VRIJE UNIVERSITEIT

Azimuthal Asymmetries in Hard Scattering Processes

ACADEMISCH PROEFSCHRIFT

ter verkrijging van de graad van doctor aan
de Vrije Universiteit te Amsterdam,
op gezag van de rector magnificus
prof.dr. T. Sminia,
in het openbaar te verdedigen ten overstaan van de
promotiecommissie natuurkunde en sterrenkunde
van de faculteit der exacte wetenschappen
op dinsdag 15 september 1998 om 13.45 uur
in het hoofdgebouw van de universiteit,
De Boelelaan 1105

door

Daniël Boer

geboren te Lopik

Promotor: prof.dr. P.J.G. Mulders

This thesis is partly based on the following publications:

1. D. Boer and R.D. Tangerman,
   Higher-twist quark-mass contributions to deep-inelastic scattering,
   Phys. Lett. B 381 (1996) 305.

2. D. Boer and R.D. Tangerman,
   Quark masses in Qiu's factorization procedure,
   Proceedings of the 12th International Symposium on High-Energy Spin Physics, Eds. C.W. de Jager *et al.*, World Scientific (1997), p. 225.

3. D. Boer, R. Jakob and P.J. Mulders,
   Asymmetries in polarized hadron production in e^+e^- annihilation up to order $1/Q$,
   Nucl. Phys. B 504 (1997) 345.

4. D. Boer, P.J. Mulders and O.V. Teryaev,
   Single spin asymmetries from a gluonic background in the Drell-Yan process,
   Phys. Rev. D 57 (1998) 3057.

5. D. Boer, P.J. Mulders and O.V. Teryaev,
   Single spin asymmetries in the Drell-Yan process,
   Proceedings of the VII Workshop on High Energy Spin Physics, Eds. A.V. Efremov, O.V. Selyugin, E2-97-413, Dubna (1997), p. 139.

6. D. Boer, P.J. Mulders and O.V. Teryaev,
   Single spin asymmetries in the Drell-Yan process,
   Proceedings of the Workshop "Deep inelastic scattering off polarized targets: theory meets experiment", Eds. J. Blümlein *et al.*, DESY 97-200, Zeuthen (1997), p. 373.

7. D. Boer and P.J. Mulders,
   Time-reversal odd distribution functions in leptoproduction,
   Phys. Rev. D 57 (1998) 5780.

8. D. Boer, R. Jakob and P.J. Mulders,
   Leading asymmetries in two-hadron production in e^+e^- annihilation at the Z pole,
   Phys. Lett. B 424 (1998) 143.

The work described in this thesis is part of the research program of the foundation of "Fundamental Research on Matter" (FOM), made possible by financial support from the "National organization for Scientific Research" (NWO).

Aan mijn ouders

The "all or nothing" philosophy is wide-spread and, unfortunately, not only in theoretical physics.

M. Shifman, in "Snapshots of Hadrons" (1998)

Contents

Chapter 1

Introduction

Quantum chromodynamics (QCD) is the theory of the strong interactions. It is the theory about quarks and gluons, which form the bound states we call hadrons, like for instance protons, neutrons, pions, etc. At low energies only bound states can be observed, a property known as *confinement*. Quarks interact with each other via gluons and the resulting force decreases in strength with increasing energy. Only in the limit of infinite energy, the coupling α_s of the gluons to the quarks tends to zero and the quarks become noninteracting. They become free particles, hence the name *asymptotic freedom* [1]. On the other hand, if the energy decreases, the coupling keeps on growing in strength, with the consequence that only above a certain energy scale ($\Lambda_{\rm QCD}$) one can treat the interactions via gluons as a perturbation of the free theory; one is then allowed to make a power series expansion in the coupling constant to approximate the effects of the perturbation. However, the hadron mass spectrum concerns the low energy region, forcing the description of states outside the realm of perturbation theory. The bound states formed by these quarks and gluons are intrinsically nonperturbative objects.

At present there is a very limited set of nonperturbative methods to study the physics of hadrons, so a first step is to investigate to what extent one can make use of the perturbative regime of the theory. The idea is that by doing high energy scattering experiments with hadrons, one can investigate situations in which one or more of the constituents of these hadrons have a very high energy. One then uses the large energy scale(s) involved to try to separate the perturbative from the nonperturbative part of the scattering. The perturbative part can be calculated and the nonperturbative part is to be parametrized in terms of some universal objects, which can be measured in one process say, but yield predictions for other processes.

These universal objects are not just parameters, they also represent information on the structure of hadrons. So the actual question is what can be learned about the structure of hadrons by doing hard scattering processes? One wants to learn how the constituents –the quarks and gluons– build up hadrons, i.e., what is their distribution inside a hadron, but also how they give rise to the properties of a hadron, in particular its energy-momentum, its spin, etc.

The naive picture of a baryon consisting of three quarks, and a meson consisting of a quark and an antiquark, is fine for certain global properties like the electric charge, but when it comes to the spin of hadrons gluons and nonvalence quarks play a crucial role. The famous spin puzzle [2] –quarks inside a proton contribute to only a fraction of its total spin– shows the nontrivial role the nonvalence constituents play. Several theoretical solutions to the puzzle have been proposed, pointing out that additional contributions to the total spin can arise from several sources, like polarized gluons and orbital angular momentum. It is not straightforward to disentangle these different effects and it will also depend on the kinematic region which of the mechanisms are dominant or negligible. Further theoretical and experimental investigations are needed.

The main goal of this thesis is to try to learn about the spin structure of hadrons by studying azimuthal asymmetries. By looking at such asymmetries in the angular distribution of produced particles in a high energy scattering experiment, one might for instance be sensitive to the distribution of polarized quarks or gluons inside hadrons. In order to describe such an asymmetry, one first has to be able to describe and define these polarized distributions. They are generalizations of the parton distributions first considered by Feynman [3]. His parton model describes hadrons as a system of free particles called *partons*, which where later on identified with quarks and gluons. In first approximation quarks can be considered as free when probed in a high energy collision and the parton model is thus only a first approximation. The corrections to this picture are of two kinds, the perturbative and the nonperturbative corrections. The perturbatively calculable corrections are conceptually straightforward although highly nontrivial from a technical viewpoint. For handling the nonperturbative corrections several methods have been proposed and developed for a number of hard scattering processes. In this thesis we will focus on one such method.

This particular framework arose from parton model ideas extended to include both polarization and the nonperturbative corrections called power corrections, which depend inversely on the large energy scale(s) involved in hard scattering processes, in contrast to the logarithmic perturbative corrections. The starting point for our specific investigations of these power corrections (also called higher twist corrections) was presented in the seminal paper by Ellis, Furmanski and Petronzio called "Unravelling Higher Twists" (1983) [4]. They investigated the process called deep inelastic scattering and later on others applied a similar approach to different processes as well (often using leading order ingredients which were already established in the late '70s by many others).

We will look at so-called inclusive and semi-inclusive processes, in which practically all produced particles go undetected. To classify the hard scattering processes that will be investigated in this thesis, we will first count the number of identified hadrons (incoming and outgoing). Hard scattering processes with one identified hadron are for instance inclusive lepton-hadron scattering, generally known as deep inelastic scattering (DIS): $\ell + H \to \ell' + X$, and inclusive one-particle production in electron-positron annihilation: $e^+ + e^- \to h + X$ (see Fig. 1.1). Hard processes with two identified hadrons are semi-inclusive DIS: $\ell + H \to \ell' + h + X$, inclusive two-particle production in electron-positron annihilation: $e^+ + e^- \to h_1 + h_2 + X$, and lepton pair production in hadron-hadron collisions, known as the Drell-Yan

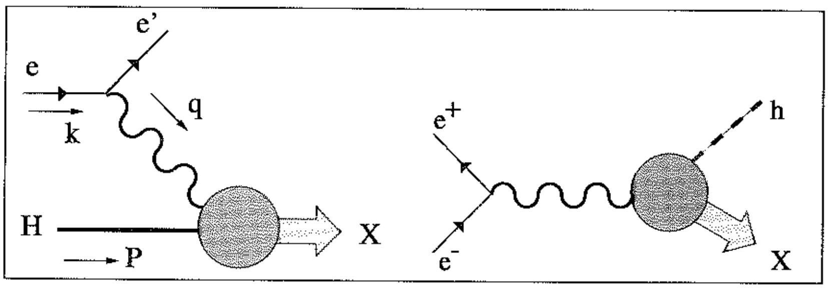


Figure 1.1: $e + H \to e' + X$ and $e^+ + e^- \to h + X$

process: $H_1 + H_2 \to \ell + \bar{\ell} + X$ (see Fig. 1.2).

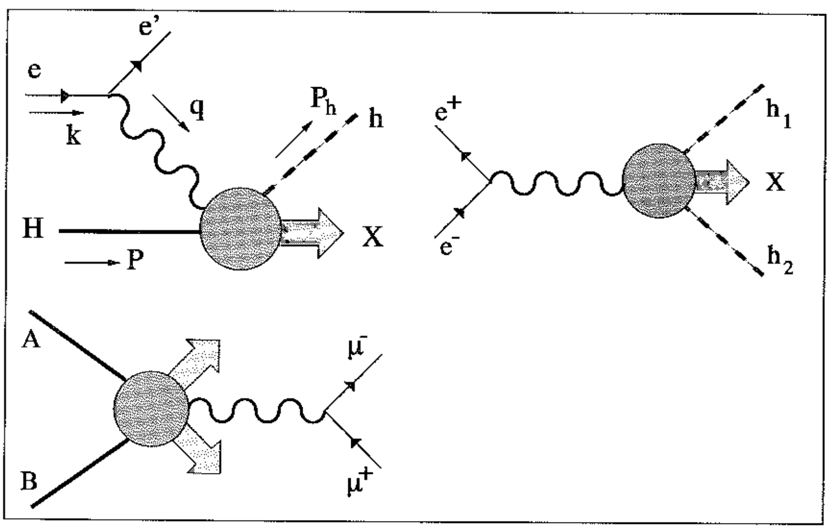


Figure 1.2: $e + H \to e' + h + X$, $e^+ + e^- \to h_1 + h_2 + X$ and $H_1 + H_2 \to \mu^+ + \mu^- + X$

These are the processes we will be concerned with, with the restriction that in case of two identified hadrons, they must be almost back-to-back (more precisely, the inner product of their momenta must be of the order of the hard scale). This last requirement excludes the case that the two hadrons connect to the same soft –nonperturbative– part, in which case the correlations between the two hadrons would allow no perturbative treatment at all (remember that hadrons are nonperturbative objects and will always be part of a soft physics region; hard scales serve to separate hard and soft regions). So rather than to count only the number of identified hadrons, we will count the number of soft parts and we will restrict to processes with one or two soft parts.

As said, we will focus on azimuthal asymmetries that might appear in these processes. For instance, if an azimuthal distribution of pions produced in semi-inclusive DIS is asymmetric with respect to the transverse polarization of the incoming proton, then one is (in first approximation) implicitly observing the distribution of longitudinally polarized quarks inside a transversely polarized hadron. This is not a straightforward conclusion, since one cannot exclude other mechanisms that also might be responsible for such an asymmetry, but this can be tested subsequently, by making predictions for asymmetries in other processes. This is what the game is all about.

The inclusive lepton-hadron scattering (DIS) is the most studied process from the theoretical as well as from the experimental side. From the theoretical point of view DIS can be described by using the operator product expansion (OPE) within the context of QCD. This description involves hadronic matrix elements of *local* operators consisting of quark and gluon fields. On the other hand, one can also describe this scattering process in terms of (hadronic matrix elements of) *non-local* operators. For the leading order in the hard scale generically denoted by Q, this was investigated by Soper and Collins [5, 6]. For treating the nonleading powers in $1/Q$, Ellis, Furmanski and Petronzio (EFP) [4] developed such a formalism for unpolarized DIS, following ideas by Politzer [7]. At tree level, i.e., order $(\alpha_s)^0$, EFP have shown the equivalence of their formalism to the OPE approach for $(1/Q)^n$ power corrections. The extension to polarized DIS was done by Efremov and Teryaev [8] and extensions to other processes have also been made. For instance, for the Drell-Yan process the parton model description was presented by Ralston and Soper [9] and the EFP-like treatment of the first subleading power in $1/Q$ was done by a number of people, most notably, Jaffe and Ji [10, 11] and Qiu and Sterman [12, 13]. Of course this brief overview of investigations is by no means complete and contributions by many others will be mentioned later on.

In the extension of EFP's approach to other processes, often only the first subleading power corrections are considered, since it became clear that further extensions can run into difficulties concerning the factorization into hard and soft parts. For practical purposes higher order power corrections are quite irrelevant, so the issues involved are mainly of academic interest. In this thesis we will restrict to the next-to-leading power corrections.

Let us give a little bit more details about the formalism. The Fourier transforms of the hadronic matrix elements of these nonlocal operators are expressed in terms of so-called *distribution functions*. The simplest example of such a distribution function is the momentum distribution appearing in the parton model. In DIS the parton model means that the photon which probes the incoming hadron actually probes a (free) parton inside the hadron. This is displayed in the first picture of Fig. 1.3 (most of the time we do not display the trivial lepton part of the process). The grey box is the nonperturbative soft part that is parametrized in terms of distribution functions. As said before this model only represents the first approximation and a generic power correction is displayed in the second picture. Its soft part can also be parametrized in terms of (multi-)parton distribution functions, however, they do not possess a clear parton interpretation in terms of *densities* of particles inside a hadron.

An example of a process with two soft parts is the Drell-Yan process, which in leading order –the parton model result– is displayed in the first picture of Fig. 1.4. A power correction

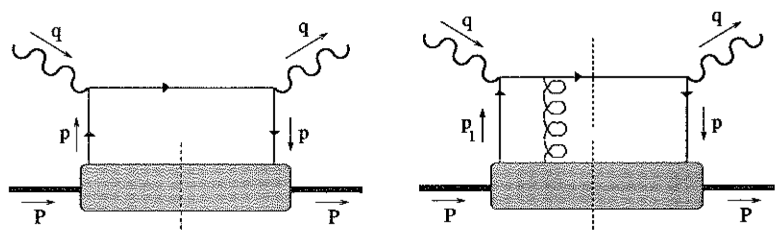


Figure 1.3: The parton model for DIS and a power correction.

to this leading order is shown in the second picture of Fig. 1.4.

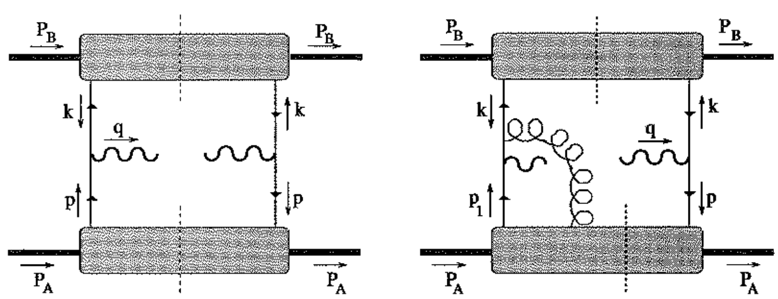


Figure 1.4: The parton model for the DY process and a power correction.

In high energy scattering experiments the particles involved are highly relativistic and move practically along the lightcone. Therefore, by doing hard scattering experiments one is sensitive to the lightcone component of the momentum of a parton with respect to that of the parent hadron. The quark and gluon distribution functions are functions of the *lightcone* momentum fraction x of a quark or a gluon inside a hadron. But this is not the end of the story, since perturbative corrections (see Fig. 1.5) induce a logarithmic dependence on the hard scale Q. Only the tree level parton model result, which is valid for very high values of Q, is independent of Q, i.e., independent of the energy scale at which the partons are probed. This property is called *scaling* and has been predicted by Bjorken [14] and observed in experiment [15]. But also the logarithmic violations of scaling due to perturbative QCD corrections have been confirmed over and over again, which belongs to the great successes of QCD as the theory of the strong interactions.

In contrast, power corrections to the leading order result in inclusive high energy scattering processes have never been experimentally established (although there is for instance room for sizeable $1/Q^2$ behavior in the fit to structure functions like F_2 [16]). The story is different in situations where the power behavior is not a correction, but the leading result (as is the case for the structure function g_2, which appears to be nonzero [17]). By studying asymmetries one hopes to subtract the leading result, which is often symmetric, and to become insensitive to the perturbative corrections, which are often suppressed by quark masses

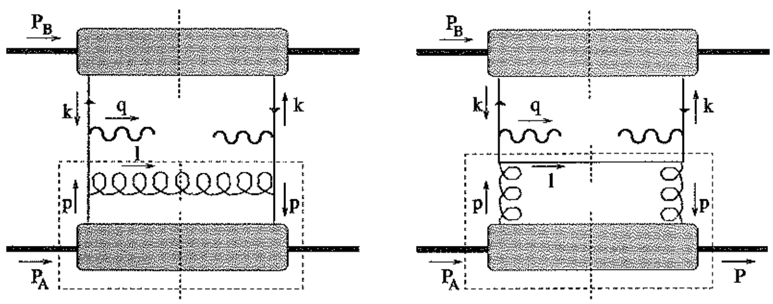


Figure 1.5: Two examples of perturbative corrections to the DY process.

or higher powers of $1/Q$ and hence, such an asymmetry might be directly sensitive to the first nonperturbative power contribution.

Another reason for looking at azimuthal distributions in processes with two soft parts is that one can also obtain information on transverse deviations from the dominant lightcone direction. This will also be discussed extensively.

Just as distribution functions describe the distribution of partons inside a hadron, one can describe the distribution of hadrons resulting from the fragmentation of a quark, by so-called *fragmentation functions*. The fragmentation of quarks and gluons into jets, i.e., showers of hadrons, is again not calculable within perturbation theory, hence fragmentation functions -like distribution functions- are nonperturbative objects and must be determined by experiment or calculated for instance with the help of models [18, 19, 20]. These fragmentation functions will appear in processes with hadrons in the final state, like in inclusive two-particle production in electron-positron annihilation, $e^+ + e^- \to h_1 + h_2 + X$, which in a parton model approximation is displayed in the first picture in Fig. 1.6 and the second picture is a generic power correction.

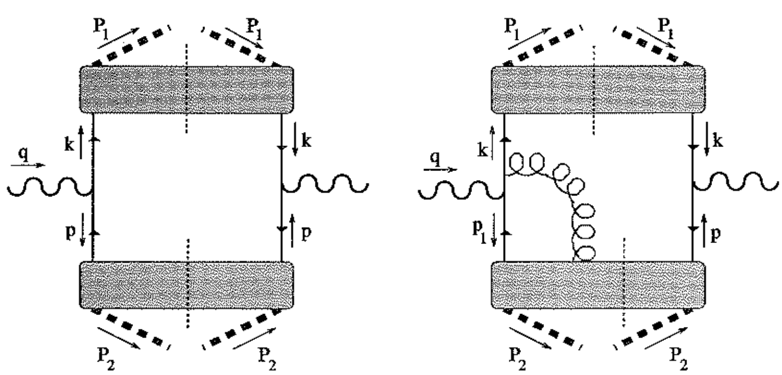


Figure 1.6: The parton model for $e^+ + e^- \to h_1 + h_2 + X$ and a power correction.

The course to be followed in this thesis is as follows. With help of symmetry properties one can first of all determine the complete set of distribution and fragmentation functions in which a process can be expressed. By expressing cross sections and asymmetries in terms of these functions one can then deduce how to measure and separate them. The idea is that all these functions are universal and once they are measured in one process they can lead to predictions in others.

In this way, one wants to find answers to questions like: What is the distribution of longitudinally polarized quarks inside a transversely polarized hadron? How does the physics or rather the description of a quark inside a hadron differ from that of a quark fragmenting into a hadron? Are nonperturbative (power) corrections sometimes more important than (logarithmic) perturbative corrections? Are the symmetries of QCD respected and how can they be used to constrain the description of processes?

We will not be able to answer all of these questions, especially since some of them are of an experimental nature, but in order to be able to answer them in the future, one needs a framework in which one can pose the questions properly (in mathematical terms that is). The work presented in this thesis is meant to contribute to the establishing of such a framework. It continues in the spirit of the work presented in two earlier theses [21, 22], but addresses many issues not discussed there.

In this thesis we will focus on the leading and next-to-leading powers in $1/Q$ and restrict to tree level. The perturbative corrections are expected to dominate at high energies, since $1/Q$ falls off faster than $1/\ln Q$. Our investigations of subleading powers require therefore hard scattering experiments with not too high energy, although it should be high enough that it factorizes into hard and soft parts and such that for instance two jets can be produced. In chapter 7 we will discuss some of these aspects concerning factorization and perturbative corrections in more detail, albeit in qualitative terms. We would like to emphasize that already the leading order result in processes with two soft parts can yield interesting azimuthal dependences due to intrinsic transverse momentum of the quarks. These might persist to higher energies, because they are not suppressed by factors of $1/Q$ (on the other hand, so-called Sudakov factors will tend to dilute these asymmetries at high energies). We will not be concerned with the specific range of values of Q and x for which the power corrections matter, because it is something one cannot calculate perturbatively.

The outline of this thesis is as follows. In chapter 2 the separation of the DIS process into perturbative and nonperturbative parts is investigated, especially looking at higher twist terms and in case polarization is taken into account. This is EFP's approach, which is sometimes called a field theoretic approach [23]. We extend a method, based on a factorization procedure due to Qiu (somewhat different from EFP's), that allows to calculate a specific order in inverse powers of the hard scale, without the necessity to calculate more dominant terms. This results in manifestly electromagnetically gauge invariant hard parts. We will include quark masses to complete the analysis. This work is partly published in [24], but also contains new aspects (especially concerning twist four).

In chapter 3 we discuss properties of correlation functions, the soft parts, which will be used in subsequent chapters. More specifically, we discuss: symmetry properties, especially time-reversal and rotational symmetry; integral relations called sum rules; relations due to the equations of motion, etc. In chapter 4 we investigate spin asymmetries in the Drell-Yan process, in particular single spin asymmetries and the mechanisms to produce these (based on the publications [25]). So-called time-reversal odd distribution functions, which are discussed in chapter 3, will play an important role. Up to this point the relevant soft parts describe how quarks and gluons are distributed inside a hadron (in the initial state), but in the remaining chapters the focus will be on similar soft parts describing hadrons produced (hence, in the final state) by a fragmenting parton. In chapter 5 we look at the process of inclusive two-hadron production in electron-positron annihilation, with the two hadrons almost back-to-back (based on the publication [26]). The focus is on azimuthal asymmetries which arise as spin asymmetries and/or as manifestations of intrinsic transverse momentum of the quarks. The energies to which the investigation applies are much lower than the mass of the Z-boson. In chapter 6 we look at the extension of the same process to energies around the Z peak and focus on an unpolarized azimuthal asymmetry (based on the publication [27]). Chapter 7 concerns some of the more technical issues, like factorization and color gauge invariance, which partly remain to be investigated or extended.

Chapter 2

Unraveling higher twist

2.1 Introduction

Hard scattering processes are processes involving one or more large energy scales, so-called hard scales. Such a hard scale serves to separate the perturbative –hard– from the nonperturbative –soft– parts of the process. This separation generally varies with the scale and is governed by corrections to the leading order result. Next to the perturbative QCD (pQCD) corrections, which depend logarithmically on the scale, there are also corrections which behave like an inverse power of the scale. The separation between hard and soft parts for those power corrections is the subject of this chapter. We will focus especially on scattering with polarized particles.

The simplest hard scattering process we will study is the so-called deep inelastic scattering (DIS) process, in which a lepton (most of the time an electron) and a hadron (often a proton) collide with high energy and the recoiled lepton is observed (ℓ+$H \to \ell'$+X), cf. Fig. 2.1. So it is a process with one identified hadron and hence with one soft part. To good approximation, the electron interacts with the hadron via one photon. This virtual photon that probes the hadron has a hard, space-like momentum q, which sets the hard scale, generically called Q, such that $Q^2 = -q^2$.

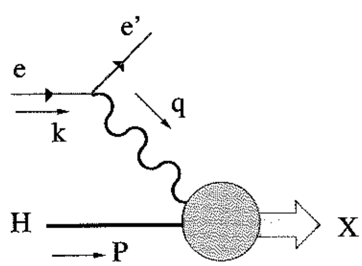


Figure 2.1: Deep inelastic scattering

The cross section for DIS can be separated into a leptonic and a hadronic part of the scattering (cf. e.g. [28]). The leptonic part is elementary and described by QED in a perturbative way. The hadronic part is a hadronic matrix element of a product of two electromagnetic currents. This product, rewritten as a commutator, is related to the time-ordered product of the two currents via the optical theorem. More specifically, we define the hadron tensor $W_{\mu\nu}$ and the forward scattering Compton amplitude $T_{\mu\nu}$ as[1]

$$W_{\mu\nu}(P,q) = \frac{1}{4\pi}\int d^4x e^{iq\cdot x}\langle P|[J_\mu(x), J_\nu(0)]|P\rangle, \tag{2.1}$$

$$T_{\mu\nu}(P,q) = i\int d^4x e^{iq\cdot x}\langle P|T\, J_\mu(x)J_\nu(0)|P\rangle. \tag{2.2}$$

From unitarity of the S-matrix, the optical theorem follows: $W_{\mu\nu} = \mathrm{Im}\, T_{\mu\nu}/2\pi$ (Fig. 2.2). Sometimes this relation is written in terms of the s-channel discontinuity $\mathrm{Disc}\,(T_{\mu\nu}) = 2i\,\mathrm{Im}\,(T_{\mu\nu})$.

