Formulas for e^+e^-

We will use just one method called A_0

$$A_0^{UL(C)}(z_1, z_2, \theta, P_{h\perp}) = \frac{\langle \sin^2\theta \rangle}{\langle 1+\cos^2\theta \rangle}\left(\frac{Z_c^U}{Z_{uu}^U} - \frac{Z_c^{L(C)}}{Z_{uu}^{L(C)}} \right)$$

$$A_0^{UL(C)}(z_1, z_2, \theta) = \frac{\langle \sin^2\theta \rangle}{\langle 1+\cos^2\theta \rangle}\left(\frac{\int dP_\perp P_\perp \ldots}{\int \ldots} - \frac{\int \ldots}{\int \ldots} \right)$$

$Z_{uu}^U \equiv Z_{uu}^{\pi^+\pi^-} + Z_{uu}^{\pi^-\pi^+}$ (unlike sign)

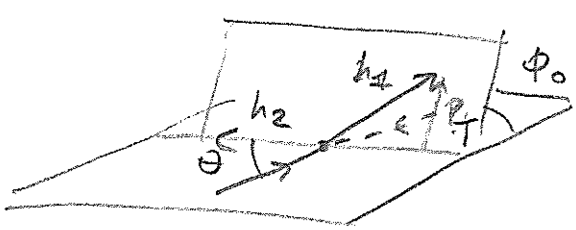
$Z_{uu}^L = Z_{uu}^{\pi^+\pi^+} + Z_{uu}^{\pi^-\pi^-}$, (like sign)

$Z_{uu}^C = Z_{uu}^U + Z_{uu}^L$, (charged hadrons)

Generic cross section

$$\frac{d^5\sigma}{dz_1\, dz_2\, d^2P_T\, d\cos\theta} = \frac{N_c \pi \alpha^2}{2Q^2}\left[(1+\cos^2\theta)\, Z_{uu} + \sin^2\theta \cos 2\phi_0\, Z_{c} \ldots \right.$$

azimuthal modulation is in $\cos 2\phi_0$

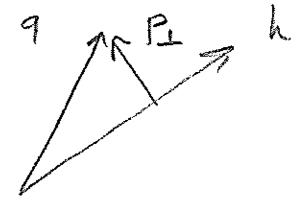


Unpolarised fragmentation function.

We start from definition of Yuan et al

$$D_1(z, p_\perp) = \int_0^\infty \frac{b\,db}{2\pi} \frac{D_1(z)}{z^2} e^{-\frac{b^2 g_u}{z^2}} J_0\left(\frac{p_\perp b}{z}\right) =$$

$$= D_1(z) \frac{e^{-p_\perp^2/4g_u}}{\pi\, 4g_u}, \quad \text{we will use } \langle p_\perp^2 \rangle \equiv 4 g_u$$



$$D_1(z, p_\perp) = D_1(z) \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

b space fragmentation

$$D_1(z, b) = \frac{D_1(z)}{z^2} e^{-\frac{b^2 g_u}{z^2}} \equiv \frac{D_1(z)}{z^2} e^{-\frac{b^2 \langle p_\perp^2 \rangle}{4 z^2}}$$

Structure function Z_{uu} in b space is:

$$\tilde{Z}_{uu}(b) \equiv \frac{D_1(z_1) D_2(z_2)}{z_1^2} e^{-b^2 \left(\frac{\langle p_\perp^2 \rangle_1}{4 z_1^2} + \frac{\langle p_\perp^2 \rangle_2}{4 z_2^2}\right)}$$

P_T space

$$Z_{uu}(P_T) = \int_0^\infty \frac{b\,db}{2\pi} J_0\left(\frac{b P_T}{z_1}\right) \tilde{Z}_{uu}(b) =$$

$$= z_2^2 D_1(z_1) D_2(z_2) \frac{e^{-\frac{P_T^2 z_2^2}{z_2^2 \langle p_\perp^2 \rangle_1 + z_1^2 \langle p_\perp^2 \rangle_2}}}{\pi (z_2^2 \langle p_\perp^2 \rangle_1 + z_1^2 \langle p_\perp^2 \rangle_2)}$$

If we call $\langle P_T^2 \rangle (z_1, z_2) = \dfrac{z_2^2 \langle p_\perp^2 \rangle_1 + z_1^2 \langle p_\perp^2 \rangle_2}{z_2^2}$

$$\mathcal{Z}_{uu}(P_T) = D_1(z_1)\, D_2(z_2)\, \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

