QGT Collaboration Notes

Topical Collaboration:
3D quark-gluon structure of hadrons: mass, spin, and tomography

Members: Ian Cloët (ANL), Adam Freese (UW), Leonard Gamberg (PSU), Wally Melnitchouk (JLab), Andreas Metz (Temple), Eric Moffat (ANL), Alexei Prokudin (PSU), Nobuo Sato (JLab)

Contents

1. Global Analysis Milestones and Timeline

A. Machine Learning Approach

The team within the global analysis working group that has a emphasis on AI/ML methods consists of Ian Cloët (Argonne), Leonard Gamberg (Penn-State), Wally Melnitchouk (Jefferson Lab), Andreas Metz (Temple), Alexei Prokudin (Penn-State), and Nobuo Sato (Jefferson Lab). The funding allocated to each institution for this effort is: Argonne = \$101K (0.64 FTE postdoc) , Penn-State = \$129K (1.92 FTE grad student), Jefferson Lab = \$148K (0.96 FTE postdoc), and Temple = \$94K (1.6 FTE grad student), giving a total of \$472K. The key deliverables, and the PI lead for each deliverable, are listed below.

Year 1: Library of theoretical models for GPDs that can be used to build observables.
Lead Institution: ANL, Lead PI: I. Cloët

Deliver RGE codes for GPD evolution.
Lead Institution: ANL, Lead PI: I. Cloët

Deliver a database consisting of mock experimental data for GPD observables.
Lead Institution: Penn-State, Lead PI: A. Prokudin

Year 2: Deliver ML models for CFFs with built-in theoretical constraints (such as dispersion relations).
Lead Institution: ANL, Lead PI: I. Cloët

Deliver first closure tests for the extraction of pure ML-based CFFs at kinematics of existing experimental facilities using different ML-based inference techniques.
Lead Institution: JLab, Lead PI: N. Sato

Year 3: Deliver ML-based GPD parameterization with built-in theoretical constraints.
Lead Institution: JLab, Lead PI: N. Sato

Deliver first closure tests for GPDs using combined experimental data with mock CFFs.
Lead Institution: JLab, Lead PI: N. Sato

Deliver sensitivity studies for GPD analysis using lattice data.
Lead Institution: Temple, Lead PI: A. Metz

Deliver a database for all available GPD related data.
Lead Institution: Penn-State, Lead PI: A. Prokudin

Deliver first global analysis of pure ML-based CFFs using all available experimental data.
Lead Institution: Penn-State, Lead PI: A. Prokudin

Year 4: Deliver DVCS/DVMP CFF codes using NLO hard coefficients.
Lead Institution: ANL, Lead PI: I. Cloët

Deliver first extraction of GPDs using extracted CFFs.
Lead Institution: JLab, Lead PI: N. Sato

Develop database for GPD/CFF visualization with user-friendly interface based on jupyter notebooks.
Lead Institution: Penn-State, Lead PI: A. Prokudin

Provide dedicated exploratory closure tests studies using new observables beyond DVCS and DVMP proposed by the theory working group. Lead Institution: Temple, Lead PI: A. Metz

Year 5: Deliver complete global analysis of GPDs using experimental and lattice data for the chiral-even GPDs E, $\tilde{H}$, and $\tilde{E}$. Lead Institution: JLab, Lead PI: W. Melnitchouk

Deliver implementation for transversity GPDs in meson production at higher twist.
Lead Institution: Penn-State, Lead PI: L. Gamberg

Budget Summary: We are requesting funds of \$500K on average per year for 5 years. We have allocated these funds according to: 1) We will support three tenure track Assistant Professor bridge positions at $200K each, so $600K total over the 5 years. 2) We will partially support 11 postdoctoral researchers, 6 graduate students, and an undergraduate program over the 5 years totaling $1,628K. 3) We will support travel for student/postdoc collaboration members and two summer schools, totaling $236K. 4) The remaining $36K of the budget will be used to support financial administration of the project, as all funds (with the exception of Lab funds) will be held and distributed by Temple University as the lead institution. In the table below, we list the funds allocated to each institution, where the purpose of the funds is indicated by BP=bridge position, PD=postdoc, GS=graduate student, and US=undergraduate student. For UC Berkeley, $80K is dedicated to a PD and $121 to a GS, Temple has $108.5K for two GS in addition to a BP, and the undergraduate program at Penn State will received $8K with the remaining funds used to support a GS. All other institutions receive funds for just one purpose.

Multi-Institutional Team Application Information ($ in thousands).

Names (Lead PI and co-PIs)	Institution	Year 1 Budget	Year 2 Budget	Year 3 Budget	Year 4 Budget	Year 5 Budget	Total Budget
Martha Constantinou*, Andreas Metz	Temple [BP, 2 GS]		100.8	83.6	70.0	54.0	$308.5
Ian Cloët, Yong Zhao	Argonne [2 PD]	117.9		94.1			$212.0
Thomas Mehen	Duke [PD]	67.6					$67.6
Alberto Accardi, Jose Goity	Hampton [PD]				81.8		$81.8
Wally Melnitchouk, David Richards, Nobuo Sato, Christian Weiss	Jefferson Lab [PD]		47.8	49.2	50.7		$147.8
Feng Yuan	LBNL [PD]	102.4					$102.4
William Detmold, John Negele, Phiala Shanahan, Iain Stewart	MIT [2 PD]				125.3	103.8	$229.0
Leonard Gamberg, Alexei Prokudin	Penn State [GS, UG]	40.2		44.7		47.4	$132.4
Edward Shuryak, Sergey Syritsyn, Ismail Zahed	Stony Brook [BP]		50.0	50.0	50.0	50.0	$200.0
Sean Fleming	U Arizona [GS]	47.3		27.5			$74.9
Peter Schweitzer	U Connecticut [PD]		76.6				$76.6
Feng Yuan	UC Berkeley [PD, GS]	57.4	68.5	29.9		44.8	$200.7
Keh-Fei Liu	U Kentucky [PD]	18.6	36.2	35.1			$89.8
Gerald Miller	U Washington [BP]			60.0	67.0	73.0	$200.0
Christopher Monahan, Konstantinos Orginos	William & Mary [GS]		50.5			54.0	$104.5
Travel, Workshops, Summer Schools		42.6	50.0	40.0	68.0	35.4	$236.0
Sub-awards processing		6.0	8.0	8.0	6.0	8.0	$36.0
Total Budget		**$500.0**	**$488.3**	**$522.2**	**$518.8**	**$470.5**	**$2,500.0**

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Key Publications: DVCS

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[2] A. V. Belitsky and D. Mueller, *Exclusive electroproduction revisited: treating kinematical effects*, Phys. Rev. D **82**, 074010 (2010), arXiv:1005.5209 [hep-ph].
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3. Deeply Virtual Compton Scattering

The exclusive lepto-production process of a real photon from a hadron/nuclear target is illustrated in Fig. 1 and reads

$$l(k,\lambda) + A(p,S) \longrightarrow l(k',\lambda') + \gamma(q',\Lambda') + A(p',S'), \tag{1}$$

where l is the incoming/outgoing lepton, A is the incoming/outgoing target hadron or nucleus, and γ is the produced real photon. The momenta of the incoming/outgoing particles are labeled by k, p, k', q', p', where momentum conservation implies

$$k + p = k' + q' + p', \tag{2}$$

and the incoming/outgoing particle helicities are labeled by λ, S, λ', Λ', S', where λ, λ', $\Lambda' \equiv \pm 1$ [1]. The target spin vectors satisfy $S^2 = -1 = S'^2$ and $S \cdot p = 0 = S' \cdot p'$, and we define the following 4-momentum variables and Lorentz scalars:

$$\Delta \equiv p' - p, \qquad q \equiv k - k', \qquad \Delta = q - q', \qquad \bar{p} = \frac{1}{2}(p' + p), \qquad \bar{q} = \frac{1}{2}(q' + q), \tag{3}$$

$$t \equiv \Delta^2, \qquad Q^2 \equiv -q^2, \qquad x_B \equiv \frac{Q^2}{2\,p\cdot q}, \qquad y \equiv \frac{p\cdot q}{p\cdot k}, \qquad \gamma = \frac{2\,M\,x_B}{Q}, \qquad s = (p+k)^2 = m_l^2 + M^2 + \frac{Q^2}{2\,x_B\,y}. \tag{4}$$

With these definitions Eq. (2) can be expressed as $p + q = p' + q'$.

The differential cross-section for this exclusive lepto-production process in the *lab frame* reads [1–4]

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}x_B\,\mathrm{d}Q^2\,\mathrm{d}|t|\,\mathrm{d}\phi\,\mathrm{d}\varphi} = \frac{\alpha_{\mathrm{em}}^3\,x_B\,y^2}{16\pi^2\,Q^4\sqrt{1+\gamma^2}}\,|\mathcal{T}_{\mathrm{BH}} + \mathcal{T}_{\mathrm{DVCS}}|^2 = \frac{\alpha_{\mathrm{em}}^3\,x_B\,y^2}{16\pi^2\,Q^4\sqrt{1+\gamma^2}}\left[|\mathcal{T}_{\mathrm{BH}}|^2 + \mathcal{T}_{\mathrm{I}} + |\mathcal{T}_{\mathrm{DVCS}}|^2\right] \tag{5}$$

where $\alpha_{\mathrm{em}} = e^2/(4\pi)$ is the electromagnetic coupling, $\phi = \phi_N - \phi_l$ is the azimuthal angle between the leptonic plane and the reaction/hadron plane, $\varphi = \Phi - \phi_N$ is the difference between the azimuthal angle Φ of the transverse part of the nucleon polarization vector $S_T = (0, \cos\Phi, \sin\Phi, 0)$ and the azimuthal angle of the reaction plane. Both of these angles are defined in the rest frame of the target [3] as illustrated in Fig. 2. [Note, the cross section in Ref. [1] lacks a y/Q^2 factor relative to the latter results, which likely needs to be accounted for in the invariant amplitude contributions.] For the invariant amplitude, $|\mathcal{T}_{\mathrm{BH}}|^2$ is the Bethe-Heitler contribution, $|\mathcal{T}_{\mathrm{DVCS}}|^2$ is the DVCS contribution, and $\mathcal{T}_{\mathrm{I}}$ is the quantum interference between these two processes:

$$\mathcal{T}_{\mathrm{I}} = \mathcal{T}_{\mathrm{BH}}^*\,\mathcal{T}_{\mathrm{DVCS}} + \mathcal{T}_{\mathrm{DVCS}}^*\,\mathcal{T}_{\mathrm{BH}} = 2\,\mathcal{T}_{\mathrm{BH}}\,\mathrm{Re}[\mathcal{T}_{\mathrm{DVCS}}]\,. \tag{6}$$

The BH and DVCS processes are illustrated in Fig. 1, where BH is purely real and DVCS has both real and imaginary pieces.

These invariant amplitude contributions can be expressed in terms of the real-valued electromagnetic form factors $F_i(Q^2)$, and complex-valued Compton form factors $\mathcal{H}_i(\xi_B, t, Q^2)$, where ξ_B is a Lorentz scalar which for reasons that will become clear we call the empirical/Bjorken skewness. Factorization theorems in QCD then relate these Compton form factors to the real-valued GPDs, which are functions of $H_i(x, \xi, t, Q^2)$. The two scaling variables are defined by

$$x \equiv \frac{\bar{\ell}\cdot n}{\bar{p}\cdot n}, \qquad\qquad \xi \equiv -\frac{\Delta\cdot n}{2\,\bar{p}\cdot n}, \tag{7}$$

where $\bar{\ell} = \frac{1}{2}(\ell' + \ell)$ is the average momentum of the active parton, ξ is the (scaling) skewness that enters the operator definition of the GPDs, and both these variables have support $-1 \leqslant x,\ \xi \leqslant 1$. In relating the GPDs to the cross section, the scaling variable x is integrated out, meaning an inverse problem needs to be solved to obtain the GPDs from DVCS data, which is a significant challenge [5]. There is another key challenge in extracting GPDs from DVCS data, which is a loss of Lorentz invariance associated with kinematical and dynamical aspects of the twist expansion that begin at twist-3. These arise from ambiguities in the twist

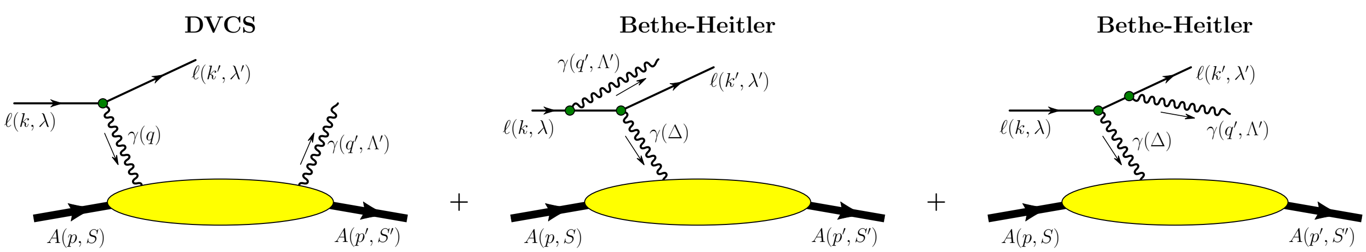

Figure 1: Exclusive lepto-production of a real photon through the DVCS and BH processes.

