Consider TMD $\tilde{f}_{a/N}(x, b_T; \overset{\overset{\zeta}{\downarrow}}{Q^2}, \overset{\overset{Q \text{ or } \mu \text{ scale}}{\downarrow}}{\mu_Q})$

one can write a solution of TMD evolution equations as

$$\tilde{f}_{a/N}(x, b_T; Q^2, \mu_Q) = \overbrace{\tilde{f}_{a/N}(x, b_T; Q_0^2, \mu_0)}^{Q_0 \text{ scale TMD}} \times$$

$$\times\, e^{-S(b_T, Q, \mu_Q, Q_0, \mu_0)/2}$$

where

$$S(b_T, Q, \mu_Q, Q_0, \mu_0) = -\tilde{K}(b_T, \mu_0) \ln \frac{Q^2}{Q_0^2} +$$

$$+ \int_{\mu_0}^{\mu_Q} \frac{d\mu'}{\mu'} \left(-2\gamma_j(\alpha_s(\mu'); 1) - \ln \frac{Q^2}{\mu'^2} \gamma_K(\alpha_s(\mu')) \right)$$

$e^{-S/2}$ gives important effect of gluon radiation

Now for DY process $AB \to e^+e^- X$ one has

$$\frac{d\sigma}{d^4q\, d\Omega} = \frac{2}{s} \sum_j \frac{d\hat{\sigma}_{j\bar{j}}(Q, \mu_Q, \alpha_s(\mu_Q))}{d\Omega}$$

$$\times \int d^2 b_T\, e^{i\bar{q}_T \cdot \bar{b}_T}\, \tilde{f}_{j/A}(x_A, b_T; Q_0^2, \mu_0)\, \tilde{f}_{\bar{j}/B}(x_B, b_T; Q_0^2, \mu_0)$$

$$\times \left(\frac{Q^2}{Q_0^2}\right)^{\tilde{K}(b_T, \mu_0)} \underbrace{\exp\left\{ \int_{\mu_0}^{\mu_Q} \frac{d\mu'}{\mu'} \left[2\gamma_j(\alpha_s(\mu'); 1) - \ln \frac{Q^2}{\mu'^2} \gamma_K(\alpha_s(\mu')) \right] \right\}}_{S_{pert}}$$

If μ' is large enough then γ_J and γ_K can be calculated perturbatively.

Suppose $\mu_0^2 \simeq 2.4\,(\mathrm{GeV})^2$, then

$$\gamma_K = 2\, C_F \frac{\alpha_s}{\pi}, \quad \gamma_J = \frac{3\, C_F}{2} \frac{\alpha_s}{\pi}$$

and

$$S_{pert} = 2\, C_F \int_{\mu_0}^{\mu_Q} \frac{d\mu'}{\mu'} \frac{\alpha_s(\mu')}{\pi} \left[\ln \frac{Q^2}{(\mu')^2} - 3/2 \right]$$

using $\alpha_s = \dfrac{1}{\beta_0 \ln Q^2/\Lambda^2}$, $\beta_0 = \dfrac{33 - 2n_f}{12\pi}$, $C = 4/3$

(and $n_f = 3$, $\Lambda = 0.25\,(\mathrm{GeV})$) we get

$$S_{pert} = -\frac{2 C_F}{\pi \beta_0} \left[\ln \frac{Q}{Q_0} - \ln \frac{Q}{\Lambda} \ln\left(\frac{\ln Q/\Lambda}{\ln Q_0/\Lambda}\right) + \frac{3}{4} \ln \left(\frac{\ln Q/\Lambda}{\ln Q_0/\Lambda} \right) \right]$$

Notice that S_{pert} does not depend on b_T and thus do not contribute to the "widening" of TMDs.

$\tilde{K}(b_T, \mu_0)$ does depend on b_T and thus contribute to widening.

$$\underbrace{f_{J/A}(x_A, b_T; Q_0^2, \mu_0)\, e^{\tilde{K}(b_T,\mu_0)\ln\frac{Q}{Q_0}}}$$

this is the TMD at scale Q in b_T space.

