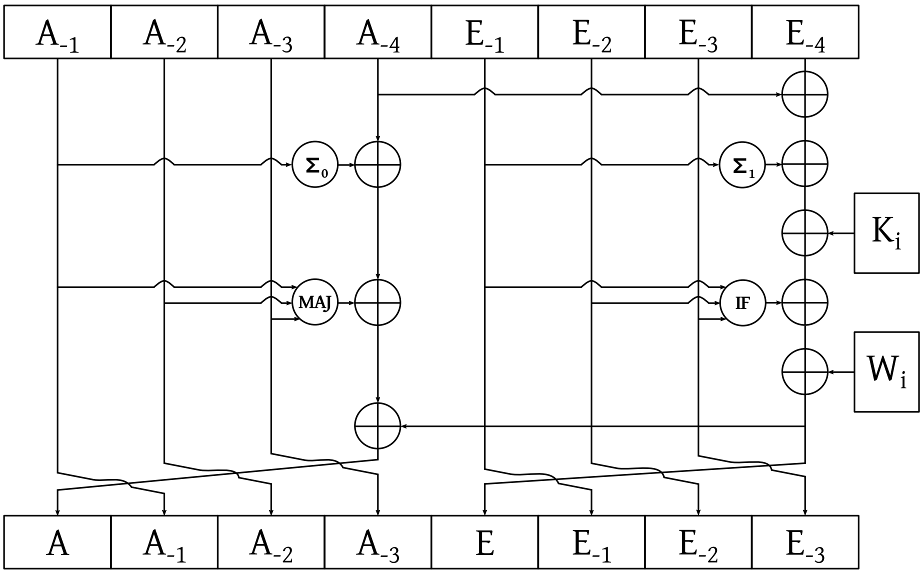


Figure 2.2: SHA-256 round function

2.2 SHA-256

SHA-256 is a hash function from the SHA-2 family designed by the National Security Agency (NSA) and published in 2001. It uses a Merkle-Damgård construction with a Davies-Meyer compression function.

block size	512 bits	as per Section 2.2 of the standard
digest size	256 bits	as per Section 1 of the standard
internal state size	256 bits	as per Section 2.2 of the standard
word size	32 bits	as per Section 2.2 of the standard

TABLE 2.2: SHA-256 hash algorithm properties

Besides MD4's two auxiliary functions `MAJ` and `IF`, another two auxiliary functions are defined:

$$\Sigma_0(X, Y, Z) = S^2(x) \oplus S^{13}(x) \oplus S^{22}(x) \tag{2.4}$$

$$\Sigma_1(X, Y, Z) = S^6(x) \oplus S^{11}(x) \oplus S^{25}(x) \tag{2.5}$$

Padding and length extension The padding and length extension scheme of MD4 is used also in SHA-256. Append bit 1 and append single bits 0 until the message reaches a length of $448 \mod 512$. Afterwards the first 64 bits of the binary representation of the original input is appended.

Initialization In a similar manner to MD4, initialization of internal state variables takes place before running the round function.

$$\begin{array}{llll} H_1^{(0)} = \texttt{6a09e667} & H_2^{(0)} = \texttt{bb67ae85} & H_3^{(0)} = \texttt{3c6ef372} & H_4^{(0)} = \texttt{a54ff53a} \\ H_5^{(0)} = \texttt{510e527f} & H_6^{(0)} = \texttt{9b05688c} & H_7^{(0)} = \texttt{1f83d9ab} & H_8^{(0)} = \texttt{5be0cd19} \end{array}$$

Round function For every block of 512 bits, the round function is applied. The round function entails a much larger internal state than MD4.

2.3 Differential notation

[2]

2.4 Addition example

2.5 Differential path

bit condition	conjunctive normal form	bit condition	conjunctive normal form
#	$(z) \wedge (\neg z)$	1	$(z) \wedge (z^*)$
0	$(\neg z) \wedge (\neg z^*)$	-	$\neg(z \oplus z^*)$
u	$(z) \wedge (\neg z^*)$	A	(z)
3	$(\neg z^*)$	B	$(z \vee \neg z^*)$
n	$(\neg z) \wedge (z^*)$	C	(z^*)
5	$(\neg z)$	D	$(\neg z \vee z^*)$
x	$(z \oplus z^*)$	E	$(z \vee z^*)$
7	$(\neg z \vee \neg z^*)$	?	

