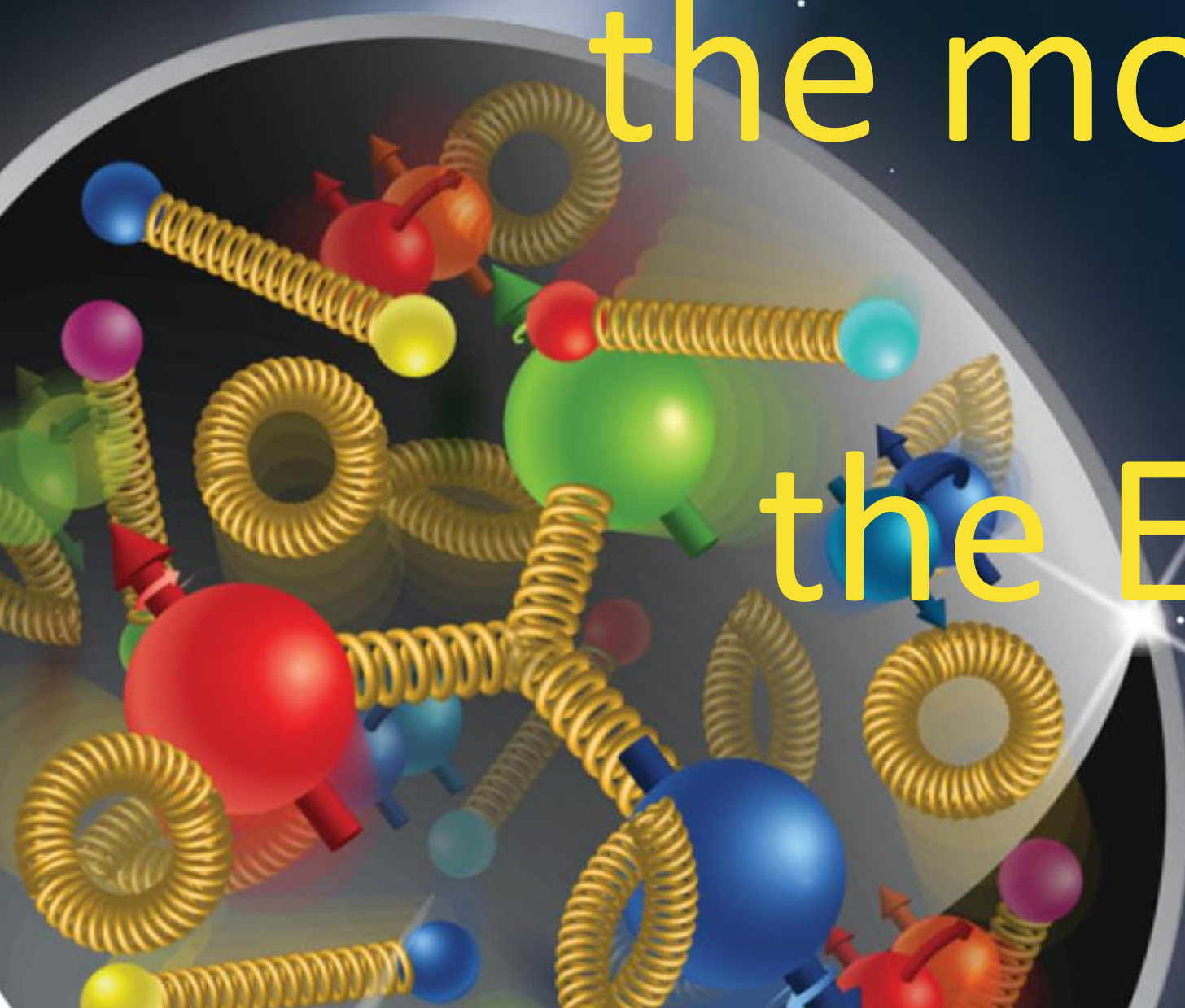


in

the momentum space and

the Electron-Ion Collider

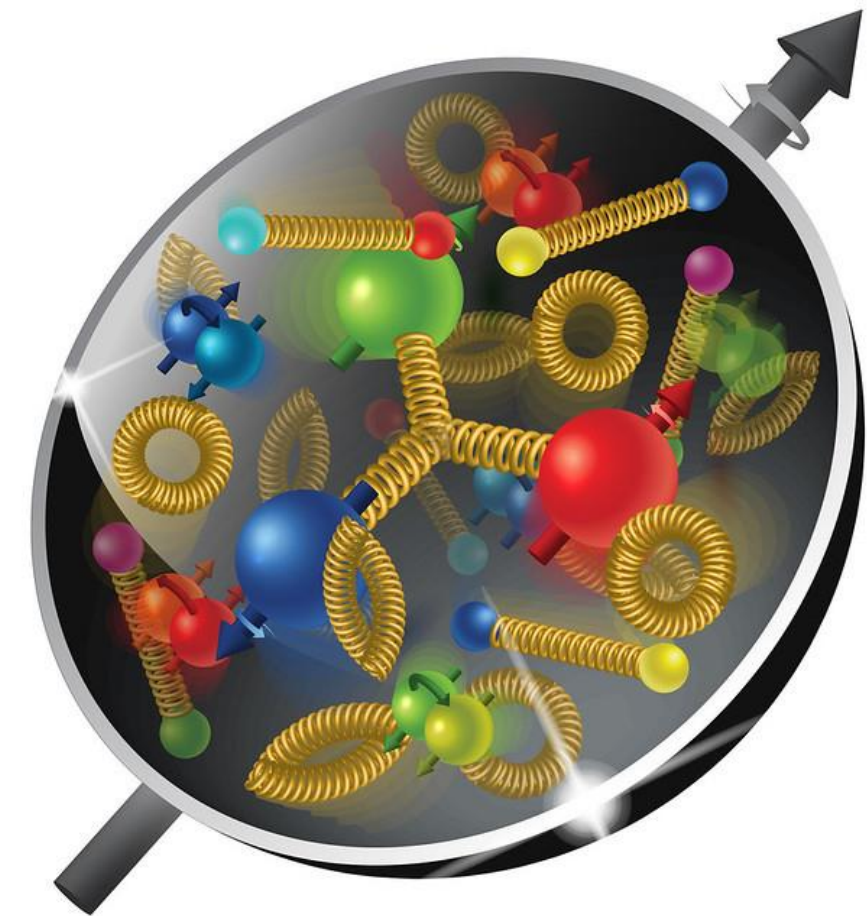


THE MENU

- ❑ What is 3D structure?
- ❑ Why to be excited about it?
- ❑ Why the Electron-Ion Collider?

Our group consists of theorists and experimentalists.

