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Master Thesis

Differential cryptanalysis with SAT solvers

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Abstract

Hash functions are ubiquitous in the modern information age. They provide preimage, second preimage and collision resistance which are needed in a wide range of applications.

In August 2006, Wang et al. showed efficient attacks against several hash function designs including MD4, MD5, HAVAL-128 and RIPEMD. With these results differential cryptanalysis has been shown useful to break collision resistance in hash functions. Over the years advanced attacks based on those differential approaches have been developed.

To find collisions like Wang et al., a cryptanalyst needs to specify a differential characteristic. Looking at the differential behavior of the underlying operations of the hash algorithm shows how differential values propagate in the algorithm. The goal is to find a differential characteristic whose differences cancel out in the output. Once such a differential characteristic was discovered, in a second step the actual values for those differences are defined yielding an actual hash collision.

Finding a differential characteristic can be a cumbersome and tedious task. Whereas propagation can be automated using dedicated tools, finding an initial differential characteristic is a difficult task as it can be specified with arbitrary levels of granularity.

SAT solvers inherently implement both tasks. They consecutively propagate values which narrow the search space. The probability to find a satisfiable assignment increases if the narrowed search space has many satisfiable assignments. And finally the assignment reveals initial values. On the other hand, SAT solvers have no notion of differential values and therefore problem encoding becomes an important topic.

In this thesis we look at differential characteristics and encode them as SAT problem. A SAT solver tells us whether a differential characteristic can represent a hash collision or not. We implemented a framework which allows us to verify differential behavior for integer operations. We then looked at the encoded problems in details and tried to change the encoding to improve the runtime of the SAT solver. We also provide a small CNF analysis library to compare an encoded problem with others.

Keywords: hash function, differential cryptanalysis, differential characteristic, MD4, SHA-256, collision resistance, satisfiability, SAT solver

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Also thanks to Roderick Bloem and Armin Biere who organized a meeting one year before submitting this work defining the main approaches involved in this thesis. Armin Biere released custom lingeling versions for us, e.g. featuring "more important" clauses in lingeling. He also provided further analysis for our testcases.

And finally I am grateful for the support by Martina, who also supported me during good and bad days with this thesis, and my parents which provided a prosperous environment to me to be able to stand where I am today.

Thank you.
どもありがとうございました。

All source code is available at lukas-prokop.at/proj/megosat and published under terms and conditions of Free/Libre Open Source Software. This document was printed with LuaLaTeX and Linux Libertine Font.

Contents

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Chapter 1

Introduction

1.1 Overview

Hash functions are used as cryptographic primitives in many applications and protocols. They take an arbitrary input message and provide a hash value. Input message and hash value are considered as byte strings in a particular encoding. The hash value is of fixed length and satisfies several properties which make it useful in a variety of applications.

In this thesis we will consider the hash algorithms MD4 and SHA-256. They use basic arithmetic functions like addition and bit-level functions such as XOR to transform an input to a hash value. We use a bit vector as input to this implementation and all operations applied to this bit vector will be represented as clauses of a SAT problem. Additionally we represent differential characteristics of hash collisions as SAT problem. If and only if satisfiability is given, the particular differential state is achievable using two different inputs leading to the same output. As far as SAT solvers return an actual model satisfying that state, we get an actual hash collision which can be verified and visualized. If the internal state of the hash algorithm is too large, the attack can be computationally simplified by modelling only a subset of steps of the hash algorithm or changing the modelled differential path.

Based on experience with these kind of problems with previous non-SAT-based tools we aim to apply best practices to a satisfiability setting. We will discuss which SAT techniques lead to best performance characteristics for our MD4 and SHA-256 testcases.

1.2 Thesis Outline

This thesis is organized as follows:

In Chapter 1 we briefly introduced the basic subjects of this thesis. We explained our high-level goal involving hash functions and SAT solvers.

In Chapter 2 we introduce the MD4 and SHA-256 hash functions. Certain design decisions imply certain properties which can be used in differential cryptanalysis. We discuss those decisions in this chapter after a formal definition of the function itself. Beginning with this chapter we developed a theoretical notion of our tools.

In Chapter 3 we discuss approaches of differential cryptanalysis. We begin with work done by Wang, et al. and followingly introduce differential notation to simplify representation of differential states. This way we can easily dump hash collisions.

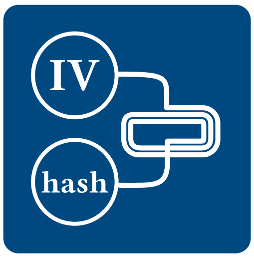
In Chapter 4 we discuss SAT solving. We give a brief overview over used SAT solvers and discuss how we can speed up SAT solvers for cryptographic problems.

In Chapter 5 we define SAT features which help us to classify SAT problems. This is a small subproject we did to look at properties of resulting DIMACS CNF files.

In Chapter 6 we discuss how we represent a problem (i.e. the hash function and a differential characteristic) as SAT problem. This ultimatively allows us to solve the problem using a SAT solver.

In Chapter 7 we show data as result of our work. Runtimes are the main part of this chapter, but also results of the SAT features project are presented.

In Chapter 8 we suggest future work based on our results.



Chapter 2

Hash algorithms

In this chapter we will define hash functions and their desired security properties. Followingly we look at SHA256 and MD4 as established hash functions. We will represent them with Boolean algebra (in chapter 6) to make reasoning about states in those hash functions possible using SAT solvers.

2.1 Preliminaries Redux

Definition 2.1 (*Hash function*)

A *hash function* is a mapping $h : X \to Y$ with $X = \{0,1\}^*$ and $Y = \{0,1\}^n$ for some fixed $n \in \mathbb{N}_{\geq 1}$.

- Let $x \in X$, then $h(x)$ is called *hash value of* x.
- Let $h(x) = y \in Y$, then x is called *preimage of* y.

Hash functions are considered as cryptographic primitives used as building blocks in cryptographic protocols. A hash function has to satisfy the following security requirements:

Definition 2.2 (*Preimage resistance*)

Given $y \in Y$, a hash function h is *preimage resistant* iff it is computationally infeasible to find $x \in X$ such that $h(x) = y$.

Definition 2.3 (*Second-preimage resistance*)

Given $x \in X$, a hash function h is *second-preimage resistant* iff it is computationally infeasible to find $x_2 \in X$ with $x \neq x_2$ such that $h(x) = h(x_2)$. x_2 is called *second preimage.*

Definition 2.4 (*Collision resistance*)

A hash function h is *collision resistant* iff it is computationally infeasible to find any two $x \in X$ and $x_2 \in X$ with $x \neq x_2$ such that $h(x) = h(x_2)$. Tuple (x, x_2) is called *collision.*

As far as hash functions accept input strings of arbitrary length, but return a fixed size output string, existence of collisions is unavoidable [20]. However, good hash functions make it very difficult to find collisions or preimages.

Any digital data can be hashed (i.e. used as input to a hash function) by considering it in binary representation. The format or encoding is not part of the hash function's specification.

2.1.1 Merkle-Damgård designs

The Merkle-Damgård design is a particular design of hash functions providing the following security guarantee:

Definition 2.5 (*Collision resistance inheritance*)

Let F_0 be a collision resistant compression function. A hash function in Merkle-Damgård design is collision resistant if F_0 is collision resistant.

