Translation operator

$$D(\vec{a}) = \underbrace{e^{-i\vec{a}\cdot\hat{P}}}_{\text{check the definition}}$$

$\hat{P}$ - total momentum of the system

if $\quad D(a)^{\dagger} H D(a) = H \;\rightarrow$ dynamics is translationally invariant

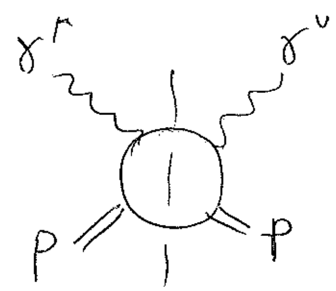
Time displacement

$$\mathcal{U}(t_0) = e^{i t_0 H}$$

Rotation

$$R(\vec{\chi}) = e^{-\vec{\chi}\,\vec{I}}$$

, $\vec{I}$ - total angular momentum



$$W^{\mu\nu} = \frac{1}{2\pi}\sum_{X}(2\pi)^4\,\delta^{(4)}(P+q-P_X)\,\langle P|J^{\mu}(0)|X\rangle\langle X|J^{\nu}(0)|P\rangle$$

$$\delta^{(4)}(P+q-P_X) = \underbrace{\int\frac{d^4z}{(2\pi)^4}\,e^{+iz\cdot(P+q-P_X)}}_{\text{check the definition!}}$$

$$W^{\mu\nu} = \frac{1}{2\pi} \sum_X \int d^4z \; e^{iz\cdot q} \langle P | \underbrace{e^{izP} J^\mu(0) e^{-izP_X}}_{J^\mu(z)} | X \rangle$$

$$\langle X | J^\nu(0) | P \rangle$$

$$= \underbrace{\frac{1}{2\pi} \cancel{\sum_X} \int d^4z \; e^{iz\cdot q} \langle P | J^\mu(z) J^\nu(0) | P \rangle}$$

otherwise we can also define

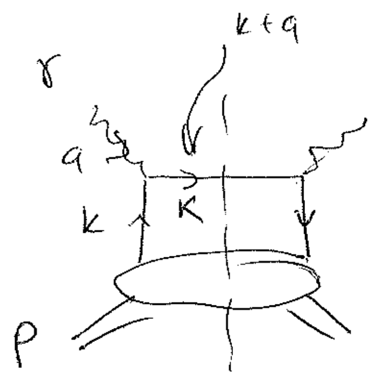
$$W^{\mu\nu} = \frac{1}{2\pi} \sum_X \int d^4z \; e^{izq} \langle P | J^\mu(0) | X \rangle \langle X | \underbrace{e^{-izP_X} J^\nu(0) e^{izP}}_{J^\nu(-z)} | P \rangle$$

$$= \frac{1}{2\pi} \cancel{\sum_X} \int d^4z \; e^{izq} \langle P | J^\mu(0) J^\nu(-z) | P \rangle \quad = (z' = -z)$$

$$= \frac{1}{2\pi} \int d^4z' \; e^{-iz'q} \langle P | J^\mu(0) J^\nu(z') | P \rangle$$

$$= \underbrace{\frac{1}{2\pi} \int d^4z \; e^{-izq} \langle P | J^\mu(0) J^\nu(z) | P \rangle}$$

Let us compare with Bacchetta et al.



$$W^{\mu\nu} = \frac{1}{2\pi} \sum_a e_a^2 \sum_X \frac{d^4k}{(2\pi)^4} \frac{d^4K}{(2\pi)^4} \delta(K^2)$$

$$\left(\bar{u}(k)\gamma^\mu \langle P| \bar{\Psi}(0) | P_X X\rangle\right)\left(\langle P_X X | \Psi(0) | PS\rangle\right) \bar{u}(k)\gamma^\nu \Big)$$

$$(2\pi)^4 \delta^{(4)}(P-k-P_X)\,(2\pi)^4(k+q-K)$$

$$\Phi_{ij}(k,P,S) = \sum_X (2\pi)^4 \delta^{(4)}(P-k-P_X) \langle PS| \psi_j(0) |X\rangle\langle X| \psi_i(0) |P\rangle$$

$$\delta^{(4)}(P-k-P_X) = \int \frac{dz}{(2\pi)^4} e^{iz(P-k-P_X)}$$

$$= (2\pi)^4 \sum_X \int \frac{dz}{(2\pi)^4} e^{-izk} \langle PS| \psi_j(0) |X\rangle \langle X| \psi_i(-z) |P\rangle$$

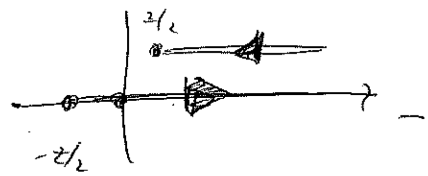
$$= \underbrace{\int dz\, e^{izk} \langle PS| \psi_j(0)\, \psi_i(z) |P\rangle}$$

as in Bacchetta et al!

