Angular dependences in inclusive two-hadron production at BELLE

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Abstract

A collection of results is presented relevant for the analysis of azimuthal asymmetries in inclusive two-hadron production at BELLE. The aim of this overview is to provide theoretical ingredients necessary to extract the Collins effect fragmentation function. The latter arises within the Collins–Soper factorization formalism, which describes both the transverse momentum and Q^2 dependence of the cross section and its angular dependences at low and moderate transverse momentum. Since the Collins effect is not the only source of angular dependences, a discussion of various other effects is included. This concerns higher twist contributions, photon–Z-boson interference effects, radiative corrections, beam polarization and weak decays. Furthermore, different frames, transverse momentum weighting and ratios of asymmetries are discussed. These issues are all of relevance for the unambiguous measurement of the Collins effect.
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1. Introduction

The BELLE experiment at KEK in Japan measures with very high luminosity the process of electron–positron annihilation in collisions of 8.0 GeV electrons and 3.5 GeV positrons. The center of mass energy is selected to be on-resonance of the $\Upsilon(4S)$ meson, which has a mass of 10.5800 ± 0.0035 GeV and which decays more than 96% of the time into $B\bar{B}$ meson pairs. The main aim of the measurements of B and $\bar{B}$ decays is to study CP violation. Besides this goal,

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there are other interesting studies that can be performed at BELLE[1] and for which also the off-resonance data are useful (which in the case of BELLE are taken 60 MeV below resonance). The acquisition of off-resonance data is mostly used for background studies, but is also of interest for physics studies that are not *b*-quark specific. This overview discusses such a case, namely the study of azimuthal asymmetries in the inclusive production of two almost back-to-back hadrons, $e^+e^- \to h_1 h_2 X$. There are several effects that can cause such asymmetries and one would like to disentangle them in order to isolate perturbative from nonperturbative effects. In the latter type of effects quark spin is expected to play a nontrivial role via the so-called Collins effect [1]. By measuring azimuthal asymmetries it may be possible to obtain trustworthy quantitative knowledge on this type of effect, for instance from the BELLE data. Recently, the first results from BELLE were published [2] and the purpose of this overview is to discuss the theoretical aspects of such type of study.

The Collins effect was first discussed in the context of semi-inclusive deep inelastic scattering (SIDIS) of leptons off transversely polarized protons, as a means to access transversity [1]. Transversity [3] describes the extent to which quarks are transversely polarized inside a proton that is polarized transversely to the probing particle, which in the case of SIDIS is a virtual photon. The Collins effect describes the angular asymmetry in the distribution of hadrons produced from a transversely polarized fragmenting quark. Via this effect the transverse polarization of the struck proton results in an asymmetric distribution of final state hadrons. If sufficiently large, the Collins effect would thus allow for a measurement of transversity and subsequently of the tensor charge, the fundamental charge that can only be measured through transversity.

The first nonzero Collins effect asymmetry in polarized SIDIS has been observed by the HERMES experiment [4] (using a deuteron target the COMPASS experiment obtained a result consistent with zero [5], presumably due to cancellations between proton and neutron contributions). The HERMES result indicates that both transversity and the Collins effect are nonzero. For an extraction of transversity from those SIDIS data a separate measurement of the Collins effect fragmentation function needs to be performed. This can best be done in the process $e^+e^- \to h_1 h_2 X$ [6] and motivates the BELLE efforts concerning the measurement of this process. Some earlier attempt to use LEP1 data has been undertaken [7], but without study of systematic effects and still remains preliminary. Moreover, as will be discussed, it is likely that the Collins effect asymmetry in $e^+e^- \to h_1 h_2 X$ has a powerlike fall-off behavior with energy, which would favor an extraction at BELLE over LEP1.

We will study various effects that could lead to azimuthal asymmetries in the process of interest, $e^+e^- \to h_1 h_2 X$, which besides the Collins effect, include electroweak $\gamma-Z$ interference effects, beam polarization effects and radiative corrections. In order to arrive at an unambiguous interpretation of the data the magnitude and scale dependence of the various effects need to be estimated, of course to the extent to which that is possible from first principles. The effects will not be treated simultaneously; combinations of effects will only be considered when the analysis requires it. As a rule we will ignore effects that are expected to be smaller than a permille, such as $Z-Z$ contributions or beam polarization in combination with $\gamma-Z$ interference. From the possible contributions considered the higher-twist effects are the least known, but unfortunately not necessarily below the percent level at BELLE. In cases where no reliable estimate can be given, such as for twist-4 effects ($\mathcal{O}(\Lambda^2/Q^2)$), additional observables or checks may need to be

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[1] We have chosen to focus on BELLE, but the observables to be discussed can of course also be studied at other e^+e^- colliders.

considered to further exclude competing interpretations of the Collins effect asymmetry. But despite some uncertainties the isolation of the Collins effect contribution does seem feasible given the possibilities that BELLE offers. The purpose of this overview, which contains several new aspects, is to assist and facilitate this endeavor. The realistic prospect of extracting the Collins effect fragmentation function and with it transversity, of which a first result has recently been obtained [8], makes the study of azimuthal asymmetries at BELLE well worth the effort.

2. General angular dependence

We consider $e^-e^+ \to h_1 h_2 X$, where the two leptons (with momentum l for the e^- and l' for the e^+) annihilate into a photon (or Z boson) with momentum $q = l + l'$. This photon momentum sets the scale Q, where $Q^2 \equiv q^2$, which is much larger than characteristic hadronic scales. Denoting the momentum of outgoing hadrons by P_h ($h = 1, 2$) we use invariants $z_h = 2P_h \cdot q/Q^2$. We will consider the case of unpolarized leptons and hadrons, although in Section 13 we will turn to the issue of transverse beam polarization due to the Sokolov–Ternov effect. We will work in the limit where Q^2 and $P_h \cdot q$ are large, keeping the ratios z_h finite. We will consider the case where the two hadronic momenta P_1 and P_2 do not belong to the same jet (i.e., $P_1 \cdot P_2$ is of order Q^2).

There is a considerable literature on angular correlations for three-jet events in electron–positron annihilation, e.g. [9–18], but here we are limiting the discussion to two-jet events exclusively.

In general, the differential cross section for the process $e^+ e^- \to h_1 h_2 X$ can be written as (see for instance [14])

$$
\begin{aligned}
\frac{dN}{d\Omega} &\equiv \left(\frac{d\sigma}{dz_1 \, dz_2 \, d^2\boldsymbol{q}_T} \right)^{-1} \frac{d\sigma}{dz_1 \, dz_2 \, d\Omega \, d^2\boldsymbol{q}_T} \\
&= F_1 \left(1 + \cos^2\theta \right) + F_2 \left(1 - 3\cos^2\theta \right) + F_3 \cos\theta \\
&\quad + F_4 \sin 2\theta \cos\phi + F_5 \sin^2\theta \cos 2\phi + F_6 \sin\theta \cos\phi \\
&\quad + F_7 \sin 2\theta \sin\phi + F_8 \sin^2\theta \sin 2\phi + F_9 \sin\theta \sin\phi.
\end{aligned}
\tag{1}
$$

The functions F_i depend on the invariants $z_h = 2P_h \cdot q/Q^2$ and on $\boldsymbol{q}_T^2 \equiv Q_T^2$, the squared transverse momentum $\boldsymbol{q}_T$ of the photon with respect to the two hadrons. The angles ϕ and θ are given in the lepton-pair center of mass frame or equivalently the photon center of mass frame. Precise definitions and explanations will be given below.

If at high Q^2 and Q_T^2 collinear factorization of the cross section is considered, then at tree level (zeroth order in α_s) only F_1, F_3 will receive nonzero contributions (F_3 only from $\gamma - Z$ interference), at first order in α_s $F_1, \dots, F_6$ receive contributions and at second order all F_i are nonzero. All this is assuming no transverse beam polarization is present. As said, the complication of transverse beam polarization will be considered in Section 13.

Below we will study the differential cross section $d\sigma/dz_1 \, dz_2 \, d\Omega \, d^2\boldsymbol{q}_T$ in much detail, also at lower values of Q_T^2 where collinear factorization is not the appropriate framework. To set the notation we will first look at the cross section expression in terms of the hadron tensor.

3. Two-particle inclusive cross section

The square of the amplitude for $e^-e^+ \to h_1 h_2 X$ can be split into a purely leptonic and a purely hadronic part,

$$|\mathcal{M}|^2 = \frac{e^4}{Q^4} L_{\mu\nu} H^{\mu\nu}, \tag{2}$$

with the helicity-conserving lepton tensor (neglecting the lepton masses) given by

$$L_{\mu\nu}(l, l') = 2l_\mu l'_\nu + 2l_\nu l'_\mu - Q^2 g_{\mu\nu}. \tag{3}$$

For the case of two observed hadrons in the final state, the product of hadronic current matrix elements is written as

$$H_{\mu\nu}(P_X; P_1; P_2) = \langle 0|J_\mu(0)|P_X; P_1; P_2\rangle\langle P_X; P_1; P_2|J_\nu(0)|0\rangle. \tag{4}$$

The cross section for two-particle inclusive e^+e^- annihilation is given by (including a factor $1/2$ from averaging over initial state polarizations)

$$\frac{P_1^0 P_2^0 \, d\sigma^{(e^+e^-)}}{d^3 P_1 \, d^3 P_2} = \frac{\alpha^2}{4Q^6} L_{\mu\nu} \mathcal{W}^{\mu\nu}, \tag{5}$$

with

$$\mathcal{W}_{\mu\nu}(q; P_1; P_2) = \int \frac{d^3 P_X}{(2\pi)^3 2P_X^0} \delta^4(q - P_X - P_1 - P_2) H_{\mu\nu}(P_X; P_1; P_2). \tag{6}$$

3.1. Frames

For the calculation of the hadron tensor it will be convenient to define *lightlike* directions using the hadronic (or jet) momenta. The two hadronic momenta P_1 and P_2 can be parameterized using the dimensionless lightlike vectors n_+ and n_- (satisfying $n_+ \cdot n_- = 1$),

$$P_1^\mu \equiv \frac{\zeta_1 \tilde{Q}}{\sqrt{2}} n_-^\mu + \frac{M_1^2}{\zeta_1 \tilde{Q}\sqrt{2}} n_+^\mu, \tag{7}$$

$$P_2^\mu \equiv \frac{M_2^2}{\zeta_2 \tilde{Q}\sqrt{2}} n_-^\mu + \frac{\zeta_2 \tilde{Q}}{\sqrt{2}} n_+^\mu, \tag{8}$$

$$q^\mu \equiv \frac{\tilde{Q}}{\sqrt{2}} n_-^\mu + \frac{\tilde{Q}}{\sqrt{2}} n_+^\mu + q_T^\mu, \tag{9}$$

where $\tilde{Q}^2 = Q^2 + Q_T^2$ with $q_T^2 \equiv -Q_T^2$. When $Q_T^2 \ll Q^2$ one has $\tilde{Q} = Q$, $\zeta_1 = z_1$ and $\zeta_2 = z_2$ up to Q_T^2/Q^2 corrections.

Vectors transverse to n_+ and n_- one obtains using the tensors

$$g_T^{\mu\nu} \equiv g^{\mu\nu} - n_+^{\{\mu} n_-^{\nu\}}, \tag{10}$$

$$\epsilon_T^{\mu\nu} \equiv \epsilon^{\mu\nu\rho\sigma} n_{+\rho} n_{-\sigma}. \tag{11}$$

The experimental analysis of the azimuthal asymmetries will usually not be performed in the frame in which the two hadrons are collinear, for which the above (Sudakov) decomposition into lightlike vectors and transverse parts is most suited. Instead it is much more common to
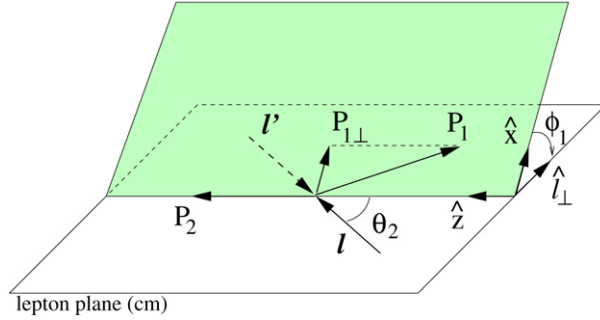
Fig. 1. Kinematics of the annihilation process in the lepton center of mass frame (the analogue of the Gottfried–Jackson frame) for a back-to-back jet situation. P_2 is the momentum of a hadron in one jet, P_1 is the momentum of a hadron belonging to the other jet.

consider angles in the lepton-pair center of mass frame or equivalently, the photon rest frame. In this case there is still freedom to select which momentum (or linear combination of momenta) is used to define the $\hat{z}$ axis, or equivalently, what determines the perpendicular plane in which the azimuthal angles lie. A few choices are common, such as the Gottfried–Jackson frame and the Collins–Soper frame.

For the most part of this overview the frame to be employed will be the e^+e^--annihilation analogue of the so-called Gottfried–Jackson frame [19]. We will first give the details of this frame. Later on we will also consider the analogue of the Collins–Soper frame [20] and a frame where the jet or thrust axis is used to fix the basis (explained separately in Section 8). We will alternate between these different basis sets depending on what is most convenient for the analysis.

In order to expand the hadron tensor in terms of independent Lorentz structures (parameterized by structure functions), it is convenient to work with vectors orthogonal to q. A normalized timelike vector is defined by q and a normalized spacelike vector is defined by $\tilde{P}_i^\mu = P_i^\mu - (P_i \cdot q/q^2)q^\mu$ for one of the outgoing momenta, say P_2,

$$\hat{t}^\mu \equiv \frac{q^\mu}{Q}, \tag{12}$$

$$\hat{z}^\mu \equiv \frac{Q}{P_2 \cdot q} \tilde{P}_2^\mu = 2\frac{P_2^\mu}{z_2 Q} - \frac{q^\mu}{Q}. \tag{13}$$

This choice of frame is the analogue of the Gottfried–Jackson frame often employed for the Drell–Yan process. This means that in the lepton-pair center of mass frame, hadron 2 is moving along the $\hat{z}$ direction (see Fig. 1). In general, in this frame hadron 1 will have momentum components orthogonal to $\hat{z}$ and $\hat{t}$.

Vectors orthogonal to $\hat{z}$ and $\hat{t}$ are obtained with help of the tensors

$$g_\perp^{\mu\nu} \equiv g^{\mu\nu} - \hat{t}^\mu \hat{t}^\nu + \hat{z}^\mu \hat{z}^\nu, \tag{14}$$

$$\epsilon_\perp^{\mu\nu} \equiv -\epsilon^{\mu\nu\rho\sigma} \hat{t}_\rho \hat{z}_\sigma = \frac{1}{(P_2 \cdot q)} \epsilon^{\mu\nu\rho\sigma} P_{2\rho} q_\sigma. \tag{15}$$

Since we have chosen hadron 2 to define the longitudinal direction, the momentum P_1 of hadron 1 can be used to express the directions orthogonal to $\hat{t}$ and $\hat{z}$. One obtains $P_{1\perp}^\mu = g_\perp^{\mu\nu} P_{1\nu}$. We define the normalized vector $\hat{h}^\mu = P_{1\perp}^\mu/|\boldsymbol{P}_{1\perp}|$ and the second orthogonal direction is given by $\epsilon_\perp^{\mu\nu} \hat{h}_\nu$.

Note that the *transverse* tensors in Eqs. (10) and (11) are not identical to the *perpendicular* ones defined above if the transverse momentum of the outgoing hadron 1 does not vanish. The lightlike directions can easily be expressed in $\hat{t}$, $\hat{z}$ and $\hat{h}$, namely, up to corrections of order Q_T^2/Q^2,

$$n_+^\mu = \frac{1}{\sqrt{2}}[\hat{t}^\mu + \hat{z}^\mu], \tag{16}$$

$$n_-^\mu = \frac{1}{\sqrt{2}}\left[\hat{t}^\mu - \hat{z}^\mu + 2\frac{Q_T}{Q}\hat{h}^\mu\right]. \tag{17}$$

This shows that the differences between $g_\perp^{\mu\nu}$ and $g_T^{\mu\nu}$ are of order $1/Q$. Especially for the treatment of azimuthal asymmetries subleading in $1/Q$ (Section 6), it is important to keep track of these differences. We will see however that taking transverse momentum into account does not automatically lead to suppression.

Thus far we considered two sets of basis vectors, the first set constructed from the two hadron momenta (P_1 and P_2), the second set from the photon momentum (q) and one of the hadron momenta (P_2). The respective frames where the momenta P_1 and P_2, or q and P_2, are collinear are the natural ones connected to these two sets. One can go from one frame to the other via a Lorentz transformation that leaves the minus components unchanged [21,22]. In the first frame q has a transverse component q_T, in the second P_1 has a perpendicular component $P_{1\perp}$. We will therefore sometimes refer to them as the "transverse basis" and the "perpendicular basis", respectively. Up to corrections of order Q_T^2/Q^2 q_T^μ and $P_{1\perp}^\mu$ are related as follows:

$$P_{1\perp}^\mu = -z_1 q_T^\mu = z_1 Q_T \hat{h}^\mu. \tag{18}$$

Azimuthal angles will lie inside the plane orthogonal to $\hat{t}$ and $\hat{z}$. In particular, ϕ^ℓ gives the orientation of $\hat{l}_\perp$, where $\hat{l}_\perp^\mu$ denotes the normalized perpendicular part of the lepton momentum l^μ. The angle ϕ_1 is between $\hat{h} \propto P_{1\perp}$ and $\hat{l}_\perp$. More specifically,

$$\hat{l}_\perp \cdot a_\perp = -|a_\perp|\cos\phi_a, \tag{19}$$

$$\epsilon_\perp^{\mu\nu}\hat{l}_{\perp\mu}a_{\perp\nu} = |a_\perp|\sin\phi_a, \tag{20}$$

for a generic vector a. The convention for the epsilon tensor used is $\epsilon^{0123} = 1$.

Sometimes it may be convenient to choose a different (rotated) set of basis vectors in the lepton center of mass frame, the Collins–Soper frame. For comparison let us denote the basis vectors of the Gottfried–Jackson (GJ) frame for $e^+e^- \to h_1 h_2 X$ as:

$$\hat{t}_{GJ}^\mu = \frac{q^\mu}{Q}, \tag{21}$$

$$\hat{z}_{GJ}^\mu = \frac{Q}{P_2 \cdot q}\tilde{P}_2^\mu, \tag{22}$$

$$\hat{x}_{GJ}^\mu = \frac{2Q}{\tilde{s}Q_T}\left((P_1 \cdot \hat{z}_{GJ})\tilde{P}_2^\mu - (P_2 \cdot \hat{z}_{GJ})\tilde{P}_1^\mu\right), \tag{23}$$

where $\tilde{s} = (P_1 + P_2)^2$. Here $\hat{x}_{GJ}$ corresponds to $\hat{h}$ and $\hat{y}_{GJ}$ is defined implicitly by requiring a right-handed basis.

The basis for the Collins–Soper (CS) frame for $e^+e^- \to h_1 h_2 X$ is defined as [23]:
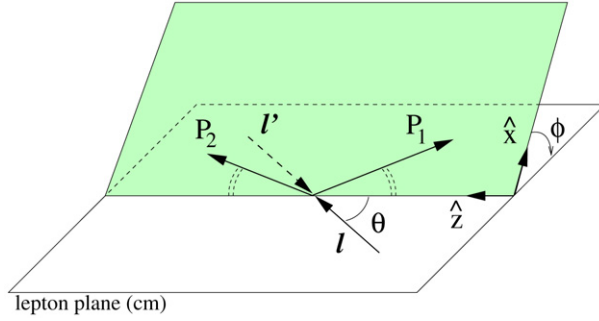
$$\hat{t}_{CS}^\mu = \frac{q^\mu}{Q}, \tag{24}$$

Fig. 2. Kinematics of the annihilation process in the lepton center of mass frame, the analogue of the Collins–Soper frame.

$$\hat{z}_{CS}^{\mu} = \frac{2}{\tilde{s}\tilde{Q}}\big((P_1 \cdot q)\tilde{P}_2^{\mu} - (P_2 \cdot q)\tilde{P}_1^{\mu}\big), \tag{25}$$

$$\hat{x}_{CS}^{\mu} = \frac{2Q}{\tilde{s}Q_T\tilde{Q}}\big((P_1 \cdot q)\tilde{P}_2^{\mu} + (P_2 \cdot q)\tilde{P}_1^{\mu}\big). \tag{26}$$

Note that we are keeping terms of order Q_T^2/Q^2, but not order M_i^2/Q^2. Throughout this paper we will neglect target mass corrections.