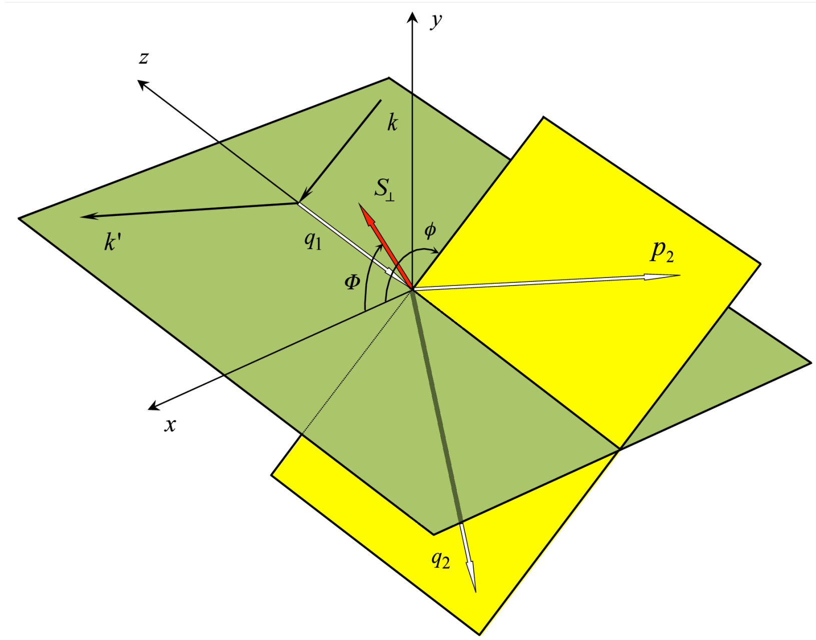

Figure 2: This figure is taken from Ref. [3], and we adopt the same conventions, just with different moment labels: $p' = p_2$, $q' = q_2$, and $q = q_1$. That is, we work in the target rest frame and the z-axis is directed counter to the photon three-momentum q, the x-component of the incoming electron momentum k is chosen to be positive. The angles parametrizing the five-fold cross section in Eq. (5) are defined as follows: ϕ is the azimuthal angle between the lepton plane and the recoiled proton momentum (reaction plane), while the difference $\varphi \equiv \Phi - \phi$ for fixed ϕ is determined by the direction of the transverse nucleon polarization vector component $S_T = (\cos\Phi, \sin\Phi)$.

expansion of the DVCS amplitude and the associated need to explicitly define a plus-direction, or equivalently the four-vector n. There are also pure kinematical higher-twist ambiguities in relating the Compton form factors to the GPDs.

A key example of a kinematical higher-twist ambiguity is the relation between the skewness variable ξ that appears in the operator definition of the GPDs, and the empirical/Bjorken skewness variable ξ_B that appears in the parametrization of the Compton form factors. To extract the GPDs from the Compton form factors it is necessary to define a mapping between ξ_B and ξ. This means that the plus direction, or equivalently, the four-vector n that appears in the definition of ξ given in Eq. (7), must be defined relative to the physical momenta in the process k, p, k', q', and p'. Having to explicitly define the plus-direction, or equivalently, the coordinate system, breaks Lorentz invariance and means that the relation between the GPDs and the cross section depends on our choice of plus-direction/coordinate system. In particular, this means the relation between the GPDs and the cross section is not invariant under rotations of our coordinate system. In practice, this means the empirical skewness ξ_B must be defined relative to ξ up to kinematical higher twist corrections:

$$\xi_B \to \xi_B(x_B, t, Q^2) = \frac{x_B}{2 - x_B} + O(1/Q^2), \tag{8}$$

where in the scaling limit these ambiguities disappear. Therefore, different choices of plus-direction result in various definitions for ξ that differ by power corrections proportional to t/Q^2. The extraction of GPDs from data therefore explicitly depends on our choice of coordinate system, and in the extraction of twist-2 GPDs it is an open question which choice of plus-direction is optimal, in that higher twist effects are minimized. These higher twist ambiguities will necessarily lead to different results for the extracted GPDs.

In the literature, various authors have chosen different conventions for the empirical skewness ξ_B and the plus-direction defined by n. For example, in Ref. [1] Belitsky, Müller, and Kirchner (BMK) define

$$\xi_B \equiv -\frac{\bar{q}^2}{2\,\bar{p}\cdot\bar{q}} = \frac{x_B + x_B\,t/(2\,Q^2)}{2 - x_B + x_B\,t/Q^2}, \qquad n \equiv q \quad \to \quad \xi = -\frac{\Delta\cdot q}{2\,\bar{p}\cdot q}, \qquad \Longrightarrow \qquad \xi_B = \xi\left(1 + \frac{t}{2\,Q^2}\right). \tag{9}$$

BMK also have an additional minus sign in their definition of ξ, which they call η. In Refs. [6, 7] Braun, Manashov, and Pirnay (BMP) define

$$n \equiv q' \qquad \xi_B \equiv \xi = -\frac{\Delta\cdot q'}{2\,\bar{p}\cdot q'} = \frac{x_B + x_B\,t/Q^2}{2 - x_B + x_B\,t/Q^2} \tag{10}$$

Another popular convention, used for example by: Kumerički and Müller [8, 9]; Vanderhaeghen, Guichon, and Guidal; and Kroll, Moutarde, and Sabatie [10] is simply to define

$$\xi_B \equiv \frac{x_B}{2 - x_B}, \qquad\qquad \xi \equiv \xi_B. \tag{11}$$

These latter definitions are motivated by a generalization of the standard DIS reference frame, where the initial photon and target momenta form the longitudinal plane. This differs from BMP, where the longitudinal plane as spanned by the two photon momenta q and q', which makes the momentum transfer to the target Δ purely longitudinal, as both the initial and final state hadron/nucleus has the same non-vanishing transverse momentum $\boldsymbol{p}_T$. A discussion of some of these different conventions can be found in Refs. [11, 12].

A question I would like to discuss is the differences between a reference frame and a coordinate system, as these terms seem to be used interchangeably in the literature.

In deriving some of the relations above, some of the following results are useful:

$$p' \cdot p = M^2 - \frac{1}{2}\,\Delta^2, \qquad q' \cdot q = -\frac{1}{2}(\Delta^2 + Q^2), \qquad k' \cdot k \simeq \frac{1}{2}\,Q^2, \tag{12}$$

$$p' \cdot q' = \frac{Q^2}{2\,x_B}(1 - x_B), \qquad p' \cdot q = \frac{Q^2}{2\,x_B}\left(1 - x_B + x_B\,\Delta^2/Q^2\right), \qquad p \cdot q' = \frac{Q^2}{2\,x_B}\left(1 + x_B\,\Delta^2/Q^2\right), \tag{13}$$

$$k \cdot q \simeq -\frac{1}{2}\,Q^2, \qquad k \cdot q' \simeq -\frac{1}{2}\,Q^2 - k \cdot \Delta, \qquad k \cdot p' = \frac{Q^2}{2\,x_B\,y} + k \cdot \Delta, \tag{14}$$

$$k' \cdot q \simeq \frac{1}{2}\,Q^2, \qquad k' \cdot q' \simeq \frac{1}{2}\,\Delta^2 - k \cdot \Delta, \qquad k' \cdot p' = \frac{Q^2}{2\,x_B\,y}\left(1 - y + x_B\,y - x_B\,y\,\frac{\Delta^2}{Q^2}\right) + k \cdot \Delta. \tag{15}$$

where we have used $q'^2 = 0$, $p \cdot q = Q^2/(2\,x_B)$, $k \cdot p = Q^2/(2\,x_B\,y)$, and $p'^2 = p^2 = M^2$. Therefore, the differential cross-section and the various contributions to the invariant amplitude can be expressed in terms of the invariants: $k \cdot \Delta$, $x_B \leftrightarrow \xi_B$, y, $t = \Delta^2$, and Q^2, which implies $\mathcal{T}_i(k, p, k', q', p') \to \mathcal{T}_i(k \cdot \Delta, \xi_B, y, \Delta^2, Q^2)$.

As discussed earlier, this lepto-production process has both BH and DVCS contributions, together with their interference. The BH process can be parameterized in terms of the electromagnetic form factors, which are defined via the electromagnetic current and for a spin-1/2 target read

$$J^\mu = \langle p', S' \,| j^\mu(0) |\, p, S\rangle = \bar{u}(p', S')\left[\gamma^\mu\,F_1(t) + \frac{i\sigma^{\mu\nu}q_\nu}{2\,M}\,F_2(t)\right]u(p, S). \tag{16}$$

The BH contribution to this lepto-production process is not plagued by any of the ambiguities discussed above. The DVCS amplitude, $T^{\mu\nu}$, has been studied for decades and can be unambiguously expressed in a manifestly Lorentz covariant manner, which for spin-1/2 targets consists of 18 Lorentz structures multiplied by scalar functions [13]:

$$T^{\mu\nu} = i\int \mathrm{d}z\, e^{i\bar{q}\cdot z}\,\langle p', S' \,| T\{j^\mu(z/2)\,j^\nu(-z/2)\} |\, p, S\rangle = \sum_{i=1}^{18} \tau_i^{\mu\nu}(q, q', \bar{p})\;A_i(q^2, q'^2, q' \cdot q, \bar{p} \cdot \bar{q}), \tag{17}$$

where this expression is valid with both initial and final photons off-shell. Using this result, it is possible to unambiguously express the lepto-production cross section in a manifestly Lorentz invariance manner using the electromagnetic form factors and the A_i scalar functions. If the goal was to simply obtain the A_i's then we would be done and the ambiguities associated with a lost of Lorentz invariance mentioned above would not play a role, however, we wish to use this process to access direct information on the quark and gluon structure of the target. To achieve this, the DVCS amplitude must be expanded in terms of amplitudes of well defined twist that can be related to the underlying quark-gluon dynamics via QCD factorization. These amplitudes of definite twist are the Compton form factors associated with DVCS, where the DVCS amplitude is expanded as

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{tw2}} + T^{\mu\nu}_{\mathrm{tw3}} + T^{\mu\nu}_{\mathrm{tw4}} + \ldots \tag{18}$$

Beyond leading-twist this expansion is not unique, which is associated with the loss of Lorentz invariance and the ambiguities in defining the plus-direction.

One decomposition of the DVCS amplitude for a spin-1/2 target up to twist-3 is given by BMK in Refs. [1, 2] which reads

$$T^{\mu\nu} = -\mathcal{P}^{\mu\sigma}g_{\sigma\lambda}\mathcal{P}^{\lambda\nu}\,\frac{\bar{q}\cdot V_1}{\bar{p}\cdot\bar{q}} + \left(\mathcal{P}^{\mu\sigma}\bar{p}_\sigma\mathcal{P}^{\lambda\nu} + \mathcal{P}^{\mu\lambda}\bar{p}_\sigma\mathcal{P}^{\sigma\nu}\right)\frac{V_{2\lambda}}{\bar{p}\cdot\bar{q}} - \mathcal{P}^{\mu\sigma}i\varepsilon_{\sigma\lambda\alpha\rho}\bar{q}^\alpha\mathcal{P}^{\lambda\nu}\,\frac{A_1^\rho}{\bar{p}\cdot\bar{q}}, \tag{19}$$

where $\mathcal{P}^{\mu\nu} = g^{\mu\nu} - q^\mu q'^\nu / q \cdot q'$, and an explicit calculation of the Compton amplitude via the operator product expansion to twist-three accuracy gives [1, 2, 14, 15]

$$V_1^\mu = \frac{1}{\bar p \cdot \bar q}\, \bar u(p', S') \left[\not{\bar q} \left[\bar p^\mu\, \mathcal{H} + \Delta_\perp^\mu\, \mathcal{H}_+^3 \right] + \frac{i\sigma^{\mu\nu} \bar q_\mu \Delta_\nu}{2\,M} \left[\bar p^\mu\, \mathcal{E} + \Delta_\perp^\mu\, \mathcal{E}_+^3 \right] + \tilde\Delta_\perp^\mu \left[\not{\bar q}\, \tilde{\mathcal{H}}_-^3 + \frac{\bar q \cdot \Delta}{2M}\, \tilde{\mathcal{E}}_-^3 \right] \gamma_5 \right] u(p, S), \tag{20}$$

$$V_2^\mu = \xi \left[V_1^\mu - \bar p^\mu\, \frac{\bar q \cdot V_1}{2\,\bar p \cdot \bar q} \right] + \frac{i\varepsilon^{\mu\sigma\alpha\beta} \Delta_\alpha \bar q_\beta}{2\,\bar p \cdot \bar q}\, A_{1\sigma}, \tag{21}$$

$$A_1^\mu = \frac{1}{\bar p \cdot \bar q}\, \bar u(p', S') \left[\not{\bar q}\gamma_5 \left[\bar p^\mu\, \tilde{\mathcal{H}} + \Delta_\perp^\mu\, \tilde{\mathcal{H}}_+^3 \right] + \frac{\bar q \cdot \Delta}{2M}\, \gamma_5 \left[\bar p^\mu\, \tilde{\mathcal{E}} + \Delta_\perp^\mu\, \tilde{\mathcal{E}}_+^3 \right] + \tilde\Delta_\perp^\mu \left[\not{\bar q}\, \mathcal{H}_-^3 + \frac{i\sigma^{\mu\nu} \bar q_\mu \Delta_\nu}{2\,M}\, \mathcal{E}_-^3 \right] \right] u(p, S). \tag{22}$$

Here

$$\Delta_\perp^\mu \equiv \Delta^\mu - \frac{\Delta \cdot \bar q}{\bar p \cdot \bar q}\, \bar p^\mu, \qquad \tilde\Delta_\perp^\mu \equiv \frac{1}{\bar p \cdot \bar q}\, i\varepsilon^{\mu\sigma\alpha\beta} \Delta_\sigma \bar p_\alpha \bar q_\beta. \tag{23}$$

The twelve Compton form factors are functions of $\mathcal{F} = \mathcal{F}(\xi, t, Q^2)$, where in DVCS kinematics $\xi \simeq \Delta \cdot \bar q / \bar p \cdot \bar q$, and analogous results for a spin-zero target are given in Refs. [16, 17].

