① 11/20/17 L.G.

Summary Compass Multiplicity definition

$$F_{UU,T}(x, z, P_{h\perp}^2) = \sum_a e_a^2 \int d^2k_\perp\, d^2P_\perp\, \delta^{(2)}(P_{h\perp} - zk_\perp - P_\perp)\, f_a(x,k_\perp)\, D_a(z,P_\perp)$$

Definition of Structure function from Anselmino et al JHEP (14) (versus Bacchetta et al JHEP (07)

$$F_{UU,T} = x \cdot \sum_a e^2(\ldots) \text{ etc} \ldots \;)$$

Gauss Model,

$$f_a(x, k_\perp^2) = \frac{1}{\pi\langle k_\perp^2\rangle_a} e^{-k_\perp^2/\langle k_a^2\rangle_a} f_a(x)$$

$$D_a(z, P_\perp^2) = \frac{1}{\pi\langle P_\perp^2\rangle_a} e^{-P_\perp^2/\langle P_\perp^2\rangle_a} D_a(z)$$

$\rightarrow$ Structure function,

$$F_{UU,T}(x, z, P_{h\perp}^2) = \sum_a e_a^2 \frac{e^{-P_{h\perp}^2/\langle P_{h\perp}^2\rangle_a}}{\pi\langle P_{h\perp}^2\rangle_a} f_a(x)\, D_a(z)$$

where,

$$\langle P_{h\perp}^2\rangle_a = z^2\langle k_\perp^2\rangle_a + \langle P_\perp^2\rangle_a$$

Cross Section, unpolarized

$$\frac{d^4\sigma}{dx\,dz\,dQ^2\,dP_{h\perp}^2} = \frac{2\pi^2\alpha_{em}^2}{Q^4}\left(1+(1-y)^2\right) F_{UU,T}(x, z, P_{h\perp}^2)$$

note, $Q^4 = (xys)^2$

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Using $\sigma_0 = \dfrac{2\pi\alpha_{em}^2}{Q^2}\dfrac{(1+(1-y)^2)}{y}$,

& $Q^2 = xyS$,

$$\frac{d^4\sigma}{dx\,dz\,dQ^2\,dP_{h\perp}^2} = \frac{\pi}{xS}\sigma_0 F_{UU,T}(x, z, P_{h\perp}^2)$$

Compass definition of multiplicity,

$$\frac{d^2 M(x, z, P_{h\perp}^2, Q^2)}{dz\,dP_{h\perp}^2} \equiv \frac{d^4\sigma^{SIDIS}}{d^2\sigma^{DIS}}$$

$$= \frac{\left(d^4\sigma/(dx\,dz\,dQ^2\,dP_{h\perp}^2)\right)}{\left(d^2\sigma/(dx\,dQ^2)\right)}$$

where

$$d^2\sigma^{DIS} = \frac{\sigma_0}{xS}\sum_a f_a(x) \quad \text{(parton model)}$$

and,

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So

compass
$$\frac{d^2M(x,z,Q^2,P_{h\perp}^2)}{dz\,dP_{h\perp}^2} = \frac{\cancel{\pi}\sum_a e_a^2 \frac{e^{-P_{h\perp}^2/\langle P_{h\perp}^2\rangle_a}}{\cancel{\pi}\langle P_{h\perp}^2\rangle_a} f_a(x) D_a(z)}{\sum_a e_a^2 f_a(x)}$$

note no factor of $P_{h\perp}$ no factor of "2" vs. HERMES case.

HERMES
$$\frac{d^2M(x,z,Q^2,P_{h\perp}^2)}{dz\,dP_{h\perp}} = 2P_{h\perp}\cdot\frac{\sum_a e_a^2 \frac{e^{-P_{h\perp}^2/\langle P_{h\perp}^2\rangle_a}}{\langle P_{h\perp}^2\rangle_a} f_a(x) D_a(z)}{\sum_a e_a^2 f_a(x)}$$

The conventions are covered on the next page

See → Anselmino et al JHEP 2015
→ Bacchetta et al JHEP 2007
note, * Boglione & Prokudin in EPJA 2016
use the conventions of Bacchetta et al.?