In the lepton-pair center of mass frame the $\hat{z}$ axis now points in the direction that bisects the three-vectors $\boldsymbol{P}_2$ and $-\boldsymbol{P}_1$ (see Fig. 2). One finds that in the limit $Q_T \to 0$: $\hat{z}_{CS}$ and $\hat{z}_{GJ}$ coincide and also $\hat{x}_{CS}$ and $\hat{x}_{GJ}$. When $Q_T \neq 0$ they differ only by a rotation:

$$\hat{z}_{GJ} = \cos\beta\,\hat{z}_{CS} + \sin\beta\,\hat{x}_{CS}, \tag{27}$$

$$\hat{x}_{GJ} = -\sin\beta\,\hat{z}_{CS} + \cos\beta\,\hat{x}_{CS}, \tag{28}$$

where

$$\cos\beta = \frac{Q}{\tilde{Q}}, \qquad \sin\beta = \frac{Q_T}{\tilde{Q}}. \tag{29}$$

We will also refer to the Collins–Soper frame vectors t, z, x, y as a perpendicular basis.

In the cross sections one will encounter the following functions of $y = P_2 \cdot l/P_2 \cdot q \approx l^-/q^-$, which in the lepton-pair center of mass frame equals $y = (1 + \cos\theta^*)/2$, where θ^* is the angle of $\hat{z}$ with respect to the momentum of the incoming lepton $\boldsymbol{l}$:

$$A(y) = \left(\frac{1}{2} - y + y^2\right) \overset{\text{cm}}{=} \frac{1}{4}(1 + \cos^2\theta^*), \tag{30}$$

$$B(y) = y(1 - y) \overset{\text{cm}}{=} \frac{1}{4}\sin^2\theta^*, \tag{31}$$

$$C(y) = (1 - 2y) \overset{\text{cm}}{=} -\cos\theta^*, \tag{32}$$

$$D(y) = \sqrt{y(1 - y)} \overset{\text{cm}}{=} \frac{1}{2}\sin\theta^*. \tag{33}$$

In the Gottfried–Jackson frame (Fig. 1) θ^* is called θ_2 and in the Collins–Soper frame θ (Fig. 2).

The cross sections are obtained from the hadron tensor after contraction with the lepton tensor. The lepton tensor for unpolarized leptons expressed in the lepton center of mass is given

Table 1
Contractions of the lepton tensor $L_{\mu\nu}$ with tensor structures appearing in the hadron tensor

$w^{\mu\nu}$	$L_{\mu\nu}w^{\mu\nu}/(4Q^2)$
$-g_\perp^{\mu\nu}$	$(\frac{1}{2} - y + y^2)$
$a_\perp^{\{\mu}b_\perp^{\nu\}} - (a_\perp \cdot b_\perp)g_\perp^{\mu\nu}$	$-y(1-y)|\boldsymbol{a}_\perp||\boldsymbol{b}_\perp|\cos(\phi_a + \phi_b)$
$\hat{z}^{\{\mu}a_\perp^{\nu\}}$	$-(1-2y)\sqrt{y(1-y)}|\boldsymbol{a}_\perp|\cos\phi_a$

by $(\hat{l}_\perp^\mu = l_\perp^\mu/(Q\sqrt{y(1-y)}))$:

$$L^{\mu\nu} = Q^2\left[-2A(y)g_\perp^{\mu\nu} + 4B(y)\hat{z}^\mu\hat{z}^\nu - 4B(y)\left(\hat{l}_\perp^\mu\hat{l}_\perp^\nu + \frac{1}{2}g_\perp^{\mu\nu}\right) - 2C(y)D(y)\hat{z}^{\{\mu}\hat{l}_\perp^{\nu\}}\right]. \tag{34}$$

For later use, the contractions of specific tensor structures in the hadron tensor are given in Table 1.

In the e^+e^- center of mass frame $d^3P_1\,d^3P_2/P_1^0P_2^0 = (dz_1/z_1)(z_2Q^2\,dz_2/4)d^2\boldsymbol{P}_{1\perp}\,d\Omega_2$, such that

$$\frac{d\sigma^{(e^+e^-)}}{dz_1\,dz_2\,d\Omega\,d^2\boldsymbol{q}_T} = \frac{\alpha^2}{16Q^4}z_1z_2L_{\mu\nu}\mathcal{W}^{\mu\nu}, \tag{35}$$

where $d\Omega = 2\,dy\,d\phi^\ell$, with ϕ^ℓ giving the orientation of $\hat{\boldsymbol{l}}_\perp$.

3.2. Structure functions

The hadron tensor $\mathcal{W}^{\mu\nu}$ can be expanded in terms of independent Lorentz structures which leads to a parameterization in terms of structure functions W_i. Ignoring lepton polarization and $\gamma-Z$ interference, the most general decomposition consists of four structure functions. Due to the similarity of the process $e^+e^- \to h_1h_2X$ with the Drell–Yan process, we will employ similar notation here as used for the latter process, i.e. we follow the notation of Lam and Tung [24,25], Collins [26], and Argyres and Lam [27].

The structure functions in the leptonic center of mass frame (or rather the perpendicular basis) are defined as

$$\mathcal{W}^{\mu\nu} = -g_\perp^{\mu\nu}W_T + \hat{z}^\mu\hat{z}^\nu W_L - \hat{z}^{\{\mu}\hat{x}^{\nu\}}W_\Delta - \left(\hat{x}^{\{\mu}\hat{x}^{\nu\}} - \hat{x}^2g_\perp^{\mu\nu}\right)W_{\Delta\Delta}, \tag{36}$$

such that $\mathcal{W}^\mu{}_\mu = -(2W_T + W_L)$. The structure functions $W_{T,L,\Delta,\Delta\Delta}$ are associated with specific polarizations of the photon [24]: $W_T = W^{1,1}$, $W_L = W^{0,0}$, $W_\Delta = (W^{0,1} + W^{1,0})/\sqrt{2}$, and $W_{\Delta\Delta} = W^{1,-1}$, where the first and second superscripts denote the photon helicity in the amplitude and its complex conjugate, respectively. In terms of these structure functions the cross section becomes

$$\frac{d\sigma(e^+e^- \to h_1h_2X)}{dz_1\,dz_2\,d\Omega\,d^2\boldsymbol{q}_T}$$
$$= \frac{3\alpha^2}{4Q^2}z_1^2z_2^2\Big\{W_T\big(1 + \cos^2\theta^*\big) + W_L\big(1 - \cos^2\theta^*\big) + W_\Delta\sin 2\theta^*\cos\phi^*$$
$$+ W_{\Delta\Delta}\sin^2\theta^*\cos 2\phi^*\Big\}, \tag{37}$$

or

$$\frac{dN}{d\Omega} \equiv \left(\frac{d\sigma}{dz_1\,dz_2\,d^2\boldsymbol{q}_T}\right)^{-1} \frac{d\sigma}{dz_1\,dz_2\,d\Omega^*\,d^2\boldsymbol{q}_T}$$

$$= \frac{3}{8\pi} \frac{1}{2W_T + W_L} \Big[W_T \big(1 + \cos^2\theta^*\big)$$

$$+ W_L \big(1 - \cos^2\theta^*\big) + W_\Delta \sin 2\theta^* \cos\phi^* + W_{\Delta\Delta} \sin^2\theta^* \cos 2\phi^* \Big]. \tag{38}$$

Here θ^*, ϕ^* indicate the polar and azimuthal angle in the e^+e^- center of mass frame (which for the BELLE experiment is not the lab frame), such that in the Gottfried–Jackson frame $\theta^* = \theta_2$ and $\phi^* = \phi_1$, the angles we have used before (see Fig. 1).

Another standard notation for the angular dependences in the Drell–Yan process can be employed here as well:

$$\frac{dN}{d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left[1 + \lambda \cos^2\theta^* + \mu \sin 2\theta^* \cos\phi^* + \frac{\nu}{2} \sin^2\theta^* \cos 2\phi^* \right]. \tag{39}$$

Expressing these parameters in terms of the structure functions W one has:

$$\lambda = \frac{W_T - W_L}{W_T + W_L}, \qquad \mu = \frac{W_\Delta}{W_T + W_L}, \qquad \nu = \frac{2W_{\Delta\Delta}}{W_T + W_L}. \tag{40}$$

One can transform from the CS frame to the GJ frame by using the following transformation matrix:

$$\begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix}_{\mathrm{GJ}} = \frac{1}{\Delta_{\mathrm{CS}}} \begin{pmatrix} 1 - \frac{1}{2}\rho^2 & -3\rho & \frac{3}{4}\rho^2 \\ \rho & 1 - \rho^2 & -\frac{1}{2}\rho \\ \rho^2 & 2\rho & 1 + \frac{1}{2}\rho^2 \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix}_{\mathrm{CS}}, \tag{41}$$

where $\rho = Q_T/Q$ and

$$\Delta = 1 + \rho^2 + \frac{1}{2}\rho^2\lambda + \rho\mu - \frac{1}{4}\rho^2\nu. \tag{42}$$

The transformation from the GJ frame to the CS frame is the same, but with the replacement $\rho \to -\rho$.

In the discussion of the contributions to the various structure functions given below, the structure function $W_{\Delta\Delta}$ and the analyzing power ν of the $\cos 2\phi$ asymmetry will receive particular attention due to its Collins effect contributions.

4. Leading order cross section

In this section we investigate the ϕ dependence that arises in leading order in α_s and $1/Q$ in the cross section of the process $e^+e^- \to h_1 h_2 X$ differential in the transverse momentum $\boldsymbol{P}_{1\perp} = -z_1\boldsymbol{q}_T$. The cross section involves products of fragmentation functions, which unlike the ordinary collinear functions include transverse momentum dependence [22]. The idea that such "intrinsic" transverse momentum will give rise to power suppression turns out not to be true, even though this was true in the pioneering studies [28,29] on azimuthal dependences due to intrinsic transverse momentum. Nontrivial quark spin effects, which require nonzero partonic transverse momenta, can arise at leading power. One such effect is the Collins effect, which gives rise to a $\cos 2\phi$ asymmetry. This was first pointed out in Refs. [6,30] and in this section we will repeat the essentials.
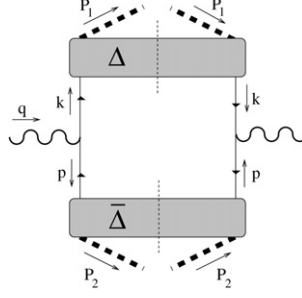
Fig. 3. Factorized diagram contributing to e^+e^- annihilation in leading order. There is a similar diagram with reversed fermion flow.

4.1. Integration over transverse photon momentum

Although we are interested in the two-hadron inclusive cross section differential in $\boldsymbol{q}_T$, we will first consider the case of integration over the transverse momentum of the photon. At tree level one needs to calculate the diagram shown in Fig. 3. It depicts the squared amplitude of the process in which the photon produces a quark and an antiquark, which subsequently fragment independently into the hadrons h_1 and h_2, respectively. The quark fragmentation correlation function $\Delta(P_1; k)$ is defined as [22]:

$$\Delta_{ij}(P_1; k) = \sum_X \int \frac{d^4x}{(2\pi)^4} e^{ik \cdot x} \langle 0|\psi_i(x)|P_1; X\rangle \langle P_1; X|\overline{\psi}_j(0)|0\rangle, \tag{43}$$

where k is the quark momentum and an averaging over color indices is left implicit. It is also understood that appropriate path-ordered exponentials should be included in order to obtain a color gauge invariant quantity, cf. e.g. [31]. In Section 5 we will briefly address the universality issue that arises from the proper gauge invariant definition.

The above matrix element as function of invariants is assumed to vanish sufficiently fast above a characteristic hadronic scale ($\mathcal{O}(M)$) which is much smaller than Q. This means that in the above matrix elements $k^2, k \cdot P_1 \ll Q^2$. Hence, we make the following Sudakov decomposition for the quark momentum k:

$$k \equiv \frac{z_1 Q}{z\sqrt{2}} n_- + \frac{z(k^2 + \boldsymbol{k}_T^2)}{z_1 Q\sqrt{2}} n_+ + k_T \approx \frac{1}{z} P_1 + k_T. \tag{44}$$

The Dirac structure of the quark correlation function can be expanded in a number of amplitudes, i.e., functions of invariants built up from the quark and hadron momenta, constrained by hermiticity and parity. In the calculation of the cross section integrated over the transverse momentum of the photon, at leading order we only encounter the integrated correlation function $\int dk^+ d^2\boldsymbol{k}_T \, \Delta(P_1; k)$, which is a function of k^- only. At leading twist this leaves only one possible Dirac structure:

$$\Delta(z) \equiv \frac{z}{4} \int dk^+ d^2\boldsymbol{k}_T \, \Delta(P_1; k)\big|_{k^- = P_1^-/z} = \frac{1}{4P_1^-} D_1(z)\not{P}_1. \tag{45}$$

The function $D_1(z)$ is the ordinary unpolarized fragmentation function.

For the fragmentation of an antiquark most things are analogous to the quark fragmentation. The main difference in the present case is that the role of the $+$ and the $-$ direction is reversed.

We will denote the antiquark correlation function by $\overline{\Delta}(P_2; p)$:

$$\overline{\Delta}_{ij}(P_2; p) = \sum_X \int \frac{d^4x}{(2\pi)^4} e^{-ip \cdot x} \langle 0|\overline{\psi}_j(0)|P_2; X\rangle \langle P_2; X|\psi_i(x)|0\rangle. \tag{46}$$

Similarly,

$$\overline{\Delta}(\bar{z}) \equiv \frac{\bar{z}}{4} \int dp^- \, d^2\boldsymbol{p}_T \, \Delta(P_2; p)\big|_{p^+ = P_2^+/\bar{z}} = \frac{1}{4P_2^+} \overline{D}_1(\bar{z})\slashed{P}_2. \tag{47}$$

The four-momentum conservation delta-function at the photon vertex is written as (neglecting $1/Q^2$ contributions)

$$\delta^4(q - k - p) = \delta(q^+ - p^+)\delta(q^- - k^-)\delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T), \tag{48}$$

fixing $P_2^+/\bar{z} = p^+ = q^+ = P_2^+/z_2$ and $P_1^-/z = k^- = q^- = P_1^-/z_1$. Eq. (48) shows why only the k^+ and p^--integrated correlation functions are relevant.

The hadron tensor as function of $\boldsymbol{q}_T$ is given by

$$\mathcal{W}^{\mu\nu} = 3 \int dp^- \, dk^+ \, d^2\boldsymbol{p}_T \, d^2\boldsymbol{k}_T \, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T) \operatorname{Tr}\big(\overline{\Delta}(p)\gamma^\mu \Delta(k)\gamma^\nu\big). \tag{49}$$

The factor 3 originates from the color summation. We have omitted the flavor indices and summation; furthermore, there is a contribution from diagrams with reversed fermion flow, which results from the above expression by replacing $\mu \leftrightarrow \nu$ and $q \rightarrow -q$ (and in the end in a summation over flavors and antiflavors). Note that the quark and antiquark transverse momentum integrations are linked, unless one integrates over $\boldsymbol{q}_T$.

After integration over the transverse momentum of the photon (or equivalently over the perpendicular momentum of hadron 1: $\boldsymbol{P}_{1\perp} = -z_1\boldsymbol{q}_T$), the integrations over $\boldsymbol{k}_T$ and $\boldsymbol{p}_T$ in the hadron tensor in Eq. (49) can be performed resulting in[2]

$$\int d^2\boldsymbol{q}_T \, \mathcal{W}^{\mu\nu} = -\frac{12}{z_1 z_2} \sum_{a,\bar{a}} e_a^2 g_\perp^{\mu\nu} D_1 \overline{D}_1. \tag{50}$$

We have now included the summation over flavor indices and e_a is the quark charge in units of e. The fragmentation functions are flavor dependent and only depend on the longitudinal momentum fractions, i.e. $D_1 \overline{D}_1 = D_1^a(z_1)\overline{D}_1^a(z_2)$.

From this hadron tensor one arrives at the following expression for the cross section at leading order in α_s and $1/Q$

$$\frac{d\sigma(e^+e^- \rightarrow h_1 h_2 X)}{dz_1 \, dz_2 \, d\Omega} = \frac{3\alpha^2}{Q^2} A(y) \sum_{a,\bar{a}} e_a^2 D_1 \overline{D}_1. \tag{51}$$

4.2. Unintegrated cross section

Now we turn to the cross section differential in the transverse momentum. In this case the correlation function Δ only integrated over k^+ needs to be considered. It can be parameterized

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[2] Eq. (50) corrects Eq. (75) of Ref. [6] by an additional factor $(z_1 z_2)^{-2}$.

in terms of transverse momentum dependent (TMD) fragmentation functions [32,33]

$$\Delta(z, \boldsymbol{k}_T) \equiv \frac{1}{4z} \int dk^+ \Delta(P_1; k)\big|_{k^- = P_1^-/z, \boldsymbol{k}_T}$$

$$= \frac{M_1}{4P_1^-} \left\{ D_1(z, \boldsymbol{k}_T^2) \frac{\not{P}_1}{M_1} + H_1^\perp(z, \boldsymbol{k}_T^2) \frac{\sigma_{\mu\nu} k_T^\mu P_1^\nu}{M_1^2} \right\}, \qquad (52)$$

where we only display the fragmentation functions that are relevant for unpolarized hadron production. The functions D_1 and $H_1^\perp$ yield contributions to the cross section that are of leading order in $1/Q$. After integration over $\boldsymbol{k}_T$ the term with $H_1^\perp$ drops out and the first term reduces[3] to the expression in Eq. (45). Strictly speaking, the TMD fragmentation functions depend on z (the lightcone momentum fraction $z = P_1^-/k^-$ of the produced hadron with respect to the fragmenting quark) and on $\boldsymbol{k}_T'^2 = z^2 \boldsymbol{k}_T^2$. Here $\boldsymbol{k}_T' \equiv -z\boldsymbol{k}_T$ is the transverse momentum of the hadron in a frame where the quark has no transverse momentum. In order to switch from quark to hadron transverse momentum a Lorentz transformation leaving k^- and P_1^- unchanged needs to be performed [21,22].

The so-called Collins effect function $H_1^\perp$ implies a correlation between the transverse polarization direction of the quark and the transverse momentum direction of the unpolarized hadron it fragments into [1]. The Collins effect correlates the azimuthal angle of the transverse spin of a fragmenting quark with that of the transverse momentum of the produced hadron (both taken around the quark momentum), via a $\sin\phi$ distribution of their difference angle ϕ. Therefore, the distribution of final state particles contains information about the spin direction of fragmenting quarks. In this sense it is the strong interaction analogue of the self-analyzing property of weak decays.

The presence of the Dirac matrix $\sigma_{\mu\nu}$ shows that the Collins effect is a chiral-odd (a quark-chirality flip) state; an interference term between opposite chirality states of the fragmenting quark. The function $H_1^\perp$ is also often referred to as 'time-reversal odd' fragmentation function, due to its behavior under time reversal. It does not imply a violation of time reversal symmetry though. For a detailed discussion cf. Ref. [31].