I will try to provide useful information for all.



THE PLAN

□ Lecture I:

Structure of the nucleon

Transverse Momentum Dependent distributions (TMDs)

Semi Inclusive Deep Inelastic Scattering (SIDIS)

Calculations of SIDIS structure functions in google colab

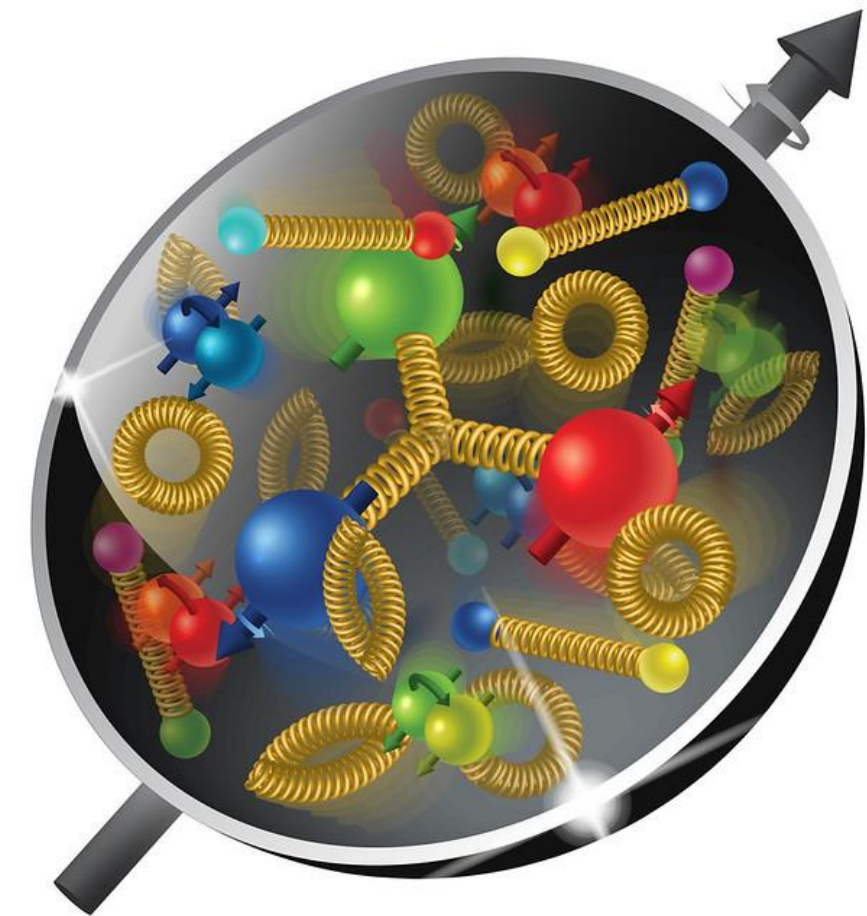
□ Lecture II:

