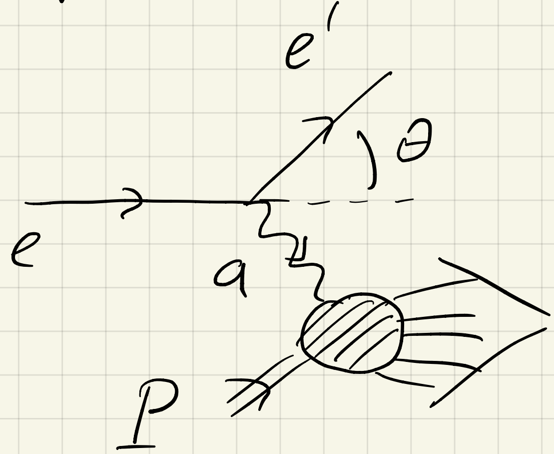
During lecture 1 we speak of a certain process. Let us look into its kinematics

Deep Inelastic Scattering (DIS)



1) Show that $q^2 < 0$

$$\ell = (E, 0, 0, E)$$

$$e' = (E', E'\sin\theta, 0, E'\cos\theta)$$

Thus $q^2 = (\ell - e')^2 \simeq -2\ell e' = -2(EE' - EE'\cos\theta)$

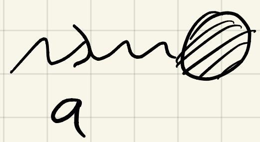
$= -2EE'(1-\cos\theta) \leq 0$.

It is customary to introduce $q^2 \equiv -Q^2$
where $Q^2 \geq 0$

Now let us explore other kinematical variables

$$x = \frac{Q^2}{2P\cdot q} \text{ - Bjorken-}x, \quad y = \frac{P\cdot q}{P\cdot e} \text{ - inelasticity}$$

These variables are constructed off scaler products which are Lorentz invariants and wherefore we can use any frame to estimate them. Let us choose target rest frame



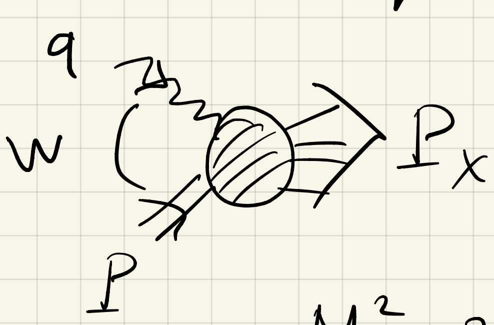
$$P = (M, \vec{0})$$

$$q = (\nu, \vec{q}), \quad \nu = E - E' > 0$$

$$2P\cdot q = 2M\nu \to \infty \quad \text{Bjorken limit}$$

$$q^2 = -Q^2, \quad Q^2 \geq 0, \quad \nu \geq 0 \Rightarrow$$

$$x = \frac{Q^2}{2P\cdot q} \geq 0$$



W is the energy of qP

$$W^2 = (P+q)^2 = P^2 + 2P\cdot q - Q^2 =$$

$$= M^2 + 2P\cdot q - Q^2 \geq \underbrace{M^2}$$

in case the proton is intact = elastic scattering

$$\Rightarrow \quad 2P\cdot q \geq Q^2 \Rightarrow \quad x = \frac{Q^2}{2P\cdot q} \leq 1$$

Inelasticity $y = \frac{P \cdot q}{P \cdot e} = \frac{M(E - E')}{ME} = 1 - E'/E$

$E' \in [0, E]$ thus $y \in [0, 1]$

$\left.\begin{array}{l} x \in [0, 1] \\ y \in [0, 1] \end{array}\right\}$ we can estimate the reach of experiments