The $\cos 2\phi$ asymmetry to be discussed below depends on a product of two Collins effect fragmentation functions $H_1^\perp$. It is an azimuthal *spin* asymmetry in the sense that the asymmetry arises due to the correlation of the transverse spin states of the quark–antiquark pair. On average the quark and antiquark will not be transversely polarized, but for each particular event the spins can have a transverse component and these components will be correlated via the photon polarization state, which in turn is determined by the lepton direction. Due to the Collins effect the directions of the produced hadrons are correlated to the quark and antiquark spin and hence, to the lepton direction. This correlation does not average out after summing over all quark polarization states.

As a transverse spin state is a helicity-flip state, one deduces that the asymmetry arises from the interference between the photon helicity ± 1 states (along the quark–antiquark axis) and hence contributes to $W_{\Delta\Delta}$ and to ν. Such a helicity-flip contribution can also arise from quark mass terms, but those are power suppressed and do not lead to an azimuthal dependence (the θ dependence will be identical though).

The Collins effect was the main reason for studying azimuthal asymmetries in the BELLE data, since it shows up at leading order (in both α_s and $1/Q$) in an azimuthal $\cos 2\phi$ asymmetry

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[3] The relation between TMD and ordinary collinear fragmentation functions is not trivial beyond leading order in α_s. For a discussion on the analogous problem for distribution functions we refer to Refs. [34,35].

in the differential cross section for unpolarized $e^+e^- \to h_1 h_2 X$ [6]:

$$\frac{d\sigma(e^+e^- \to h_1 h_2 X)}{dz_1\, dz_2\, d\Omega\, d^2\boldsymbol{q}_T}$$

$$= \frac{3\alpha^2}{Q^2} z_1^2 z_2^2 \Bigg\{ A(y)\mathcal{F}[D_1 \overline{D}_1]$$

$$+ B(y)\cos 2\phi_1 \mathcal{F}\Bigg[\Big(2\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T \hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T - \boldsymbol{k}_T \cdot \boldsymbol{p}_T\Big)\frac{H_1^\perp \overline{H}_1^\perp}{M_1 M_2}\Bigg] \Bigg\}, \tag{53}$$

where we use the convolution notation

$$\mathcal{F}[D\overline{D}] \equiv \sum_{a,\bar{a}} e_a^2 \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{p}_T\, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T) D^a\big(z_1, z_1^2 \boldsymbol{k}_T^2\big)\overline{D}^a\big(z_2, z_2^2 \boldsymbol{p}_T^2\big). \tag{54}$$

The angle ϕ_1 is the azimuthal angle of $\hat{\boldsymbol{h}} = \hat{\boldsymbol{x}}$, see Fig. 1. So we find that in the lepton-pair center of mass frame:

$$\frac{dN}{d\Omega} \equiv \Bigg(\frac{d\sigma}{dz_1\, dz_2\, d^2\boldsymbol{q}_T}\Bigg)^{-1} \frac{d\sigma}{dz_1\, dz_2\, d\Omega\, d^2\boldsymbol{q}_T}$$

$$= \frac{3}{16\pi}\Bigg\{ (1+\cos^2\theta_2)\mathcal{F}[D_1\overline{D}_1]$$

$$+ \sin^2\theta_2 \cos 2\phi_1 \mathcal{F}\Bigg[\Big(2\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T \hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T - \boldsymbol{k}_T \cdot \boldsymbol{p}_T\Big)\frac{H_1^\perp \overline{H}_1^\perp}{M_1 M_2}\Bigg] \Bigg\}\Big/ \mathcal{F}[D_1\overline{D}_1]. \tag{55}$$

This shows that

$$W_T = \mathcal{F}[D_1\overline{D}_1], \tag{56}$$

$$W_L = W_\Delta = 0, \tag{57}$$

$$W_{\Delta\Delta} = \mathcal{F}\Bigg[\Big(2\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T \hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T - \boldsymbol{k}_T \cdot \boldsymbol{p}_T\Big)\frac{H_1^\perp \overline{H}_1^\perp}{M_1 M_2}\Bigg], \tag{58}$$

or equivalently, that at tree level $\lambda = 1$, $\mu = 0$ and

$$\nu = 2\frac{\mathcal{F}[(2\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T \hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T - \boldsymbol{k}_T \cdot \boldsymbol{p}_T)H_1^\perp \overline{H}_1^\perp]}{M_1 M_2 \mathcal{F}[D_1\overline{D}_1]}. \tag{59}$$

We emphasize that measuring ν does not involve a measurement of the polarization of the produced hadrons nor of the incoming leptons. Also, the result is not suppressed by factors of $1/Q$, in contrast to the $\cos 2\phi$ asymmetry discussed by Berger [29], which is $1/Q^2$-suppressed. Eq. (59) applies to the GJ frame, but upon neglecting Q_T^2/Q^2 power suppressed terms it is the same expression in the CS frame, as can be seen using Eq. (41).

We will often assume Gaussian $\boldsymbol{k}_T$-dependence of the various functions, since in that case the convolutions can be explicitly evaluated. Eq. (53) then becomes

$$\frac{d\sigma(e^+e^- \to h_1 h_2 X)}{dz_1\, dz_2\, d\Omega\, d^2\boldsymbol{q}_T} = \frac{3\alpha^2}{Q^2}\mathcal{G}(Q_T; R)\sum_{a,\bar{a}} e_a^2 \Bigg\{ A(y)D_1^a(z_1)\overline{D}_1^a(z_2)$$

$$+ B(y)\cos 2\phi_1 \frac{Q_T^2 R^4}{M_1 M_2 R_1^2 R_2^2} H_1^{\perp a}(z_1)\overline{H}_1^{\perp a}(z_2) \Bigg\}, \tag{60}$$

where $R^2 = R_1^2 R_2^2 / (R_1^2 + R_2^2)$ and

$$D_1(z_1, \mathbf{k}_T'^2) = D_1(z_1) R_1^2 \exp(-R_1^2 \mathbf{k}_T^2) / \pi z_1^2 \equiv D_1(z_1) \mathcal{G}(|\mathbf{k}_T|; R_1) / z_1^2, \tag{61}$$

and similarly for $\overline{D}_1, H_1^\perp, \overline{H}_1^\perp$ with obvious replacements (for details cf. Ref. [33]). For simplicity we have assumed the same Gaussian width for D_1 and $H_1^\perp$, which is not expected to be realistic. Later on we will drop this assumption. Rather, we will often assume $R_1 = R_2 = R$ which means equal widths for $H_1^\perp$ and $\overline{H}_1^\perp$, $R_{1u} = R_{2u} = R_u$ which means equal widths for D_1 and $\overline{D}_1$, and moreover, $M_1 = M_2 = M$, which should be a reasonable assumption when the two produced hadrons are charged pions. This leads to

$$\frac{d\sigma(e^+e^- \to h_1 h_2 X)}{dz_1 \, dz_2 \, d\Omega \, d^2\mathbf{q}_T} = \frac{3\alpha^2}{Q^2} \sum_{a,\bar{a}} e_a^2 \left\{ A(y) \mathcal{G}(Q_T; R_u/2) D_1^a(z_1) \overline{D}_1^a(z_2) \right.$$

$$\left. + B(y) \cos 2\phi_1 \frac{Q_T^2}{4M^2} \mathcal{G}(Q_T; R/2) H_1^{\perp a}(z_1) \overline{H}_1^{\perp a}(z_2) \right\}. \tag{62}$$

4.3. Weighted cross sections

The expressions in the previous subsection contain convolutions, which are not the objects of interest, rather one wants to learn about the fragmentation functions depending on z and $\mathbf{k}_T^2$. This may not be possible without further assumptions about the type of $\mathbf{k}_T^2$ dependence, such as assuming Gaussian transverse momentum dependence. As a way out, it has been suggested [6,36] to consider specific integrated, weighted asymmetries that probe instead of the full transverse momentum dependence, the $\mathbf{k}_T^2$-moments of the functions. These so-called transverse moments are defined as:

$$F^{(n)}(z_1) = \int d^2\mathbf{k}_T' \left(\frac{\mathbf{k}_T^2}{2M_1^2} \right)^n F(z_1, \mathbf{k}_T'^2), \tag{63}$$

for a generic fragmentation function F. In particular, the first transverse moment of the Collins fragmentation function

$$H_1^{\perp(1)}(z) = z^2 \int d^2\mathbf{k}_T \frac{\mathbf{k}_T^2}{2M^2} H_1^\perp(z, z^2 \mathbf{k}_T^2) \tag{64}$$

has been considered frequently in the literature.

In Section 4.1 we have presented the hadron tensor and cross section integrated over transverse momentum of the photon. A number of structures averaged out to zero, which are retained when the integration is weighted with an appropriate number of factors of $\mathbf{q}_T$. By constructing such weighted cross sections *at tree level* the convolutions become simply products of such $\mathbf{k}_T^2$-moments. The $\mathbf{k}_T^2$-moments can be used in other processes where they also occur.

To shorten the notation we define weighted cross sections as follows

$$\langle W \rangle = \int d^2\mathbf{q}_T \, W \frac{d\sigma(e^+e^- \to h_1 h_2 X)}{dz_1 \, dz_2 \, d\Omega \, d^2\mathbf{q}_T}, \tag{65}$$

where $W = W(Q_T, \phi_1)$. In this way one finds:

$$\langle 1 \rangle = \frac{3\alpha^2}{Q^2} A(y) \sum_{a,\bar{a}} e_a^2 D_1^a(z_1) \overline{D}_1^a(z_2), \tag{66}$$

$$\langle Q_T^2 \rangle = \frac{3\alpha^2}{Q^2} 2A(y) \sum_{a,\bar{a}} e_a^2 \big(M_1^2 D_1^{(1)a}(z_1) \overline{D}_1^a(z_2) + M_2^2 D_1^a(z_1) \overline{D}_1^{(1)a}(z_2) \big), \tag{67}$$

$$\left\langle \frac{Q_T^2}{4M_1 M_2} \cos 2\phi_1 \right\rangle = \frac{3\alpha^2}{Q^2} B(y) \sum_{a,\bar{a}} e_a^2 H_1^{\perp(1)a}(z_1) \overline{H}_1^{\perp(1)a}(z_2). \tag{68}$$

The $\boldsymbol{k}_T^2$-moment $H_1^{\perp(1)}$ that arises in the above e^+e^--annihilation expression also appears in the Q_T-weighted $\sin(\phi_h + \phi_S)$ asymmetry in semi-inclusive lepton-hadron scattering, in that case multiplied by the transversity distribution function [37]. This illustrates the purpose of considering such weighted cross sections.

Next we will discuss an estimate of the tree level weighted expressions of interest for the Collins effect. In Section 11 we will address the effect of radiative corrections on such weighted asymmetries.

4.4. Estimate of the Q_T^2-weighted Collins effect asymmetry

A natural question to ask is what one expects for the magnitude of the Collins effect asymmetry. This runs immediately into the problem that the Collins fragmentation function is a nonperturbative quantity that is at least as hard to calculate from first principles as the ordinary unpolarized fragmentation function D_1. In this subsection we will discuss a rough estimate of the weighted cross section defined in Eq. (68), appropriately normalized.

For an order of magnitude estimate of the weighted asymmetry, we consider the situation of the produced hadrons being a π^+ and a π^- and only consider up and down quarks. Furthermore, we assume $D_1^{u \to \pi^+}(z) = D_1^{\bar{d} \to \pi^+}(z)$, $D_1^{d \to \pi^-}(z) = D_1^{\bar{u} \to \pi^-}(z)$ and neglect unfavored fragmentation functions like $D^{d \to \pi^+}(z)$, etc.; and similarly for the time-reversal odd functions $H_1^\perp, \overline{H}_1^\perp$. These equalities seem quite safe for the D_1 functions on grounds of isospin and charge conjugation. The same assumptions might be nontrivial for the $H_1^\perp$ functions and remain to be tested. Studies of the HERMES data seem to indicate that the unfavored Collins functions can be of the same magnitude as the favored ones [4]. It remains to be seen whether this also holds true at higher energy scales, when the multiplicity of hadrons in the final state is significantly higher and momentum conservation would not correlate the hadron momenta as much as at low multiplicities.

With these assumptions we obtain

$$\left\langle \frac{Q_T^2}{4M_\pi^2} \cos 2\phi_1 \right\rangle = F(y) \frac{H_1^{\perp(1)}(z_1)}{D_1(z_1)} \frac{H_1^{\perp(1)}(z_2)}{D_1(z_2)} \langle 1 \rangle, \tag{69}$$

where

$$F(y) = \frac{B(y)}{A(y)} \overset{\text{cm}}{=} \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2}. \tag{70}$$

An upper bound on the ratio $H_1^{\perp(1)}(z_1)/D_1(z_1)$ could be obtained using the Soffer type of bound [38,39]

$$|\boldsymbol{k}_T| \big| H_1^\perp \big(z, |\boldsymbol{k}_T| \big) \big| \leqslant M_h D_1 \big(z, |\boldsymbol{k}_T| \big), \tag{71}$$

which should hold for all $|\boldsymbol{k}_T|$. It implies:

$$\frac{|H_1^{\perp(1)}(z)|}{D_1(z)} \leqslant \frac{\langle |\boldsymbol{k}_T| \rangle(z)}{2M_h}. \tag{72}$$

Alternatively, several model calculations of the Collins function have been performed and could be used to obtain an estimate of the weighted asymmetry, rather than of an upper bound on it. A review of models has been presented in Ref. [40]. The first model was given by Collins [1] and employed in Ref. [41] to estimate the Collins effect asymmetry in SIDIS, which led to the conclusion that $H_1^{\perp(1)}(z_1)/D_1(z_1) = \mathcal{O}(1)$. Here we will use this result and not worry about the sign of the Collins function. Also, we will use that the average value of $F(y)$ is approximately 0.5.

In order to get an estimate of the true asymmetry without artificial enhancement due to the weight, one should compare Eq. (68) with the weighted cross section $\langle Q_T^2/4M_\pi^2 \rangle$, rather than with $\langle 1 \rangle$. From Eqs. (66) and (67) one obtains:

$$\left\langle \frac{Q_T^2}{4M_\pi^2} \right\rangle = \frac{1}{2}\left(\frac{D_1^{\perp(1)}(z_1)}{D_1(z_1)} + \frac{D_1^{\perp(1)}(z_2)}{D_1(z_2)} \right)\langle 1 \rangle. \tag{73}$$

This we estimate by using that $D_1^{(1)}/D_1 = \langle k_T'^2 \rangle(z)/(2z^2 M^2)$. Ref. [42] presented a fit to LEP data to find that the average transverse momentum squared of pions inside a jet can be parameterized well as

$$\langle k_T'^2 \rangle^{\frac{1}{2}}(z) = 0.61 z^{0.27}(1-z)^{0.20} \text{ GeV}/c. \tag{74}$$

This leads at $z_1 = z_2 = 1/2$ (where the average transverse momentum squared is maximal approximately) to $\langle Q_T^2/4M_\pi^2 \rangle \approx 20\langle 1 \rangle$ and hence to an estimate at the few percent level for the ratio $\langle (Q_T^2/4M_\pi^2)\cos 2\phi_1 \rangle/\langle Q_T^2/4M_\pi^2 \rangle$. Of course, this should not be viewed as more than a crude estimate.

Note that $H_1^{\perp(1)}(z_1)/D_1(z_1) = \mathcal{O}(1)$ is consistent with the bound in Eq. (72), if one uses the maximum of $\langle k_T'^2 \rangle^{\frac{1}{2}}(z) \approx 0.44$ (around $z = 1/2$) for $z\langle |k_T| \rangle(z)$, which leads to $H_1^{\perp(1)}(z_1)/D_1(z_1) \lesssim 3$.

Another model study [39] has also obtained a prediction for the twice-weighted asymmetry (cf. Eqs. (67) and (68) with $M_1 = M_2$)

$$\langle P_{h\perp}^2 \cos 2\phi_1 \rangle(\theta_2, z_1, z_2) = \frac{\int d^2 P_{h\perp} P_{h\perp}^2 \cos 2\phi_1 \, d^5\sigma}{\int d^2 P_{h\perp} P_{h\perp}^2 \, d^5\sigma}$$
$$= \frac{2B(y)H_1^{\perp(1)}\overline{H}_1^{\perp(1)}}{A(y)(D_1\overline{D}_1^{(1)} + D_1^{(1)}\overline{D}_1)}, \tag{75}$$

in $e^+e^- \to h_1 h_2 X$. The calculation is based on the Manohar–Georgi model [43] and reproduces the unpolarized fragmentation function reasonably well. It leads for fixed z_2 bins to an asymmetry that is almost linearly rising as function of z_1. For $0.5 \leqslant z_2 \leqslant 0.8$ and $z_1 \sim 0.5$ the weighted asymmetry is 3–4%. This is similar in size to the estimate obtained above. This indicates that measuring the weighted asymmetry should be feasible for the present-day high luminosity electron–positron scattering experiments.

5. Universality of the Collins effect

Applying the results for the Collins function from BELLE in the Collins asymmetry in semi-inclusive DIS, assumes the Collins function is universal, i.e. that it is the same for all processes in which it occurs. This assumption is not as obvious as it may seem. For T-odd *distribution*

functions it has been shown [44,45] that they are process dependent. This follows from their gauge invariant definition as matrix elements of operators involving path-ordered exponentials that are nonlocal off the lightcone. For T-odd fragmentation functions, such as the Collins fragmentation function, a similar conclusion was drawn [31,46]. However, for the particular case of $e^+e^- \to h_1 h_2 X$ and SIDIS, it has been argued using a model, but later also with more general arguments, that the Collins fragmentation functions are identical [47,48]. Recently this conclusion was extended to the process $pp \to h$ jet X [49] and argued to be a generic and model-independent result. For the use of the Collins fragmentation functions in hadronic collisions it is essential though that factorization holds, which is not established however [50–53]. A further indication for the universality of the Collins function comes from a spectral analysis of the fragmentation correlation function within a spectator model calculation [54].

Assuming the universality of the Collins fragmentation function for the processes $e^+e^- \to h_1 h_2 X$ and SIDIS, a simultaneous fit to the Collins effect asymmetry data has been performed and a first extraction of transversity was obtained [8]. This demonstrates the feasibility of using the Collins effect to access transversity and why it is worth doing the Collins effect asymmetry measurement at BELLE. For this reason it becomes also important to study other potential contributions to the asymmetry that arises from the Collins effect. This we will do in the following sections.

6. Higher twist

In this section we will study the terms that arise when going beyond leading order in $1/Q$. Insertion of the leading order parameterization Eq. (52) of Δ in the calculation of the diagram shown in Fig. 3 also produces $1/Q$ contributions. Such $1/Q$ contribution can already be generated by simply transforming to a different frame. This contribution is not electromagnetically gauge invariant and the full calculation at order $1/Q$ [6] requires first of all, that the correlation function Δ is parameterized further to include higher twist fragmentation functions [33]

$$\Delta(z, \boldsymbol{k}_T) = \frac{M_1}{4P_1^-}\left\{ D_1 \frac{\not{P}_1}{M_1} + H_1^\perp \frac{\sigma_{\mu\nu}k_T^\mu P_1^\nu}{M_1^2} + E\mathbf{1} + D^\perp \frac{\not{k}_T}{M_1} + H\sigma_{\mu\nu}n_-^\mu n_+^\nu \right\}, \tag{76}$$

and second, inclusion of the diagrams in Fig. 4 [21]. These four diagrams involve one gluon which connects to one of the two soft hadronic matrix elements.