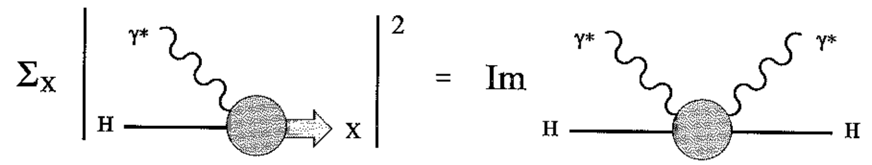


Figure 2.2: The optical theorem

In the case of unpolarized scattering (via vector currents) one has a symmetric hadron tensor, which one can decompose in terms of tensor structures using the available vectors, as

$$W^{\mu\nu}(P,q) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)F_1 + \frac{\tilde{P}^\mu\tilde{P}^\nu}{P\cdot q}F_2, \tag{2.3}$$

where $\tilde{P}^\mu = P^\mu - (P\cdot q)q^\mu/q^2$ and P is the hadron momentum with $P^2 = M^2$. For DIS off a polarized spin-1/2 hadron characterized by a spin vector S, the hadron tensor also has an antisymmetric part:

$$W^A_{\mu\nu}(P,S,q) = i\varepsilon_{\mu\nu\rho\sigma}\frac{M\,q^\rho}{P\cdot q}\left[S^\sigma g_1 + \left(S^\sigma - \frac{S\cdot q}{P\cdot q}P^\sigma\right)g_2\right]. \tag{2.4}$$

The *structure functions* F_1, F_2, g_1 and g_2 are dimensionless and functions of the Bjorken variable $x_B = Q^2/(2P\cdot q)$ and the logarithm of Q^2. This holds in the so-called Bjorken limit, in which Q^2 and $P\cdot q$ become large at fixed x_B and in fact the dependence on Q^2 disappears (for a discussion of this property called scaling cf. e.g. [29]).

[1]States are normalized as $\langle P|P'\rangle = (2\pi)^3\, 2P^0\,\delta^3(P-P')$, such that the dimension of a one-particle state is -1, hence $W_{\mu\nu}$ and $T_{\mu\nu}$ are dimensionless.

One wants to learn as much as possible about these structure functions and one well-known theoretical tool is the operator product expansion, which provides a separation of the hadron tensor into products of hard and soft parts. In this chapter we will focus on this separation using a different, but related approach. In order to explain that relation we first discuss the OPE.

2.2 Operator product expansion

One of the techniques used to describe hard scattering processes involving hadrons is the operator product expansion (OPE). The processes which are considered are highly inelastic, such that the structure of the hadrons can be probed. The underlying theory to describe this structure is QCD and therefore the operators in the expansion are built up from QCD fields. The OPE itself is viewed as an operator identity, but in fact it is only proven in perturbation theory. The assumption that it is also valid when used inside hadronic matrix elements is implicit. Since a hadron is a bound state of quarks and gluons, the general proof (if it exists) is necessarily non-perturbative in nature. Even so, the OPE cannot be used in all processes of interest, because some of these lack a small parameter in which to expand. In those cases we will resort to another method, which is less rigorous (at least some aspects have not been proven fully), but is quite useful nevertheless and is equivalent to the OPE in case it applies.

The OPE will be applied in case of a product of two electromagnetic currents. We will first investigate the time-ordered product versus the commutator of two such currents. We consider the normal-ordered current $J_\mu(x) =: \overline{\psi}(x)\gamma_\mu\psi(x) :$. Using Wick's theorem we find:

$$\begin{aligned} T(:\overline{\psi}_\alpha\psi_\beta :: \overline{\psi}_\sigma\psi_\rho :) &= :\overline{\psi}_\alpha\psi_\beta\overline{\psi}_\sigma\psi_\rho : -\langle 0|T\overline{\psi}_\alpha\psi_\rho|0\rangle : \overline{\psi}_\sigma\psi_\beta : + \langle 0|T\psi_\beta\overline{\psi}_\sigma|0\rangle : \overline{\psi}_\alpha\psi_\rho : \\ &+ \langle 0|T\overline{\psi}_\alpha\psi_\rho|0\rangle\langle 0|T\psi_\beta\overline{\psi}_\sigma|0\rangle, \end{aligned} \tag{2.5}$$

where for convenience we have left out the position of the fields and take fields with an index α or β at point x and with ρ or σ at point y. The Feynman propagator is [30]

$$\begin{aligned} \langle 0|T\psi_\beta\overline{\psi}_\sigma|0\rangle &\equiv iS^F_{\beta\sigma}(x-y) = \frac{i}{2\pi}(\gamma_\mu)_{\beta\sigma}\frac{(x-y)^\mu}{((x-y)^2 - i\varepsilon)^2} \\ &= -\not{\partial}_{\beta\sigma}\left[\frac{i}{4\pi^2}\frac{1}{(x-y)^2 - i\varepsilon}\right]. \end{aligned} \tag{2.6}$$

These last two expressions are exact in the case $m = 0$, in which case one uses

$$\int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik\cdot x}}{k^2 + i\varepsilon} = \frac{i}{4\pi^2}\frac{1}{x^2 - i\varepsilon} \tag{2.7}$$

to arrive at expressions for the Green's function in momentum space.

It follows that:

$$\begin{aligned} T\,J_\mu(x)J_\nu(0) &= :(\overline{\psi}\gamma_\mu\psi)(x)(\overline{\psi}\gamma_\nu\psi)(0): + :\overline{\psi}(x)\gamma_\mu iS^F(x)\gamma_\nu\psi(0): \\ &+ :\overline{\psi}(0)\gamma_\nu iS^F(-x)\gamma_\mu\psi(x): - \mathrm{Tr}\left[iS^F(-x)\gamma_\mu iS^F(x)\gamma_\nu\right]. \end{aligned} \tag{2.8}$$

This is the expression which is commonly used to arrive at the OPE result for DIS. One expands Eq. (2.8) inside the expression for $T^{\mu\nu}$ and uses the optical theorem to arrive at $W^{\mu\nu}$. On the other hand, we observe the following identity, which follows solely from the definition of the time-ordering operator and the hermiticity of the electromagnetic currents:

$$T\,J_\mu(x)J_\nu(0) - (T\,J_\mu(x)J_\nu(0))^\dagger = \varepsilon(x_0)[J_\mu(x), J_\nu(0)]. \tag{2.9}$$

This identity could be used to arrive at an OPE for the commutator and hence for $W^{\mu\nu}$ directly [31].

As a side remark we note that for the commutator the normal ordering of the currents is inessential, since commutators of c-numbers vanish. But for the time-ordered product it is essential. For comparison we consider:

$$\begin{aligned} T(\overline{\psi}_\alpha\psi_\beta\overline{\psi}_\sigma\psi_\rho) &= T(:\overline{\psi}_\alpha\psi_\beta::\overline{\psi}_\sigma\psi_\rho:) + \langle 0|T\overline{\psi}_\alpha\psi_\beta|0\rangle :\overline{\psi}_\sigma\psi_\rho: + \langle 0|T\overline{\psi}_\sigma\psi_\rho|0\rangle :\overline{\psi}_\alpha\psi_\beta: \\ &+ \langle 0|T\overline{\psi}_\alpha\psi_\beta|0\rangle\langle 0|T\overline{\psi}_\sigma\psi_\rho|0\rangle. \end{aligned} \tag{2.10}$$

Note that $\langle 0|T\overline{\psi}_\alpha\psi_\beta|0\rangle = -iS^F_{\beta\alpha}(0)$ and $\langle 0|T\overline{\psi}_\sigma\psi_\rho|0\rangle = -iS^F_{\rho\sigma}(0)$ and therefore are infinite constants, hence the necessity of considering normal-ordered currents is apparent. Upon taking the imaginary part of the time-ordered product, which we will always do, the infinite constants cancel. One can view it in a different way: in order for the derivation of Eq. (2.9) to be well-defined one needs normal ordering.

In any case we will consider connected matrix elements, such that the last term in Eq. (2.8) is irrelevant anyway.

If one puts Eq. (2.8) into Eq. (2.2) and neglecting the disconnected and four-fermion term (which is not a leading contribution), one arrives at the so-called parton model result

$$T^{\mu\nu} = \int d^4k\,\mathrm{Tr}\left[S^{\mu\nu}(k)\Phi(k)\right], \tag{2.11}$$

where

$$S^{\mu\nu}_{ij}(k) = i\int d^4y\,e^{ik\cdot y}\left(\gamma^\mu iS^F(x)\gamma^\nu\right)_{ij} = -i\left(i\gamma^\mu\frac{i\not{k}}{k^2+i\epsilon}i\gamma^\nu\right)_{ij}, \tag{2.12}$$

$$\Phi_{ij}(k) = \int\frac{d^4z}{(2\pi)^4}e^{ik\cdot z}\langle P|:\overline{\psi}_j(0)\,\psi_i(z):|P\rangle_c = \int\frac{d^4z}{(2\pi)^4}e^{ik\cdot z}\langle P|T\,\overline{\psi}_j(0)\,\psi_i(z)|P\rangle_c, \tag{2.13}$$

where the subscript c indicates that we consider connected matrix elements only (from here onwards this c is not indicated anymore). Graphically this expression is depicted in Fig. 2.3.

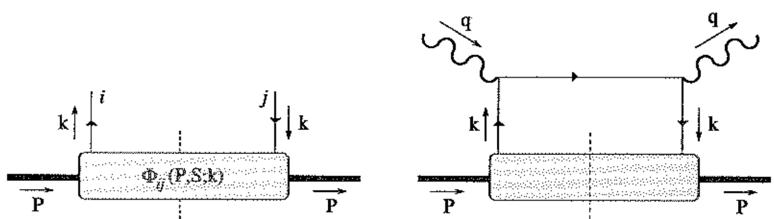


Figure 2.3: The correlation function $\Phi(p)$ and the parton model result for $T^{\mu\nu}$

Since the currents are operator-valued distributions, the product of two such currents will in general acquire (additional) singularities if the currents are at the same space-time point. The OPE of the product will be a sum of products of so-called coefficient functions and local operators, where the coefficient functions contain the singularities. The OPE displays a property called factorization, meaning that there is a separation of short and long distance parts.

To arrive at a possible OPE of the time-ordered product of two currents, one can Taylor expand in their separation if it happens to be a small parameter. So for example it might be justified to Taylor expand the r.h.s. of Eq. (2.8) around $x = 0$, which is called a short-distance expansion, or around $x^2 = 0$, which is the so-called lightcone expansion. In the former case only a limited number of composite operators contribute to terms with the dominant short-distance singularity; in the latter case this number will be infinite and the OPE will take the following form (suppressing the Lorentz indices of the currents):

$$T\,J(x)J(0) = \sum_{i,n} C_n^{(i)}(x^2)\, x^{\mu_1} \ldots x^{\mu_n}\, O^{(i)}_{\mu_1 \cdots \mu_n}(0), \tag{2.14}$$

where the C's are the coefficient functions and the O's are the local operators, which are symmetric and traceless, in order to be in an irreducible representation of the Lorentz group. The index i denotes the species of operators, because in general the OPE is not just a Taylor expansion and other operators are present. For instance, next to the quark operators

$$O^Q_{\mu_1 \cdots \mu_n} = i^{n-1}\overline{\psi}\gamma_{\{\mu_1} D_{\mu_2} \ldots D_{\mu_n\}}\psi, \tag{2.15}$$

there are also gluon operators

$$O^G_{\mu_1 \cdots \mu_n} = i^{n-2} S F_{\lambda\{\mu_1} D_{\mu_2} \ldots D_{\mu_{n-1}} F^{\lambda}_{\mu_n\}}, \tag{2.16}$$

where $\{\ldots\}$ indicates symmetrization in the indices μ_i; D_{μ_i} and $F_{\mu\nu}$ are the covariant derivative and the field strength, respectively.

For physical matrix elements of the operators one finds the following form:

$$\langle P|O^{(i)}_{\mu_1 \cdots \mu_n}(0)|P\rangle = \Theta_n^{(i)}[P_{\mu_1} \ldots P_{\mu_n} - \text{traces}]. \tag{2.17}$$

After Fourier transforming one finds for the forward scattering Compton amplitude T (still suppressing the Lorentz indices of the currents):

$$T = \sum_{i,n} \tilde{C}_n^{(i)}(Q^2, \mu^2)\, \Theta_n^{(i)}(M^2, \mu^2) \left[\left(\frac{1}{2x}\right)^n + \mathcal{O}\left(\frac{M^2}{Q^2}\right) \right]. \tag{2.18}$$

The main point of the OPE is that it factorizes the expression into products of coefficient functions, depending on the hard scale Q, and operator matrix elements, depending on the soft hadronic scale, taken to be M^2 (sometimes people use $\Lambda_{\rm QCD}$). Both depend on the factorization and renormalization scale, denoted generically as μ.

The power corrections $\mathcal{O}(M^2/Q^2)$ arise due to the traces, which were required to make the operators traceless. This kind of corrections is called kinematic power corrections or target mass corrections [32]. These have a different origin than the so-called dynamical power corrections which we will investigate extensively. On dimensional grounds one can deduce that the Fourier transform $\tilde{C}$ of the coefficient function C depends on the scale as follows:

$$\tilde{C}_n^{(i)}(q^2; g, \mu^2) = c_n^{(i)}\left(\ln(q^2/\mu^2)\right) \cdot (q^2)^{2+n-d[O_n^{(i)}]}, \tag{2.19}$$

where $c_n^{(i)}$ is a dimensionless function of $\ln(q^2/\mu^2)$. So depending on the *twist* t of the operator $O_n^{(i)}$, defined as its dimension $d[O_n^{(i)}]$ minus its spin[2] n, its dominant contribution is of order $1/Q^{t-2}$. So for example the twist of $O^Q_{\mu_1\ldots\mu_n}$ (Eq. (2.15)) and $O^G_{\mu_1\ldots\mu_n}$ (Eq. (2.16)) is two, so their dominant contribution is in principle at order 1. The leading twist contribution comes from such twist-two operators. Examples of twist three operators are

$$O'^{Q}_{\mu_1\ldots\mu_{n-1}} = i^n \overline{\psi}\gamma_\sigma D_{\{\mu_1} \ldots D^\sigma \ldots D_{\mu_{n-1}\}}\psi, \tag{2.20}$$

For examples of such higher twist operators in the case polarized scattering, see [33, 23, 34, 35].

Operators of twist three and higher in general will provide power corrections to the leading contribution and these will be called dynamical power corrections. As said we will not concern ourselves with kinematical power corrections, but we *will* in some cases look at quark mass corrections.

2.3 Nonlocal operators

In the previous section we discussed the OPE and illustrated what it means for the time-ordered product and for the commutator of two currents. Using the OPE the hadron tensor, and hence the structure functions, can be expanded in terms of physical matrix elements of local operators. The relation between specific operators in the OPE and a structure function

[2] The spin of a symmetric, traceless operator is equal to the number of indices, so $O^{(i)}_{\mu_1\ldots\mu_n}$ is of spin n.

is obtained by taking moments. The n-th moment (or Mellin transform) of a function F is defined as[3]

$$F^{(n)} \equiv \int_0^1 dx\, x^{n-1}\, F(x). \tag{2.21}$$

By taking such moments one can project out specific operators from the OPE description of the function F.

The idea is now to write the local operator matrix elements as (multiple) moment integrals over new functions. These new functions are then expressed in terms of *nonlocal* operator matrix elements. For the local operators at twist two this leads to the well-known parton picture, where one has so-called parton distribution functions, which upon taking (single) moments are directly related to (single) moments of structure functions (if the latter are restricted to the leading twist). In this case there are also simple relations between the structure function itself and the sum over flavors of the distribution functions, cf. e.g. [28],

$$F_2(x) = 2x\, F_1(x) = \sum_a e_a^2\, x\, [q_a(x) + \bar{q}_a(x)]\,, \tag{2.22}$$

where $q_a(x)$ is interpreted as the probability of finding a parton/quark of flavor a in the proton with a (lightcone) momentum fraction x and e_a is its charge; similarly, $\bar{q}_a(x)$ denotes the antiquark distribution function.

The distribution function $q_a(x)$ has a simple representation in terms of a nonlocal operator matrix element (in a lightcone gauge $n \cdot A = 0$, where $n^2 = 0$):

$$q(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P|T\,\overline{\psi}(0)\, \not{n} \psi(\lambda n)|P\rangle, \tag{2.23}$$

as will be elaborated upon later. By a Taylor expansion of $\psi(\lambda n)$,

$$\psi(\lambda n) = \psi(0) + (-i\lambda) n^\mu D_\mu \psi(0) + \frac{1}{2}(-i\lambda)^2 n^\mu n^\nu D_\mu D_\nu \psi(0) + \ldots, \tag{2.24}$$

one finds

$$q(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P|\overline{\psi}(0)\, \not{n} \psi(0)|P\rangle - \frac{\partial}{\partial x} \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P|\overline{\psi}(0)\, \not{n}\, n\cdot D\, \psi(0)|P\rangle + \ldots, \tag{2.25}$$

such that for instance the first moment projects out the local operator $\overline{\psi}(0)\, \not{n} \psi(0)$,

$$\int_{-1}^{1} dx\, q(x) = \int_0^1 dx\, [q(x) - \bar{q}(x)] = \frac{1}{2} \langle P|\overline{\psi}(0)\, \not{n} \psi(0)|P\rangle, \tag{2.26}$$

[3]The inverse Mellin transform is used to arrive at a function $F(x)$ from its moments $F^{(n)}$ and is given by

$$F(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dn\, x^{-n}\, F^{(n)},$$

where $F^{(n)}$ has no singularities to the right of the line $\mathrm{Re}(n) = c$ in the complex n plane.

where we exploited the symmetry property $\bar{q}(x) = -q(-x)$ between the antiquark and quark distribution function (see section 3.5). The support properties of distribution functions will be discussed in the next chapter, App. 3.C. So the first moment involves the valence quark distribution, as do all the other odd moments. On the other hand, the even moments involve the sum of quark and antiquark distribution functions and thus are related to the even moments of the structure functions F_1 and F_2, cf. Eq. (2.22). The valence distribution appears in the odd moments of the charged current structure function F_3, cf. [28].

For higher twist operators this simple parton picture can be extended as Politzer proposed [7]. This has been done for tree-level unpolarized deep inelastic scattering (DIS) by Ellis, Furmanski and Petronzio (EFP) [4]. For instance, consider the following hadron matrix element of the local operator in Eq. (2.20):

$$\begin{aligned} A_{nm} &\equiv \langle P|\overline{\psi}(i\,n\cdot D)_1 \ldots (i\,n\cdot D)_n\,(i\not{D})\,(i\,n\cdot D)_{n+1} \cdots (i\,n\cdot D)_{n+m}\psi|P\rangle \\ &= \langle P|O'^{Q}_{\mu_1\cdots\mu_{n-1}}|P\rangle n^{\mu_1}\ldots n^{\mu_{n-1}}. \end{aligned} \tag{2.27}$$

The function A_{nm} one now writes as a double moment of a new two-argument function f:

$$A_{nm} \equiv \int_0^1 dx_1 \int_0^1 dx_2\, x_1^n x_2^m\, f(x_1, x_2). \tag{2.28}$$

The natural interpretation (extension of the parton model) of this function f is that it parametrizes a three-parton (quark-gluon-antiquark) correlation function in the hadron as a function of the momentum fractions of two of the partons. The function $f(x_1, x_2)$ is (a double Fourier transform of) a matrix element of the nonlocal operator $\overline{\psi}(0)i\not{D}(y)\psi(x)$.

Such functions f cannot be measured as a function of these two variables. Experimentally one only has a handle on the structure functions and its single moments (if some extrapolation is done to small x). Nevertheless, these new generalized distribution functions are helpful in the analysis of the structure functions. They represent whole classes/towers of local operators of the OPE. The definition of twist of such nonlocal operators differs from that used for the operators in the OPE and will be defined below.

Next we focus on the formalism by EFP. They studied the case of unpolarized DIS, without taking into account pQCD corrections or quark masses. We will look at polarized DIS with massless quarks first and later on include quark masses. After that the unpolarized case will be investigated in the case of twist four including quark masses.

2.4 Twist three polarized DIS with massless quarks

In this section we look at polarized DIS up to and including order $1/Q$ (see also Ref. [23] and the references therein). The EFP approach in this context was first applied by Efremov and Teryaev [8]. The major ingredient is the so-called collinear expansion [4]. We will also discuss the additional technique due to Qiu [36], i.e., the use of the so-called special propagator [37, 38, 36], leading to a different factorized form of the hadron tensor than the one in EFP's approach. We will first consider the massless quark case and study the massive case in the next section.

2.4.1 EFP's factorization

EFP proposed a diagrammatic expansion of the forward scattering Compton amplitude $T^{\mu\nu}$ in DIS, that is inspired by the parton model and is a concrete realization of Politzer's ideas [7] on how to treat dynamical power corrections in a parton model way. The justification of the expansion is best given with hindsight by showing that upon taking moments one arrives at the OPE result.

Following EFP, we start with the diagrammatic expansion of the forward scattering amplitude $T^{\mu\nu}$ in DIS (we consider only quarks of one flavor for the moment):

$$T^{\mu\nu} = \int d^4k \, \mathrm{Tr}\left[S^{\mu\nu}(k)\Phi(k)\right] + \int d^4k_1 d^4k_2 \, \mathrm{Tr}\left[S^{\mu\nu}_{\alpha}(k_1,k_2)\Phi^{\alpha}_{A}(k_1,k_2)\right] + \ldots, \tag{2.29}$$

keeping only the terms contributing up to and including order $1/Q$ (see Fig. 2.4). Here,

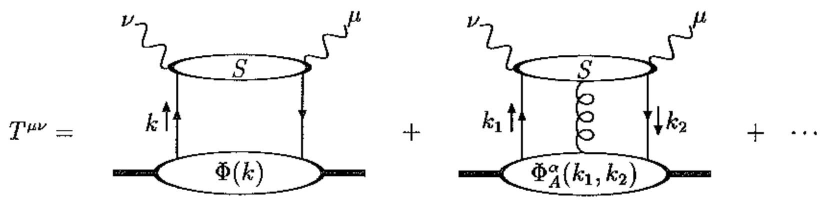


Figure 2.4: The forward scattering amplitude.

S and Φ are the hard and soft scattering parts, respectively. One recognizes that the first term is the parton model term Eq. (2.11). We will not consider loop corrections, so the hard parts consist of the forward parton-photon scattering tree graphs which are one-particle-irreducible (1PI) in the $\gamma\gamma$-channel, or as Politzer puts it, the diagrams that are 1PI in the hanging legs. We indicate explicitly the dependence on the parton momenta $k_{..}$, but not on the photon momentum q ($q^2 = -Q^2$) or the hadron momentum P, but note that actually $S(k) = S(q,k)$ and $\Phi(k) = \Phi(P,k)$. The soft parts are defined as Fourier transforms of hadronic matrix elements of nonlocal operator products:

$$\Phi_{ij}(k) = \int \frac{d^4z}{(2\pi)^4} e^{ik\cdot z} \langle P,S|T\overline{\psi}_j(0)\psi_i(z)|P,S\rangle, \tag{2.30}$$

$$\Phi^{\alpha}_{Aij}(k_1,k_2) = \int \frac{d^4z}{(2\pi)^4}\frac{d^4z'}{(2\pi)^4} e^{ik_1\cdot z} e^{i(k_2-k_1)\cdot z'} \langle P,S|T\overline{\psi}_j(0)gA^{\alpha}(z')\psi_i(z)|P,S\rangle. \tag{2.31}$$

We have included a color identity and g times t^a from the hard into the soft parts, respectively. Thereby the hard parts effectively become QED graphs with unit charge [4]. Since the soft parts are matrix elements of elementary fields of the QCD Lagrangian, one can use them as (nonlocal) composite operator insertions in Feynman diagrams.

The hadron is characterized by its momentum, satisfying $P^2 = M^2$, and its spin vector S (in case of a spin-1/2 particle), satisfying $P \cdot S = 0$ and $S^2 = -1$. We make a Sudakov decomposition of the relevant vectors with respect to two lightlike vectors p and n, satisfying $p \cdot n = 1$,

$$q = -x_B p + \frac{Q^2}{2x_B} n, \tag{2.32}$$

$$P = p + \frac{M^2}{2} n, \tag{2.33}$$

$$S = \frac{\lambda}{M}\left(p - \frac{M^2}{2} n\right) + S_T, \tag{2.34}$$

$$k = xp + \frac{k^2 - k_T^2}{2x} n + k_T. \tag{2.35}$$

Here, λ is the hadron's helicity, $S_T^\rho = g_T^{\rho\sigma} S_\sigma \equiv (g^{\rho\sigma} - p^\rho n^\sigma - n^\rho p^\sigma) S_\sigma$ its transverse spin. Also, x is the quark's longitudinal momentum fraction, k_T its transverse momentum.

The target hadron momentum is predominantly in the p-direction. As said before we will ignore target mass corrections arising from the M^2 part of the momentum P.

The next step is performing a collinear expansion of the hard parts $S^{\mu\nu}$ and $S_\alpha^{\mu\nu}$ around p, which is allowed since one can use the hard scale Q for the expansion. It yields (keeping only relevant terms):

$$S^{\mu\nu}(k) = S^{\mu\nu}(xp) + (k - xp)^\alpha \left.\frac{\partial S^{\mu\nu}(k)}{\partial k^\alpha}\right|_{k=xp} + \ldots, \tag{2.36}$$

where xp is the collinear part of the momentum k and similarly:

$$S_\alpha^{\mu\nu}(k_1, k_2) = S_\alpha^{\mu\nu}(x_1 p, x_2 p) + \ldots \tag{2.37}$$

We insert these two expansions into Eq. (2.29) and we will show that the terms arising from the second term in Eq. (2.36) and the (first) term in Eq. (2.37) can be combined to result in one term with a matrix element containing a covariant derivative.

First consider the term in Eq. (2.29) arising from the first term in Eq. (2.36), which can be rewritten with help of

$$\int \frac{d^4k}{(2\pi)^4} e^{ik\cdot z} \delta(x - k\cdot n) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \delta^4(z - \lambda n) \tag{2.38}$$

yielding

$$\int d^4k \, \mathrm{Tr}\left[S^{\mu\nu}(xp)\Phi(k)\right] = \int dx \, \mathrm{Tr}\left[S^{\mu\nu}(x)\Phi(x)\right], \tag{2.39}$$

where

$$\Phi_{ij}(x) \equiv \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P, S | T\, \overline{\psi}_j(0) \psi_i(\lambda n) | P, S\rangle. \tag{2.40}$$

Note that $q(x) = \mathrm{Tr}(\Phi \not{n})/2$, cf. Eq. (2.23).

