REPLY TO THE EDITOR

Dear Editor,

We would like to thank the Referee for careful reading, constructive criticism, and helpful recommendations. We revised the manuscript closely following the Referee's suggestions in all points.

Sincerely,

Prokudin's Eleven

REPLY TO THE REFEREE

We would like to thank the Referee for careful reading, constructive criticism, and the many helpful recommendations. We revised the manuscript closely following the Referee's suggestions as detailed out in the attached "Summary of Changes." Below we respond to several specific points in the report.

- We agree that the Q^2 evolution is an important limiting factor, and added a brief comment in the Introduction as well as a more detailed paragraph in Sec. 3.8.

- The preliminary COMPASS data shown in Fig. 12 and several subsequent figures have been released and can be shown in our work under the condition that we show these Figures exactly as provided to us by the COMPASS Collaboration. This means that at this point we are not allowed to include the results of our calculations in these plots, and must show them separately. These data will presumably be unconditionally released at some later time in the summer of this year. We added explanatory remarks in Fig. 12 and other figures where this applies. On the one hand, this makes the presentation of the results and the formatting of the figures cumbersome in many cases. On the other hand, we are grateful to the COMPASS Collaboration for giving us this possibility to show their preliminary data. We are also grateful to the HERMES Collaboration for making their preliminary data available and allowing us to include our curves in those plots. We added a remark in the acknowledgments.

- We understand the concerns of the Referee regarding the subpercent results shown e.g. in Fig. 15. We had similar concerns but decided to show these results, even though subpercent asymmetries are not realistic predictions for COMPASS. But the smallness of the effect shown in Fig. 15 is not only due to the (in our approximation) small involved TMDs but also due to the kinematical suppression in the COMPASS kinematics. The figure still gives insights e.g. on the relative flavor dependence for the production of positive vs negative pions in the WW-type approximation.

- Finally, we followed the recommendations of the Referee and removed App. C on the mathematica package, unified the notation for the masses of produced hadrons m_h, specified the Mellin moments before Eq. (3.8), fixed the literal repetition on pages 16/17, improved non-optimal wording and fixed typos. It is inexplicable how some of these things could have escaped our attention during proof-reading, and we are grateful for these corrections.

SUMMARY OF CHANGES

- Sec.1, page 3, the following paragraph and the new references [1–4] are added:

  "The WW-type approximation is not preserved under Q^2 evolution. Some intuition can be obtained from the collinear case [1–4]. However, much less is known about the $k_\perp$-evolution especially at subleading twist. More theoretical work is required here."

- Sec.1, page 3, we replaced:

  "study of all SIDIS structure functions up to twist-3 in a unique approach." $\rightarrow$
  "study of all SIDIS structure functions up to twist-3 evaluated within one common systematic theoretical guideline."

- Sec.1, page 3, we replaced the paragraph:

  In App. C we describe an open-source package implemented in `Mathematica` [5] (already available) and `Python` (to be released in the near future) that is made publicly available on `github.com`: https://github.com/prokudin/WW-SIDIS

  by:

  An open-source package is available which allows one to visualize and reproduce the results presented in this work, and may easily be adapted by interested colleagues for their purposes [5].

  and we removed App. C.

- Sec. 3.3, page 14: The sentence before Eq. (3.8) is modified as:

  For the $n = 3$ Mellin moments (i.e. the lowest non-trivial ones for these tilde-functions) it was found ...

- Sec. 3.5, page 17: The first instance of the repetitive sentence starting with "This assumption holds ..." is removed.

- Sec. 3.8, page 20: The paragraph is added:

  As it was mentioned in the Introduction one important limitation concerns the fact that the WW-type approximations are not preserved under Q^2 evolution. Still some intuition can be obtained from the collinear case: the evolution equations for $g_T^a(x)$ and $h_L^a(x)$ exhibit complicated mixing patterns typical for higher twist functions which simplify to DGLAP-type evolutions in the limit of a large number of colors N_c and in the limit of large-x [1–4]. These evolution equations differ from those of

the leading-twist functions $g_1^a(x)$ and and $h_1^a(x)$. This point is of not much practical relevance here, because the Q^2 varies only moderately between JLab, HERMES, and COMPASS (except for the largest x-bins). However, if for some reason the $\bar{q}gq$-terms were found to be very small at one renormalization scale, it is not guaranteed the WW-type approximation will work equally well also at other scales. More theoretical work is required, especially also in order to understand $k_\perp$-evolution effects at subleading twist.