Solution of TMD evolution equations

Collins-Soper-Sterman (CSS) formalism

□ Lecture III: Giuseppe Bozzi

Phenomenology of unpolarized TMDs



MATERIAL

https://inspirehep.net/literature/2650019

TMD Handbook

A modern introduction to the physics of Transverse Momentum Dependent distributions

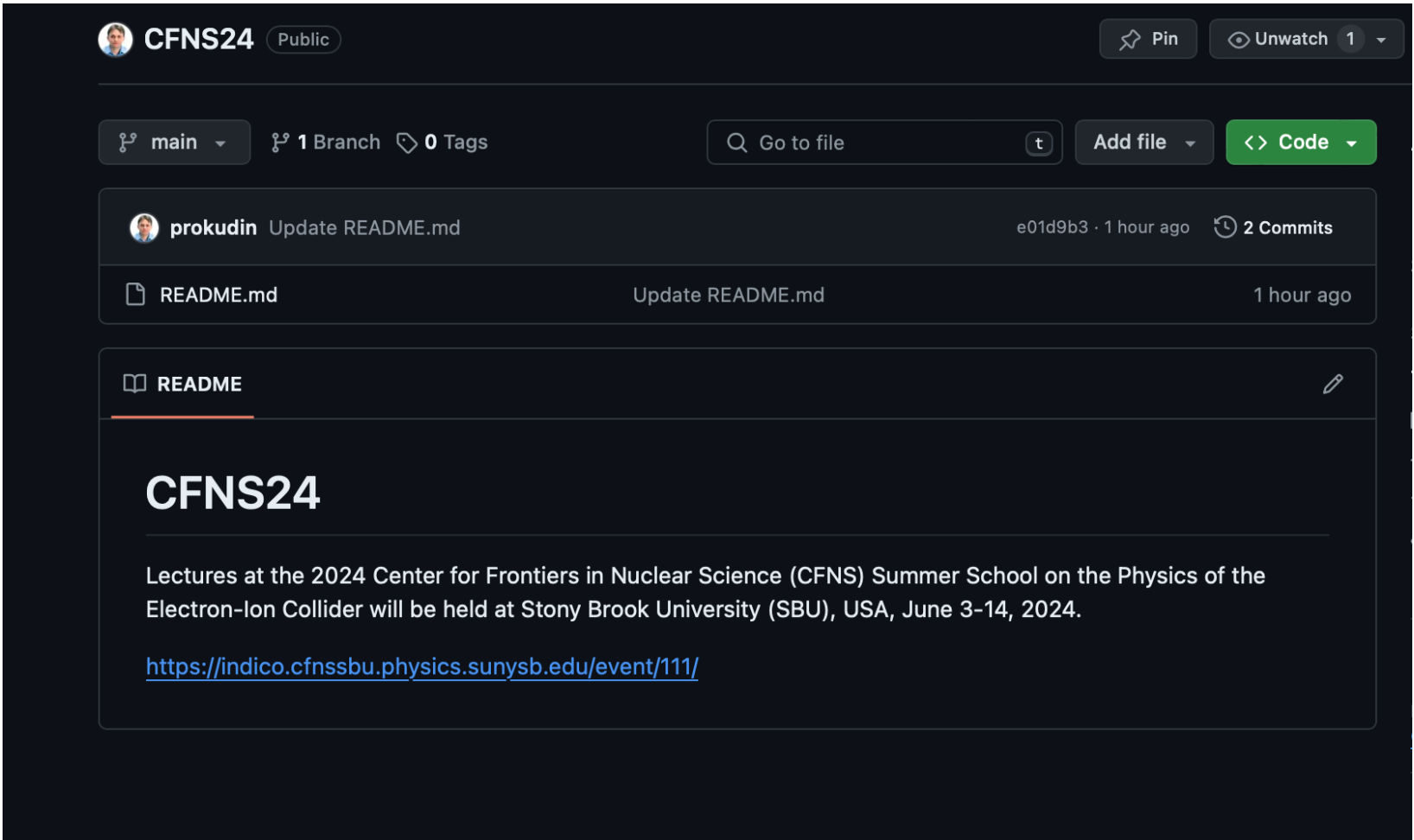
April 6, 2023



- Renaud Boussarie
- Matthias Burkardt
- Martha Constantinou
- William Detmold
- Markus Ebert
- Michael Engelhardt
- Sean Fleming
- Leonard Gamberg
- Xiangdong Ji
- Zhong-Bo Kang
- Christopher Lee
- Keh-Fei Liu
- Simonetta Liuti
- Thomas Mehen *
- Andreas Metz
- John Negele
- Daniel Pitonyak
- Alexei Prokudin
- Jian-Wei Qiu
- Abha Rajan
- Marc Schlegel
- Phiala Shanahan
- Peter Schweitzer
- Iain W. Stewart *
- Andrey Tarasov
- Raju Venugopalan
- Ivan Vitev
- Feng Yuan
- Yong Zhao

* - Editors

https://github.com/prokudin/CFNS24



Clone the repository, or download files from github

TMD TABLE

Leading Quark TMDPDFs  Nucleon Spin  Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \text{Unpolarized}$		$h_1^\perp = \text{Boer-Mulders}$