Next consider the second term in Eq. (2.36). By using the Ward identity (which also holds for pQCD corrections):

$$\frac{\partial S^{\mu\nu}(k)}{\partial k^\alpha} = S^{\mu\nu}_\alpha(k,k), \tag{2.41}$$

where the r.h.s. is effectively obtained from $S^{\mu\nu}(k)$ by insertion of a zero-momentum gluon, with coupling $i\gamma^\rho_{ij}$ to the fermions, and by using the fact that $(k-xp)^\alpha = \omega^\alpha{}_\beta k^\beta$, where $\omega^\alpha{}_\beta = g^\alpha{}_\beta - p^\alpha n_\beta$, hence also $A^\alpha = \omega^\alpha{}_\beta A^\beta$ in the lightcone gauge $n\cdot A = 0$, one finds that:

$$\begin{aligned}\int d^4k \,\mathrm{Tr}\left[(k-xp)^\alpha \left.\frac{\partial S^{\mu\nu}(k)}{\partial k^\alpha}\right|_{k=xp} \Phi(k)\right] &+ \int d^4k_1 d^4k_2 \,\mathrm{Tr}\left[S^{\mu\nu}_\alpha(x_1p,x_2p)\Phi^\alpha_A(k_1,k_2)\right] \\ = \int dx_1 dx_2 \,\mathrm{Tr}&\left[S^{\mu\nu}_\alpha(x_1,x_2)\omega^\alpha{}_\beta\left(\Phi^\beta_k(x_1,x_2)+\Phi^\beta_A(x_1,x_2)\right)\right],\end{aligned} \tag{2.42}$$

where

$$\Phi^\alpha_k(x_1,x_2) \equiv \delta(x_2-x_1)\int d^4k_1 \delta(x_1 - k_1\cdot n)k_1^\alpha \Phi(k_1), \tag{2.43}$$

$$\Phi^\alpha_A(x_1,x_2) \equiv \int d^4k_1 d^4k_2 \delta(x_1-k_1\cdot n)\delta(x_2-k_2\cdot n)\Phi^\alpha_A(k_1,k_2). \tag{2.44}$$

Next one observes that

$$\begin{aligned}\Phi^\alpha_k(x_1,x_2)+\Phi^\alpha_A(x_1,x_2) &= \int \frac{d\lambda}{2\pi}\frac{d\eta}{2\pi} e^{i\lambda x_1} e^{i\eta(x_2-x_1)} \langle P,S|T\overline{\psi}_j(0) iD^\alpha(\eta n)\psi_i(\lambda n)|P,S\rangle \\ &\equiv \Phi^\alpha_{Dij}(x_1,x_2)\end{aligned} \tag{2.45}$$

where we have used Eq. (2.38) and $iD^\alpha(\eta n) = i\partial^\alpha + gA^\alpha(\eta n)$. Note that this is the function $f(x_1,x_2)$ in Eq. (2.28) upon tracing with a γ-matrix.

To summarize: we can now rewrite $T^{\mu\nu}$ as follows (cf. Eq. (2.39) and Eq. (2.42) in combination with Eq. (2.45)):

$$T^{\mu\nu} = \int dx \,\mathrm{Tr}\left[S^{\mu\nu}(x)\Phi(x)\right] + \int dx_1 dx_2 \,\mathrm{Tr}\left[S^{\mu\nu}_\alpha(x_1,x_2)\omega^\alpha{}_\beta \Phi^\beta_D(x_1,x_2)\right] + \cdots \tag{2.46}$$

This is EFP's factorized result for polarized DIS up to order $1/Q$. The leading part of the first term is the (QCD-improved) parton model and is well-established. Upon taking moments of the *full* expression one arrives back at the OPE result, showing the equivalence.

2.4.2 Qiu's factorization

The main idea of EFP's factorization is that transverse momentum in the hard part gives rise to suppression by powers of the hard photon momentum, therefore is of higher twist and should be included in higher twist matrix elements.

Not just transverse momentum of partons is considered to be of higher twist, off-shellness also is. One wants to express off-shellness of a parton in terms of additional on-shell partons.

This was achieved by Qiu. Here we will repeat the method and supplement it, by noting that these on-shell partons are in fact so-called good fields only.

As said before both EFP and Qiu have considered unpolarized DIS at first subleading order in $1/Q$, which is order $1/Q^2$. The new ingredient due to Qiu [36] is basically the splitting of the different terms in EFP's factorization into parts which contribute at a specific order in $1/Q$. The sets of diagrams of a particular order in $1/Q$ constitute gauge invariant sets. To illustrate this we will apply Qiu's factorization method to *polarized* DIS at subleading order in $1/Q$.

We start by noting that the first term in Eq. (2.46) contributes at order $(1/Q)^0$ and at higher powers, whereas the latter term contributes at order $(1/Q)^1$ and higher. So we want to separate the order $1/Q$ contribution from the first term in Eq. (2.46) in a diagrammatic way. Moreover, we will introduce projectors to make Qiu's procedure completely algorithmic.

Let us illustrate how all this can be done; the result can be found in Eq. (2.67) and further. The Dirac trace in the first term in Eq. (2.46) can be Fierz decomposed according to

$$\mathrm{Tr}\left[S^{\mu\nu}(x)\Phi(x)\right] = \frac{1}{4}\mathrm{Tr}\left[S^{\mu\nu}(x)\gamma_\rho\right]\mathrm{Tr}\left[\gamma^\rho\Phi(x)\right] + \frac{1}{4}\mathrm{Tr}\left[S^{\mu\nu}(x)\gamma_5\gamma_\rho\right]\mathrm{Tr}\left[\gamma^\rho\gamma_5\Phi(x)\right], \quad (2.47)$$

where we used the fact that $m = 0$, such that chirality is conserved and the Fierz decomposition is restricted. The vector term contributes to the unpolarized scattering only, so we discard it, because it will be analogous to the polarized case. We make a Sudakov decomposition of the axial-vector projection of the soft part,

$$\mathrm{Tr}\left[\gamma^\rho\gamma_5\Phi(x)\right] = p^\rho\mathrm{Tr}\left[\not{n}\,\gamma_5\Phi(x)\right] + \mathrm{Tr}\left[\gamma_T^\rho\gamma_5\Phi(x)\right] + n^\rho\mathrm{Tr}\left[\not{p}\,\gamma_5\Phi(x)\right], \quad (2.48)$$

where $\gamma_T^\rho = g_T^{\rho\sigma}\gamma_\sigma$. The only dimensionful quantity in the hard part is Q, whereas the soft parts are assumed not to contain large scales. This, in combination with the fact that p has dimension 1, and n dimension -1, leads to the observation that the first term is the leading one, while the second and third are $1/Q$ and $1/Q^2$ suppressed, respectively.

We will rewrite the different terms in Eq. (2.48) and for this we first investigate the quantity $\mathrm{Tr}\left[\Phi(x)\not{p}\right]$. One can write:

$$\Phi(k)\not{p} = \Phi(k)\frac{\not{n}\not{p}+\not{p}\not{n}}{2}\not{p} = -\Phi(k)\left((\not{k} - x\not{p}) - \not{k}\right)\frac{\not{n}}{2x}\not{p}. \quad (2.49)$$

If we define $\hat{k} = k - k^2\,n/(2x)$, which is the on-shell part of the momentum k, i.e., $\hat{k}^2 = 0$, then the term $\not{n}/(2x)$ is the off-shell part of the propagator and is referred to as the special propagator or contact term (since it does not propagate). With help of the classical e.o.m. (cf. Appendix 2.A) we rewrite:

$$\Phi(k)\not{k} = -\int d^4k_1 d^4k_2\,\Phi_A^\alpha(k_1,k_2)\gamma_\alpha\delta^4(k-k_1), \quad (2.50)$$

such that Eq. (2.49) becomes

$$\Phi(k)\not{p} = -\Phi(k)k^\beta\omega^\alpha{}_\beta\gamma_\alpha\frac{\not{n}}{2x}\not{p} - \int d^4k_1 d^4k_2\,\Phi_A^\alpha(k_1,k_2)\gamma_\alpha\frac{\not{n}}{2x}\not{p}\delta^4(k-k_1), \quad (2.51)$$

hence

$$
\begin{aligned}
\Phi(x)\not{p} &= \int d^4k\, \delta(x - k\cdot n)\Phi(k)\not{p} \\
&= -\int dx_2\, \left(\Phi_A^\beta(x,x_2) + \Phi_k^\beta(x,x_2)\right) \omega^\alpha{}_\beta \gamma_\alpha \frac{\not{n}}{2x}\not{p} \\
&= -\int dx_2\, \Phi_D^\beta(x,x_2)\omega^\alpha{}_\beta \gamma_\alpha \frac{\not{n}}{2x}\not{p}.
\end{aligned}
\tag{2.52}
$$

So the $\not{p}$ projection of the two-parton amplitude $\Phi(x)$ is expressed in terms of the three-parton amplitude $\Phi_D^\beta(x,x_2)$. Actually, the $\not{p}$ projection yields a $1/Q^2$ contribution and corresponds to a four-parton amplitude, which can be achieved by letting the $\not{p}$ act on the other side of Φ_D^β as well, yielding:

$$
\mathrm{Tr}\left[\Phi(x)\not{p}\right] = \int dx_2 dx_3\, \mathrm{Tr}\left[\frac{\not{n}}{2x}\gamma_\beta \omega^\beta{}_\rho \Phi_{DD}^{\rho\sigma}(x,x_2,x_3)\omega^\alpha{}_\sigma \gamma_\alpha \frac{\not{n}}{2x}\not{p}\right],
\tag{2.53}
$$

where $\Phi_{DD}^{\rho\sigma}(x,x_2,x_3)$ stands for the amplitude with two covariant derivatives instead of one, an amplitude which in fact we do not take into account, because we will retain only terms of at most order $1/Q$.

Obviously, the above holds also if one replaces $\not{p}$ by $\not{p}\gamma_5$. The reason we gave the example for the $\not{p}$ projection is that it can be used analogously for the $\gamma_T^\rho\gamma_5$ projection as well, by noting that:

$$
\begin{aligned}
\mathrm{Tr}\left[\Phi(x)\gamma_T^\rho\gamma_5\right] &= \mathrm{Tr}\left[\Phi(x)\left(\frac{\not{p}\gamma_T^\rho\gamma_5\not{n}}{2} + \frac{\not{n}\gamma_T^\rho\gamma_5\not{p}}{2}\right)\right] \\
&= -\int dx_2\, \mathrm{Tr}\left[\Phi_D^\beta(x,x_2)\omega^\alpha{}_\beta\gamma_\alpha\frac{\not{n}}{2x}\left(\frac{\not{p}\gamma_T^\rho\gamma_5\not{n}}{2}\right) + \frac{\not{n}}{2x}\gamma_\alpha\omega^\alpha{}_\beta\Phi_D^\beta(x,x_2)\left(\frac{\not{n}\gamma_T^\rho\gamma_5\not{p}}{2}\right)\right] \\
&= -\int dx_2\, \mathrm{Tr}\left[\left(\Phi_D^\beta(x,x_2)\omega^\alpha{}_\beta\gamma_\alpha\frac{\not{n}}{2x} + \frac{\not{n}}{2x}\gamma_\alpha\omega^\alpha{}_\beta\Phi_D^\beta(x,x_2)\right)\gamma_T^\rho\gamma_5\right].
\end{aligned}
\tag{2.54}
$$

Thus we have rewritten the $\gamma_T^\rho\gamma_5$ projection of the two-parton amplitude $\Phi(x)$ in terms of the sum of two three-parton amplitudes. The parts containing the special propagator we are going to include in the hard scattering part.

So we arrive at the following result (from Eq. (2.48) and Eq. (2.47)):

$$
\begin{aligned}
\int dx\, \mathrm{Tr}\left[S^{\mu\nu}(x)\Phi(x)\right] &= \int dx\, \mathrm{Tr}\left[S^{\mu\nu}(x)\Phi(x)\right]\Big|_{LO} - \int dx dx_2\, \mathrm{Tr}\left[\omega^\alpha{}_\beta\gamma_\alpha\frac{\not{n}}{2x}S^{\mu\nu}(x)\right. \\
&\left.\times\Phi_D^\beta(x,x_2) + S^{\mu\nu}(x)\frac{\not{n}}{2x}\gamma_\alpha\omega^\alpha{}_\beta\Phi_D^\beta(x,x_2)\right]\Big|_{LO} + \mathcal{O}(1/Q^2),
\end{aligned}
\tag{2.55}
$$

where the subscript LO means one only calculates the quantity to leading order in $1/Q$ (this will be made explicit with help of projectors below). The final result for $T^{\mu\nu}$ (Eq. (2.46)) becomes:

$$T^{\mu\nu} = \int dx \,\mathrm{Tr}\left[S^{\mu\nu}(x)\Phi(x)\right]\Big|_{LO} + \int dx dx_2 \,\mathrm{Tr}\left[\hat{H}^{\mu\nu}_{\alpha}(x,x_2)\omega^{\alpha}{}_{\beta}\Phi^{\beta}_{D}(x,x_2)\right]\Big|_{LO} + \mathcal{O}(1/Q^2), \tag{2.56}$$

where the first (second) term contributes at order 1 ($1/Q$) only and the new hard scattering part $\hat{H}^{\mu\nu}_{\alpha}$ is given by

$$\hat{H}^{\mu\nu}_{\alpha}(x,x_2) \equiv S^{\mu\nu}_{\alpha}(x,x_2) + i\gamma_\alpha \frac{i\not{n}}{2x} S^{\mu\nu}(x) + S^{\mu\nu}(x_2)\frac{i\not{n}}{2x_2} i\gamma_\alpha. \tag{2.57}$$

2.4.3 Projection onto good fields

As said before, one should only calculate the separate quantities in Eq. (2.56) to leading order in $1/Q$. One would like to use projectors for this purpose however. We will introduce such projectors before undoing the Fierz decomposition Eq. (2.47).

Consider the leading $\not{n}\gamma_5$ projection of $\Phi(x)$ first. Introducing the projectors

$$P_+ = \not{p}\not{n}/2, \tag{2.58}$$
$$P_- = \not{n}\not{p}/2, \tag{2.59}$$

which project onto 'good' and 'bad' quark fields, respectively [37], and using the relations

$$\not{n} = P_-\not{n} = \not{n}P_+, \tag{2.60}$$

it can be diagrammatically represented as in Fig. 2.5. Consider next the trace of $\Phi(x)$ with

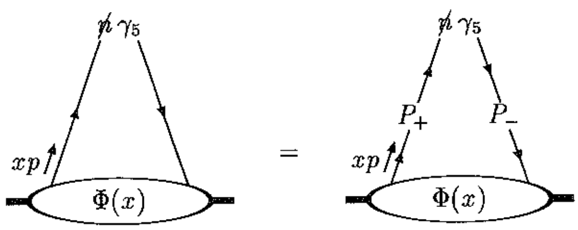


Figure 2.5: The leading axial-vector projection of $\Phi(x)$.

$\gamma_T\gamma_5$. Due to the special propagator terms and the ω projectors appearing in this case, we will use

$$\omega^{\alpha}{}_{\beta}\gamma_\alpha\not{n} = P_-\, g_T{}^{\alpha}{}_{\beta}\gamma_\alpha\not{n}, \tag{2.61}$$
$$\not{n}\gamma_\alpha\omega^{\alpha}{}_{\beta} = \not{n}\gamma_\alpha g_T{}^{\alpha}{}_{\beta}P_+, \tag{2.62}$$

along with Eq. (2.60), to project out the good quark and good (i.e., transverse) gluon fields. Putting the pieces together, we can write the trace diagrammatically as a sum of two gluon

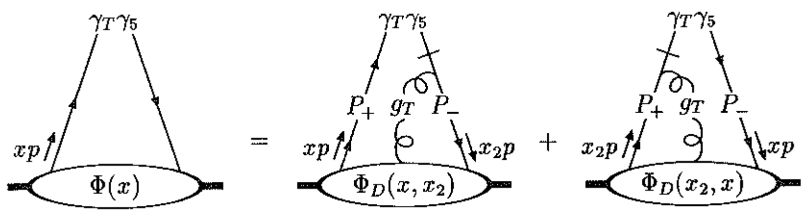
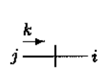


Figure 2.6: The subleading axial-vector projection of $\Phi(x)$.

diagrams. They are depicted in Fig. 2.6. We have used the *special propagator* [37, 38, 36], denoted by a slashed fermion propagator,



$$= \frac{i\not n_{ij}}{2k \cdot n}. \tag{2.63}$$

The essence of the above derivation is that an $\not n$ pulls out a good quark (or antiquark) from the soft part, whereas a $\not p$ pulls out a special propagator, a good quark, along with a good gluon.

In order to undo the Fierz decomposition, Eq. (2.47), we use the following relations:

$$P_- p^\rho \not n P_+ = P_- \gamma^\rho P_+, \tag{2.64}$$
$$P_- \not n \gamma_T^\rho P_+ = P_- \not n \gamma^\rho P_+, \tag{2.65}$$
$$P_- \gamma_T^\rho \not n P_+ = P_- \gamma^\rho \not n P_+. \tag{2.66}$$

Returning to Eq. (2.46), the second term, from dimensional arguments one can again infer that only good fields contribute at leading order. So we may include projectors in between the hard and soft parts.

In summary: the leading and subleading forward scattering amplitudes become

$$T^{\mu\nu}_{\mathrm{twist-2}} = \int dx \, \mathrm{Tr}\left[P_- S^{\mu\nu}(xp) P_+ \Phi(x)\right], \tag{2.67}$$
$$T^{\mu\nu}_{\mathrm{twist-3}} = \int dx_1 \, dx_2 \, \mathrm{Tr}\left[P_- H^{\mu\nu}_\alpha(x_1 p, x_2 p) P_+ g_T{}^\alpha{}_\beta \Phi_D^\beta(x_1, x_2)\right] \tag{2.68}$$

The projectors $P_\pm$ are given in Eqs. (2.58) and (2.59), the new hard scattering part $\hat{H}^{\mu\nu}_\alpha$ in Eq. (2.57). Due to the projectors (not used by Qiu) the above terms are of a specific order in $1/Q$. That is, 'twist-t' contributes only at order Q^{2-t}. Also, t always equals the number of partons connecting the hard and soft parts.

To arrive at the DIS hadronic tensor, $W^{\mu\nu} = \mathrm{Disc}[T^{\mu\nu}]/(4M\pi i)$, one has to cut the hard diagrams. The uncrossed twist-three cut diagrams are depicted in Fig. 2.7 (cf. Eq. (2.63)). Diagrams (c) and (d) are due to Qiu.

So, if one uses the *special propagator* in the above way, one can factorize such that a complete separation of the hard parts between different orders in $1/Q$ can be obtained. Hence, the electromagnetic gauge invariance of these hard parts is manifest. Moreover, only

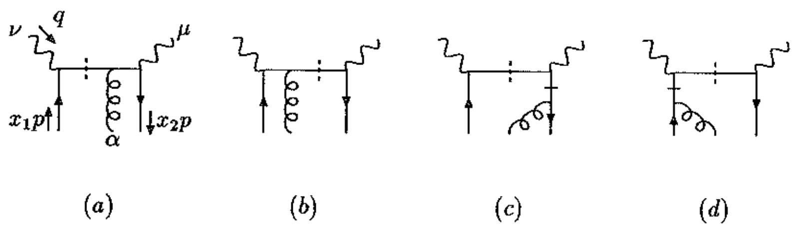


Figure 2.7: Uncrossed twist-three gluonic diagrams.

matrix elements with a fixed number of partons contribute to a particular order in $1/Q$. This fact also enables a clear parton model picture of the higher twist terms. These partons are described by good fields only, as can be seen from our inclusion of good and bad field projectors.

The algorithm can be summarized as follows. To obtain the massless contributions to the order Q^{2-t} DIS hadron tensor, one has to (1) write down all possible forward 2-photon t-parton cut diagrams, where partons can be quarks, antiquarks or gluons; (2) replace all propagators that do not lie between the photon vertices by special propagators; (3) project out the good fields and couple them with t-parton soft matrix elements.

2.4.4 Color gauge invariance

In the above investigations we have chosen the lightcone gauge $n \cdot A = 0$ (see App. 7.A), which does not affect the results of electromagnetic gauge invariance, but one also likes to have color gauge invariance. Since we deal with nonlocal operators, one can use the following path-ordered exponential, also called link operator,

$$\mathcal{L}(0,x) = \mathcal{P}\exp\left(-ig\int_0^x dy^\mu A_\mu(y)\right). \tag{2.69}$$

In the gauge $n{\cdot}A = 0$, for instance Φ_D^α can be written as:

$$\begin{aligned}\Phi^\alpha_{Dij}(x_1,x_2) \equiv \int \frac{d\lambda}{2\pi}\frac{d\eta}{2\pi} e^{i\lambda x_1} e^{i\eta(x_2-x_1)} \\ \times \langle P,S|T\,\overline{\psi}_j(0)\mathcal{L}(0,\eta n)iD^\alpha(\eta n)\mathcal{L}(\eta n,\lambda n)\psi_i(\lambda n)|P,S\rangle,\end{aligned} \tag{2.70}$$

where both paths are chosen along the n-direction entirely, such that the link operators are unity. But also without choosing this gauge the above is now a color gauge invariant matrix element.

However, introducing link operators in this *ad hoc* way, does not mean that these are in fact the correlation functions appearing in the factorization in case another or no gauge is chosen. By using a Ward identity one can show that the link in Eq. (2.69) indeed is the appropriate one [39, 12]. The Ward identity to consider at the order we investigate is[4]

$$p^\alpha\, S_\alpha^{\mu\nu}(x_1, x_2) = \frac{S^{\mu\nu}(x_2) - S^{\mu\nu}(x_1)}{x_2 - x_1}, \tag{2.71}$$

of which Eq. (2.41) is the infinitesimal form. In a general gauge one writes $A^\alpha = p^\alpha\,(n \cdot A) + \omega^\alpha{}_\beta\, A^\beta$, such that the term proportional to p^α will yield, due to the above Ward identity,

$$\int dx_1 dx_2 \,\mathrm{Tr}\left[S_\alpha^{\mu\nu}(x_1,x_2)\, p^\alpha\, n_\beta \Phi_A^\beta(x_1,x_2)\right] = \\ \int dx \,\mathrm{Tr}\left[S^{\mu\nu}(x) \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P|T\,\overline{\psi}(0)\left(-ig\int_0^\lambda d\eta\,(n\cdot A)(\eta)\right)\psi(\lambda)|P\rangle\right]. \tag{2.72}$$

Combined with matrix elements containing arbitrary numbers of $(n \cdot A)$–gluons (all contributing at the same order, which in this case is order 1), this will exponentiate into the path-ordered link operator in $\Phi(x)$, with a straight path along the n-direction. Hence, color gauge invariance of the factorized hadron tensor is manifest and a lightcone gauge can be chosen without problems.

Another observation is that in the lightcone gauge the gluon propagator also has a non-propagating part. One can define a gluon special propagator in the same way as for the fermions. The algorithm as given before is unchanged, if special propagator diagrams for both fermions and gluons are taken into account. This will become relevant beyond twist four [36].

2.5 Twist three polarized DIS with massive quarks

In this section we will extend the above sketched factorization procedure of the deep inelastic scattering hadron tensor, developed by Qiu, to include nonzero quark masses.

The masses of the light quarks are small as compared to the typical hadronic mass scales. Therefore, in most calculations of DIS structure functions they are neglected. In some cases, however, this is not allowed. A notorious (but academic) example is g_2 of a free quark target at tree level [40]. In that case the quark mass term exactly cancels the regular contribution, yielding $g_2 = 0$. In general, if the ratio m/M is not negligible, one is forced to keep the quark mass contributions.

Both EFP and Qiu neglect quark masses. Our goal is to include them in such a fashion that the electromagnetic gauge invariance remains manifest. Since the quark masses arise from two sources, namely from the hard parts and from the use of the equations of motion in the soft parts, this is a nontrivial question.

The basics of the method are as follows: in addition to the collinear expansion, employed by EFP, we include a quark mass expansion. Qiu's factorization can be extended as well.

[4] The prescription for the pole is irrelevant, but if one uses this Ward identity for the discontinuities directly, then one has to take care that conjugated diagrams get opposite prescriptions.

In order to preserve the parton picture we employ an auxiliary parton, named *spurion* [41], which couples only to the fermions. The spurions will generate the mass contributions (they are essentially mass insertions). One maintains the advantages of combining EFP and Qiu's techniques, namely, manifest electromagnetic gauge invariance of the hard scattering parts and a clear parton picture at all times, also beyond leading order, i.e., each order of power suppression means taking into account correlation functions with one parton more, where only the *good* fields [37] contribute. Another advantage of the method is that it allows to separately calculate the quark mass contributions.

2.5.1 Factorization

Again the starting point is the familiar diagrammatic expansion of the forward scattering amplitude $T^{\mu\nu}$:

$$T^{\mu\nu} = \int d^4k\, \mathrm{Tr}\,[S^{\mu\nu}(k;m)\Phi(k)] + \int d^4k_1 d^4k_2\, \mathrm{Tr}\,[S^{\mu\nu}_{\alpha}(k_1,k_2;m)\Phi^{\alpha}_{A}(k_1,k_2)] + \ldots, \tag{2.73}$$

keeping only the terms contributing up to order $1/Q$. Again we will not consider pQCD corrections. The soft parts are defined as in the previous section. The hard parts have now an explicit dependence on the quark mass m.