Another decomposition of the DVCS amplitude in terms of helicity amplitudes, which includes the complete kinematic power corrections up to $\sim t/Q^2$ and $\sim m/Q^2$, is given by BMP in Ref. [7]:

$$\mathcal{A}_{\mu\nu} = \varepsilon_\mu^+ \varepsilon_\nu^-\, \mathcal{A}_{\mu\nu}^{++} + \varepsilon_\mu^- \varepsilon_\nu^+\, \mathcal{A}_{\mu\nu}^{--} + \varepsilon_\mu^0 \varepsilon_\nu^-\, \mathcal{A}_{\mu\nu}^{0+} + \varepsilon_\mu^0 \varepsilon_\nu^+\, \mathcal{A}_{\mu\nu}^{0-} + \varepsilon_\mu^+ \varepsilon_\nu^+\, \mathcal{A}_{\mu\nu}^{+-} + \varepsilon_\mu^- \varepsilon_\nu^-\, \mathcal{A}_{\mu\nu}^{-+} + q'_\nu, \mathcal{A}_\mu^{(3)}. \tag{24}$$

Here, the two photon momenta q and q' are used to define the two light-like vectors

$$n = q', \qquad \tilde n = -q + (1 - \tau)\, q', \qquad \tau = t/(t + Q^2), \qquad n \cdot \tilde n = \ldots \tag{25}$$

which define the longitudinal plane. The photon polarization vectors are chosen to be

$$\varepsilon_\mu^0 = -\left(q_\mu - \frac{q'_\mu\, q^2}{q \cdot q'} \right) \frac{1}{\sqrt{-q^2}}, \qquad \varepsilon_\mu^\pm = \left(P_\mu^\perp \pm i \bar P_\mu^\perp \right) \frac{1}{2\,|P_\mu^\perp|}, \qquad P_\mu^\perp = g_{\mu\nu}^\perp\, \bar p^\nu, \qquad \bar P_\mu^\perp = \varepsilon_{\mu\nu}^\perp\, \bar p^\nu, \tag{26}$$

where

$$g_{\mu\nu}^\perp =, \qquad \varepsilon_{\mu\nu}^\perp =, \tag{27}$$

and BMP use $\varepsilon_{0123} = 1$. In the BMP convention, each helicity amplitude involves the sum over quark flavors $\mathcal{A} = \sum e_q^2\, \mathcal{A}_q$, where e_q is the quark electromagnetic charge, and each $\mathcal{A}_q$ can be expressed in terms of the leading-twist GPDs H_q, E_q, $\tilde H_q$, $\tilde E_q$ via QCD factorization. For the GPD definitions BMP follows Diehl [18]. Expressions for the helicity amplitudes in terms of the GPDs are given in Ref. [7].

In the BMK conventions, the twist-2 and twist-3 Compton form factors are related to twist-2 and twist-3 GPDs via convolution with perturbatively calculable coefficient functions [1]:

$$\left\{ \mathcal{H},\, \mathcal{E},\, \mathcal{H}_+^3,\, \mathcal{E}_+^3,\, \tilde{\mathcal{H}}_-^3,\, \tilde{\mathcal{E}}_-^3 \right\} (\xi_B, t, Q^2) = \int_{-1}^{1} \mathrm{d}x\, C^{(-)}(\xi_B, x) \left\{ H,\, E,\, H_+^3,\, E_+^3,\, \tilde H_-^3,\, \tilde E_-^3 \right\} (x, \xi, t, Q^2), \tag{28}$$

$$\left\{ \tilde{\mathcal{H}},\, \tilde{\mathcal{E}},\, \tilde{\mathcal{H}}_+^3,\, \tilde{\mathcal{E}}_+^3,\, \mathcal{H}_-^3,\, \mathcal{E}_-^3 \right\} (\xi_B, t, Q^2) = \int_{-1}^{1} \mathrm{d}x\, C^{(+)}(\xi_B, x) \left\{ \tilde H,\, \tilde E,\, \tilde H_+^3,\, \tilde E_+^3,\, H_-^3,\, E_-^3 \right\} (x, \xi, t, Q^2), \tag{29}$$

where the vector and axial quark operator definitions of the GPDs $\{H, \ldots, \tilde E_-^3\}$ and $\{\tilde H, \ldots, E_-^3\}$, and explicit expressions for the coefficient functions $C^{(\pm)}$ will be given later. The DVCS amplitude also receives contributions from Compton form factors associated with twist-2 gluon transversity GPDs which are related by [1]. These GPDs are defined by

$$\begin{aligned} G_{\mu\nu}^T(x, \xi, t) &\equiv \frac{4}{\bar p \cdot n} \int \frac{d\kappa}{2\,\pi}\, \mathrm{e}^{ix\,\kappa\,\bar p \cdot n} \left\langle p' \left| G_{+\rho}(\kappa_2 n)\, \tau_{\mu\nu;\rho\sigma}^\perp\, G_{\sigma+}(\kappa_1 n) \right| p \right\rangle \\ &= \frac{\tau_{\mu\nu;\alpha\beta}^\perp}{2\,M}\, \Delta^\alpha\, \bar u(p') \left[H_T(x, \xi, t)\, \frac{i\sigma^{\gamma\beta} q_\gamma}{\bar p \cdot q} + \tilde H_T(x, \xi, t)\, \frac{\Delta^\beta}{2\,M^2} + E_T(x, \xi, t)\, \frac{1}{2\,M} \left(\frac{\gamma \cdot q}{\bar p \cdot q} \Delta^\beta - \eta\, \gamma^\beta \right) - \tilde E_T(x, \xi, t)\, \frac{\gamma^\beta}{2\,M} \right] u(p), \end{aligned} \tag{30}$$

where

$$\tau_{\mu\nu;\rho\sigma}^\perp = \frac{1}{2} \left(g_{\mu\rho}^\perp g_{\nu\sigma}^\perp + g_{\mu\sigma}^\perp g_{\nu\rho}^\perp - g_{\mu\nu}^\perp g_{\rho\sigma}^\perp \right), \qquad g_{\mu\nu}^\perp = g_{\mu\nu} - n_\mu \bar n_\nu - \bar n_\mu n_\nu, \qquad n^2 = \bar n^2 = 0, \qquad n \cdot \bar n = 1. \tag{31}$$

A one-loop calculation then gives the following result for the real final-state photon DVCS amplitude

$$T_{\mu\nu} = \frac{\alpha_s}{2\pi} T_F \sum_{i=u,d,s} \int_{-1}^{1} \mathrm{d}x\, C^{(+)}_{(0)i}(x,\xi)\, G^{T}_{\mu\nu}(x,\xi,t), \tag{32}$$

with $T_F = 1/2$ and the coefficient function $C^{(+)}_{(0)i}$ will be given later. The Compton form factors are then defined as

$$\left\{\mathcal{H}_T,\, \mathcal{E}_T,\, \tilde{\mathcal{H}}_T,\, \tilde{\mathcal{E}}_T\right\}(\xi,t,Q^2) = \frac{\alpha_S}{2\,\pi}\, T_F \sum_{i=u,d,s} \int_{-1}^{1} \mathrm{d}x\, C^{(+)}_{(0)i}(\xi,x)\left\{H_T,\, E_T,\, \tilde{H}_T,\, \tilde{E}_T\right\}(x,\xi,t,Q^2), \tag{33}$$

where we can adopt the unifying notation $\mathcal{F}_T = \left[\mathcal{H}_T, \ldots, \tilde{\mathcal{E}}_T\right]$.

All twist-three GPDs can be decomposed as $F^3_\pm = F^{\mathrm{WW}}_\pm + F^{qgq}_\pm$, where the so-called Wandzura–Wilczek (WW) term $F^{\mathrm{WW}}_\pm$ can be expressed solely in terms of the twist-two GPDs $F = H,\, E,\, \tilde{H},\, \tilde{E}$ and the term $F^{qgq}_\pm$ that contains new dynamical information arising from antiquark–gluon–quark correlations. The WW relations read [1]:

$$F^{\mathrm{WW}}_+(x,\xi) = \int_{-1}^{1} \mathrm{d}y\, \frac{1}{\xi}\, W_+\!\left(\frac{x}{\xi},\frac{y}{\xi}\right)\left(y\,\frac{\overleftarrow{\partial}}{\partial y} - \xi\,\frac{\overrightarrow{\partial}}{\partial \xi}\right) F(y,\xi) - \frac{4\,M^2\,F^\perp_+(x,\xi)}{(1-\xi^2)(t-t_{\min})} - \frac{1}{\xi}\,F(x,\xi), \tag{34}$$

$$F^{\mathrm{WW}}_-(x,\xi) = -\int_{-1}^{1} \mathrm{d}y\, \frac{1}{\xi}\, W_-\!\left(\frac{x}{\xi},\frac{y}{\xi}\right)\left(y\,\frac{\overleftarrow{\partial}}{\partial y} - \xi\,\frac{\overrightarrow{\partial}}{\partial \xi}\right) F(y,\xi) - \frac{4\,M^2\,F^\perp_-(x,\xi)}{(1-\xi^2)(t-t_{\min})}, \tag{35}$$

where the W-kernels are given by

$$W_\pm\!\left(\frac{x}{\xi},\frac{y}{\xi}\right) = \frac{1}{2\,\xi}\left[W\!\left(\frac{x}{\xi},\frac{y}{\xi}\right) \pm W\!\left(-\frac{x}{\xi},-\frac{y}{\xi}\right)\right], \qquad W(x,y) = \frac{\theta(1+x)-\theta(x-y)}{1+y}, \tag{36}$$

and the minimum value of the momentum transfer squared $t_{\min}$ is given by [1]

$$t_{\min} = -Q^2\,\frac{2(1-x_B)\left[1-\sqrt{1+\epsilon^2}\right]+\epsilon^2}{4\,x_B\,(1-x_B)+\epsilon^2} \simeq \frac{M^2\,x_B}{1-x_B+x_B\,M^2/Q^2}. \tag{37}$$

The functions $F^\perp_\pm$ first appear for spin-1/2 targets and read

$$H^\perp_\pm(x,\xi) = \mp\frac{t}{4\,M^2}\int_{-1}^{1} \mathrm{d}y\left[\xi\,W_\pm\!\left(\frac{x}{\xi},\frac{y}{\xi}\right)[H(y,\xi)+E(y,\xi)] - W_\mp\!\left(\frac{x}{\xi},\frac{y}{\xi}\right)\tilde{H}(y,\xi)\right], \tag{38}$$

$$E^\perp_\pm(x,\xi) = \pm\int_{-1}^{1} \mathrm{d}y\left[\xi\,W_\pm\!\left(\frac{x}{\xi},\frac{y}{\xi}\right)[H(y,\xi)+E(y,\xi)] - W_\mp\!\left(\frac{x}{\xi},\frac{y}{\xi}\right)\tilde{H}(y,\xi)\right], \tag{39}$$

$$\tilde{H}^\perp_\pm(x,\xi) = \pm\int_{-1}^{1} \mathrm{d}y\left[\xi\left(1-\frac{t}{4\,M^2}\right)W_\pm\!\left(\frac{x}{\xi},\frac{y}{\xi}\right)\tilde{H}(y,\xi) + \frac{t}{4\,M^2}\,W_\mp\!\left(\frac{x}{\xi},\frac{y}{\xi}\right)[H(y,\xi)+E(y,\xi)]\right], \tag{40}$$

$$\tilde{E}^\perp_\pm(x,\xi) = \pm\frac{1}{\xi}\int_{-1}^{1} \mathrm{d}y\left[W_\pm\!\left(\frac{x}{\xi},\frac{y}{\xi}\right)\tilde{H}(y,\xi) + \xi\,W_\mp\!\left(\frac{x}{\xi},\frac{y}{\xi}\right)[H(y,\xi)+E(y,\xi)]\right]. \tag{41}$$

These WW relations are clearly much more involved than those these appear for the parton distribution functions.