Hence, up to and including order $1/Q$ the quark fragmentation is described with help of two types of correlation functions: the quark correlation function $\Delta(P_1; k)$ discussed before and the quark–gluon correlation function $\Delta_A^\alpha(P_1; k, k_1)$ [21]:

$$\Delta_{Aij}^\alpha(P_1; k, k_1) = \sum_X \int \frac{d^4x}{(2\pi)^4} \frac{d^4y}{(2\pi)^4} e^{ik\cdot y + ik_1\cdot(x-y)}$$

$$\times \langle 0|\psi_i(x)gA_T^\alpha(y)|P_1; X\rangle\langle P_1; X|\overline{\psi}_j(0)|0\rangle, \tag{77}$$

where k, k_1 are the quark momenta and again inclusion of path-ordered exponentials and an averaging over color indices are understood. Note that the definition of Δ_A^α includes one power of the strong coupling constant g and $A_T^\alpha \equiv g_T^{\alpha\beta} A_\beta$.

In a calculation up to subleading order, we only encounter the partly integrated correlation functions $\int dk^+ \Delta(P_1; k)$ and $\int dk^+ d^4k_1 \Delta_A^\alpha(P_1; k, k_1)$. This allows to express the quark–gluon
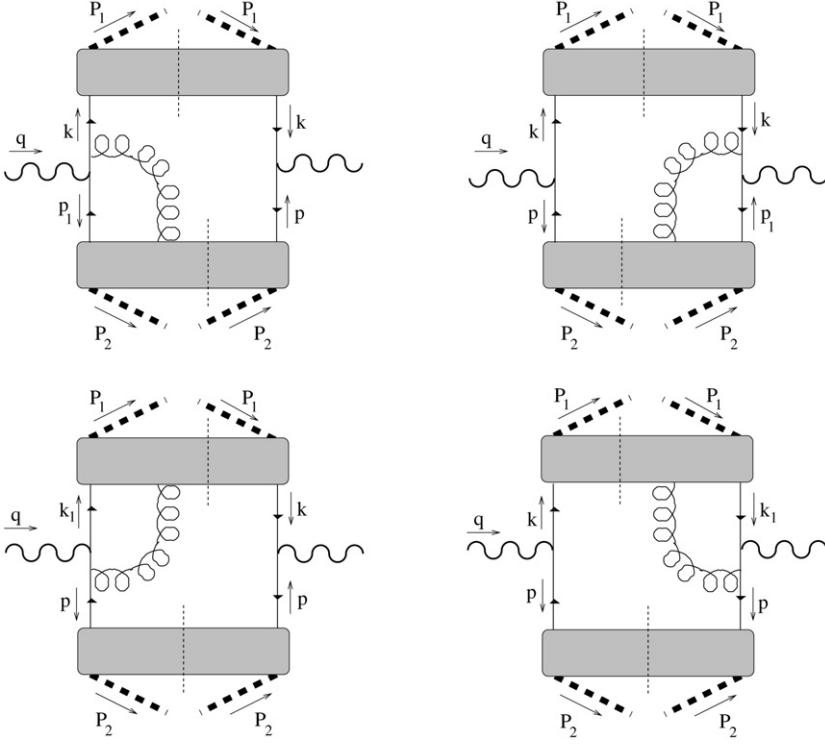
Fig. 4. Diagrams contributing to e^+e^- annihilation at order $1/Q$.

correlation functions in terms of the quark correlation functions with help of the classical equations of motion (e.o.m.) [55]. In the subleading terms of the cross section one encounters functions indicated with a tilde ($\tilde{E}$, $\tilde{H}$, ...), which differ from the corresponding twist-3 functions (H, E, ...) by a twist-2 part, namely

$$E = \frac{m}{M_1} z D_1 + \tilde{E}, \tag{78}$$

$$D^\perp = z D_1 + \tilde{D}^\perp, \tag{79}$$

$$H = -\frac{\boldsymbol{k}_T^2}{M_1^2} z H_1^\perp + \tilde{H}. \tag{80}$$

For more details we refer to [6].

The five diagrams lead to the following expression for the hadron tensor up to and including order $1/Q$:

$$\mathcal{W}^{\mu\nu} = 3 \int dp^- \, dk^+ \, d^2\boldsymbol{p}_T \, d^2\boldsymbol{k}_T \, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T) \bigg\{ \mathrm{Tr}\big(\overline{\Delta}(p)\gamma^\mu \Delta(k)\gamma^\nu\big)$$

$$- \mathrm{Tr}\bigg(\overline{\Delta}_A^\alpha(p)\gamma^\mu \Delta(k)\gamma_\alpha \frac{\slashed{n}_+}{Q\sqrt{2}}\gamma^\nu\bigg) - \mathrm{Tr}\bigg((\gamma_0 \bar{\Delta}_A^{\alpha\dagger}(p)\gamma_0)\gamma^\mu \frac{\slashed{n}_+}{Q\sqrt{2}}\gamma_\alpha \Delta(k)\gamma^\nu\bigg)$$

$$+ \mathrm{Tr}\bigg(\overline{\Delta}(p)\gamma^\mu (\gamma_0 \Delta_A^{\alpha\dagger}(k)\gamma_0)\gamma^\nu \frac{\slashed{n}_-}{Q\sqrt{2}}\gamma_\alpha\bigg)$$

$$+ \mathrm{Tr}\left(\overline{\Delta}(p)\gamma_\alpha \frac{\not{n}_-}{Q\sqrt{2}}\gamma^\mu \Delta_A^\alpha(k)\gamma^\nu\right)\right\}\Bigg|_{p^+,k^-}. \tag{81}$$

The terms with $\not{n}_\pm$ arise from the fermion propagators in the hard part neglecting contributions that will appear suppressed by powers of Q^2,

$$\frac{\not{q} - \not{p}_1 + m}{(q - p_1)^2 - m^2} \approx \frac{(q^+ - p_1^+)\gamma^-}{2(q^+ - p_1^+)q^-} = \frac{\gamma^-}{2q^-} = \frac{\not{n}_+}{Q\sqrt{2}}, \tag{82}$$

$$\frac{\not{k}_1 - \not{q} + m}{(k_1 - q)^2 - m^2} \approx \frac{(k_1^- - q^-)\gamma^+}{-2(k_1^- - q^-)q^+} = \frac{\gamma^+}{-2q^+} = -\frac{\not{n}_-}{Q\sqrt{2}}. \tag{83}$$

As mentioned, one can always integrate out one of the momenta of $\Delta_A^\alpha(k, k_1)$ or $\overline{\Delta}_A^\alpha(p, p_1)$ and apply the e.o.m. immediately. The quantities $\Delta_A^\alpha(k)$ and $\gamma_0 \Delta_A^{\alpha\dagger}(k)\gamma_0$ arise from integrating out the second and first argument of $\Delta_A^\alpha(k, k_1)$, respectively:

$$\int d^4k_1 \, \Delta_{Aij}^\alpha(k, k_1) = \sum_X \int \frac{d^4x}{(2\pi)^4} e^{ik\cdot x} \langle 0|\psi_i(x) g A_T^\alpha(x)|P_1; X\rangle\langle P_1; X|\overline{\psi}_j(0)|0\rangle$$

$$= \Delta_{Aij}^\alpha(k), \tag{84}$$

$$\int d^4k_1 \, \Delta_{Aij}^\alpha(k_1, k) = \sum_X \int \frac{d^4x}{(2\pi)^4} e^{ik\cdot x} \langle 0|\psi_i(x)|P_1; X\rangle\langle P_1; X|g A_T^\alpha(0)\overline{\psi}_j(0)|0\rangle$$

$$= \left(\gamma_0 \Delta_A^{\alpha\dagger}\gamma_0\right)_{ij}(k), \tag{85}$$

and similarly for $\overline{\Delta}_A^\alpha(p)$ and $\gamma_0 \overline{\Delta}_A^{\alpha\dagger}(p)\gamma_0$.

To obtain the expressions for the symmetric and antisymmetric parts of the hadron tensor we expand all vectors in $\Delta, \overline{\Delta}, \Delta_A^\alpha$ and $\overline{\Delta}_A^\alpha$ in the perpendicular basis ($\hat{t}, \hat{z}$ and $\perp$ directions). In particular, we re-express the momenta k_T and p_T in terms of their perpendicular parts and a part along $\hat{t}$ and $\hat{z}$. For this we need

$$g_T^{\mu\nu} = g_\perp^{\mu\rho} g_{T\rho}^\nu - \frac{Q_T}{Q}(\hat{t}^\mu + \hat{z}^\mu)\hat{h}^\nu. \tag{86}$$

We will refer to these perpendicular projections as e.g. $k_\perp$ rather than $k_{T\perp}$. Thus

$$k_\perp^\mu \equiv g_\perp^{\mu\nu} k_{T\nu} = k_T^\mu + \frac{\boldsymbol{q}_T \cdot \boldsymbol{k}_T}{Q}(\hat{t}^\mu + \hat{z}^\mu), \tag{87}$$

and similarly for $p_\perp$. We note that for these four vectors the two-component perpendicular parts are the same as the two-component transverse parts, i.e., $\boldsymbol{k}_\perp = \boldsymbol{k}_T$, etc. The full expression for the hadron tensor is then

$$\mathcal{W}^{\mu\nu} = 12z_1 z_2 \int d^2\boldsymbol{k}_T \, d^2\boldsymbol{p}_T \, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T)$$

$$\times \left\{ -g_\perp^{\mu\nu} D_1 \overline{D}_1 - \frac{k_\perp^{\{\mu} p_\perp^{\nu\}} + g_\perp^{\mu\nu} \boldsymbol{k}_\perp \cdot \boldsymbol{p}_\perp}{M_1 M_2} H_1^\perp \overline{H}_1^\perp \right.$$

$$\left. + 2\frac{\hat{z}^{\{\mu} k_\perp^{\nu\}}}{Q}\left[\frac{\tilde{D}^\perp}{z_1}\overline{D}_1 - \frac{M_2}{M_1}H_1^\perp \frac{\overline{H}}{z_2}\right] - 2\frac{\hat{z}^{\{\mu} p_\perp^{\nu\}}}{Q}\left[D_1 \frac{\overline{D}^\perp}{z_2} - \frac{M_1}{M_2}\frac{\tilde{H}}{z_1}\overline{H}_1^\perp\right]\right\}. \tag{88}$$

The cross section at leading order in α_s but including twist-3 contributions becomes

$$
\frac{d\sigma(e^+e^- \to h_1 h_2 X)}{dz_1\, dz_2\, d\Omega\, d^2\boldsymbol{q}_T}
$$

$$
= \frac{3\alpha^2}{Q^2} z_1^2 z_2^2 \left\{ A(y)\mathcal{F}[D_1 \overline{D}_1] + B(y)\cos 2\phi_1 \mathcal{F}\left[\left(2\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T \hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T - \boldsymbol{k}_T\cdot\boldsymbol{p}_T\right) \frac{H_1^\perp \overline{H}_1^\perp}{M_1 M_2} \right] \right.
$$

$$
- C(y)D(y)\cos\phi_1 \left(\frac{M_1}{Q}\mathcal{F}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_1} \frac{\tilde{D}^\perp}{z_1}\overline{D}_1 \right] - \frac{M_2}{Q}\mathcal{F}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_1} H_1^\perp \frac{\overline{H}}{z_2} \right] \right.
$$

$$
\left. \left. - \frac{M_2}{Q}\mathcal{F}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M_2} D_1 \frac{\overline{D}^\perp}{z_2} \right] + \frac{M_1}{Q}\mathcal{F}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M_2} \frac{\tilde{H}}{z_1}\overline{H}_1^\perp \right] \right) \right\}. \tag{89}
$$

This is the result in Eq. (53) plus an additional $\cos\phi$ asymmetry of order M/Q. The above expression is given in the GJ frame. When transforming to the CS frame the $\cos 2\phi$ asymmetry remains unchanged up to terms of order MQ_T/Q^2, as can be seen from Eq. (41).

The function E only contributes in the case of polarized electrons at $1/Q$ and for unpolarized electrons at the $1/Q^2$ level. The latter would be the analogue of the contribution considered by Jaffe and Ji [56] for the Drell–Yan process after the replacement $\delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T) \to \delta^2(\boldsymbol{q}_T)$.

We have neglected dynamic twist-4 effects, which are order M^2/Q^2 corrections. At BELLE these effects are expected to be at most at the percent level, but nevertheless could be relevant to include. To investigate this further theoretically requires an extensive study due to the many possible sources of $1/Q^2$ effects [28,29,56,57], which thus far has not been undertaken. One reason for this is that factorization has not been considered yet in this case (actually not even for the twist-3 case thus far). One particular type of higher twist effect was studied by Berger [29] for $e^+e^- \to \pi X$, which is mostly relevant at large values of z_h, towards the exclusive limit. This is like the higher twist contributions to the Drell–Yan azimuthal asymmetries studied in Refs. [58, 59], which contribute mainly at large x. However, it remains to be seen whether this specific z_h dependence holds true for all types of higher twist effects. They *are* generally expected to lead to $\mu > \nu$. Therefore, if μ is found to be much smaller than ν, it would be a strong indication that one is not dealing with higher twist effects. Experimentally one could also test whether such effects are relevant by allowing in the fits to asymmetries for additional $1/Q^2$ dependence, as has been done in studies of DIS data [60–62].

7. Electroweak interference effects

In the previous sections we have presented the results of the tree-level calculation of inclusive two-hadron production in electron–positron annihilation via one photon up to and including order $1/Q$, where the scale Q is defined by the (timelike) photon momentum q (with $Q^2 \equiv q^2$) and given by $Q = \sqrt{s}$. The quantity Q is required to be much larger than characteristic hadronic scales, a requirement satisfied by the BELLE experiment for which $\sqrt{s} \sim 10.5$ GeV. Although this energy is well below the Z-boson mass, we will now consider γ–Z interference effects, which could lead to percent level contributions ($\mathcal{O}(s/M_Z^2)$).

Only leading order $(1/Q)^0$ effects are discussed, since the combination of power corrections of order $1/Q$ and γ–Z interference effects is expected to be negligible (permille level). Here we will only focus on tree level, i.e., order $(\alpha_s)^0$. This may be improved upon at a later stage, following e.g. Ref. [63].

Table 2
Contractions of the lepton tensor $L_{\mu\nu}^{ij}$ with tensor structures appearing in the hadron tensor

$w^{\mu\nu}$	$L_{\mu\nu}^{ij} w^{\mu\nu}/(4Q^2)$
$-g_\perp^{\mu\nu}$	$C^{ij}(\frac{1}{2} - y + y^2)$
$a_\perp^{\{\mu} b_\perp^{\nu\}} - (a_\perp \cdot b_\perp) g_\perp^{\mu\nu}$	$-C^{ij} y(1-y)|\boldsymbol{a}_\perp||\boldsymbol{b}_\perp|\cos(\phi_a + \phi_b)$
$\frac{1}{2}(a_\perp^{\{\mu} \epsilon_\perp^{\nu\}\rho} b_{\perp\rho} + b_\perp^{\{\mu} \epsilon_\perp^{\nu\}\rho} a_{\perp\rho}) = a_\perp^{\{\mu} \epsilon_\perp^{\nu\}\rho} b_{\perp\rho} - (\epsilon_\perp^{\rho\sigma} a_{\perp\rho} b_{\perp\sigma}) g_\perp^{\mu\nu}$	$C^{ij} y(1-y)|\boldsymbol{a}_\perp||\boldsymbol{b}_\perp|\sin(\phi_a + \phi_b)$
$i\epsilon_\perp^{\mu\nu}$	$-D^{ij}(\frac{1}{2} - y)$
$ia_\perp^{[\mu} b_\perp^{\nu]}$	$-D^{ij}(\frac{1}{2} - y)|\boldsymbol{a}_\perp||\boldsymbol{b}_\perp|\sin(\phi_b - \phi_a)$

To leading order the expression for the hadron tensor, including quarks and antiquarks, is

$$\mathcal{W}^{\mu\nu} = 3 \int dp^- dk^+ d^2 \boldsymbol{p}_T d^2 \boldsymbol{k}_T \, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T) \operatorname{Tr}\big(\overline{\Delta}(p) V^\mu \Delta(k) V^\nu\big)\big|_{p^+, k^-}$$
$$+ \begin{pmatrix} q \leftrightarrow -q \\ \mu \leftrightarrow \nu \end{pmatrix}, \tag{90}$$

where for a photon $V^\mu = e\gamma^\mu$ and for a Z boson $V^\mu = g_V \gamma^\mu + g_A \gamma_5 \gamma^\mu$. We have omitted flavor indices and summation. The vector and axial-vector couplings to the Z boson are given by:

$$g_V^j = T_3^j - 2Q^j \sin^2 \theta_W, \tag{91}$$

$$g_A^j = T_3^j, \tag{92}$$

where Q^j denotes the charge and T_3^j the weak isospin of particle j (i.e., $T_3^j = +1/2$ for $j = u$ and $T_3^j = -1/2$ for $j = e^-, d, s$).

The lepton tensor is given by (neglecting lepton masses and polarization)

$$L_{\mu\nu}^{ij}(l, l') = C^{ij}\big[2l_\mu l_\nu' + 2l_\nu l_\mu' - Q^2 g_{\mu\nu}\big] + D^{ij} 2i\epsilon_{\mu\nu\rho\sigma} l^\rho l'^\sigma, \tag{93}$$

where we have defined

$$C^{\gamma\gamma} = 1, \qquad C^{\gamma Z} = C^{Z\gamma} = e^l g_V^l, \qquad C^{ZZ} = g_V^{l\,2} + g_A^{l\,2}, \tag{94}$$

$$D^{\gamma\gamma} = 0, \qquad D^{\gamma Z} = D^{Z\gamma} = -e^l g_A^l, \qquad D^{ZZ} = -2g_V^l g_A^l, \tag{95}$$

where e^l denotes the coupling of the photon to the leptons in units of the positron charge; g_V^l, g_A^l denote the vector and axial-vector couplings of the Z boson to the leptons, respectively.

The cross section is obtained from the hadron tensor after contraction with the lepton tensor, here given in the perpendicular basis,

$$L_{ij}^{\mu\nu} = C^{ij} Q^2 \Big[-(1 - 2y + 2y^2) g_\perp^{\mu\nu} + 4y(1-y)\hat{z}^\mu \hat{z}^\nu$$
$$- 4y(1-y)\Big(\hat{l}_\perp^\mu \hat{l}_\perp^\nu + \frac{1}{2} g_\perp^{\mu\nu}\Big) - 2(1 - 2y)\sqrt{y(1-y)}\hat{z}^{\{\mu} \hat{l}_\perp^{\nu\}} \Big]$$
$$+ D^{ij} Q^2 \big[i(1-2y)\epsilon_\perp^{\mu\nu} - 2i\sqrt{y(1-y)}\hat{l}_{\perp\rho} \epsilon_\perp^{\rho[\mu} \hat{z}^{\nu]} \big]. \tag{96}$$

Here only unsuppressed results arise when both indices μ, ν are in the perpendicular directions.

