In this project we would like to study the shape of helicity TMD distribution $g_1(x, k_T^2)$.
We would like to find out if the width of this distribution is different from that of unpolarised $f_1(x, k_T^2)$

The relevant part of the lepton-proton cross-section reads

$$\frac{d\sigma(S_{\parallel}, \lambda_e)}{dx\,dQ^2 dz\, dP_T^2\, d\phi} = 2\pi \frac{1}{sx}\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)\left\{F_{UU} + S_{\parallel}\lambda_e\sqrt{1-\epsilon^2}F_{LL}\right\}$$

where

$$\frac{y^2}{2(1-\epsilon)} = \frac{1}{1+\gamma^2}\left(1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2 y^2\right) \simeq \left(1-y+\frac{1}{2}y^2\right) = \frac{1}{2}\left(1+(1-y)^2\right)$$

$$\frac{y^2}{2(1-\epsilon)}\sqrt{1-\epsilon^2} \simeq y\left(1-y/2\right) = \frac{1}{2}y\,(2-y)$$

so that

$$\frac{d\sigma(S_{\parallel}, \lambda_e)}{dx\,dQ^2 dz\, dP_T^2\, d\phi} = \frac{\pi\alpha^2}{sx^2yQ^2}\left\{(1+(1-y)^2)F_{UU} + y(2-y)S_{\parallel}\lambda_e F_{LL}\right\}$$

$Q^2 = sxy$

$$\Rightarrow \frac{d\sigma(S_{\parallel}, \lambda_e)}{dx\,dQ^2 dz\, dP_T^2\, d\phi} = \frac{\pi\alpha^2}{xQ^4}\left\{(1+(1-y)^2)F_{UU} + y(2-y)S_{\parallel}\lambda_e F_{LL}\right\}$$

$$F_{UU} = x\sum_a e_a^2 \int d^2k_T\, f_1^a(x, k_T^2)\, D_{1\,a/h}(z, (\vec{P}_T - z\vec{k}_T)^2)$$

$$F_{LL} = x\sum_a e_a^2 \int d^2k_T\, g_1^a(x, k_T^2)\, D_{1\,a/h}(z, (\vec{P}_T - z\vec{k}_T)^2)$$

The asymmetry is defined as follows.

$$A_{LL} \equiv \frac{d\sigma(S_{\parallel}=+1) - d\sigma(S_{\parallel}=-1)}{d\sigma(S_{\parallel}=+1) + d\sigma(S_{\parallel}=-1)}$$

e.g. for $\lambda_e = +1$:

$$d\sigma(+1) - d\sigma(-1) = \frac{2\pi\alpha^2}{x Q^4}\, y(1-y)\, F_{LL}$$

$$d\sigma(+1) + d\sigma(-1) = \frac{2\pi\alpha^2}{x Q^4}\left(1+(1-y)^2\right) F_{UU}$$

so that

$$A_{LL} = \frac{\frac{2\pi\alpha^2}{x Q^4}\, y(1-y)\, F_{LL}}{\frac{2\pi\alpha^2}{x Q^4}\left(1+(1-y)^2\right) F_{UU}}$$

Sometimes the so-called depolarisation factor is extracted out

$$D_{LL} \equiv \frac{y(1-y)}{1+(1-y)^2}$$

$$A_{LL}^{exp} \equiv \frac{1}{D_{LL}} A_{LL}$$

A_{LL}^{exp} is presented in PRL 105, 262002 (2010)

the data from Fig 1 suggest that $\langle k_T^2\rangle_{g_1} < \langle k_T^2\rangle_{f_1}$

The authors estimated

$$\frac{\langle k_T^2\rangle_{g_1}}{\langle k_T^2\rangle_{f_1}} = 0.7 \pm 0.1\,, \quad \chi^2/dof = 1.9$$

Can we do better?