TABLE 2.3: Set bit condition value clauses in Simple Evaluation added for a bit condition. A bit condition corresponds to two boolean variables z and z^*.

> “What idiot called them logic errors rather than bool shit?”
> —Unknown

Chapter 3

Satisfiability

3.1 The DIMACS de-facto standard

Definition 3.1

A *conjunction* is a sequence of Boolean functions combined using a logical OR. A *disjunction* is a sequence of Boolean functions combined using a logical AND. A *literal* is a Boolean variable (*positive*) or its negation (*negative*).

A SAT problem is given in *Conjunctive Normal Form* (CNF) if the problem is defined as conjunction of disjunctions of literals.

A simple example for a SAT problem in CNF is the exclusive OR (XOR). It takes two Boolean values a and b as arguments and returns true if and only if the two arguments differ.

$$(a \lor b) \land (\neg a \lor \neg b) \tag{3.1}$$

Display 3.1 shows one conjunction (denoted $\land$) of two disjunctions (denoted $\lor$) of literals (denoted a and b where prefix $\neg$ represents negation). This structure constitutes a CNF.

Analogously we define a *Disjunctive Normal Form* (DNF) as disjunction of conjunctions of literals. The negation of a CNF is in DNF, because literals are negated and conjunctions become disjunctions, vice versa.

Theorem 3.1

Every Boolean function can be represented as CNF.

Theorem 3.1 is easy to prove. Consider the truth table of an arbitrary Boolean function f with k input arguments and j rows of output value false. We represent f as CNF.

Consider Boolean variables $b_{i,l}$ with $0 \leqslant i \leqslant j$ and $0 \leqslant l \leqslant k$. For every row i of the truth table with assignment (r_i), add one disjunction to the CNF. This disjunction contains $b_{i,l}$ if $r_{i,l}$ is false. The disjunction contains $b_{i,l}$ if $r_{i,l}$ is true.

As far as f is an arbitrary k-ary Boolean function, we have proven that any function can be represented as CNF.

SAT problems are usually represented in the DIMACS de-facto standard. Consider a SAT problem in CNF with *nbclauses* clauses and enumerate all variables from 1 to *nbvars*. A DIMACS file is an ASCII text file. Lines starting with “`c`” are skipped (comment lines). The first remaining line has to begin with “`p cnf`” followed by *nbclauses* and *nbvars* separated by spaces (header line). All following non-comment lines are space-separated indices of Boolean variables optionally prefixed by a minus symbol. Then one line represents one clause and must be terminated with a zero symbol after a space. All lines are conjuncted to form a CNF.

Variations of the DIMACS de-facto standard also allow multiline clauses (the zero symbol constitutes the end of a clause) or arbitrary whitespace instead of spaces. The syntactical details are individually published on a per competition basis.

Listing 3.1: Display 3.1 represented in DIMACS format

```
p cnf 2 2
a b
-a -b
```

Definition 3.2

A *clause* is a disjunction of literals. A *k-clause* is a clause consisting of exactly k literals. A *unit clause* is a 1-clause. A *Horn clause* is a clause with at most one positive literal. A *definite clause* is a clause with exactly one positive literal.

3.2 SAT features and CNF analysis

At the very beginning I was very intrigued with the question "What is an 'average' SAT problem?". Answers to this question can help to optimize SAT solver memory layouts. But originally I was wondering whether our differential cryptanalysis SAT problems distinguish from "average" SAT problems in some very basic properties. First of all, we need to elaborate on the question itself.

Definition 3.3 (*SAT feature*)

A *SAT feature* is a statistical value (named *feature value*) retrievable from some given SAT problem in some well-defined encoding.

A SAT feature is called *performance-driven* if the runtime of any computation contributes to the feature value.

The most basic example of a SAT feature is the number of variables and clauses of a given SAT problem. This SAT feature is stored in the CNF header of a SAT problem encoded in the DIMACS format.

It should be computationally easy to evaluate SAT features of a given SAT problem. The general goal is to write a tool which evaluates several SAT features at the same time and retrieve them for comparison with other problems. A SAT feature is expected to be computable in linear time and memory with the number of variables and number of clauses. But a suggested limit is only given with polynomial complexity for evaluation algorithms. Sticking to this convention implies that evaluation of satisfiability must not be necessary to evaluate a SAT feature under the assumption that $\mathscr{P} \neq \mathscr{NP}$. Hence the number of valid models cannot be a SAT feature as far as satisfiability needs to be determined. But no actual hard boundary for runtime requirements is given. Previous work has shown that expensive algorithms can provide useful data in a small time frame if they are limited to a constant subproblem size.