This motivates thorough research of collisions in compression functions. The design was found independently by Ralph C. Merkle and Ivan B. Damgård. It was described by Merkle in his PhD thesis [13, p. 13–15] and followingly used in popular hash functions such as MD4, MD5 and the SHA2 hash function family. The single-pipe design works as follows:

1. Split the input into blocks of uniform block size. If necessary, apply padding to the last block to achieve full block size.
2. Compression function F_0 is applied iteratively using the output y_{i-1} of the previous iteration and the next input block x_i, denoted $y_i = F_0(y_{i-1}, x_i)$.
3. An optional postprocessing function is applied.

2.1.2 Padding and length extension attacks

Hash functions of single-piped Merkle-Damgård design inherently suffer from length extension attacks. MD4 and SHA256 apply padding to their input to normalize their input size to a multiple of its block size. The compression function is applied afterwards. This design is vulnerable to length extensions.

Consider some collision (x_0, x_1) with $F_0(x_0) = y = F_0(x_1)$ where x_0 and x_1 have a size of one block. Let p be a suffix with size of one block. Then also $(x_0 \| p, x_1 \| p)$ (where $\|$ denotes concatenation) represents a collision in single-piped Merkle-Damgård designs, because it holds that:

$$F_0(F_0(x_0), p) = F_0(F_0(x_1), p) \iff F_0(y, p) = F_0(y, p)$$

Hence $(x_0 \| p, x_1 \| p)$ is a collision as well. As far as F_0 is applied recursively to every block, p can be of arbitrary size and (x_0, x_1) can be of arbitrary uniform size.

Because of this vulnerability, cryptanalysts only need to find a collision in compression functions. In our tests will only consider input of one block and padding will be neglected.

2.1.3 Example usage

One example showing the use of hash functions as primitives are JSON Web Tokens (JWT) specified in RFC 7519 [11]. Its application allows web developers to represent claims to be transferred between two parties.

Section 8 defines implementation requirements and refers to RFC 7518 [7], which specifies cryptographic algorithms to be implemented. "HMAC SHA-256" (besides "none") is the only signature and MAC algorithm required to be implemented. Hence SHA-256 as hash algorithm is used as primitive in this configuration.

2.2 MD4

MD4 is a cryptographic hash function originally described in RFC 1186 [17], updated in RFC 1320 [18] and declared obsolete by RFC 6150 [22]. It was invented by Ronald Rivest in 1990 with properties given in Table 2.1. In 1995 [4] successful full-round attacks have been found to break collision resistance. Followingly preimage and second-preimage resistance in MD4 have been broken as well. Some of those attacks are described in [19] and [14]. We derived a Python 3 implementation based on a Python 2 implementation and made it available on github [16].

MD4 uses three auxiliary Boolean functions:

Definition 2.6

The Boolean `IF` function is defined as follows: If the first argument is true, the second argument is returned. Otherwise the third argument is returned.

block size	512 bits	namely variable `block` in RFC 1320 [18]
digest size	128 bits	as per Section 3.5 in RFC 1320 [18]
internal state size	128 bits	namely variables A, B, C and D
word size	32 bits	as per Section 2 in RFC 1320 [18]

TABLE 2.1: MD4 hash algorithm properties

The Boolean `MAJ` function returns true if the number of Boolean values true in arguments is at least 2. The Boolean `XOR` function returns true if the number of Boolean values true in arguments is odd.

Using the logical operators $\wedge$ (AND), $\vee$ (OR) and $\neg$ (NEG) we can define them as (see section 4.1 for a thorough discussion of these operators):

$$\texttt{IF}(X,Y,Z) = (X \wedge Y) \vee (\neg X \wedge Z) \tag{2.1}$$

$$\texttt{MAJ}(X,Y,Z) = (X \wedge Y) \vee (X \wedge Z) \vee (Y \wedge Z) \tag{2.2}$$

$$\begin{aligned}\texttt{XOR}(X,Y,Z) = {} & (X \wedge \neg Y \wedge \neg Z) \vee (\neg X \wedge Y \wedge \neg Z) \\ & \vee (\neg X \wedge \neg Y \wedge Z) \vee (X \wedge Y \wedge Z)\end{aligned} \tag{2.3}$$

In the following a brief overview over MD4's design is given.

Padding and length extension. First of all, padding is applied. A single bit 1 is appended to the input. As long as the input does not reach a length congruent 448 modulo 512, bit 0 is appended. Followingly, length appending takes place. Represent the length of the input (without the previous modifications) in binary and take its first 64 less significant bits. Append those 64 bits to the input.

Initialization. The message is split into 512-bit blocks (i.e. 16 32-bit words). Four state variables A_i with $-4 \leq i < 0$ are initialized with these hexadecimal values:

$[\texttt{A}_{-4}]$ `01234567` $[\texttt{A}_{-1}]$ `89abcdef` $[\texttt{A}_{-2}]$ `fedcba98` $[\texttt{A}_{-3}]$ `76543210`

Round function with state variable updates. We also need an auxiliary matrix $(i_{k,l})$ which stores indices. Let $i_{k,l}$ be the value in the k-th row and l-th column of matrix $(i_{k,l})$. Analogously $j_{k,l}$ is defined for matrix $(j_{k,l})$.

$$(i_{k,l}) = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 0 & 4 & 8 & 12 & 1 & 5 & 9 & 13 & 2 & 6 & 10 & 14 & 3 & 7 & 11 & 15 \\ 0 & 8 & 4 & 12 & 2 & 10 & 6 & 14 & 1 & 9 & 5 & 13 & 3 & 11 & 7 & 15 \end{pmatrix}$$

$$(j_{k,l}) = \begin{pmatrix} 3 & 7 & 9 & 11 \\ 3 & 5 & 9 & 13 \\ 3 & 9 & 11 & 15 \end{pmatrix}$$

Then the round function is applied to this block in three rounds with 16 iterations each. Let $1 \leq k \leq 3$ be the round counter and $1 \leq l \leq 16$ be the iteration counter. For every round, for every iteration apply the following function:

The values of state variable A_{-1}, A_{-2} and A_{-3} are taken as arguments for function F where F is `IF` in the first 16 iterations, `MAJ` in the following 16 iterations and finally `XOR` in the last 16 iterations. This return value is added to the value of state variable A_{-4}, the current message block M and $X_{i_{k,l}}$. This sum modulo 2^{32} is then left-rotated (see Definition 2.7) by $j_{k,l \ mod \ 4}$ bits and stored in value A_{-1}. State variables A_{-1}, A_{-2} and A_{-3} update variables A_{-2}, A_{-3} and A_{-4} respectively.

This round function design is visualized in Figure 2.1.

2.3 SHA-256

SHA-256 is a hash function from the SHA-2 family designed by the National Security Agency (NSA) and published in 2001 [6]. It uses a Merkle-Damgård construction with a Davies-Meyer compression function. The best known preimage attack was found in 2011 and breaks preimage resistance for 52 rounds [8]. The best known collision attack breaks collision resistance for 31 rounds of SHA-256 [12] and pseudo-collision resistance for 46 rounds [9].

block size	512 bits	as per Section 1 of the standard [6]
digest size	256 bits	mentioned as Message Digest size [6]
internal state size	256 bits	as per Section 1 of the standard [6]
word size	32 bits	as per Section 1 of the standard [6]

Table 2.2: SHA-256 hash algorithm properties

Definition 2.7 (*Shifts, rotations and a notational remark*)

Consider a 32-bit word X with 32 binary values b_i with $0 \leq i \leq 31$. b_0 refers to the least significant bit. Shifting ($\ll$ and $\gg$) and rotation ($\lll$ and $\ggg$) creates a