Following Ref. [19], the BH, DVCS, and interference contributions to the differential cross-section of Eq. (5) can be expressed in terms of structure functions:

$$|\mathcal{T}_{\mathrm{BH}}|^2 = \frac{1}{t}\left[F^{\mathrm{BH}}_{UU} + \lambda\,S_L\,F^{\mathrm{BH}}_{LL} + \lambda\,|S_T|\,F^{\mathrm{BH}}_{LT}\right], \tag{42}$$

$$\mathcal{T}_{\mathrm{I}} = \frac{e_l}{Q^2|t|}\left[F^{\mathrm{I}}_{UU} + \lambda\,F^{\mathrm{I}}_{LU} + S_L\,F^{\mathrm{I}}_{UL} + |S_T|\,F^{\mathrm{I}}_{UT} + \lambda\,S_L\,F^{\mathrm{I}}_{LL} + \lambda\,|S_T|\,F^{\mathrm{I}}_{LT}\right], \tag{43}$$

$$|\mathcal{T}_{\mathrm{DVCS}}|^2 = \frac{1}{Q^2(1-\epsilon)}\Bigg[F_{UU,T} + \epsilon\,F_{UU,L} + \epsilon\,\cos 2\phi\,F^{\cos 2\phi}_{UU} + \sqrt{\epsilon(1+\epsilon)}\,\cos\phi\,F^{\cos\phi}_{UU}$$
$$+ \lambda\sqrt{2\epsilon(1-\epsilon)}\,\sin\phi\,F^{\sin\phi}_{LU} + S_L\left[\sqrt{\epsilon(1+\epsilon)}\,\sin\phi\,F^{\sin\phi}_{UL} + \epsilon\,\sin 2\phi\,F^{\sin 2\phi}_{UL}\right]$$

$$
\begin{aligned}
&+ |S_T| \left[\sin(\phi - \phi_S) \left[F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon\, F_{UT,L}^{\sin(\phi-\phi_S)} \right] + \epsilon\, \sin(\phi + \phi_S)\, F_{UT}^{\sin(\phi+\phi_S)} + \epsilon\, \sin(3\phi - \phi_S)\, F_{UT}^{\sin(3\phi-\phi_S)} \right. \\
&\qquad\qquad \left. \sqrt{2\epsilon(1+\epsilon)} \left[\sin\phi_S\, F_{UT}^{\sin\phi_S} + \sin(2\phi - \phi_S)\, F_{UT}^{\sin(2\phi-\phi_S)} \right] \right] \\
&+ \lambda\, S_L \left[\sqrt{1-\epsilon^2}\, F_{LL} + 2\sqrt{\epsilon(1-\epsilon)}\, \cos\phi\, F_{LL}^{\cos\phi} \right] \\
&+ \lambda\, |S_T| \left[\sqrt{1-\epsilon^2}\, \cos(\phi - \phi_S)\, F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\, \cos\phi_S\, F_{LT}^{\cos\phi_S} + \sqrt{2\epsilon(1-\epsilon)}\, \cos(2\phi - \phi_S)\, F_{LT}^{\cos(2\phi-\phi_S)} \right] \Bigg], \qquad (44)
\end{aligned}
$$

where we have adapted to use our normalization of the lepton and target helicities, and the subscripts on the structure functions refer to the polarization of the beam $P_{\text{beam}} = U,\, L$, polarization of the target $P_{\text{target}} = U,\, L,\, T$, and if there is a third subscript it specifies the polarization of the virtual photon $P_{\gamma^*} = L,\, T$. The invariant ϵ is the ratio of longitudinal to transverse virtual photon flux and is given by [19]

$$
\epsilon = \frac{1 - y - \frac{1}{4}\, y^2\, \gamma^2}{1 - y + \frac{1}{2}\, y^2 + \frac{1}{4}\, y^2\, \gamma^2}. \qquad (45)
$$

In this study we choose the incoming 3-momentum of the target in the z-direction and denote the Cartesian components of the unit vector $\boldsymbol{S}$ by $\boldsymbol{S} = \left(S^1,\, S^2,\, S^3\right) = (\boldsymbol{S}_T, S_L) = (S_T \cos\varphi, S_T \sin\varphi, S_L)$, where $\boldsymbol{S}_T$ is normal to the hadron momentum. In terms of $\boldsymbol{S}_T$ and S_L the spin 4-vector is expressed as

$$
S^\mu(p) = \left[S^+,\, S^-,\, \boldsymbol{S}_T \right] = \left(\frac{p^+}{M}\, S_L,\, -\frac{p^-}{M}\, S_L,\, \boldsymbol{S}_T \right), \qquad (46)
$$

where for any given spin direction $\boldsymbol{S}$, the target can have the spin projections $\lambda = \pm 1$ onto this direction. Therefore, longitudinal polarization means that $\boldsymbol{S}_T = 0$ and $|S_L| = 1$, and transverse polarization implies $S_L = 0$ and $|\boldsymbol{S}_T| = 1$. The spin-vector has the properties

$$
S(p) \cdot S(p) = -1 \qquad \Longrightarrow \qquad S_L^2 + \boldsymbol{S}_T^2 = 1, \qquad\qquad S(p) \cdot p = 0. \qquad (47)
$$

The structure functions that parameterize $|\mathcal{T}_{\text{BH}}|^2$, $\mathcal{T}_{\text{I}}$, and $|\mathcal{T}_{\text{DVCS}}|^2$ can now be expressed in terms of the two electromagnetic form factors $\{F_1(t),\, F_2(t)\}$, the eight twist-2 Compton form factors $\left\{\mathcal{H},\, \mathcal{E},\, \tilde{\mathcal{H}},\, \tilde{\mathcal{E}},\, \mathcal{H}_T,\, \mathcal{E}_T,\, \tilde{\mathcal{H}}_T,\, \tilde{\mathcal{E}}_T\right\}$, and the eight twist-3 Compton form factors $\left\{\mathcal{H}_+^3,\, \mathcal{E}_+^3,\, \tilde{\mathcal{H}}_-^3,\, \tilde{\mathcal{E}}_-^3,\, \tilde{\mathcal{H}}_+^3,\, \tilde{\mathcal{E}}_+^3,\, \mathcal{H}_-^3,\, \mathcal{E}_-^3\right\}$.

The benchmark result for the BH contribution was determined by BMK in Ref. [1] and is given by

$$
|\mathcal{T}_{BH}|^2 = \frac{1}{x_B^2\, y\, (1+\epsilon^2)\, t\, \mathcal{P}_1(\phi)\, \mathcal{P}_2(\phi)} \left[c_0^{\text{BH}} + c_1^{\text{BH}} \cos(\phi) + c_2^{\text{BH}} \cos(2\phi) + s_1^{\text{BH}} \sin(\phi) \right], \qquad (48)
$$

where [1]

$$
\begin{aligned}
c_{0,UU}^{\text{BH}} &= 8\, K^2 \left[\frac{1}{\bar{\tau}}\, (2 + 3\,\epsilon^2)\, (F_1^2 - \tau\, F_2^2) + 2\, x_B^2\, (F_1 + F_2)^2 \right] \\
&\quad + (2-y)^2 \Bigg[(2+\epsilon^2) \left[\frac{x_B^2}{\tau}\, (1+\bar{\tau})^2 + 4\, (1-x_B)\, (1 + x_B\, \bar{\tau}) \right] \left(F_1^2 - \tau\, F_2^2 \right) \\
&\qquad\qquad + 4\, x_B^2 \left[x_B + \left(1 - x_B + \frac{\epsilon^2}{2} \right) (1-\bar{\tau})^2 - x_B\, (1 - 2\, x_B)\, \bar{\tau}^2 \right] (F_1 + F_2)^2 \Bigg] \\
&\quad + 2\, (1+\epsilon^2) \left(4 - 4\, y - \epsilon^2 y^2 \right) \left[2\, \epsilon^2\, (1-\tau) \left(F_1^2 - \tau\, F_2^2 \right) - x_B^2\, (1-\bar{\tau})^2\, (F_1 + F_2)^2 \right], \qquad (49)
\end{aligned}
$$

$$
c_{1,UU}^{\text{BH}} = 8\, K\, (2-y) \left[\left(\frac{x_B^2}{\tau} - 2\, x_B - \epsilon^2 \right) \left(F_1^2 - \tau\, F_2^2 \right) + 2\, x_B^2\, [1 - (1 - 2\, x_B)\, \bar{\tau}]\, (F_1 + F_2)^2 \right], \qquad (50)
$$

$$
c_{2,UU}^{\text{BH}} = 8\, x_B^2\, K^2 \left[\frac{1}{\tau} \left(F_1^2 - \tau\, F_2^2 \right) + 2\, (F_1 + F_2)^2 \right], \qquad (51)
$$

$$
\begin{aligned}
c_{0,LL}^{\text{BH}} &= 8\, \lambda\, \Lambda\, x_B\, (2-y)\, y\, \frac{\sqrt{1+\epsilon^2}}{1-\tau}\, (F_1 + F_2) \\
&\quad \times \left[\frac{1}{4}\, [x_B\, (1-\bar{\tau}) - 2\, \tau] \left[2 - x_B - 2\, (1-x_B)^2 \bar{\tau} + \epsilon^2\, (1-\bar{\tau}) - x_B\, (1 - 2\, x_B)\, \bar{\tau}^2 \right] (F_1 + F_2) \right.
\end{aligned}
$$

$$+ \left[1 - (1 - x_B)\,\bar{\tau}\right] \left[\frac{x_B^2\, M^2}{t}\,(1+\bar{\tau})^2 + (1-x_B)\,(1+x_B\bar{\tau})\right] (F_1 + \tau\, F_2) \Bigg], \tag{52}$$

$$c_{1,LL}^{\mathrm{BH}} = -8\,\lambda\,\Lambda\,x_B\,y\,K \frac{\sqrt{1+\epsilon^2}}{1-\tau}\,(F_1+F_2) \Bigg[\left[\frac{t}{2\,M^2} - x_B\,(1-\bar{\tau})\right](1 - x_B + x_B\,\bar{\tau})\,(F_1+F_2)$$
$$+ \left[1 + x_B - (3 - 2\,x_B)\,(1 + x_B\,\bar{\tau}) - \frac{4\,x_B^2\,M^2}{t}\,(1+\bar{\tau}^2)\right](F_1 + \tau\,F_2)\Bigg], \tag{53}$$

$$c_{0,LT}^{\mathrm{BH}} = -8\,\lambda\,\cos(\varphi) \frac{2\,(2-y)\,y\,Q\,K}{M} \frac{\sqrt{1+\epsilon^2}}{\sqrt{4 - 4\,y - \epsilon^2 y^2}}\,(F_1+F_2)$$
$$\times \left[\frac{x_B^3 M^2}{Q^2}\,(1-\bar{\tau})\,(F_1+F_2) + [1-(1-x_B)\bar{\tau}]\left[\frac{x_B^2\,M^2}{t}\,(1-\bar{\tau})\,F_1 + \frac{x_B}{2} F_2\right]\right], \tag{54}$$

$$c_{1,LT}^{\mathrm{BH}} = -16\,\lambda\,\cos(\varphi)\,\frac{x_B\,y\,M}{2\,Q}\sqrt{(4-4\,y-\epsilon^2 y^2)(1+\epsilon^2)}\,(F_1+F_2)$$
$$\times \left[\frac{8\,K^2\,Q^2}{t\,(4-4\,y-\epsilon^2 y^2)}\,[x_B\,(1-\bar{\tau})\,F_1 + \tau\,F_2] + (1+\epsilon^2) x_B\,(1-\bar{\tau})\,(F_1 + \tau\,F_2)\right], \tag{55}$$

$$s_{1,LT}^{\mathrm{BH}} = 16\,\lambda\,\sin(\varphi)\,\frac{y\,x_B^2\,M}{2\,Q}\sqrt{(4-4\,y-\epsilon^2 y^2)(1+\epsilon^2)^3}\,(1-\bar{\tau})\,(F_1+F_2)\,(F_1+\tau\,F_2)\,, \tag{56}$$

where $\tau = t/(4\,M^2)$ and $\bar{\tau} = t/Q^2$. The factors $\mathcal{P}_1(\phi)$ and $\mathcal{P}_2(\phi)$ are associated with the lepton propagators in the BH terms and are defined by [1]

$$Q^2\,\mathcal{P}_1(\phi) \equiv (k - q'^2) = Q^2 + 2\,k\cdot\Delta, \qquad Q^2\,\mathcal{P}_1(\phi) \equiv (k - \Delta) = \Delta^2 - 2\,k\cdot\Delta, \tag{57}$$

where

$$k\cdot\Delta = -\frac{Q^2}{2\,y\,(1+\epsilon^2)}\left[1 + 2\,K\cos\phi - \frac{t}{Q^2}\left[1 - x_B(2-y) + \frac{y\,\epsilon^2}{2}\right] + \frac{y\,\epsilon^2}{2}\right], \tag{58}$$

and the $1/Q$ power suppressed kinematical factor K is given by

$$K = -\frac{t}{Q^2}(1-x_B)\left(1 - y - \frac{y^2\epsilon^2}{4}\right)\left(1 - \frac{t_{\min}}{t}\right)\left[\sqrt{1+\epsilon^2} + \frac{4\,x_B\,(1-x_B)+\epsilon^2}{4(1-x_B)}\,\frac{t - t_{\min}}{Q^2}\right]. \tag{59}$$

This gives [1]

$$\mathcal{P}_1(\phi) = -\frac{1}{y(1+\epsilon^2)}\,[J + 2\,K\cos(\phi)]\,, \qquad \mathcal{P}_2(\phi) = 1 + \frac{t}{Q^2} + \frac{1}{y(1+\epsilon^2)}\,[J + 2\,K\cos(\phi)]\,, \tag{60}$$

where

$$J = \left(1 - y - \frac{y\,\epsilon^2}{2}\right)\left(1 + \frac{t_{\min}}{t}\right) - (1-x)\,(1-y)\,\frac{t}{Q^2}. \tag{61}$$

To have a compact notation, Ref. [1] expressed the cross section for a polarized target as

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{UU} + \cos(\theta)\ \mathrm{d}\sigma_{LL}(\Lambda) + \sin(\theta)\ \mathrm{d}\sigma_{LT}(\varphi), \tag{62}$$

where the polar angle θ appears in the decomposition of the spin vector $S = \cos(\theta)\,S_L(\Lambda) + \sin(\theta)\,S_T(\Phi)$.