The next step is performing a collinear and mass expansion of the hard parts (keeping only relevant terms):

$$S^{\mu\nu}(k;m) = S^{\mu\nu}(xp;0) + (k-xp)^{\alpha}\left.\frac{\partial S^{\mu\nu}(k;m)}{\partial k^{\alpha}}\right|_{\substack{k=xp\\ m=0}} + m\left.\frac{\partial S^{\mu\nu}(k;m)}{\partial m}\right|_{\substack{k=xp\\ m=0}} + \ldots, \tag{2.74}$$

$$S^{\mu\nu}_{\alpha}(k_1,k_2;m) = S^{\mu\nu}_{\alpha}(x_1p,x_2p;0) + \ldots. \tag{2.75}$$

We use the following Ward identities to rewrite the partial derivatives:

$$\frac{\partial S^{\mu\nu}(k;m)}{\partial k^{\alpha}} = S^{\mu\nu}_{\alpha}(k,k;m), \tag{2.76}$$

$$\frac{\partial S^{\mu\nu}(k;m)}{\partial m} = S^{\mu\nu}_{\mathrm{spur}}(k,k;m), \tag{2.77}$$

where the r.h.s. of Eq. (2.77) follows from the insertion of a zero-momentum (scalar) spurion [41] (denoted by a dashed line) which couples to the fermions through the vertex $-i\mathbf{1}_{ij}$ (see Fig. 2.8). Inserting the Ward identities, one arrives at

$$\begin{aligned} T^{\mu\nu} &= \int dx\, \mathrm{Tr}\,[S^{\mu\nu}(xp;0)\Phi(x)] + \int dx_1 dx_2\, \mathrm{Tr}\left[S^{\mu\nu}_{\alpha}(x_1p,x_2p;0)\omega^{\alpha}{}_{\beta}\Phi^{\beta}_{D}(x_1,x_2)\right] \\ &\quad + \int dx_1 dx_2\, \mathrm{Tr}\left[S^{\mu\nu}_{\mathrm{spur}}(x_1p,x_2p;0)\Phi_m(x_1,x_2)\right] + \ldots \end{aligned} \tag{2.78}$$

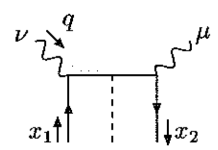
$$S^{\mu\nu}_{\rm spur}(x_1,x_2) =$$ 

Figure 2.8: Tree-level spurion hard part.

The soft parts are defined as in Eqs. (2.40) and (2.45) and

$$\begin{aligned}\Phi_{mij}(x_1,x_2) &= \int \frac{d\lambda}{2\pi}\frac{d\eta}{2\pi} e^{i\lambda x_1} e^{i\eta(x_2-x_1)} \langle P,S|T\,\overline{\psi}_j(0) m\psi_i(\lambda n)|P,S\rangle \\ &= m\,\delta(x_2-x_1)\,\Phi_{ij}(x_1). \end{aligned} \tag{2.79}$$

The first term in Eq. (2.78) contributes to order $(1/Q)^0$ and at higher powers, whereas the latter terms contribute to order $(1/Q)^1$ and higher. As before we are going to split the different terms into parts which contribute at a specific order in $1/Q$. The sets of diagrams of a particular order in $1/Q$, which constitute gauge invariant sets, can only be identified by splitting off the mass contributions.

The Dirac trace in the first term in Eq. (2.78) is Fierz decomposed according to

$$\begin{aligned}\mathrm{Tr}\left[S^{\mu\nu}(xp;0)\Phi(x)\right] = \frac{1}{4}\mathrm{Tr}\left[S^{\mu\nu}(xp;0)\gamma_\rho\right]\mathrm{Tr}\left[\gamma^\rho\Phi(x)\right] \\ +\frac{1}{4}\mathrm{Tr}\left[S^{\mu\nu}(xp;0)\gamma_5\gamma_\rho\right]\mathrm{Tr}\left[\gamma^\rho\gamma_5\Phi(x)\right], \end{aligned} \tag{2.80}$$

where we used the fact that in the hard part $m=0$, such that chirality is conserved. Again we make a Sudakov decomposition of the axial-vector projection of the soft part, relevant for polarized scattering,

$$\mathrm{Tr}\left[\gamma^\rho\gamma_5\Phi(x)\right] = p^\rho \mathrm{Tr}\left[\not n\gamma_5\Phi(x)\right] + \mathrm{Tr}\left[\gamma_T^\rho\gamma_5\Phi(x)\right] + n^\rho\mathrm{Tr}\left[\not p\gamma_5\Phi(x)\right]. \tag{2.81}$$

From dimensional arguments it was noted that the first term is the leading one, while the second and third are $1/Q$ and $1/Q^2$ suppressed, respectively, so we discard the last one.

For the $\not p$ projection we now use the following relation, which follow from the massive equations of motion $i\not D\psi = m\psi$,

$$\Phi(x)\not p = \int dx_2 \left[\Phi_D^\beta(x,x_2)\omega^\alpha{}_\beta(i\gamma_\alpha) + \Phi_m(x,x_2)(-i\mathbf{1})\right]\frac{i\not n}{2x}\not p, \tag{2.82}$$

$$\not p\,\Phi(x) = \not p\frac{i\not n}{2x}\int dx_2 \left[(i\gamma_\alpha)\omega^\alpha{}_\beta\Phi_D^\beta(x_2,x) + (-i\mathbf{1})\Phi_m(x_2,x)\right]. \tag{2.83}$$

We again introduce projectors $P_\pm$, in order to undo the Fierz decomposition, Eq. (2.80).

Returning to the second and third terms in Eq. (2.78), from dimensional arguments one can infer that only good fields contribute at leading order. So we may include projectors in between the hard and soft parts. The modified hard gluonic and spurionic parts read

$$H_\alpha^{\mu\nu}(x_1p, x_2p; 0) = S_\alpha^{\mu\nu}(x_1p, x_2p; 0) + i\gamma_\alpha \frac{i\not{n}}{2x_1} S^{\mu\nu}(x_1p; 0) + S^{\mu\nu}(x_2p; 0)\frac{i\not{n}}{2x_2} i\gamma_\alpha, \quad (2.84)$$

$$H_{\text{spur}}^{\mu\nu}(x_1p, x_2p; 0) = S_{\text{spur}}^{\mu\nu}(x_1p, x_2p; 0) + (-i\mathbf{1})\frac{i\not{n}}{2x_1} S^{\mu\nu}(x_1p; 0) + S^{\mu\nu}(x_2p; 0)\frac{i\not{n}}{2x_2}(-i\mathbf{1}). \quad (2.85)$$

The leading and subleading forward scattering amplitudes become

$$T_{\text{twist}-2}^{\mu\nu} = \int dx \, \text{Tr}\left[P_- S^{\mu\nu}(xp; 0) P_+ \Phi(x)\right], \quad (2.86)$$

$$\begin{aligned} T_{\text{twist}-3}^{\mu\nu} &= \int dx_1\, dx_2 \, \text{Tr}\left[P_- H_\alpha^{\mu\nu}(x_1p, x_2p; 0) P_+ g_T{}^\alpha{}_\beta \Phi_D^\beta(x_1, x_2)\right] \\ &\quad + \int dx_1\, dx_2 \, \text{Tr}\left[P_- H_{\text{spur}}^{\mu\nu}(x_1p, x_2p; 0) P_+ \Phi_m(x_1, x_2)\right], \quad (2.87) \end{aligned}$$

which are of a specific order in $1/Q$. The uncrossed twist-three cut diagrams are depicted in Fig. 2.7 and Fig. 2.9. Diagrams (e)–(h) generate the mass terms.

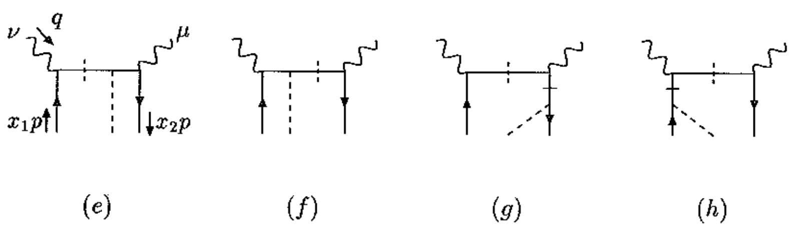


Figure 2.9: Uncrossed twist-three spurionic diagrams.

In summary: to obtain the massless contributions to the order Q^{2-t} DIS hadron tensor, one applies the algorithm as summarized in the previous section. The quark mass terms follow in the same way, except that one (or more) of the gluon legs that connect hard and soft parts is replaced by a spurion. In the hard parts the partons are kept massless. By the identification of the electromagnetic gauge invariant sets of hard scattering diagrams in the massive case, we have now completed the unraveling of higher twist contributions in the diagrammatic approach.

2.5.2 Application to g_1 and g_2

Explicitly, one finds for the (projected) discontinuities (displaying only the antisymmetric parts in $\mu \leftrightarrow \nu$ relevant for polarized scattering):

$$\text{Disc}\left[P_- S^{\mu\nu}(xp; 0) P_+\right] = \pi i e^2 \delta(x - x_B)\, i\epsilon_T{}^{\mu\nu} \not{n}\, \gamma_5, \quad (2.88)$$

$$\text{Disc}\left[P_- H_\alpha^{\mu\nu}(x_1 p, x_2 p; 0) P_+ g_T{}^\alpha{}_\beta\right] = \frac{\pi i e^2}{Q^2} i\epsilon^{\mu\nu\rho\sigma} q_\rho \Big\{ i\epsilon_{T\sigma\beta} \not{n} \left[\delta(x_1 - x_B) - \delta(x_2 - x_B)\right] + g_{T\sigma\beta} \not{n}\, \gamma_5 \left[\delta(x_1 - x_B) + \delta(x_2 - x_B)\right]\Big\}, \tag{2.89}$$

$$\text{Disc}\left[P_- H_{\text{spur}}^{\mu\nu}(x_1 p, x_2 p; 0) P_+\right] = \frac{\pi i e^2}{Q^2} i\epsilon^{\mu\nu\rho\sigma} q_\rho \not{n}\, \gamma_{T\sigma} \gamma_5 \left[\delta(x_1 - x_B) + \delta(x_2 - x_B)\right]. \tag{2.90}$$

where $\epsilon_T^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} p_\rho n_\sigma$. The calculation of these discontinuities is especially simple since the partons are massless, the effects of nonzero m being already taken into account by including Eq. (2.90). At this point the electromagnetic gauge invariance is manifest. That is, contraction of the modified hard parts with q gives zero identically. Compare this with the much more elaborate demonstration of electromagnetic gauge invariance in Ref. [23, App. C], of which it is not at all clear how it extends to even higher twist.

For the soft parts we use the following parametrizations [10, 11] (cf. next chapter)

$$\begin{aligned}
\Phi(x) &= \frac{1}{2} g_1(x)\, \lambda \gamma_5 \not{p} + \frac{1}{2} h_1(x)\, \gamma_5 \not{S}_T \not{p} + \frac{M}{2} g_T(x)\, \gamma_5 \not{S}_T + \ldots, & (2.91)\\
\Phi_D^\alpha(x_1, x_2) &= \frac{M}{2} G(x_1, x_2)\, i\epsilon_T^{\alpha\beta} S_{T\beta} \not{p} + \frac{M}{2} \tilde{G}(x_1, x_2)\, S_T^\alpha \gamma_5 \not{p} + \ldots, & (2.92)\\
\Phi_m(x_1, x_2) &= \frac{M}{2} H_1(x_1, x_2)\, \gamma_5 \not{S}_T \not{p} + \ldots, & (2.93)
\end{aligned}$$

where we listed only those Dirac structures that are relevant here. The functions of x are called distribution functions and will be discussed in much detail in the next chapter. The function $g_1(x)$ for instance is the distribution of longitudinally polarized quarks inside a longitudinally polarized hadron. It is the helicity distribution analogue of the unpolarized function $q(x)$ encountered in Eq. (2.22), which is called $f_1^a(x)$ in the rest of this thesis.

The functions $\tilde{G}$ and H_1 are symmetric, G is antisymmetric under interchange of the two arguments. The functions are not all independent. One has the following relations

$$\begin{aligned}
2x_1 g_T(x_1) &= \int dx_2 \Big[G(x_1, x_2) - G(x_2, x_1) + \tilde{G}(x_1, x_2) + \tilde{G}(x_2, x_1) \\
&\qquad + H_1(x_1, x_2) + H_1(x_2, x_1) \Big], & (2.94)\\
H_1(x_1, x_2) &= \frac{m}{M} \delta(x_2 - x_1) h_1(x_1), & (2.95)
\end{aligned}$$

where the first equation follows from the equations of motion, the second from Eq. (2.79). Contracting the hard parts, Eqs. (2.88)-(2.90), with the soft parts, Eqs. (2.91)-(2.93), gives for the antisymmetric hadron tensor

$$\begin{aligned}
W_{\text{twist}-2}^{A\mu\nu} &= \frac{e^2\, \lambda}{2} i\epsilon_T^{\mu\nu} g_1(x_B), & (2.96)\\
W_{\text{twist}-3}^{A\mu\nu} &= \frac{e^2\, M}{2\, P \cdot q} i\epsilon^{\mu\nu\rho\sigma} q_\rho S_{T\sigma} g_T(x_B), & (2.97)
\end{aligned}$$

where we used Eq. (2.94) to eliminate the two-argument functions. Comparing with the expression for the hadron tensor in terms of structure functions Eq. (2.4), we identify the tree-level results for the structure functions (l.h.s.) in terms of distribution functions (r.h.s.)

$$g_1(x_B, Q^2) = \frac{1}{2} \sum_{a,\bar{a}} e_a^2 g_1^a(x_B), \tag{2.98}$$

$$g_1(x_B, Q^2) + g_2(x_B, Q^2) = \frac{1}{2} \sum_{a,\bar{a}} e_a^2 g_T^a(x_B), \tag{2.99}$$

where we reinstalled the flavor sum and added the crossed (antiquark) diagrams, such that the sum is over antiflavors $\bar{a}$ also. These results are similar to Eq. (2.22). These results are in agreement with the standard EFP-type of calculations [8, 23]. It is the way they are derived that makes the difference.

For completeness, we give here the expression for the DIS cross section to the order we are considering. The incoming lepton is longitudinally polarized (λ_e) and the incoming hadron can be longitudinally as well as transversely polarized. The cross section is (cf. [42])

$$\frac{d\sigma(\vec{\ell}\vec{H} \to \ell' X)}{dx_B\, dy} = \frac{4\pi\alpha^2\, s}{Q^4} \sum_{a,\bar{a}} e_a^2 \left\{ \left(\frac{y^2}{2} + 1 - y\right) x_B f_1^a(x_B) \right.$$
$$\left. + y\left(1 - \frac{y}{2}\right) \lambda_e\, \lambda\, x_B g_1^a(x_B) - 2y\sqrt{1-y}\lambda_e\, |\boldsymbol{S}_T|\, \cos(\phi_s)\, \frac{M}{Q}\, x_B^2\, g_T^a(x_B) \right\}. \tag{2.100}$$

The meaning of the variable y and the azimuthal angle ϕ_s of the transverse spin $\boldsymbol{S}_T$ with respect to some scattering plane will be discussed in chapter 4. Here we just like to point out that the subleading twist term proportional to g_T^a appears as an azimuthal spin asymmetry.

We have also applied the above discussed method to one-hadron production in e^+e^--annihilation, which essentially only differs from the above by the fact that the photon momentum is now timelike and the distribution functions have to be replaced by fragmentation functions (often indicated by a hat, e.g., $\hat{g}_T^a$, but we will use the notation G_T^a later on). We can again express the subleading structure function (denoted by $\hat{g}_1 + \hat{g}_2$) in terms of $\hat{g}_T^a$ only, that is, without explicit mass terms. This result differs from Eq. (38) of Ref. [43] (in which Qiu's factorization is also applied) by quark mass terms (accompanied by the fragmentation function $\hat{h}_1^a$); our claim is that these terms should not be present.

To summarize: we have extended the formalism by Qiu, leading to a factorization of hard and soft scattering parts in a manifestly electromagnetic gauge invariant way, to include nonzero quark masses. We have illustrated the method for polarized DIS at leading and first subleading order.

2.6 Twist four unpolarized DIS with massive quarks

The method described in the previous section has been demonstrated for the well-known case of polarized DIS up to subleading order in $1/Q$. In this section we will apply it to the subleading order in unpolarized DIS, that is order $1/Q^2$. EFP [4] and Qiu [36] have investigated this situation for the massless case. We will use the spurion method to calculate separately the quark mass corrections of order $1/Q^2$.

In the notation of EFP the unpolarized DIS hadron tensor is given by

$$W^{\mu\nu}(P,q) = e_L^{\mu\nu} F_L + e_T^{\mu\nu} F_T \equiv \frac{1}{2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) F_L - \frac{1}{2} g_T^{\mu\nu} F_T. \tag{2.101}$$

One finds for the massless quark case

$$F_L(x_B, Q^2) = \frac{8M^2}{Q^2} T_1(x_B) + \mathcal{O}(1/Q^4), \tag{2.102}$$

$$F_T(x_B, Q^2) = f_1(x_B) + \frac{2M^2}{Q^2}\Big[4T_1(x_B) - x_B \int dx_2\, dx_1 \frac{\delta(x_2 - x_B) - \delta(x_1 - x_B)}{x_2 - x_1} T_2(x_2, x_1)\Big] + \mathcal{O}(1/Q^4), \tag{2.103}$$

where

$$M^2\, T_1(x_B) = \int \frac{d\lambda}{2\pi}\, e^{i\lambda x_B} \langle P|\, T\, \overline{\psi}(0) \not{D}_T(0) \frac{\not{n}}{4} \not{D}_T(\lambda) \psi(\lambda) |P\rangle, \tag{2.104}$$

$$M^2\, T_2(x_2, x_1) = \int \frac{d\lambda}{2\pi} \frac{d\eta}{2\pi}\, e^{i\lambda x_1}\, e^{i\eta(x_2 - x_1)} \langle P|\, T\, \overline{\psi}(0) \gamma_\alpha \frac{\not{n}}{4} \gamma_\beta D_T^\beta(\eta) D_T^\alpha(\eta) \psi(\lambda) |P\rangle. \tag{2.105}$$

For the massive quark case in EFP's approach one needs the following parametrizations [11, 4] [5]:

$$\Phi(x) = \frac{1}{2} f_1(x) \not{p} + \frac{M}{2} e(x) \mathbf{1} + \frac{M^2}{2} f_3(x) \not{n}, \tag{2.106}$$

$$\Phi_D^\alpha(x_1, x_2) = \frac{M}{2} E(x_1, x_2) \gamma_T^\alpha \not{p}, \tag{2.107}$$

$$\Phi_{DD}^{\alpha\beta}(x_1, x, x_2) = \frac{M^2}{2} C_1(x_1, x, x_2) g_T^{\alpha\beta} + \frac{M^2}{2} \tilde{C}_1(x_1, x, x_2) i\epsilon_T^{\alpha\beta} \gamma_5 \not{p}, \tag{2.108}$$

where

$$\Phi_{DD}^{\alpha\beta}(x_1, x, x_2) = \int \frac{d\lambda}{2\pi} \frac{d\eta}{2\pi} \frac{d\zeta}{2\pi}\, e^{i\lambda x_1}\, e^{i\eta(x - x_1)}\, e^{i\zeta(x_2 - x)} \times \langle P|\, T\, \overline{\psi}(0)\, iD_T^\alpha(\zeta)\, iD_T^\beta(\eta)\, \psi(\lambda) |P\rangle. \tag{2.109}$$

If one defines $C_L = C_1 - \tilde{C}_1$ and $C_R = C_1 + \tilde{C}_1$, then one finds

$$M^2\, T_1(x) = -2 \int dx_1\, dx_2\, C_L(x_1, x, x_2), \tag{2.110}$$

$$M^2\, T_2(x_1, x_2) = -2 \int dx\, C_R(x_1, x, x_2). \tag{2.111}$$

[5] Unlike Ref. [11] we use f_3 instead of f_4.

For the method we are proposing one would need $\Phi_{DD}^{\alpha\beta}, \Phi_{Dm}^{\alpha}, \Phi_{mD}^{\alpha}$ and Φ_{mm}, where the latter three are derived from $\Phi_{DD}^{\alpha\beta}$ by replacing one or two of the iD_T's by $m\mathbf{1}$. Hence,

$$\Phi_{Dm}^{\alpha}(x_1, x, x_2) = m\ \delta(x - x_1)\ \Phi_D^{\alpha}(x_1, x_2), \tag{2.112}$$

$$\Phi_{mD}^{\alpha}(x_1, x, x_2) = m\ \delta(x_2 - x)\ \Phi_D^{\alpha}(x_1, x_2), \tag{2.113}$$

$$\Phi_{mm}(x_1, x, x_2) = m^2\ \delta(x - x_1)\ \delta(x_2 - x)\Phi(x_1). \tag{2.114}$$

One only needs the leading order terms (having the lowest power in M) in the parametrizations of Φ, Φ_D^{α} and $\Phi_{DD}^{\alpha\beta}$.

In both methods one will make use of the e.o.m.

$$\int dx_2 \left[E(x_1, x_2) - E(x_2, x_1)\right] = xe(x) - \frac{m}{M} f_1(x). \tag{2.115}$$

Since our method allows to calculate the quark mass terms separately and since they have not been given in the literature (to the best of our knowledge), except for second [44] and other specific moments [35] of the structure functions, we give them here. We find the following additional terms in case of quark mass corrections

$$F_L^{m\neq 0}(x_B, Q^2) = F_L(x_B, Q^2) + \frac{4mM}{Q^2} x_B e(x_B), \tag{2.116}$$

$$F_T^{m\neq 0}(x_B, Q^2) = F_T(x_B, Q^2) + \frac{2mM}{Q^2} x_B e(x_B). \tag{2.117}$$

Of course this is a rather academic result since these terms are likely to be extremely small, as an estimate of the second moment indicates [44]. The second moment of the above given additional quark mass term in F_L is (reinstalling flavor indices and charges[6])

$$\int dx\, x\, (F_L^{m\neq 0}(x) - F_L(x)) = \sum_{a,\bar{a}} e_a^2 \frac{4mM}{Q^2} \int dx\, x^2\, e^a(x) \equiv -\frac{2}{Q^2}\, C, \tag{2.118}$$

where $C = \sum_{a,\bar{a}} e_a^2 \langle P|\overline{\psi}^a(0)(D\cdot n)^2 m\psi^a(0)|P\rangle \approx -0.0008\ \mathrm{GeV}^2$ for the proton, according to the estimate in [44], indicating that the quark mass power corrections from up and down quarks are very small indeed.

Terms proportional to $m^2 f_1$ have dropped out in the end result, but this is only fictitious, since they are hidden inside T_1 and hence in F_L. Usually upon encountering integrals over some of the arguments of a distribution function, one uses the e.o.m. to find an expression in terms of functions with less arguments. For instance, in Eq. (2.115) one expresses a combination of integrals over $E(x_1, x_2)$ in terms of already defined functions $e(x_1)$ and $f_1(x_1)$. In case of the encountered integral over C_L one has defined the resulting one-argument expression as the function T_1, cf. Eq. (2.110). One would rather want to use the e.o.m. for this purpose. This will yield an expression in terms of f_1 and f_3, which together form T_1: $M^2 T_1(x_B) = -x^2 M^2 f_3(x_B) + m^2 f_1(x_B)/2$. In case of zero quark mass there would only be

[6]The charge e_a should not be confused with the distribution function $e^a(x)$ of a quark with flavor a.

a relation between f_3 and T_1. Since the encountered expression involving T_2 has no obvious replacement by use of the e.o.m., we have kept T_1 also.

Another note of warning concerns the use of massless propagators in a collinear expansion, which sometimes results in additional, nonphysical divergences. The way to prevent this artifact is to keep the masses in denominators (but take cut propagators as massless) until the end of the calculation, which does not imply a loss of the calculational advantage which massless quarks provide, since that arises from the numerators. So the quark masses in denominators only play the role of regulators, to be removed in the end. In the twist-three calculation this problem did not arise, but in this twist-four case it does. Two additional divergences cancel exactly. This can be seen as a drawback of the method, but since the solution to this problem is easily established, it does not complicate the calculation in any way.

We also like to comment on the fact that the two expansions, the collinear and the mass expansion, need not commute. In those cases the order depends on the physical situation at hand.

We have refrained from considering the four fermion matrix elements also contributing to this order, since they are straightforward to obtain and have been given in the literature, [4, pages 70–71] and [36].

2.7 Summary

We have considered the DIS process using methods (mainly) due to EFP and Qiu, which resulted in a factorization of the cross section into hard and soft parts. This factorization was established at each order in $1/Q$ separately and the resulting hard parts were manifestly electromagnetic gauge invariant. The soft parts were described by nonlocal operator matrix elements, where the operators consist of good fields only. A short conceptual comparison to the OPE was made. The polarized twist-three and unpolarized twist-four dynamical power corrections were investigated in much detail, also taking into account quark masses.

In the following chapters EFP's approach will be applied to the Drell-Yan process and to inclusive two-hadron production in electron-positron annihilation, again at subleading twist including polarization. Both are processes for which the OPE cannot be applied.

2.A Application of the equations of motion inside physical matrix elements

In Ref. [7], Politzer gave a proof of the validity of naive use of the classical equations of motion inside physical matrix elements. Here we will repeat and supplement the arguments.

Local operators of the form

$$\theta = F(\phi)\frac{d\mathcal{L}}{d\phi}, \tag{2.A1}$$

where F is an arbitrary local function of a field generically denoted as $\phi(x)$, vanish upon use of the classical equations of motion, i.e., by construction due to the Euler-Lagrange equation $d\mathcal{L}/d\phi = 0$. For example, θ could be $F(i\not{D} - m)\psi$. When θ is inserted into a general Green's function, with possibly off-shell legs, this property need not hold any longer, due to quantum effects. However, a hadronic, i.e., physical, matrix element of such an operator does indeed vanish as will be shown.