Using Table 2 one finds for the leading order unpolarized cross section, taking into account both photon and Z-boson contributions [64],

$$
\begin{aligned}
\frac{d\sigma(e^+e^- \to h_1 h_2 X)}{dz_1\, dz_2\, d\Omega\, d^2\boldsymbol{q}_T} \\
= \frac{3\alpha^2}{Q^2} z_1^2 z_2^2 \sum_{a,\bar{a}} \bigg\{ & K_1^a(y)\mathcal{F}[D_1 \overline{D}_1] + \big[K_3^a(y)\cos(2\phi_1) + K_4^a(y)\sin(2\phi_1) \big] \\
& \times \mathcal{F}\bigg[\Big(2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\,\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - \boldsymbol{p}_T\cdot\boldsymbol{k}_T \Big) \frac{H_1^\perp \overline{H}_1^\perp}{M_1 M_2} \bigg] \bigg\}.
\end{aligned}
\tag{97}
$$

The functions $K_i^a(y)$ (as before, a is the flavor index) are defined as

$$
K_1^a(y) = A(y)\big[e_a^2 + 2e_l g_V^l e_a g_V^a \chi_1 + c_1^l c_1^a \chi_2 \big] - \frac{C(y)}{2}\big[2e_l g_A^l e_a g_A^a \chi_1 + c_3^l c_3^a \chi_2 \big],
\tag{98}
$$

$$
K_3^a(y) = B(y)\big[e_a^2 + 2e_l g_V^l e_a g_V^a \chi_1 + c_1^l c_2^a \chi_2 \big],
\tag{99}
$$

$$
K_4^a(y) = B(y)\big[2e_l g_V^l e_a g_A^a \chi_3 \big],
\tag{100}
$$

which contain the combinations of the couplings

$$
c_1^j = \big(g_V^{j\,2} + g_A^{j\,2} \big),
\tag{101}
$$

$$
c_2^j = \big(g_V^{j\,2} - g_A^{j\,2} \big),
\tag{102}
$$

$$
c_3^j = 2 g_V^j g_A^j.
\tag{103}
$$

The propagator factors are given by

$$
\chi_1 = \frac{1}{\sin^2(2\theta_W)} \frac{Q^2(Q^2 - M_Z^2)}{(Q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2},
\tag{104}
$$

$$
\chi_2 = \frac{1}{\sin^2(2\theta_W)} \frac{Q^2}{Q^2 - M_Z^2}\chi_1,
\tag{105}
$$

$$
\chi_3 = \frac{-\Gamma_Z M_Z}{Q^2 - M_Z^2}\chi_1.
\tag{106}
$$

Using $M_Z = 91.1876 \pm 0.0021$ GeV ≈ 91.19 GeV and $\Gamma_Z = 2.4952 \pm 0.0023$ GeV ≈ 2.50 GeV, and using $\sin^2\theta_W(M_Z)|_{\overline{\mathrm{MS}}} = 0.23113(15) \approx 0.231$ (and assuming very slow running of this quantity [65]), we obtain for $Q = 10.50$ GeV

$$
\chi_1 \approx 0.019, \qquad \chi_2 \approx 0.00036, \qquad \chi_3 \approx -0.00052.
\tag{107}
$$

This shows that we can neglect the $\sin 2\phi_1$ asymmetry in Eq. (97) and also the ZZ contributions in K_1 and K_3. This leads to the following approximations for K_i^a in the lepton center of mass frame:

$$
K_1^u(y) \approx \frac{e_u^2}{4}\big[1.0005\big(1 + \cos^2\theta_2 \big) + 0.03\cos\theta_2 \big],
\tag{108}
$$

$$
K_1^d(y) \approx \frac{e_d^2}{4}\big[1.002\big(1 + \cos^2\theta_2 \big) + 0.06\cos\theta_2 \big],
\tag{109}
$$

$$
K_3^u(y) \approx \frac{e_u^2}{4}\big[1.0004\sin^2\theta_2 \big],
\tag{110}
$$

$$K_3^d(y) \approx \frac{e_d^2}{4} \big[1.001 \sin^2 \theta_2 \big], \tag{111}$$

$$K_4^u(y) \approx \frac{e_u^2}{4} \big[0.00003 \sin^2 \theta_2 \big] \approx 0, \tag{112}$$

$$K_4^d(y) \approx \frac{e_d^2}{4} \big[0.00006 \sin^2 \theta_2 \big] \approx 0. \tag{113}$$

In general, the γ–Z interference corrections are small (permille level), except for the forward–backward asymmetry ($\sim \cos\theta_2$), which is on the few percent level. The latter agrees with the results of Ref. [66], taking into account that their angle θ is defined as the angle between the incoming electron and the outgoing quark, leading to an additional minus sign for the asymmetry.

In order to estimate the size of the hadron level forward–backward asymmetry, we will again consider the case of π^+ and π^- production from only u and d quarks. Furthermore, we assume $D_1^{u\to\pi^+}(z) = D_1^{\bar{d}\to\pi^+}(z)$, $D_1^{d\to\pi^-}(z) = D_1^{\bar{u}\to\pi^-}(z)$ and neglect unfavored fragmentation functions like $D^{d\to\pi^+}(z)$, etc. If we define

$$d\sigma \sim \big[1 + \cos^2\theta_2 + A_{\text{FB}} \cos\theta_2 + \cdots \big], \tag{114}$$

then these assumptions lead to $A_{\text{FB}} \approx 4\%$.

In conclusion, γ–Z interference does not lead to any significant additional ϕ dependence at $\sqrt{s} \sim 10.5$ GeV. It only modifies the θ distribution with a forward–backward asymmetry term of a few percent.

8. Jet frame asymmetry

In an unpublished study [67] a transverse spin correlation similar to the $\cos 2\phi$ in back-to-back jets was experimentally investigated to some extent using LEP's DELPHI data. The following angular dependence of the differential cross section for correlated hadron production in opposite jets was studied:

$$\frac{d\sigma}{d\cos\theta \, d\phi \, d\phi'} \propto 1 + \cos^2\theta + c_{TT} S \sin^2\theta \cos(\phi + \phi'). \tag{115}$$

Here $c_{TT} = (|v_q|^2 - |a_q|^2)/(|v_q|^2 + |a_q|^2)$, with v_q and a_q the vector and axial-vector couplings of quarks to the Z boson, respectively. So in our notation $c_{TT} = c_2^a/c_1^a$. S is the analyzing power of the asymmetry to be determined. The azimuthal angles ϕ and ϕ' are of the leading particles in the two jets with respect to the $q\bar{q}$ axis in the lepton pair center of mass frame.

Such a $\cos(\phi + \phi')$ asymmetry differs from the $\cos 2\phi_1$ asymmetry discussed thus far in that now *three* momenta in the final state need to be determined, namely besides two hadron momenta also the jet axis. Hence there are two azimuthal angles, ϕ and ϕ', that need to be measured. Furthermore, as we will show, the analyzing power S is a different expression of the Collins functions. We find that it involves moments of the functions $H_1^\perp$ and $\overline{H}_1^\perp$, different from the ones in the correlation Eq. (68) that we considered thus far.

Unlike Ref. [67] we will not only include the Z boson, but also the photon contributions, including photon-Z interference terms (except for the interference term analogous to $K_4^a(y) \sin 2\phi_1$, which was found to be very small).

To derive a tree level expression for the analyzing power S in terms of fragmentation functions, we start with the hadron tensor expressed in the transverse basis

$$\mathcal{W}_{ij}^{\mu\nu} = 12 z_1 z_2 \int d^2\boldsymbol{k}_T \, d^2\boldsymbol{p}_T \, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T)$$

$$\times \left\{ -\left[g_T^{\mu\nu} C_{ij}^a - i\epsilon_T^{\mu\nu} D_{ij}^a \right] D_1 \overline{D}_1 - \frac{k_T^{\{\mu} p_T^{\nu\}} + g_T^{\mu\nu} \boldsymbol{k}_T \cdot \boldsymbol{p}_T}{M_1 M_2} E_{ij}^a H_1^\perp \overline{H}_1^\perp \right\}, \quad (116)$$

where in analogy to the lepton tensor Eq. (93) we have defined

$$C_{\gamma\gamma}^a = e_a^2, \qquad C_{\gamma Z}^a = C_{Z\gamma}^a = e^a g_V^a, \qquad C_{ZZ}^a = c_1^a, \qquad (117)$$

$$D_{\gamma\gamma}^a = 0, \qquad D_{\gamma Z}^a = D_{Z\gamma}^a = -e^a g_A^a, \qquad D_{ZZ}^a = -c_3^a, \qquad (118)$$

$$E_{\gamma\gamma}^a = e_a^2, \qquad E_{\gamma Z}^a = E_{Z\gamma}^a = e^a g_V^a, \qquad E_{ZZ}^a = c_2^a. \qquad (119)$$

In previous sections we have transformed such hadron tensor expressions into the perpendicular basis of the GJ frame, after which the contraction with the lepton tensor yields the cross section. Now we will follow the frame choice of Ref. [67]. If one would determine the jet-axis, which is identified with the $q\bar{q}$ axis, or as an approximation to it the thrust-axis, then a measurement of the transverse momenta of the leading particles in the two jets compared to the jet momentum is a determination of the transverse momenta of the quarks compared to the leading hadrons they fragment into. One can then keep the cross section differential in the azimuthal angles of the transverse momentum of the quarks, after which the $\boldsymbol{q}_T$ integration can be safely done (as opposed to the case of the $\cos 2\phi_1$ asymmetry) and it will not average to zero unless one integrates over the azimuthal angles. In this way one will arrive at an expression involving the moments

$$F^{[n]}(z_i) \equiv \int d|\boldsymbol{k}_T|^2 \left[\frac{|\boldsymbol{k}_T|}{M_i} \right]^n F\left(z_i, |\boldsymbol{k}_T|^2\right), \qquad (120)$$

for $n = 0$ and $n = 1$. The latter is often referred to as the "half-moment" and also written as $F^{(1/2)}$.

To make the transformation from the frame in which P_1 and P_2 are collinear (for which we employed the transverse basis) into the lepton pair center of mass frame where the $q\bar{q}$ axis defines the $\hat{z}$ axis, which we will refer to as the jet frame, a new perpendicular basis will be defined (the new perpendicular directions will also be indicated by $\perp$). We will choose:

$$\hat{t}^\mu \equiv \frac{q^\mu}{Q}, \qquad (121)$$

$$\hat{z}^\mu \equiv \frac{k^\mu - p^\mu}{Q}. \qquad (122)$$

We find that

$$g_\perp^{\mu\nu} = g_T^{\mu\nu} - \frac{\sqrt{2}}{Q} \left(p_T^{\{\mu} n_+^{\nu\}} + k_T^{\{\mu} n_-^{\nu\}} \right) - \frac{2}{Q^2} p_T^{\{\mu} k_T^{\nu\}}. \qquad (123)$$

Hence, $P_{1\perp} = -z_1 k_T$ and $P_{2\perp} = -z_2 p_T$, up to $1/Q^2$ corrections.

Since the $\hat{z}$ axis is defined differently now, the contraction of the hadron tensor with the lepton tensor yields somewhat different expressions. We give the relevant contractions in Table 3. The functions A, B and C are the same functions of $y = P_2 \cdot l / P_2 \cdot q$ as before. If one expresses the lepton momentum l in terms of $\hat{t}$ and $\hat{z}$, i.e., $l = Q\hat{t}/2 + \sqrt{Q^2/4 - \boldsymbol{l}_\perp^2}\,\hat{z} + l_\perp$, one finds that $A(y) = 1/2 - B(y)$ and $C(y) = -\sqrt{1 - 4B(y)}$, with $B(y) = \boldsymbol{l}_\perp^2/Q^2$.

Table 3
Contractions of the lepton tensor $L_{\mu\nu}$ with tensor structures appearing in the hadron tensor, for the jet frame

$w^{\mu\nu}$	$L_{\mu\nu}w^{\mu\nu}/(4Q^2)$
$-g_T^{\mu\nu}$	$c_1^l A(y)$
$k_T^{\{\mu} p_T^{\nu\}} + (\boldsymbol{k}_T \cdot \boldsymbol{p}_T)g_T^{\mu\nu}$	$-c_1^l B(y)|\boldsymbol{P}_{1\perp}||\boldsymbol{P}_{2\perp}|\cos(\phi+\phi')/(z_1 z_2)$
$i\epsilon_T^{\mu\nu}$	$c_3^l C(y)/2$

Using Table 3 we obtain in leading order in $1/Q$ and α_s the following expression for the cross section differential in ϕ and ϕ', in case of unpolarized final state hadrons:

$$
\begin{aligned}
\frac{d\sigma(e^+e^- \to h_1 h_2 X)}{dz_1\, dz_2\, d\Omega\, d\phi\, d\phi'} \\
= \sum_{a,\bar{a}} \frac{3\alpha^2}{Q^2} z_1^2 z_2^2 \big\{ K_1^a(y) D_1^{[0]a}(z_1) \overline{D}_1^{[0]a}(z_2) \\
+ K_3^a(y) \cos(\phi+\phi') H_1^{\perp[1]a}(z_1) \overline{H}_1^{\perp[1]a}(z_2) \big\}.
\end{aligned}
\tag{124}
$$

This is to be compared with Eq. (53). For simplicity of comparison we will now consider the one-flavor case (u-quark dominance). Hence, we find for the analyzing power S of Eq. (115)

$$
S = \frac{H_1^{\perp[1]}(z_1) \overline{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1) \overline{D}_1^{[0]}(z_2)}.
\tag{125}
$$

This can be compared to the expression for the weighted $\cos 2\phi_1$ asymmetry (cf. Eq. (69)):

$$
\frac{\langle \frac{Q_T^2}{4M_1 M_2} \cos 2\phi_1 \rangle}{\langle 1 \rangle} = \frac{K_3(y)}{K_1(y)} S'
\tag{126}
$$

where

$$
S' = \frac{H_1^{\perp(1)}(z_1) \overline{H}_1^{\perp(1)}(z_2)}{D_1(z_1) \bar{D}_1(z_2)}.
\tag{127}
$$

If one now simply assumes a Gaussian $\boldsymbol{k}_T$-dependence of the functions, i.e.,

$$
H_1^{\perp}(z, \boldsymbol{k}_T'^2) = H_1^{\perp}(z) R^2 \exp(-R^2 \boldsymbol{k}_T^2)/(\pi z^2)
\tag{128}
$$

and similarly for $D_1(z, \boldsymbol{k}_T'^2)$, and equal masses and widths, then one finds that

$$
S = \frac{\pi}{4R^2 M^2} \frac{H_1^{\perp}(z_1) \overline{H}_1^{\perp}(z_2)}{D_1(z_1) \overline{D}_1(z_2)}
\tag{129}
$$

and

$$
S' = \frac{1}{4R^4 M^4} \frac{H_1^{\perp}(z_1) \overline{H}_1^{\perp}(z_2)}{D_1(z_1) \overline{D}_1(z_2)}.
\tag{130}
$$

Note that in these expressions only the Gaussian width of $H_1^{\perp}$ appears. Therefore, one finds:

$$
S' = \frac{1}{\pi R^2 M^2} S.
\tag{131}
$$

For a pion we assume $RM \approx 0.5$, such that $S' \approx 4S/\pi$. As S' was expected to be on the few percent level, we also expect the analyzing power of the $\cos(\phi + \phi')$ asymmetry in the jet frame to be of that magnitude.

If one considers Gaussian transverse momentum dependence one does not need to resort to weighting to deal with the convolutions. One can consider simply the Q_T^2-integrated $\cos 2\phi_1$ asymmetry (numerator and denominator integrated separately). One arrives at:

$$S'' \equiv \frac{\int d\boldsymbol{q}_T^2 \, \mathcal{F}[(2\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T \hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T - \boldsymbol{k}_T \cdot \boldsymbol{p}_T)H_1^\perp \overline{H}_1^\perp]}{M^2 \int d\boldsymbol{q}_T^2 \, \mathcal{F}[D_1 \overline{D}_1]}$$

$$= \frac{1}{2R^2 M^2} \frac{H_1^\perp(z_1)\overline{H}_1^\perp(z_2)}{D_1(z_1)\overline{D}_1(z_2)} = \frac{2}{\pi}S, \tag{132}$$

which is somewhat smaller than the analyzing power S of the $\cos(\phi + \phi')$ asymmetry. All this indicates that changing from the GJ frame to the jet frame does not modify the analyzing power of the Collins effect asymmetry much.

9. Scale dependence of the Collins effect asymmetry

Thus far we have only discussed the azimuthal dependences that arise at tree level. This means that the expressions are valid in the region where the observed transverse momentum Q_T is small compared to the hard scale Q, applicable only in the region of intrinsic transverse momentum. After the extraction of the Collins function at BELLE, which is performed at one particular Q^2 value, the extracted functions will be used in asymmetries at different energies, for instance at lower Q^2 for SIDIS or higher Q^2 for a comparison to LEP1 data. For this one would need to know the scale dependence of the Collins function and asymmetry. However, the study of the scale dependence of expressions involving TMDs is not far developed. Here we will follow (and improve) the discussion of this issue given in Ref. [64].

It is clear that collinear factorization is not the right framework to address the question of scale dependence of the Collins effect asymmetry. Rather, for small Q_T the so-called Collins–Soper (CS) factorization theorem [68] is of relevance (see also Ref. [69] for a discussion of factorization and transverse momentum in two-hadron inclusive e^+e^- annihilation). It applies to the differential cross section:

$$\frac{d\sigma}{dz_1 \, dz_2 \, d\Omega d^2 \boldsymbol{q}_T}. \tag{133}$$

Collins and Soper [68] proved their factorization theorem for the process of interest here: $e^+e^- \to h_1 h_2 X$, but without inclusion of quark spin effects. A similar factorization for SIDIS and Drell–Yan, including polarization, was discussed more recently by Ji et al. [70], with some small differences w.r.t. Ref. [68].

In this section we will study the scale dependence of the Collins effect asymmetry taking CS factorization as our starting point. It allows us to obtain the dominant Q^2 dependence of the asymmetry at small Q_T^2, but the analysis also extends the range of applicability of the asymmetry results from the region of intrinsic transverse momentum to the region of moderate Q_T values (still under the restriction that $Q_T^2 \ll Q^2$). The consequences discussed below affect all transverse momentum dependent azimuthal spin asymmetries. It turns out that such azimuthal asymmetries generally suffer from Sudakov suppression with increasing Q^2 in the region where the transverse momentum Q_T is much smaller than the large energy scale Q. This Sudakov suppression stems from *soft* gluon radiation. In Ref. [1] Collins remarks that Sudakov factors will

have the effect of diluting the (Collins effect) single spin asymmetry in semi-inclusive DIS, due to broadening of the transverse momentum distribution by soft gluon emission. Here we will study this effect in a quantitative way for the $\cos 2\phi$ Collins-effect asymmetry in $e^+e^- \to h_1 h_2 X$. It shows that tree level estimates of such asymmetries tend to yield overestimates and increasingly so with rising energy.

9.1. Collins–Soper factorization

In the CS formalism the differential cross section at small Q_T^2/Q^2 is written as

$$\frac{d\sigma}{dz_1 \, dz_2 \, d\Omega \, d^2\boldsymbol{q}_T} = \int d^2 b \, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}_T} \, \tilde{W}(\boldsymbol{b}, Q; z_1, z_2) + \mathcal{O}\left(\frac{Q_T^2}{Q^2}\right), \tag{134}$$

where

$$\tilde{W}(\boldsymbol{b}, Q; z_1, z_2) = \sum_a \tilde{D}_1^a\big(z_1, \boldsymbol{b}; 1/b, \alpha_s(1/b)\big) \sum_b \tilde{D}_1^b\big(z_2, \boldsymbol{b}; 1/b, \alpha_s(1/b)\big)$$

$$\times \, e^{-S(b,Q)} H_{ab}\big(Q; \alpha_s(Q)\big) \tilde{U}\big(b; 1/b, \alpha_s(1/b)\big). \tag{135}$$

Here $\tilde{D}_1(z, \boldsymbol{b})$ is the Fourier transform of the transverse momentum dependent fragmentation function $D_1(z, z^2 \boldsymbol{k}_T^2)$; $e^{-S(b,Q)}$ is the Sudakov form factor; H is the partonic hard scattering part; and, $\tilde{U}$ is called the soft factor. The fragmentation functions and the soft factor are taken at the scale $\mu = 1/b$. Furthermore, note that there are no integrals over momentum fractions, those appear in the large Q_T or equivalently, small-b limit only. Hence, the z_i in Eq. (135) are the observed momentum fractions.