The three Bethe-Heitler structure functions are given by [19]

$$F_{UU}^{\rm BH} = \frac{1}{t}\,\frac{8\,M^2}{(k\cdot q')(k'\cdot q')}\Big[2\,\tau\,G_M^2\,\big[(k\cdot\Delta)^2+(k'\cdot\Delta)^2\big] + (F_1^2+\tau F_2^2)\Big[4\,\tau\,\big[(k\cdot P)^2+(k'\cdot P)^2\big]-(1+\tau)\,\big[(k\cdot\Delta)^2+(k'\cdot\Delta)^2\big]\Big]\Big], \quad (63)$$

$$F_{LL}^{\rm BH} = \frac{2\,\lambda}{t^2}\,\frac{8\,M^2}{(k\cdot q')(k'\cdot q')}\Bigg[G_M^2\Big[\frac{p'\cdot S_L}{M}\,\big[(k'\cdot\Delta)^2-(k\cdot\Delta)^2\big]+\frac{k\cdot S_L}{M}\,\Delta^2\,k\cdot\Delta-\frac{k'\cdot S_L}{M}\,\Delta^2\,k'\cdot\Delta\Big] - F_2\,G_M\Big[\frac{p'\cdot S_L}{M}\,\big[\big[(k'\cdot\Delta)^2-(k\cdot\Delta)^2\big]-2\tau\,[k'\cdot\Delta\,k'\cdot p-k\cdot\Delta\,k\cdot p]\big] + \frac{k\cdot S_L}{M}\,(1+\tau)\,\Delta^2\,k\cdot\Delta-\frac{k'\cdot S_L}{M}\,(1+\tau)\,\Delta^2\,k'\cdot\Delta\Big]\Bigg], \quad (64)$$

$$F_{LT}^{\rm BH} = F_{LL}^{\rm BH}(S_L\to S_T), \quad (65)$$

The six interference structure functions up to twist-3 are given by [19]

$$F_{UU}^{I} = F_{UU}^{I,tw2}+\frac{K}{\sqrt{Q^2}}F_{UU}^{I,tw3} \quad (66)$$

$$F_{UU}^{\mathcal{I},tw2} = A_{UU}^{\mathcal{I}}\,\Re\,(F_1\mathcal{H}+\tau F_2\mathcal{E})+B_{UU}^{\mathcal{I}}G_M\Re\,(\mathcal{H}+\mathcal{E})+C_{UU}^{\mathcal{I}}G_M\Re\tilde{\mathcal{H}} \quad (67)$$

$$F_{LU}^{I,tw2} = A_{LU}^{\mathcal{I}}\,\Im\,(F_1\mathcal{H}+\tau F_2\mathcal{E})+B_{LU}^{\mathcal{I}}G_M\,\Im\,(\mathcal{H}+\mathcal{E})+C_{LU}^{\mathcal{I}}G_M\Im\tilde{\mathcal{H}}, \quad (68)$$

$$F_{UL}^{I,tw2} = A_{UL}^{\mathcal{I}}\Im\Big(F_1(\tilde{\mathcal{H}}-\xi\tilde{\mathcal{E}})+\tau F_2\tilde{\mathcal{E}}\Big)+B_{UL}^{\mathcal{I}}G_M\Im\tilde{\mathcal{H}}+C_{UL}^{\mathcal{I}}G_M\Im\,(\mathcal{H}+\mathcal{E})\,, \quad (69)$$

$$F_{LL}^{\mathcal{I},tw2} = A_{LL}^{\mathcal{I}}\Re\Big(F_1(\tilde{\mathcal{H}}-\xi\tilde{\mathcal{E}})+\tau F_2\tilde{\mathcal{E}}\Big)+B_{LL}^{\mathcal{I}}G_M\Re\tilde{\mathcal{H}}+C_{LL}^{\mathcal{I}}G_M\Re\,(\mathcal{H}+\mathcal{E})\,, \quad (70)$$

$$F_{UU}^{\mathcal{I},tw3} = \Re\Big\{A_{UU}^{(3)\mathcal{I}}\Big[F_1(2\tilde{\mathcal{H}}_{2T}+\mathcal{E}_{2T})+F_2(\mathcal{H}_{2T}+\tau\tilde{\mathcal{H}}_{2T})\Big]+B_{UU}^{(3)\mathcal{I}}G_M\,\tilde{E}_{2T}+C_{UU}^{(3)\mathcal{I}}G_M\,\Big[2\xi H_{2T}-\tau(\tilde{E}_{2T}-\xi E_{2T})\Big]\Big\} + \Re\Big\{\tilde{A}_{UU}^{(3)\mathcal{I}}\Big[F_1(2\tilde{H}'_{2T}+E'_{2T})+F_2(H'_{2T}+\tau\tilde{H}'_{2T})\Big]+\tilde{B}_{UU}^{(3)\mathcal{I}}G_M\tilde{E}'_{2T}+\tilde{C}_{UU}^{(3)\mathcal{I}}G_M\Big[2\xi H'_{2T}-\tau(\tilde{E}'_{2T}-\xi E'_{2T})\Big]\Big\}, \quad (71)$$

$$F_{LU}^{I,tw3} = \Im\Big\{A_{LU}^{(3)\mathcal{I}}\Big[F_1(2\tilde{\mathcal{H}}_{2T}+\mathcal{E}_{2T})+F_2(\mathcal{H}_{2T}+\tau\tilde{\mathcal{H}}_{2T})\Big]+B_{LU}^{(3)\mathcal{I}}G_M\,\tilde{E}_{2T}+C_{LU}^{(3)\mathcal{I}}G_M\,\Big[2\xi H_{2T}-\tau(\tilde{E}_{2T}-\xi E_{2T})\Big]\Big\} + \Im\Big\{\tilde{A}_{LU}^{(3)\mathcal{I}}\Big[F_1(2\tilde{H}'_{2T}+E'_{2T})+F_2(H'_{2T}+\tau\tilde{H}'_{2T})\Big]+\tilde{B}_{LU}^{(3)\mathcal{I}}G_M\tilde{E}'_{2T}+\tilde{C}_{LU}^{(3)\mathcal{I}}G_M\Big[2\xi H'_{2T}-\tau(\tilde{E}'_{2T}-\xi E'_{2T})\Big]\Big\}, \quad (72)$$

$$F_{UL}^{\mathcal{I},tw3} = \Im\Big\{A_{UL}^{(3)\mathcal{I}}G_M\mathcal{H}_{2T}+B_{UL}^{(3)\mathcal{I}}F_2\mathcal{H}_{2T}+C_{UL}^{(3)\mathcal{I}}G_M\mathcal{E}_{2T}+D_{UL}^{(3)\mathcal{I}}G_M(\xi\mathcal{E}_{2T}-\tilde{\mathcal{E}}_{2T})\Big\} + \Im\Big\{\tilde{A}_{UL}^{(3)\mathcal{I}}G_M\mathcal{H}'_{2T}+\tilde{B}_{UL}^{(3)\mathcal{I}}F_2\mathcal{H}'_{2T}+\tilde{C}_{UL}^{(3)\mathcal{I}}G_M\mathcal{E}'_{2T}+\tilde{D}_{UL}^{(3)\mathcal{I}}G_M(\xi\mathcal{E}'_{2T}-\tilde{\mathcal{E}}'_{2T})\Big\}, \quad (73)$$

$$F_{LL}^{\mathcal{I},tw3} = \Re\Big\{A_{LL}^{(3)\mathcal{I}}G_M\mathcal{H}_{2T}+B_{LL}^{(3)\mathcal{I}}F_2\mathcal{H}_{2T}+C_{LL}^{(3)\mathcal{I}}G_M\mathcal{E}_{2T}+D_{LL}^{(3)\mathcal{I}}G_M(\xi\mathcal{E}_{2T}-\tilde{\mathcal{E}}_{2T})\Big\} + \Re\Big\{\tilde{A}_{LL}^{(3)\mathcal{I}}G_M\mathcal{H}'_{2T}+\tilde{B}_{LL}^{(3)\mathcal{I}}F_2\mathcal{H}'_{2T}+\tilde{C}_{LL}^{(3)\mathcal{I}}G_M\mathcal{E}'_{2T}+\tilde{D}_{LL}^{(3)\mathcal{I}}G_M(\xi\mathcal{E}'_{2T}-\tilde{\mathcal{E}}'_{2T})\Big\}, \quad (74)$$

$$F_{UT_x}^{\mathcal{I}} = \Big(\Sigma_S^\rho\cos\phi+\Sigma_A^\rho\sin\phi\Big)\Big\{A_{T_x,\rho}^{\mathcal{I}}G_M\Im\mathcal{E}+B_{T_x\,\rho}^{\mathcal{I}}F_2\Im(\mathcal{H}+\mathcal{E})+C_{T_x\,\rho}^{\mathcal{I}}G_M\Im(\mathcal{H}+\mathcal{E})+\tilde{A}_{T_x}^{\mathcal{I}}G_M\Im\tilde{\mathcal{H}}+\tilde{B}_{T_x}^{\mathcal{I}}F_2\Im\tilde{\mathcal{H}}+\tilde{C}_{T_x}^{\mathcal{I}}G_M\Im\tilde{\mathcal{E}}\Big\}, \quad (75)$$

$$F_{UT_y}^{\mathcal{I}} = -\Big(\Sigma_S^\rho\cos\phi+\Sigma_A^\rho\sin\phi\Big)\Big\{A_{T_y,\rho}^{\mathcal{I}}G_M\Im\mathcal{E}+B_{T_y\,\rho}^{\mathcal{I}}F_2\Im(\mathcal{H}+\mathcal{E})+C_{T_y\,\rho}^{\mathcal{I}}G_M\Im(\mathcal{H}+\mathcal{E})+\tilde{A}_{T_y}^{\mathcal{I}}G_M\Im\tilde{\mathcal{H}}+\tilde{B}_{T_y}^{\mathcal{I}}F_2\Im\tilde{\mathcal{H}}+\tilde{C}_{T_y}^{\mathcal{I}}G_M\Im\tilde{\mathcal{E}}\Big\}, \quad (76)$$

$$F_{LT_x}^{\mathcal{I}} = \Big(D_S^\rho\cos\phi+D_A^\rho\sin\phi\Big)\Big\{A_{T_x,\rho}^{\mathcal{I}}G_M\Im\mathcal{E}+B_{T_x\,\rho}^{\mathcal{I}}F_2\Im(\mathcal{H}+\mathcal{E})+C_{T_x\,\rho}^{\mathcal{I}}G_M\Im(\mathcal{H}+\mathcal{E})+\tilde{A}_{T_x}^{\mathcal{I}}G_M\Im\tilde{\mathcal{H}}+\tilde{B}_{T_x}^{\mathcal{I}}F_2\Im\tilde{\mathcal{H}}+\tilde{C}_{T_x}^{\mathcal{I}}G_M\Im\tilde{\mathcal{E}}\Big\}, \quad (77)$$

$$F_{LT_y}^{\mathcal{I}} = -\left(D_S^\rho\cos\phi+D_A^\rho\sin\phi\right)\,\Im\Big\{A_{T_y,\rho}^{\mathcal{I}}G_M\mathcal{E}+B_{T_y\,\rho}^{\mathcal{I}}F_2(\mathcal{H}+\mathcal{E})+C_{T_y\,\rho}^{\mathcal{I}}G_M(\mathcal{H}+\mathcal{E})+\tilde{A}_{T_y}^{\mathcal{I}}G_M\tilde{\mathcal{H}}+\tilde{B}_{T_y}^{\mathcal{I}}F_2\tilde{\mathcal{H}}+\tilde{C}_{T_y}^{\mathcal{I}}G_M\tilde{\mathcal{E}}\Big\}. \quad (78)$$