	L		$g_1 = \text{Helicity}$	$h_{1L}^\perp = \text{Worm-gear}$
	T	$f_{1T}^\perp = \text{Sivers}$	$g_{1T}^\perp = \text{Worm-gear}$	$h_1 = \text{Transversity}$ $h_{1T}^\perp = \text{Pretzelosity}$

Figure 2.5: Leading power quark parton distribution functions for the proton or a spin-1/2 hadron.

TRANSVERSE MOMENTUM DEPENDENT DISTRIBUTIONS

$$\begin{aligned}
 f_{i/p_S}^{[\gamma^+]}(x, \mathbf{k}_T, \mu, \zeta) &= f_1(x, k_T) - \frac{\epsilon_T^{\rho\sigma} k_{T\rho} S_{T\sigma}}{M} \kappa f_{1T}^\perp(x, k_T), \\
 f_{i/p_S}^{[\gamma^+ \gamma_S]}(x, \mathbf{k}_T, \mu, \zeta) &= S_L g_1(x, k_T) - \frac{k_T \cdot S_T}{M} g_{1T}^\perp(x, k_T), \\
 f_{i/p_S}^{[i\sigma^{\alpha+} \gamma_S]}(x, \mathbf{k}_T, \mu, \zeta) &= S_T^a h_1(x, k_T) + \frac{S_L k_T^\alpha}{M} h_{1L}^\perp(x, k_T) \\
 &\quad - \frac{\mathbf{k}_T^2}{M^2} \left(\frac{1}{2} g_T^{\alpha\rho} + \frac{k_T^\alpha k_T^\rho}{\mathbf{k}_T^2} \right) S_{T\rho} h_{1T}^\perp(x, k_T) - \frac{\epsilon_T^{\alpha\rho} k_{T\rho}}{M} \kappa h_1^\perp(x, k_T).
 \end{aligned} \tag{2.123}$$