The Sudakov form factor arises due to summation of soft gluon contributions. This is in contrast to more inclusive cross sections for which there is often a cancellation of such soft gluon contributions. At values $b^2 = \boldsymbol{b}^2 \ll 1/\Lambda^2$, the Sudakov form factor is perturbatively calculable and of the form

$$S(b, Q) = \int\limits_{b_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A\big(\alpha_s(\mu)\big) \ln \frac{Q^2}{\mu^2} + B\big(\alpha_s(\mu)\big) \right], \tag{136}$$

where $b_0 = 2 \exp(-\gamma_E) \approx 1.123$ (here we use the usual CS factorization constants $C_1 = b_0$, $C_2 = 1$). One can expand the functions A and B in α_s/π and the first few coefficients are known, cf. e.g. [71,72]. At the leading logarithm (LL) level one needs to take into account only the first term in the expansion of A: $A^{(1)} = C_F/\pi$, which is the same for unpolarized and polarized scattering. It leads to the double leading logarithmic approximation (DLLA) result [73]:

$$S(b, Q) = C_F \frac{\alpha_s(Q)}{2\pi} \log^2\big(b^2 Q^2\big). \tag{137}$$

Including the running of α_s leads to the expression [74]

$$S(b, Q) = -\frac{16}{33 - 2n_f} \left[\log\left(\frac{b^2 Q^2}{b_0^2}\right) + \log\left(\frac{Q^2}{\Lambda^2}\right) \log\left[1 - \frac{\log(b^2 Q^2/b_0^2)}{\log(Q^2/\Lambda^2)} \right] \right]. \tag{138}$$

We will take for the number of flavors $n_f = 5$, since we are interested in energies just below the $\Upsilon(4S)$, which is above the $b\bar{b}$ threshold. Furthermore, we take $\Lambda_{\text{QCD}} = 200$ MeV.

Using only the perturbative expression for the Sudakov factor in the cross section expression (134) is valid for Q^2 very large, when the restriction $b^2 \ll 1/\Lambda^2$ is justified. If also $b^2 \gtrsim 1/\Lambda^2$ contributions are important ($\mu^2 \lesssim \Lambda^2$), for example at small Q_T, then one needs to include a nonperturbative Sudakov factor. This can be done for instance via the introduction of a b-regulator [75][4]: $b \to b_* = b/\sqrt{1 + b^2/b_{\max}^2}$, such that b_* stays always smaller than $b_{\max}$. Usually $b_{\max} = 0.5$ GeV^{-1}, such that $\alpha_s(b_0/b_* \gtrsim 2) \lesssim 0.3$. For the function $\tilde{W}(b_*)$ a perturbative expression for the Sudakov factor can thus be used.

One can rewrite $\tilde{W}(b)$ as:

$$\tilde{W}(b) \equiv \tilde{W}(b_*)e^{-S_{\mathrm{NP}}(b)}. \tag{139}$$

In general the nonperturbative Sudakov factor is of the form [75]

$$S_{\mathrm{NP}}(b, Q/Q_0) = g_1(z_1, b) + g_2(z_2, b) + \ln(Q^2/Q_0^2)g_3(b), \tag{140}$$

where $Q_0 \approx 1/b_{\max}$ is the lowest scale for which one trusts perturbation theory. The g_i functions ($i = 1, 2, 3$) are not calculable in perturbation theory and need to be fitted to data. In fact, for the Drell–Yan process they have been shown to be necessary to include in order to describe available data [77]. Note that S_{NP} is in general Q^2 dependent, unlike what one may at first thought expect for a nonperturbative quantity.

Due to the lack of knowledge of the nonperturbative Sudakov factor for the case of interest here, the quantitative results obtained below about the size and Q^2 dependence of the suppression should not be taken too literally. Nevertheless, one can draw conclusions about what determines the Q^2 dependence of the transverse momentum distribution of the asymmetries and about the size of the suppression for generic nonperturbative Sudakov factors.

Concerning the other factors in the CS factorization expression, it should be noted that the one-loop expression for H and $\tilde{U}$ were obtained in Ref. [68]. At the scale $\mu = 1/b$ the b-dependence of $\tilde{U}$ disappears at order α_s in the $\overline{\mathrm{MS}}$ scheme.

Due to the integration over b, the CS factorization expression explicitly requires knowledge of the scale dependence of the b-dependent fragmentation functions. In Ref. [64] this dependence was assumed to be weak (logarithmic) and neglected, but here we will include it and use that it is determined by the quark field renormalization. As already mentioned by Collins and Soper [68] in some cases it may be more convenient to take instead of the varying scale $\mu = 1/b$, a fixed scale $M_0 \gtrsim 1$ GeV. That scale was called μ_L by Ji et al. [70,78] and here we will call it Q_0. Using the renormalization group equations given in Ref. [68], the CS factorization expression (135) can be rewritten as:

$$\tilde{W}(\boldsymbol{b}, Q; z_1, z_2) = \sum_a \tilde{D}_1^a(z_1, \boldsymbol{b}; Q_0, \alpha_s(Q_0)) \sum_b \tilde{D}_1^b(z_2, \boldsymbol{b}; Q_0, \alpha_s(Q_0))$$
$$\times e^{-S(b, Q, Q_0)} H_{ab}(Q; \alpha_s(Q)) \tilde{U}(b; Q_0, \alpha_s(Q_0)). \tag{141}$$

This has the advantage that the b-dependent fragmentation functions are always considered at the same scale Q_0 when integrating over b.

We first consider the above expression in the perturbative regime: $b \leqslant 1/Q_0$. The one-loop expressions for H, $\tilde{U}$ and S are

$$H_{ab}(Q; \alpha_s(Q)) \propto \delta_{b\bar{a}}e_a^2(1 + \alpha_s(Q^2)F_1 + \mathcal{O}(\alpha_s^2)), \tag{142}$$

---

[4] An alternative method has been put forward in Ref. [76].

$$\tilde{U}\big(b; Q_0, \alpha_s(Q_0)\big) = 1 - \frac{\alpha_s(Q_0^2)}{\pi} C_F \big(\log Q_0^2 b^2 + F_2\big) + \mathcal{O}\big(\alpha_s^2\big), \tag{143}$$

$$S(b, Q, Q_0) = C_F \int\limits_{Q_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s(\mu)}{\pi} \left[\log \frac{Q^2}{\mu^2} + \log Q_0^2 b^2 + F_3 \right]. \tag{144}$$

Here F_i denote renormalization-scheme-dependent finite terms which will be neglected in the following. Following Ref. [78], one might also consider dropping in the expression between brackets in Eq. (144) the second term with respect to the first one in order to arrive at the double leading log expression. This is because $\log Q_0^2 b^2$ is in general not a large log, since the small-b^2 ($\sim 1/Q^2$) region contributes little for small Q_T. In that approximation S is not a function of b anymore. It leads to the double log term in $S \propto \log^2 Q^2/Q_0^2$. This means there is no coupling between Q^2 and Q_T^2 (ignoring for simplicity the Q^2 dependence of $S_{NP}(b, Q/Q_0)$), such that the Sudakov factor $\exp(-S)$ will cancel in asymmetry ratios. As a consequence, azimuthal asymmetries will be Q^2-independent in this approximation.

However, the $\log Q_0^2 b^2$ term in S is important to keep in order to obtain the leading Q^2-dependence of azimuthal asymmetries. If one keeps this term, then one obtains (ignoring the running of α_s for the moment):

$$S(b, Q, Q_0) = C_F \frac{\alpha_s(Q)}{2\pi} \big[\log^2 Q^2 b^2 - \log^2 Q_0^2 b^2\big]. \tag{145}$$

In this expression the second term *can* safely be ignored, if one is interested in the leading Q^2-dependence of asymmetries only. The average b^2 probed is of order $1/Q_T^2$, such that in the small-Q_T^2 region (of order Q_0^2): $\log^2 Q^2 b^2 \gg \log^2 Q_0^2 b^2$. The result then coincides with the well-known DLLA expression (137). Upon dropping all other small logarithms and finite terms, this leads to:

$$\tilde{W}(\boldsymbol{b}, Q; z_1, z_2) \propto \sum_a e_a^2 \tilde{D}_1^a\big(z_1, \boldsymbol{b}; Q_0, \alpha_s(Q_0)\big) \tilde{D}_1^{\bar{a}}\big(z_2, \boldsymbol{b}; Q_0, \alpha_s(Q_0)\big)$$

$$\times \exp\left(-C_F \frac{\alpha_s(Q)}{2\pi} \log^2 Q^2 b^2\right). \tag{146}$$

This expression, which we will refer to below as the DLLA of $\tilde{W}$, should capture the dominant Q^2-dependence of the differential cross section, for $Q^2 \gg Q_0^2$ and bearing in mind that this becomes a worse approximation as Q^2 increases. In the numerical results presented below we will include also the subleading logarithms explicitly given above and the one-loop running of α_s.

9.2. Numerical study of the Q^2 dependence of the Collins effect asymmetry

In order to study the expression Eq. (134) with $\tilde{W}$ of Eq. (141), we will assume again a Gaussian transverse momentum dependence for $D_1(z, z^2 \boldsymbol{k}_T^2)$:

$$D_1\big(z, z^2 \boldsymbol{k}_T^2\big) = D_1(z) R_u^2 \exp\big(-R_u^2 \boldsymbol{k}_T^2\big)/\pi z^2 \equiv D_1(z) \mathcal{G}\big(|\boldsymbol{k}_T|; R_u\big)/z^2, \tag{147}$$

such that the Fourier transform is

$$\tilde{D}_1\big(z, b^2\big) = D_1(z) \exp\left(-\frac{b^2}{4R_u^2}\right)/z^2. \tag{148}$$

We do not need to worry about any scale dependence of the width R_u, since by construction the fragmentation functions are always at scale Q_0. Here we will not aim to connect to the large-Q_T behavior of the cross section, where the Gaussian behavior is certainly not correct anymore. This fact and the assumed factorization of b and z dependence implies however that it is not guaranteed that the function $D_1(z)$ that appears here is indeed exactly equal to the integrated fragmentation function $D_1(z; Q_0^2)$ at the scale Q_0^2. Therefore, the safest thing may be to construct ratios of asymmetries where the functions $D_1(z)$ drop out, cf. Section 12. After these words of caution we will make some further pragmatic assumptions in order to be able to numerically investigate the Q^2 dependence of the Collins effect asymmetry.

The b-dependent piece of the fragmentation function $\tilde{D}_1(z, b^2)$, taken to be a Gaussian here, can be viewed as the Q^2-independent part of S_{NP}, because they are simply indistinguishable (ignoring the subtlety of using b here instead of b_*). Any remaining z dependence that does not factorize can in principle also be included in S_{NP}. We further assume that S_{NP} is flavor and spin independent, i.e. it is the same for all fragmentation functions. The assumption of spin independence will get better as Q^2 becomes larger. Refinements can be included at a later stage if the accuracy of the data demands it. Here we will take for the nonperturbative Sudakov factor the parameterization by Ladinsky and Yuan, which was fitted to the transverse momentum distribution of W/Z production in pp ($p\bar{p}$) scattering [79]. We will use it with the additional simplifying choice of $x_1 x_2 = 10^{-2}$:

$$S_{NP}(b, Q/Q_0) = g_1 b^2 + g_2 b^2 \ln\left(\frac{Q}{2Q_0}\right), \tag{149}$$

with $g_1 = 0.11 \text{ GeV}^2$, $g_2 = 0.58 \text{ GeV}^2$, $Q_0 = 1.6 \text{ GeV}$ and $b_{max} = 0.5 \text{ GeV}^{-1}$. Note that for $Q = 10 \text{ GeV}$ the Q^2-independent part is negligible, which justifies ignoring the above-mentioned subtlety.

Although quantitatively the results will depend considerably on this choice of S_{NP} (cf. Ref. [64] for a detailed discussion), the Q^2 dependence of the asymmetry turns out not to be very sensitive to it. Nevertheless, our results underline the importance of a good determination of S_{NP} for the proper extraction of the Collins function, when going beyond tree level, see Section 9.4.

The nonperturbative Sudakov factor allows us to integrate $\tilde{W}$ safely over the large b region, but one also has to deal with the region of very small b. $S(b, Q, Q_0)$ at *very* small $b^2 < 1/Q^2$ requires regularization in order to ensure the correct limiting behavior. At small Q_T this issue becomes less important as Q^2 increases. Usually S is regulated by the replacement [73]:

$$\log^2(Q^2 b^2) \to \log^2(Q^2 b^2 + 1). \tag{150}$$

In summary, we consider the following pragmatic simplifying steps: we assume a factorized Gaussian dependence of the fragmentation functions; we ignore flavor and spin dependence of this Gaussian dependence; we take a generic nonperturbative Sudakov factor known from pp scattering; we do not worry about matching to high Q_T; and, we drop the finite terms F_i. All this leads to a manageable expression for $\tilde{W}$, which in DLLA becomes:

$$\tilde{W}(\boldsymbol{b}, Q; z_1, z_2) \propto \sum_a e_a^2 D_1^a(z_1) D_1^{\bar{a}}(z_2)$$

$$\times \exp\left(-C_F \frac{\alpha_s(Q)}{2\pi} \log^2(Q^2 b_*^2 + 1)\right) \exp(-S_{NP}(b, Q/Q_0)). \tag{151}$$

But as said, below we will also include the single logarithms and the running of α_s, and doing so leads to qualitatively similar, but quantitatively different results compared to earlier results of [64], where $\mu = 1/b$ was used.

The final step to be taken is to include the Collins fragmentation function. This can be done via the replacement of $\tilde{D}(z, b^2) \to \tilde{\Delta}(z, \boldsymbol{b})$, where for unpolarized hadron production:

$$\tilde{\Delta}(z, \boldsymbol{b}) = \frac{M}{4} \left\{ \tilde{D}_1(z, b^2) \frac{\not{P}}{M} + \left(\frac{\partial}{\partial b^2} \tilde{H}_1^\perp(z, b^2) \right) \frac{2 \not{b} \not{P}}{M^2} \right\}, \tag{152}$$

which is the Fourier transform of Eq. (52) (we have included a factor P^- into the definition of Δ). Since the second term is $\boldsymbol{b}$-odd, it leads to a different Q^2-dependence than the first term. This leads to the Q^2 dependence of the asymmetry.

A model for the transverse momentum dependence of the function $H_1^\perp$ is needed. As for D_1 we will simply assume a Gaussian form: $H_1^{\perp a}(z, z^2 \boldsymbol{k}_T^2) = H_1^{\perp a}(z) \mathcal{G}(|\boldsymbol{k}_T|; R)/z^2$. Strictly speaking, the radius R should be taken larger than R_u of the unpolarized function D_1, such as to satisfy the bound [38]

$$|\boldsymbol{k}_T| \left| H_1^\perp(z, |\boldsymbol{k}_T|) \right| \leqslant M_h D_1(z, |\boldsymbol{k}_T|), \tag{153}$$

for all $|\boldsymbol{k}_T|$. But since we will include this (in principle spin dependent) Gaussian dependence into S_{NP}, we will approximate R by R_u, because the (spin independent) Q^2 dependent part of S_{NP} dominates anyway. Note that equating R by R_u is not allowed in tree level treatments of the asymmetry, as no fall-off with Q_T will be obtained otherwise.

As a further simplification, we will assume that the fragmentation functions for both hadrons are Gaussians of equal width, i.e. we take $R_{u1} = R_{u2} = R_u$ and $R_1 = R_2 = R = R_u$. Also we take $M_1 = M_2 = M$. All these simplifications can be easily undone when needed.

With all these steps in place, we can write the Collins effect asymmetry in the CS formalism in a form similar to that obtained at tree level in Section 4.2. If we define the asymmetry $A(\boldsymbol{q}_T)$ as the analyzing power of the $\cos 2\phi_1$ asymmetry:

$$\frac{d\sigma(e^+ e^- \to h_1 h_2 X)}{dz_1 \, dz_2 \, d\Omega \, d^2 \boldsymbol{q}_T} \propto \left\{ 1 + \cos 2\phi_1 A(\boldsymbol{q}_T) \right\}, \tag{154}$$

then at tree level we obtained

$$A(\boldsymbol{q}_T) = \frac{\sum_a K_3^a(y) \mathcal{F}[(2\boldsymbol{q}_T \cdot \boldsymbol{p}_T \boldsymbol{q}_T \cdot \boldsymbol{k}_T - \boldsymbol{q}_T^2 \boldsymbol{p}_T \cdot \boldsymbol{k}_T) H_1^\perp \overline{H}_1^\perp]}{Q_T^2 M_1 M_2 \sum_b K_1^b(y) \mathcal{F}[D_1 \overline{D}_1]}. \tag{155}$$

Here we taken the flavor summation out of the definition of $\mathcal{F}[\cdots]$ and replaced $e_a^2 A(y) \to K_1^a(y)$ and $e_a^2 B(y) \to K_3^a(y)$ compared to Section 4.2, in order to include γ–Z interference effects, although we do ignore the $\sin 2\phi_1$ Collins effect asymmetry that was estimated to be small in Section 4.4. We have also multiplied the asymmetry by a trivial factor Q_T^2/Q_T^2 in order to be able to replace $\hat{\boldsymbol{h}} \to \boldsymbol{q}_T$. This might seem problematic at $Q_T^2 = 0$, but that turns out not to be a problem as the asymmetry has a kinematic zero at $Q_T = 0$, because $\hat{\boldsymbol{h}}$ cannot be defined in that case.

At tree level

$$\mathcal{F}[D\overline{D}] \equiv \int d^2 \boldsymbol{p}_T \, d^2 \boldsymbol{k}_T \, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T) D^a(z_1, z_1^2 \boldsymbol{p}_T^2) \overline{D}^a(z_2, z_2^2 \boldsymbol{k}_T^2). \tag{156}$$

In order to apply the CS factorization expression we can replace in Eq. (156) [64]

$$\delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T) \to \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{b}\cdot(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T)} e^{-S}\tilde{U}, \tag{157}$$

leading to (suppressing the flavor indices and the arguments of S and $\tilde{U}$)

$$\mathcal{F}[D_1\overline{D}_1] \equiv \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T} e^{-S}\tilde{U}\tilde{D}_1(z_1, b^2)\tilde{\overline{D}}_1(z_2, b^2)$$

$$= \frac{1}{2\pi}\int_0^\infty db\, b\, J_0(bQ_T) e^{-S}\tilde{U}\tilde{D}(z_1, b)\tilde{\overline{D}}(z_2, b). \tag{158}$$

The numerator in Eq. (155),

$$\mathcal{F}\big[(2\boldsymbol{q}_T\cdot\boldsymbol{p}_T\boldsymbol{q}_T\cdot\boldsymbol{k}_T - q_T^2\boldsymbol{p}_T\cdot\boldsymbol{k}_T)H_1^\perp\overline{H}_1^\perp\big]$$

$$\equiv \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T} e^{-S}\tilde{U}\int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\,(2\boldsymbol{q}_T\cdot\boldsymbol{p}_T\boldsymbol{q}_T\cdot\boldsymbol{k}_T - q_T^2\boldsymbol{p}_T\cdot\boldsymbol{k}_T)$$

$$\times e^{-i\boldsymbol{b}\cdot(\boldsymbol{p}_T+\boldsymbol{k}_T)}H_1^\perp(z_1, z_1^2\boldsymbol{p}_T^2)\overline{H}_1^\perp(z_2, z_2^2\boldsymbol{k}_T^2), \tag{159}$$

cannot be treated exactly like the denominator. But for Gaussian transverse momentum dependence of the fragmentation functions the transverse momentum integrals can be performed. One finds

$$\int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\,(2\boldsymbol{q}_T\cdot\boldsymbol{p}_T\boldsymbol{q}_T\cdot\boldsymbol{k}_T - q_T^2\boldsymbol{p}_T\cdot\boldsymbol{k}_T)e^{-i\boldsymbol{b}\cdot(\boldsymbol{p}_T+\boldsymbol{k}_T)}\mathcal{G}(\boldsymbol{p}_T^2; R)\mathcal{G}(\boldsymbol{k}_T^2; R)$$

$$= -\frac{1}{4R^4}\big[2(\boldsymbol{q}_T\cdot\boldsymbol{b})^2 - q_T^2 b^2\big]\exp\left(-\frac{b^2}{2R^2}\right), \tag{160}$$

which after application to Eq. (159) yields

$$\mathcal{F}\big[(2\boldsymbol{q}_T\cdot\boldsymbol{p}_T\boldsymbol{q}_T\cdot\boldsymbol{k}_T - q_T^2\boldsymbol{p}_T\cdot\boldsymbol{k}_T)H_1^\perp\overline{H}_1^\perp\big]$$

$$= \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{b}\cdot\boldsymbol{q}_T}\left(-\frac{1}{4R^4}\right)\big[2(\boldsymbol{q}_T\cdot\boldsymbol{b})^2 - q_T^2 b^2\big]$$

$$\times e^{-S}\tilde{U}\tilde{H}_1^\perp(z_1, b^2)\tilde{\overline{H}}_1^\perp(z_2, b^2). \tag{161}$$

It is important to note that the factor $1/R^4$ stems from the Gaussian width of the functions $H_1^\perp(z, \boldsymbol{k}_T^2)$ to be used in this expression. Therefore, it is not dependent on the scale Q as discussed above already.