Let us start from analysis of unpolarised data from PRC 85, 015202 (2012)

The data presented is for

$$\frac{d\sigma}{\underbrace{d\Omega_e dE_e}_{\sim dx\, dQ^2 \text{ (calculate it!)}} dz\, dP_T^2\, d\phi} \left[\frac{nb}{GeV^3 c^2 sr} \right]$$

Note that ϕ dependence is <u>not</u> integrated out,

generically

$$\frac{d\sigma}{d\phi} \sim \frac{\pi\alpha^2}{xQ^4}(1+(1-y)^2)\left\{ F_{UU} + F_{UU}^{\cos\phi}\cos\phi + F_{UU}^{\cos 2\phi}\cos 2\phi \right\}$$

$F_{UU}^{\cos 2\phi}$ is related to the so-called Boer-Mulders function and is usually small

$F_{UU}^{\cos\phi}$ is twist-3 and usually can be large

(see my notes on $F_{UU}^{\cos\phi}$ approximate calculations)

Generically we could parametrize

$$F_{UU}^{\cos\phi} = \frac{P_T}{Q} F_{UU} \cdot \text{const}$$

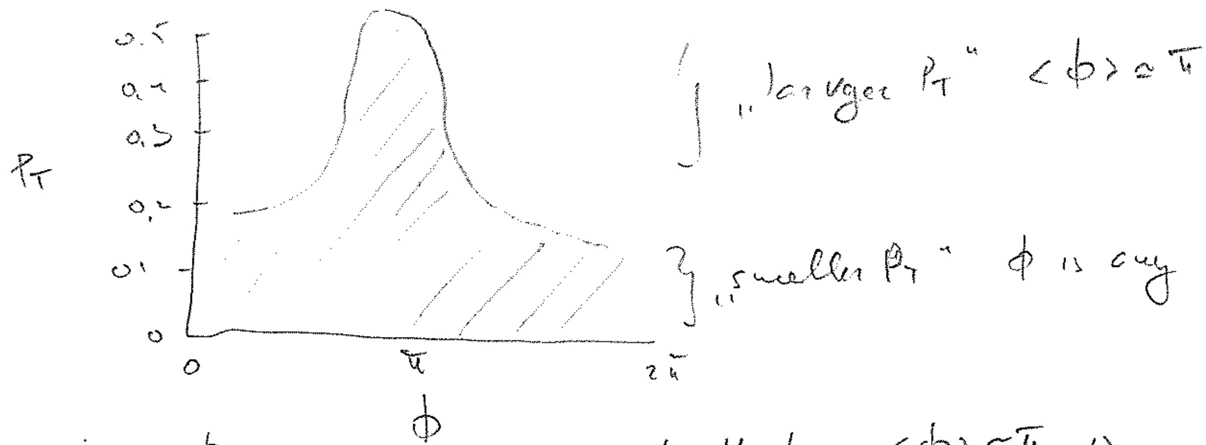
because it is twist-3

because $P_T = 0$ means ϕ cannot be resolved

PRC gives the following

$$\frac{F_{UU}^{\cos\phi}}{F_{UU}} (P_T = 0.05 \text{ GeV}) \simeq 0.02 \pm 0.02$$, see page 15 of PRC

Fig 4 of PRC shows ϕ distribution:



as $F_{UU}^{\cos\phi} \propto P_T$ we conclude that $\langle\phi\rangle \simeq \pi$ is a good approximation as at low P_T the contribution is small for any ϕ and at larger P_T the distribution of ϕ peaks around π

$\Rightarrow$ $\langle\cos\phi\rangle \sim -1$ and we could then

just parametrise

$$F_{UU}^{\cos\phi}\cos\phi \sim -\frac{P_T}{Q}\,\text{const}\,F_{UU}$$

Let us start with $\frac{d\sigma}{dP_T^2 d\phi}$ from PRC.

HERMES data indicates that

$$\langle k_T^2 \rangle_{f_1} \simeq 0.57 \ (GeV^2)$$

$$\langle p_T^2 \rangle_{D_1} \simeq 0.12 \ (GeV^2)$$

Let us fit the data for $\gamma^* P \to \pi^\pm X$, $\gamma^* D \to \pi^\pm X$

using 4 normalization coefficients for $P+\pi^+$, $P+\pi^-$, $D+\pi^+$, $D+\pi^-$

and fixing $\langle k_T^2 \rangle$ $\langle p_T^2 \rangle$ by Hermes values

Alexei_fit_v0.py does it.

$$\chi^2/_{dof} = 2.72, \quad \# dof = 100$$

One can see that the result is not very reliable.

$$\langle x \rangle = 0.32$$

$$\langle z \rangle = 0.55$$

$$\langle Q^2 \rangle = 2.3 \ (GeV^2)$$

$$\langle y \rangle = ?$$

F_{LL} measurements are at

$$\langle x \rangle = 0.23$$

$$\langle z \rangle = 0.53$$

$$\langle Q^2 \rangle = ?$$

$$\langle y \rangle = ?$$