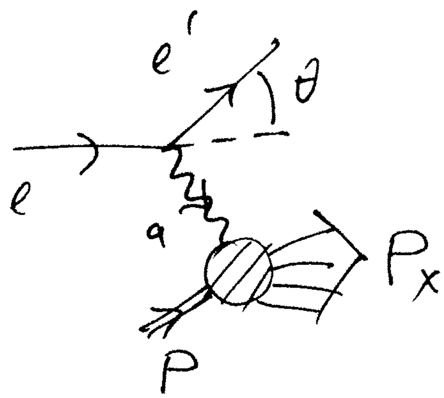
Some notes on kinematics



1) Show that $q^2 < 0$

$$e = (E, 0, 0, E)$$

$$e' = (E', E'\sin\theta, 0, E'\cos\theta)$$

$$q^2 = (e-e')^2 \simeq -2ee' = -2(EE' - EE'\cos\theta) =$$

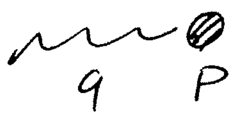
$$= -2EE'(1-\cos\theta) \leq 0$$

q^2 is negative $\quad q^2 = -Q^2$

$$x = \frac{Q^2}{2P\cdot q} \quad , \quad y = \frac{P\cdot q}{P\cdot e}$$

Let's find possible values.

Scalar products are scalars => lorentz invariant can be calculated in any frame. We choose target rest frame



$P = (M, \vec{0})$

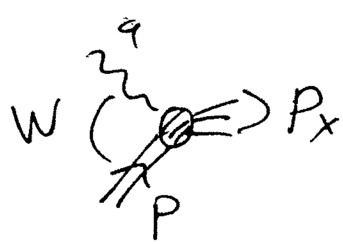
$$q = (\nu, \vec{q}) \, , \quad \nu = E - E' > 0$$

$$2P\cdot q = 2M\nu \to \infty \quad \text{Bjorken limit}$$

$$q^2 = -Q^2 \, , \quad Q^2 \geq 0 \, , \quad \nu \geq 0 \quad \Rightarrow$$

$$x = \frac{Q^2}{2P\cdot q} \gtrsim 0$$

$$W^2 = (P+q)^2$$



$$W^2 = P^2 + 2P\cdot q - Q^2 = M^2 + 2P\cdot q - Q^2 \gtrsim M^2$$

$$\Rightarrow \quad 2P\cdot q \gtrsim Q^2$$

$$\Rightarrow \quad Q^2/2P\cdot q \leq 1$$

$$y = \frac{P\cdot q}{P\cdot e} = \frac{M(E-E')}{ME} = 1 - E'/E$$

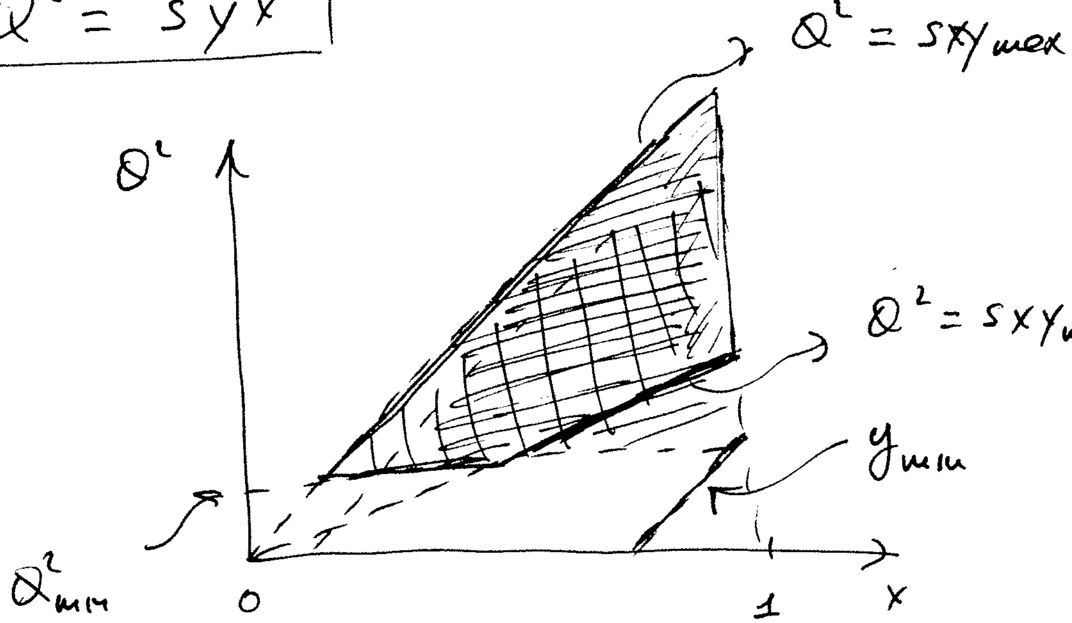
$$E' \in [0, E] \Rightarrow \underline{y \in [0,1]}$$

