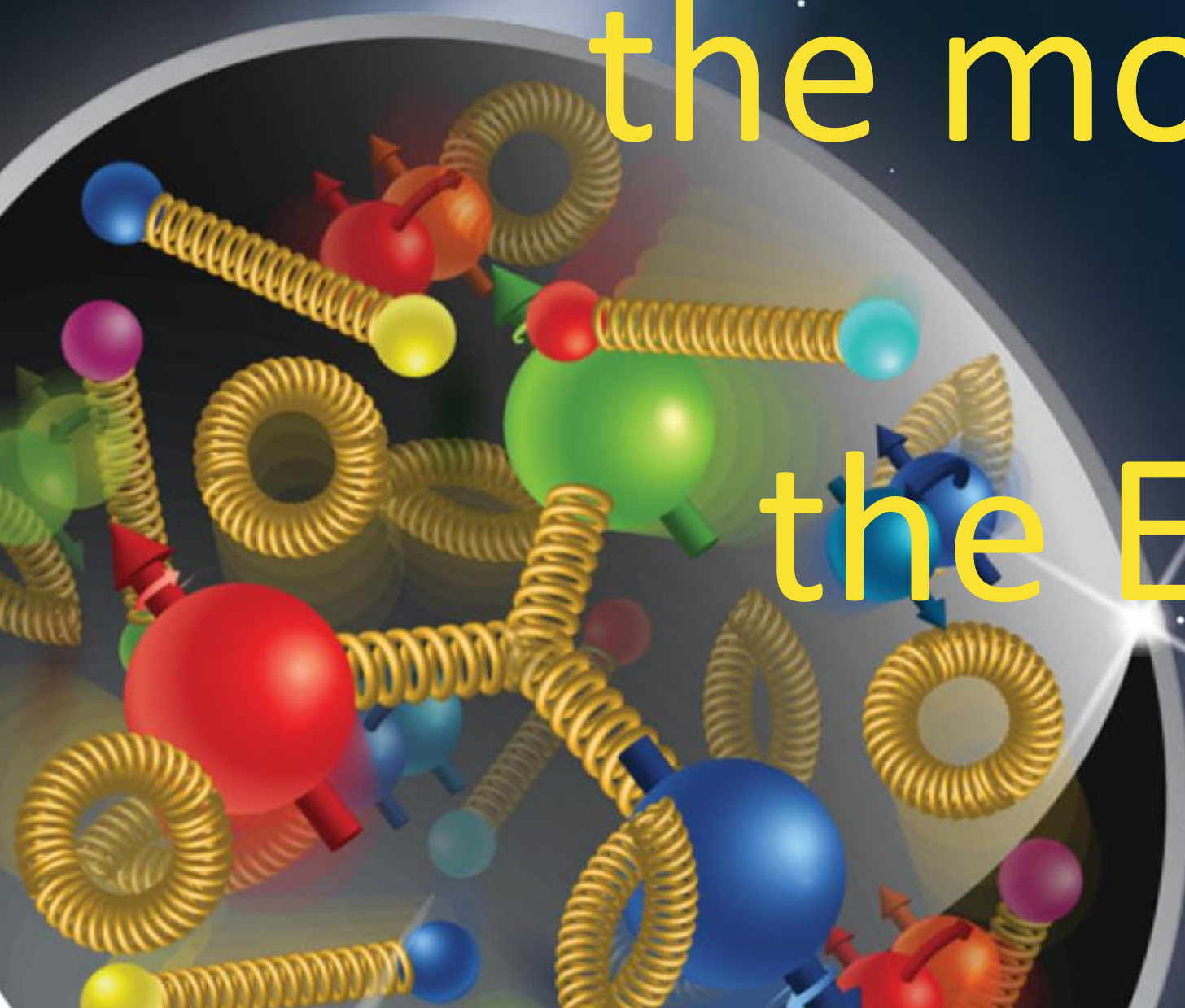in

the momentum space and

the Electron-Ion Collider
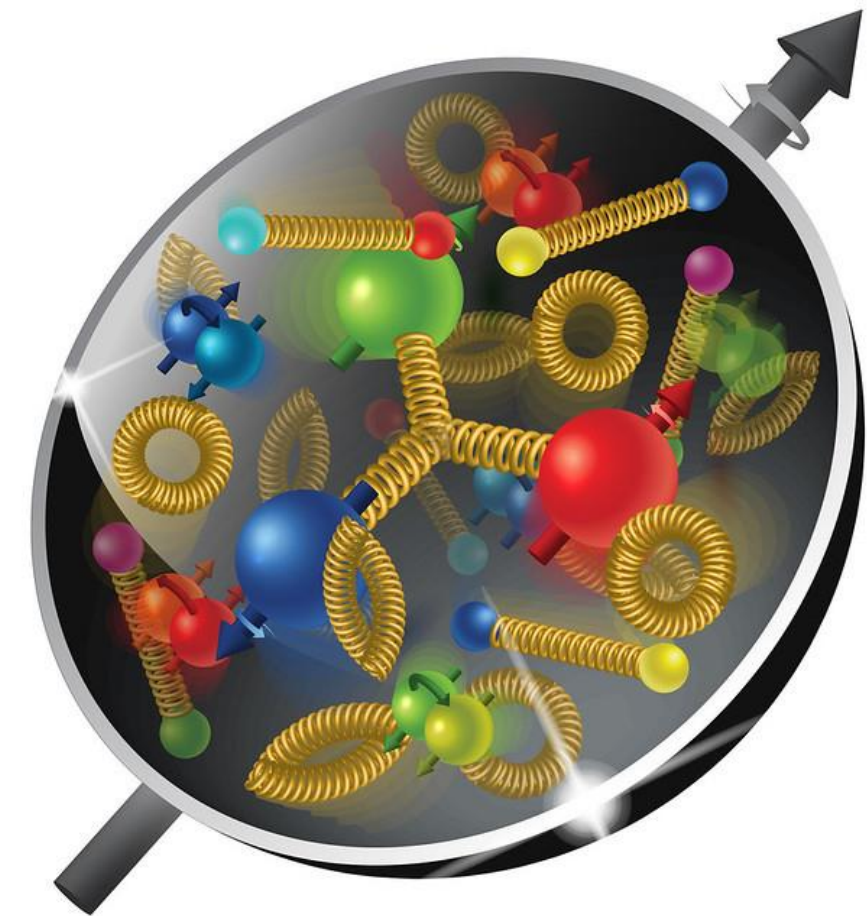
THE MENU

- ## What is 3D structure?

- ## Why to be excited about it?

- ## Why the Electron-Ion Collider?

Our group consists of theorists and experimentalists.

I will try to provide useful information for all.

THE PLAN

❑ ## Lecture I:

Structure of the nucleon

Transverse Momentum Dependent distributions (TMDs)

Semi Inclusive Deep Inelastic Scattering (SIDIS)