- Caption of Fig.7, page 31, is modified as follows:

  "(where $(-1)A_{UT}^{\sin(\phi_h+\phi_S)}$ is shown since COMPASS uses an opposite sign in its Collins asymmetry convention)."

- Sec. 5.6, page 32 (our correction): we added $\langle \dots \rangle$ in the following in-line formula $A_{UT,\langle y\rangle}^{\sin(3\phi_h-\phi_S)} = \langle (1-y)F_{UT}^{\sin(3\phi_h-\phi_S)}\rangle / \langle (1-y+y^2/2)F_{UU}\rangle$ to indicate that the kinematic variable y is averaged over.

- Captions of Figs. 9, 11, 12, 14, 15, 16, 19: we added explanations why our curves cannot be added on the plots provided by the COMPASS Collaboration.

- Fig. 18 (our correction): the previous Fig. 18c was showing an estimate of the asymmetry from an earlier (less consistent and superseded) way of using the Gaussian model and applying the WW-type approximation, and we removed this plot. In the presently adopted scheme this asymmetry vanishes as described in the caption of Fig. 18 and in Sec. 7.6.

- Sec. 5.6, page 33, the text is removed:

  "A notable exception is COMPASS, where the largest x–bins (where Q^2 is largest) bear the best hints on this TMD, see Fig. 9."

- Conclusions, page 47, the text is removed:

  "The classic WW approximation for $g_2(x)$ works with a relative accuracy of $\pm\,40\,\%$ or better. This is remarkable."

  The following phrase is modified:

  "on the positive site we also observe no alarming hints" $\rightarrow$
  "on the positive side we also observe no hints"

- The misprints are fixed, the consistent notation m_h is used, and several figures are rearranged and placed more accurately within the sections where they are discussed. We did minor editing in several places (purely stylistic improvements).

- Acknowledgments, we added the remark:

  We thank the COMPASS and HERMES Collaborations for the permissions to show their preliminary data on several figures.

- **Harut:** Do we show something preliminary from JLab?
  If so, we should acknowledge JLab too! ;-)
- **Alexei:** please eloborate on Ref. [5] as needed.

References

[1] A. Ali, V. M. Braun and G. Hiller, *Asymptotic solutions of the evolution equation for the polarized nucleon structure function* $g_2(x, Q^2)$, *Phys. Lett.* **B266** (1991) 117–125.

[2] Y. Koike and K. Tanaka, Q^2*evolution of nucleon's chiral odd twist-three structure function:* $h_L(x, Q^2)$, *Phys. Rev.* **D51** (1995) 6125–6138, [hep-ph/9412310].

[3] I. I. Balitsky, V. M. Braun, Y. Koike and K. Tanaka, Q^2 *evolution of chiral odd twist-three distributions* $h_L(x, Q^2)$ *and* $e(x, Q^2)$ *in the large* N_c *limit*, *Phys. Rev. Lett.* **77** (1996) 3078–3081, [hep-ph/9605439].

[4] A. V. Belitsky and D. Müller, *Scale dependence of the chiral odd twist-three distributions* $h_L(x)$ *and* $e(x)$, *Nucl. Phys.* **B503** (1997) 279–308, [hep-ph/9702354].

[5] A. Prokudin and K. Tezgin, "Open-source packages with implementations of SIDIS structure functions in the WW-type approximation are publicly available on `github.com`:
in `Mathematica, Version 11.3` on https://github.com/prokudin/WW-SIDIS,
in `Python` on https://jeffersonlab.github.io/jam3d/_build/html/index.html."