Putting everything together Eq. (155) can be transformed into

$$A(Q_T) = \frac{\sum_a K_3^a(y)H_1^{\perp a}(z_1)\overline{H}_1^{\perp a}(z_2)}{4M^4 R^4 \sum_b K_1^b(y)D_1^b(z_1)\overline{D}_1^b(z_2)}\mathcal{A}(Q_T)$$

$$= \frac{\sum_a K_3^a(y)H_1^{\perp(1)a}(z_1)\overline{H}_1^{\perp(1)a}(z_2)}{\sum_b K_1^b(y)D_1^b(z_1)\overline{D}_1^b(z_2)}\mathcal{A}(Q_T), \tag{162}$$

where

$$\mathcal{A}(Q_T) \equiv M^2\frac{\int_0^\infty db\, b^3 J_2(bQ_T)\tilde{U}(b_*; Q_0, \alpha_s(Q_0))\exp(-S(b_*, Q, Q_0) - S_{\mathrm{NP}}(b, Q/Q_0))}{\int_0^\infty db\, b\, J_0(bQ_T)\tilde{U}(b_*; Q_0, \alpha_s(Q_0))\exp(-S(b_*, Q, Q_0) - S_{\mathrm{NP}}(b, Q/Q_0))}. \tag{163}$$
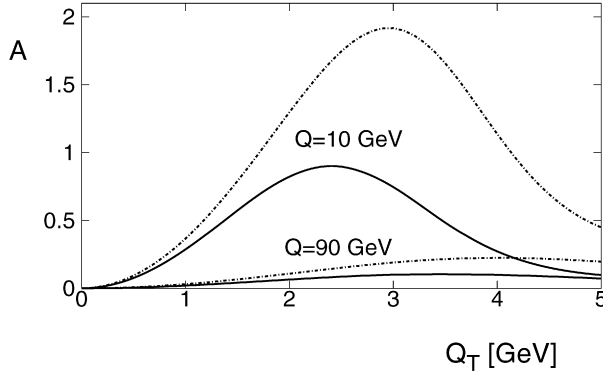
Fig. 5. The asymmetry factor $\mathcal{A}(Q_T)$ (in units of M^2) at $Q = 10$ GeV and $Q = 90$ GeV. The solid curves are obtained with the method explained in the text; the dashed-dotted curves are from the earlier analysis of Ref. [64].

We will employ this expression in combination with Eqs. (143), (144) and (149), dropping the finite terms F_i, but including the one-loop running of α_s. As the replacement of Eq. (150) turns out to have only a minor effect in this expression, in contrast to the DLLA expression, it will not be included.

In Fig. 5 the asymmetry factor $\mathcal{A}(Q_T)$ is given at the scales $Q = 10$ GeV and $Q = 90$ GeV, in order to compare the results for the BELLE and LEP1 scales. The solid curves are obtained with the method explained here, whereas the dashed-dotted curves are the results obtained from the analysis of Ref. [64], where $\mu = 1/b$ was employed and the scale dependence of the fragmentation functions was ignored. One observes a reduction w.r.t. earlier results, but the large Sudakov suppression with increasing Q remains equally strong, i.e. approximately $1/Q$. The DLLA result (not displayed) decreases more slowly, but as said it becomes a worse approximation as Q^2 increases.

The $\cos 2\phi$ asymmetry has been studied using DELPHI data ($\sqrt{s} = M_Z$) and the magnitude was found to be small [7]. This may have several reasons, one of which could be the Sudakov suppression discussed here. In that case a small result at LEP1 energy does not imply that also at BELLE the asymmetry has to be small. In Ref. [80] a comparison is made of the Collins functions extracted from the DELPHI and BELLE data (here it should be emphasized that the DELPHI data analysis remains preliminary and does not consider possible systematic effects). At higher Q^2 the extraction of the Collins function from a tree level expression becomes less accurate, as we will discuss in the next subsection. Therefore, it would be interesting to also compare the asymmetries directly (rather than the extracted Collins functions). This could serve as a check of the Q^2 dependence of the asymmetry as a whole and thus of the CS formalism.

Of course, one also wants to compare to Collins effect asymmetries in SIDIS at lower energies (e.g. for HERMES $\langle Q^2 \rangle = 2.41$ GeV^2). The question is what to do at smaller values of Q^2? The logarithms that are resummed in the CS formalism are not that large to begin with and neither is the relevant b-range (set by Q_0 and Q). In this case one can set $Q_0 = Q$, which means $S(b, Q, Q) = 1$ and $S_{\mathrm{NP}}(b)$ Q^2-independent, and consider $Q_T \sim M$. A reduction to the tree level form occurs up to small logarithmic corrections of order $\alpha_s(Q^2) \log Q_T^2/Q^2$. Tree level analyses should yield reasonable results in this case.
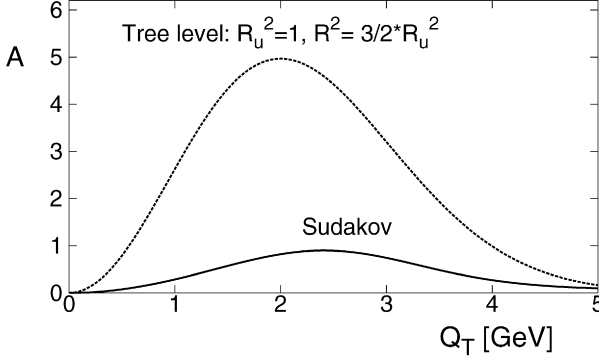
Fig. 6. The asymmetry factor $\mathcal{A}(Q_T)$ at $Q = 10$ GeV (solid curve) and the tree level quantity $\mathcal{A}^{(0)}(Q_T)$ using $R_u^2 = 1$ GeV^{-2} and $R^2/R_u^2 = 3/2$. Both factors are given in units of M^2.

9.3. Comparison to tree level

We will compare the above result for $Q = 10$ GeV with the tree level result (cf. Eq. (62)). In the tree level expression for the asymmetry it is important to keep Gaussians in numerator and denominator different, in order to ensure the bound given in Eq. (153) is satisfied and an asymmetry is obtained that falls off at larger Q_T. The tree level expressions for $A(Q_T)$ and $\mathcal{A}(Q_T)$ will be denoted by $A^{(0)}(Q_T)$ and $\mathcal{A}^{(0)}(Q_T)$. They are given by (ignoring electroweak interference effects for simplicity)

$$A^{(0)}(Q_T) = \frac{Q_T^2 R^2 \exp(-R^2 Q_T^2/2)}{4M^2 R_u^2 \exp(-R_u^2 Q_T^2/2)} \frac{\sin^2\theta_2}{1+\cos^2\theta_2} \frac{\sum_a e_a^2 H_1^{\perp a}(z_1)\overline{H}_1^{\perp a}(z_2)}{\sum_b e_b^2 D_1^b(z_1)\overline{D}_1^b(z_2)}, \tag{164}$$

and

$$\mathcal{A}^{(0)}(Q_T) = \exp\left[-\left(R^2 - R_u^2\right)Q_T^2/2\right]M^2 Q_T^2 R^6/R_u^2. \tag{165}$$

In Fig. 6 we have displayed the comparison of $\mathcal{A}(Q_T)$ at $Q = 10$ GeV and the tree level quantity $\mathcal{A}^{(0)}(Q_T)$ using the values $R_u^2 = 1$ GeV^{-2} and $R^2/R_u^2 = 3/2$, which were chosen such as to minimize the magnitude of $\mathcal{A}^{(0)}(Q_T)$, cf. [64] for further discussion. We conclude that inclusion of Sudakov factors has the effect of suppressing the tree level result roughly by a factor of 5, whereas for $Q = 90$ GeV it is more than an order of magnitude. Tree level extractions of the Collins function at large Q^2 therefore can significantly underestimate its actual magnitude (roughly by the square-root of the Sudakov suppression factor of the asymmetry). It is important to keep this in mind when comparing predictions or fits of transverse momentum dependent azimuthal spin asymmetries based on tree level expressions applied at different energies.

The above also shows that upon including Sudakov factors one retrieves parton model or tree level characteristics (also noted in Ref. [75]), but with transverse momentum spreads that are significantly larger than would be expected from intrinsic transverse momentum (this is supported by the presently available parameterizations of S_{NP} in various processes, which usually have Gaussian b-dependence with widths that increase with Q^2, cf. e.g. [81]).

9.4. Nonperturbative Sudakov factor from BELLE

Since the previous results depend on the input for the nonperturbative Sudakov factors S_{NP}, which (as a function of z_1, z_2) is not determined for the process $e^+ e^- \to h_1 h_2 X$, the numerical conclusions about the size and Q^2 dependence of the suppression should be viewed as generic, not as precise predictions. Therefore, we would like to stress the need for an extraction of the nonperturbative Sudakov factor from the process $e^+ e^- \to h_1 h_2 X$. Considering the wealth of data from BELLE this should pose no problem. Here we will give a brief outline of how this could be done.

Thus far the nonperturbative Sudakov factor has only been obtained from older $e^+ e^- \to h_1 h_2 X$ data for the energy–energy correlation function [82], $\frac{1}{8} \sum_{h_1,h_2} \int dz_1 z_1 dz_2 z_2 Q^2 d\sigma / dz_1 dz_2 dQ_T^2$, at various values of Q^2 [83]. A method based on CSS factorization [84] was used as discussed in detail in Ref. [75]. CSS factorization can be obtained from CS factorization for the ϕ-integrated cross section which receives only contributions from unpolarized quarks (cf. the discussion in Ref. [35] on the relation between CSS and CS factorization for SIDIS). In CSS factorization only collinear fragmentation functions appear, which do not affect the energy–energy correlation function on account of the momentum sum rule $\sum_h \int dz \, z D_1^{a \to h}(z) = 1$.

It needs to be mentioned however that the Q_T distribution for the Drell–Yan process requires an S_{NP} that is dependent on the momentum fractions [77]. It would therefore be better to extract S_{NP} from $e^+ e^- \to h_1 h_2 X$ data at given z_1 and z_2, rather than from the energy–energy correlation function.

To obtain the nonperturbative Sudakov factor the cross section differential in z_1, z_2 and Q_T needs to be fitted (as said, one can integrate over ϕ_1). Either one uses the CSS formalism as explained in Ref. [75] or one can use the CS formalism with a suitable b dependence of the fragmentation functions. Here we will do the latter and stay within the pragmatic approach adopted before, employing a Gaussian form (148). In this way we arrive at (leaving the Q_0 dependence implicit)

$$\frac{d\sigma(e^+ e^- \to h_1 h_2 X)}{dz_1 dz_2 dQ_T^2}$$
$$= \frac{\alpha^2}{Q^2} \sum_{a,\bar{a}} e_a^2 D_1^a(z_1) \overline{D}_1^a(z_2) \int_0^\infty db \, b J_0(bQ_T) \exp\bigl(-S(b_*, Q) - S_{NP}(b, Q)\bigr) \tilde{U}(b_*). \quad (166)$$

This expression together with Eq. (138) for $S(b)$ and the general parameterization of S_{NP} in Eq. (140) can be used to obtain a fit of the parameters g_i to the data at small Q_T ($\ll Q^2$, in order to avoid inclusion of $\mathcal{O}(Q_T^2/Q^2)$ terms that were dropped in Eq. (134)). Such an extraction would be of more general interest as well, since there is a relation between the dominant Q^2-dependent part of nonperturbative Sudakov factors in the Drell–Yan process, SIDIS and $e^+ e^- \to h_1 h_2 X$ [81], which would be interesting to test.

10. Radiative corrections

Thus far we have restricted the discussion to $Q_T^2 \ll Q^2$. In this section we will look at the high-Q_T region, where one expects collinear factorization and fixed order perturbation theory to yield a good description of the cross section.
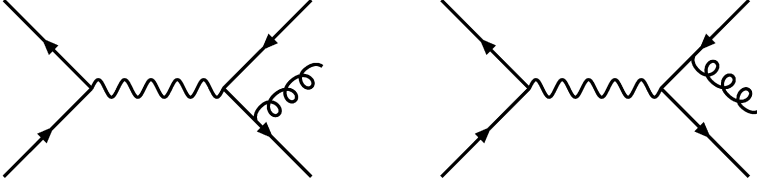
Fig. 7. Lowest order gluon bremsstrahlung diagrams.

It is well known that gluon bremsstrahlung also leads to angular asymmetries, as the radiating quark or antiquark is recoiling away from the original quark–antiquark axis. In this section we will look at this effect at order α_s, i.e. the radiation of an additional gluon into the final state: $e^+e^- \to q\bar{q}g$ (we will not include γ–Z interference terms; this can be done at a later stage, when required), see Fig. 7. As it turns out the expressions are the simplest in the Collins–Soper frame. This frame is the lepton-pair center of mass frame with the $\hat{z}$ axis defined as pointing in the direction that bisects the three-vectors $\boldsymbol{P}_2$ and $-\boldsymbol{P}_1$ and was discussed in Section 3.1.

In the collinear factorization approach contributions to W_T, W_L, W_Δ and $W_{\Delta\Delta}$ will be generated at order α_s. In the CS frame certain ratios of these structure functions will be independent of the subsequent fragmentation of the quark and antiquark. The expressions we will give here are completely analogous to the Drell–Yan expressions obtained in Ref. [26]. They are valid for large Q_T ($\gg M$), but still restricting to two jet events. Here we will not consider the case where one of the hadrons comes from the gluon fragmenting, which becomes relevant when $Q_T \sim Q$. For pronounced three jet events it would certainly have to be taken into account.

At large Q_T at least one of the hadron momenta is deviating considerably from the $\hat{z}$ direction. The hadron momenta are again P_1 and P_2. Upon neglecting M_i^2/Q^2 corrections, the photon momentum is parameterized as $q = P_1/\zeta_1 + P_2/\zeta_2 + q_T$ (cf. Eqs. (7)–(9)), such that $\tilde{Q}^2 = Q^2 + Q_T^2 = \tilde{s}/(\zeta_1\zeta_2)$, where $\tilde{s} = (P_1 + P_2)^2$. It follows that $z_i = \zeta_i \tilde{Q}^2/Q^2$. The calculation is done in collinear factorization, which means that parton transverse momenta w.r.t. the produced hadrons are neglected. The parton momenta are defined such that $p_i = P_i/\xi_i$.

Following the calculation of [26], but now applied to process $e^+e^- \to q\bar{q}g \to h_1h_2X$ (that is, one has to calculate the contribution of the two diagrams in Fig. 7), one finds *in the CS frame*

$$\frac{dN}{d\Omega} = \frac{3}{16\pi}\left[\frac{Q^2 + \frac{3}{2}Q_T^2}{Q^2 + Q_T^2} + \frac{Q^2 - \frac{1}{2}Q_T^2}{Q^2 + Q_T^2}\cos^2\theta_{\rm CS}\right.$$
$$\left. + \frac{Q_T Q}{Q^2 + Q_T^2}K\left(\zeta_1, \zeta_2, Q_T^2/\tilde{s}\right)\sin 2\theta_{\rm CS}\cos\phi_{\rm CS} + \frac{1}{2}\frac{Q_T^2}{Q^2 + Q_T^2}\sin^2\theta_{\rm CS}\cos 2\phi_{\rm CS}\right],$$
(167)

where the function $K(\zeta_1, \zeta_2, Q_T^2/\tilde{s})$ is given by

$$K\left(\zeta_1, \zeta_2, Q_T^2/\tilde{s}\right)$$
$$= \frac{\int d\xi_1 \int d\xi_2\, \delta((1/\xi_1 - 1/\zeta_1)(1/\xi_2 - 1/\zeta_2) - Q_T^2/\tilde{s})\sum_a e_a^2 D_1^a(\xi_1, \mu)\overline{D}_1^a(\xi_2, \mu)(\zeta_2^2/\xi_2^2 - \zeta_1^2/\xi_1^2)}{\int d\xi_1 \int d\xi_2\, \delta((1/\xi_1 - 1/\zeta_1)(1/\xi_2 - 1/\zeta_2) - Q_T^2/\tilde{s})\sum_a e_a^2 D_1^a(\xi_1, \mu)\overline{D}_1^a(\xi_2, \mu)(\zeta_2^2/\xi_2^2 + \zeta_1^2/\xi_1^2)}.$$
(168)

This expression means that only the partonic asymmetry $\propto \hat{W}_\Delta$ depends on the lightcone momentum fractions of partons and therefore that only the hadronic asymmetry $\propto W_\Delta$ depends on the fragmentation functions D_1 and $\overline{D}_1$. This is only the case in the Collins–Soper frame. In the

Gottfried–Jackson frame all angular dependences will depend on the fragmentation functions. Note that α_s drops out in the above asymmetry ratio expressions.

Using the other standard notation for the angular dependences Eq. (39) one finds that the above yields:

$$\lambda_{CS} = \frac{Q^2 - \frac{1}{2}Q_T^2}{Q^2 + \frac{3}{2}Q_T^2}, \tag{169}$$

$$\nu_{CS} = \frac{Q_T^2}{Q^2 + \frac{3}{2}Q_T^2}. \tag{170}$$

We see that also in $e^+e^- \to h_1 h_2 X$ the so-called Lam–Tung relation holds [24,25]:

$$1 - \lambda_{CS} - 2\nu_{CS} = 0, \tag{171}$$

or equivalently, $W_L = 2W_{\Delta\Delta}$. Using Eq. (41) one sees that the relation also holds in the GJ frame and as shown for the Drell–Yan process, it is expected that it continues to hold if one includes the gluon fragmentation contribution, although in that case both λ and ν *will* depend on the fragmentation functions (cf. Ref. [85] for a discussion of these aspects for the Drell–Yan process, including the effects of resummation). We also note that the Lam–Tung relation is purely a $\mathcal{O}(\alpha_s)$ result, i.e. it does not apply beyond leading order. Surprisingly, in the Drell–Yan process the Lam–Tung relation is known to be violated by much more than the $\mathcal{O}(\alpha_s^2)$ contribution [86]. The distribution function analogue of the Collins effect has been suggested as an explanation for this large deviation from the NLO pQCD result [87]. It would be very interesting to check experimentally in $e^+e^- \to h_1 h_2 X$ whether the Lam–Tung relation is violated as much as in the Drell–Yan process.

A natural question is how to combine this collinear factorization fixed order perturbative result, which is valid for $M^2 \ll Q_T^2$, with the CS factorization Collins effect result, which is valid for $Q_T^2 \ll Q^2$. This question was addressed recently in Ref. [35] for the analogous situation in semi-inclusive DIS. There it was pointed out that the two results may simply be added, which gives an expression for the asymmetry that is correct up to power suppressed corrections (of order Q_T^2/Q^2 in the low Q_T region and of order M^2/Q_T^2 in the high Q_T region). It is based on the fact that the denominators of the asymmetry expressions in Eqs. (59) and (170) coincide in the intermediate Q_T region ($M^2 \ll Q_T^2 \ll Q^2$) (a fact that is also used in the derivation of the CSS formalism [84] from CS factorization). In addition, one uses the following observations. The above calculation shows that the fixed order perturbative expression for ν is of order Q_T^2/Q^2 when $Q_T^2 \ll Q^2$, hence power suppressed w.r.t. the Collins asymmetry expression given in Eq. (59). Conversely, considering the latter in the large Q_T ($\gg M$) limit beyond tree level (within CS factorization) leads to a result of order M^2/Q_T^2 [35], which is power suppressed w.r.t. ν in Eq. (170). This implies that upon neglecting power suppressed terms the sum of the two contributions reduces to Eq. (59) at low Q_T and to Eq. (170) at high Q_T. In the intermediate region the results can be added because they have a common denominator. For further details we refer to [35].