First we look at the unrenormalized situation. Let S be the source of ϕ and J be the source of θ, then the generating functional of connected Green's functions W is given by:

$$e^{iW(S,J)} = \int [\mathcal{D}\phi] \exp\left(i \int dx \left[\mathcal{L}(\phi) + S\phi + JF\frac{d\mathcal{L}}{d\phi}\right]\right). \tag{2.A2}$$

We now perform a change of variables and write everything in terms of the field $\phi' = \phi + JF$. We need only to consider the terms linear in J, because we are only interested in Green's functions with one θ, i.e., we act with $\delta/\delta iJ$ on the generating functional once and put all sources equal to zero afterwards. In terms of ϕ'

$$e^{iW(S,J)} = \int [\mathcal{D}\phi'] \left|1 - J\frac{dF}{d\phi'}\right| \exp\left(i \int dx \left[\mathcal{L}(\phi') + S\phi' - JSF\right]\right). \tag{2.A3}$$

With this we find the following relation:

$$\begin{aligned}\langle 0|T\,\theta(x)\phi(x_1)\ldots\phi(x_n)|0\rangle &= -\langle 0|T\,\frac{dF}{d\phi}(x)\phi(x_1)\ldots\phi(x_n)|0\rangle\,\delta^4(0)\\ &-\sum_i \delta^4(x-x_i)\langle 0|T\,F\left(\phi(x_i)\right)\phi(x_1)\ldots\widehat{\phi(x_i)}\ldots\phi(x_n)|0\rangle,\end{aligned} \tag{2.A4}$$

where the hat means that the field is deleted. The first term arises from differentiating the Jacobian of Eq. (2.A3) and here one must note that

$$\frac{dF(x)}{d\phi(y)} = \frac{dF}{d\phi}(y)\,\delta^4(x-y). \tag{2.A5}$$

This term will be absent after normal ordering; it is a type of tadpole (the vertex connects to itself). Thus normal ordering of θ is needed, next to ordinary renormalization of the operator, for which operator mixing can occur.

The second term on the r.h.s. of Eq. (2.A4) does survive renormalization and in general modifies the classical e.o.m. given in terms of Green's functions, however not when physical matrix elements are concerned. We start with the most simple case, namely that of a single hadron state $|P\rangle$, say a proton. Let $\mathcal{H}(x)$ be a (normalized) local interpolating field for this hadron state: $\langle P|\mathcal{H}(x)|0\rangle = 1$. The LSZ reduction formula states (cf. [45, pages 205–207]) that

$$\begin{aligned}\langle P|\theta(x)|P'\rangle &= \int d^4z d^4z'\, e^{iPz}e^{iPz'}\left(P^2 - m_P^2\right)\left(P'^2 - m_P^2\right)\langle 0|T\,\mathcal{H}^*(z)\,\theta(x)\,\mathcal{H}(z')|0\rangle\\ &+\langle P|P'\rangle\langle 0|\theta(x)|0\rangle.\end{aligned} \tag{2.A6}$$

The disconnected, vacuum term vanishes as can be seen from Eq. (2.A4). If we use Eq. (2.A4) for the Green's function of the first term, we get a function of the form $\delta^4(x-z)\,f(z,z')$ and $\delta^4(x-z')\,f(z,z')$. The function f is a two-point function, albeit not a proton-proton two-point function, but even if it was, the singular structure would be such that there is at most a single pole at the proton mass and a cut from the pion threshold beginning at $(m_P + m_\pi)$ onward. So one can never have a double pole at the proton mass needed to survive the integration, cf. also [45, page 212]. Hence, we conclude: $\langle P|\theta(x)|P'\rangle = 0$.

The only remaining issue is the renormalization of θ. Since only operators which also vanish due to the (classical) e.o.m. can mix with θ under renormalization, the conclusion cannot be affected. This was the simplest case. We are actually interested in the general quantity $\langle P_1 \ldots P_n|\theta(x)\phi(x_1)\ldots\phi(x_n)|P'_1 \ldots P'_n\rangle$. In principle, it is possible to relate this quantity to a series of quantities $\langle P|\theta_n(x)|P'\rangle$ by Taylor expansion of the elementary and interpolating fields and include the local products of these fields and their derivatives in F, now labeled by an index n. For each θ_n we can reach the above conclusion and therefore we find also:

$$\langle P_1 \ldots P_n|\theta(x)\phi(x_1)\ldots\phi(x_n)|P'_1 \ldots P'_n\rangle = 0. \tag{2.A7}$$

The only obstacle now is that each θ_n will require additional renormalization, but as before only other operators which vanish due to the (classical) e.o.m. can mix with θ_n under renormalization, so again the conclusion is unaffected.

We like to point out that the underlying assumption of the above considerations (and also later on to show the support properties of the distribution functions) is analyticity of the Green's functions involved.

Chapter 3

Properties of correlation functions

In this chapter we will study the properties of the soft parts. In the previous chapter we encountered various *correlation functions*, as the soft parts $\Phi, \Phi_A^\alpha, \Phi_D^\alpha, \ldots$, are called. The symmetries of QCD impose restrictions on these correlation functions. First of all, the symmetries restrict the number of independent structures, but often they also give rise to restrictions on such structures themselves. One type of such restriction, so-called sum rules, is particularly interesting, since it provides possibilities to check whether the theoretical assumptions are valid or not. These sum rules arise from reduction to local matrix elements by integration. For example, the Burkhardt-Cottingham (BC) sum rule expresses that the integral of the function g_2 vanishes, which implies that g_2 must switch sign at least once somewhere along the x range $(0 \leq x \leq 1)$. However, most of the sum rules, including the BC sum rule, are based on assumptions not derived from QCD, like assumptions on small x behavior, invertibility of Fourier transforms, interchanging of integrals, choosing boundary conditions, neglecting higher order corrections, etc. We will discuss these matters for several sum rules, focusing especially on the BC, Burkardt and Efremov-Leader-Teryaev sum rules and derive some new sum rules in addition.

Another type of restriction is formed by bounds. One can have that a particular function must be smaller than some constant or that it must be positive, but more sophisticated bounds can be considered, like functions bounding other functions, of which a famous example is Soffer's inequality. This subject will only very briefly be discussed.

The importance of these considerations is that violations of such sum rules and bounds might have severe consequences for the underlying theory, but most likely will just confirm that some of the questionable, not firmly QCD-based assumptions are indeed wrong.

In the first section we will discuss some parametrizations of the quark correlation function Φ, use symmetries, including time reversal symmetry, to restrict the number of structures, discuss nomenclature and interpretations of some of the structures. In the second and third section we discuss sum rules and bounds. In the fourth section consequences of dropping time reversal symmetry as a constraint are investigated. Finally, the antiquark and quark-gluon correlation functions $\overline{\Phi}$ and Φ_A^α (Φ_D^α) are discussed.

3.1 Quark correlation functions

Up to and including order $1/Q$ the correlation functions to consider are [5, 9, 6, 4]

$$\Phi_{ij}(P,S;k) = \int \frac{d^4z}{(2\pi)^4} e^{ik\cdot z} \langle P,S|\overline{\psi}_j(0)\psi_i(z)|P,S\rangle, \tag{3.1}$$

$$\Phi^{\alpha}_{Aij}(P,S;k_1,k_2) = \int \frac{d^4z}{(2\pi)^4}\frac{d^4z'}{(2\pi)^4} e^{ik_1\cdot z} e^{i(k_2-k_1)\cdot z'} \langle P,S|\overline{\psi}_j(0) g A^{\alpha}(z')\psi_i(z)|P,S\rangle. \tag{3.2}$$

As in the previous chapter we include a color identity and g times t^a from the hard into the soft parts Φ and Φ^{α}_A, respectively. The inclusion of path-ordered exponentials, which are needed in order to render the correlation functions gauge invariant, is implicit. Also, we left out the time-ordering, since it turns out to be fictitious, as is explained in App. 3.C.

3.1.1 The k_T-dependent distribution functions

The quark correlation function $\Phi_{ij}(k)$ can be expanded in a number of invariant amplitudes according to Dirac structure [9]. The available vectors are the momentum and spin vectors P and S of the incoming hadron (spin-1/2), such that $P\cdot S = 0$, and the quark momentum k. In the case of a hard scattering process the momentum of the struck quark is predominantly along the direction of the hadron momentum, which itself is chosen to be predominantly along a lightlike direction given by the vector n_+. Another lightlike direction n_- is chosen such that $n_+\cdot n_- = 1$; both vectors are dimensionless. Note that this choice differs from that of the previous chapter, where dimensionful lightlike vectors p, n were used[1]. The reason for this change is that in the previous chapter we exploited the fact that the vectors were chosen dimensionful and in this chapter we simply use the direction of the vectors and anticipate the case where two hadrons are scattered, such that one is predominantly in the n_+ and the other in the n_- direction, therefore the vectors are treated on an equal footing. We will often refer to the $\pm$ components of a momentum p, which are defined as $p^{\pm} = p\cdot n_{\mp}$.

We make the following Sudakov decompositions:

$$P^{\mu} \equiv \frac{Q}{x_1\sqrt{2}} n_+^{\mu} + \frac{x_1 M^2}{Q\sqrt{2}} n_-^{\mu}, \tag{3.3}$$

$$q^{\mu} \equiv \frac{Q}{\sqrt{2}} n_+^{\mu} + \frac{Q}{\sqrt{2}} n_-^{\mu} + q_T^{\mu}, \tag{3.4}$$

for $Q_T^2 \equiv -q_T^2 \ll Q^2$. Furthermore, we decompose the parton momentum k and the spin vector S of the hadron as

$$k \equiv \frac{xQ}{x_1\sqrt{2}} n_+ + \frac{x_1(k^2 + \boldsymbol{k}_T^2)}{xQ\sqrt{2}} n_- + k_T \approx xP + k_T, \tag{3.5}$$

$$S \equiv \frac{\lambda Q}{x_1 M\sqrt{2}} n_+ - \frac{x_1\lambda M}{Q\sqrt{2}} n_- + S_T \approx \frac{\lambda}{M} P + S_T, \tag{3.6}$$

[1]The vectors p, n are related to n_+, n_- via the rescaling: $p = Qn_+/(x_1\sqrt{2}), n = x_1\sqrt{2}n_-/Q$.

where $\boldsymbol{k}_T^2 = -k_T^2$. In general, we will use boldface vectors to denote Euclidean vectors. In the above approximations the $-$ components ($\propto 1/Q$) are neglected, as these are irrelevant compared to the $-$ components of the momenta in the hard part ($\propto Q$).

The parametrization of $\Phi(k)$ should be consistent with requirements imposed on Φ following from hermiticity, parity and time reversal invariance (see App. 3.D for a discussion of time reversal symmetry in general),

$$\Phi^\dagger(P,S;k) = \gamma_0\,\Phi(P,S;k)\,\gamma_0 \qquad \text{[Hermiticity]} \tag{3.7}$$

$$\Phi(P,S;k) = \gamma_0\,\Phi(\bar{P},-\bar{S};\bar{k})\,\gamma_0 \qquad \text{[Parity]} \tag{3.8}$$

$$\Phi^*(P,S;k) = \gamma_5 C\,\Phi(\bar{P},\bar{S};\bar{k})\,C^\dagger\gamma_5 \qquad \text{[Time reversal]} \tag{3.9}$$

where $\bar{k} = (k^0, -\boldsymbol{k})$, etc. For the validity of Eq. (3.9) it is essential that the incoming hadron is a plane wave state.

The Dirac structure of the quark correlation function can be expanded into a number of amplitudes, i.e., functions of invariants built up from the quark and hadron momenta, constrained by hermiticity and parity [9, 42]. The most general expression of the correlation function Φ in terms of these amplitudes is due to the requirement of parity invariance:

$$\begin{aligned}\Phi(P,S;k) = {} & M\,A_1 + A_2\,\not{P} + A_3\,\not{k} + (A_4/M)\,\sigma^{\mu\nu}P_\mu k_\nu + i\,A_5\,(k\cdot S)\gamma_5 + M\,A_6\,\not{S}\gamma_5 \\ & +(A_7/M)\,(k\cdot S)\,\not{P}\gamma_5 + (A_8/M)\,(k\cdot S)\,\not{k}\gamma_5 + i\,A_9\,\sigma^{\mu\nu}\gamma_5\,S_\mu P_\nu + i\,A_{10}\,\sigma^{\mu\nu}\gamma_5\,S_\mu k_\nu \\ & +i\,(A_{11}/M^2)\,(k\cdot S)\,\sigma^{\mu\nu}\gamma_5\,k_\mu P_\nu + (A_{12}/M)\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho S^\sigma. \end{aligned} \tag{3.10}$$

Hermiticity requires the amplitudes $A_i = A_i(k\cdot P, k^2)$ to be real. The amplitudes A_4, A_5 and A_{12} vanish in case time reversal invariance applies. Parity violating pieces are expected to be suppressed by order of $G_F \propto 1/M_W^2$, so they are neglected.

In calculations up to and including order $1/Q$, we encounter the correlation function integrated over k^-, which is parametrized in terms of so-called *distribution functions* as [42]

$$\begin{aligned}\Phi(x,\boldsymbol{k}_T) \equiv {} & \int dk^-\;\Phi(P,S;k)\Big|_{k^+=xP^+,\boldsymbol{k}_T} = \frac{M}{2P^+}\Bigg\{e(x,\boldsymbol{k}_T) + f_1(x,\boldsymbol{k}_T)\,\frac{\not{P}}{M} \\ & +f^\perp(x,\boldsymbol{k}_T)\,\frac{\not{k}_T}{M} - g_{1s}(x,\boldsymbol{k}_T)\,\frac{\not{P}\gamma_5}{M} - g_T'(x,\boldsymbol{k}_T)\,\not{S}_T\gamma_5 - g_s^\perp(x,\boldsymbol{k}_T)\,\frac{\not{k}_T\gamma_5}{M} \\ & -h_{1T}(x,\boldsymbol{k}_T)\,\frac{i\sigma_{\mu\nu}\gamma_5 S_T^\mu P^\nu}{M} - h_T^\perp(x,\boldsymbol{k}_T)\,\frac{i\sigma_{\mu\nu}\gamma_5 S_T^\mu k_T^\nu}{M} - h_{1s}^\perp(x,\boldsymbol{k}_T)\,\frac{i\sigma_{\mu\nu}\gamma_5 k_T^\mu P^\nu}{M^2} \\ & -h_s(x,\boldsymbol{k}_T)\,i\sigma_{\mu\nu}\gamma_5 n_+^\mu n_-^\nu\Bigg\}. \end{aligned} \tag{3.11}$$

For the moment we have restricted to distribution functions that are allowed by time reversal symmetry, the so-called time-reversal even distribution functions. We used the shorthand notation

$$g_{1s}(x,\boldsymbol{k}_T) \equiv \lambda\,g_{1L}(x,\boldsymbol{k}_T^2) + \frac{(\boldsymbol{k}_T\cdot\boldsymbol{S}_T)}{M}\,g_{1T}(x,\boldsymbol{k}_T^2), \tag{3.12}$$

etc.

We identify leading and subleading functions, which in principle start contributing at order 1 and $1/Q$, respectively. The order at which a function first can contribute depends on the power of M/P^+ in front of the function. Each factor M/P^+ leads to a suppression with a power of M/Q in cross sections. We will refer to the function multiplying a power $(M/P^+)^{t-2}$ as being of 'twist' t.

The names of the functions are assigned according to the following scheme. All functions obtained after tracing with a scalar ($\mathbf{1}$) or pseudoscalar ($i\gamma_5$) Dirac matrix are given the name $e_{..}$, those traced with a vector matrix (γ^μ) are given the name $f_{..}$, those traced with an axial vector matrix ($\gamma^\mu\gamma_5$) are given the name $g_{..}$ and finally those traced with the second rank tensor $i\sigma^{\mu\nu}\gamma_5$ are given the name $h_{..}$. A subscript '1' is given to the twist-two functions, subscripts 'L' or 'T' refer to the connection with the hadron spin being longitudinal or transverse and a superscript '$\perp$' signals the explicit presence of transverse momenta with a noncontracted index.

3.1.2 The $\boldsymbol{k}_T$-integrated distribution functions

The expressions for the $\boldsymbol{k}_T$-dependent distribution functions in terms of the amplitudes $A_i(\sigma,\tau)$, where $\sigma = 2\,k\cdot P$ and $\tau = k^2$, is given in App. 3.A. For example, one has that

$$f_1(x,\boldsymbol{k}_T^2) = \int d\sigma d\tau\,\delta(\boldsymbol{k}_T^2 + x^2M^2 + \tau - x\sigma)\ \left[A_2 + xA_3\right]. \tag{3.13}$$

Since there are less amplitudes than distribution functions (due to the fact that after integration over one momentum component k^-, there is effectively an additional vector in which one can expand), there exist several relations between distribution functions. If we define

$$f^{(n)}(x,\boldsymbol{k}_T^2) \equiv \int d\sigma d\tau\,\delta(\boldsymbol{k}_T^2 + x^2M^2 + \tau - x\sigma)\left(\frac{\boldsymbol{k}_T^2}{2M^2}\right)^n F(x,\sigma,\tau), \tag{3.14}$$

$$f'^{(n)}(x,\boldsymbol{k}_T^2) \equiv \int d\sigma d\tau\,\delta(\boldsymbol{k}_T^2 + x^2M^2 + \tau - x\sigma)\left(\frac{\boldsymbol{k}_T^2}{2M^2}\right)^n \frac{\partial F}{\partial x}(x,\sigma,\tau), \tag{3.15}$$

$$g^{(n)}(x,\boldsymbol{k}_T^2) \equiv -\int d\sigma d\tau\,\delta(\boldsymbol{k}_T^2 + x^2M^2 + \tau - x\sigma)\left(\frac{\boldsymbol{k}_T^2}{2M^2}\right)^n \frac{\sigma - 2x\,M^2}{2M^2} F(x,\sigma,\tau), \tag{3.16}$$

one can prove the relation

$$\frac{d}{dx}f^{(1)}(x,\boldsymbol{k}_T^2) = -g(x,\boldsymbol{k}_T^2) + f'^{(1)}(x,\boldsymbol{k}_T^2) + 2M^2\frac{d}{d\boldsymbol{k}_T^2}g^{(1)}(x,\boldsymbol{k}_T^2), \tag{3.17}$$

Schematically the proof goes as follows:

$$\begin{aligned}
&\frac{d}{dx}\int d\sigma d\tau\,\delta(\tau - A(x,\sigma,\tau))\left(\frac{\boldsymbol{k}_T^2}{2M^2}\right)^n F(x,\sigma,\tau)\\
&\quad = \int d\sigma\left(\frac{\boldsymbol{k}_T^2}{2M^2}\right)^n\left(\frac{\partial F(x,\sigma,A(x,\sigma,\tau))}{\partial x} + \frac{\partial F(x,\sigma,A(x,\sigma,\tau))}{\partial A}\frac{dA}{dx}\right).
\end{aligned} \tag{3.18}$$

Since $dA/d\boldsymbol{k}_T^2 = -1$, one has:

$$\frac{\partial F(x,\sigma,A(x,\sigma,\tau))}{\partial A} = -\frac{dF(x,\sigma,A(x,\sigma,\tau))}{d\boldsymbol{k}_T^2}. \tag{3.19}$$

One can do a partial integration with respect to $d\boldsymbol{k}_T^2$ and reinstall the τ integration with a delta function and the relation Eq. (3.17) follows.

The relations following from Eq. (3.17) we will express in terms of $\boldsymbol{k}_T$-integrated distribution functions. Let us first set the notation before discussing the relations. In general, the superscript (n) on a function $f(x)$ stands for the n-th $\boldsymbol{k}_T^2$-moment of the $\boldsymbol{k}_T$-dependent distribution function $f(x,\boldsymbol{k}_T^2)$,

$$f^{(n)}(x) = \int d^2\boldsymbol{k}_T \left(\frac{\boldsymbol{k}_T^2}{2M^2}\right)^n f(x,\boldsymbol{k}_T^2). \tag{3.20}$$

In case of $n = 0$, the superscript (0) is not indicated. For the following few exceptions the name of the function changes upon integration over $\boldsymbol{k}_T$, in order to connect to the commonly used parametrization of the $\boldsymbol{k}_T$-integrated correlation function:

$$g_1(x) \equiv \int d^2\boldsymbol{k}_T\, g_{1L}(x,\boldsymbol{k}_T^2), \tag{3.21}$$

$$h_1(x) \equiv \int d^2\boldsymbol{k}_T \left[h_{1T}(x,\boldsymbol{k}_T^2) + \frac{\boldsymbol{k}_T^2}{2M^2}\, h_{1T}^\perp(x,\boldsymbol{k}_T^2)\right], \tag{3.22}$$

$$g_T(x) \equiv \int d^2\boldsymbol{k}_T \left[g_T'(x,\boldsymbol{k}_T^2) + \frac{\boldsymbol{k}_T^2}{2M^2}\, g_T^\perp(x,\boldsymbol{k}_T^2)\right]. \tag{3.23}$$

The $\boldsymbol{k}_T$-integrated correlation function is hence parametrized as:

$$\Phi(x) \equiv P^+ \int dk^-\, d^2\boldsymbol{k}_T\, \Phi(P,S;k)\Big|_{k^+=xP^+} = \frac{M}{2}\Big\{e(x) + f_1(x)\,\frac{\not{P}}{M} - \lambda\, g_1(x)\,\frac{\not{P}\gamma_5}{M}$$
$$-g_T(x)\,\not{S}_T\gamma_5 - h_1(x)\,\frac{i\sigma_{\mu\nu}\gamma_5 S_T^\mu P^\nu}{M} - \lambda\, h_L(x)\, i\sigma_{\mu\nu}\gamma_5 n_+^\mu n_-^\nu\Big\}. \tag{3.24}$$

Under the assumption that $g^{(1)}(x,0)$ vanishes[2], one then finds the following five relations (there are 14 $\boldsymbol{k}_T$-dependent distribution functions depending on 9 amplitudes):

$$g_T(x) = g_1(x) + \frac{d}{dx}\, g_{1T}^{(1)}(x), \tag{3.25}$$

$$g_L^\perp(x) = -\frac{d}{dx}\, g_T^{\perp(1)}(x), \tag{3.26}$$

[2]This assumption is made to avoid singular behavior. For example, in the first relation Eq. (3.25), $g^{(1)}(x,0)$ is proportional to $[g_{1L}(x,\boldsymbol{k}_T^2) - g_T'(x,\boldsymbol{k}_T^2)] \times \boldsymbol{k}_T^2$ at the point $\boldsymbol{k}_T^2 = 0$. In order for this to be (nonzero and) finite, $[g_{1L}(x,\boldsymbol{k}_T^2) - g_T'(x,\boldsymbol{k}_T^2)]$ should behave as $1/\boldsymbol{k}_T^2$ for *small* $\boldsymbol{k}_T^2$, yielding a divergent result for the $\boldsymbol{k}_T$-integrated functions $g_1(x)$ and/or $g_T(x)$.

$$h_L(x) = h_1(x) - \frac{d}{dx}\, h_{1L}^{\perp(1)}(x), \tag{3.27}$$

$$h_T(x) = -\frac{d}{dx}\, h_{1T}^{\perp(1)}(x), \tag{3.28}$$

$$h_{1L}^{\perp}(x, \boldsymbol{k}_T^2) = h_T(x, \boldsymbol{k}_T^2) - h_T^{\perp}(x, \boldsymbol{k}_T^2), \tag{3.29}$$

where we used $\boldsymbol{k}_T$-integrated functions. Notice that upon integrated over $\boldsymbol{k}_T$ one is left with only six functions; no further relations due to overparametrization arise.

One can perform one more integration, namely over x, which sets the normalization of f_1,

$$\int_{-1}^{1} dx\, f_1^a(x) = \langle P|\overline{\psi}_a(0)\gamma^+\psi_a(0)|P\rangle/(2P^+) = \langle P|j_a^+(0)|P\rangle/(2P^+) \equiv n_a, \tag{3.30}$$

where n_a is the number of valence quarks with flavor a. The second moment of f_1 is known as the momentum sum rule (it must be smaller or equal to 1). All moment integrals run from $x = -1$ to $x = 1$ as is explained in App. 3.C.

The helicity distribution function $g_1(x)$ is often denoted as $\Delta q(x)$ and appears in the Ellis-Jaffe sum rule, which is known to be violated considerably [17], without consequences for QCD because of unjustified assumptions about the quark content of the proton (only polarized u and d quarks inside the proton). The idea behind the sum rule is that all contributions to the spin of the proton should add up to 1/2.

The first moment of the *transversity* distribution $h_1(x)$ (often denoted as $\delta q(x)$ or $\Delta_T q(x)$) is called the tensor charge (often denoted as δq), which is not a conserved quantity [46]. It will show up in the discussion on the Efremov-Leader-Teryaev sum rule (which involves g_1 and g_2) below.

3.1.3 Chirality

The functions $e_{..}$ and $h_{..}$ are called chiral-odd distribution functions [10, 11], since they couple quarks with opposite chirality in the quark correlation function. One defines the chirality right and left components of ψ by $\psi_{R/L} = (1 \pm \gamma_5)\psi/2$, such that

$$\overline{\psi}\psi = \overline{\psi}_R\psi_L + \overline{\psi}_L\psi_R, \tag{3.31}$$

$$\overline{\psi}i\sigma^{\mu\nu}\gamma_5\psi = \overline{\psi}_R i\sigma^{\mu\nu}\gamma_5\psi_L + \overline{\psi}_L i\sigma^{\mu\nu}\gamma_5\psi_R, \tag{3.32}$$

couple left- and righthanded spinors, whereas

$$\overline{\psi}\gamma^\mu\psi = \overline{\psi}_R\gamma^\mu\psi_R + \overline{\psi}_L\gamma^\mu\psi_L, \tag{3.33}$$

$$\overline{\psi}\gamma^\mu\gamma_5\psi = \overline{\psi}_R\gamma^\mu\gamma_5\psi_R + \overline{\psi}_L\gamma^\mu\gamma_5\psi_L, \tag{3.34}$$

couple spinors which have the same chirality. From this we also infer that the QCD interactions conserve chirality, but the quark mass term flips chirality. In cross sections chiral-odd functions arise either accompanied by a quark mass term (in general small and suppressed) or by another chiral-odd function, like is the case for the Drell-Yan process (cf. next chapter), such that the product is again chiral-even.