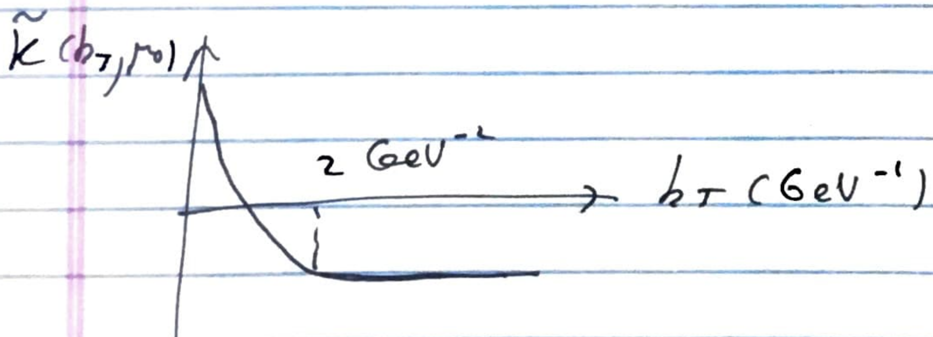
commonly used $f_{J/A}(x_A, b_T; Q_0^2, \mu_0^2) \simeq f_{J/A}(x_A, Q_0^2)\cdot e^{-b_T^2 \frac{\langle k_{TA}^2 \rangle}{4}}$

$\tilde{K}(b_T, \mu_0)$ has perturbative expansion if $b_T \to 0$:

$$\tilde{K}(b_T, \mu_0) = -\frac{\alpha_s(\mu_0)}{\pi}\left[\ln\frac{b_T^2 \mu_0^2}{4} + 2\gamma_E\right] + \mathcal{O}(\alpha_s^2)$$

but this approximation fails when b_T is <u>large</u>

Rogers, Collins PRD 91 hypothesize $\tilde{K}(b_T) \to$ <u>const</u> at large b_T



They also provide a simple formula for $\tilde{K}$ (Eq(66))

$$\tilde{K}(b_T, \mu_b) = \tilde{K}(b_*, \mu_b) - g_K(b_T, b_{max})$$

$$\tilde{K}(b_T, \mu_0) = \tilde{K}(b_T, \mu_b) - \int_{\mu_{b*}}^{\mu_0} \frac{d\mu'}{\mu'} \gamma_K(\alpha_s(\mu')) \quad \text{(Eq(64))}$$

Thus:

$$\tilde{K}(b_T, \mu_0) = \tilde{K}(b_*, \mu_b) - g_K(b_T, b_{max}) - \int_{\mu_{b_*}}^{\mu_0} \frac{d\mu'}{\mu'} \gamma_K(\alpha_s(\mu'))$$

$$b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \ , \ \mu_b = \frac{2e^{-\gamma_E}}{b_*}$$

we will choose $b_{max} = 1 \ (GeV^{-1})$

now g_K proposed by Rogers, Collins is

$$g_K(b_T, b_{max}) = g_0\left(1 - \exp\left[-\frac{C_F \alpha_s(\mu_{b_*}) b_T^2}{\pi g_0 b_{max}^2}\right]\right)$$

$g_0 \simeq 0.3$

Now we put all together in Mathematica file:

$$\int_{\mu_{b*}}^{\mu_0} \frac{d\mu'}{\mu'} \gamma_K = \int_{\mu_{b*}}^{\mu_0} \frac{d\mu'}{\mu'} 2 C_F \frac{\alpha_s(\mu')}{\pi} =$$

$$= \frac{2 C_F}{\pi} \int_{\mu_{b*}}^{\mu_0} \frac{d\mu'}{\mu'} \frac{1}{\beta_0 \ln \mu'^2/\Lambda^2} =$$

$$= \frac{2 C_F}{2 \pi \beta_0} \int_{\ln \mu_{b*}}^{\ln \mu_0} \frac{d \ln \mu'}{\ln \mu' - \ln \Lambda} = \frac{C_F}{\pi \beta_0} \int_{\ln \mu_{b*}/\Lambda}^{\ln \mu_0/\Lambda} \frac{d \ln \mu'/\Lambda}{\ln \mu'/\Lambda}$$

$$= \frac{C_F}{\pi \beta_0} \ln \left(\frac{\ln \mu_0/\Lambda}{\ln \mu_{b*}/\Lambda} \right)$$

To reconstruct $f(x, k_T)$ we will use

$$f(x, k_T) = \int \frac{db\, b}{2\pi} J_0(k_\perp b) \tilde{f}(x, b)$$