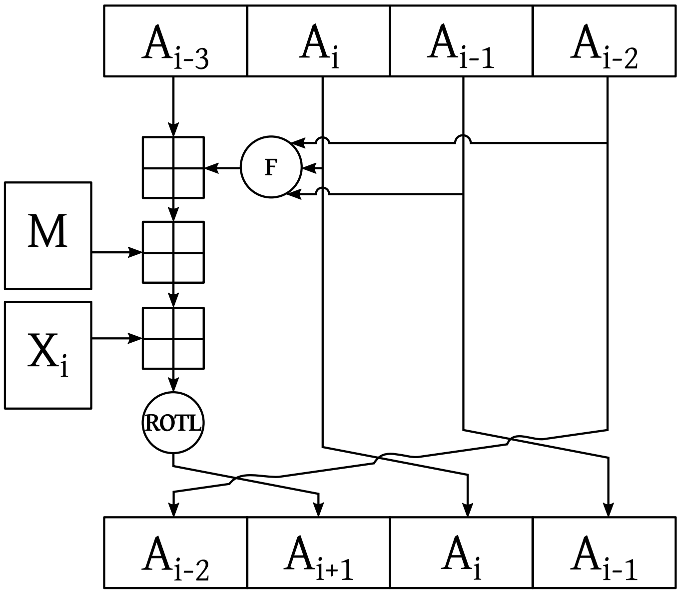


Figure 2.1: MD4 round function updating state variables A, B, C and D

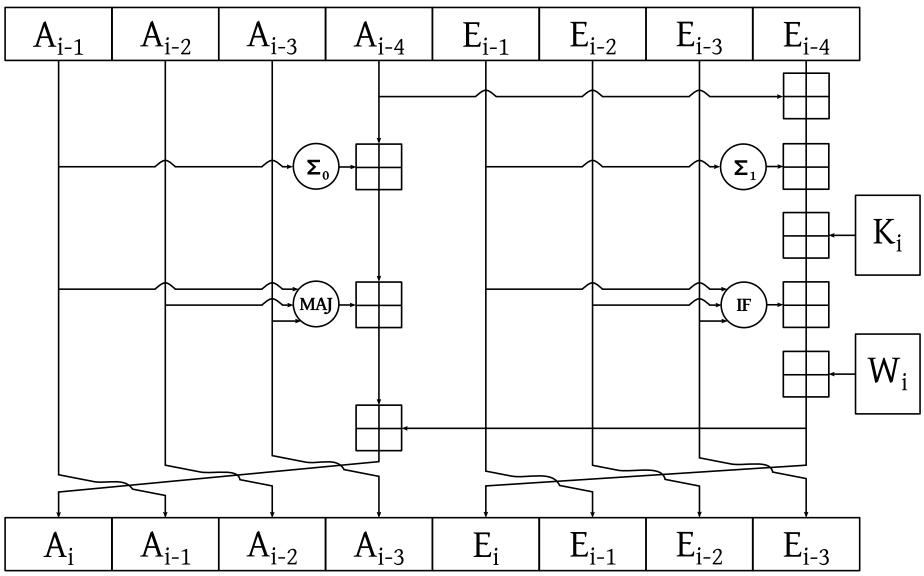


Figure 2.2: SHA-256 round function as characterized in [5]

new 32-bit word Y with 32 binary values a_i. We define the following notations:

$$
\begin{aligned}
Y := X \ll n &\iff a_i := b_{i-n} \text{ if } 0 \leq i - n < 32 \text{ and } 0 \text{ otherwise} \\
Y := X \gg n &\iff a_i := b_{i+n} \text{ if } 0 \leq i + n < 32 \text{ and } 0 \text{ otherwise} \\
Y := X \lll n &\iff a_i := b_{i-n \; mod \; 32} \\
Y := X \ggg n &\iff a_i := b_{i+n \; mod \; 32}
\end{aligned}
$$

Furthermore $X \oplus Y$ denotes `XOR` with arguments X and Y.

Besides MD4's two auxiliary functions `MAJ` and `IF`, another four auxiliary functions are defined. Be aware that $\oplus$ denotes the XOR functions whereas $\boxplus$ denotes addition modulo 2^{32}.

$$
\begin{array}{rcccccc}
\Sigma_0(X) & := & (X \ggg 2) & \oplus & (X \ggg 13) & \oplus & (X \ggg 22) \\
\Sigma_1(X) & := & (X \ggg 6) & \oplus & (X \ggg 11) & \oplus & (X \ggg 25) \\
\sigma_0(X) & := & (X \ggg 7) & \oplus & (X \ggg 18) & \oplus & (X \gg 3) \\
\sigma_1(X) & := & (X \ggg 17) & \oplus & (X \ggg 19) & \oplus & (X \gg 10)
\end{array}
$$

Padding and length extension The padding and length extension scheme of MD4 is used also in SHA-256. Append bit 1 and followed by a sequence of bit 0 until the message reaches a length of 448 modulo 512 bits. Afterwards the first 64 bits of the binary representation of the original input are appended.

Initialization In a similar manner to MD4, initialization of internal state variables (called "working variables" in [6, Section 6.2.2]) takes place before running the round function. The eight state variables are initialized with the following hexadecimal values:

A_{-1} = `6a09e667`	A_{-2} = `bb67ae85`	A_{-3} = `3c6ef372`	A_{-4} = `a54ff53a`
E_{-1} = `510e527f`	E_{-2} = `9b05688c`	E_{-3} = `1f83d9ab`	E_{-4} = `5be0cd19`

Furthermore SHA-256 uses 64 constant values in its round function. We initialize step constants K_i for $0 \leq i < 64$ with the following hexadecimal values (which must be read left to right and top to bottom):

428a2f98	71374491	b5c0fbcf	e9b5dba5	3956c25b	59f111f1
923f82a4	ab1c5ed5	d807aa98	12835b01	243185be	550c7dc3
72be5d74	80deb1fe	9bdc06a7	c19bf174	e49b69c1	efbe4786
0fc19dc6	240ca1cc	2de92c6f	4a7484aa	5cb0a9dc	76f988da
983e5152	a831c66d	b00327c8	bf597fc7	c6e00bf3	d5a79147
06ca6351	14292967	27b70a85	2e1b2138	4d2c6dfc	53380d13
650a7354	766a0abb	81c2c92e	92722c85	a2bfe8a1	a81a664b
c24b8b70	c76c51a3	d192e819	d6990624	f40e3585	106aa070
19a4c116	1e376c08	2748774c	34b0bcb5	391c0cb3	4ed8aa4a
5b9cca4f	682e6ff3	748f82ee	78a5636f	84c87814	8cc70208
90befffa	a4506ceb	bef9a3f7	c67178f2		

Precomputation of W Let W_i for $0 \leq i < 16$ be the sixteen 32-bit words of the padded input message. Then compute W_i for $16 \leq i < 64$ the following way:

$$W_i \coloneqq \sigma_1(W_{i-2}) + W_{i-7} + \sigma_0(W_{i-15}) + W_{i-16}$$

Round function For every block of 512 bits, the round function is applied. The eight state variables are updated iteratively for i from 0 to 63.

$$E_i \coloneqq A_{i-4} + E_{i-4} + \Sigma_1(E_{i-1}) + \mathtt{IF}\ (E_{i-1}, E_{i-2}, E_{i-3}) + K_i + W_i$$
$$A_i \coloneqq E_i - A_{i-4} + \Sigma_0(A_{i-1}) + \mathtt{MAJ}\ (A_{i-1}, A_{i-2}, A_{i-3})$$

W_i and K_i refer to the previously initialized values.