This gives us a tool to estimate reach of experiments

$$s = (P+e)^2 = M^2 + 2P.e \simeq 2P.e \quad \text{(high energy)}$$

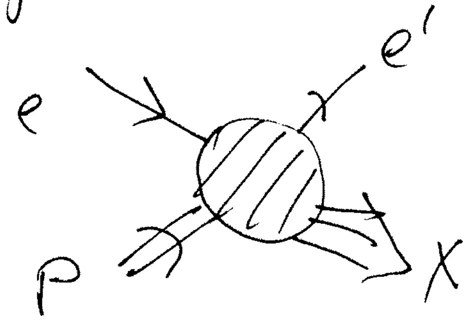
$$\Rightarrow \quad x = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{2P\cdot q}\cdot\frac{P\cdot e}{P\cdot e} = \frac{Q^2}{ys}$$

$$\boxed{Q^2 = syx}$$



$Q^2_{min} \simeq 1 (GeV^2)$ to ensure DIS regime

We want to calculate cross-section of this process:



$$e + P \to e' + X$$

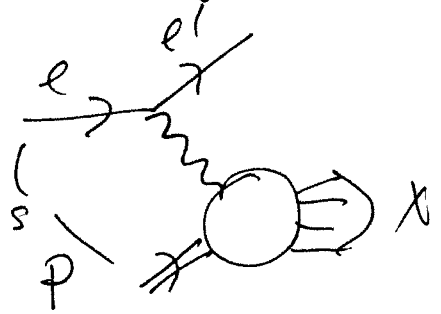
Deep Inelastic Scattering

$\downarrow$ $\qquad\qquad$ $\searrow$

$q^2 = +(e-e')^2$ $\qquad$ $P \to X$

big

We use one photon exchange approximation:

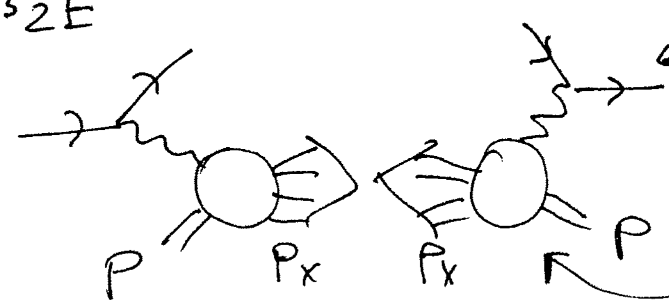


$$\sigma = \frac{1}{\mathcal{F}} |M|^2 \, d\,PS \quad , \; \mathcal{F} \simeq 2s = 2\,(e+P)^2 \text{ flux}$$

$$d\,PS = \frac{d^3 e'}{(2\pi)^3\, 2E'}$$

Leptonic tensor

$$|M|^2 =$$



$$\equiv L_{\mu\nu} W^{\mu\nu} \frac{1}{Q^4}$$

Hadronic tensor

Some useful formulae

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\psi$$

γ^μ - gamma matrices

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu}\,, \quad \bar{\psi} \equiv \psi^+ \gamma^0$$

Global gauge transformations

$$\psi'(x) = e^{i\alpha}\psi(x)$$

$$\bar{\psi}'(x) = \bar{\psi}(x)\, e^{-i\alpha}$$

$\Rightarrow$ current

$$j^\mu(x) = \bar{\psi}(x)\,\gamma^\mu \psi(x)$$

Local gauge transformations

$$\begin{cases} \psi'(x) = e^{i\alpha(x)}\psi(x) \\ \bar{\psi}'(x) = \bar{\psi}(x)\, e^{-i\alpha(x)} \end{cases}$$

$$\partial_\mu \psi(x) = e^{-i\alpha(x)}(\partial_\mu - i\partial_\mu \alpha(x))\,\psi'(x)$$

$$\Rightarrow \quad \mathcal{L} = \bar{\Psi}'(x)\left(i\gamma^\mu(\partial_\mu - i\partial_\mu\alpha(x)) - m\right)\psi'(x)$$

We can restore gauge invariance if we use

$$(\partial_\mu + ieA_\mu(x))\psi(x)$$

$$(\partial_\mu + ieA_\mu(x))\psi(x) = e^{-i\alpha(x)}(\partial_\mu + ieA'_\mu(x))\psi'(x)$$

where

$$A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x)$$

$\partial_\mu + ieA_\mu(x) \rightarrow$ covariant derivative. "D_μ"

$$\mathcal{L} = \bar{\Psi}(x)\left(i\gamma^\mu(\partial_\mu + ieA_\mu(x)) - m\right)\psi(x)$$

$\uparrow$

invariant also under local gauge transformations.

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{Interaction}}$$

$$\mathcal{L}_I = -e\, j^\mu A_\mu \quad \text{where} \quad \underbrace{j^\mu = \bar{\Psi}(x)\gamma^\mu\psi(x)}_{\text{current.}}$$

charge

Construction $\rightarrow$ local gauge invariance

~~Conservation~~ $\cancel{\partial_\mu j^\mu = 0 \Rightarrow \partial_\mu}$

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu \equiv \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

$$\gamma^0(\gamma^\mu)^\dagger\gamma^0 = \gamma^\mu \Rightarrow (\gamma^0)^\dagger = \gamma^0, \ (\gamma^k)^\dagger = -\gamma^k$$

Independent fields $\psi(x)$ & $\bar{\psi}(x) = \psi^\dagger_{(x)}\gamma^0$

$$\left.\begin{aligned} &\frac{\partial\mathcal{L}}{\partial\psi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi} = 0 \\ &\frac{\partial\mathcal{L}}{\partial\bar{\psi}} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\bar{\psi}} = 0 \end{aligned}\right\} \text{Euler-Lagrange equations}$$

$$\frac{\partial\mathcal{L}}{\partial\bar{\psi}} = (i\gamma^\mu\partial_\mu - m)\psi, \quad \frac{\partial\mathcal{L}}{\partial\partial_\mu\bar{\psi}} = 0$$

$$\frac{\partial\mathcal{L}}{\partial\psi} = -m\bar{\psi}, \quad \frac{\partial\mathcal{L}}{\partial\partial_\mu\psi} = i\bar{\psi}\gamma^\mu$$

$$\Rightarrow \begin{cases} (i\gamma^\mu\partial_\mu - m)\psi(x) = 0 \\ i\partial_\mu\bar{\psi}(x)\gamma^\mu + m\bar{\psi}(x) = 0 \end{cases}$$

