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Derivation of 6.4 and 6.7

We have the following positivity bounds

$$\frac{k_\perp^2}{M^2}\left(\left(g_{1T}^{\perp}(x,k_\perp^2)\right)^2+\left(f_{1T}^{\perp}(x,k_\perp^2)\right)^2\right)\le\left(f_1(x,k_\perp^2)\right)^2$$

as far as RHS > 0 => LHS > 0 => we can safely integrate $\int d^2k_\perp$ both sides.

We have
$$f_{1T}^{\perp}(x,k_\perp^2)=f_{1T}^{\perp(1)}(x)\frac{2M^2}{\pi\langle k_\perp^2\rangle_{f_{1T}^{\perp}}}e^{-k_\perp^2/\langle k_\perp^2\rangle_{f_{1T}^{\perp}}}$$

$$g_{1T}^{\perp}(x,k_\perp^2)=g_{1T}^{\perp(1)}(x)\frac{2M^2}{\pi\langle k_\perp^2\rangle_{g_{1T}^{\perp}}}e^{-k_\perp^2/\langle k_\perp^2\rangle_{g_{1T}^{\perp}}}$$

$$f_1(x,k_\perp^2)=f_1(x)\frac{1}{\pi\langle k_\perp^2\rangle_{f_1}}e^{-k_\perp^2/\langle k_\perp^2\rangle_{f_1}}$$

thus we obtain

$$\int d^2k_\perp\left(\frac{k_\perp^2}{2M^2}g_{1T}^{\perp}(x,k_\perp^2)\right)^2=\frac{\left(g_{1T}^{\perp(1)}(x)\right)^2}{4\langle k_\perp^2\rangle_{g_{1T}^{\perp}}\pi}$$

$$\int d^2k_\perp\left(\frac{k_\perp^2}{2M^2}f_{1T}^{\perp}(x,k_\perp^2)\right)^2=\frac{\left(f_{1T}^{\perp(1)}(x)\right)^2}{4\langle k_\perp^2\rangle_{f_{1T}^{\perp}}\pi}$$

$$\int d^2k_\perp\frac{k_\perp^2}{4M^2}\left(f_1(x,k_\perp^2)\right)^2=\frac{\left(f_1(x)\right)^2}{16M^2\pi}$$

from which we obtain

$$\frac{\left(f_1(x)\right)^2}{16\pi M^2}-\frac{\left(f_{1T}^{\perp(1)}(x)\right)^2}{4\pi\langle k_\perp^2\rangle_{f_{1T}^{\perp}}}-\frac{\left(g_{1T}^{\perp(1)}(x)\right)^2}{4\pi\langle k_\perp^2\rangle_{g_{1T}^{\perp}}}\ge 0$$

which is (6.4).

6.7 is proven analogously.