Computation of intermediate hash values Intermediate hash values for the Davies-Meyer construction are initialized with the following values:

$$\begin{array}{llll} H_0^{(0)} \coloneqq A_{-1} & H_1^{(0)} \coloneqq A_{-2} & H_2^{(0)} \coloneqq A_{-3} & H_3^{(0)} \coloneqq A_{-4} \\ H_4^{(i)} \coloneqq E_{-1} & H_5^{(i)} \coloneqq E_{-2} & H_6^{(i)} \coloneqq E_{-3} & H_7^{(i)} \coloneqq E_{-4} \end{array}$$

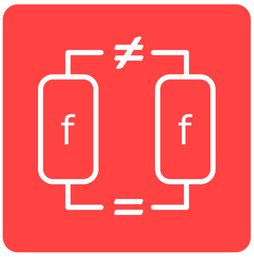
Every block creates its own E_i and A_i values for $60 \leq i < 64$. These are used to compute the next intermediate values:

$$\begin{array}{ll} H_0^{(j)} \coloneqq A_{63} + H_0^{(i-1)} & H_4^{(j)} \coloneqq E_{63} + H_4^{(i-1)} \\ H_1^{(j)} \coloneqq A_{62} + H_1^{(i-1)} & H_5^{(j)} \coloneqq E_{62} + H_5^{(i-1)} \\ H_2^{(j)} \coloneqq A_{61} + H_2^{(i-1)} & H_6^{(j)} \coloneqq E_{61} + H_6^{(i-1)} \\ H_3^{(j)} \coloneqq A_{60} + H_3^{(i-1)} & H_7^{(j)} \coloneqq E_{60} + H_7^{(i-1)} \end{array}$$

Finalization The final hash digest of size 256 bits is provided as

$$H_0^{(N)} \,||\, H_1^{(N)} \,||\, H_2^{(N)} \,||\, H_3^{(N)} \,||\, H_4^{(N)} \,||\, H_5^{(N)} \,||\, H_6^{(N)} \,||\, H_7^{(N)}$$

where N denotes the index of the last block and operator $||$ denotes concatenation. Hence $H_0^{(N)}$ are the four least significant bytes of the digest.



"Just because it's automatic doesn't mean it works."
—Daniel J. Bernstein

Chapter 3

Differential cryptanalysis

In chapter 2 we defined two hash functions. In this chapter we consider such functions from a differential perspective. Differential considerations will turn out to yield successful collision attacks on hash functions. We introduce a notation to easily represent differential characteristics.

3.1 Preliminaries Redux

TODO

3.2 Cryptanalysis of Hash Functions

In August 2004, Wang et al. published results at Crypto'04 [23] which revealed that MD4, MD5, HAVAL-128 and RIPEMD can be broken practically using differential cryptanalysis. Their work is based on preliminary work by Hans Dobbertin [4]. On an IBM P690 machine, an MD5 collision can be computed in about one hour using this approach. Collisions for HAVAL-128, MD4 and RIPEMD were found as well. Patrick Stach's `md4coll.c` program [21] implements Wang's approach and can find MD4 collisions in few seconds on my Thinkpad x220 setup specified in Appendix C.

Let n denote the digest size, i.e. the size of the hash value $h(x)$ in bits. Due to the birthday paradox, a collision attack has a generic complexity of $2^{n/2}$ whereas preimage and second preimage attacks have generic complexities of 2^n. In other

Message 1			
4d7a9c83	**d6cb927a**	**29d5a578**	57a7a5ee
de748a3c	dcc366b3	b683a020	3b2a5d9f
c69d71b3	f9e99198	d79f805e	a63bb2e8
45dc8e31	97e31fe5	2794bf08	b9e8c3e9
Message 2			
4d7a9c83	**56cb927a**	**b9d5a578**	57a7a5ee
de748a3c	dcc366b3	b683a020	3b2a5d9f
c69d71b3	f9e99198	d79f805e	a63bb2e8
45dd8e31	97e31fe5	2794bf08	b9e8c3e9
Hash value of Message 1 and Message 2			
5f5c1a0d	71b36046	1b5435da	9b0d807a

TABLE 3.1: One of two MD4 hash collisions provided in [23]. Values are given in hexadecimal, message words are enumerated from left to right, top to bottom. Differences are highlighted in bold for illustration purposes. For comparison the first bits of Message 1 are `11000001`… and the last bits are …`10011101`. A message represents one block of 512 bits.

words it is computationally easier to find any two colliding hash values than the preimage or second preimage for a given hash value.

Following results by Wang et al., differential cryptanalysis was shown as powerful tool for cryptanalysis of hash algorithms. This thesis applies those ideas to satisfiability approaches.

3.3 Differential cryptanalysis

Definition 3.1 (*Hash collision*)

Given a hash function h, a hash collision is a pair (x, x_2) with $x \neq x_2$ such that $h(x) = h(x_2)$.

Differential cryptanalysis is based on the idea to consider two execution states of hash algorithms for slightly different input messages. We trace those difference to learn about the propagation of message differences.

Hash algorithms consume input values as blocks of bits. As far as the length of the input must not conform to the block size, padding is applied. Now consider such a block of input values and another copy of it. We use those two blocks as inputs for two hash algorithm implementations, but provide slight modifications in few bits. MD4 has 48 round function applications in 3 rounds. Differential cryptanalysis considers the difference in the evaluation state between the two instances (compare with Figure 3.1).

Visualizing those differences helps the cryptanalyst to find modifications yielding a small number of differences in the evaluation state. The cryptanalyst consecutively modifies the input values to eventually receive a collision in the output value. If the number of differences in the evaluation state is small, this trail is expected to result in a hash collision with higher probability.

3.4 Differential notation

Differential notation helps us to visualize differential characteristics by defining so-called *generalized bit conditions*. It was introduced by Christian Rechberger and Christophe de Cannière in 2006 [2, Section 3.2], inspired by *signed differences* by Wang et al. and is shown in Table 3.3.

Consider two hash algorithm implementations. Let x_i be some bit from the first implementation and let x_i^* be the corresponding bit from the second implementation. Differences are computed using a `XOR` and commonly denoted as $\Delta x = x_i \oplus x_i^*$. Bit conditions allow us to encode possible relations between bits x_i and x_i^*.

For example, let us take a look at the original Wang et al. hash collision in MD4 provided in Table 3.1. We extract all values with differences and represent them using differential notation. This gives us Table 3.2.

The following properties hold for bit conditions:

- If $x_i = x_i^*$ holds and some value is known, $\{0, 1\}$ contains its bit condition.
- If $x_i \neq x_i^*$ holds and some value is known, $\{u, n\}$ contains its bit condition.
- If $x_i = x_i^*$ holds and the values are unknown, its bit condition is `-`.