The chiral-odd functions are matrix elements of operators which are off-diagonal in the chirality basis. One can also choose a basis in which they are diagonal, but then the chiral-even operators will be off-diagonal. For this purpose one uses the spinors $\psi_{\uparrow/\downarrow} = \frac{1}{2}(1 \pm \gamma^i \gamma_5)\psi$, where i is a transverse index.

In the limit of massless quarks chirality equals helicity, so sometimes the chiral-odd matrix elements are referred to as helicity-flip amplitudes.

Often one writes the transverse spin states of the hadron $|\uparrow\rangle$ and $|\downarrow\rangle$ in the helicity basis via

$$|\uparrow\rangle = [\,|+\rangle + i\,|-\rangle\,]/\sqrt{2}, \tag{3.35}$$

$$|\downarrow\rangle = -[\,|+\rangle - i\,|-\rangle\,]/\sqrt{2}, \tag{3.36}$$

such that

$$|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| = |+\rangle\langle+| + |-\rangle\langle-|, \tag{3.37}$$

$$|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow| = -i\,|+\rangle\langle-| + i\,|-\rangle\langle+|, \tag{3.38}$$

where the last equation is associated with helicity flip. If one characterizes Φ_{ij} by the helicity of the incoming (outgoing) hadron $H(H')$ and that of the quark (antiquark) $h(h')$, then helicity conservation requires $H+h = H'+h'$. Parity flips all helicities and time reversal switches $(H,h) \leftrightarrow (H',h')$ [46]. Imposing all these requirements there are three possibilities left for the set $(H,h;H',h')$:

$$(1) \qquad (+,+;+,+), \tag{3.39}$$

$$(2) \qquad (+,-;+,-), \tag{3.40}$$

$$(3) \qquad (+,-;-,+). \tag{3.41}$$

The functions $f_{..}, g_{..}$ and $h_{..}$ correspond to the combinations (1) + (2), (1) − (2) and (3), respectively.

3.1.4 Interpretation of leading twist functions

The leading twist functions [9, 47, 48] have natural interpretations in terms of momentum densities, which pictorially can be displayed as in Fig. 3.1. The arrow on the parton (black dot) is its spin vector, where horizontal and vertical means it is longitudinally or transversely polarized, respectively. The other arrow denotes the hadron's spin vector. The partons in principle have a transverse momentum, which is not indicated in the pictures.

The leading twist functions can be written in terms of densities $\mathcal{P}$ (recall that $\psi_\pm = \not{n}_\pm \not{n}_\mp \psi/2$, cf. Eqs. (2.58) and (2.59)):

$$\frac{1}{2}\mathrm{Tr}\,[\Phi(x,\boldsymbol{k}_T)\,\not{n}_-] = f_1(x,\boldsymbol{k}_T) \equiv \mathcal{P}(x,\boldsymbol{k}_T) \propto \langle P|\overline{\psi}\gamma^+\psi|P\rangle = \langle P|\psi_+^\dagger\psi_+|P\rangle, \tag{3.42}$$

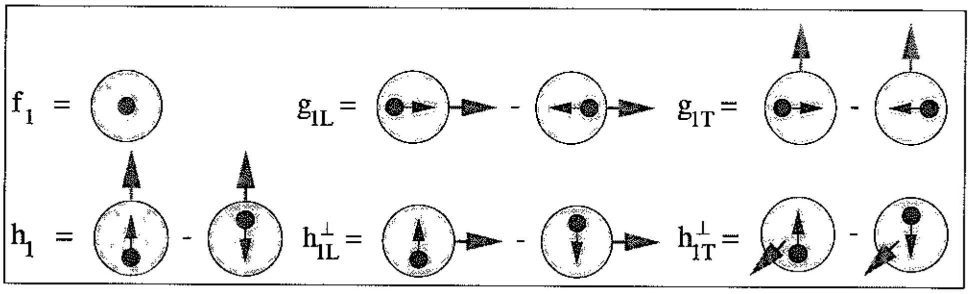


Figure 3.1: The leading twist distribution functions.

$$\frac{1}{2}\mathrm{Tr}\left[\Phi(x,\boldsymbol{k}_T)\, \not{n}_-\gamma_5\right] \equiv \mathcal{P}_R(x,\boldsymbol{k}_T) - \mathcal{P}_L(x,\boldsymbol{k}_T) \propto \langle P|\psi_{+R}^\dagger\psi_{+R} - \psi_{+L}^\dagger\psi_{+L}|P\rangle, \tag{3.43}$$

$$\frac{1}{2}\mathrm{Tr}\left[\Phi(x,\boldsymbol{k}_T)\, i\sigma^{i\alpha}n_{-\alpha}\gamma_5\right] \equiv \mathcal{P}_\uparrow(x,\boldsymbol{k}_T) - \mathcal{P}_\downarrow(x,\boldsymbol{k}_T) \propto \langle P|\psi_{+\uparrow}^\dagger\psi_{+\uparrow} - \psi_{+\downarrow}^\dagger\psi_{+\downarrow}|P\rangle. \tag{3.44}$$

The function g_{1T} for instance is the distribution of longitudinally polarized quarks (the density of right- minus lefthanded quarks) inside a transversely polarized hadron. One can also observe that the function h_1 is indeed associated with helicity flip.

Note that $g_{1L} \neq h_1$ and $g_{1T} \neq h_{1L}^\perp$, just like $g_1(x) \neq h_1(x)$, as opposed to what one might naively expect from rotational invariance. The point is that by rotating the spin the dominant direction rotates accordingly. Clearly this difference between dominant and transverse directions is a relativistic effect. Moreover, even if the functions would be equal at a certain energy scale, they would not be equal at other energies, since it is known that the (LO and NLO, i.e. α_s^2) evolution kernels of the functions g_1 [49] and h_1 [50, 51, 52] are different.

3.2 The Burkhardt-Cottingham sum rule

The Burkhardt-Cottingham (BC) sum rule states that the first moment of g_2 vanishes. The sum rule as originally derived by Burkhardt and Cottingham [53] was about the *structure* function g_2, which then yields a sum rule for the *distribution* function g_2^a (at least up to order $1/Q^2$ corrections), since $g_2 = 1/2\sum_{a,\bar{a}} e_a^2 g_2^a$, with $g_2^a \equiv g_T^a - g_1^a$, as we saw in the previous chapter (for a discussion on the interpretation of g_2 and many other aspects, see [40, 54, 29]).

On the other hand, the relation Eq. (3.25) implies for the distribution function g_2 (dropping the flavor index)

$$\int_0^1 dx\, g_2(x) = -g_{1T}^{(1)}(0). \tag{3.45}$$

We have assumed and further on always will assume that distributions in the endpoints $x = \pm 1$ vanish. The support of the correlation functions is from $-1 < x < 1$ (cf. App. 3.C). We will now look at the e.o.m. in order to be able to conclude some more from this relation.

But first we need to split the twist-three functions into a part which can be expressed in terms of twist-two functions and a left-over part denoted by a tilde. The splitting is achieved by using the free quark target results, which give solely the twist-two parts, or equivalently the results for quark-gluon correlation functions using the QCD equations of motion. The tilde function piece is the part which is interaction dependent and hence is absent for the free quark target. The tilde function $\tilde{g}_T$ is the one we need in this section (the others can be found in App. 3.B) and is defined as

$$g_T(x,\boldsymbol{k}_T) = \frac{1}{x}\, g_{1T}^{(1)}(x,\boldsymbol{k}_T) + \frac{m}{Mx}\, h_1(x,\boldsymbol{k}_T) + \tilde{g}_T(x,\boldsymbol{k}_T). \tag{3.46}$$

One finds due to the e.o.m. for the $\boldsymbol{k}_T$-integrated functions (cf. Eqs. (3.21)-(3.23)):

$$g_T(x) = \frac{g_{1T}^{(1)}(x)}{x} + \frac{m}{M}\frac{h_1(x)}{x} + \tilde{g}_T(x), \tag{3.47}$$

from which one obtains, when combined with the relation Eq. (3.25),

$$x^2\,\frac{d}{dx}\left(\frac{g_{1T}^{(1)}(x)}{x}\right) = -x\, g_1(x) + \frac{m}{M}\, h_1(x) + x\, \tilde{g}_T(x), \tag{3.48}$$

which can be used to eliminate $g_{1T}^{(1)}$,

$$\frac{d}{dx}\, g_2(x) = -\frac{1}{x}\frac{d}{dx}\,(x g_1(x)) + \frac{m}{M}\frac{1}{x}\frac{d}{dx}\, h_1(x) + \frac{1}{x}\frac{d}{dx}\,(x\tilde{g}_T(x)), \tag{3.49}$$

or provided the distribution functions vanish at the endpoint $x=1$:

$$\begin{aligned} g_2(x) = &-\left[g_1(x) - \int_x^1 dy\, \frac{g_1(y)}{y}\right] + \frac{m}{M}\left[\frac{h_1(x)}{x} - \int_x^1 dy\, \frac{h_1(y)}{y^2}\right] \\ &+ \left[\tilde{g}_T(x) - \int_x^1 dy\, \frac{\tilde{g}_T(y)}{y}\right]. \end{aligned} \tag{3.50}$$

Upon neglecting the interaction dependent functions and quark mass terms, leaving the first term in brackets, one arrives at the so-called Wandzura-Wilczek relation [55]. From Eq. (3.50) one arrives at

$$\int_0^1 dx\, g_2(x) = \int_0^1 dx\, G(x) - \int_0^1 dx \int_x^1 dy\, \frac{G(y)}{y}, \tag{3.51}$$

where $G(x) \equiv -g_1(x) + m h_1(x)/(Mx) + \tilde{g}_T(x)$. The r.h.s. will vanish if one is allowed to interchange the two integrations. This is not allowed if $G(x)$ behaves as $1/x$ as $x \to 0$. In a spectator model calculation [20] $G(x)$ behaves as $m/(Mx)$ as $x \to 0$, since it results in finite $g_1(0), h_1(0)$ ($\tilde{g}_T$ is absent in the model). Hence, the BC sum rule would be violated by a term proportional to the mass, which is indeed the case in the model.

Differently put one finds from Eq. (3.48):

$$g_{1T}^{(1)}(x) = -x \int_x^1 dy \frac{G(y)}{y}. \tag{3.52}$$

From which we can also draw the conclusion that if $xG(x)$ is finite as $x \to 0$, then $g_{1T}^{(1)}(0)$ is not zero. So on the basis of the most general Dirac structure of the correlation function and the e.o.m.'s, one cannot exclude that a BC sum rule for the distribution function g_2 is violated.

There is on the other hand a different observation which is based on Lorentz invariance and which does yield a BC sum rule for the distribution function g_2 [3, 56]. We will repeat it. We consider the following projection of the $\boldsymbol{k}_T$-integrated correlation function:

$$\begin{aligned}
\frac{1}{2}\mathrm{Tr}\,[\Phi(x)\gamma^\mu\gamma_5] &= \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P,S|\overline{\psi}(0)\gamma^\mu\gamma_5\psi(\lambda n_-)|P,S\rangle \\
&= \lambda n_+^\mu g_1(x) + \frac{M}{P+} S_T^\mu g_T(x) + \frac{\lambda M^2}{(P+)^2} n_-^\mu g_3(x).
\end{aligned} \tag{3.53}$$

For completeness we have included the twist-four distribution function g_3. If we take the first moment this leads to:

$$\frac{1}{2}\langle P,S|\overline{\psi}(0)\gamma^\mu\gamma_5\psi(0)|P,S\rangle = \int_{-1}^1 dx \left[\lambda n_+^\mu g_1(x) + \frac{MS_T^\mu}{P+} g_T(x) + \frac{\lambda M^2 n_-^\mu}{(P+)^2} g_3(x)\right]. \tag{3.54}$$

Since the l.h.s. must be proportional to S^μ, one can easily calculate the proportionality factor, since we know that $P \cdot S = 0$. This leads to:

$$\int_{-1}^1 dx\, g_1(x) = \int_{-1}^1 dx\, g_T(x), \tag{3.55}$$

$$\int_{-1}^1 dx\, g_1(x) = -2\int_{-1}^1 dx\, g_3(x), \tag{3.56}$$

hence the first equation implies the BC sum rule. Note that the integral is now from -1 to 1, since that is the support of the correlation function. This does not imply that the sum rule holds if the integral runs from 0 to 1 as we will show below. So the above arguments are not conclusive either.

Relations between moments of distribution functions of different twist, like Eq. (3.55), are expected due to Lorentz invariance of matrix elements of *local* operators. This does not mean that Lorentz invariance implies that the functions of different twist themselves are equal. Look for instance at the second moment:

$$\int_{-1}^1 dx\, x\, \mathrm{Tr}\,[\Phi(x)\gamma^\mu\gamma_5] = i\langle P,S|\overline{\psi}(0)\gamma^\mu\gamma_5 \left(n_- \frac{\partial\psi(\lambda n_-)}{\partial\lambda n_-}\right)\bigg|_{\lambda=0} |P,S\rangle. \tag{3.57}$$

One can arrive at the above relation by first expanding $\psi(\lambda n_-)$ around zero. The derivative introduces a direction (in this case in the n_- direction), such that Lorentz invariance cannot be applied as before. The quantity will be proportional to $S^\mu(P \cdot n_-)$ and $P^\mu(S \cdot n_-)$ with independent coefficients. So, only for the first moment one can use that contraction with P_μ gives zero. The fact that the anomalous dimensions for g_1 and g_T are different (so not all moments can be equal) confirms this observation.

To continue with the discussion on the BC sum rule: on the one hand we have that

$$\int_0^1 dx\, g_2(x) = -g_{1T}^{(1)}(0) = \frac{m}{M} h_1(0), \tag{3.58}$$

where the second equality follows from Eq. (3.48), and on the other hand

$$\int_{-1}^1 dx\, g_2(x) = 0, \tag{3.59}$$

from Lorentz invariance.

These two conclusions can be compatible, by requiring that the function $g_{1T}^{(1)}$ is continuous but still nonzero at $x=0$. This can be deduced by noting that $g_2^a(-x) = g_2^{\bar{a}}(x)$ (as will be explained in section 3.5 about the antiquark correlation function), such that

$$\int_{-1}^1 dx\, g_2^a(x) = \int_0^1 dx\, [g_2^a(x) + g_2^{\bar{a}}(x)] = g_{1T}^{(1)a}(0^-) - g_{1T}^{(1)a}(0^+), \tag{3.60}$$

due to $g_{1T}^a(x) = -g_{1T}^{\bar{a}}(-x)$, which gives $g_{1T}^{(1)a}(0^+) = -g_{1T}^{(1)\bar{a}}(0^-)$. The r.h.s. is only zero in case the function g_{1T} is continuous at $x=0$, but $g_{1T}^{(1)}(0)$ can still be nonzero without contradiction. Since $g_{1T}^{(1)}(0) = -mh_1(0)/M$, it is however expected to be very small. Larger contributions from the point $x=0$ can arise from $\delta(x)$ contributions, as discussed in [56, 29]. One cannot exclude such violations of the BC sum rule experimentally, since one certainly cannot measure distributions at the point $x=0$. In [56] it is claimed that such contributions should be identified with zero-modes that appear in light-front quantization.

Other objections to the BC sum rule have been given [57, 23]: one cannot arrive at the BC sum rule from the OPE, moreover Regge considerations cast doubt on the small x assumptions on the function g_2 and the integral might not even converge.

3.3 Other sum rules and bounds

3.3.1 The Burkardt sum rule

Similar conclusions like for g_2 can be drawn for h_2. We obtain

$$h_L(x) = \frac{m}{M}\frac{g_1(x)}{x} - 2\frac{h_{1L}^{\perp(1)}(x)}{x} + \tilde{h}_L(x), \tag{3.61}$$

from which one obtains a relation for $h_2 \equiv 2(h_L - h_1)$:

$$\frac{d}{dx}\left(\frac{h_2(x)}{2x}\right) = -\frac{1}{x^2}\frac{d}{dx}\left(xh_1(x)\right) + \frac{m}{M}\frac{1}{x^2}\frac{d}{dx}\, g_1(x) + \frac{1}{x^2}\frac{d}{dx}\left(x\tilde{h}_L(x)\right), \tag{3.62}$$

or provided the distribution functions vanish at the endpoint $x = 1$:

$$\frac{1}{2}\,h_2(x) = -\left[h_1(x) - 2x\int_x^1 dy\,\frac{h_1(y)}{y^2}\right] + \frac{m}{M}\left[\frac{g_1(x)}{x} - 2x\int_x^1 dy\,\frac{g_1(y)}{y^3}\right]$$
$$+\left[\tilde{h}_L(x) - 2x\int_x^1 dy\,\frac{\tilde{h}_L(y)}{y^2}\right]. \tag{3.63}$$

The integral would vanish in case one can interchange the two integrations, but because of the relation

$$h_2(x) = -\frac{1}{2}\frac{d}{dx}\,h_{1L}^{\perp(1)}(x), \tag{3.64}$$

one rather finds

$$\int_0^1 dx\,h_2(x) = 2h_{1L}^{\perp(1)}(0) = \frac{m}{M}g_1(0). \tag{3.65}$$

Again the r.h.s. is proportional to a quark mass.

Now one also looks at (for completeness we include the twist-four function h_3)

$$\frac{1}{2}\mathrm{Tr}\,[\Phi i\sigma^{\mu\nu}\gamma_5] = S_T^{[\mu}n_+^{\nu]}h_1(x) + \lambda\frac{M}{P^+}n_+^{[\mu}n_-^{\nu]}h_L(x) + \frac{M^2}{(P^+)^2}S_T^{[\mu}n_-^{\nu]}h_3(x), \tag{3.66}$$

where $[\mu\nu]$ indicates antisymmetrization. One requires Lorentz invariance on its first moment which must be proportional to $P^{[\mu}S^{\nu]}$. This leads to two constraints

$$\int_{-1}^1 dx\,h_1(x) = \int_{-1}^1 dx\,h_L(x), \tag{3.67}$$
$$\int_{-1}^1 dx\,h_1(x) = 2\int_{-1}^1 dx\,h_3(x), \tag{3.68}$$

where the first one leads to the Burkardt sum rule for h_2 [58, 56]:

$$\int_{-1}^1 dx\,h_2(x) = 0. \tag{3.69}$$

It is important to note that this sum rule holds for the *distribution* function h_2 and not for a *structure* function. In this respect it is not similar to the BC sum rule.

One does not obtain a first moment sum rule for the remaining $\boldsymbol{k}_T$-integrated twist-three distribution function $e(x)$ in this way.

3.3.2 The Efremov-Leader-Teryaev sum rule

One can also look at sum rules for higher moments of distribution functions. Let us from now on assume that one can interchange integrations in for instance Eq. (3.51), even though this remains unjustified. Then one finds

$$\int_0^1 dx\, x^{n-1} \left[f(x) - k\, x^{k-1} \int_x^1 dy \frac{f(y)}{y^k}\right] = \frac{n-1}{n+k-1} \int_0^1 dx\, x^{n-1}\, f(x), \tag{3.70}$$

from which we see that for $n=1$ the r.h.s. is zero and for $f(x) = G(x)$ and $k=1$ the BC sum rule results. For arbitrary n we find:

$$\int_0^1 dx\, x^{n-1} \left[\frac{n-1}{n} g_1(x) + g_2(x)\right] = \frac{n-1}{n} \int_0^1 dx\, x^{n-1} \left[\frac{m}{Mx} h_1(x) + \tilde{g}_T(x)\right], \tag{3.71}$$

such that upon neglecting the twist-three contribution proportional to $\tilde{g}_T$ and taking $m=0$ one arrives at the Wandzura-Wilczek (WW) sum rules [55], which are clearly approximations. For odd n these sum rules follow also from the OPE, but the same approximations have to be done. For even n one can use the OPE to derive the sum rules for the *valence* distribution functions $(g^a_{..}(x) - g^{\bar{a}}_{..}(x))$. In particular, for $n=2$ one arrives at the sum rule

$$\int_0^1 dx\, x\, [g_1(x) + 2\, g_2(x)] = \int_0^1 dx\, \left[\frac{m}{M} h_1(x) + x\tilde{g}_T(x)\right]. \tag{3.72}$$

The first term on the r.h.s. is proportional to the tensor charge and the second term can be investigated with help of the e.o.m. given in Eq. (3.135) below. One can show that (cf. section 3.6)

$$\begin{aligned}\int_0^1 dx\, x\, \tilde{g}_T(x) &= \int_0^1 dx\, dy\, \mathrm{Re}\, \tilde{G}_A(x,y) \\ &= \mathrm{Re}\langle P,S|\overline{\psi}(0)\gamma^\mu\gamma_5\, gA^\nu(0)\, \psi(0)|P,S\rangle \frac{n_{-\mu} S_{T\nu}}{M\, S_T^2}.\end{aligned} \tag{3.73}$$

In [59] it is claimed that $\langle P,S|\overline{\psi}(0)\gamma^\mu\gamma_5\, gA^\nu(0)\, \psi(0)|P,S\rangle = 0$, such that when $m=0$ the r.h.s. of Eq. (3.72) vanishes *without* the neglect of twist-three contributions. The resulting sum rule for valence distribution functions is then called the Efremov-Leader-Teryaev (ELT) sum rule. Let us repeat their argument. Imposing the gauge condition $n_- \cdot A = 0$ on the most general form

$$\begin{aligned}\frac{1}{M}\langle P,S|\overline{\psi}(0)\gamma^\mu\gamma_5\, gA^\nu(\zeta n_-)\, \psi(\zeta n_-)|P,S\rangle &= \zeta(S\cdot n_-)[B_1 P^\mu P^\nu + \zeta B_2 P^\mu n_-^\nu \\ +\zeta B_3 n_-^\mu P^\nu + \zeta^2 B_4 n_-^\mu n_-^\nu] + B_5 S^\mu P^\nu &+ B_6 P^\mu S^\nu + \zeta B_7 S^\mu n_-^\nu + \zeta B_8 n_-^\mu S^\nu,\end{aligned} \tag{3.74}$$

which must be linear in S, yields

$$\begin{aligned}\frac{1}{M}\langle P,S|\overline{\psi}(0)\gamma^\mu\gamma_5\, gA^\nu(0)\, \psi(0)|P,S\rangle &= \zeta B_1 \left[(S\cdot n_-)P^\mu P^\nu - P^\mu S^\nu\right] \\ +\zeta(S\cdot n_-)\left[B_2 P^\mu n_-^\nu + \zeta B_4 n_-^\mu n_-^\nu\right] &+ \zeta^2 B_3\left[(S\cdot n_-)n_-^\mu P^\nu - n_-^\mu S^\nu\right] + \zeta B_7 S^\mu n_-^\nu.\end{aligned} \tag{3.75}$$

In Ref. [59] it is assumed that all scalar functions behave like $\zeta B(\zeta) \to 0$ as $\zeta \to 0$, to be in accordance with the expectation from the OPE, since the limit $\zeta \to 0$ in the lightcone operators will make them into local operators. This is not only an assumption that is based on requiring ζ independence, since one can achieve that also by letting $\zeta B(\zeta) \to c$, where c is some constant. It rather is chosen such that in the limit of $\zeta \to 0$ the matrix element of the local operator should become independent of n_-, which can only be achieved by requiring $\zeta B(\zeta) \to 0$ as $\zeta \to 0$, but in that case the r.h.s. vanishes completely. So it follows that the ELT sum rule is not based on the neglect of twist-three contributions, but upon the reasoning that Lorentz invariance yields $\int_0^1 dx\, x\, \tilde{g}_T(x) = 0$. Extension of these arguments to higher moments is not possible however, since higher moments yield derivatives of the fields (cf. Eq. (3.57)) and thereby introduce a direction and hence, n_- dependence.

Hence, the Efremov-Leader-Teryaev (ELT) sum rule states that without the neglect of twist-three contributions (but taking $m = 0$)

$$\int_0^1 dx\, x \left[g_1^V(x) + 2\, g_2^V\right] = 0. \tag{3.76}$$

Contrary to the BC sum rule, convergence of the sum rule is not an issue since small x divergences will cancel. It should be mentioned that the derivation in [59] was also within the context of EFP's field theoretic approach (but with zero quark mass). Moreover, in [35, 60][3] the ELT sum rule has been derived by means of the OPE (including quark mass terms). In case of nonzero quark mass the r.h.s. of the sum rule is not equal to zero, but proportional to the (valence component of the) tensor charge.

3.3.3 New sum rules

Using Eq. (3.70) for $f(x) = -h_1(x) + m g_1(x)/(Mx) + \tilde{h}_L$ one finds:

$$\int_0^1 dx\, x^{n-1} \left[\frac{n-1}{n+1} h_1(x) + \frac{1}{2} h_2(x)\right] = \frac{n-1}{n+1} \int_0^1 dx\, x^{n-1} \left[\frac{m}{Mx} g_1(x) + \tilde{h}_L(x)\right]. \tag{3.77}$$

For $n = 1$ one arrives at the Burkardt sum rule. Upon neglecting twist-three contributions proportional to $\tilde{h}_L$ and taking $m = 0$, one gets analogues of the WW sum rules. For $n = 2$ we find

$$\int_0^1 dx\, x\, [2\, h_1(x) + 3\, h_2(x)] = \int_0^1 dx\, x\, 2 \left[\frac{m}{Mx} g_1(x) + \tilde{h}_L(x)\right], \tag{3.78}$$

with (cf. section 3.6)

$$\begin{aligned} \int_0^1 dx\, x\, \tilde{h}_L(x) &= \int_0^1 dx\, dy\, 2\mathrm{Re}\, H_A(x,y) \\ &= \mathrm{Re} \frac{\lambda}{M} \langle P, S | \overline{\psi}(0) \gamma^\mu \gamma_T^\nu \gamma_5\, g A_{T\nu}(0)\, \psi(0) | P, S \rangle n_{-\mu}, \end{aligned} \tag{3.79}$$

[3]Ref. [60] contains a comprehensive study of sum rules for spin-dependent electroweak structure functions including twist-three contributions.

which as argued in the previous section must vanish (like all the second moments of the tilde functions). Hence, we conclude that the following sum rule holds *without* neglecting twist-three contributions:

$$\int_0^1 dx\, x\, [2\, h_1(x) + 3\, h_2(x)] = \int_0^1 dx\, 2\frac{m}{M} g_1(x). \tag{3.80}$$

Upon taking $m = 0$, one finds the analogue of the ELT sum rule, which as far as we know has received no attention in the literature. The valence part of this sum rule is not a result that can be derived by means of the OPE, however.