Despite the fact that the asymmetry for all Q_T can be described by the sum of the Collins effect expression and the fixed order collinear factorization expression, the latter complicates the extraction of the Collins function and the weighting with Q_T^2, as explained next. Therefore, one wants to construct quantities in which the hard gluon radiation component of the $\cos 2\phi$ asymmetry is (largely) absent. One such option is to consider ratios of asymmetries where the

hard gluon radiation component of the $\cos 2\phi$ asymmetry drops out. The other would be to exploit the Lam–Tung relation. Both options will be discussed below.

11. Weighted asymmetry beyond tree level

In this section we return to the issue of integrating over Q_T and weighting with Q_T^2. In the cross section kept differential in the transverse momentum $\boldsymbol{q}_T$ the Collins effect gives rise to a $\cos 2\phi_1$ asymmetry (both in the GJ and CS frames). In principle one can consider this quantity integrated over the length Q_T of $\boldsymbol{q}_T$, but this yields a quantity that does not arise in other processes in the same way. Without further assumptions about the Collins fragmentation function, such a quantity would only be useful to test whether it has the expected Q^2 behavior. This formed the main reason for the suggestion to weight the $\boldsymbol{q}_T$ integration with an additional factor of Q_T^2. At tree level this yields an expression involving the first transverse moment of the Collins function. Beyond tree level but considering only the low-Q_T expression of the CS formalism this remains true and moreover, a quantity results that does not suffer from Sudakov suppression,[5] as was pointed out in Ref. [64]. These nice properties are spoiled however when the perturbative high-Q_T contribution is included. Unfortunately the weighted cross section becomes sensitive mainly to the large Q_T contribution (note that we are discussing the asymmetry in the weighted cross section, i.e. we integrate numerator and denominator of the Q_T-dependent asymmetry separately). Because the latter is calculable perturbatively and can simply be added to the low-Q_T result, it can in principle be subtracted from the experimental result before extracting $H_1^{\perp(1)}$.

One way to subtract it experimentally is to consider ratios of asymmetries where the hard gluon radiation component of the $\cos 2\phi$ asymmetry drops out. This is discussed in the next section. The other possibility would be to exploit the Lam–Tung relation. Here the idea is to use the structure functions themselves, as opposed to the ratios of structure functions λ and ν. We observe that the quantity

$$\int d^2\boldsymbol{q}_T \, \boldsymbol{q}_T^2 (W_L - 2W_{\Delta\Delta}) = \frac{8M_1M_2}{z_1^2 z_2^2} \sum_{a,\bar{a}} e_a^2 H_1^{\perp(1)a}(z_1) \overline{H}_1^{\perp(1)a}(z_2) \tag{172}$$

does not receive radiative corrections at order α_s. This applies in both the GJ and CS frames and will hold true even upon inclusion of the gluon fragmentation contribution.

Once the first transverse moment of the Collins function has been extracted there is another potentially interesting test of the Collins effect in the CS formalism, which may turn out to be scale independent. This exploits the so-called Schäfer–Teryaev sum rule [88]

$$\sum_h \int dz \, z H_1^{\perp(1)}(z) = 0. \tag{173}$$

If we define for each hadron type h the function C_h that is a function of M_h and scale μ only, as

$$C_h(M_h, \mu) \equiv \int dz \, z H_1^{\perp(1)}(z), \tag{174}$$

then one can conclude that for each hadron type this has the same μ dependence (if any). This is because there is no gluon contribution, such that there is no mixing among gluon and quark

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[5] One probes the Sudakov factor at the point $b = 0$ where it vanishes, although that cannot be seen from the perturbative expression for $S(b)$ in Eq. (136), which is only valid for $b > b_0/Q$.

functions. For each quark flavor one has the same evolution properties. Assuming autonomous evolution of the function $H_1^{\perp(1)}(z)$, this implies that ratios of C_h's for different hadrons types, e.g. C_π/C_K, will be scale independent. This could then be tested experimentally without the need to know the scale dependence of the Collins asymmetries. Measuring the ratios at BELLE, HER-MES, LEP1, etc., should then all give the same answer. The only experimental difficulty would be to obtain a sufficient coverage in z and Q_T to be able to extract the integrated quantities C_h reliably. In Ref. [89] an autonomous evolution equation for $zH_1^{\perp(1)}(z)$ has been obtained in the large N_c limit. However, since its derivation employed so-called Lorentz invariance relations the correctness of which has afterwards been disputed [90], the assumption of autonomous evolution may turn out to be unwarranted. This issue should first be settled before any conclusions can be drawn from the ratios C_{h_1}/C_{h_2}. But if scale independence of the ratios C_{h_1}/C_{h_2} can be established, it would offer an experimental consistency check of the extracted Collins fragmentation functions.

12. Ratios of asymmetries

There are several reasons why it may be useful to consider taking ratios of asymmetries for different sets of hadrons. These may be experimental reasons such as to cancel systematic effects, but also theoretical reasons namely that effects drop out that are independent of the type of hadron considered.

If the $\cos 2\phi$ asymmetry itself is not that large, the asymmetry in a ratio of two $\cos 2\phi$ asymmetries is to good approximation also a $\cos 2\phi$ asymmetry:

$$\frac{\frac{d\sigma(e^+e^-\to h_1 h_2 X)}{dz_1\, dz_2\, d\Omega\, d^2\boldsymbol{q}_T}}{\frac{d\sigma(e^+e^-\to h_3 h_4 X)}{dz_1\, dz_2\, d\Omega\, d^2\boldsymbol{q}_T}} \propto \frac{1+\cos 2\phi_1\, A^{h_1 h_2}(\boldsymbol{q}_T)}{1+\cos 2\phi_1\, A^{h_3 h_4}(\boldsymbol{q}_T)}$$
$$\approx 1+\cos 2\phi_1 \big\{ A^{h_1 h_2}(\boldsymbol{q}_T) - A^{h_3 h_4}(\boldsymbol{q}_T) \big\}. \tag{175}$$

As can be seen, effects that are independent of the type of hadrons h_i cancel in the difference $A^{h_1 h_2} - A^{h_3 h_4}$. Since fragmentation functions generally do depend on the type of hadron, only effects that are independent of the fragmentation functions will cancel. In Section 10 it was discussed that in the Collins–Soper frame the perturbative order-α_s contribution to ν (Eq. (170)) is independent of the fragmentation functions. This assumes that the hadrons arise from quark fragmentation, which should be a good approximation for Q_T not too close to Q. Hence, under these conditions (in the CS frame for moderate Q_T) the order-α_s radiative correction term cancels in the ratio Eq. (175). If in addition $\mu_{CS} \approx 0$, then the conclusion also extends to other frames that are rotations of the CS frame, such as the GJ frame or the jet frame discussed in Section 8.

Since at moderate Q_T the dominant contribution to ν from radiative corrections (Eq. (170)) is already small itself, any uncanceled remainder from higher order corrections or from the gluon fragmentation contribution is expected to be negligible. This can be verified via explicit prediction of the radiative correction to the cross section, e.g. via a Monte Carlo study. Due to the cancellation of the dominant radiative correction, the ratio method presents a good way to access the Collins effect without taking into account the radiative corrections explicitly. The use of different types of hadron pairs, such as $\pi^+\pi^-$, $\pi^\pm\pi^\pm$ and $\pi^\pm\pi^0$, moreover allows one to learn about favored versus unfavored fragmentation functions [80].

Predictions for the ratio of jet frame $\cos(\phi + \phi')$ asymmetries (cf. Eq. (124) or Ref. [8]) for unlike-sign pion pairs to like-sign pairs have been given in Ref. [91] employing a spectator

model. Asymmetry ratio's of the order of the experimental data could be obtained, but with considerable uncertainties. Interestingly, kaon asymmetries were predicted to be larger than the pion asymmetries.

13. Beam polarization

Thus far we have assumed that no transverse beam polarization is present. However, it is well known that charged particles circulating in a magnetic field become polarized transversely to the beam direction due to emission of spin-flipping synchrotron radiation: the Sokolov–Ternov effect [92]. This effect can be significant for electrons and positrons due to their small mass. The transverse polarization of particles circulating in a uniform magnetic field is the following function of time:

$$P(t) = \frac{1}{a}\left\{1 - \exp\left(-a\frac{e^2\hbar\gamma^5}{m^2c^2\rho^3}t\right)\right\}, \qquad (176)$$

where $\gamma = E/m$ is the Lorentz factor, ρ is the bending radius, and $a = 5\sqrt{3}/8$. More general situations are reviewed in Ref. [93]. The polarization has a strong dependence on the mass in the exponent: the lighter the particle, the faster it becomes polarized.

It is also well known [63,94–96] that beam polarization can affect the angular distribution of produced hadrons in $e^+e^- \to hX$. As explained in [94] it leads to a $\cos 2\phi$ asymmetry, where the angle ϕ is defined w.r.t. the beam and beam spin directions. This asymmetry has in common with the Collins effect asymmetry that both are transverse spin asymmetries: the former concerns lepton spins, the latter quark spins. Both asymmetries arise due to interference between contributions of ± 1 photon helicity states. In the former the magnetic field determines the electron and positron spin directions, which then determine the photon polarization state and hence, the subsequent decay of the photon into quark–antiquark pairs, which is visible in the angular distribution of final state hadrons. In the Collins asymmetry case, the angular distribution of two final state hadrons w.r.t. each other filters or selects the quark and anti-quark spin directions, which via the photon are correlated to the electron–positron angular distribution. The Collins effect does not contribute to the angular distribution of produced hadrons in $e^+e^- \to hX$, but transverse beam polarization may affect the angular distribution in $e^+e^- \to h_1h_2X$.

Turning to the situation at BELLE: one should learn whether the beams are polarized due to the Sokolov–Ternov effect, because it may lead to angular asymmetries that require correcting for. From the beam energies and bending radii of the KEK accelerator one can try to estimate the polarization build-up time from Eq. (176). Here one should keep in mind that the accelerator consists mostly of straight sections. For the low (high) energy ring the bending radius is 16.3 m (104.5 m), but the length of bending is 0.915 m (5.86 m), whereas the circumference of the rings is a little over 3000 m. Therefore, the polarization build-up time interval is much reduced compared to simply using the quoted bending radii in Eq. (176). Moreover, depolarization effects should be very important, especially since the interaction point is in the middle of a straight section. Negligible beam polarization at the collision point is thus expected.

The degree of polarization P can be measured experimentally via the process $e^+e^- \to \mu^+\mu^-$ for which the cross section is [94]

$$\frac{d\sigma(e^+e^- \to \mu^+\mu^-)}{d\Omega} = \frac{1}{2}\sigma_T\left(1 + \cos^2\theta + P^2\sin^2\theta\cos 2\phi\right), \qquad (177)$$

where $\sigma_L/\sigma_T = 4m_\mu^2/Q^2 \approx 0$ is used. Experimental investigations of this cross section can show the presence or absence of any significant transverse beam polarization at BELLE.

14. Weak decays background

Another test of systematic effects is offered by the process $e^+e^- \to \tau^+\tau^- \to \pi^+\bar{\nu}_\tau\pi^-\nu_\tau$ (and other such weak decays). In this process a $\cos 2\phi$ asymmetry analogous to the Collins effect asymmetry can arise. In this case it is calculable within the Standard Model, using the parity violation parameters (usually denoted by α) for the tau decays ($\alpha_{\pi^\pm} = \mp 1$ for $\tau^\pm \to \pi^\pm \overset{(-)}{\nu}_\tau$). The self-analyzing parity-violating weak decay of the τ's plays a role similar to the Collins effect. It correlates the spatial distribution of the produced pions to the spins of the τ's. The asymmetry does not depend on the τ's being transversely polarized *on average* (in that case the effect would have been of order m_τ^2/s).

The analyzing power of the $\cos 2\phi$ asymmetry is not a very simple expression, because it depends on the decay kinematics, for details we refer to Refs. [97–99]. The asymmetry is proportional to $\alpha_{\pi^+}\alpha_{\pi^-}C_{TT} = -C_{TT}$, where C_{TT} is given in terms of the axial and vector coupling of the τ lepton to the neutral gauge boson:

$$C_{TT} = \frac{|a_\tau|^2 - |v_\tau|^2}{|a_\tau|^2 + |v_\tau|^2}, \tag{178}$$

which for the Z boson is close to $+1$ and for the photon is -1. This change of sign implies a sign difference between the asymmetry measured at LEP [100–102] and what would be obtained at BELLE. The LEP results are in good agreement with the Standard Model prediction, therefore, assuming the same applies to BELLE, it could provide a way to check for systematic effects. The same would apply to heavy-quark weak decays.

15. Summary

We have given an overview of the Collins effect asymmetry in the process $e^+e^- \to h_1 h_2 X$, where the two final state hadrons belong to opposite jets. We restricted the discussion to two jet events and discussed the particular situation of BELLE, but many results also are applicable at other e^+e^- collider facilities. The Collins effect—a spin effect in the quark fragmentation process—gives rise to an azimuthal asymmetry, which in the Gottfried–Jackson or Collins–Soper frames is a $\cos 2\phi$ asymmetry. It was our main objective to study to which extent the analyzing power of this angular asymmetry is a measure of the Collins effect.

At low values of the observed transverse momentum Q_T of the photon we employ Collins–Soper factorization, which leads to expressions involving parton transverse momentum dependent fragmentation functions. It is within this framework that the Collins effect fragmentation function arises. At tree level and leading twist it leads to the only contribution to the $\cos 2\phi$ asymmetry. Twist-3 effects (order M/Q terms) were shown in the Gottfried–Jackson frame to only contribute to μ, i.e. to give rise to a $\cos\phi$ asymmetry. Throughout this paper we have neglected twist-4 effects that are of order M^2/Q^2. They can contribute to the $\cos 2\phi$ asymmetry, but are expected to lead to $\nu < \mu$, i.e. a larger $\cos\phi$ than $\cos 2\phi$ asymmetry. This property allows to estimate their importance at BELLE.

Beyond tree level, but still at low Q_T, the Collins–Soper factorization expression also dictates the scale Q^2 dependence of the Collins effect asymmetry. This was studied extensively and it was demonstrated that the Collins effect asymmetry suffers from considerable Sudakov suppression as Q^2 increases. This was already pointed out in Ref. [64], but the analysis presented here improves on that treatment and leads to quantitatively different but qualitatively very similar

results. We conclude that inclusion of the dominant double and single logarithmic terms at order α_s has the effect of suppressing the tree level result. In the example considered this suppression was roughly by a factor of 5 for $Q = 10$ GeV as relevant for the BELLE experiment. Therefore, tree level extractions of the Collins function can significantly underestimate its actual magnitude. It is important to keep this in mind when comparing predictions or fits of transverse momentum dependent azimuthal spin asymmetries based on tree level expressions but applied at different energies. To properly describe the cross section beyond tree level within the Collins–Soper formalism, a good experimental determination of the nonperturbative Sudakov factor is required. An explanation of how this can be done at BELLE was given.

In the Collins–Soper formalism the Collins fragmentation functions enter in a transverse momentum convolution expression. Such convolutions do not arise in other processes in the same way, therefore one has to resort to other means of extracting information that can be applied in other processes. Often a Gaussian transverse momentum dependence of the Collins function is assumed because in that case all integrations can be performed analytically. But this introduces a model dependence in the results. Another way suggested is to consider weighted integration over the observed transverse momentum Q_T. In this way the Q_T^2-weighted cross section *at tree level* probes the first transverse moment of the Collins fragmentation function, which also arises in other processes identically. This would allow for model independent predictions once the Collins functions are extracted from the BELLE data. Another advantage of the Q_T^2-weighted cross section is that it does not suffer from the Sudakov suppression mentioned above. However, considering the integral over all Q_T requires that one also describes the asymmetry well at high Q_T. This complicates matters considerably.

For the $\cos 2\phi$ asymmetry at high Q_T collinear factorization can be used to describe it. This is most conveniently done in the Collins–Soper frame, where the dependence on the fragmentation functions drops out to a large extent. The asymmetry arises from hard gluon emission and is not related to the Collins effect. Therefore, it becomes important to describe this dependence, that behaves as Q_T^2/Q^2, as best as possible in order to distinguish it from the Collins effect contribution that at large Q_T behaves as M^2/Q_T^4 (as opposed to Gaussian transverse momentum). Due to this specific Q_T^2 dependence of the two effects, it is possible (upon ignoring twist-4 contributions) to simply add the two contributions (cf. Ref. [35]). The Collins effect then dominates at low Q_T and the hard gluon emission at large Q_T. Unfortunately this implies that the Q_T^2-weighted integration is sensitive mostly to the latter effect and not to the Collins effect of interest. So one first has to subtract the perturbative collinear factorization result at large Q_T. This can be done to a large extent automatically by considering ratios of asymmetries, but a new way exploiting the analogue of the Lam–Tung relation was pointed out. This relation was shown to be violated strongly in the Drell–Yan process w.r.t. the NLO pQCD result. It would be very interesting to see whether this also holds in e^+e^- collisions.

To get an idea about the size of the Collins effect asymmetry in $e^+e^- \rightarrow h_1 h_2 X$, a crude estimate of the Q_T^2-weighted Collins asymmetry at tree level was made. It shows the asymmetry can be on the level of a few percent. This is in agreement with a model prediction in the literature. These estimates employ tree level expressions, but they should remain reasonable estimates beyond tree level (due to the absence of Sudakov suppression), provided the dominant high-Q_T gluon emission contribution is subtracted.

A comparison to the Collins asymmetry in the so-called jet frame (defined with help of the $q\bar{q}$ axis or approximately by the thrust axis) was given. In this frame the Collins asymmetry becomes a $\cos(\phi + \phi')$ asymmetry. It was shown to probe so-called half-moments of the Collins fragmenta-

tion functions, as opposed to the first transverse moments appearing in the Q_T^2-weighted Collins asymmetry. It is estimated to be similar in size as the asymmetry in the GJ frame.

Other potential contributions to the Collins effect asymmetry have been investigated, such as from γ–Z interference and the effect of beam polarization. Both effects should be negligible at BELLE (well below the percent level). Electroweak contributions can in principle lead to a $\sin 2\phi_1$ asymmetry, but also that was estimated to be very small. Therefore, we conclude that γ–Z interference does not lead to any significant additional ϕ dependence. It only modifies the θ distribution with a forward–backward asymmetry term of a few percent.

A $\cos 2\phi$ asymmetry analogous to the Collins effect asymmetry arises in $e^+e^- \to \tau^+\tau^- \to \pi^+\bar{\nu}_\tau\pi^-\nu_\tau$ (and other such weak decays). Using the known τ weak decay parameters this asymmetry can be calculated entirely within the electroweak sector of the Standard Model. A comparison of the data to this Standard Model result could provide a good way to check for systematic effects.

Based on all these considerations it appears feasible to arrive at an actual measurement of the Collins effect and an extraction of the Collins fragmentation function to a reasonable extent. Based on arguments in favor of the universality of the Collins fragmentation function for the processes $e^+e^- \to h_1h_2X$ and SIDIS, a simultaneous fit to the Collins effect asymmetry data can be and already has been performed, leading to a first extraction of transversity [8]. This demonstrates clearly and explicitly the merit of the Collins effect asymmetry measurement at BELLE.

Acknowledgements

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