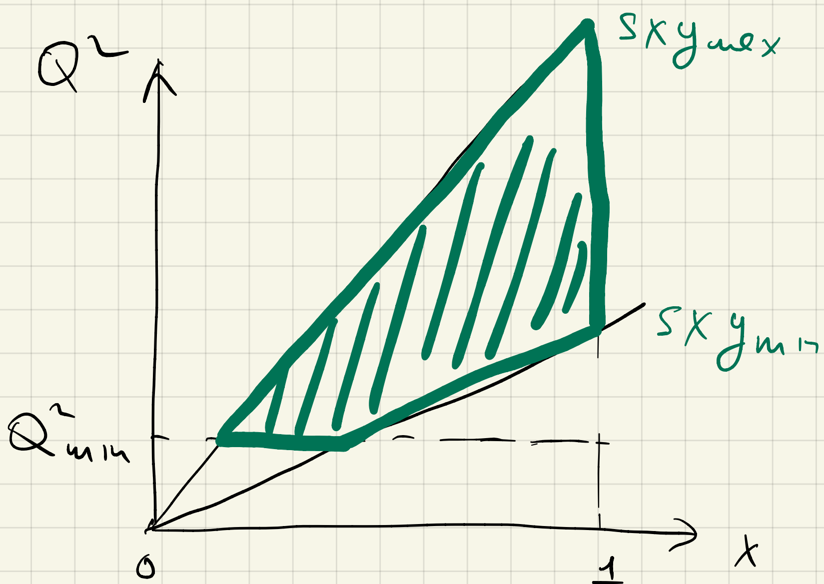
$$s = (P + e)^2 \simeq M^2 + 2P \cdot e \simeq 2P \cdot e$$

$\Rightarrow$

$$x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2P \cdot q} \cdot \frac{P \cdot e}{P \cdot e} = \frac{Q^2}{ys}$$

Therefore

$$Q^2 \simeq s y x$$

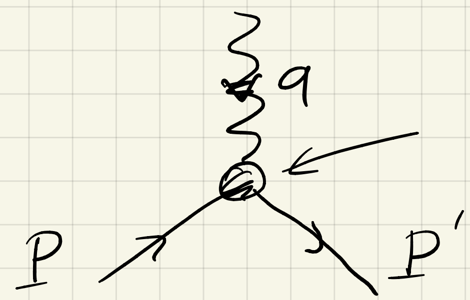


$Q^2_{min} \simeq 1\,(\mathrm{GeV}^2)$ to ensure DIS regime

$y \in \underbrace{[y_{min}, y_{max}]}_{\text{experimental resolution}}$

Why do we need x_{Bj}?

Let us consider form factors → elastic scattering



$$\delta^{(4)}(P+q-P')$$

$$\Rightarrow q = P' - P$$

$$q^2 = -Q^2 = (P'-P)^2 = P'^2 + P^2 - 2P'\cdot P =$$

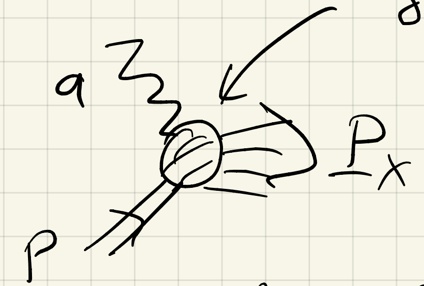
$$= 2M^2 - 2P'\cdot P \to -2P'\cdot P$$

$$P\cdot q = P(P'-P) = P\cdot P' - M^2 \to P\cdot P'$$

therefore

$$x = \frac{Q^2}{2P\cdot q} \to 1$$ not an independent variable

$$\delta^{(4)}(P+q-P_X)$$



We integrate over P_X

$$\left.\begin{array}{l} q^2 = -Q^2 \to \infty \\ 2P\cdot q \to \infty \end{array}\right\} \text{independently}$$

$$x_{Bj} \equiv \frac{Q^2}{2P\cdot q} \in [0,1].$$

Experiments : fixed target vs collider

$$s = (\underline{P}+\ell)^2 \quad \text{cm energy}$$

Fixed target

$$\ell = (P_{lab}, 0, 0, -P_{lab})$$

$$\underline{P} = (M_p, 0, 0, 0)$$

$$s = (\underline{P}+\ell)^2 = \underline{P}^2 + 2\underline{P}\cdot\ell + \ell^2 \simeq 2 M_p P_{lab}$$

Collider

$$\ell = (E_e, 0, 0, -E_e)$$

$$\underline{P} \simeq (E_p, 0, 0, E_p) \quad \text{(neglect the mass)}$$

$$s = (\underline{P}+\ell)^2 \simeq (E_e + E_p)^2 - (E_p - E_e)^2 = 4 E_p E_e$$

The energy increased easily in collider