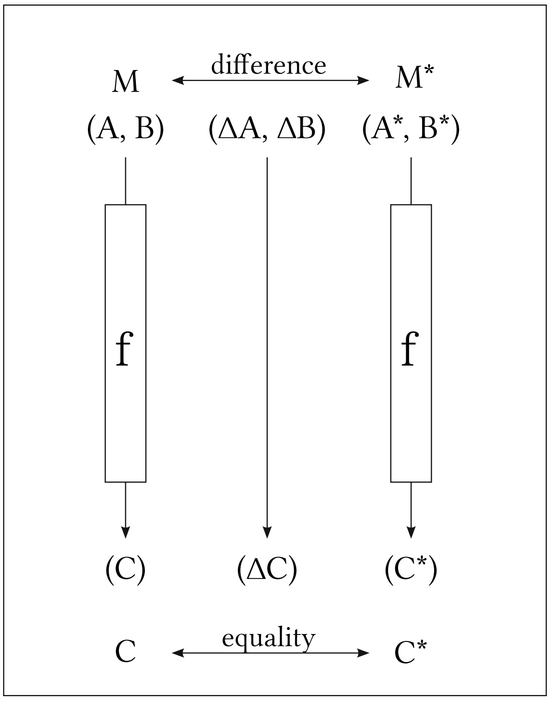


Figure 3.1: Typical attack setting for a collision attack: Hash function f is applied to two inputs M and M^* which differ by some predefined bits. $\triangle M$ describes the difference between these values. A hash collision is given if and only if output values C and C^* show the same value. In differential cryptanalysis we observe the differences between two instances applying function f to inputs M and M^*.

bit	binary	hexadecimal representation / differential notation
x_0	d6cb927a	11010110110010111001001001111010
x_1	29d5a578	00101001110101011010010101111000
x_2	45dc8e31	01000101110111001000111000110001
x_0^*	56cb927a	01010110110010111001001001111010
x_1^*	b9d5a578	10111001110101011010010101111000
x_2^*	45dd8e31	01000101110111011000111000110001
Δx		u1010110110010111001001001111010
		n01n1001110101011010010101111000
		010001011101110n1000111000110001

TABLE 3.2: The three words different between Message 1 and Message 2 of the original MD4 hash collision. The last three lines show how differences can be written down using bit conditions. As far as 4 symbols are not from the set $\{0, 1\}$ it holds that the messages differ by 4 bits.

- If $x_i \neq x_i^*$ holds and the values are unknown, its bit condition is x.

Applying this notation to hash collisions means that arbitrary bit conditions (expect for #) can be specified for the input values. In one of the intermediate iterations, we enforce a difference using one of the bit conditions $\{u, n, x\}$. This excludes trivial solutions with no differences from the set of possible solutions. And the final values need to lack differences thus are represented using -.

3.5 Addition example

TODO:

- illustrate how differences propagate by an addition example illustrated in

(x_i, x_i^*)	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$	(x_i, x_i^*)	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$
?	✓	✓	✓	✓	3	✓	✓		
-	✓			✓	5	✓		✓	
x		✓	✓		7	✓	✓	✓	
0	✓				A		✓		✓
u		✓			B	✓	✓		✓
n			✓		C			✓	✓
1				✓	D	✓		✓	✓
#					E		✓	✓	✓

TABLE 3.3: Differential notation as introduced in [2]. The left-most column specifies a symbol called "bit condition" and right-side columns indicate which bit configurations are possible for two given bits x_i and x_i^*.

differential notation

- reference to Magnus Daum's thesis

3.6 Differential path

TODO:

- refer to some testcase which shows a differential path with many unresolved differences.
- Then show the corresponding testcase where `?` became `-` and `x`.
- Illustrate how MD4 and SHA-256 descriptions maps to matrix representation.

Δx	conjunctive normal form	Δx	conjunctive normal form
`#`	$(x) \wedge (\neg x)$	`1`	$(x) \wedge (x^*)$
`0`	$(\neg x) \wedge (\neg x^*)$	`-`	$\neg(x \oplus x^*)$
`u`	$(x) \wedge (\neg x^*)$	`A`	(x)
`3`	$(\neg x^*)$	`B`	$(x \vee \neg x^*)$
`n`	$(\neg x) \wedge (x^*)$	`C`	(x^*)
`5`	$(\neg x)$	`D`	$(\neg x \vee x^*)$
`x`	$(x \oplus x^*)$	`E`	$(x \vee x^*)$
`7`	$(\neg x \vee \neg x^*)$	`?`	

TABLE 3.4: All bit conditions represented as CNF using two boolean variables x and x^* to represent two bits.

“What idiot called them logic errors rather than bool shit?”
—Unknown

Chapter 4

Satisfiability

Boolean algebra allows us to describe functions over two-valued variables. Satisfiability is the question for an assignment such that a function evaluates to true. Satisfiability problems are solved by SAT solvers. We discuss the basic theory behind satisfiability. We will learn that any computation can be represented as satisfiability problem. In Chapter 6 we will represent a differential cryptanalysis problem such that it is solvable iff the corresponding SAT problem is satisfiable.

4.1 Basic notation and definitions

Definition 4.1 (*Boolean function*)

A *Boolean function* is a mapping $h : X \to Y$ with $X = \{0, 1\}^n$ for $n \in \mathbb{N}_{\geq 1}$ and $Y = \{0, 1\}$.

Definition 4.2 (*Assignment*)

A k-*assignment* is an element of $\{0, 1\}^k$.

Let f be some k-ary Boolean function. An *assignment for function* f is any k-assignment.

Definition 4.3 (*Truth table*)

Let f be some k-ary Boolean function. The *truth table of Boolean function* f assigns truth value 0 or 1 to any assignment of f.

Boolean functions are characterized by their corresponding truth table.

x_1	x_2	$f(x_1, x_2)$
1	1	1
1	0	0
0	1	0
0	0	0

(A) AND

x_1	x_2	$f(x_1, x_2)$
1	1	1
1	0	1
0	1	1
0	0	0

(B) OR

v	$f(v)$
1	0
0	1

(C) NOT

TABLE 4.1: Truth tables for AND, OR and NOT

Table 4.1 shows example truth tables for the Boolean AND, OR and NOT functions. A different definition of the three functions is given the following way:

Definition 4.4

Let AND, OR and NOT be three Boolean functions.

- AND maps $X = \{0, 1\}^2$ to 1 if all values of X are 1.
- OR maps $X = \{0, 1\}^2$ to 1 if any value of X is 1.
- NOT maps $X = \{0, 1\}^1$ to 1 if the single value of X is 0.

All functions return 0 in the other case.

Those functions are denoted $a_0 \wedge a_1$, $a_0 \vee a_1$ and $\neg a_0$ respectively, for input parameters a_0 and a_1.

It is interesting to observe, that any Boolean function can be represented using only these three operators. This can be proven by complete induction over the number of arguments k of the function.

Let $k = 1$. Then we consider any possible 2-assignment for one input variable x_1 and one value of $f(x_1)$. Then four truth tables are possible listed in Table 4.2. The description shows the corresponding definition of f using AND, OR and NOT only.

Now let g be some k-ary function. Let $(a_0, a_1, \dots, a_k)$ be the k input arguments to g and $x_1 := g(a_0, a_1, \dots, a_k)$. Then we can again look at Table 4.2 to discover that 4 cases are possible: 2 cases where the return value of our new $(k+1)$-ary function depends on value x_1 and 2 cases where the return value is constant.

This completes our proof.

x_1	$f(x_1)$
1	1
0	1

(A) $f : x \mapsto 1$

x_1	$f(x_1)$
1	1
0	0

(B) $f : x \mapsto x$

x_1	$f(x_1)$
1	0
0	1

(C) $f : x \mapsto \neg x$

x_1	$f(x_1)$
1	0
0	0

(D) $f : x \mapsto 0$

TABLE 4.2: Unary f and its four possible cases

Boolean functions have an important property which is described in the following definition:

Definition 4.5

A Boolean function f is *satisfiable* iff there exists at least one input $x \in X$ such that $f(x) = 1$. Every input $x \in X$ satisfying this property is called *model*.

The corresponding tool to determine satisfiability is defined as follows:

Definition 4.6

A *SAT solver* is a tool to determine satisfiability (SAT or UNSAT) of a Boolean function. If satisfiability is given, it returns some model.

4.1.1 Computational considerations

The generic complexity of SAT determination is given by 2^n for n Boolean variables.

Let n be the number of variables of a Boolean function. No known algorithm exists to determine satisfiability in polynomial runtime. This means no algorithm solves the SAT problem with runtime behavior which depends polynomially on the growth of n.