The 18 structure functions that appear in the DVCS piece of the differential cross section are, up to twist-3, given by [19]

$$
\begin{aligned}
F_{UU,T} &= 4\Big[(1-\xi^2)\Big[(\Re\mathcal{H})^2+(\Im\mathcal{H})^2+(\Re\tilde{\mathcal{H}})^2+(\Im\tilde{\mathcal{H}})^2\Big]+\frac{t_o-t}{2M^2}\left[(\Re\mathcal{E})^2+(\Im\mathcal{E})^2+\xi^2(\Re\tilde{\mathcal{E}})^2+\xi^2(\Im\tilde{\mathcal{E}})^2\right]\\
&\quad-\frac{2\xi^2}{1-\xi^2}\Big(\Re\mathcal{H}\,\Re\mathcal{E}+\Im\mathcal{H}\Im\mathcal{E}+\Re\tilde{\mathcal{H}}\,\Re\tilde{\mathcal{E}}+\Im\tilde{\mathcal{H}}\Im\tilde{\mathcal{E}}\Big)\Big]\\
F_{LL} &= 4\Big[2(1-\xi^2)\Big(\Re\mathcal{H}\,\Re\tilde{\mathcal{H}}+\Im\mathcal{H}\,\Im\tilde{\mathcal{H}}\Big)+2\,\frac{t_o-t}{2M^2}\Big(\Re\mathcal{E}\,(\xi\Re\tilde{\mathcal{E}})+\Im\mathcal{E}\,(\xi\Im\tilde{\mathcal{E}})\Big)\\
&\quad+\frac{2\xi^2}{1-\xi^2}\Big(\Re\mathcal{H}\Re\tilde{\mathcal{E}}+\Im\mathcal{H}\Im\tilde{\mathcal{E}}+\Re\tilde{\mathcal{H}}\Re\mathcal{E}+\Im\tilde{\mathcal{H}}\Im\mathcal{E}\Big)\Big]
\end{aligned}
\tag{79}
$$

$$
F_{UT,T}^{\sin(\phi-\phi_S)} = \frac{\sqrt{t_0-t}}{M}\Big[\Im m\mathcal{H}\Re e\mathcal{E}-\Re\mathcal{H}I\Im m\mathcal{E}+\Re\tilde{\mathcal{H}}\,(\xi\Im\tilde{\mathcal{E}})-\Im m\tilde{\mathcal{H}}\,(\xi\Re\tilde{\mathcal{E}})\Big]
\tag{80}
$$

$$
F_{LT}^{\cos(\phi-\phi_S)} = \frac{\sqrt{t_0-t}}{M}\Big[-\Re\tilde{\mathcal{H}}\Re e\mathcal{E}-\Im m\tilde{\mathcal{H}}\Im m\mathcal{E}+\Re\mathcal{H}\,(\xi\Re\tilde{\mathcal{E}})+\Im m\mathcal{H}\,(\xi\Im\tilde{\mathcal{E}})+\frac{\xi^2}{1-\xi^2}\Big(\Re\tilde{\mathcal{E}}\Re e\mathcal{E}+\Im\tilde{\mathcal{E}}\Im m\mathcal{E}\Big)\Big],
\tag{81}
$$

$$
\begin{aligned}
F_{UU}^{\cos\phi} &= -\frac{2K}{\sqrt{Q^2}}(1-\xi^2)\\
&\quad\times\Re\mathrm{e}\Big\{\Big(2\tilde{\mathcal{H}}_{2T}+\mathcal{E}_{2T}+2\tilde{\mathcal{H}}'_{2T}+\mathcal{E}'_{2T}\Big)^*\Big(\mathcal{H}-\frac{\xi^2}{1-\xi^2}\mathcal{E}\Big)\\
&\quad+\Big(\mathcal{H}_{2T}+\frac{t_0-t}{4M^2}\tilde{\mathcal{H}}_{2T}+\mathcal{H}'_{2T}+\frac{t_0-t}{4M^2}\tilde{\mathcal{H}}'_{2T}\Big)^*\Big(\mathcal{E}-\xi\tilde{\mathcal{E}}\Big)\\
&\quad-2\xi\,\big(\tilde{\mathcal{E}}_{2T}+\tilde{\mathcal{E}}'_{2T}\big)^*\Big(\tilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}\tilde{\mathcal{E}}\Big)+\frac{\xi}{1-\xi^2}\Big(\tilde{\mathcal{E}}_{2T}-\xi\mathcal{E}_{2T}+\tilde{\mathcal{E}}'_{2T}-\xi\mathcal{E}'_{2T}\Big)^*\Big(\mathcal{E}-\xi\tilde{\mathcal{E}}\Big)\\
&\quad+\frac{t_0-t}{16M^2}\Big(\tilde{\mathcal{H}}_{2T}+\tilde{\mathcal{H}}'_{2T}\Big)^*\Big(\mathcal{E}+\xi\tilde{\mathcal{E}}\Big)\Big\}
\end{aligned}
\tag{82}
$$

$$
\begin{aligned}
F_{LU}^{\sin\phi} &= -\frac{2K}{\sqrt{Q^2}}(1-\xi^2)\Im\mathrm{m}\Big\{\Big(2\tilde{\mathcal{H}}_{2T}+\mathcal{E}_{2T}+2\tilde{\mathcal{H}}'_{2T}+\mathcal{E}'_{2T}\Big)^*\Big(\mathcal{H}-\frac{\xi^2}{1-\xi^2}\mathcal{E}\Big)\\
&\quad+\Big(\mathcal{H}_{2T}+\frac{t_0-t}{4M^2}\tilde{\mathcal{H}}_{2T}+\mathcal{H}'_{2T}+\frac{t_0-t}{4M^2}\tilde{\mathcal{H}}'_{2T}\Big)^*\Big(\mathcal{E}-\xi\tilde{\mathcal{E}}\Big)\\
&\quad-2\xi\,\big(\tilde{\mathcal{E}}_{2T}+\tilde{\mathcal{E}}'_{2T}\big)^*\Big(\tilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}\tilde{\mathcal{E}}\Big)+\frac{\xi}{1-\xi^2}\Big(\tilde{\mathcal{E}}_{2T}-\xi\mathcal{E}_{2T}+\tilde{\mathcal{E}}'_{2T}-\xi\mathcal{E}'_{2T}\Big)^*\Big(\mathcal{E}-\xi\tilde{\mathcal{E}}\Big)\\
&\quad+\frac{t_0-t}{16M^2}\Big(\tilde{\mathcal{H}}_{2T}+\tilde{\mathcal{H}}'_{2T}\Big)^*\Big(\mathcal{E}+\xi\tilde{\mathcal{E}}\Big)\Big\}
\end{aligned}
\tag{83}
$$

$$
\begin{aligned}
F_{UL}^{\sin\phi} &= -\frac{2K}{\sqrt{Q^2}}(1-\xi^2)\Im\Big\{\Big(\mathcal{E}_{2T}-\xi\tilde{\mathcal{E}}_{2T}+\mathcal{E}'_{2T}-\xi\tilde{\mathcal{E}}'_{2T}\Big)^*\Big(\tilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}\tilde{\mathcal{E}}\Big)\\
&\quad+\frac{t_0-t}{16M^2}\Big(\tilde{\mathcal{H}}_{2T}+\tilde{\mathcal{H}}'_{2T}\Big)^*\Big(\mathcal{E}+\xi\tilde{\mathcal{E}}\Big)\\
&\quad+\Big(\mathcal{H}_{2T}+\frac{t_0-t}{4M^2}\tilde{\mathcal{H}}_{2T}+\mathcal{H}'_{2T}+\frac{t_0-t}{4M^2}\tilde{\mathcal{H}}'_{2T}\Big)^*\Big(\mathcal{E}-\xi\tilde{\mathcal{E}}\Big)\\
&\quad+\frac{\xi}{1-\xi^2}\Big(\tilde{\mathcal{E}}_{2T}-\xi\mathcal{E}_{2T}+\tilde{\mathcal{E}}'_{2T}-\xi\mathcal{E}'_{2T}\Big)^*\Big(\mathcal{E}-\xi\tilde{\mathcal{E}}\Big)\Big\}
\end{aligned}
\tag{84}
$$

$$
\begin{aligned}
F_{LL}^{\cos\phi} &= -\frac{2K}{\sqrt{Q^2}}(1-\xi^2)\Re\Big\{\Big(\mathcal{E}_{2T}-\xi\tilde{\mathcal{E}}_{2T}+\mathcal{E}'_{2T}-\xi\tilde{\mathcal{E}}'_{2T}\Big)^*\Big(\tilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}\tilde{\mathcal{E}}\Big)\\
&\quad+\frac{t_0-t}{16M^2}\Big(\tilde{\mathcal{H}}_{2T}+\tilde{\mathcal{H}}'_{2T}\Big)^*\Big(\mathcal{E}+\xi\tilde{\mathcal{E}}\Big)\\
&\quad+\Big(\mathcal{H}_{2T}+\frac{t_0-t}{4M^2}\tilde{\mathcal{H}}_{2T}+\mathcal{H}'_{2T}+\frac{t_0-t}{4M^2}\tilde{\mathcal{H}}'_{2T}\Big)^*\Big(\mathcal{E}-\xi\tilde{\mathcal{E}}\Big)\\
&\quad+\frac{\xi}{1-\xi^2}\Big(\tilde{\mathcal{E}}_{2T}-\xi\mathcal{E}_{2T}+\tilde{\mathcal{E}}'_{2T}-\xi\mathcal{E}'_{2T}\Big)^*\Big(\mathcal{E}-\xi\tilde{\mathcal{E}}\Big)\Big\}
\end{aligned}
\tag{85}
$$

$$
\begin{aligned}
F_{UT}^{\sin(\phi_S)} &= \frac{K}{\sqrt{2Q^2}}\sqrt{1-\xi^2}\,\Im\Big[\Big(\mathcal{H}_{2T}+\mathcal{H}'_{2T}\Big)+\frac{t_0-t}{4M^2}\Big(\tilde{\mathcal{H}}_{2T}+\tilde{\mathcal{H}}'_{2T}\Big)+\frac{\xi}{1-\xi^2}\Big(\tilde{\mathcal{E}}_{2T}+\tilde{\mathcal{E}}'_{2T}\Big)\\
&\quad-\frac{\xi^2}{1-\xi^2}\Big(\mathcal{E}_{2T}+\mathcal{E}'_{2T}\Big)\Big]\Big(\mathcal{H}+\tilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}\Big(\mathcal{E}+\tilde{\mathcal{E}}\Big)\Big)^*
\end{aligned}
$$

$$+\frac{K}{\sqrt{2}Q^2}\frac{t_0-t}{4M^2}\Im\left[2\tilde{\mathcal{H}}_{2T}+(1+\xi)\left(\mathcal{E}_{2T}-\tilde{\mathcal{E}}_{2T}\right)+2\tilde{\mathcal{H}}'_{2T}+(1+\xi)\left(\mathcal{E}'_{2T}-\tilde{\mathcal{E}}'_{2T}\right)\right]\left(\mathcal{E}+\xi\tilde{\mathcal{E}}\right)^* \tag{86}$$

$$F_{UT}^{\sin(2\phi-\phi_S)}=-\frac{K}{\sqrt{2}Q^2}\frac{t_0-t}{4M^2}\Im\left[2\tilde{\mathcal{H}}_{2T}+(1-\xi)\left(\mathcal{E}_{2T}+\tilde{\mathcal{E}}_{2T}\right)+2\tilde{\mathcal{H}}'_{2T}+(1-\xi)\left(\mathcal{E}'_{2T}+\tilde{\mathcal{E}}'_{2T}\right)\right]$$
$$\times\left(\mathcal{E}-\xi\tilde{\mathcal{E}}\right)^*$$
$$-\frac{K}{\sqrt{2}Q^2}\sqrt{1-\xi^2}\frac{t_0-t}{8M^2}\Im\left[\tilde{\mathcal{H}}_{2T}+\tilde{\mathcal{H}}'_{2T}\right]\left(\mathcal{H}-\tilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}\left(\mathcal{E}-\tilde{\mathcal{E}}\right)\right)^* \tag{87}$$

$$F_{LT}^{\cos(\phi_S)}=\frac{K}{\sqrt{2}Q^2}\sqrt{1-\xi^2}\Re\mathrm{e}\left[\left(\mathcal{H}_{2T}+\mathcal{H}'_{2T}\right)+\frac{t_0-t}{4M^2}\left(\tilde{\mathcal{H}}_{2T}+\tilde{\mathcal{H}}'_{2T}\right)+\frac{\xi}{1-\xi^2}\left(\tilde{\mathcal{E}}_{2T}+\tilde{\mathcal{E}}'_{2T}\right)\right.$$
$$\left.-\frac{\xi^2}{1-\xi^2}\left(\mathcal{E}_{2T}+\mathcal{E}'_{2T}\right)\right]\left(\mathcal{H}+\tilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}\left(\mathcal{E}+\tilde{\mathcal{E}}\right)\right)^*$$
$$+\frac{K}{\sqrt{2}Q^2}\frac{t_0-t}{4M^2}\Re\mathrm{e}\left[2\tilde{\mathcal{H}}_{2T}+(1+\xi)\left(\mathcal{E}_{2T}-\tilde{\mathcal{E}}_{2T}\right)+2\tilde{\mathcal{H}}'_{2T}+(1+\xi)\left(\mathcal{E}'_{2T}-\tilde{\mathcal{E}}'_{2T}\right)\right]\left(\mathcal{E}+\xi\tilde{\mathcal{E}}\right)^* \tag{88}$$

$$F_{LT}^{\cos(2\phi-\phi_S)}=\frac{K}{\sqrt{2}Q^2}\frac{t_0-t}{4M^2}\Re\mathrm{e}\left[2\tilde{\mathcal{H}}_{2T}+(1-\xi)\left(\mathcal{E}_{2T}+\tilde{\mathcal{E}}_{2T}\right)+2\tilde{\mathcal{H}}'_{2T}+(1-\xi)\left(\mathcal{E}'_{2T}+\tilde{\mathcal{E}}'_{2T}\right)\right]\left(\mathcal{E}-\xi\tilde{\mathcal{E}}\right)^*$$
$$+\frac{K}{\sqrt{2}Q^2}\sqrt{1-\xi^2}\frac{t_0-t}{8M^2}\Re\mathrm{e}\left[\tilde{\mathcal{H}}_{2T}+\tilde{\mathcal{H}}'_{2T}\right]\left(\mathcal{H}-\tilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}\left(\mathcal{E}-\tilde{\mathcal{E}}\right)\right)^*, \tag{89}$$