The most similar resource I found looking at SAT features was the SATzilla project [9, 18] in 2012. The authors systematically defined 138 SAT features categorized in 12 groups. The features themselves are not defined formally, but an implementation is provided bundled with example data.

POSNEG_RATIO_CLAUSE_mean	ratio of positive to negative clauses, mean

Many SAT solvers collect feature values to improve algorithm selection, restart strategies and estimate problem sizes. Recent trends to apply Machine Learning to SAT solving imply feature evaluation. SAT features and the resulting satisfiability runtime are used as training data for Machine Learning. One example using SAT features for algorithm selection is ASlib [1].

However, most of these SAT features are performance-driven. Examples for performance-driven SAT features include the number of restarts within a certain time frame or evaluation of local minima.

POSNEG-RATIO-CLAUSE-mean

In the following section we want to evaluate SAT features and compare test cases.

3.3 SAT features in comparison

Proposition 3.1

The set of public benchmarks in SAT competitions between 2008 and 2015 represent average SAT problems

Define a large set of SAT features. Present data. Categorize data.

3.4 Basic SAT solving techniques

3.5 SAT solvers in use

3.6 Encodings

3.6.1 STP approach

Given a set of clauses, return a subset of clauses satisfying given criterion	
`clauses_allLitsNeg`	all literals are negative
`clauses_oneLitNeg`	exactly one literal is negative
`clauses_geqOneLitNeg`	more than one literal is negative
`clauses_allLitsPos`	all literals are positive
`clauses_oneLitPos`	exactly one literal is positive
`clauses_geqOneLitPos`	more than one literal is positive
`clauses_length1`	clause contains exactly one literal ("unit clause")
`clauses_length2`	clause contains exactly two literals
`clauses_unique`	clause did not yet occur
`clauses_tautological`	clause contains some literal and its negation
Given a set of literals/variables, return Boolean property	
`literals_existential`	literal does not occur negated
`literals_unit`	literal occurs in clause of length 1
`literals_contradiction`	literal occurs with its negation on one clause
`literals_1occ`	literal occurs only in one clause once
`literals_2occs`	literal occurs two times in clauses
`literals_3occs`	literal occurs three times in clauses
`variables_unit`	variable occurs in clause of length 1
Given a set of clauses, return real number based on this clause	
`clauses_mapLength`	number of literals in clause
`clauses_mapRatioPosNeg`	number of positive literals divided by total number of literal
`clauses_mapNumPos`	number of positive literals in clause
Given one clause, return Boolean property	
`clauselits_someEx`	any is literal existential
`clauselits_allEx`	all literals are existential
`clauselits_someUnit`	contains unit variable
`clauselits_someContra`	contains contradiction variable
`clauselits_all1occ`	all variables occur only once in all clauses
`clauselits_all12occ`	all variables occur only once or twice in all clauses
Given all clauses, return the following property	
`concomp_variable`	number of connected components where two variables are in the same component iff they occur in at least one clause together
`concomp_literal`	number of connected components where two literals are in the same component iff they occur in at least one clause together
`xor2_count`	Number of clause pairs ($a \vee b$, $\neg a \vee b$) for two variables a and b