3.3.4 Bounds

There are also restrictions on the distribution functions in the form of bounds, like $|g_1(x)| \leq f_1(x)$ and Soffer's inequality $|h_1(x)| \leq [f_1(x) + g_1(x)]/2$, derived from the parton model, which can be used as a test of the underlying assumptions. Recently, it has been shown that Soffer's inequality is also satisfied at next-to-leading order in α_s [52, 61]. More complicated bounds can be derived for structure functions from the semi-positive-definiteness of the hadron tensor $W^{\mu\nu} a^*_\mu a_\nu \geq 0$, which holds for an arbitrary vector a^μ [62, 22].

3.4 Time-reversal odd distribution functions

In the beginning of this chapter it is stated that the amplitudes (and hence the distribution functions) are real due to hermiticity. This can however also be inferred from the e.o.m., because one can write down the real and imaginary parts of the e.o.m. and it turns out that each distribution function appears only in one of the two. This implies that either a distribution function is real or imaginary. By putting in appropriate factors of i in the starting parametrization, one can always arrive at real distribution functions. The functions appearing in the real and imaginary parts of the e.o.m. are the time-reversal even and odd distribution functions, respectively. Nonzero time-reversal odd distribution functions require a violation of time reversal symmetry, unless the hadron states are not plane-wave states (cf. next chapter). Here we will just make an inventory of the time-reversal odd part of the correlation function Φ.

The time-reversal odd part of the correlation function Φ is given in terms of amplitudes as follows (cf. Eq. (3.10))

$$\Phi(P,S;k)|_{\text{T-odd}} = (A_4/M)\, \sigma^{\mu\nu} P_\mu k_\nu + i\, A_5\, (k \cdot S)\gamma_5 + (A_{12}/M)\, \epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho S^\sigma. \tag{3.81}$$

In calculations up to and including order $1/Q$, we encounter the correlation function integrated over k^-, of which the time-reversal (T) odd part is parametrized as follows [63]

$$\int dk^- \; \Phi(P,S;k)\Big|_{k^+ = xP^+,\; \boldsymbol{k}_T}\Big|_{\text{T-odd}} = \frac{M}{2P^+}\left\{ f_{1T}^\perp(x,\boldsymbol{k}_T)\, \epsilon_{\mu\nu\rho\sigma}\gamma^\mu \frac{P^\nu k_T^\rho S_T^\sigma}{M^2}\right.$$

$$+f_T(x,\boldsymbol{k}_T)\,\epsilon_{\mu\nu\rho\sigma}n_-^\mu n_+^\nu\gamma^\rho S_T^\sigma + \lambda\,f_L^\perp(x,\boldsymbol{k}_T)\,\epsilon_{\mu\nu\rho\sigma}n_-^\mu n_+^\nu\gamma^\rho\frac{k_T^\sigma}{M}$$

$$\left.-e_s(x,\boldsymbol{k}_T)\,i\gamma_5 + h_1^\perp(x,\boldsymbol{k}_T)\,\frac{\sigma_{\mu\nu}k_T^\mu P^\nu}{M^2} + h(x,\boldsymbol{k}_T)\,\sigma_{\mu\nu}n_+^\mu n_-^\nu\right\}. \tag{3.82}$$

Again the leading twist functions have simple interpretations in terms of densities. The

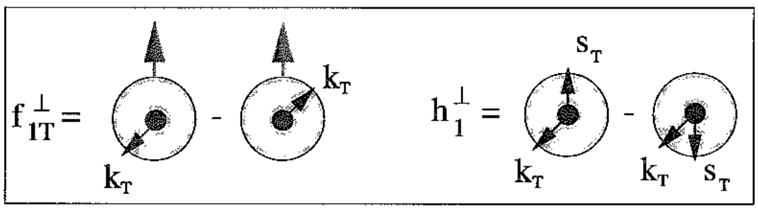


Figure 3.2: The leading twist T-odd distribution functions.

function $f_{1T}^\perp$ corresponds to the so-called Sivers effect [64] and is related to the function $\Delta^N f$ of Refs. [65, 66]. It is interpreted as the distribution of an unpolarized quark with nonzero transverse momentum inside a transversely polarized nucleon (the first picture in Fig. 3.2), while the function $h_1^\perp$ (the second picture in Fig. 3.2) is interpreted as the distribution of a transversely polarized quark with nonzero transverse momentum inside an unpolarized hadron (the analogue of the Collins effect, to be discussed in Chaps. 5 and 6). In both cases the polarization is orthogonal to the transverse momentum of the quark.

In App. 3.A one can find the expressions of these T-odd distribution functions in terms of the amplitudes defined in the beginning of this chapter. One then finds the following four relations (7 distribution functions depending on 3 amplitudes)

$$f_T(x) = -\frac{d}{dx}\,f_{1T}^{\perp(1)}, \tag{3.83}$$

$$h(x) = -\frac{d}{dx}\,h_1^{\perp(1)}, \tag{3.84}$$

$$e_L(x) = -\frac{d}{dx}\,e_T^{(1)}, \tag{3.85}$$

$$f_{1T}^\perp(x,\boldsymbol{k}_T^2) = -f_L^\perp(x,\boldsymbol{k}_T^2). \tag{3.86}$$

The following (T-odd) distribution functions remain after the $\boldsymbol{k}_T$ integration (absorbing a factor P^+ in the definition of $\Phi(x)$, cf. Eq. (3.24)):

$$\Phi(x)\big|_{\text{T-odd}} = \frac{M}{2}\left[f_T(x)\epsilon_T^{\mu\nu}S_{T\mu}\gamma_{T\nu} - e_L(x)\lambda i\gamma_5 + h(x)\frac{i}{2}\,[\not{n}_+,\not{n}_-]\right]. \tag{3.87}$$

Note that these are all subleading functions as opposed to $f_{1T}^\perp(x,\boldsymbol{k}_T)$ and $h_1^\perp(x,\boldsymbol{k}_T)$. The consequences of these T-odd distribution functions will be discussed in the next chapter for the case of the Drell-Yan process (for the case of semi-inclusive leptoproduction see [63]).

From Lorentz invariance required on the first moments of $\mathrm{Tr}\,[\Phi\gamma^\mu]$, $\mathrm{Tr}\,[\Phi i\sigma^{\mu\nu}\gamma_5]$ and $\mathrm{Tr}\,[\Phi\gamma_5]$, one arrives at the sum rules

$$\int_{-1}^{1} dx\, f_T(x) = 0, \tag{3.88}$$

$$\int_{-1}^{1} dx\, h(x) = 0, \tag{3.89}$$

$$\int_{-1}^{1} dx\, e_L(x) = 0. \tag{3.90}$$

The first sum rule was already given in [67] (its function c_V is proportional to our function f_T).

3.5 Antiquark correlation functions

Some of the above considerations were based on the symmetry relations between quark and antiquark correlation functions, like $g_2^a(x) = g_2^{\bar{a}}(-x)$. One can arrive at them in the following way. The correlation function for antiquarks in a hadron is given by the matrix element [9]

$$\overline{\Phi}_{ij}(P,S;k) \;=\; \int \frac{d^4z}{(2\pi)^4}\, e^{-ik\cdot z}\langle P,S|\psi_i(z)\overline{\psi}_j(0)|P,S\rangle. \tag{3.91}$$

Again this is parametrized by a set of distribution functions. This parametrization should however be consistent with charge conjugation. We parametrize Φ^c, which is defined as Φ, but containing the conjugate spinors $\psi^c = C\,\overline{\psi}^T$, where $C\,\gamma_\mu^T\,C^\dagger = -\gamma_\mu$,

$$\Phi^c_{ij}(P,S;k) = \int \frac{d^4z}{(2\pi)^4}\, e^{ik\cdot z}\langle P,S|\overline{\psi}^c_j(0)\psi^c_i(z)|P,S\rangle, \tag{3.92}$$

in exactly the same way as Φ, but now we overline the functions $(\overline{f}_1,\ldots)$. Φ^c is related to the antiquark correlation function $\overline{\Phi}$, via $\Phi^c = -C\,\overline{\Phi}^T C^\dagger$. This relation yields the parametrization of $\overline{\Phi}$ in terms of the antiquark distribution functions. So there can be a relative sign between $\overline{\Phi}^{[\Gamma]}$ and $\Phi^{c[\Gamma]}$ depending on $\Gamma = \mp C\Gamma^T C^\dagger$. Explicitly,

$$\overline{\Phi}^{[\Gamma]} = +\Phi^{c[\Gamma]} \qquad \text{for } \Gamma = \gamma_\mu,\ \sigma_{\mu\nu},\ i\sigma_{\mu\nu}\gamma_5, \tag{3.93}$$

$$\overline{\Phi}^{[\Gamma]} = -\Phi^{c[\Gamma]} \qquad \text{for } \Gamma = 1,\ \gamma_\mu\gamma_5,\ i\gamma_5. \tag{3.94}$$

Furthermore, the anticommutation relations can be used to obtain the symmetry relation

$$\overline{\Phi}_{ij}(P,S;k) = -\Phi_{ij}(P,S;-k). \tag{3.95}$$

For the distribution functions this gives the symmetry relations

$$\overline{f}_1(x,\boldsymbol{k}_T^2) = -f_1(-x,\boldsymbol{k}_T^2) \tag{3.96}$$

and similarly for g_{1T}, h_{1T}, $h_{1T}^\perp$, e_T, f_T, $g_L^\perp$, h and h_L, while

$$\overline{g}_{1L}(x,\boldsymbol{k}_T^2) = g_{1L}(-x,\boldsymbol{k}_T^2) \tag{3.97}$$

and similarly for $f_{1T}^\perp$, $h_1^\perp$, $h_{1L}^\perp$, e, e_L, $f^\perp$, $f_L^\perp$, g_T', $g_T^\perp$, $h_T^\perp$ and h_T.

For completeness we give the T-even parametrization for $\overline{\Phi}$:

$$\begin{aligned}\int dk^+\, \overline{\Phi}(P,S;k)\Big|_{k^-=xP^-,\ \boldsymbol{k}_T} &= \frac{M}{2P^-}\Big\{-\overline{e}(x,\boldsymbol{k}_T) + \overline{f}_1(x,\boldsymbol{k}_T)\,\frac{\not{P}}{M} + \overline{f}^\perp(x,\boldsymbol{k}_T)\,\frac{\not{k}_T}{M} \\ &+\overline{g}_{1s}(x,\boldsymbol{k}_T)\,\frac{\not{P}\gamma_5}{M} + \overline{g}_T'(x,\boldsymbol{k}_T)\,\not{S}_T\gamma_5 + \overline{g}_s^\perp(x,\boldsymbol{k}_T)\,\frac{\not{k}_T\gamma_5}{M} - \overline{h}_{1T}(x,\boldsymbol{k}_T)\,\frac{i\sigma_{\mu\nu}\gamma_5 S_T^\mu P^\nu}{M} \\ &-\overline{h}_T^\perp(x,\boldsymbol{k}_T)\,\frac{i\sigma_{\mu\nu}\gamma_5 S_T^\mu k_T^\nu}{M} - \overline{h}_{1s}^\perp(x,\boldsymbol{k}_T)\,\frac{i\sigma_{\mu\nu}\gamma_5 k_T^\mu P^\nu}{M^2} - \overline{h}_s(x,\boldsymbol{k}_T)\,i\sigma_{\mu\nu}\gamma_5 n_-^\mu n_+^\nu\Big\}\end{aligned} \tag{3.98}$$

and its T-odd parametrization:

$$\begin{aligned}\int dk^+\, \overline{\Phi}(P,S;k)\Big|_{k^-=xP^-,\ \boldsymbol{k}_T}\Big|_{\text{T-odd}} &= \frac{M}{2P^-}\Big\{\overline{f}_{1T}^\perp(x,\boldsymbol{k}_T)\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\frac{P^\nu k_T^\rho S_T^\sigma}{M^2} \\ &+\overline{f}_T(x,\boldsymbol{k}_T)\,\epsilon_{\mu\nu\rho\sigma}n_+^\mu n_1^\nu\gamma^\rho S_T^\sigma + \lambda\,\overline{f}_L^\perp(x,\boldsymbol{k}_T)\,\epsilon_{\mu\nu\rho\sigma}n_+^\mu n_-^\nu\gamma^\rho\frac{k_T^\sigma}{M} \\ &+\overline{e}_s(x,\boldsymbol{k}_T)\,i\gamma_5 + \overline{h}_1^\perp(x,\boldsymbol{k}_T)\,\frac{\sigma_{\mu\nu}k_T^\mu P^\nu}{M^2} + \overline{h}(x,\boldsymbol{k}_T)\,\sigma_{\mu\nu}n_-^\mu n_+^\nu\Big\}.\end{aligned} \tag{3.99}$$

3.6 Quark-gluon correlation functions

Next to the quark and antiquark correlation functions Φ and $\overline{\Phi}$, one encounters the quark-gluon correlation function Φ_A^α, but for reasons of color gauge invariance one actually wants to deal with Φ_D^α (see previous chapter). For Φ_D^α, and similarly for Φ_A^α, hermiticity, parity and time reversal invariance yield the following relations:

$$[\Phi_D^\alpha(P,S;p_1,p_2)]^\dagger = \gamma_0\,\Phi_D^\alpha(P,S;p_2,p_1)\,\gamma_0 \qquad \text{[Hermiticity]} \tag{3.100}$$

$$\Phi_D^\alpha(P,S;p_1,p_2) = \gamma_0\,\Phi_{D\alpha}(\bar{P},-\bar{S};\bar{p}_1,\bar{p}_2)\,\gamma_0 \qquad \text{[Parity]} \tag{3.101}$$

$$[\Phi_D^\alpha(P,S;p_1,p_2)]^* = \gamma_5 C\,\Phi_{D\alpha}(\bar{P},\bar{S};\bar{p}_1,\bar{p}_2)\,C^\dagger\gamma_5 \qquad \text{[Time reversal]} \tag{3.102}$$

The two parton momenta p_1 and p_2 are decomposed as

$$p_1 \equiv \frac{xQ}{x_1\sqrt{2}}\,n_+ + \frac{x_1(p_1^2+\boldsymbol{p}_{1T}^2)}{xQ\sqrt{2}}\,n_- + p_{1T} \approx xP + p_{1T}, \tag{3.103}$$

$$p_2 \equiv \frac{yQ}{x_1\sqrt{2}}\,n_+ + \frac{x_1(p_2^2+\boldsymbol{p}_{2T}^2)}{yQ\sqrt{2}}\,n_- + p_{2T} \approx yP + p_{2T}. \tag{3.104}$$

In the calculations up to and including order $1/Q$, one encounters $\Phi_D^\alpha(p_1, p_2)$ integrated over all, but the $+$ components, for which we use the following parametrization in terms of two-argument distribution functions [10, 11]:

$$\Phi_D^\alpha(x,y) \equiv (P^+)^2 \int dp_1^- d^2\boldsymbol{p}_{1T}\, dp_2^- d^2\boldsymbol{p}_{2T}\, \Phi_D^\alpha(P,S;p_1,p_2)\Big|_{\substack{p_1^+=xP^+ \\ p_2^+=yP^+}} \tag{3.105}$$

$$= \frac{M}{2}\Big[G_D(x,y)\, i\epsilon_T^{\alpha\beta} S_{T\beta} \not{P} + \tilde{G}_D(x,y)\, S_T^\alpha \gamma_5 \not{P} + H_D(x,y)\lambda\gamma_5\gamma_T^\alpha \not{P} + E_D(x,y)\gamma_T^\alpha \not{P}\Big],$$

where $\epsilon_T^{\mu\nu} = \epsilon^{\alpha\beta\mu\nu} n_{+\alpha} n_{-\beta}$. We make a similar expansion for $\Phi_A^\alpha(x,y)$ with the functions $G_D, \ldots$ replaced by $G_A, \ldots$, while the rest stays the same.

Hermiticity then gives for the two-argument functions in Eq. (3.105) the following constraints:

$$G_D(x,y) = -G_D^*(y,x), \tag{3.106}$$
$$\tilde{G}_D(x,y) = \tilde{G}_D^*(y,x), \tag{3.107}$$
$$H_D(x,y) = H_D^*(y,x), \tag{3.108}$$
$$E_D(x,y) = -E_D^*(y,x). \tag{3.109}$$

Hence, the real and imaginary parts of these two-argument functions have definite symmetry properties under the interchange of the two arguments. If we would impose time reversal invariance all four functions must be real and $\tilde{G}_D$ and H_D are then symmetric and G_D and E_D are antisymmetric under interchange of the two arguments, such that at $x = y$ only $\tilde{G}_D$ and H_D survive. In the remainder of this chapter we do not impose time reversal invariance and hence allow for imaginary parts of these functions.

Also in accordance with Eq. (3.11) and the form of Eq. (3.105) we write:

$$\Phi_\partial^\alpha(x) \equiv \int d^2\boldsymbol{k}_T\, k_T^\alpha\, \Phi(x,k_T) = -\frac{M}{2}\Big[i f_{1T}^{\perp(1)}(x)\, i\epsilon_T^{\alpha\beta} S_{T\beta}\not{P} - g_{1T}^{(1)}(x)\, S_T^\alpha\gamma_5\not{P} + h_{1L}^{\perp(1)}(x)\lambda\gamma_5\gamma_T^\alpha\not{P} + i h_1^{\perp(1)}(x)\gamma_T^\alpha\not{P}\Big]. \tag{3.110}$$

One must take into account the difference between $\Phi_A^\alpha(x,y)$ and $\Phi_D^\alpha(x,y)$, which is proportional to $\Phi_\partial^\alpha(x)\,\delta(x-y)$. This difference is only zero, in case there are no transverse polarization vectors present. Similarly for the difference between $\overline{\Phi}_A^\alpha$ and $\overline{\Phi}_D^\alpha$. Recall that $f_{1T}^{\perp(1)}(x)$ and $h_1^{\perp(1)}(x)$ are T-odd.

We observe (since $iD^\alpha = i\partial^\alpha + gA^\alpha$):

$$\int dy\, [G_D(x,y) + G_D(y,x)] = \int dy\, [G_A(x,y) + G_A(y,x)] - 2i f_{1T}^{\perp(1)}(x), \tag{3.111}$$
$$\int dy\, \Big[\tilde{G}_D(x,y) + \tilde{G}_D(y,x)\Big] = \int dy\, \Big[\tilde{G}_A(x,y) + \tilde{G}_A(y,x)\Big] + 2g_{1T}^{(1)}(x), \tag{3.112}$$
$$\int dy\, [H_D(x,y) + H_D(y,x)] = \int dy\, [H_A(x,y) + H_A(y,x)] - 2h_{1L}^{\perp(1)}(x), \tag{3.113}$$
$$\int dy\, [E_D(x,y) + E_D(y,x)] = \int dy\, [E_A(x,y) + E_A(y,x)] - 2ih_1^{\perp(1)}(x), \tag{3.114}$$

while for the 'differences' no k_T^2-moments appear:

$$\int dy\, \left[G_D(x,y) - G_D(y,x)\right] = \int dy\, \left[G_A(x,y) - G_A(y,x)\right], \tag{3.115}$$

etc.

3.7 Equations of motion relations

The two-argument functions and the one-argument functions are related by the classical e.o.m., which hold inside hadronic matrix elements [7] (also cf. App. 2.A.) We will now derive these e.o.m. relations explicitly [8, 11, 22].

We will project out Dirac structures as follows:

$$\begin{aligned} \Phi_D^\alpha\,[\Gamma]\,(x) &= \int dy\ \mathrm{Tr}(\Phi_D^\alpha(y,x)\,\Gamma) \\ &= \int \frac{d\lambda}{2\pi}\, e^{i\lambda x}\, \langle P,S|\overline{\psi}(0)\,\Gamma\, iD^\alpha(\lambda n_-)\,\psi(\lambda n_-)|P,S\rangle \end{aligned} \tag{3.116}$$

and note that its hermitian conjugate is given by

$$(\Phi_D^\alpha\,[\Gamma])^\dagger\,(x) = \int dy\ \mathrm{Tr}\left(\Phi_D^\alpha(x,y)\,\gamma_0\Gamma^\dagger\gamma_0\right). \tag{3.117}$$

Due to the classical e.o.m. the following holds:

$$\langle P,S|\overline{\psi}(0)\,\not{n}_-\gamma_T^\nu\gamma_5\,[i\not{D}(\lambda n_-) - m]\,\psi(\lambda n_-)|P,S\rangle = 0. \tag{3.118}$$

By using the relation

$$\not{n}_-\gamma_T^\nu\gamma^\rho = \not{n}_- g_T^{\nu\rho} - n_-^\rho\gamma_T^\nu + i\epsilon^{\mu\alpha\rho\sigma} g_{T\alpha}{}^\nu\gamma_\sigma\gamma_5 n_{-\mu}, \tag{3.119}$$

one derives

$$\begin{aligned} &\langle P,S|\overline{\psi}(0)\,[g_T^{\nu\rho}\not{n}_-\gamma_5 + i\epsilon_T^{\rho\nu}\not{n}_-]\,iD_\rho(\lambda n_-)\psi(\lambda n_-)|P,S\rangle = \\ &\qquad \langle P,S|\overline{\psi}(0)\,[(n_-^\rho\gamma_T^\nu\gamma_5 - i\epsilon_T^{\nu\sigma}n_-^\rho\gamma_\sigma)\,iD_\rho(\lambda n_-) - m\not{n}_-\gamma_T^\nu\gamma_5]\,\psi(\lambda n_-)|P,S\rangle. \end{aligned} \tag{3.120}$$

One can do similar things for the identities (analogues of Eq. (3.118))

$$\langle P,S|\overline{\psi}(0)\,\not{n}_-\gamma_5\,[i\not{D}(\lambda n_-) - m]\,\psi(\lambda n_-)|P,S\rangle = 0, \tag{3.121}$$

$$\langle P,S|\overline{\psi}(0)\,\not{n}_-\,[i\not{D}(\lambda n_-) - m]\,\psi(\lambda n_-)|P,S\rangle = 0. \tag{3.122}$$

In this way we find for the real and imaginary parts of the e.o.m.:

$$\frac{1}{2}\Phi_D^\rho\left[g_{T\rho\nu}\not{n}_-\gamma_5 + i\epsilon_{T\rho\nu}\not{n}_-\right] + \text{h.c.} = x\Phi\left[\gamma_{T\nu}\gamma_5\right] - m\Phi\left[\not{n}_-\gamma_{T\nu}\gamma_5\right], \tag{3.123}$$

$$\frac{1}{2}\Phi_D^\rho\left[g_{T\rho\nu}\not{n}_-\gamma_5 + i\epsilon_{T\rho\nu}\not{n}_-\right] - \text{h.c.} = x\Phi\left[i\epsilon_{T\sigma\nu}\gamma_T^\sigma\right], \tag{3.124}$$

$$\frac{1}{2}\Phi_D^\rho\left[\not{n}_-\gamma_{T\rho}\gamma_5\right] + \text{h.c.} = \frac{x}{2}\Phi\left[\left(\not{n}_+\not{n}_- - \not{n}_-\not{n}_+\right)\gamma_5\right] - m\Phi\left[\not{n}_-\gamma_5\right], \tag{3.125}$$

$$\frac{1}{2}\Phi_D^\rho\left[\not{n}_-\gamma_{T\rho}\gamma_5\right] - \text{h.c.} = -x\Phi\left[\gamma_5\right], \tag{3.126}$$

$$\frac{1}{2}\Phi_D^\rho\left[\gamma_{T\rho}\not{n}_-\right] + \text{h.c.} = x\Phi\left[\mathbf{1}\right] - m\Phi\left[\not{n}_-\right], \tag{3.127}$$

$$\frac{1}{2}\Phi_D^\rho\left[\gamma_{T\rho}\not{n}_-\right] - \text{h.c.} = -\frac{x}{2}\Phi\left[\left(\not{n}_+\not{n}_- - \not{n}_-\not{n}_+\right)\right], \tag{3.128}$$

where h.c. stands for hermitian conjugate. Using the above parametrizations one has the following relations:

$$\int dy\left[G_D(x,y) - G_D(y,x) + \tilde{G}_D(x,y) + \tilde{G}_D(y,x)\right] = 2xg_T(x) - 2\frac{m}{M}h_1(x), \tag{3.129}$$

$$\int dy\left[G_D(x,y) + G_D(y,x) + \tilde{G}_D(x,y) - \tilde{G}_D(y,x)\right] = 2ixf_T(x), \tag{3.130}$$

$$\int dy\left[H_D(x,y) + H_D(y,x)\right] = xh_L(x) - \frac{m}{M}g_1(x), \tag{3.131}$$

$$\int dy\left[H_D(x,y) - H_D(y,x)\right] = -ixe_L(x), \tag{3.132}$$

$$\int dy\left[E_D(x,y) - E_D(y,x)\right] = xe(x) - \frac{m}{M}f_1(x), \tag{3.133}$$

$$\int dy\left[E_D(x,y) + E_D(y,x)\right] = ixh(x). \tag{3.134}$$

From this we see that the (T-odd) imaginary parts of the two-argument functions are related to the T-odd one-argument functions, as one expects. So if time reversal invariance is imposed, the imaginary parts of the e.o.m. Eqs. (3.130), (3.132) and (3.134) become three trivial equalities. We like to point out that if one integrates Eqs. (3.129) and (3.130) over x, weighted with some test-function $\sigma(x)$, one arrives at the sum rules discussed in [8, 67].

