Now let us write $f(x,b_T,\mu,\zeta)$ in terms of $f(x,b_T,\mu_0,\zeta_0)$:

$$\tilde f(x,b_T,\mu,\zeta) = \tilde f(x,b_*,\mu,\zeta)\left[\frac{\tilde f(x,b_T,\mu,\zeta)}{\tilde f(x,b_*,\mu,\zeta)}\right] =$$

$$= \tilde f(x,b_*,\mu,\zeta_0)\exp\left[\tilde K(b_*,\mu)\ln\sqrt{\frac{\zeta}{\zeta_0}}\right]\underbrace{\left[\frac{\tilde f(x,b_T,\mu,\zeta_0)}{\tilde f(x,b_*,\mu,\zeta_0)}\right]}_{\exp[-g(x,b_T)]}$$

$$\cdot \exp\left[\ln\sqrt{\frac{\zeta}{\zeta_0}}\,\underbrace{(\tilde K(b_T,\mu)-\tilde K(b_*,\mu))}_{g_K(b_T)}\right]$$

np behav. of TMD

$$= \tilde f(x,b_*,\mu_0,\zeta_0)\exp\left[\int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\left(\gamma_F(\mu';1)-\ln\sqrt{\frac{\zeta_0}{\mu'^2}}\,\gamma_K(\mu')\right)\right]$$

$$\cdot \exp\left[\ln\sqrt{\frac{\zeta}{\zeta_0}}\,\tilde K(b_*,\mu_0) - \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\ln\sqrt{\frac{\zeta}{\zeta_0}}\,\gamma_K(\mu')\right]$$

$$\cdot \exp\left[-g(x,b_T)-\ln\sqrt{\frac{\zeta}{\zeta_0}}\,g_K(b_T)\right]$$

$$= \tilde f(x,b_*,\mu_0,\zeta_0)\exp\left[\ln\sqrt{\frac{\zeta}{\zeta_0}}\,\tilde K(b_*,\mu_0)+\int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F(\mu';1)-\ln\sqrt{\frac{\zeta}{\mu'^2}}\,\gamma_K(\mu')\right]\right]$$

$$\cdot \exp\left[-g(x,b_T)-\ln\sqrt{\frac{\zeta}{\zeta_0}}\,g_K(b_T)\right]$$

Let us use $\mu_0 = \mu_b = \frac{2e^{-\gamma_E}}{b_*}$

$$\zeta_0 = Q_0^2 \sim 1-2 \ (\mathrm{GeV}^2)$$

Then: , $\zeta = Q^2$ (the scale)

$$\tilde{f}(x, b_T, Q, Q^2) = \tilde{f}(x, b_*, \mu_b, Q_0^2) \left(\frac{Q}{Q_0}\right)^{\tilde{K}(b_*, \mu_b) - g_K(b)} \cdot \exp\left[\int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left(\gamma_F(\mu', 1) - \ln \frac{Q}{\mu'} \gamma_K(\mu') \right) \right] \cdot \exp\left[-g(x, b_T) \right]$$

$$\downarrow$$

Sudakov form factor

$$\exp[S]$$

Contains result of gluon radiation

$$\underbrace{\tilde{f}(x, b_T, Q, Q^2) = \tilde{f}(x, b_*, \mu_b, Q_0^2)\, e^{-g(x, b_T)}\, e^{S}}_{\text{almost like GPM!}}$$

$$\approx \tilde{f}(x, \mu_b)\, e^{-g(x, b_T)}\, e^{S} \quad \text{if} \begin{cases} g(x, b_T) \simeq \dfrac{b_T^2 \langle k_T^2 \rangle}{4} \\ S \simeq 0 \end{cases}$$

study in Mathematica.

We will use Mathematica to study how it differs from GPM at higher scales.