Solutions $\to$ 4, 2 with $p_0 > 0$, 2 with $p_0 < 0$

Lets consider only positive energy:

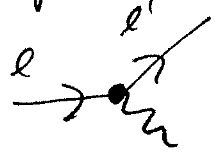
$$\psi(x) = u(p,s)\,e^{-ip\cdot x}, \quad p^2 = m^2, \ p_0 > 0$$

We have

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0$$

$$\Rightarrow \quad (\gamma^\mu p_\mu - m)\, u(p) = 0 \quad , \quad \gamma^\mu p_\mu = \not p$$

$(\not p - m)\, u(p) = 0$, $u(p)$ is called spinor



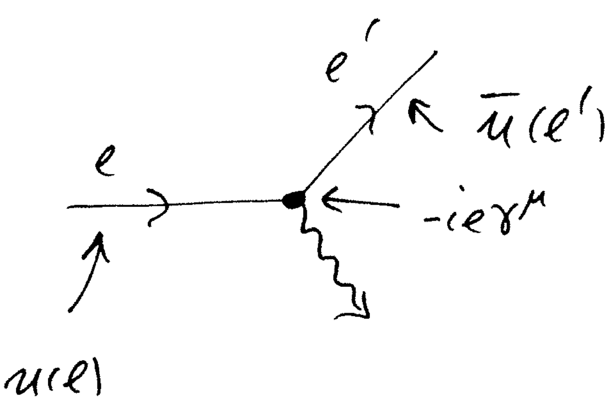
$q = e - e'$

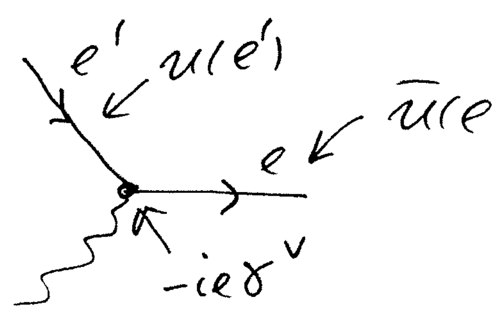
$$\bar u(p)(\not p - m) = 0$$

Current conservation: $\partial_\mu j^\mu = 0 \Rightarrow j^\mu = \bar u(e')\gamma^\mu u(e)\, e^{-i(e-e')x}$

$$\partial_\mu j^\mu = -i q_\mu j^\mu = 0 \Rightarrow \boxed{q_\mu j^\mu(x) = 0}$$

Now we have all ingredients to construct Feynman diagrams. For example $L^{\mu\nu}$:





$$L^{\mu\nu}_{\beta} = \frac{1}{2s+1} \sum_{s'} \bar{u}_\alpha(\ell,s)(-ie\gamma^\nu)_{\alpha\beta} u(\ell',s')_\beta \bar{u}_a(\ell',s')(-ie\gamma^\mu)_{ab} u(\ell)_b$$

Spin projector

$$\sum_{s'} u_\beta(\ell',s')\bar{u}_a(\ell',s') = (\not{\ell}' + m)_{\beta a}$$

$$u_b(\ell,s)\bar{u}_\alpha(\ell,s) = \left[\frac{(\not{\ell}+m)(1+\gamma_5 \not{s})}{2}\right]_{b\alpha}$$

where $\gamma_5 = +i\gamma^0\gamma^1\gamma^2\gamma^3$, $\gamma_5^\dagger = \gamma_5$, $(\gamma_5)^2 = 1$, $\{\gamma_5, \gamma^\mu\} = 0$

Let's sum over s and obtain symmetric part of $L^{\mu\nu}$:

$$L^{\mu\nu} = \frac{e^2}{2} \cancel{\sum_{s,s'}} \underbrace{(\not{\ell}+m)_{b\alpha}(\gamma^\nu)_{\alpha\beta}(\not{\ell}'+m)_{\beta a}(\gamma^\mu)_{ab}}_{\text{Trace}}$$

neglect m and

$$L^{\mu\nu} = \frac{e^2}{2} T_2(\not{\ell}\gamma^\nu\not{\ell}'\gamma^\mu)$$

Traces:

$$Tr(\text{odd } \# \gamma) = 0$$

$$Tr(\not{a}\not{b}) = 4\, a\cdot b$$

$$Tr(\not{a}\not{b}\not{c}\not{d}) = 4\left[(a\cdot b)(c\cdot d) - (a\cdot c)(b\cdot d) + (a\cdot d)(c\cdot d)\right]$$

$$Tr(\gamma^a\gamma^b\gamma^c\gamma^d) = 4\left[g^{ab}g^{cd} - g^{ac}g^{bd} + g^{ad}g^{cd}\right]$$

Thus we get

$$L^{\mu\nu} = 2e^2(\ell_\mu \ell'_\nu + \ell_\nu \ell'_\mu - g_{\mu\nu}(\ell\cdot\ell'))$$

Now we go from leptonic to hadronic tensor

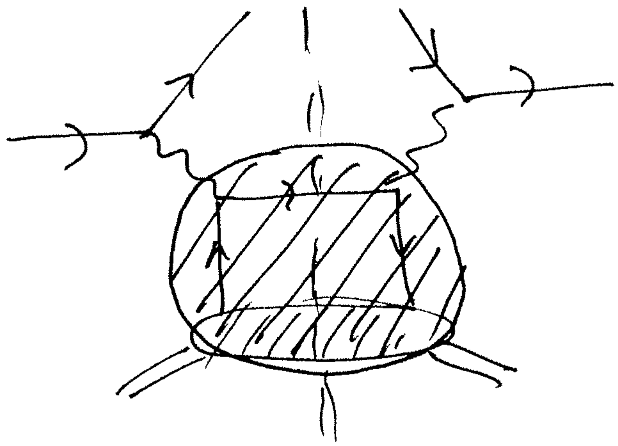
γ matrices can have different representations,