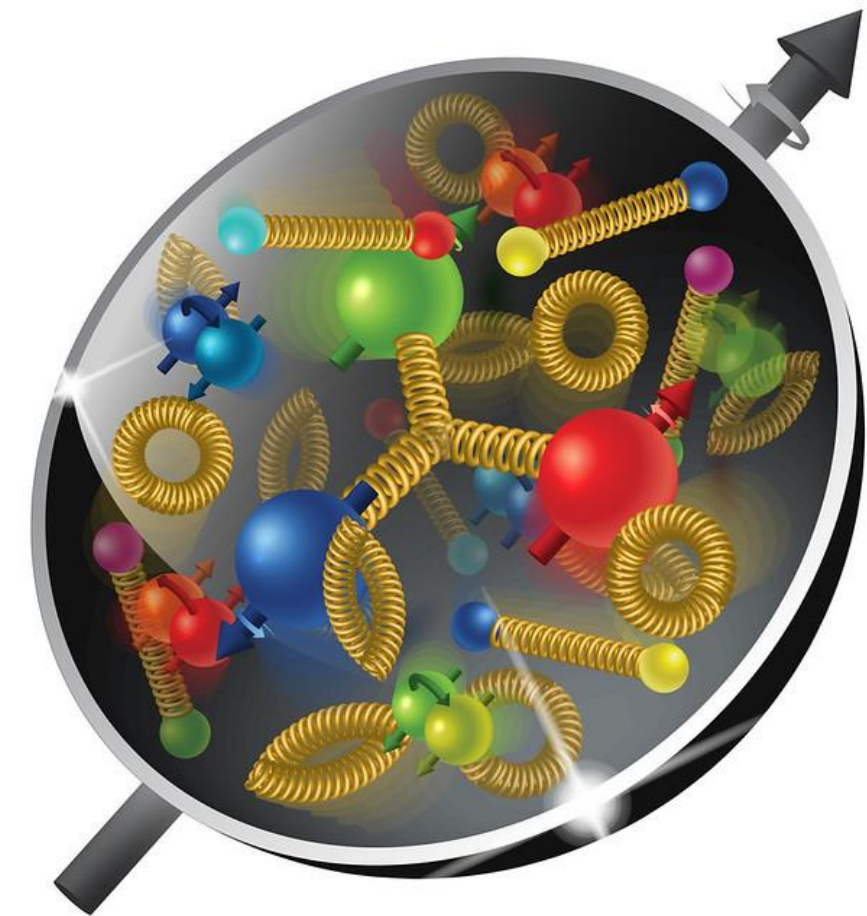Calculations of SIDIS structure functions in google colab

❑ ## Lecture II:

Solution of TMD evolution equations

Collins-Soper-Sterman (CSS) formalism

❑ ## Lecture III: Giuseppe Bozzi

Phenomenology of unpolarized TMDs

MATERIAL

https://inspirehep.net/literature/2650019

https://github.com/prokudin/CFNS24
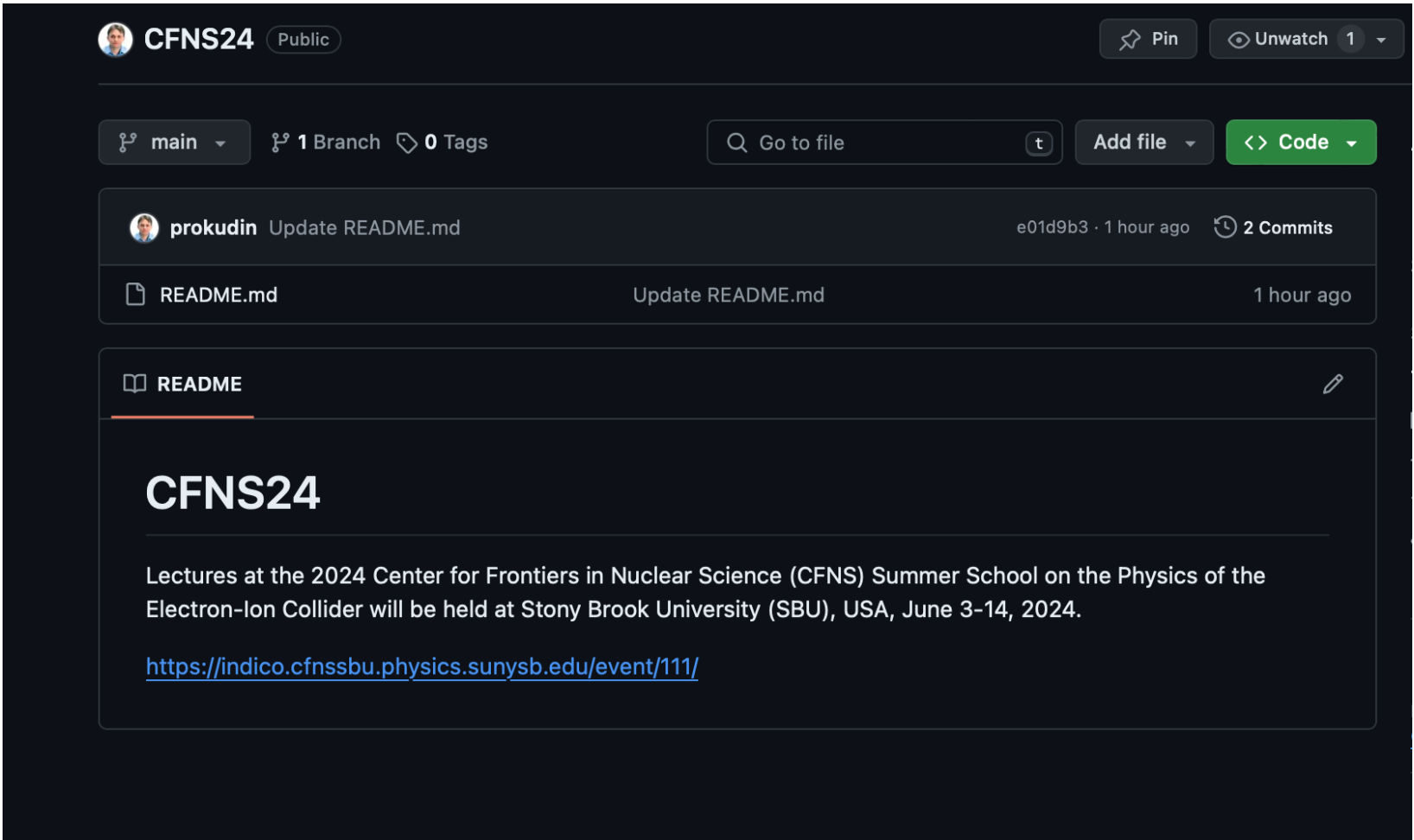
Clone the repository, or download files from github

