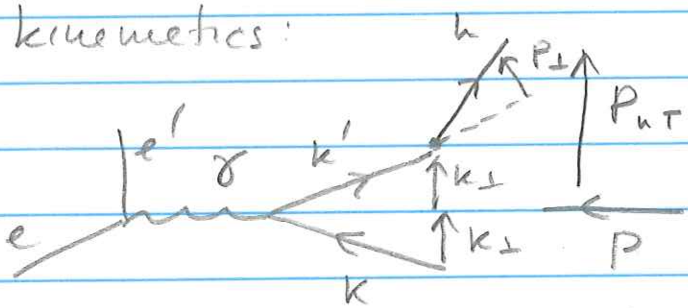
For John, notes on TMDs in b space

In TMD formalism all structure functions can be expressed in terms of convolutions of TMDs. For instance, the unpolarised SIDIS structure function F_{uu} is

$$F_{uu} = C[f_1 D_1] \text{ where}$$

$$C[\,] \equiv x \sum_a e_a^2\, d^2k_\perp\, d^2p_\perp\, \delta^{(2)}(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{hT})\, f_1^a(x, k_\perp^2)\, D_1(z, p_\perp^2)$$

and kinematics:



$$P_h \simeq z k' + p_\perp \Rightarrow \vec{P}_{hT} \simeq z\vec{k}_\perp + \vec{p}_\perp$$

now let us write F_{uu} in b space:

$$F_{uu} = x \sum_a e_a^2\, d^2k_\perp\, d^2p_\perp \frac{1}{z^2} \underbrace{\delta^{(2)}\left(\vec{k}_\perp + \frac{\vec{p}_\perp}{z} - \frac{\vec{P}_{hT}}{z}\right)}_{\int \frac{d^2b}{(2\pi)^2} e^{i\vec{b}\left(\vec{k}_\perp + \frac{\vec{p}_\perp}{z} - \frac{\vec{P}_{hT}}{z}\right)}} f_1(x, k_\perp^2)\, D_1(z, p_\perp^2)$$

$$F_{uu} = x \sum_a e_a^2 \int d^2b\, e^{-i\vec{b}\vec{P}_{hT}/z} \int \frac{d^2k_\perp}{2\pi} e^{i\vec{b}\vec{k}_\perp} f_1(x, k_\perp^2) \int \frac{d^2p_\perp}{2\pi z^2} e^{i\vec{b}\vec{p}_\perp/z} D_1(z, p_\perp^2)$$

Now let me <u>define</u>

$$\tilde{f}_1(x, b^2) \equiv \int \frac{d^2 k_\perp}{2\pi} e^{i\vec{b}\vec{k}_\perp} f_1(x, k_\perp^2)$$

$$\tilde{D}_1(z, b^2) \equiv \int \frac{d^2 p_\perp}{2\pi z^2} e^{i\vec{b}\vec{P}_\perp/z} D_1(z, p_\perp^2)$$

notice that I have $1/2\pi$ in the definition which is an unusual choice for F.T. but I do it in order to have symmetry in formulas for F.T. and anti-F.T

$$\tilde{f}_1(x, b^2) = \int \frac{k_\perp dk_\perp}{2\pi} \underbrace{\int d\varphi\, e^{ibk_\perp \cos\varphi}}_{2\pi J_0(k_\perp b)} f_1(x, k_\perp^2) =$$

$$= \int k_\perp dk_\perp J_0(k_\perp b) f_1(x, k_\perp^2)$$

$$\tilde{D}_1(z, b^2) = \int p_\perp dp_\perp J_0\left(\frac{p_\perp b}{z}\right) \frac{1}{z^2} D_1(z, p_\perp^2)$$

and we finally have

$$F_{UU} = 2\pi x \sum_a e_a^2 \int b\, db\, J_0\left(\frac{b P_{hT}}{z}\right) \tilde{f}_1(x, b^2) \tilde{D}_1(z, b^2)$$

How do we parametrise f_1 and D_1?
Usual choice is gaussian parametrisation

$$f_1(x,k_\perp) = f_1(x)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

(notice $\int d^2k_\perp\, f_1(x,k_\perp^2) = f_1(x) \leftarrow$ collinear PDF)

$$\text{so,}\quad \tilde{f}(x,b^2) = \frac{f_1(x)}{2\pi}\,e^{-\frac{b^2\langle k_\perp^2\rangle}{4}}$$

$$D_1(z,p_\perp^2) = D_1(z)\,\frac{1}{\pi\langle p_\perp^2\rangle}\,e^{-p_\perp^2/\langle p_\perp^2\rangle}$$

$$\text{so,}\quad \tilde{D}_1(z,b^2) = \frac{D_1(z)}{2\pi z^2}\,e^{-\frac{b^2\langle p_\perp^2\rangle}{4z^2}}$$

It is known that $\tilde{f}_1$ and $\tilde{D}_1$ obey Collins-Soper evolution equations, so that the Sudakov form factor should be added to above formulae, i.e

$$\tilde{f}_1(x,b^2) = \frac{f_1(x)}{2\pi}\,e^{-\frac{b^2\langle k_\perp^2\rangle}{4}}\,e^{-S_{sud}(b)/2}$$

$$\tilde{D}_1(z,b^2) = \frac{D_1(z)}{2\pi z^2}\,e^{-\frac{b^2\langle p_\perp^2\rangle}{4z^2}}\,e^{-S_{sud}(b)/2}$$

(notice that Sudakov is partitioned ½ & ½ between $\tilde{f}_1$ and $\tilde{D}_1$)