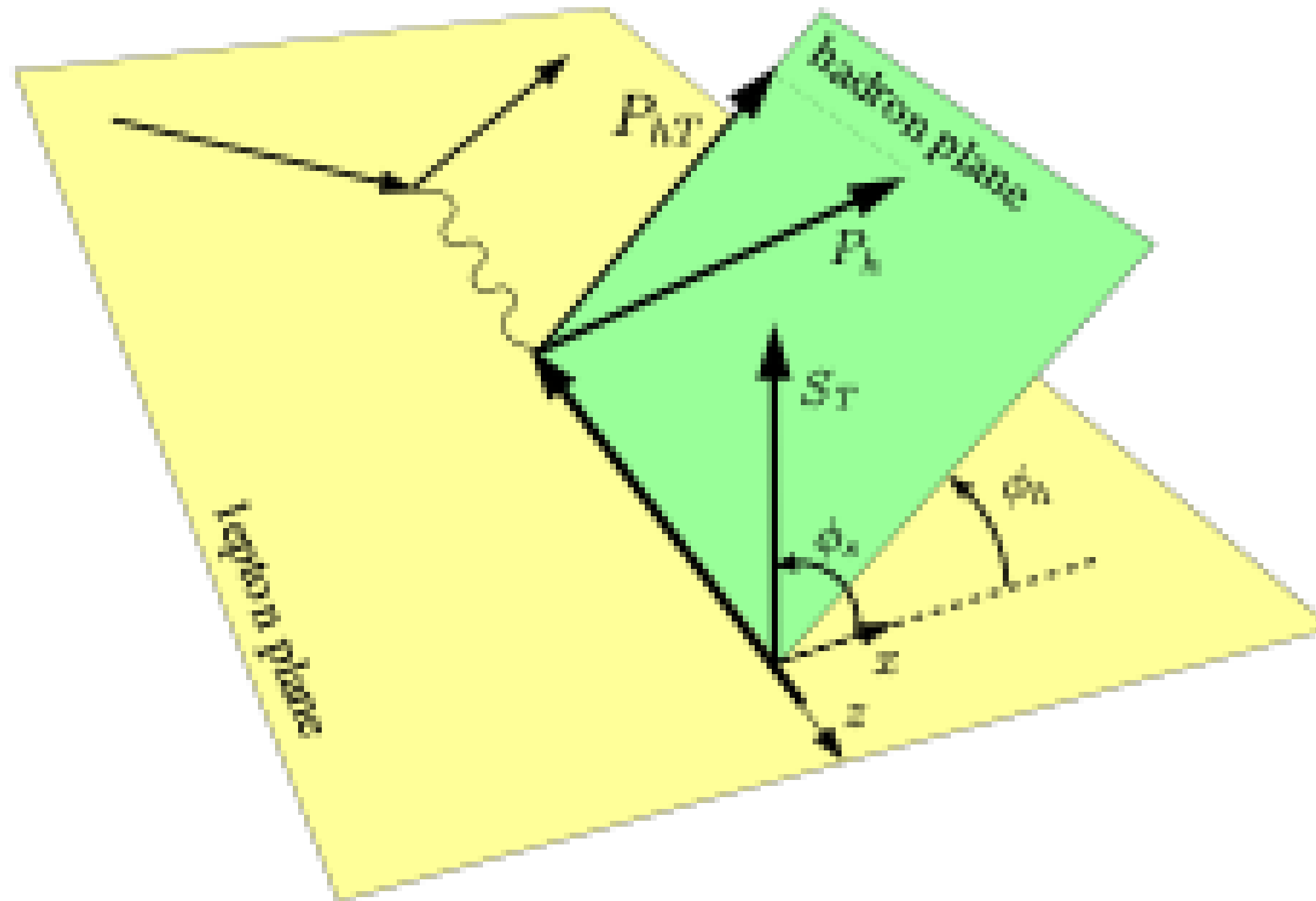
$$\kappa = \begin{cases} +1 & \text{(Drell-Yan)} \\ -1 & \text{(SIDIS)} \end{cases}$$

- $f_1(x, k_T)$ describes an unpolarized quark inside an unpolarized hadron, similar to the unpolarized collinear distribution $f_1(x)$.
- $g_1(x, k_T)$ is the helicity distribution which describes a longitudinally polarized quark inside a longitudinally polarized hadron, similar to the collinear helicity distribution $g_1(x)$.
- $h_1(x, k_T)$ is the transversity distribution which describes a transversely polarized quark inside a transversely polarized hadron, similar to the collinear transversity distribution $h_1(x)$.
- $f_{1T}^\perp(x, k_T)$ is the Sivers function [135] which describes an unpolarized quark inside a transversely polarized hadron. Since it is T -odd, it was originally believed to vanish due to symmetry arguments [61]. It was later clarified that it is non-vanishing when correctly taking the Wilson lines in the definition of the unsubtracted TMD PDF and soft function into account [38, 62, 136].
- The function $g_{1T}^\perp(x, k_T)$ describes longitudinally polarized quarks in a transversely polarized hadron, and vice versa $h_{1L}^\perp(x, k_T)$ describes transversely polarized quarks in a longitudinally polarized hadron [137]. They are referred in the literature as “worm-gear” T and L functions or Kotzinian-Mulders [64, 138] functions.
- $h_1^\perp(x, k_T)$ is the Boer-Mulders function [63] which describes a transversely polarized quark in an unpolarized hadron. Like the Sivers function f_{1T}^\perp , it is time-reversal odd.
- $h_{1T}^\perp(x, k_T)$ is the pretzelosity function, which contributes to the distribution of a transversely polarized quark in a transversely polarized hadron [132], in addition to the transversity $h_1(x, k_T)$. Curiously, the name of this function stems from its expected shape [139] published by G. Miller, which was also highlighted in the New York Times [140], exhibiting the unusual shape of the proton due to the presence of this function.