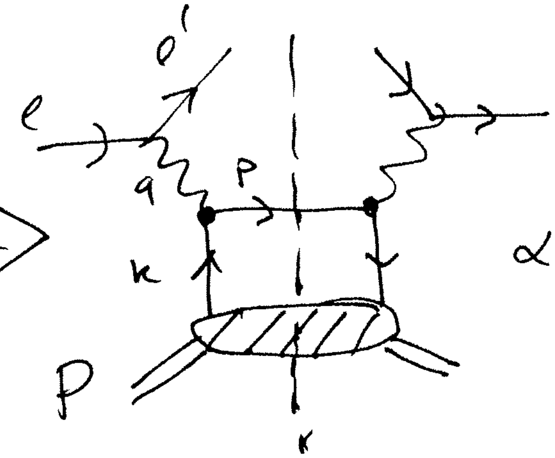
for example Weyl representation

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix}$$

σ^i – Pauli matrices

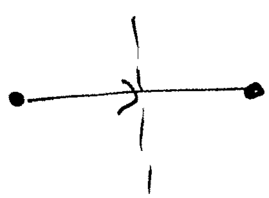
What about hadron?





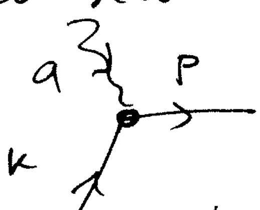
$\propto (e_q)^2$

Square of the amplitude,
but also imaginary part
„optical theorem"



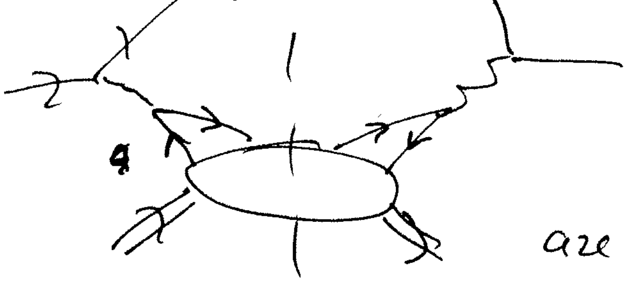
$= \operatorname{Im} \frac{1}{p^2 + i\epsilon} \propto \delta(p^2)$

Conservation



$\delta^{(4)}(k+q-p) \Rightarrow \underline{\underline{p = k+q}}$

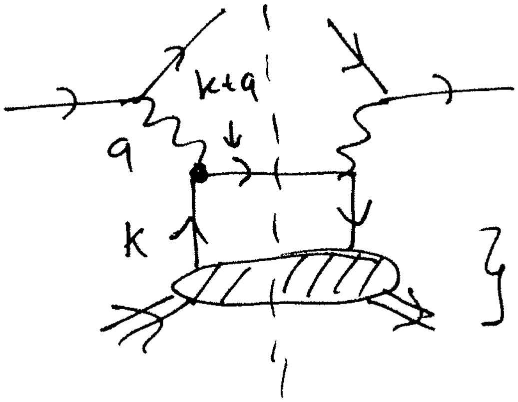
Other diagrams „CATs EARS"



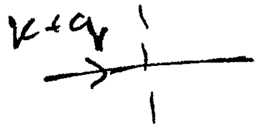
$\propto e_{q_1} e_{q_2}$

are suppressed by $\left(\frac{1}{Q^2}\right)^2$

Why x_{Bj} ?



} $f(x)$ probability to find a quark $\underline{\underline{k = xP}}$.



$\equiv \quad \delta((k+q)^2)$

$$(k+q)^2 = k^2 + q^2 + 2kq = \not{0} - Q^2 + 2k\cdot q = \not{0}$$

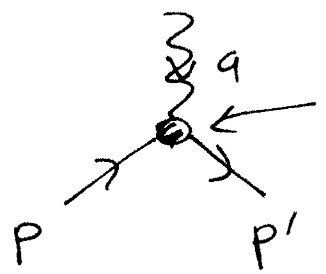
Let us assume that $k = xP$ where $x \in [-\infty, \infty)$

now we get

$$-Q^2 + 2x\,P\cdot q = 0 \Rightarrow \quad x = \frac{Q^2}{2P\cdot q} = x_{Bj}$$

thus $x = x_{Bj}$ – interaction is important!

Form factors vs distributions



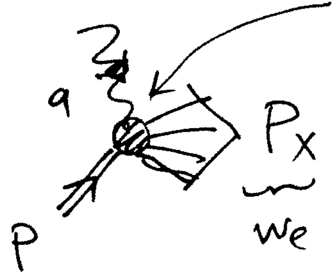
$\delta^{(4)}(p+q-p') \Rightarrow \underbrace{q = p'-p}$

$$q^2 = -Q^2 = (p'-p)^2 = p'^2 + p^2 - 2p'\cdot p = 2M^2 - 2p'p \to -2P'\cdot P$$

$$P\cdot q = P(P'-P) = P\cdot P' - M^2 \to P\cdot P'$$

$$\Rightarrow \quad x = \frac{Q^2}{2P\cdot q} = 1 \quad \text{not an independent variable}$$