This is known as the famous $\mathscr{P} \overset{?}{\neq} \mathscr{NP}$ problem.

However, SAT solver can take advantage of the problem's description. For example consider function f in Display 4.1.

$$f(x_0, x_1, x_2) = x_0 \wedge (\neg x_1 \vee x_2) \tag{4.1}$$

Instead of trying all possible 8 cases for 3 Boolean variables, we can immediately see that x_0 is required to be 1. So we don't need to test $x_0 = 0$ and can skip 4 cases. This particular strategy is called *unit propagation*.

4.1.2 SAT competitions

SAT research is heavily concerned with finding good heuristics to find some model for a given SAT problem as fast as possible. Biyearly SAT competitions take place to challenge SAT solvers in a set of benchmarks. The committee evaluates the most successful SAT solvers solving the most problems within a given time frame.

SAT 2016 is currently ongoing, but in 2014 lingeling by Armin Biere has won first prize in the Application benchmarks track and second prize in the Hard Combinatorial benchmarks track for SAT and UNSAT instances respectively. Its parallelized sibling plingeling and Cube & Conquer sibling treengeling have won prizes in parallel settings.

In chapter 7 we will look at runtime results shown by (but not limited to) those SAT solvers.

4.2 The DIMACS de-facto standard

Definition 4.7

> A *conjunction* is a sequence of Boolean functions combined using a logical OR. A *disjunction* is a sequence of Boolean functions combined using a logical AND. A *literal* is a Boolean variable (*positive*) or its negation (*negative*).
>
> A SAT problem is given in *Conjunctive Normal Form* (CNF) if the problem is defined as conjunction of disjunctions of literals.

A simple example for a SAT problem in CNF is the exclusive OR (XOR). It takes two Boolean values a and b as arguments and returns true if and only if the two arguments differ.

$$(a \vee b) \wedge (\neg a \vee \neg b) \tag{4.2}$$

Display 4.2 shows one conjunction (denoted $\wedge$) of two disjunctions (denoted $\vee$) of literals (denoted a and b where prefix $\neg$ represents negation). This structure constitutes a CNF.

Analogously we define a *Disjunctive Normal Form* (DNF) as disjunction of conjunctions of literals. The negation of a CNF is in DNF, because literals are negated and conjunctions become disjunctions, vice versa.

Theorem 4.1

> Every Boolean function can be represented as CNF.

Theorem 4.1 is easy to prove. Consider the truth table of an arbitrary Boolean function f with k input arguments and j rows of output value false. We represent f as CNF.

Consider Boolean variables $b_{i,l}$ with $0 \leq i \leq j$ and $0 \leq l \leq k$. For every row i of the truth table with assignment (r_i), add one disjunction to the CNF. This disjunction contains $b_{i,l}$ if $r_{i,l}$ is false. The disjunction contains $b_{i,l}$ if $r_{i,l}$ is true.

As far as f is an arbitrary k-ary Boolean function, we have proven that any function can be represented as CNF.

SAT problems are usually represented in the DIMACS de-facto standard. Consider a SAT problem in CNF with *nbclauses* clauses and enumerate all variables from 1 to *nbvars*. A DIMACS file is an ASCII text file. Lines starting with "`c`" are skipped (comment lines). The first remaining line has to begin with "`p cnf`" followed by *nbclauses* and *nbvars* separated by spaces (header line). All following non-comment lines are space-separated indices of Boolean variables optionally prefixed by a minus symbol. Then one line represents one clause and must be terminated with a zero symbol after a space. All lines are conjuncted to form a CNF.

Variations of the DIMACS de-facto standard also allow multiline clauses (the zero symbol constitutes the end of a clause) or arbitrary whitespace instead of spaces. The syntactical details are individually published on a per competition basis.

LISTING 4.1: Display 4.2 represented in DIMACS format

```
p cnf 2 2
a b
-a -b
```

Definition 4.8

A *clause* is a disjunction of literals. A *k-clause* is a clause consisting of exactly *k* literals. A *unit clause* is a 1-clause. A *Horn clause* is a clause with at most one positive literal. A *definite clause* is a clause with exactly one positive literal.

4.3 Basic SAT solving techniques

4.4 SAT solvers in use

4.4.1 STP approach

“What idiot called them logic errors rather than bool shit?”
—Unknown

Chapter 5

SAT features

5.1 SAT features and CNF analysis

At the very beginning I was very intrigued with the question “What is an ‘average’ SAT problem?”. Answers to this question can help to optimize SAT solver memory layouts. But originally I was wondering whether our differential cryptanalysis SAT problems distinguish from “average” SAT problems in some very basic properties. First of all, we need to elaborate on the question itself.

Definition 5.1 (*SAT feature*)

A *SAT feature* is a statistical value (named *feature value*) retrievable from some given SAT problem in some well-defined encoding.

A SAT feature is called *performance-driven* if the runtime of any computation contributes to the feature value.

The most basic example of a SAT feature is the number of variables and clauses of a given SAT problem. This SAT feature is stored in the CNF header of a SAT problem encoded in the DIMACS format.

It should be computationally easy to evaluate SAT features of a given SAT problem. The general goal is to write a tool which evaluates several SAT features at the same time and retrieve them for comparison with other problems. A SAT feature is expected to be computable in linear time and memory with the number of variables and number of clauses. But a suggested limit is only given with polynomial complexity for evaluation algorithms. Sticking to this convention implies that evaluation of satisfiability must not be necessary to evaluate a SAT feature under the assumption that $\mathscr{P} \neq \mathscr{NP}$. Hence the number of valid models cannot be a SAT feature as far as satisfiability needs to be determined. But no actual hard boundary for runtime requirements is given. Previous work has shown that expensive algorithms can provide useful data in a small time frame if they are limited to a constant subproblem size.

The most similar resource I found looking at SAT features was the SATzilla project [15, 24] in 2012. The authors systematically defined 138 SAT features categorized in 12 groups. The features themselves are not defined formally, but an implementation is provided bundled with example data.

POSNEG_RATIO_CLAUSE_mean	ratio of positive to negative clauses, mean

Table 5.1: SAT features defined by SATzilla

Many SAT solvers collect feature values to improve algorithm selection, restart strategies and estimate problem sizes. Recent trends to apply Machine Learning to SAT solving imply feature evaluation. SAT features and the resulting satisfiability runtime are used as training data for Machine Learning. One example using SAT features for algorithm selection is ASlib [1].

However, most of these SAT features are performance-driven. Examples for performance-driven SAT features include the number of restarts within a certain time frame or evaluation of local minima.

POSNEG-RATIO-CLAUSE-mean

In the following section we want to evaluate SAT features and compare test cases.

5.2 SAT features in comparison

Proposition 5.1

The set of public benchmarks in SAT competitions between 2008 and 2015 represent average SAT problems

Define a large set of SAT features. Present data. Categorize data.