TRANSVERSE MOMENTUM DEPENDENT DISTRIBUTIONS

$$\begin{aligned}
 \tilde{f}_{i/p_S}^{[\gamma^+]}(x, \mathbf{b}_T, \mu, \zeta) &= \tilde{f}_1(x, b_T) + i\epsilon_{\rho\sigma} b_T^\rho S_T^\sigma M \tilde{f}_{1T}^\perp(x, b_T), \\
 \tilde{f}_{i/p_S}^{[\gamma^+ \gamma_S]}(x, \mathbf{b}_T, \mu, \zeta) &= S_L \tilde{g}_1(x, b_T) + i b_T \cdot S_T M \tilde{g}_{1T}^\perp(x, b_T), \\
 \tilde{f}_{i/p_S}^{[i\sigma^{\alpha+} \gamma_S]}(x, \mathbf{b}_T, \mu, \zeta) &= S_T^\alpha \tilde{h}_1(x, b_T) - i S_L b_T^\alpha M \tilde{h}_{1L}^\perp(x, b_T) + i\epsilon^{\alpha\rho} b_{\perp\rho} M \tilde{h}_1^\perp(x, b_T) \\
 &\quad + \frac{1}{2} \mathbf{b}_T^2 M^2 \left(\frac{1}{2} g_T^{\alpha\rho} + \frac{b_T^\alpha b_T^\rho}{\mathbf{b}_T^2} \right) S_{\perp\rho} \tilde{h}_{1T}^\perp(x, b_T).
 \end{aligned} \tag{2.126}$$

SEMI INCLUSIVE DEEP INELASTIC SCATTERING



$$\ell(l) + p(P) \rightarrow \ell(l') + h(P_h) + X$$

SEMI INCLUSIVE DEEP INELASTIC SCATTERING

$$\begin{aligned}
 \frac{d^6\sigma}{dx dy dz_h d\phi_S d\phi_h dP_{hT}^2} = & \frac{\alpha_{em}^2}{x y Q^2} \left(1 - y + \frac{1}{2} y^2 \right) \left[F_{UU,T} + \cos(2\phi_h) p_1 F_{UU}^{\cos(2\phi_h)} \right. \\
 & + S_L \sin(2\phi_h) p_1 F_{UL}^{\sin(2\phi_h)} + S_L \lambda p_2 F_{LL} \\
 & + S_T \sin(\phi_h - \phi_S) F_{UT,T}^{\sin(\phi_h - \phi_S)} \\
 & + S_T \sin(\phi_h + \phi_S) p_1 F_{UT}^{\sin(\phi_h + \phi_S)} + \lambda S_T \cos(\phi_h - \phi_S) p_2 F_{LT}^{\cos(\phi_h - \phi_S)} \\
 & \left. + S_T \sin(3\phi_h - \phi_S) p_1 F_{UT}^{\sin(3\phi_h - \phi_S)} \right], \quad (2.186)
 \end{aligned}$$

$$p_1 = \frac{1-y}{1-y+\frac{1}{2}y^2}, \quad p_2 = \frac{y(1-\frac{1}{2}y)}{1-y+\frac{1}{2}y^2}, \quad p_3 = \frac{(2-y)\sqrt{1-y}}{1-y+\frac{1}{2}y^2}, \quad p_4 = \frac{y\sqrt{1-y}}{1-y+\frac{1}{2}y^2}$$