TMD TABLE

Figure 2.5: Leading power quark parton distribution functions for the proton or a spin-1/2 hadron.

$$f_{i/PS}^{[\gamma^+]}(x, \mathbf{k}_T, \mu, \zeta) = f_1(x, k_T) - \frac{\epsilon_T^{\rho\sigma} k_{T\rho} S_{T\sigma}}{M} \kappa f_{1T}^{\perp}(x, k_T),$$

$$f_{i/PS}^{[\gamma^+ \gamma_5]}(x, \mathbf{k}_T, \mu, \zeta) = S_L g_1(x, k_T) - \frac{k_T \cdot S_T}{M} g_{1T}^{\perp}(x, k_T),$$

$$f_{i/PS}^{[i\sigma^{\alpha+} \gamma_5]}(x, \mathbf{k}_T, \mu, \zeta) = S_T^{\alpha} h_1(x, k_T) + \frac{S_L k_T^{\alpha}}{M} h_{1L}^{\perp}(x, k_T)$$

$$- \frac{\mathbf{k}_T^2}{M^2}\left(\frac{1}{2} g_T^{\alpha\rho} + \frac{k_T^{\alpha} k_T^{\rho}}{\mathbf{k}_T^2}\right) S_{T\rho} h_{1T}^{\perp}(x, k_T) - \frac{\epsilon_T^{\alpha\rho} k_{T\rho}}{M} \kappa h_1^{\perp}(x, k_T).$$

(2.123)

$$\kappa = \begin{cases} +1 & \text{(Drell-Yan)} \\ -1 & \text{(SIDIS)} \end{cases}$$

- $f_1(x, k_T)$ describes an unpolarized quark inside an unpolarized hadron, similar to the unpolarized collinear distribution $f_1(x)$.

- $g_1(x, k_T)$ is the helicity distribution which describes a longitudinally polarized quark inside a longitudinally polarized hadron, similar to the collinear helicity distribution $g_1(x)$.

- $h_1(x, k_T)$ is the transversity distribution which describes a transversely polarized quark inside a transversely polarized hadron, similar to the collinear transversity distribution $h_1(x)$.

- $f_{1T}^{\perp}(x, k_T)$ is the Sivers function [135] which describes an unpolarized quark inside a transversely polarized hadron. Since it is T-odd, it was originally believed to vanish due to symmetry arguments [61]. It was later clarified that it is non-vanishing when correctly taking the Wilson lines in the definition of the unsubtracted TMD PDF and soft function into account [38, 62, 136].