$\delta^{(4)}(p+q-P_X)$

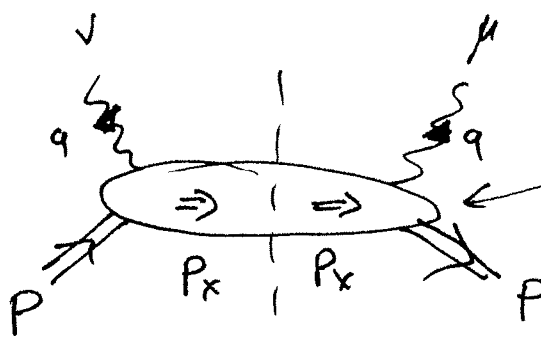


we integrate over P_X !

$$\left.\begin{array}{l} q^2 = -Q^2 \to \infty \\ 2P\cdot q \to \infty \end{array}\right\} \text{independently}$$

$$x_{Bj} \equiv \frac{Q^2}{2P\cdot q}$$

Hadronic tensor



$\delta^{(4)}(P_X - P - q)$

Let's use

$$\delta^{(4)}(k) = \int \frac{d^4\xi}{(2\pi)^4} e^{-ik\cdot\xi}$$

$$\oint_X \equiv \int \frac{d^3P_X}{2E_X(2\pi)^3} = \int \frac{d^4P_X}{(2\pi)^4}\theta(E_X)$$

$$W^{\mu\nu} = \oint_X \delta^{(4)}(P_X - P - q) \langle P|J^\mu(0)|X\rangle\langle X|J^\nu(0)|P\rangle$$

$$= \oint_X \int \frac{d^4\xi}{(2\pi)^4} e^{-i(P_X - P - q)\cdot\xi} \langle P|J^\mu(0)|X\rangle\langle X|J^\nu(0)|P\rangle =$$

$$= \oint_X \int \frac{d^4\xi}{(2\pi)^4} e^{iq\xi} \underbrace{\langle P|e^{iP\cdot\xi}}_{\langle P|e^{i\hat{P}\cdot\xi}} J^\mu(0) \underbrace{e^{-iP_X\cdot\xi}|X\rangle}_{e^{-i\hat{P}\cdot\xi}|X\rangle} \langle X|J^\nu(0)|P\rangle$$

momentum operator

$$e^{i\hat{P}\cdot\xi} J^\mu(0)\, e^{-i\hat{P}\cdot\xi} = J^\mu(\xi)$$ translation of fields

$$= \sum_X \int \frac{d^4\xi}{(2\pi)^4} e^{iq\cdot\xi} \langle P| J^\mu(\xi) |X\rangle\langle X| J^\nu(0)|P\rangle$$

now we use $\sum_X |X\rangle\langle X| = \mathbb{1}$ completeness of states

and obtain

$$W^{\mu\nu} = \int \frac{d^4\xi}{(2\pi)^4} e^{iq\cdot\xi} \underbrace{\langle P| J^\mu(\xi) J^\nu(0)|P\rangle}_{\text{coordinate space.}}$$

Again Bjorken limit

$P = (M, \vec{0})$

$q = (\nu, 0, 0, \sqrt{\nu^2+Q^2})$

$$x = \frac{Q^2}{2Pq} = \frac{Q^2}{2M\nu}, \quad \underline{\underline{Q^2\to\infty,\ \nu\to\infty}}$$

$$q\cdot\xi = q^0\cdot\xi^0 - \vec{q}\cdot\vec{\xi} = \frac{(q^0+q^3)}{\sqrt{2}}\frac{(\xi^0-\xi^3)}{\sqrt{2}} + \frac{(q^0-q^3)}{\sqrt{2}}\frac{(\xi^0+\xi^3)}{\sqrt{2}} - q_T\cdot\xi_T$$

By the way $A^\pm \equiv \frac{A^0 \pm A^3}{\sqrt{2}}$ light-cone coordinates

$$A\cdot B = A^+B^- + A^-B^+ - \vec{A}_T\cdot\vec{B}_T, \quad \vec{A}_T = (A^1, A^2)$$