$$\langle k_T^i \rangle = \int d^2k_T\, k_T^i\, \Phi(x, \vec{k}_T, S) = \frac{1}{2}\int d^2k_T\, k_T^i \big(\Phi(x, \vec{k}_T, \vec{S})$$

$$- \Phi(x, k_T, -\vec{S})\big)$$

Φ can be also written as



$$\Phi = \int \frac{d^4z}{(2\pi)^4} e^{ikz} \langle P | \bar{\Psi}(-z/2)\gamma^+ W(-z/2)(z/2)\, \Psi(z/2) | P\rangle \Big|_{z^+=0}$$

$$= \int \frac{dz^-}{2\pi} \frac{dz_\perp}{(2\pi)^2} e^{ik^+z^- - i\vec{k}_\perp \vec{z}_T} \langle P, -S | \bar{\Psi}(-z/2)\, \gamma^+ W(-z/2, z/2)\Psi(z/2)|P, S\rangle$$

Parity and time reversal

$$= \int \frac{dz^-}{(2\pi)} \frac{dz_\perp}{(2\pi)^2} e^{ik^+z^- - i\vec{k}_\perp \vec{z}_\perp} \langle P, S | \bar{\Psi}(-z/2)\, \gamma^+ W_{+\infty}(-z/2, z/2)\Psi(z/2)|P, S\rangle$$

$$\Rightarrow \langle k_T^i \rangle = \frac{1}{2}\int d^2k_\perp\, k_T^i \int \frac{dz^-}{(2\pi)} \frac{dz_\perp}{(2\pi)^2} e^{ik^+z^- - i\vec{k}_\perp \vec{z}_\perp}$$

$$\langle P, S | \bar{\Psi}(-z/2)\, \gamma^+ \big(W_{-\infty}(-z/2, z/2) - W_{+\infty}(-z/2, z/2)\big)\Psi(z/2)|P, S\rangle$$

$$k_T^i = i\frac{\partial}{\partial z_i} e^{ikz}, \quad \int \frac{d^2k_\perp}{(2\pi)^2} e^{-i\vec{k}_\perp \vec{z}_\perp} = \delta^{(2)}(z_\perp)$$

$$\langle k_T^i \rangle = \frac{i}{2} \int \frac{dz^-}{2\pi} e^{ikz} \frac{\partial}{\partial z_i}\big(\langle P, S | \bar{\Psi}(-z/2)\gamma^+$$

$$\times \big(W_{+\infty}(-z/2, z/2) - W_{-\infty}(-z/2, z/2)\big)\, \Psi(z/2)|P, S\rangle \Big|_{\substack{z^+=0 \\ \vec{z}_\perp = \vec{0}}}$$

Note that in Ji we have,

$$\underbrace{\phi \propto \int dz\, e^{-izk} \langle PS| \psi_j(z)\, \psi(0) |P\rangle}$$

The same object!

We can use

$$\phi = \int \frac{d^4z}{(2\pi)^4} e^{ikz} \langle P| \bar{\psi}_j(0) W(0,z)\, \psi_i(z) |P,S\rangle$$

as definition.

From M. Burkardt

$$\phi = \int \frac{d^4z}{(2\pi)^4} e^{ikz} \langle P| \psi_j(0)\, W(0,z)\, \psi_i(z) |P,S\rangle$$

$$W(0,z) = \mathcal{P} \exp\left[ig \int_{x[C]}^{y} dz^\mu A_\mu^a(z) t^a \right]$$

Gluon field strength tensor

$$\underbrace{F_a^{\mu\nu} = \partial^\mu A_a^\mu(x) - \partial^\nu A_a^\mu(x) + g f_{abc} A_b^\mu(x) A_c^\nu(x)}$$

Definition by Ji:

$$F = \frac{1}{2}\int \frac{d\lambda}{2\pi}\, e^{i\lambda x} \langle P' |\, \bar{\Psi}\left(-\frac{\lambda}{2}n\right) \gamma\, P e^{ig\int_{\lambda/2}^{-\lambda/2} d\alpha\, n\cdot A(\alpha n)}$$

$$\Psi\left(\frac{\lambda n}{2}\right) |P\rangle =$$

$$= H\, \frac{1}{2}\bar{u}(p')\, \not{n}\, u(p) + E\, \frac{1}{2}\bar{u}(p')\, \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M}\, u(P)$$

$$n\cdot A(\alpha n) = A_+(\alpha n)$$

$$\int_{\lambda/2}^{-\lambda/2} d\alpha\, A_+(\alpha n) = F(-\lambda/2) - F(\lambda/2)$$

$$\frac{d}{d\lambda}\left(\quad\right) = F'(-\lambda/2)\Big| - F'(\lambda/2).$$

$$= A_+(-\lambda/2\cdot n) - A_+(\lambda/2\cdot n)$$