- The function $g_{1T}^{\perp}(x, k_T)$ describes longitudinally polarized quarks in a transversely polarized hadron, and vice versa $h_{1L}^{\perp}(x, k_T)$ describes transversely polarized quarks in a longitudinally polarized hadron [137]. They are referred in the literature as "worm-gear" T and L functions or Kotzinian-Mulders [64, 138] functions.

- $h_1^{\perp}(x, k_T)$ is the Boer-Mulders function [63] which describes a transversely polarized quark in an unpolarized hadron. Like the Sivers function $f_{1T}^{\perp}$, it is time-reversal odd.

- $h_{1T}^{\perp}(x, k_T)$ is the pretzelosity function, which contributes to the distribution of a transversely polarized quark in a transversely polarized hadron [132], in addition to the transversity $h_1(x, k_T)$. Curiously, the name of this function stems from its expected shape [139] published by G. Miller, which was also highlighted in the New York Times [140], exhibiting the unusual shape of the proton due to the presence of this function.

$$\tilde{f}_{i/p_s}^{[\gamma^+]}(x, \mathbf{b}_T, \mu, \zeta) = \tilde{f}_1(x, b_T) + i\epsilon_{\rho\sigma}b_T^\rho S_T^\sigma M \tilde{f}_{1T}^\perp(x, b_T),$$

$$\tilde{f}_{i/p_s}^{[\gamma^+\gamma_5]}(x, \mathbf{b}_T, \mu, \zeta) = S_L \tilde{g}_1(x, b_T) + i\mathbf{b}_T \cdot S_T M \tilde{g}_{1T}^\perp(x, b_T),$$

$$\tilde{f}_{i/p_s}^{[i\sigma^{\alpha+}\gamma_5]}(x, \mathbf{b}_T, \mu, \zeta) = S_T^\alpha \tilde{h}_1(x, b_T) - iS_L b_T^\alpha M \tilde{h}_{1L}^\perp(x, b_T) + i\epsilon^{\alpha\rho} b_{\perp\rho} M \tilde{h}_1^\perp(x, b_T)$$