Chapter 4

Results

4.1 Benchmark results

4.2 Related work

4.3 Conclusion

Chapter 5

Summary and Future Work

5.1 Summary of results

5.2 Future work

Appendices

Appendix A

Illustration

i		$\nabla S_{i,0}$	$\nabla S_{i,1}$	$\nabla S_{i,2}$
-4	A:	01100111010001010010001100000001		
-3	A:	00010000000110010010101000111 0110		
-2	A:	10011000101110101101110011111110		
-1	A:	11101111111001101101010111000 1001		
0	A:	01101011110101001110010000010010	W:	01001101011110101001110010000011
1	A:	0111011001001111111011100u110001	W:	u1010110110010111001001001111010
2	A:	1010101101000000001110u01n1110010	W:	n01n1001110101011010010101111000
3	A:	101011u1001111010101001001010001	W:	01010111101001111010010111101110
4	A:	00101100011000110101010111110010	W:	11011110011101001000101000111100
5	A:	0001101001100010 10u1101000000001	W:	11011100110000110110011010110011
6	A:	0001101100unuu110001000001111010	W:	10110110100000111010000000100000
7	A:	0010101110000001 0unn011001010000	W:	00111011001010100101110110011111
8	A:	011100110010001u1111111110110000	W:	11000110100111010111000110110011
9	A:	101011n01unnu0001111100110011111	W:	11111001111010011001000110011000
10	A:	10n00100100001010100000001010 1110	W:	11010111100111111000000001011110
11	A:	u1000110101101100100101011111111	W:	10100110001110111011001011101000
12	A:	001011u00u1010111111110001111011	W:	0100010111011 10n1000111000110001
13	A:	10un1n01001100010100000111100101	W:	10010111111000110001111111100101
14	A:	00001010010100011000100011010110	W:	00100111100101001011111100001000
15	A:	000111101010 1u010110011011010100	W:	10111001111010001100001111101001
16	A:	n00n0un0110100101001101101011111		
17	A:	00011111001110100001001000011110		
18	A:	01010111000011010000000001001010 0		
19	A:	u1n10000000101111001101011000100		
20	A:	n1un1001111111011101000000110100		
21	A:	11110011101100000101111111010100		
22	A:	01011101110011010011001100111010		
23	A:	01010000111011101100011110001111		
24	A:	00000010000100100011011100011010		
25	A:	10110000100101100001010011101010		
26	A:	00001010100010010111011101000001		
27	A:	00000110111011110101101010110011		
28	A:	10110110010111010110110000100101		
29	A:	10100010000011010100100001101001		
30	A:	00101001110101111100011101100011		
31	A:	11111100100100101101011110110110		
32	A:	01001111110100100110100000101111		
33	A:	00111000000111101011011101110010 0		
34	A:	00100000001110101111010000000101 01		
35	A:	n0100000000110011000000100011100 10		
36	A:	n0000111111010111101111001011001		
37	A:	11001000000110100100001100001100		
38	A:	10110000011001111110100110101100		
39	A:	00010010000001010000110110001110 0		
40	A:	11000000010010000111000110000101		
41	A:	00000110100001101111010100100110		
42	A:	01001110110111011111111010000110		
43	A:	01010000011000111101000001101101		
44	A:	11111000000101101111011100001100		
45	A:	10001010110110110010110000000100		
46	A:	10000010100110010101100011011100		
47	A:	10000001111001011011010010111101		

Table A.1: One of the original MD4 collision given by Wang, et al.

Appendix B

Testcases

Figures B.1, B.2, B.3 and B.4 show testcases used to test performance measures.

i		$\nabla S_{i,0}$	$\nabla S_{i,1}$	$\nabla S_{i,2}$
-4	A:	01100111010001010010001100000001		
-3	A:	00010000001100100101010001110110		
-2	A:	10011000101110101101110011111110		
-1	A:	11101111110011011010101110001001		
0	A:	x-------------------------------	W:	---x----------------------------
1	A:	--------------------------------	W:	--------------------------------
2	A:	---------------------x----------	W:	x-------------------------------
3	A:	xxx-----------------------------	W:	--------------------------------
4	A:	------------------------------xx	W:	x-------------------------------
5	A:	-------xxxxxxxxxxxxxxxx-x-------	W:	--------------------------------
6	A:	x--------x-----------x-x-xxxx--x	W:	--------------------------------
7	A:	---------x-----x----------------	W:	--------------------------------
8	A:	---------x-------x-x-x----------	W:	x-------------------------------
9	A:	-----------------x---x----------	W:	--------------------------------
10	A:	------x-----x--xxx-xxx----------	W:	--------------------------------
11	A:	x----------------xxx-x----------	W:	--------------------------------
12	A:	---x--x-------------------------	W:	x-------------------------------
13	A:	--------------------------------	W:	--------------------------------
14	A:	-x------------------------------	W:	--------------------------------
15	A:	x-x----------x------------------	W:	--------------------------------
16	A:	-xxx----------------------------		
17	A:	--------------------------------		
18	A:	--------------------------------		
19	A:	x-------------------------------		
20	A:	x-------------------------------		
21	A:	--------------------------------		
22	A:	--------------------------------		
23	A:	--------------------------------		
24	A:	--------------------------------		
25	A:	--------------------------------		
26	A:	--------------------------------		
27	A:	--------------------------------		
28	A:	--------------------------------		
29	A:	--------------------------------		
30	A:	--------------------------------		
31	A:	--------------------------------		
32	A:	x-------------------------------		
33	A:	--------------------------------		
34	A:	--------------------------------		
35	A:	--------------------------------		
36	A:	--------------------------------		
37	A:	--------------------------------		
38	A:	--------------------------------		
39	A:	--------------------------------		
40	A:	--------------------------------		
41	A:	--------------------------------		
42	A:	--------------------------------		
43	A:	--------------------------------		
44	A:	--------------------------------		
45	A:	--------------------------------		
46	A:	--------------------------------		
47	A:	--------------------------------		

