REPLY TO THE EDITOR

Dear Editor,

We would like to thank the Referee for careful reading, constructive criticism, and helpful recommendations. We revised the manuscript closely following the Referee's suggestions in all points.

Sincerely,

Authors

REPLY TO THE REFEREE

We would like to thank the Referee for careful reading, constructive criticism, and the many helpful recommendations. We revised the manuscript closely following the Referee's suggestions as detailed out in the attached "Summary of Changes." Below we respond to several specific points in the report.

- We agree that the Q^2 evolution is an important limiting factor, and added a brief comment in the Introduction as well as a more detailed paragraph in Sec. 3.8.

- The preliminary COMPASS data shown in Fig. 12 and several subsequent figures have been released and can be shown in our work under the condition that we show these Figures exactly as provided to us by the COMPASS Collaboration. This means that at this point we are not allowed to include the results of our calculations in these plots, and must show them separately. These data will presumably be unconditionally released at some later time in the summer of this year. We added explanatory remarks in Fig. 12 and other figures where this applies. On the one hand, this makes the presentation of the results and the formatting of the figures cumbersome in many cases. On the other hand, we are grateful to the COMPASS Collaboration for giving us this possibility to show their preliminary data. We are also grateful to the HERMES Collaboration for making their preliminary data available and allowing us to include our curves in those plots. We added a remark in the Acknowledgments.

- We understand the concerns of the Referee regarding the subpercent results shown e.g. in Fig. 15. We had similar concerns but decided to show these results, even though subpercent asymmetries are not realistic predictions for COMPASS. But the smallness of the effect shown in Fig. 15 is not only due to the (in our approximation) small involved TMDs but also due to the kinematical suppression in the COMPASS kinematics. The figure still gives insights e.g. on the relative flavor dependence for the production of positive vs negative pions in the WW-type approximation.

- Finally, we followed the recommendations of the Referee and removed App. C on the mathematica package, unified the notation for the masses of produced hadrons m_h, specified the Mellin moments before Eq. (3.8), fixed the literal repetition on pages 16/17, improved non-optimal wording and fixed typos. It is inexplicable how some of these things could have escaped our attention during proof-reading, and we are grateful for these corrections.

SUMMARY OF CHANGES

- Sec.1, page 3, the following paragraph and the new references [1–4] are added:

  "The WW-type approximation is not preserved under Q^2 evolution. Some intuition can be obtained from the collinear case [1–4]. However, much less is known about the $k_\perp$-evolution especially at subleading twist. More theoretical work is required here."

- Sec.1, page 3, we replaced:

  "study of all SIDIS structure functions up to twist-3 in a unique approach." $\rightarrow$
  "study of all SIDIS structure functions up to twist-3 evaluated within one common systematic theoretical guideline"

- Sec.1, page 3, we replaced the paragraph:

  "In App. C we describe an open-source package implemented in `Mathematica` [14] (already available) and `Python` (to be released in the near future) that is made publicly available on `github.com`: https://github.com/prokudin/WW-SIDIS"

  by:

  "An open-source package is available which allows one to visualize and reproduce the results presented in this work, and may easily be adapted by interested colleagues for their purposes [14]."

  and we removed App. C.

- Sec. 2.1, Eq. 2.2: we absorbed the definition of the unpolarized lepton-quark subprocess cross section in the overall cross section formulas.

- Sec. 3.3, page 14: The sentence before Eq. (3.8) is modified as:

  "For the $n = 3$ Mellin moments (i.e. the lowest non-trivial ones for these tilde-functions) it was found ..."

- Sec. 3.5, page 16: In the second paragraph we added a sentence refering to the latest developments concerning lattice:

  "For the latest developments we refer the interested reader to Refs. [7078]"

- Sec. 3.5, page 17: The first instance of the repetitive sentence starting with "This assumption holds ..." is removed.