Given a set of clauses, return a subset of clauses satisfying given criterion	
`clauses_allLitsNeg`	all literals are negative
`clauses_oneLitNeg`	exactly one literal is negative
`clauses_geqOneLitNeg`	more than one literal is negative
`clauses_allLitsPos`	all literals are positive
`clauses_oneLitPos`	exactly one literal is positive
`clauses_geqOneLitPos`	more than one literal is positive
`clauses_length1`	clause contains exactly one literal ("unit clause")
`clauses_length2`	clause contains exactly two literals
`clauses_unique`	clause did not yet occur
`clauses_tautological`	clause contains some literal and its negation
Given a set of literals/variables, return Boolean property	
`literals_existential`	literal does not occur negated
`literals_unit`	literal occurs in clause of length 1
`literals_contradiction`	literal occurs with its negation on one clause
`literals_1occ`	literal occurs only in one clause once
`literals_2occs`	literal occurs two times in clauses
`literals_3occs`	literal occurs three times in clauses
`variables_unit`	variable occurs in clause of length 1
Given a set of clauses, return real number based on this clause	
`clauses_mapLength`	number of literals in clause
`clauses_mapRatioPosNeg`	number of positive literals divided by total number of literal
`clauses_mapNumPos`	number of positive literals in clause
Given one clause, return Boolean property	
`clauselits_someEx`	any is literal existential
`clauselits_allEx`	all literals are existential
`clauselits_someUnit`	contains unit variable
`clauselits_someContra`	contains contradiction variable
`clauselits_all1occ`	all variables occur only once in all clauses
`clauselits_all12occ`	all variables occur only once or twice in all clauses
Given all clauses, return the following property	
`concomp_variable`	number of connected components where two variables are in the same component iff they occur in at least one clause together
`concomp_literal`	number of connected components where two literals are in the same component iff they occur in at least one clause together
`xor2_count`	Number of clause pairs $(a \vee b, \neg a \vee b)$ for two variables a and b

"What idiot called them logic errors rather than bool shit?"
—Unknown

Chapter 6

Problem encoding

Chapter 7

Results

7.1 Benchmark results

7.2 Related work

7.3 Conclusion

Chapter 8

Summary and Future Work

8.1 Summary of results

8.2 Future work

Appendices

Appendix A

Illustration

i		$\nabla S_{i,0}$	$\nabla S_{i,1}$	$\nabla S_{i,2}$
-4	A:	01100111010001010010001100000001		
-3	A:	00010000001100100101010001110110		
-2	A:	10011000101110101101110011111110		
-1	A:	11101111110011011010101110001001		
0	A:	01101011110101001110010000010010	W:	01001101011110101001110010000011
1	A:	0111011001001111111011100u110001	W:	u1010110110010111001001001111010
2	A:	10101011010000000001110u01n1110010	W:	n01n10011101010110100101011111000
3	A:	101011u1001111010101001001010001	W:	01010111101001111010010111101110
4	A:	00101100011000110101010111110010	W:	11011110011101001000101000111100
5	A:	0001101001100010 10u1101000000001	W:	11011100110000110110011010110011
6	A:	0001101100unuu110001000001111010	W:	10110110100000111010000000100000
7	A:	0010101110000001 0unn011001010000	W:	00111011001010100101110110011111
8	A:	011100110010001u1111111110110000	W:	11000110100111010111000110110011
9	A:	101011n01unnu0001111100110011111	W:	11111001111010011001000110011000
10	A:	10n00100100001010100000010101110	W:	11010111100111111000000001011110
11	A:	u1000110101101100100101011111111	W:	10100110001110111011001011101000
12	A:	001011u00u1010111111110001111011	W:	0100010111011110n1000111000110001
13	A:	10un1n01001100010100000111100101	W:	10010111111000110001111111100101
14	A:	00001010010100011000100011010110	W:	00100111100101001011111100001000
15	A:	000111101010 1u010110011011010100	W:	10111001111010001100001111101001
16	A:	n00n0un0110100101001101101011111		
17	A:	00011111001110100001001000011110		
18	A:	01010111000011010000000000010010100		
19	A:	u1n10000000101111001101011000100		
20	A:	n1un1001111111011101000000110100		
21	A:	11110011101100000101111111010100		
22	A:	01011101110011010011001100111010		
23	A:	01010000111011101100011110001111		
24	A:	00000010000100100011011100011010		
25	A:	10110000100101100001010011101010		
26	A:	00001010100010010111011101000001		
27	A:	00000110111011110101101010110011		
28	A:	10110110010111010110110000100101		
29	A:	10100010000011010100100001101001		
30	A:	00101001110101111100011101100011		
31	A:	11111100100100101101011110110110		
32	A:	01001111110100100110100000101111		
33	A:	00111000000111101011011101110 0100		
34	A:	00100000011101011110100000010101		
35	A:	n0100000000110011000001000111001 0		
36	A:	n0000111111010111101111001011001		
37	A:	11001000000110100100001100001100		
38	A:	10110000011001111110100110101100		
39	A:	00010010000010100001101100011100		
40	A:	11000000010010000111000110000101		
41	A:	00000110100001101111010100100110		
42	A:	01001110110111011111111010000110		
43	A:	01010000011000111101000001101101		
44	A:	11111000000101101111011100001100		
45	A:	10001010110110110010110000000100		
46	A:	10000010100110010101100011011100		
47	A:	10000001111001011011010010111101		

Table A.1: One of the original MD4 collision given by Wang, et al.

Appendix B

Testcases

Figures B.1, B.2, B.3 and B.4 show testcases used to test performance measures.

i		$\nabla S_{i,0}$	$\nabla S_{i,1}$	$\nabla S_{i,2}$
-4	A:	01100111010001010010001100000001		
-3	A:	000100000001100100101010001110110		
-2	A:	10011000101110101101110011111110		
-1	A:	11101111110011011010101110001001		
0	A:	x-------------------------------	W:	---x----------------------------
1	A:	--------------------------------	W:	--------------------------------
2	A:	---------------------x----------	W:	x-------------------------------
3	A:	xxx-----------------------------	W:	--------------------------------
4	A:	------------------------------xx	W:	x-------------------------------
5	A:	-------xxxxxxxxxxxxxxxx-x-------	W:	--------------------------------
6	A:	x--------x-----------x-x-xxxx--x	W:	--------------------------------
7	A:	---------x-----x----------------	W:	--------------------------------
8	A:	---------x-------x-x-x----------	W:	x-------------------------------
9	A:	-----------------x---x----------	W:	--------------------------------
10	A:	------x-----x--xxx-xxx----------	W:	--------------------------------
11	A:	x----------------xxx-x----------	W:	--------------------------------
12	A:	---x--x-------------------------	W:	x-------------------------------
13	A:	--------------------------------	W:	--------------------------------
14	A:	-x------------------------------	W:	--------------------------------
15	A:	x-x----------x------------------	W:	--------------------------------
16	A:	-xxx----------------------------		
17	A:	--------------------------------		
18	A:	--------------------------------		
19	A:	x-------------------------------		
20	A:	x-------------------------------		
21	A:	--------------------------------		
22	A:	--------------------------------		
23	A:	--------------------------------		
24	A:	--------------------------------		
25	A:	--------------------------------		
26	A:	--------------------------------		
27	A:	--------------------------------		
28	A:	--------------------------------		
29	A:	--------------------------------		
30	A:	--------------------------------		
31	A:	--------------------------------		
32	A:	x-------------------------------		
33	A:	--------------------------------		
34	A:	--------------------------------		
35	A:	--------------------------------		
36	A:	--------------------------------		
37	A:	--------------------------------		
38	A:	--------------------------------		
39	A:	--------------------------------		
40	A:	--------------------------------		
41	A:	--------------------------------		
42	A:	--------------------------------		
43	A:	--------------------------------		
44	A:	--------------------------------		
45	A:	--------------------------------		
46	A:	--------------------------------		
47	A:	--------------------------------		