TABLE B.1: TODO description

i		$\nabla S_{i,0}$	$\nabla S_{i,1}$	$\nabla S_{i,2}$
-4	A:	01100111010001010010001100000001		
-3	A:	00010000001100100101010001110110		
-2	A:	10011000101110101101110011111110		
-1	A:	11101111110011011010101110001001		
0	A:	????????????????????????????????	W:	---x----------------------------
1	A:	????????????????????????????????	W:	--------------------------------
2	A:	????????????????????????????????	W:	x-------------------------------
3	A:	????????????????????????????????	W:	--------------------------------
4	A:	????????????????????????????????	W:	x-------------------------------
5	A:	????????????????????????????????	W:	--------------------------------
6	A:	????????????????????????????????	W:	--------------------------------
7	A:	????????????????????????????????	W:	--------------------------------
8	A:	????????????????????????????????	W:	x-------------------------------
9	A:	????????????????????????????????	W:	--------------------------------
10	A:	????????????????????????????????	W:	--------------------------------
11	A:	????????????????????????????????	W:	--------------------------------
12	A:	??????????????------------------	W:	x-------------------------------
13	A:	??????????????------------------	W:	--------------------------------
14	A:	??????????????------------------	W:	--------------------------------
15	A:	??????????????------------------	W:	--------------------------------
16	A:	???x----------------------------		
17	A:	?-------------------------------		
18	A:	?-------------------------------		
19	A:	?-------------------------------		
20	A:	x-------------------------------		
21	A:	--------------------------------		
22	A:	--------------------------------		
23	A:	--------------------------------		
24	A:	--------------------------------		
25	A:	--------------------------------		
26	A:	--------------------------------		
27	A:	--------------------------------		
28	A:	--------------------------------		
29	A:	--------------------------------		
30	A:	--------------------------------		
31	A:	--------------------------------		
32	A:	x-------------------------------		
33	A:	--------------------------------		
34	A:	--------------------------------		
35	A:	--------------------------------		
36	A:	--------------------------------		
37	A:	--------------------------------		
38	A:	--------------------------------		
39	A:	--------------------------------		
40	A:	--------------------------------		
41	A:	--------------------------------		
42	A:	--------------------------------		
43	A:	--------------------------------		
44	A:	--------------------------------		
45	A:	--------------------------------		
46	A:	--------------------------------		
47	A:	--------------------------------		

Table B.2: TODO description

i		$\nabla S_{i,0}$	$\nabla S_{i,1}$	$\nabla S_{i,2}$
-4	A:	01100111010001010010001100000001		
-3	A:	00010000001100100101010001110110		
-2	A:	10011000101110101101110011111110		
-1	A:	11101111110011011010101110001001		
0	A:	????????????????????????????????	W:	---x----------------------------
1	A:	????????????????????????????????	W:	--------------------------------
2	A:	????????????????????????????????	W:	x-------------------------------
3	A:	????????????????????????????????	W:	--------------------------------
4	A:	????????????????????????????????	W:	x-------------------------------
5	A:	????????????????????????????????	W:	--------------------------------
6	A:	????????????????????????????????	W:	--------------------------------
7	A:	????????????????????????????????	W:	--------------------------------
8	A:	????????????????????????????????	W:	x-------------------------------
9	A:	????????????????????????????????	W:	--------------------------------
10	A:	????????????????????????????????	W:	--------------------------------
11	A:	????????????????????????????????	W:	--------------------------------
12	A:	????????????????????????????????	W:	x-------------------------------
13	A:	????????????????????????????????	W:	--------------------------------
14	A:	????????????????????????????????	W:	--------------------------------
15	A:	????????????????????????????????	W:	--------------------------------
16	A:	????????????????????????????????		
17	A:	????????????????????????????????		
18	A:	????????????????????????????????		
19	A:	????????????????????????????????		
20	A:	????????????????????????????????		
21	A:	--------------------------------		
22	A:	--------------------------------		
23	A:	--------------------------------		
24	A:	--------------------------------		
25	A:	--------------------------------		
26	A:	--------------------------------		
27	A:	--------------------------------		
28	A:	--------------------------------		
29	A:	--------------------------------		
30	A:	--------------------------------		
31	A:	--------------------------------		
32	A:	x-------------------------------		
33	A:	--------------------------------		
34	A:	--------------------------------		
35	A:	--------------------------------		
36	A:	--------------------------------		
37	A:	--------------------------------		
38	A:	--------------------------------		
39	A:	--------------------------------		
40	A:	--------------------------------		
41	A:	--------------------------------		
42	A:	--------------------------------		
43	A:	--------------------------------		
44	A:	--------------------------------		
45	A:	--------------------------------		
46	A:	--------------------------------		
47	A:	--------------------------------		