- Sec. 3.8, page 22: The paragraph is added:

  "As it was mentioned in the Introduction one important limitation concerns the fact that the WW-type approximations are not preserved under Q^2 evolution. Still some intuition can be obtained from the collinear case: the evolution equations for $g_T^a(x)$ and $h_L^a(x)$ exhibit complicated mixing patterns typical for higher twist functions, which simplify to DGLAP-type evolutions in the limit of a large number of colors N_c and in the limit of large-x [1–4]. These evolution equations differ from those of the leading-twist functions $g_1^a(x)$ and $h_1^a(x)$. However, since Q^2 varies moderately in the considered experiments (e.g. for common values of x the Q^2 at COMPASS is only about a factor 2-3 larger than at HERMES), this point is not a major uncertainty in our study. More theoretical work will be required to understand $k_\perp$-evolution effects of subleading twist TMDs in future experiments (EIC) covering kinematic regions that vary by orders of magnitude in Q^2."

- Sec. 5.2, page 29: "We remark that HERMES and COMPASS data also show flat P_{hT}-distributions [11, 12]."

- Sec. 5.3, page 30: the sign change of the Sivers asymmetry in hadron-hadron collisions is pointed out, including references to recent experimental results by STAR and COMPASS:

  "The Sivers function is predicted to enter the description of hadron-hadron collisions (with transversely polarized protons) with an opposite sign compared to SIDIS [15–17]. Recent results on single-spin asymmetries in weak-boson production from RHIC [18] and Drell–Yan from COMPASS [19, 20] are consistent with this prediction."

- Sec. 5.4, page 30: "Transversity can also be accessed as a PDF in Drell–Yan or dihadron production [5–10]."

- (Caption of) Fig. 7, page 31, removed the debated statement about the different sign conventions and plotted the COMPASS data using the conventions used here:

  "Collins asymmetry for a proton target vs x based on the fit [21]. (a) $A_{UT,\langle y\rangle}^{\sin(\phi_h+\phi_S)}$ in comparison to HERMES [22] data. (b) $A_{UT}^{\sin(\phi_h+\phi_S)}$ in comparison to COMPASS [23] data"

- Sec. 5.6, page 33 (our correction): we added $\langle \dots \rangle$ in the following in-line formula $A_{UT,\langle y\rangle}^{\sin(3\phi_h-\phi_S)} = \langle (1-y) F_{UT}^{\sin(3\phi_h-\phi_S)}\rangle / \langle (1-y+y^2/2) F_{UU}\rangle$ to indicate that the kinematic variable y is averaged over.

- Sec. 5.6, page 33, the text is removed:

  "A notable exception is COMPASS, where the largest x–bins (where Q^2 is largest) bear the best hints on this TMD, see Fig. 9."

- Captions of Figs. 9, 11, 12, 15, 16, 17, 19: we added explanations why our curves cannot be added on the plots provided by the COMPASS Collaboration.

" [We remark that in this and several subsequent figures we have the permission to show the preliminary data [13] only in the official figures provided by the COMPASS collaboration in (a,b), and have to display our results separately in (d). Notice also the different scale on the y-axis in panel (d) as compared to (a,b).] "

- Sect. 7.2, caption of Fig. 14: added "where a different scale is chosen to better visualize the theory curves."
- Sect. 7.4, third line (our correction): $F_{UL}^{\cos\phi_h}$ changed to $F_{LL}^{\cos\phi_h}$
- Fig. 18 (our correction): the previous Fig. 18c was showing an estimate of the asymmetry from an earlier (less consistent and superseded) way of using the Gaussian model and applying the WW-type approximation, and we removed this plot. In the presently adopted scheme this asymmetry vanishes as described in the caption of Fig. 18 and in Sec. 7.6.
- Fig. 20 (our correction): HEPDATA were used for the COMPASS results, which we realized have a bug. The plot got replaced using the original data tables from the COMPASS paper.
- Conclusions, page 47, the text is removed:

  "The classic WW approximation for $g_2(x)$ works with a relative accuracy of $\pm\,40\,\%$ or better. This is remarkable."

  The following phrase is modified:

  "on the positive site we also observe no alarming hints" $\rightarrow$
  "on the positive side we also observe no hints"
- The misprints are fixed, the consistent notation m_h is used, and several figures are rearranged and placed more accurately within the sections where they are discussed. We did minor editing in several places (purely stylistic improvements), and used a $\log x$ scale also for the HERMES data at several places for consistency with COMPASS plots shown in the same figure.
- Acknowledgments, we added the remark:

  We thank the COMPASS and HERMES Collaborations for the permissions to show their preliminary data on several figures.

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