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Some details for unpolarized Cross Section
Anselmino & Bacchetta Convention:

$$d^5\sigma \equiv \frac{d^5\sigma}{dx\,dy\,dz\,dP_{h\perp}^2\,d\phi_h} = \int d\psi \frac{d^6\sigma}{dx\,dy\,dz\,dP_{h\perp}^2\,d\phi_h\,d\psi}$$

Bacchetta JHEP(07)

$$= \frac{2\pi\alpha_{em}^2}{xyQ^2}\underbrace{\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)}_{\left(1-y+\frac{y^2}{2}\right)=\frac{1+(1-y)^2}{2}} F_{UU,T}(x,z,P_{h\perp}^2)$$

$$\frac{d^4\sigma}{dx\,dy\,dz\,dP_{h\perp}^2} = \int d^5\sigma\,d\phi_h$$

$$= \frac{2\pi^2\alpha_{em}^2}{xyQ^2}\left(1+(1-y)^2\right) F_{UU,T}(x,z,P_{h\perp}^2)$$

&

$$\frac{d^4\sigma}{dx\,dz\,dQ^2\,dP_{h\perp}^2} = \frac{2\pi^2\alpha_{em}^2}{(xS)\,xyQ^2}\left(1+(1-y)^2\right) F_{UU,T}(x,z,P_{h\perp}^2)$$

where,

$$F_{UU,T} = x\sum_a e_a^2\int d^2p_\perp\,d^2k_\perp\, f_a(x,p_\perp^2)\,D_a(z,z^2k_\perp^2)$$

$$\cdot\,\delta^{(2)}\!\left(P_{h\perp}/z-(p_\perp-k_\perp)\right)$$

note

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$$= \frac{2\pi^2\alpha^2}{xs\; xy\; xys}\left(1+(1-y)^2\right) F_{uu,T}$$

$$= \pi\,\frac{2\pi\alpha^2}{(xs)^2}\,\frac{(1+(1-y)^2)}{y^2}\,F_{uu,T}$$

$$= \pi\,\frac{1}{x}\,\underbrace{\frac{2\pi\alpha^2}{xys}}_{Q^2}\,\frac{1}{\boxed{xs}}\,\frac{(1+(1-y)^2)}{y}\,F_{uu,T}$$

$$= \pi\,\frac{1}{x}\,\frac{1}{xs}\,\underbrace{\frac{2\pi\alpha^2}{Q^2}\,\frac{(1+(1-y)^2)}{y}}_{\sigma_0}\,F_{uu,T}$$

$$= \frac{\pi}{xs}\,\frac{1}{x}\,\sigma_0\,F_{uu,T}^{\text{Bacchetta}}$$

$$= \frac{\pi}{xs}\,\sigma_0\,F_{uu,T}^{\text{Anselmino}}$$

ok consistent

Note, Boglione & Prokudin in EPJA 52 (16) use Bacchetta convention, $F_{uu,T} = x\sum_a e_a^2 \ldots$, Anselmino et al. including Boglione & Prokudin use $F_{uu,T} = \sum_a e_a^2 \ldots$ what a mess ...!

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Note:

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{(xs)^2}\frac{(1+(1-y)^2)}{y^2}\sum_a e_a^2 f_a(x)$$

$$\sigma_0 \equiv \frac{2\pi\alpha^2}{Q^2}\frac{(1+(1-y)^2)}{y}$$

$$= \frac{1}{xs}\left(\frac{2\pi\alpha^2}{xys}\frac{(1+(1-y)^2)}{y}\right)\sum_a e_a^2 f_a(x)$$

$$= \frac{1}{xs}\left(\frac{2\pi\alpha^2}{Q^2}\frac{(1+(1-y)^2)}{y}\right)\sum_a e_a^2 f_a(x)$$

$$= \frac{\sigma_0}{xs}\sum_a e_a^2 f_a(x)$$