SEMI INCLUSIVE DEEP INELASTIC SCATTERING

$$\begin{aligned}
 F_{UU,T} &= C[f_1 D_1], \\
 F_{UU}^{\cos 2\phi_h} &= C \left[\frac{2 (\hat{h} \cdot p_T) (\hat{h} \cdot k_T) - p_T \cdot k_T}{z_h M_N M_h} h_1^\perp H_1^\perp \right], \\
 F_{UL}^{\sin 2\phi_h} &= C \left[\frac{2 (\hat{h} \cdot p_T) (\hat{h} \cdot k_T) - p_T \cdot k_T}{z_h M_N M_h} h_{1L}^\perp H_1^\perp \right], \\
 F_{LL} &= C[g_1 D_1], \\
 F_{LT}^{\cos(\phi_h - \phi_S)} &= C \left[\frac{\hat{h} \cdot k_T}{M_N} g_{1T}^\perp D_1 \right], \\
 F_{UT}^{\sin(\phi_h + \phi_S)} &= C \left[\frac{\hat{h} \cdot p_T}{z_h M_h} h_1 H_1^\perp \right], \\
 F_{UT}^{\sin(\phi_h - \phi_S)} &= C \left[-\frac{\hat{h} \cdot k_T}{M_N} f_{1T}^\perp D_1 \right], \\
 F_{UT}^{\sin(3\phi_h - \phi_S)} &= C \left[\frac{4 (\hat{h} \cdot p_T) (\hat{h} \cdot k_T)^2 - 2 (\hat{h} \cdot k_T) (k_T \cdot p_T) - (\hat{h} \cdot p_T) k_T^2}{2 z_h M_N^2 M_h} h_{1T}^\perp H_1^\perp \right], \quad (2.188)
 \end{aligned}$$

$$\begin{aligned}
 C[\omega f D] &= x \sum_i H_{ii}(Q^2, \mu) \int d^2 k_T d^2 p_T \delta^{(2)}(z_h k_T + p_T - P_{hT}) \\
 &\quad \times \omega f_{i/p_S}(x, k_T, \mu, \zeta_1) D_{h/i}(z_h, p_T, \mu, \zeta_2),
 \end{aligned}$$

SEMI INCLUSIVE DEEP INELASTIC SCATTERING

$$\begin{aligned}
 F_{UU}(x, z_h, P_{hT}, Q^2) &= \mathcal{B} \left[\tilde{f}_1^{(0)} \tilde{D}_1^{(0)} \right], \\
 F_{UU}^{\cos 2\phi_h}(x, z_h, P_{hT}, Q^2) &= M_N M_h \mathcal{B} \left[\tilde{h}_1^{\perp(1)} \tilde{H}_1^{\perp(1)} \right], \\
 F_{UL}^{\sin 2\phi_h}(x, z_h, P_{hT}, Q^2) &= M_N M_h \mathcal{B} \left[\tilde{h}_{1L}^{\perp(1)} \tilde{H}_1^{\perp(1)} \right], \\
 F_{LL}(x, z_h, P_{hT}, Q^2) &= \mathcal{B} \left[\tilde{g}_1^{(0)} \tilde{D}_1^{(0)} \right], \\
 F_{LT}^{\cos(\phi_h - \phi_s)}(x, z_h, P_{hT}, Q^2) &= M_N \mathcal{B} \left[\tilde{g}_{1T}^{\perp(1)} \tilde{D}_1^{(0)} \right], \\
 F_{UT}^{\sin(\phi_h + \phi_s)}(x, z_h, P_{hT}, Q^2) &= M_h \mathcal{B} \left[\tilde{h}_1^{(0)} \tilde{H}_1^{\perp(1)} \right], \\
 F_{UT}^{\sin(\phi_h - \phi_s)}(x, z_h, P_{hT}, Q^2) &= -M_N \mathcal{B} \left[\tilde{f}_{1T}^{\perp(1)} \tilde{D}_1^{(0)} \right], \\
 F_{UT}^{\sin(3\phi_h - \phi_s)}(x, z_h, P_{hT}, Q^2) &= \frac{M_N^2 M_h}{4} \mathcal{B} \left[\tilde{h}_{1T}^{\perp(2)} \tilde{H}_1^{\perp(1)} \right],
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{B}[\tilde{f}^{(m)} \tilde{D}^{(n)}] &\equiv x \sum_i H_{ii}(Q^2, \mu) \int_0^\infty \frac{db_T}{2\pi} b_T b_T^{m+n} J_{m+n}(q_T b_T) \\
 &\times \tilde{f}_{i/N}^{(m)}(x, b_T, \mu, \zeta_1) \tilde{D}_{h/i}^{(n)}(z_h, b_T, \mu, \zeta_2).
 \end{aligned}$$

NOTEBOOK EXAMPLE