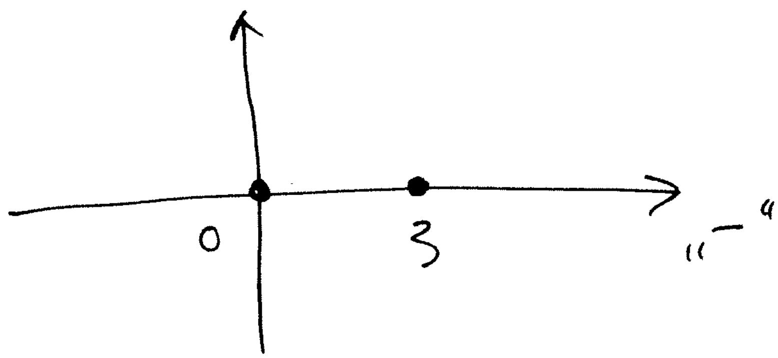
Bjorken limit

$$q^0 + q^3 \simeq 2\nu$$

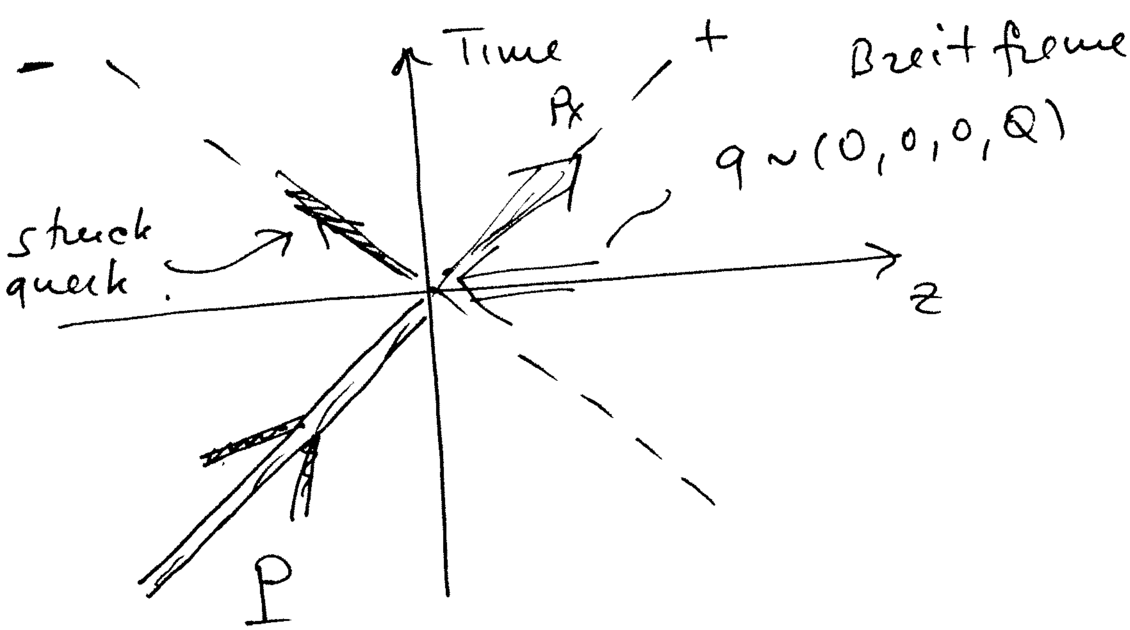
$$q^0 - q^3 \simeq \frac{Q^2}{2\nu}$$

main part of $e^{iq\cdot\xi}$ comes from region of less rapid oscillations $\Rightarrow$ $q\cdot\xi = O(1)$

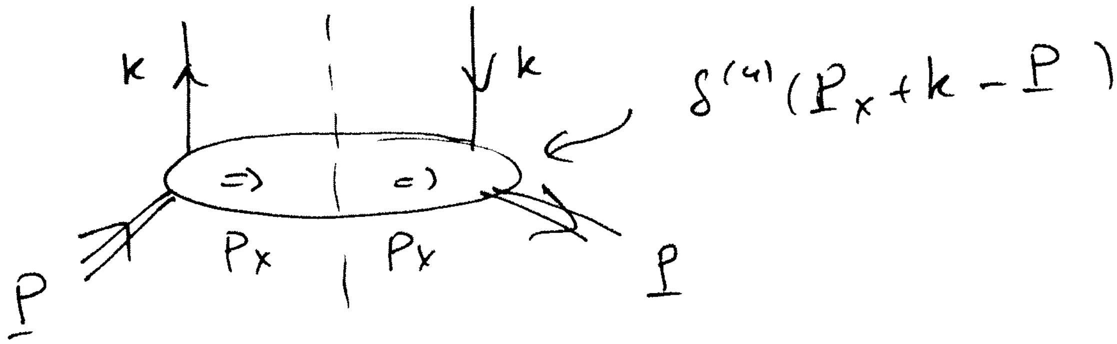
$$\Rightarrow \quad \underbrace{\xi^0+\xi^3}_{\xi^+} \sim O(1/\nu), \quad \underbrace{\xi^0-\xi^3}_{\xi^-} \sim O(1/xM)$$



DIS $\Longleftrightarrow$ Light cone behaviour



What about quarks?



$$\Phi(k, P) = \sum_X \!\!\!\!\!\!\int \; \delta^{(4)}(P_X + k - P) \langle P | \bar\psi(0) | X \rangle \langle X | \psi(0) | P \rangle$$

$$= \int \frac{d^4 z}{(2\pi)^4} e^{ikz} \underbrace{\langle P | \bar\psi(z) \psi(0) | P \rangle}_{\text{contain all distributions}}$$

What is k?

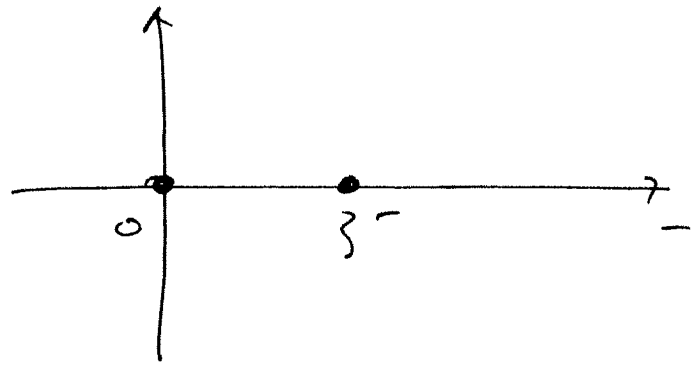
$$k^\mu = k^+ n_+^\mu + k^- n_-^\mu + \vec{k}_\perp^\mu$$

quark goes with the proton $\underbrace{k^+ = x P^+}$ light-cone fraction

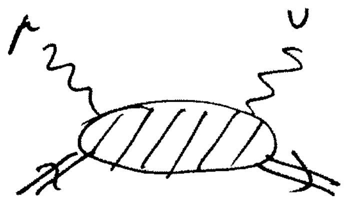
$$k^\mu = x P^+ n_+^\mu + \underbrace{\frac{+\vec{k}_\perp^2 + k^2}{2 x P^+}}_{\text{small}} n_-^\mu + \underbrace{\vec{k}_\perp^\mu}_{\text{may not be small}}$$

if we neglect k^- & $\vec{k}_\perp$ then $k^\mu \simeq x P^+ n_+^\mu$

and we recover collinear picture



Hadronic tensor



can be parametrized as

$$W^{\mu\nu} = -\left(g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2}\right) W_1(x,Q^2) + \left(p^\mu + \frac{q^\mu}{2x}\right)\left(p^\nu + \frac{q^\nu}{2x}\right) W_2(x,Q^2) \qquad (*)$$

(here we write only symmetric part of the tensor and do not take into account spin of the hadron)

Home work:

one can write
$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^\mu p^\nu + \frac{W_4}{M^2} q^\mu q^\nu + \frac{W_5}{M^2}\left(p^\mu q^\nu + q^\mu p^\nu\right) \qquad (**)$$

show that (*) can be derived from (**) using current conservation $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$.