$$F_{UU,L}=\frac{K^2}{Q^2}\left(\left|2\tilde{\mathcal{H}}_{2T}+(1-\xi)(\mathcal{E}_{2T}+\tilde{\mathcal{E}}_{2T})+2\tilde{\mathcal{H}}'_{2T}+(1-\xi)(\mathcal{E}'_{2T}+\tilde{\mathcal{E}}'_{2T})\right|^2\right.$$
$$\left.+\left|2\tilde{\mathcal{H}}_{2T}+(1+\xi)(\mathcal{E}_{2T}-\tilde{\mathcal{E}}_{2T})+2\tilde{\mathcal{H}}'_{2T}+(1+\xi)(\mathcal{E}'_{2T}-\tilde{\mathcal{E}}'_{2T})\right|^2\right), \tag{90}$$

$$F_{UT,L}=\frac{K^2}{Q^2}\left(\left[2\tilde{\mathcal{H}}_{2T}+(1-\xi)(\mathcal{E}_{2T}+\tilde{\mathcal{E}}_{2T})+2\tilde{\mathcal{H}}'_{2T}+(1-\xi)(\mathcal{E}'_{2T}+\tilde{\mathcal{E}}'_{2T})\right]^*\left[(\mathcal{H}_{2T}+\mathcal{H}'_{2T})\right.\right.$$
$$\left.+\frac{t_0-t}{4M^2}\left(\tilde{\mathcal{H}}_{2T}+\tilde{\mathcal{H}}'_{2T}\right)+\frac{\xi}{1-\xi^2}\left(\tilde{\mathcal{E}}_{2T}+\tilde{\mathcal{E}}'_{2T}\right)-\frac{\xi^2}{1-\xi^2}\left(\mathcal{E}_{2T}+\mathcal{E}'_{2T}\right)\right]+\left[2\tilde{\mathcal{H}}_{2T}+(1+\xi)(\mathcal{E}_{2T}-\tilde{\mathcal{E}}_{2T})\right.$$
$$\left.\left.+2\tilde{\mathcal{H}}'_{2T}+(1+\xi)(\mathcal{E}'_{2T}-\tilde{\mathcal{E}}'_{2T})\right]\left[\tilde{\mathcal{H}}_{2T}+\tilde{\mathcal{H}}'_{2T}\right]^*\right), \tag{91}$$

$$F_{UU}^{\cos 2\phi}=-2\frac{\alpha_S}{2\pi}\sqrt{1-\xi^2}\frac{t_0-t}{4M^2}\Re\mathrm{e}\left[\sqrt{1-\xi^2}\left(\tilde{\mathcal{H}}_T^g+(1-\xi)\frac{\mathcal{E}_T^g+\tilde{\mathcal{E}}_T^g}{2}\right)\left(\mathcal{H}+\tilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}(\mathcal{E}+\tilde{\mathcal{E}}\right)^*\right.$$
$$+\sqrt{1-\xi^2}\left(\tilde{\mathcal{H}}_T^g+(1+\xi)\frac{\mathcal{E}_T^g-\tilde{\mathcal{E}}_T^g}{2}\right)\left(\mathcal{H}-\tilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}(\mathcal{E}+\tilde{\mathcal{E}}\right)^*$$
$$+\frac{\sqrt{t_0-t}}{2M}\left(\tilde{\mathcal{H}}_T^g+(1+\xi)\frac{\mathcal{E}_T^g-\tilde{\mathcal{E}}_T^g}{2}\right)\left(\mathcal{E}+\xi\tilde{\mathcal{E}}\right)^*$$
$$\left.-\sqrt{1-\xi^2}\left(\mathcal{H}_T^g+\frac{t_0-t}{M^2}\tilde{\mathcal{H}}_T^g-\frac{\xi^2}{1-\xi^2}\mathcal{E}_T^g+\frac{\xi}{1-\xi^2}\tilde{\mathcal{E}}_T^g\right)\left(\mathcal{E}-\xi\tilde{\mathcal{E}}\right)^*\right], \tag{92}$$

$$F_{UL}^{\sin 2\phi}=-2\frac{\alpha_S}{2\pi}\sqrt{1-\xi^2}\frac{t_0-t}{4M^2}\Im\mathrm{m}\left[\sqrt{1-\xi^2}\left(\tilde{\mathcal{H}}_T^g+(1-\xi)\frac{\mathcal{E}_T^g+\tilde{\mathcal{E}}_T^g}{2}\right)\left(\mathcal{H}+\tilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}(\mathcal{E}+\tilde{\mathcal{E}}\right)^*\right.$$
$$+\sqrt{1-\xi^2}\left(\tilde{\mathcal{H}}_T^g+(1+\xi)\frac{\mathcal{E}_T^g-\tilde{\mathcal{E}}_T^g}{2}\right)\left(\mathcal{H}-\tilde{\mathcal{H}}-\frac{\xi^2}{1-\xi^2}(\mathcal{E}+\tilde{\mathcal{E}}\right)^*$$
$$+\frac{\sqrt{t_0-t}}{2M}\left(\tilde{\mathcal{H}}_T^g+(1+\xi)\frac{\mathcal{E}_T^g-\tilde{\mathcal{E}}_T^g}{2}\right)\left(\mathcal{E}+\xi\tilde{\mathcal{E}}\right)^*$$

$$- \sqrt{1-\xi^2}\Big(\mathcal{H}_T^g + \frac{t_0 - t}{M^2}\tilde{\mathcal{H}}_T^g - \frac{\xi^2}{1-\xi^2}\mathcal{E}_T^g + \frac{\xi}{1-\xi^2}\tilde{\mathcal{E}}_T^g\Big)\Big(\mathcal{E} - \xi\tilde{\mathcal{E}}\Big)^*\Bigg], \tag{93}$$

$$F_{UT}^{\sin(\phi+\phi_S)} = 0, \tag{94}$$

$$F_{UT}^{\sin(3\phi-\phi_S)} = -4\frac{\alpha_S}{2\pi}\sqrt{1-\xi^2}\frac{\sqrt{t_0-t}^3}{8M^3}\Im\Bigg[(1-\xi^2)\Big(\tilde{\mathcal{H}}_T^g\Big)\Big(\mathcal{H} - \tilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2}(\mathcal{E}-\tilde{\mathcal{E}})\Big)^* + \Big(\tilde{\mathcal{H}}_T^g + (1-\xi)\frac{\mathcal{E}_T^g + \tilde{\mathcal{E}}_T^g}{2}\Big)\Big(\mathcal{E} - \xi\tilde{\mathcal{E}}\Big)^*\Bigg]. \tag{95}$$

Therefore, the Compton form factors that appear in the interference and DVCS structure functions are

$$\text{Twist 2: } \mathcal{H},\ \mathcal{E},\ \tilde{\mathcal{H}},\ \tilde{\mathcal{E}},\ \mathcal{H}_T^g,\ \mathcal{E}_T^g,\ \tilde{\mathcal{H}}_T^g,\ \tilde{\mathcal{E}}_T^g, \qquad \text{Twist 3: } \mathcal{H}_{2T},\ \mathcal{E}_{2T},\ \tilde{\mathcal{H}}_{2T},\ \tilde{\mathcal{E}}_{2T},\ \mathcal{H}'_{2T},\ \mathcal{E}'_{2T},\ \tilde{\mathcal{H}}'_{2T},\ \tilde{\mathcal{E}}'_{2T} \tag{96}$$

where these Compton form factors are functions of $\mathcal{F} = \mathcal{F}(\xi, t, Q^2)$. In addition, the following GPDs appear in the interference structure functions:

$$\text{Twist 3: } H_{2T},\ E_{2T},\ \tilde{E}_{2T},\ H'_{2T},\ E'_{2T},\ \tilde{E}'_{2T}. \tag{97}$$

4. Deeply Virtual Meson Production

5. Double Deeply Virtual Compton Scattering

6. Other Deeply Virtual Exclusive Processes

7. Generalized Parton Distributions

7.1 Spin-half Targets

The GPDs are formally defined through matrix elements of quark and gluon operators at a light-like separation. For a spin-half target like the nucleon the leading-twist unpolarized quark and gluon GPDs are defined by [18, 20]

$$\bar{p}\cdot n\int\frac{\mathrm{d}\lambda}{2\pi}\,e^{ixP\cdot n\lambda}\left\langle p',\lambda'\left|\bar{\psi}^q(-\tfrac{1}{2}\lambda n)\,\not{n}\,\psi^q(\tfrac{1}{2}\lambda n)\right|p,\lambda\right\rangle=\bar{u}(p',\lambda')\left[\quad H^q\,\not{n}+\quad E^q\,\frac{i\sigma^{n\Delta}}{2M}\right]u(p,\lambda),\tag{98}$$

$$n_\mu n_\nu\int\frac{\mathrm{d}\lambda}{2\pi}\,e^{ixP\cdot n\lambda}\left\langle p',\lambda'\left|G^{\mu\alpha}(-\tfrac{1}{2}\lambda n)\,G_\alpha{}^{\nu}(\tfrac{1}{2}\lambda n)\right|p,\lambda\right\rangle=\bar{u}(p',\lambda')\left[x\,H^g\,\not{n}+x\,E^g\,\frac{i\sigma^{n\Delta}}{2M}\right]u(p,\lambda).\tag{99}$$

The leading-twist quark and gluon helicity-dependent GPDs for a spin-half target read [18, 20]

$$\bar{p}\cdot n\int\frac{\mathrm{d}\lambda}{2\pi}\,e^{ixP\cdot n\lambda}\left\langle p',\lambda'\left|\bar{\psi}^q(-\tfrac{1}{2}\lambda n)\,\not{n}\gamma_5\,\psi^q(\tfrac{1}{2}\lambda n)\right|p,\lambda\right\rangle=\bar{u}(p',\lambda')\left[\quad\tilde{H}^q\,\not{n}\gamma_5+\quad\tilde{E}^q\,\frac{\Delta\cdot n\,\gamma_5}{2M}\right]u(p,\lambda),\tag{100}$$

$$-i\,n_\mu n_\nu\int\frac{\mathrm{d}\lambda}{2\pi}\,e^{ixP\cdot n\lambda}\left\langle p',\lambda'\left|G^{\mu\alpha}(-\tfrac{1}{2}\lambda n)\,\tilde{G}_\alpha{}^{\nu}(\tfrac{1}{2}\lambda n)\right|p,\lambda\right\rangle=\bar{u}(p',\lambda')\left[x\,\tilde{H}^g\,\not{n}\gamma_5+x\,\tilde{E}^g\,\frac{\Delta\cdot n\,\gamma_5}{2M}\right]u(p,\lambda),\tag{101}$$

The leading-twist quark and gluon transversity GPDs for a spin-half target read [21–23]

$$\begin{aligned}\bar{p}\cdot n\int\frac{\mathrm{d}\lambda}{2\pi}\,e^{ixP\cdot n\lambda}&\left\langle p',\lambda'\left|\bar{\psi}^q(-\tfrac{1}{2}\lambda n)\,i\sigma^{ni}\,\psi^q(\tfrac{1}{2}\lambda n)\right|p,\lambda\right\rangle=\\&\bar{u}(p',\lambda')\left[\quad H_T^q\,i\sigma^{ni}+\quad\tilde{H}_T^q\,\frac{\bar{p}\cdot n\,\Delta^i-\Delta\cdot n\,\bar{p}^i}{M^2}+\quad E_T^q\,\frac{\not{n}\,\Delta^i-\Delta\cdot n\,\gamma^i}{2M}+\quad\tilde{E}_T^q\,\frac{\not{n}\,\bar{p}^i-\bar{p}\cdot n\,\gamma^i}{M}\right]u(p,\lambda),\end{aligned}\tag{102}$$

$$\begin{aligned}n_\mu n_\nu\int\frac{\mathrm{d}\lambda}{2\pi}\,e^{ixP\cdot n\lambda}&\left\langle p',\lambda'\left|S\,G^{\mu i}(-\tfrac{1}{2}z)\,G^{\nu j}(\tfrac{1}{2}z)\right|p,\lambda\right\rangle=S\,\frac{\bar{p}\cdot n\,\Delta^j-\bar{p}\cdot\Delta\,\bar{p}^j}{2M\,\bar{p}\cdot n}\\&\bar{u}(p',\lambda')\left[x\,H_T^g\,i\sigma^{ni}+x\,\tilde{H}_T^g\,\frac{\bar{p}\cdot n\,\Delta^i-\Delta\cdot n\,\bar{p}^i}{M^2}+x\,E_T^g\,\frac{\not{n}\,\Delta^i-\Delta\cdot n\,\gamma^i}{2M}+x\,\tilde{E}_T^g\,\frac{\not{n}\,\bar{p}^i-\bar{p}\cdot n\,\gamma^i}{M}\right]u(p,\lambda).\end{aligned}\tag{103}$$

The GPDs are functions of four variables $F^{q/g}=F^{q/g}(x,\xi,t;\mu^2)$, are defined relative to the vector n which is light-like $n^2=0$, the gluon dual field strength tensor is given by $\tilde{G}^{\mu\nu}=\frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}G_{\alpha\beta}$, and for legibility we do not display the color degrees of freedom in the field operators, which should be implicitly understood as containing gauge links and color sums. We use the notation $\sigma^{n\Delta}\equiv\sigma^{\mu\nu}n_\mu\Delta_\nu$, M is the target mass, $\bar{p}=\frac{1}{2}(p'+p)$ is the average of the initial and final target momenta, $\Delta=p'-p$ is the momentum transferred to the target, $t=\Delta^2$, $x=\ell\cdot n/\bar{p}\cdot n$ is the average light cone momentum fraction of the active parton where k is the average of the initial and final parton momenta, and $\xi=-\Delta\cdot n/(2\,\bar{p}\cdot n)$ is a measure of the longitudinal momentum transfer to the target (skewness). The GPDs also depend on the renormalization scale μ^2 in accordance with the renormalization group equations [24–26]. GPDs have support in the region $-1\leqslant x\leqslant 1$ and $-1\leqslant\xi\leqslant 1$, with the constraint that for a given ξ the maximal value of $t\leqslant t_{\min}\leqslant 0$ for a physical process is $t_{\min}=-4\xi^2M^2/(1-\xi^2)$. We use the Ji convention [20, 21] for the gluon GPDs, which for the spin-independent and helicity gluon GPDs differs from the Diehl convention [18, 23] by $F^g_{\text{Diehl}}=2\,x\,F^g_{\text{Ji}}$, and for the gluon transversity GPDs there is an additional minus sign.