$$+ \frac{1}{2}\mathbf{b}_T^2 M^2 \left(\frac{1}{2}g_T^{\alpha\rho} + \frac{b_T^\alpha b_T^\rho}{\mathbf{b}_T^2}\right) S_{\perp\rho} \tilde{h}_{1T}^\perp(x, b_T). \tag{2.126}$$
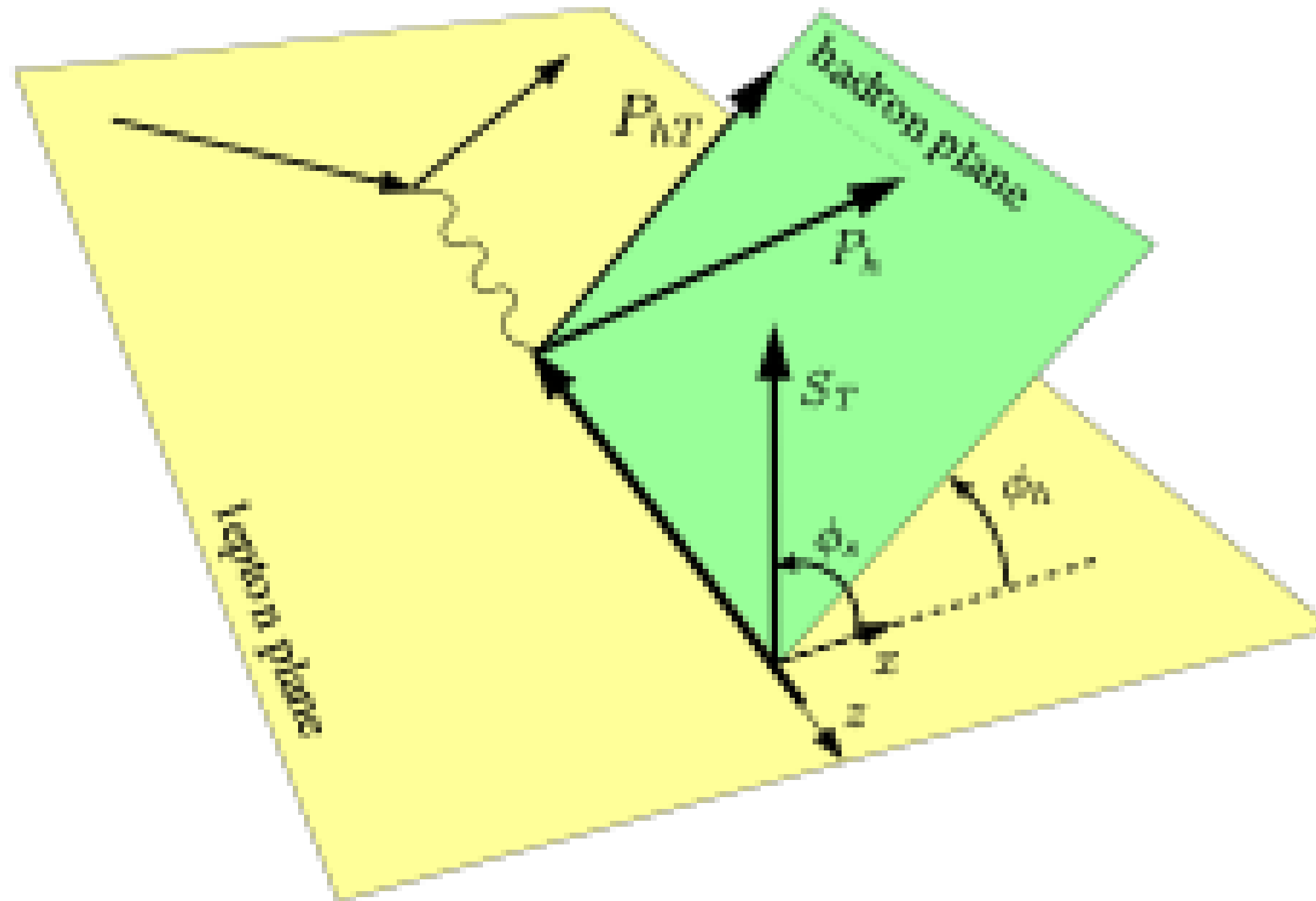
$$\ell(l) + p(P) \rightarrow \ell(l') + h(P_h) + X$$

$$\frac{d^6\sigma}{dx\,dy\,dz_h\,d\phi_S\,d\phi_h\,dP_{hT}^2} = \frac{\alpha_{em}^2}{x\,y\,Q^2}\left(1-y+\frac{1}{2}y^2\right)\left[F_{UU,T} + \cos(2\phi_h)\,p_1\,F_{UU}^{\cos(2\phi_h)}\right.$$

$$+ S_L\sin(2\phi_h)\,p_1\,F_{UL}^{\sin(2\phi_h)} + S_L\,\lambda p_2\,F_{LL}$$

$$+ S_T\sin(\phi_h-\phi_S)\,F_{UT,T}^{\sin(\phi_h-\phi_S)}$$

$$+ S_T\sin(\phi_h+\phi_S)\,p_1\,F_{UT}^{\sin(\phi_h+\phi_S)} + \lambda\,S_T\cos(\phi_h-\phi_S)\,p_2\,F_{LT}^{\cos(\phi_h-\phi_S)}$$

$$\left. + S_T\sin(3\phi_h-\phi_S)\,p_1\,F_{UT}^{\sin(3\phi_h-\phi_S)}\right], \tag{2.186}$$

$$p_1 = \frac{1-y}{1-y+\frac{1}{2}y^2}, \quad p_2 = \frac{y(1-\frac{1}{2}y)}{1-y+\frac{1}{2}y^2}, \quad p_3 = \frac{(2-y)\sqrt{1-y}}{1-y+\frac{1}{2}y^2}, \quad p_4 = \frac{y\sqrt{1-y}}{1-y+\frac{1}{2}y^2}$$