$S_{sud}(b)$ can be calculated perturbatively

$$S_{sud}(b) \equiv \int_{\mu_b}^{Q} \frac{d\mu^2}{\mu^2}\left(\underset{\downarrow}{A} \ln Q^2/\mu^2 + \underset{|}{B}\right)$$

$$\sum\left(\frac{\alpha_s}{2\pi}\right)^n A^{(n)} \qquad \sum\left(\frac{\alpha_s}{2\pi}\right)^n B^{(n)}$$

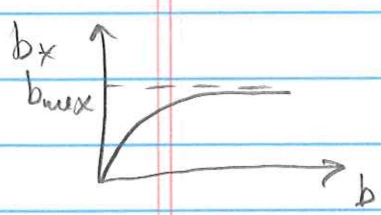
$$\mu_b = \frac{b_0}{b}, \quad b_0 = 2e^{-\gamma_E}$$

(notice that it also means that $f_1(x)$ and $D_1(z)$ should be evaluated at the scale of μ_b)

The problem is now with $b \to \infty$ as $\mu_b \to 0$ and we will have to deal with the Landau pole in α_s or $\equiv$ non-perturbative physics.

A usual solution is to introduce a b_* prescription

$b_*(b)$ is a smooth function that maps all b onto $[0, b_{max}]$ region. A possible form



is $$b_*(b) = \frac{b}{\sqrt{1 + b^2/b_{max}^2}}$$

$$b_*(0) = 0, \quad b_*(\infty) = b_{max}$$

One can now write

$$e^{-S_{sud}(b)} = e^{-S_{sud}(b_*)} \underbrace{e^{-S_{NP}(b)}}_{\text{non perturbative Sudakov formfactor}}$$

We know that $S_{NP}(0) = 0$ and a possible parametrisation will be

$$S_{NP}(b) = g_2 \ln(b/b_*) \ln Q/Q_0$$

g_2: a parameter

$b_* = \sqrt{1 + b^2/b_{max}^2}$

$$S_{sud}(b_*) = \int_{\mu_{b_*}}^{Q} \frac{d\mu^2}{\mu^2} \left(A \ln Q^2/\mu^2 + B\right)$$

$$\mu_{b*} = \frac{b_0}{b_*}$$

So that

$$\tilde{f}_1(x, b^2) = \frac{f_1(x, \mu_{b_*}^2)}{2\pi} e^{-\frac{b^2\langle k_\perp^2\rangle}{4} - \frac{g_2}{2} \ln b/b_* \ln Q/Q_0} e^{-\frac{S_{sud}(b_*)}{2}}$$

$$\tilde{D}_1(z, b^2) = \frac{D_1(z, \mu_{b_*}^2)}{2\pi\, z^2} e^{-\frac{b^2\langle p_\perp^2\rangle}{4z^2} - \frac{g_2}{2} \ln \frac{b}{b_*} \ln \frac{Q}{Q_0}} e^{-\frac{S_{sud}(b_*)}{2}}$$

$\langle k_\perp^2\rangle$, $\langle p_\perp^2\rangle$, g_2 are free parameters

Now lets talk of the data

We have multiplicities from Hermes & Compass

if we define $F_2 \equiv x \sum_q e_q^2 f(x, Q^2)$

Hermes measures $M_n^h(x, Q^2, z, P_{hT}) \equiv \frac{1}{\frac{d\sigma^{DIS}}{dx\, dQ^2}} \frac{d^4\sigma}{dx\, dQ^2\, dz\, dP_{hT}}$

and

$$M_n^h(x, Q^2, z, P_{hT}) = 2\pi P_{hT} \frac{F_{UU}}{F_2}$$

compass measures $\frac{d^2 n^h(x, Q^2, z, P_{hT}^2)}{dz\, dP_{hT}} = \pi \frac{F_{UU}}{F_2}$

The measurement is at low Q thus there should be little phase space for $S_{NP}(b_*)$ and we will drop it assuming $S_{NP}(b_*) \simeq 0$ (at this energy)

We have a program that fits the data, what we will need is a program that will perform numerically the b integration, see next page

$$F_{UU}(P_{hT}) \equiv 2\pi x \sum_a e_a^2 \int b\, db\, J_0\left(\frac{b P_{hT}}{z}\right) \tilde{f}_1(x,b^2)\tilde{D}_1(z,b^2)$$

where

$$\tilde{f}_1(x,b^2) = \frac{f_1(x,\mu_{b*})}{2\pi} e^{-\frac{b^2\langle k_\perp^2\rangle}{4} - \frac{g_2}{2}\ln\frac{b}{b_*}\ln\frac{Q}{Q_0}}$$

$$\tilde{D}_1(z,b^2) = \frac{D_1(z,\mu_{b*})}{2\pi z^2} e^{-\frac{b^2\langle p_\perp^2\rangle}{4z^2} - \frac{g_2}{2}\ln\frac{b}{b_*}\ln\frac{Q}{Q_0}}$$

$$b_* = \frac{b}{\sqrt{1 + b^2/b_{max}^2}}, \quad \underbrace{b_{max} = 1\ (\mathrm{GeV}^{-1})}_{\text{parameter}}$$

$$\mu_{b*} = \frac{b_0}{b_*}, \quad b_0 = 2e^{-\gamma_E}$$

In total we will have

$\langle k_\perp^2\rangle$, $\langle p_\perp^2\rangle$, b_{max}, g_2 $\rightarrow$ 4 parameters

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can be flavour dependent