TABLE B.3: TODO description

i		$\nabla S_{i,0}$	$\nabla S_{i,1}$	$\nabla S_{i,2}$
-4	A:	01100111010001010010001100000001		
-3	A:	00010000001100100101010001110110		
-2	A:	10011000101110101101110011111110		
-1	A:	11101111110011011010101110001001		
0	A:	????????????????????????????????	W:	????????????????????????????????
1	A:	????????????????????????????????	W:	????????????????????????????????
2	A:	????????????????????????????????	W:	????????????????????????????????
3	A:	????????????????????????????????	W:	????????????????????????????????
4	A:	????????????????????????????????	W:	????????????????????????????????
5	A:	????????????????????????????????	W:	????????????????????????????????
6	A:	????????????????????????????????	W:	????????????????????????????????
7	A:	????????????????????????????????	W:	????????????????????????????????
8	A:	????????????????????????????????	W:	????????????????????????????????
9	A:	????????????????????????????????	W:	????????????????????????????????
10	A:	????????????????????????????????	W:	????????????????????????????????
11	A:	????????????????????????????????	W:	????????????????????????????????
12	A:	????????????????????????????????	W:	????????????????????????????????
13	A:	????????????????????????????????	W:	????????????????????????????????
14	A:	????????????????????????????????	W:	????????????????????????????????
15	A:	????????????????????????????????	W:	????????????????????????????????
16	A:	????????????????????????????????		
17	A:	????????????????????????????????		
18	A:	????????????????????????????????		
19	A:	????????????????????????????????		
20	A:	????????????????????????????????		
21	A:	--------------------------------		
22	A:	--------------------------------		
23	A:	--------------------------------		
24	A:	--------------------------------		
25	A:	--------------------------------		
26	A:	--------------------------------		
27	A:	--------------------------------		
28	A:	--------------------------------		
29	A:	--------------------------------		
30	A:	--------------------------------		
31	A:	--------------------------------		
32	A:	x???????????????????????????????		
33	A:	--------------------------------		
34	A:	--------------------------------		
35	A:	--------------------------------		
36	A:	--------------------------------		
37	A:	--------------------------------		
38	A:	--------------------------------		
39	A:	--------------------------------		
40	A:	--------------------------------		
41	A:	--------------------------------		
42	A:	--------------------------------		
43	A:	--------------------------------		
44	A:	--------------------------------		
45	A:	--------------------------------		
46	A:	--------------------------------		
47	A:	--------------------------------		

TABLE B.4: TODO description

Appendix C

Hardware setup

In the following we introduce two hardware setups which were used to run our testcases. The first setup is referred to as “Thinkpad x220” throughout the document whereas the second setup is referred to as “Cluster”.

Type model	Thinkpad Lenovo x220 tablet, 4299-2P6
Processor	Intel i5-2520M, 2.50 GHz, dual-core, Hyperthreaded
RAM	16 GB (extension to common retail setup)
Memory	160 GB SSD
L3 cache size	3072 KB

Table C.1: Thinkpad x220 Tablet specification [6]

Processor	Intel Xeon X5690, 3.47 GHz, 6 cores, Hyperthreaded
RAM	192 GB
L3 cache size	12288 KB

Table C.2: Cluster node nehalem192go specification [3]

Index

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