Instead $W_{1,2}$ one usually introduces:

$$\begin{cases} F_1(x,Q^2) = W_1(x,Q^2) \\ F_2(x,Q^2) = \nu W_2(x,Q^2) \\ F_L(x,Q^2) = F_2 - 2x F_1 \end{cases}$$

Proton structure functions

Note that Z exchange & neutrino DIS is not included here!

Let us try to calculate those functions in parton model

We introduce vectors:

$$p^\mu = (P,0,0,P)$$

$$n^\mu = \left(\frac{1}{2P},0,0,-\frac{1}{2P}\right)$$

$$p^2 = n^2 = 0$$

$$p \cdot n = 1$$

Let's choose the frame such that

$$q^\mu = q_\perp^\mu + \nu n^\mu$$

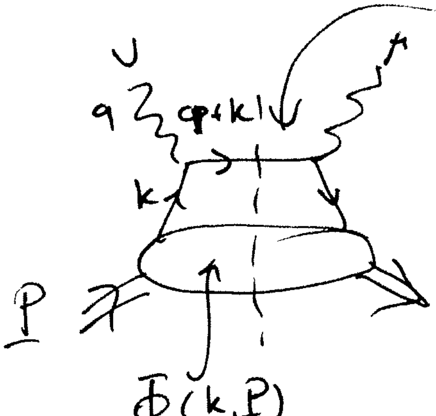
such that $P \cdot q = \nu$, $(q)^2 = (q_\perp^\mu)^2 = -\vec{q}_\perp^{\,2} = -Q^2$

Then

$$p^\mu p^\nu W_{\mu\nu} = -\frac{\nu^2}{Q^2} W_1 + \frac{\nu^2}{4x^2} W_2 = \frac{\nu}{4x^2} F_L$$

$$\underbrace{n^\mu n^\nu W_{\mu\nu}}_{\text{projections}} = W_2 = \frac{1}{\nu} F_2$$

$$\Rightarrow \quad \begin{cases} F_2 = \nu\, n^\mu n^\nu W_{\mu\nu} \\ F_L = \dfrac{4x^2}{\nu} p^\mu p^\nu W_{\mu\nu} \end{cases}$$



from quark propagator

$$W^{\mu\nu} = e_a^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left(\gamma^\mu (\not q + \not k) \gamma^\nu \Phi(k,P)\right) \times \delta\left((k+q)^2\right)$$

$$k^\mu = x\, p^\mu + \frac{k^2 + k_\perp^2}{2x} n^\mu + k_\perp^\mu$$

(note that here I use different set of light cone vectors with respect to page 16.)

$$\delta\left((k+q)^2\right) = \delta\left(k^2 - Q^2 + 2x\nu - 2\vec{k}_\perp \cdot \vec{q}_\perp\right)$$

$$\approx \delta\left(2x\nu - Q^2\right) = \frac{1}{2\nu}\, \delta\left(x - x_{Bj}\right)$$

$$F_2 = \nu\, n^\mu n^\nu W_{\mu\nu} = \frac{1}{2} e_q^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\big(\not n \underbrace{\underbrace{\not k \not n}_{-\not n \not k + 2n\cdot k} \Phi(k,P)\big)\delta(x - x_{Bj})}_{2x\, \mathrm{Tr}(\not n \Phi)}$$

We can define:

$$\underbrace{f(x) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\big(\not n\, \Phi(k,P)\big)\,\delta(x - x_{Bj})}_{\text{Parton distribution}}$$

$\Rightarrow$ $F_2(x) = e_q^2\, x\, f(x)$ or summing over quarks

$$F_2(x) = \sum_q e_q^2\, x \left(f(x) + \bar f(x)\right)$$

$F_2(x)$ depends <u>only</u> on x $\rightarrow$ Bjorken scaling!

now lets calculate

$$F_L = \frac{4x^2}{\nu} p^\mu p^\nu W_{\mu\nu} =$$

$$= \frac{4x^2}{2\nu^2} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left(\not{p} \underbrace{(\not{k}+\not{q}) \not{p}}_{-\not{p}(\not{k}+\not{q}) + 2\underbrace{p\cdot(k+q)}_{\frac{k^2+k_\perp^2}{x}+2\nu}} \Phi(k,P)\right) \delta(x - x_{Bj})$$

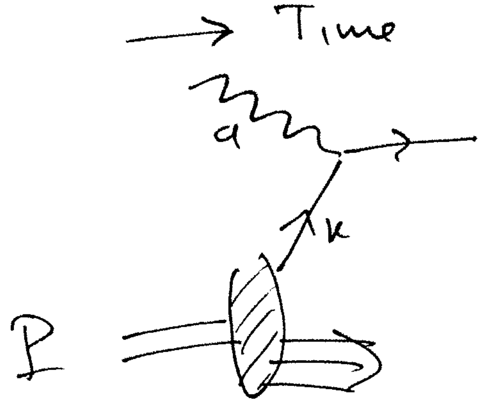
$$\simeq \frac{4x^2}{\nu} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left(\not{p}\, \Phi(k,P)\right) \delta(x - x_{Bj})$$

$\nu \to \infty \Rightarrow F_L$ is suppressed! $\Rightarrow$

$$\boxed{F_2 = 2x F_1}$$ Callan-Gross relation:

tells us that quarks are spin-½ fermions.