TABLE B.1: TODO description

i		$\nabla S_{i,0}$	$\nabla S_{i,1}$	$\nabla S_{i,2}$
-4	A:	01100111010001010010001100000001		
-3	A:	00010000001100100101010001110110		
-2	A:	10011000101110101101110011111110		
-1	A:	11101111110011011010101110001001		
0	A:	????????????????????????????????	W:	---x----------------------------
1	A:	????????????????????????????????	W:	--------------------------------
2	A:	????????????????????????????????	W:	x-------------------------------
3	A:	????????????????????????????????	W:	--------------------------------
4	A:	????????????????????????????????	W:	x-------------------------------
5	A:	????????????????????????????????	W:	--------------------------------
6	A:	????????????????????????????????	W:	--------------------------------
7	A:	????????????????????????????????	W:	--------------------------------
8	A:	????????????????????????????????	W:	x-------------------------------
9	A:	????????????????????????????????	W:	--------------------------------
10	A:	????????????????????????????????	W:	--------------------------------
11	A:	????????????????????????????????	W:	--------------------------------
12	A:	??????????????------------------	W:	x-------------------------------
13	A:	??????????????------------------	W:	--------------------------------
14	A:	??????????????------------------	W:	--------------------------------
15	A:	??????????????------------------	W:	--------------------------------
16	A:	???x----------------------------		
17	A:	?-------------------------------		
18	A:	?-------------------------------		
19	A:	?-------------------------------		
20	A:	x-------------------------------		
21	A:	--------------------------------		
22	A:	--------------------------------		
23	A:	--------------------------------		
24	A:	--------------------------------		
25	A:	--------------------------------		
26	A:	--------------------------------		
27	A:	--------------------------------		
28	A:	--------------------------------		
29	A:	--------------------------------		
30	A:	--------------------------------		
31	A:	--------------------------------		
32	A:	x-------------------------------		
33	A:	--------------------------------		
34	A:	--------------------------------		
35	A:	--------------------------------		
36	A:	--------------------------------		
37	A:	--------------------------------		
38	A:	--------------------------------		
39	A:	--------------------------------		
40	A:	--------------------------------		
41	A:	--------------------------------		
42	A:	--------------------------------		
43	A:	--------------------------------		
44	A:	--------------------------------		
45	A:	--------------------------------		
46	A:	--------------------------------		
47	A:	--------------------------------		

TABLE B.2: TODO description

i		$\nabla S_{i,0}$	$\nabla S_{i,1}$	$\nabla S_{i,2}$
-4	A:	01100111010001010010001100000001		
-3	A:	00010000001100100101010001110110		
-2	A:	10011000101110101101110011111110		
-1	A:	11101111110011011010101110001001		
0	A:	????????????????????????????????	W:	---x----------------------------
1	A:	????????????????????????????????	W:	--------------------------------
2	A:	????????????????????????????????	W:	x-------------------------------
3	A:	????????????????????????????????	W:	--------------------------------
4	A:	????????????????????????????????	W:	x-------------------------------
5	A:	????????????????????????????????	W:	--------------------------------
6	A:	????????????????????????????????	W:	--------------------------------
7	A:	????????????????????????????????	W:	--------------------------------
8	A:	????????????????????????????????	W:	x-------------------------------
9	A:	????????????????????????????????	W:	--------------------------------
10	A:	????????????????????????????????	W:	--------------------------------
11	A:	????????????????????????????????	W:	--------------------------------
12	A:	????????????????????????????????	W:	x-------------------------------
13	A:	????????????????????????????????	W:	--------------------------------
14	A:	????????????????????????????????	W:	--------------------------------
15	A:	????????????????????????????????	W:	--------------------------------
16	A:	????????????????????????????????		
17	A:	????????????????????????????????		
18	A:	????????????????????????????????		
19	A:	????????????????????????????????		
20	A:	????????????????????????????????		
21	A:	--------------------------------		
22	A:	--------------------------------		
23	A:	--------------------------------		
24	A:	--------------------------------		
25	A:	--------------------------------		
26	A:	--------------------------------		
27	A:	--------------------------------		
28	A:	--------------------------------		
29	A:	--------------------------------		
30	A:	--------------------------------		
31	A:	--------------------------------		
32	A:	x-------------------------------		
33	A:	--------------------------------		
34	A:	--------------------------------		
35	A:	--------------------------------		
36	A:	--------------------------------		
37	A:	--------------------------------		
38	A:	--------------------------------		
39	A:	--------------------------------		
40	A:	--------------------------------		
41	A:	--------------------------------		
42	A:	--------------------------------		
43	A:	--------------------------------		
44	A:	--------------------------------		
45	A:	--------------------------------		
46	A:	--------------------------------		
47	A:	--------------------------------		

TABLE B.3: TODO description

i		$\nabla S_{i,0}$	$\nabla S_{i,1}$	$\nabla S_{i,2}$
-4	A:	01100111010001010010001100000001		
-3	A:	00010000001100100101010001110110		
-2	A:	10011000101110101101110011111110		
-1	A:	11101111110011011010101110001001		
0	A:	????????????????????????????????	W:	????????????????????????????????
1	A:	????????????????????????????????	W:	????????????????????????????????
2	A:	????????????????????????????????	W:	????????????????????????????????
3	A:	????????????????????????????????	W:	????????????????????????????????
4	A:	????????????????????????????????	W:	????????????????????????????????
5	A:	????????????????????????????????	W:	????????????????????????????????
6	A:	????????????????????????????????	W:	????????????????????????????????
7	A:	????????????????????????????????	W:	????????????????????????????????
8	A:	????????????????????????????????	W:	????????????????????????????????
9	A:	????????????????????????????????	W:	????????????????????????????????
10	A:	????????????????????????????????	W:	????????????????????????????????
11	A:	????????????????????????????????	W:	????????????????????????????????
12	A:	????????????????????????????????	W:	????????????????????????????????
13	A:	????????????????????????????????	W:	????????????????????????????????
14	A:	????????????????????????????????	W:	????????????????????????????????
15	A:	????????????????????????????????	W:	????????????????????????????????
16	A:	????????????????????????????????		
17	A:	????????????????????????????????		
18	A:	????????????????????????????????		
19	A:	????????????????????????????????		
20	A:	????????????????????????????????		
21	A:	--------------------------------		
22	A:	--------------------------------		
23	A:	--------------------------------		
24	A:	--------------------------------		
25	A:	--------------------------------		
26	A:	--------------------------------		
27	A:	--------------------------------		
28	A:	--------------------------------		
29	A:	--------------------------------		
30	A:	--------------------------------		
31	A:	--------------------------------		
32	A:	x???????????????????????????????		
33	A:	--------------------------------		
34	A:	--------------------------------		
35	A:	--------------------------------		
36	A:	--------------------------------		
37	A:	--------------------------------		
38	A:	--------------------------------		
39	A:	--------------------------------		
40	A:	--------------------------------		
41	A:	--------------------------------		
42	A:	--------------------------------		
43	A:	--------------------------------		
44	A:	--------------------------------		
45	A:	--------------------------------		
46	A:	--------------------------------		
47	A:	--------------------------------		

Table B.4: TODO description

Appendix C

Hardware setup

In the following we introduce two hardware setups which were used to run our testcases. The first setup is referred to as "Thinkpad x220" throughout the document whereas the second setup is referred to as "Cluster".

Type model	Thinkpad Lenovo x220 tablet, 4299-2P6
Processor	Intel i5-2520M, 2.50 GHz, dual-core, Hyperthreaded
RAM	16 GB (extension to common retail setup)
Memory	160 GB SSD
L3 cache size	3072 KB

TABLE C.1: Thinkpad x220 Tablet specification [10]

Processor	Intel Xeon X5690, 3.47 GHz, 6 cores, Hyperthreaded
RAM	192 GB
L3 cache size	12288 KB

TABLE C.2: Cluster node nehalem192go specification [3]

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