Using the relation

$$\sigma^{\mu\nu}\gamma_5=-\frac{1}{2}\,i\varepsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta},\tag{104}$$

the transversity quark GPDs can be expressed as [23]

$$\begin{aligned}\bar{p}\cdot n\int\frac{\mathrm{d}\lambda}{2\pi}\,e^{ixP\cdot n\lambda}&\left\langle p',\lambda'\left|\bar{\psi}^q(-\tfrac{1}{2}\lambda n)\,i\sigma^{ni}\gamma_5\,\psi^q(\tfrac{1}{2}\lambda n)\right|p,\lambda\right\rangle=\\&\bar{u}(p',\lambda')\left[H_T^q\,i\sigma^{ni}\gamma_5+\tilde{H}_T^q\,\frac{i\varepsilon^{ni\alpha\beta}\Delta_\alpha\bar{p}_\beta}{M^2}+E_T^q\,\frac{i\varepsilon^{ni\alpha\beta}\Delta_\alpha\gamma_\beta}{2M}+\quad\tilde{E}_T^q\,\frac{i\varepsilon^{ni\alpha\beta}\bar{p}_\alpha\gamma_\beta}{M}\right]u(p,\lambda).\end{aligned}\tag{105}$$

The gluon transversity GPDs involve the gluon field-strength tensor operator $S\,G^{ni}(-\frac{1}{2}z)\,G^{nj}(\frac{1}{2}z)$, where S denotes symmetrization in i and j and subtraction of the trace. Explicity, these GPDs can be expressed as [1, 22]

$$\begin{aligned}G^T_{\mu\nu}(x,\xi,t)&\equiv\frac{4}{\bar{p}\cdot n}\int\frac{d\kappa}{2\,\pi}\,\mathrm{e}^{ix\,\kappa\,\bar{p}\cdot n}\left\langle p'\left|G_{+\rho}(\kappa_2 n)\,\tau^{\perp}_{\mu\nu;\rho\sigma}\,G_{\sigma+}(\kappa_1 n)\right|p\right\rangle\\&=\frac{\tau^{\perp}_{\mu\nu;\alpha\beta}}{2M}\,\Delta^\alpha\,\bar{u}(p')\left[H_T(x,\xi,t)\,\frac{i\sigma^{\gamma\beta}q_\gamma}{\bar{p}\cdot q}+\tilde{H}_T(x,\xi,t)\,\frac{\Delta^\beta}{2M^2}+E_T(x,\xi,t)\,\frac{1}{2M}\left(\frac{\gamma\cdot q}{\bar{p}\cdot q}\Delta^\beta-\eta\,\gamma^\beta\right)-\tilde{E}_T(x,\xi,t)\,\frac{\gamma^\beta}{2M}\right]u(p),\end{aligned}\tag{106}$$

where

$$\tau^{\perp}_{\mu\nu;\rho\sigma} = \frac{1}{2}\left(g^{\perp}_{\mu\rho}g^{\perp}_{\nu\sigma} + g^{\perp}_{\mu\sigma}g^{\perp}_{\nu\rho} - g^{\perp}_{\mu\nu}g^{\perp}_{\rho\sigma}\right), \qquad g^{\perp}_{\mu\nu} = g_{\mu\nu} - n_\mu \bar{n}_\nu - \bar{n}_\mu n_\nu, \qquad n^2 = \bar{n}^2 = 0, \qquad n\cdot\bar{n} = 1. \tag{107}$$

Under the transformation $\xi \to -\xi$ the leading-twist GPDs satisfy

The GPDs exhibit several interesting properties, such as polynomiality [18], which is a consequence of Lorentz covariance and implies the x-weighted moments of GPDs are even polynomials in ξ. In the forward limit ($\xi \to 0, t \to 0$) the H^a and $\tilde{H}^a$ GPDs reduce to [1]

$$H^q(x,0,0) = q(x)\,\Theta(x) - \bar{q}(-x)\,\Theta(-x), \qquad \tilde{H}^q(x,0,0) = \Delta q(x)\,\Theta(x) + \Delta\bar{q}(-x)\,\Theta(-x), \tag{108}$$

$$2\,H^g(x,0,0) = g(x)\,\Theta(x) - g(-x)\,\Theta(-x), \qquad 2\,\tilde{H}^g(x,0,0) = \Delta g(x)\,\Theta(x) + \Delta g(-x)\,\Theta(-x), \tag{109}$$

where we have dropped the μ^2 dependence, the factor 2 for gluons avoids double counting because gluons are their own anti-particle, Θ is the Heaviside step function, and q, $\bar{q}$, and g are the PDFs for the quarks, anti-quarks, and gluons, respectively.

The tensor form factors are parameterized as [27]

The first moments of quark GPDs are related to the quark contributions to the Dirac, Pauli, axial, and pseudoscalar form factors:

$$\int_{-1}^{1} \mathrm{d}x\,[H^q, E^q, \tilde{H}^q, \tilde{E}^q](x,\xi,t;\mu^2) = [F_1^q(t), F_2^q(t), G_A^q(t), G_P^q(t)]. \tag{110}$$

An important reason for the interest in GPDs, is that their second moments are related to the quark and gluon gravitational form factors. For the nucleon this implies [28]

$$\int_{-1}^{1} \mathrm{d}x\; x\,H^a(x,\xi,t;\mu^2) = A^a(t;\mu^2) + \xi^2\,C^a(t;\mu^2), \qquad \int_{-1}^{1} \mathrm{d}x\; x\,E^a(x,\xi,t;\mu^2) = B^a(t;\mu^2) - \xi^2\,C^a(t;\mu^2), \tag{111}$$

where the form factors are defined with respect to matrix elements of the energy-momentum tensor as

$$\left\langle p',\lambda'\left|T_a^{\mu\nu}(x)\right|p,\lambda\right\rangle = \bar{u}(p',\lambda')\left[A^a(t)\,\frac{\gamma^{\{\mu}P^{\nu\}}}{2} + B^a(t)\,\frac{P^{\{\mu}i\sigma^{\nu\}\Delta}}{4\,M} + C^a(t)\,\frac{\Delta^\mu\Delta^\nu - \Delta^2\,g^{\mu\nu}}{4\,M} + M\,\bar{C}^a(t)\,g^{\mu\nu}\right]u(p,\lambda). \tag{112}$$

We have introduced the notation $a^{\{\mu}b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$. The total quark and gluon angular moment is then given by the Ji sum rule [29] as $J^a(\mu^2) = \frac{1}{2}[A^a(0,\mu^2) + B^a(0,\mu^2)]$ and $J = \frac{1}{2} = \sum_a J^a(\mu^2)$. The C^a form factors are related to internal stresses within the nucleon [28, 30–34], with $D = C(0) = \sum_a C^a(0;\mu^2)$ known as the nucleon D-term, and $\sum_a \bar{C}^a(t,\mu^2) = 0$.

A remarkable property of GPDs is known as polynomiality, which means that the Mellin moments of the GPDs are even polynomials in ξ. For the nucleon GPDs, polynomiality states that moments of the quark and gluon GPDs satisfy [18]

$$\int_{-1}^{1} \mathrm{d}x\,x^s H^a(x,\xi,t;\mu^2) = \sum_{i=0\,\text{(even)}}^{s} (2\xi)^i\,A^a_{s+1,i}(t,\mu^2) + \mathrm{mod}(s,2)\,(2\xi)^{s+1}\,C^a_{s+1}(t,\mu^2), \tag{113a}$$

$$\int_{-1}^{1} \mathrm{d}x\,x^s E^a(x,\xi,t;\mu^2) = \sum_{i=0\,\text{(even)}}^{s} (2\xi)^i\,B^a_{s+1,i}(t,\mu^2) - \mathrm{mod}(s,2)\,(2\xi)^{s+1}\,C^a_{s+1}(t,\mu^2), \tag{113b}$$

$$\int_{-1}^{1} \mathrm{d}x\,x^s \tilde{H}^a(x,\xi,t;\mu^2) = \sum_{i=0\,\text{(even)}}^{s} (2\xi)^i\,\tilde{A}^a_{s+1,i}(t,\mu^2), \tag{113c}$$

$$\int_{-1}^{1} \mathrm{d}x\,x^s \tilde{E}^a(x,\xi,t;\mu^2) = \sum_{i=0\,\text{(even)}}^{s} (2\xi)^i\,\tilde{B}^a_{s+1,i}(t,\mu^2), \tag{113d}$$

where mod$(s, 2)$ gives 1 if s is odd and zero otherwise. Note, because we use the Ji definition of the gluon GPDs, the polynomiality relations for the quarks and gluons take the same form. The quark and gluon total angular momentum is related to the second moments of the GPDs by the Ji sum rule [29]:

$$J^a(\mu^2) = \frac{1}{2}\left[A^a_{20}(0,\mu^2) + B^a_{20}(0,\mu^2)\right] = \frac{1}{2}\int_{-1}^{1} \mathrm{d}x\, x\left[H^a(x,0,0;\mu^2) + E^a(x,0,0;\mu^2)\right], \tag{114}$$

where A^a_{20} and B^a_{20} are the $s = 1, i = 0$ terms from Eqs. (113a) and (113b).

GPDs formally satisfy numerous positivity conditions [35–44], with an example for the nucleon including [39]

$$\left|H^q(x,\xi,t;\mu^2) - \frac{\xi^2}{1-\xi^2}\,E^q(x,\xi,t;\mu^2)\right|^2 + \left|\frac{\sqrt{t_{\text{min}} - t}}{2M\sqrt{1-\xi^2}}\,E^q(x,\xi,t;\mu^2)\right|^2 \leqslant \frac{q(x_{\text{in}};\mu^2)\,q(x_{\text{out}};\mu^2)}{1-\xi^2}, \tag{115}$$

where $q(x;\mu^2)$ are the familiar collinear PDFs, $x_{\text{in}} = (x+\xi)/(1+\xi)$, $x_{\text{out}} = (x-\xi)/(1-\xi)$, and this positivity constraint applies in the region $|x| > |\xi|$.

For spin-half targets such as the nucleon, there are four complex-valued CFFs ($\mathcal{H}$, $\mathcal{E}$, $\tilde{\mathcal{H}}$, $\tilde{\mathcal{E}}$) that enter the DVCS cross section at leading-twist, which are related to several quark and gluon leading-twist GPDs (H^a, E^a, $\tilde{H}^a$, $\tilde{E}^a$) by [1]

$$\mathcal{F}(\xi,t,Q^2) = \int_{-1}^{1}\mathrm{d}x\,\sum_a C^a(x,\xi,Q^2,\mu^2)\,F^a(x,\xi,t;\mu^2), \tag{116}$$

where $\mathcal{F} = \mathcal{H},\ \mathcal{E}$ are associated with the leading-twist spin-independent GPDs $F = H^a,\ E^a$, respectively, the sum is over all active parton flavors ($a = q,\ g$), and C^a is a hard-scattering coefficient function. An analogous relation holds between the $\tilde{\mathcal{H}}$ and $\tilde{\mathcal{E}}$ CFFs and the leading-twist spin-dependent GPDs $\tilde{H}^a$ and $\tilde{E}^a$, where in this case different hard-scattering coefficient functions ($\tilde{C}^a$) enter [1].

It is made clear by Eq. (116) that inferring GPDs from DVCS data involves solving several inverse problems, subject to some or all of the constraints given by Eqs. (115)–(111) [45–48]. The first step in this procedure is to obtain the CFFs from DVCS data, which at leading twist is a closed problem [1] and several leading-twist CFF extractions have been reported in the literature [49–54]. The challenge of extracting GPDs from the CFFs lies in the fact that the x dependence of the GPDs is completely integrated out and does not appear in the CFFs. Nevertheless, GPD extractions are beginning to become available; see, for instance, Ref. [48].

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APPENDIX 1. Identities

We define

$$g^{\mu\nu} = \mathrm{diag}[1, -1, -1, -1], \qquad\qquad \varepsilon_{0123} \equiv 1, \qquad \varepsilon_{0123} = -\varepsilon^{0123}. \tag{117}$$

We have [55]

$$\int \mathrm{d}\xi \, \frac{f(\xi)\, e^{i\lambda\,\xi}}{\xi - x \pm i\varepsilon} = \mp\, 2\pi i \; \Theta(\mp\lambda)\; f(x)\; e^{i\lambda\, x}, \tag{118}$$

where f should be sufficiently well behaved.