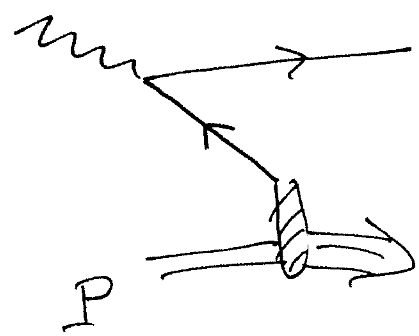
What do we actually measure in experiments?



Structure of the proton

?

or



Structure of vacuum fluctuations

Let's consider a plane wave fn's

$$e^{-ik\cdot\xi}, \quad k = (k^0, 0, 0, k_z)$$

$$k\cdot\xi = k^0\xi^0 - \vec{k}\cdot\vec{\xi} = k^0\xi^0 - k_z\xi^3 =$$

$$= \frac{(k^0+k^3)(\xi^0-\xi^3)}{2} + \frac{(k^0-k^3)(\xi^0+\xi^3)}{2}$$

$$k \cdot z = k^+ z^- + k^- z^+$$

if we define z^+ as our new time then

if $k^- \simeq 0$ then the "time" is "frozen" for this wave.

$z^- \to$ new spacial coordinate.

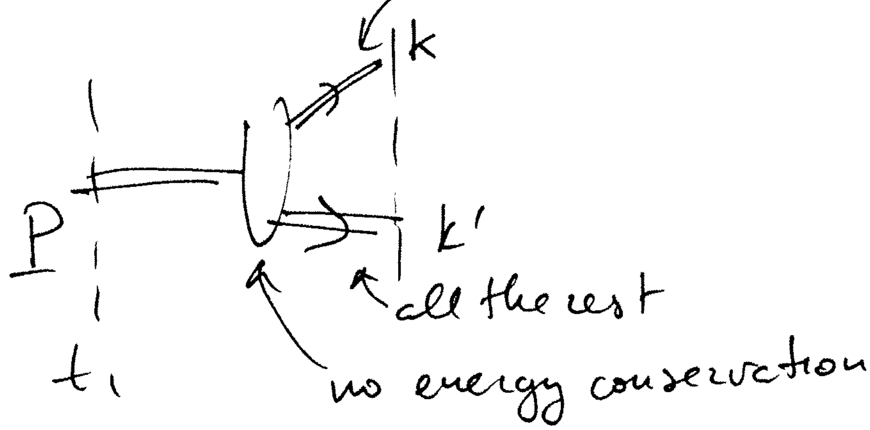
What is the advantage of Infinite Momentum Frame?

$$\underline{P}^\mu = \left(\sqrt{P_z^2 + M^2}, 0, 0, P_z \right) \quad , P_z \to \infty$$

$$\Rightarrow \underline{P}^\mu \simeq \left(P_z, 0, 0, P_z \right) \quad , \underline{\underline{\underline{P}^- = 0}}.$$

Let us evaluate characteristic times for vacuum fluctuations and partons in this frame

Partons :



$$k_3 = x P_3 \; , \qquad P_3 \approx P_0$$

$$k'_3 = (1-x) P_3 \, ,$$

$$k_0 = \sqrt{k_\perp^2 + (k_3)^2} \approx x P_3 \left(1 + \frac{1}{2} \frac{k_\perp^2}{x^2 P_3^2} \right)$$

$$k'_0 = \sqrt{k_\perp^2 + (k'_3)^2} \approx (1-x) P_3 \left(1 + \frac{1}{2} \frac{k_\perp^2}{(1-x)^2 P_3^2} \right)$$

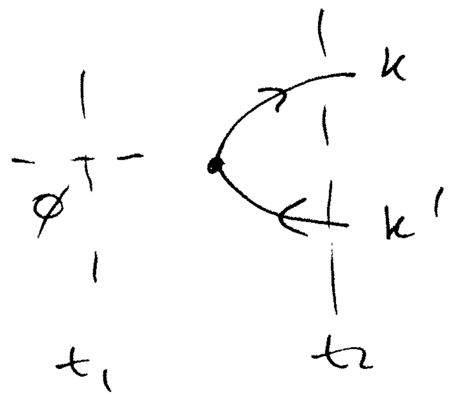
Energy at t_1 : $\quad E_1 = P_0$

Energy at t_2 : $\quad E_2 = k_0 + k'_0 = P_0 + \dfrac{k_\perp^2}{(1-x) x P_0}$

$$\Delta E = E_2 - E_1 = \frac{k_\perp^2}{(1-x) x P_0}$$

$$\Delta t \sim \frac{1}{\Delta E} = \frac{(1-x) x P_0}{k_\perp^2} \to \infty \text{ if } P_0 \to \infty$$

Vacuum fluctuation:



$k_3 = x P_3 \Rightarrow k_0 \simeq x P_3 \approx x P_0$

Energy at t_1: $\quad E_1 = 0$

Energy at t_2: $\quad E_2 > k_0 = x P_0$

$\Delta E = E_2 - E_1 = x P_0$

$\Rightarrow \Delta t \approx \dfrac{1}{\Delta E} = \dfrac{1}{x P_0} \to 0$ if $P_0 \to \infty$

$\Rightarrow$ quantum fluctuations are suppressed

and we study structure of the proton!