Inverse Fourier definition

$$D(z, b^2) = \frac{1}{z^2} \int d^2 p_\perp \, e^{-\vec{p}_\perp \cdot \vec{b}/z} D(z, p_\perp^2) =$$

$$= \frac{2\pi}{z^2} \int dp_\perp \, p_\perp \, J_0(p_\perp b/z)\, D(z, p_\perp) =$$

$$= \frac{1}{z^2} D_1(z)\, e^{-\frac{b^2 \langle p_\perp^2 \rangle}{4 z^2}}$$

Contribution from Collins function.

$$Z_C = \frac{1}{z_1^2}\,\frac{1}{4 z_1 z_2}\int_0^\infty \frac{db\, b^3}{2\pi}\, J_2\!\left(\frac{P_T b}{z_1}\right) e^{-\left(\frac{g_{h_1}-g_{c_1}}{z_1^2}+\frac{g_{h_2}-g_{c_2}}{z_2^2}\right)b^2}$$

$$\times \sum_q e_q^2\, \hat{H}^{(3)}(z_1)\,\hat{H}^{(3)}(z_2) =$$

$$= \sum_q e_q^2\, \hat{H}^{(3)}(z_1)\,\hat{H}^{(3)}(z_2)\, \frac{e^{-\frac{P_T^2 z_2^2}{4(g_{h_2}-g_{c_2})z_1^2+4(g_{h_1}-g_{c_1})z_2^2}}}{64\pi\left((g_{h_2}-g_{c_2})z_1^2+(g_{h_1}-g_{c_1})z_2^2\right)^3}\, P_T^2 z_1 z_2^5$$

Relation of $\hat{H}^{(3)}$ and Collins FF

$$\hat{H}^{(3)}(z) = \int d^2p_\perp \frac{p_\perp^2}{M_h}\left(-\frac{1}{z}\right) H_1^\perp(z, p_\perp^2)\Big|_{\text{Trento}}$$

We use the following parametrisation

$$H_1^\perp(z, p_\perp) = H_1^{\perp(1)}(z)\,\frac{2 z^2 M_h^2}{\pi \langle p_\perp^2\rangle_{H_1^\perp}^2}\, e^{-p_\perp^2/\langle p_\perp^2\rangle_{H_1^\perp}}$$

so that:

$$\hat{H}^{(3)}(z) = -2 z M_h H_1^{\perp(1)}(z)$$

or

$$\hat{H}^{(3)}(z)\,\frac{e^{-p_\perp^2/4(g_h-g_c)}}{\pi(4(g_h-g_c))^2}\, p_\perp = \frac{p_\perp}{M_h}\left(-\frac{1}{z}\right) H_1^\perp(z, p_\perp^2)\Big|_{\text{Trento}}$$

$$\Rightarrow \quad 4(g_h-g_c) \equiv \langle p_\perp^2\rangle_{H_1^\perp}$$

So that:

$$z_c = \sum_q e_q^2 \; H_1^{\perp(1)}(z_1)\, H_1^{\perp(1)}(z_2) \; \frac{e^{-\frac{P_T^2 z_2^2}{\langle p_{\perp 2}^2\rangle_{H_1^\perp} z_1^2 + \langle p_{\perp 1}^2\rangle_{H_1^\perp} z_2^2}}}{\pi\left(\langle p_{\perp 2}^2\rangle_{H_1^\perp} z_1^2 + \langle p_{\perp 1}^2\rangle_{H_1^\perp} z_2^2\right)^3}$$

$$\times \; P_\perp^2 \, z_1 z_2^5 \; 4 z_1 M_h z_2 M_h$$

Finally

$$z_c = \sum_q e_q^2 \; H_1^{\perp(1)}(z_1)\, H_1^{\perp(2)}(z_2) \; \frac{e^{-\frac{P_T^2 z_2^2}{(\langle p_{\perp 2}^2\rangle_{H_1^\perp} z_1^2 + \langle p_{\perp 1}^2\rangle_{H_1^\perp} z_2^2)}}}{\pi\left(\langle p_{\perp 2}^2\rangle_{H_1^\perp} z_1^2 + \langle p_{\perp 1}^2\rangle_{H_1^\perp} z_2^2\right)^3}$$

$$\times \; P_\perp^2 \; 4 M_h^2 z_1^2 z_2^6$$

If we define $\langle P_T^2\rangle_{H_1^\perp} = \dfrac{\langle p_{\perp 2}^2\rangle_{H_1^\perp} z_1^2 + \langle p_{\perp 1}^2\rangle_{H_1^\perp} z_2^2}{z_2^2}$

then

$$z_c = \sum_q e_q^2 \; H_1^{\perp(1)}(z_1)\, H_1^{\perp(2)}(z_2) \; \frac{e^{-P_T^2/\langle P_T^2\rangle_{H_1^\perp}}}{\pi \langle P_T^2\rangle_{H_1^\perp}^3}$$

$$\times \; P_\perp^2 \; 4 M_h^2 z_1^2 z_2^6$$