$$F_{UU,T} = C\left[f_1 D_1\right],$$

$$F_{UU}^{\cos 2\phi_h} = C\left[\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T) - \boldsymbol{p}_T\cdot\boldsymbol{k}_T}{z_h M_N M_h} h_1^\perp H_1^\perp\right],$$

$$F_{UL}^{\sin 2\phi_h} = C\left[\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T) - \boldsymbol{p}_T\cdot\boldsymbol{k}_T}{z_h M_N M_h} h_{1L}^\perp H_1^\perp\right],$$

$$F_{LL} = C\left[g_1 D_1\right],$$

$$F_{LT}^{\cos(\phi_h-\phi_S)} = C\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_N} g_{1T}^\perp D_1\right],$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = C\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{z_h M_h} h_1 H_1^\perp\right],$$

$$F_{UT}^{\sin(\phi_h-\phi_S)} = C\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_N} f_{1T}^\perp D_1\right],$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = C\left[\frac{4(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)^2 - 2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\boldsymbol{k}_T\cdot\boldsymbol{p}_T) - (\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\boldsymbol{k}_T^2}{2z_h M_N^2 M_h} h_{1T}^\perp H_1^\perp\right], \qquad (2.188)$$

$$C\left[\omega f D\right] = x\sum_i H_{ii}(Q^2,\mu)\int d^2\boldsymbol{k}_T\, d^2\boldsymbol{p}_T\,\delta^{(2)}(z_h\boldsymbol{k}_T + \boldsymbol{p}_T - \boldsymbol{P}_{hT})$$
$$\times \omega\, f_{i/PS}(x, k_T, \mu, \zeta_1)\, D_{h/i}(z_h, p_T, \mu, \zeta_2),$$

$$F_{UU}(x, z_h, P_{hT}, Q^2) = \mathcal{B}\left[\tilde{f}_1^{(0)} \tilde{D}_1^{(0)}\right],$$

$$F_{UU}^{\cos 2\phi_h}(x, z_h, P_{hT}, Q^2) = M_N M_k \, \mathcal{B}\left[\tilde{h}_1^{\perp(1)} \tilde{H}_1^{\perp(1)}\right],$$

$$F_{UL}^{\sin 2\phi_h}(x, z_h, P_{hT}, Q^2) = M_N M_k \, \mathcal{B}\left[\tilde{h}_{1L}^{\perp(1)} \tilde{H}_1^{\perp(1)}\right],$$

$$F_{LL}(x, z_h, P_{hT}, Q^2) = \mathcal{B}\left[\tilde{g}_1^{(0)} \tilde{D}_1^{(0)}\right],$$

$$F_{LT}^{\cos(\phi_h-\phi_S)}(x, z_h, P_{hT}, Q^2) = M_N \, \mathcal{B}\left[\tilde{g}_{1T}^{\perp(1)} \tilde{D}_1^{(0)}\right],$$

$$F_{UT}^{\sin(\phi_h+\phi_S)}(x, z_h, P_{hT}, Q^2) = M_k \, \mathcal{B}\left[\tilde{h}_1^{(0)} \tilde{H}_1^{\perp(1)}\right],$$

$$F_{UT}^{\sin(\phi_h-\phi_S)}(x, z_h, P_{hT}, Q^2) = -M_N \, \mathcal{B}\left[\tilde{f}_{1T}^{\perp(1)} \tilde{D}_1^{(0)}\right],$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)}(x, z_h, P_{hT}, Q^2) = \frac{M_N^2 M_k}{4} \, \mathcal{B}\left[\tilde{h}_{1T}^{\perp(2)} \tilde{H}_1^{\perp(1)}\right],$$

$$\mathcal{B}[\tilde{f}^{(m)} \tilde{D}^{(n)}] \equiv x \sum_i H_{ii}(Q^2, \mu) \int_0^\infty \frac{db_T}{2\pi} \, b_T \, b_T^{m+n} \, J_{m+n}(q_T b_T)$$

$$\times \tilde{f}_{i/N}^{(m)}(x, b_T, \mu, \zeta_1) \, \tilde{D}_{k/i}^{(n)}(z_h, b_T, \mu, \zeta_2).$$

11

NOTEBOOK EXAMPLE