In order to observe the role of intrinsic transverse momentum, we will use some specific combinations of distribution functions, indicated by a tilde on the function (cf. App. 3.B). As mentioned before the tilde functions are the true interaction-dependent twist-three parts of subleading functions which often also contain twist-two parts called Wandzura-Wilczek parts [55] (in analogy to the case of g_2). They are defined such that in the analogues of Eqs. (3.129) - (3.134) for $G_A, \ldots$, only tilde functions appear,

$$\int dy\left[\operatorname{Re} G_A(x,y) + \operatorname{Re}\tilde{G}_A(x,y)\right] = xg_T(x) - \frac{m}{M}h_1(x) - g_{1T}^{(1)}(x) \equiv x\tilde{g}_T(x), \tag{3.135}$$

$$\int dy\left[\operatorname{Im} G_A(x,y) + \operatorname{Im}\tilde{G}_A(x,y)\right] = xf_T(x) + f_{1T}^{\perp(1)}(x) \equiv x\tilde{f}_T(x), \tag{3.136}$$

$$\int dy\,\Big[2\,\mathrm{Re}\,H_A(x,y)\Big] = xh_L(x) - \frac{m}{M}g_1(x) + 2\,h_{1L}^{\perp(1)}(x) \equiv x\tilde{h}_L(x), \tag{3.137}$$

$$\int dy\,\Big[2\,\mathrm{Im}\,H_A(x,y)\Big] = -xe_L(x) \equiv -x\tilde{e}_L(x), \tag{3.138}$$

$$\int dy\,\Big[2\,\mathrm{Re}\,E_A(x,y)\Big] = xe(x) - \frac{m}{M}f_1(x) \equiv x\tilde{e}(x), \tag{3.139}$$

$$\int dy\,\Big[2\,\mathrm{Im}\,E_A(x,y)\Big] = xh(x) + 2\,h_1^{\perp(1)}(x) \equiv x\tilde{h}(x). \tag{3.140}$$

3.8 Summary

In this chapter we studied the properties of correlation functions, like $\Phi, \overline{\Phi}, \Phi_A^\alpha$ and Φ_D^α. The symmetries of QCD impose restrictions on these correlation functions, namely, they restrict the number of independent structures, but they often also give rise to sum rules for these structures and to bounds.

Next to the standard objections to the possible validity of the BC sum rule, like convergence of the integral, symmetry arguments cannot be used to proof that the integral of g_2 vanishes if the integration runs from 0 to 1. Of course one cannot exclude that there exist an additional argument why $g_{1T}^{(1)}(0) = -mh_1(0)/M$ should vanish, but since the verification of the sum rule needs extrapolation to small x values anyway, at least no conclusive answer can be reached from experiment.

We also discussed the Burkardt and ELT sum rules and the underlying assumptions, like interchanging of integrals. Some new sum rules were derived. We also made an inventory of the T-odd distribution functions, which will play an important role in the next chapter.

3.A Distribution functions in terms of amplitudes

The expressions for the time-reversal even distribution functions in terms of the amplitudes $A_i(\sigma,\tau)$, where $\sigma = 2\,k\cdot P$ and $\tau = k^2$, is as follows:

$$f_1(x,\boldsymbol{k}_T^2) = \int \ldots\ [A_2 + xA_3]\,, \tag{3.A1}$$

$$g_{1L}(x,\boldsymbol{k}_T^2) = \int \ldots\ \left[-A_6 - \left(\frac{\sigma - 2xM^2}{2M^2}\right)(A_7 + xA_8)\right], \tag{3.A2}$$

$$g_{1T}(x,\boldsymbol{k}_T^2) = \int \ldots\ [A_7 + xA_8]\,, \tag{3.A3}$$

$$h_{1T}(x,\boldsymbol{k}_T^2) = \int \ldots\ [-(A_9 + xA_{10})]\,, \tag{3.A4}$$

$$h_{1L}^\perp(x,\boldsymbol{k}_T^2) = \int \ldots\ \left[A_{10} - \left(\frac{\sigma - 2xM^2}{2M^2}\right)A_{11}\right], \tag{3.A5}$$

$$h_{1T}^\perp(x,\boldsymbol{k}_T^2) = \int \ldots\ [A_{11}]\,, \tag{3.A6}$$

$$
\begin{aligned}
e(x, \boldsymbol{k}_T^2) &= \int \ldots \,[A_1]\,, & (3.A7)\\
f^{\perp}(x, \boldsymbol{k}_T^2) &= \int \ldots \,[A_3]\,, & (3.A8)\\
g'_T(x, \boldsymbol{k}_T^2) &= \int \ldots \,[-A_6]\,, & (3.A9)\\
g_L^{\perp}(x, \boldsymbol{k}_T^2) &= \int \ldots \left[-\left(\frac{\sigma - 2xM^2}{2M^2}\right) A_8\right], & (3.A10)\\
g_T^{\perp}(x, \boldsymbol{k}_T^2) &= \int \ldots \,[A_8]\,, & (3.A11)\\
h_T^{\perp}(x, \boldsymbol{k}_T^2) &= \int \ldots \,[-A_{10}]\,, & (3.A12)\\
h_L(x, \boldsymbol{k}_T^2) &= \int \ldots \left[-(A_9 + xA_{10}) - \left(\frac{\sigma - 2xM^2}{2M^2}\right) A_{10} + \left(\frac{\sigma - 2xM^2}{2M^2}\right)^2 A_{11}\right], & (3.A13)\\
h_T(x, \boldsymbol{k}_T^2) &= \int \ldots \left[-\left(\frac{\sigma - 2xM^2}{2M^2}\right) A_{11}\right], & (3.A14)
\end{aligned}
$$

where

$$
\int \ldots = \int d\sigma d\tau\, \delta(\boldsymbol{k}_T^2 + x^2M^2 + \tau - x\sigma). \qquad (3.A15)
$$

For the time-reversal odd distribution function one finds

$$
\begin{aligned}
f_{1T}^{\perp}(x, \boldsymbol{k}_T^2) &= \int \ldots \,[A_{12}]\,, & (3.A16)\\
h_1^{\perp}(x, \boldsymbol{k}_T^2) &= \int \ldots \,[-A_4]\,, & (3.A17)\\
e_L(x, \boldsymbol{k}_T^2) &= \int \ldots \left[-\left(\frac{\sigma - 2xM^2}{2M^2}\right) A_5\right], & (3.A18)\\
e_T(x, \boldsymbol{k}_T^2) &= \int \ldots \,[A_5]\,, & (3.A19)\\
f_L^{\perp}(x, \boldsymbol{k}_T^2) &= \int \ldots \,[-A_{12}]\,, & (3.A20)\\
f_T(x, \boldsymbol{k}_T^2) &= \int \ldots \left[-\left(\frac{\sigma - 2xM^2}{2M^2}\right) A_{12}\right], & (3.A21)\\
h(x, \boldsymbol{k}_T^2) &= \int \ldots \left[\left(\frac{\sigma - 2xM^2}{2M^2}\right) A_4\right]. & (3.A22)
\end{aligned}
$$

3.B Interaction dependent parts of distribution functions

A distribution function supplemented by a tilde is that part of a distribution function that is interaction dependent. The tilde functions are defined as follows:

$$e(x, \boldsymbol{k}_T) = \frac{m}{Mx} f_1(x, \boldsymbol{k}_T) + \tilde{e}(x, \boldsymbol{k}_T), \tag{3.B1}$$

$$f^\perp(x, \boldsymbol{k}_T) = \frac{1}{x} f_1(x, \boldsymbol{k}_T) + \tilde{f}^\perp(x, \boldsymbol{k}_T), \tag{3.B2}$$

$$g'_T(x, \boldsymbol{k}_T) = \frac{m}{Mx} h_{1T}(x, \boldsymbol{k}_T) + \tilde{g}'_T(x, \boldsymbol{k}_T), \tag{3.B3}$$

$$g_L^\perp(x, \boldsymbol{k}_T) = \frac{1}{x} g_{1L}(x, \boldsymbol{k}_T) + \frac{m}{Mx} h_{1L}^\perp(x, \boldsymbol{k}_T) + \tilde{g}_L^\perp(x, \boldsymbol{k}_T), \tag{3.B4}$$

$$g_T^\perp(x, \boldsymbol{k}_T) = \frac{1}{x} g_{1T}(x, \boldsymbol{k}_T) + \frac{m}{Mx} h_{1T}^\perp(x, \boldsymbol{k}_T) + \tilde{g}_T^\perp(x, \boldsymbol{k}_T), \tag{3.B5}$$

$$g_T(x, \boldsymbol{k}_T) = \frac{\boldsymbol{k}_T^2}{2M^2x} g_{1T}(x, \boldsymbol{k}_T) + \frac{m}{Mx} h_1(x, \boldsymbol{k}_T) + \tilde{g}_T(x, \boldsymbol{k}_T), \tag{3.B6}$$

$$h_T^\perp(x, \boldsymbol{k}_T) = \frac{1}{x} h_{1T}(x, \boldsymbol{k}_T) + \tilde{h}_T^\perp(x, \boldsymbol{k}_T), \tag{3.B7}$$

$$h_L(x, \boldsymbol{k}_T) = \frac{m}{Mx} g_{1L}(x, \boldsymbol{k}_T) - \frac{\boldsymbol{k}_T^2}{M^2x} h_{1L}^\perp(x, \boldsymbol{k}_T) + \tilde{h}_L(x, \boldsymbol{k}_T), \tag{3.B8}$$

$$h_T(x, \boldsymbol{k}_T) = \frac{m}{Mx} g_{1T}(x, \boldsymbol{k}_T) - \frac{h_{1T}(x, \boldsymbol{k}_T)}{x} - \frac{\boldsymbol{k}_T^2}{M^2x} h_{1T}^\perp(x, \boldsymbol{k}_T) + \tilde{h}_T(x, \boldsymbol{k}_T). \tag{3.B9}$$

3.C Support properties and time ordering

Here we will briefly repeat the arguments given in Ref. [68] why the time ordering of the operators inside the connected hadronic matrix elements is fictitious (first argued by Jaffe in [69], see also the comments on this in [70]). One also finds the support properties of the distribution functions.

We will look at the correlation function $\Phi(x, \boldsymbol{k}_T) = \int dk^- \Phi(k)$, where

$$\Phi(k) = \int \frac{d^4z}{(2\pi)^4} e^{ik\cdot z} \langle P, S|T\overline{\psi}(0)\psi(z)|P, S\rangle, \tag{3.C1}$$

is viewed as an amplitude describing the scattering of an off-shell antiquark with momentum $-k$ off an on-shell hadron with momentum P. It is a function of the invariants s, u and k^2 and the k^--component can be written in three ways:

$$k^- = \frac{s + \boldsymbol{k}_T^2}{2P^+(x-1)} + P^- = \frac{u + \boldsymbol{k}_T^2}{2P^+(x+1)} - P^- = \frac{k^2 + \boldsymbol{k}_T^2}{2P^+ x}. \tag{3.C2}$$

Assuming that $\Phi(k)$ has the standard analyticity properties of an amplitude, it will have cuts for nonnegative $\mathrm{Re}\, s, \mathrm{Re}\, u$ and a pole for nonnegative $\mathrm{Re}\, k^2$. By convention these are all slightly below the real axis.

For $|x| \geq 1$ the singularities in the complex k^- plane will then all be on the same side, such that the contour can be closed without encircling them, hence the amplitude vanishes. This results in a support of the distribution functions in the range $-1 \leq x \leq 1$.

For $0 \leq x \leq 1$ the s cut is in the upper half plane of k^- as opposed to the other singularities. Hence,

$$\int_{-\infty}^{\infty} dk^- \, \Phi(k) = \int dk^- \mathrm{Disc}_s \Phi(k). \tag{3.C3}$$

This is equivalent to insertion of a complete set of states in the amplitude that is not time ordered. For $-1 \leq x \leq 0$ a similar argument applies and yields Disc_u in Eq. (3.C3).

Next to the assumption of analyticity, there is also implicitly the assumption that the amplitude falls off fast enough at k^- infinity. For a discussion of these details see [68].

3.D Time reversal symmetry

Time reversal symmetry plays a central role in this thesis. QCD is time reversal invariant, hence we expect the observables under investigation also to be invariant under time reversal. How this symmetry poses constraints will be discussed in later chapters, but here we want to simply review how the time reversal operation acts on different objects (fields and Fock states).

Let us first look at this operation in ordinary quantum mechanics, cf. e.g. [71]. The time reversal operator $\mathcal{T}$ is an antiunitary operator. On a state vector it acts as $\mathcal{T} = UK$, where K is the complex conjugation operator, which is antilinear, and U is a unitary operator (due to preservation of normalization), such that $\mathcal{T}$ itself is antilinear and satisfies $\mathcal{T}^\dagger \mathcal{T} = \mathcal{T}\mathcal{T}^\dagger = I$, hence is called an antiunitary operator.

If Ψ is an arbitrary state and $\Psi'(-t) = \mathcal{T}\Psi(t)$ its time-reversed state, than their respective time developments are given by:

$$\begin{aligned} \Psi(t) &= e^{-iHt/\hbar}\Psi(0), & (3.D1)\\ \Psi'(t) &= e^{-iHt/\hbar}\Psi'(0), & (3.D2) \end{aligned}$$

hence $e^{-iHt/\hbar}\mathcal{T} = \mathcal{T}e^{+iHt/\hbar}$. If $\mathcal{T}$ would be unitary, then one would find $\{\mathcal{T}, H\} = 0$, implying that every stationary state with energy E would be accompanied by another stationary state –the time-reversed state– with energy $-E$. Hence, $\mathcal{T}$ cannot be unitary and one finds for the antiunitary $\mathcal{T}$, that $[\mathcal{T}, H] = 0$, which does not mean that it is a constant of motion, since $i\hbar\, d\langle A\rangle/dt = \langle [A, H]\rangle$ only holds for linear operators.

The transition amplitude from a state $|\Psi_i\rangle$ at time t_i to the state $|\Psi_f\rangle$ at time t_f is equal to the corresponding amplitude from $|\mathcal{T}\Psi_f\rangle$ at time t_i to the state $|\mathcal{T}\Psi_i\rangle$ at time t_f:

$$\langle\Psi_f|e^{-iH(t_f-t_i)}|\Psi_i\rangle = \langle\mathcal{T}\Psi_f|e^{-iH(t_i-t_f)}|\mathcal{T}\Psi_i\rangle^* = \langle\mathcal{T}\Psi_i|e^{-iH(t_f-t_i)}|\mathcal{T}\Psi_f\rangle. \tag{3.D3}$$

One also finds that $\mathcal{T}^2\Psi = \pm\Psi$; depending on the nature of the system one has either $+$ or $-$ for all Ψ. For instance, one has

$$\mathcal{T}|\alpha jm\rangle = e^{i\delta(\alpha,j)}(-1)^m|\alpha j\ -m\rangle, \tag{3.D4}$$

where α represents all quantum numbers apart from j, m, and moreover,

$$\mathcal{T}^2|\alpha jm\rangle = (-1)^{2j}|\alpha jm\rangle. \tag{3.D5}$$

So $\mathcal{T}^2 = 1$ for integer j and $\mathcal{T}^2 = -1$ for half-integer j. The first result Eq. (3.D4) is derived by using the fact that $\mathcal{T}\boldsymbol{J}\mathcal{T}^{-1} = -\boldsymbol{J}$ and the second, Eq. (3.D5), by using the antilinearity of $\mathcal{T}$. Also note that in case $\mathcal{T}^2\Psi = -\Psi$, one has $\langle\mathcal{T}\Psi|\Psi\rangle = 0$.

All this information can be used to derive the transformation properties of fields, cf. e.g. [45, 72]. One requires that annihilation operators transform as $\mathcal{T}a(\boldsymbol{p})\mathcal{T}^{-1} = e^{i\alpha(\boldsymbol{p})}a(-\boldsymbol{p})$ (and hence $\mathcal{T}a^\dagger(\boldsymbol{p})\mathcal{T}^{-1} = e^{-i\alpha(\boldsymbol{p})}a^\dagger(-\boldsymbol{p})$). The result is that a field that satisfies the Dirac equation transforms according to

$$\mathcal{T}\psi(\boldsymbol{r},t)\mathcal{T}^{-1} = \mathbf{T}\psi(\boldsymbol{r},-t), \quad \mathcal{T}\psi^\dagger(\boldsymbol{r},t)\mathcal{T}^{-1} = \psi^\dagger(\boldsymbol{r},-t)\mathbf{T}^\dagger, \tag{3.D6}$$

where $\mathbf{T}$ is a unitary matrix (which follows from the closure relation for the time-reversed spinors), that satisfies $\mathbf{T}\mathbf{T}^* = -\mathbf{1}$. One can also prove that due to the last property (from which one derives $e^{i\alpha(-\boldsymbol{p})-i\alpha(\boldsymbol{p})} = -1$), that $\mathcal{T}^2a(\boldsymbol{p})\mathcal{T}^{-2} = -a(\boldsymbol{p})$ and this means that $\mathcal{T}^2$ acts like $+I$ $(-I)$ on a state with an even (odd) number of fermions, just as one expects from the above-mentioned result for an integer (half-integer) j system.

In the standard representation $\mathbf{T} = i\gamma^1\gamma^3 = -i\gamma^5 C$, where $C = i\gamma^2\gamma^0$ is the charge conjugation matrix, which satisfies $C\gamma_\mu^T C^{-1} = -\gamma_\mu$. Furthermore, note that $(\gamma^5 C)^{-1} = -\gamma^5 C$.

Also time reversal acts on a scalar and vector field, called ϕ and A^μ respectively, as follows (modulo phases):

$$\mathcal{T}\phi(t,\boldsymbol{x})\mathcal{T}^{-1} = \phi(-t,\boldsymbol{x}), \tag{3.D7}$$

$$\mathcal{T}A^\mu(t,\boldsymbol{x})\mathcal{T}^{-1} = A_\mu(-t,\boldsymbol{x}). \tag{3.D8}$$

Chapter 4

Spin asymmetries in the Drell-Yan process

In Chap. 2 we discussed the DIS process and its separation into soft and hard parts. In this chapter we will look at the Drell-Yan process (DY): $p + p \to l + \bar{l} + X$ [73]. Unlike DIS, DY is a process with *two* soft parts. This implies that intrinsic transverse momentum of the quarks will play an important role, even at leading order. From the results in Chap. 2 it can be concluded that in DIS transverse momentum of the quarks always leads to suppression by factors of $1/Q$ and that it can be absorbed into the definition of the quark-gluon correlation functions, such that only correlation functions depending on lightcone momentum fractions are relevant (i.e., only $\Phi(x), \Phi_D^\alpha(x,y)$ appear).

In DY the transverse momentum of the quarks of both incoming hadrons are linked via momentum conservation, which prevents that correlation functions integrated over transverse momenta appear exclusively. Only if one integrates over the transverse momentum of the photon (or Z-boson) producing the lepton pair, can one describe the process in terms of correlation functions depending only on lightcone momentum fractions. But even in that case the effect of intrinsic transverse momentum cannot be completely combined with Φ_A^α to yield Φ_D^α solely.

Several studies of DY at the leading twist have been performed [74, 9, 47, 75], which often include polarization and/or intrinsic transverse momentum. At subleading twist [11, 12, 76, 22] similar investigations using EFP-like approaches have been done (note that the OPE is not applicable, since the process is not lightcone dominated, cf. e.g. [29]). In this chapter we study DY at subleading twist, integrated over the transverse momentum of the photon and focus on spin asymmetries arising in that case. In particular we investigate single spin asymmetries, like the one discussed in Ref. [77].

In the regular description of DY in terms of quark and antiquark distribution functions time reversal symmetry, or rather the absence of time-reversal odd (T-odd) distribution functions, implies the absence of single spin asymmetries at tree level (but including subleading order in $1/Q$) [76, 22]. However, T-odd distribution functions could be present without violating time reversal symmetry if the incoming hadrons cannot be treated as plane-wave states. This may occur due to some factorization breaking mechanism [65, 78]. We will show that, even apart from such mechanisms, the contributions of T-odd distribution functions may effectively arise due to the presence of so-called gluonic poles in the twist-three hadronic matrix elements [79, 80, 81]. The resulting single spin asymmetries in DY (like the one of Ref. [77]) cannot be distinguished from those stemming from T-odd distribution functions, although time reversal invariance is not broken by the presence of such poles.

The outline of this chapter is as follows. We will first discuss how DY is described in terms of correlation functions, which themselves are parametrized in terms of distribution functions (see previous chapter). In the next section we present the DY cross section integrated over the transverse momentum of the photon, with emphasis on the effects of properly taking into account intrinsic transverse momentum. Next we investigate the behavior of the quark-gluon correlation function in case it has a pole when the gluon has zero momentum. We will show that such poles will effectively contribute to the imaginary part of the equations of motion (e.o.m.) and hence, to T-odd distribution functions (see previous chapter). The large distance nature of gluonic poles is elaborated upon, in particular, boundary conditions are investigated. In the final section we present those contributions to the DY cross section which arise due to the effective T-odd distribution functions.

4.1 The Drell-Yan process in terms of correlation functions

As in the case of DIS we restrict to tree level, but include $1/Q$ power corrections. The photon producing the lepton pair has a timelike momentum q, which sets the scale Q, such that $Q^2 = q^2$. We do not take Z bosons into account, since the asymmetries of interest are all suppressed by a factor of $1/Q$ and are likely to be negligible at such high energies.

The Drell-Yan process consists of two soft parts (depicted in Fig. 4.1 is the leading order diagram [9]) and one of the soft parts is described by the quark correlation functions $\Phi(P_1, S_1; p)$ and $\Phi_A^\alpha(P_1, S_1; p, p_1)$, and the other soft part by the antiquark correlation functions, denoted by $\overline{\Phi}(P_2, S_2; k)$ and $\overline{\Phi}_A^\alpha(P_2, S_2; k, k_1)$.

The momenta of the quarks, which annihilate into the photon, are predominantly along the direction of the parent hadrons. One hadron momentum (P_1) is chosen to be predominantly along the lightlike direction given by the vector n_+. The second hadron with momentum P_2 is predominantly in the n_- direction, such that $P_1 \cdot P_2 = \mathcal{O}(Q^2)$. We make the following Sudakov decompositions:

$$P_1^\mu \equiv \frac{Q}{x_1\sqrt{2}}\, n_+^\mu + \frac{x_1 M_1^2}{Q\sqrt{2}}\, n_-^\mu, \tag{4.1}$$

$$P_2^\mu \equiv \frac{x_2 M_2^2}{Q\sqrt{2}}\, n_+^\mu + \frac{Q}{x_2\sqrt{2}}\, n_-^\mu, \tag{4.2}$$

$$q^\mu \equiv \frac{Q}{\sqrt{2}}\, n_+^\mu + \frac{Q}{\sqrt{2}}\, n_-^\mu + q_T^\mu, \tag{4.3}$$

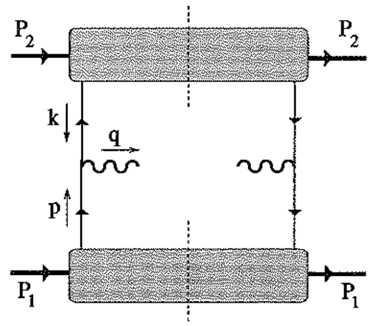


Figure 4.1: The leading order contribution to the Drell-Yan process

for $Q_T^2 \equiv -q_T^2 \ll Q^2$. Furthermore, we decompose the parton momenta p, p_1 and the spin vector S_1 of hadron one as

$$p \equiv \frac{xQ}{x_1\sqrt{2}} n_+ + \frac{x_1(p^2 + \boldsymbol{p}_T^2)}{xQ\sqrt{2}} n_- + p_T \approx xP_1 + p_T, \tag{4.4}$$

$$p_1 \equiv \frac{yQ}{x_1\sqrt{2}} n_+ + \frac{x_1(p_1^2 + \boldsymbol{p}_{1T}^2)}{yQ\sqrt{2}} n_- + p_{1T} \approx yP_1 + p_{1T}, \tag{4.5}$$

$$S_1 \equiv \frac{\lambda_1 Q}{x_1 M_1\sqrt{2}} n_+ - \frac{x_1\lambda_1 M_1}{Q\sqrt{2}} n_- + S_{1T} \approx \frac{\lambda_1}{M_1} P_1 + S_{1T}. \tag{4.6}$$

The vectors in $\overline{\Phi}$ and $\overline{\Phi}_A^\alpha$ are also decomposed in $n_\pm$,

$$k \equiv \frac{\bar{x}Q}{x_2\sqrt{2}} n_- + \frac{x_2(k^2 + \boldsymbol{k}_T^2)}{\bar{x}Q\sqrt{2}} n_+ + k_T \approx \bar{x}P_2 + k_T, \tag{4.7}$$

$$k_1 \equiv \frac{\bar{y}Q}{x_2\sqrt{2}} n_- + \frac{x_2(k_1^2 + \boldsymbol{k}_{1T}^2)}{\bar{y}Q\sqrt{2}} n_+ + k_{1T} \approx \bar{y}P_2 + k_{1T}, \tag{4.8}$$

$$S_2 \equiv \frac{\lambda_2 x_2 Q}{M_2\sqrt{2}} n_- - \frac{\lambda_2 M_2}{x_2 Q\sqrt{2}} n_+ + S_{2T} \approx \frac{\lambda_2}{M_2} P_2 + S_{2T}. \tag{4.9}$$

The four-momentum conservation delta-function at the photon vertex is written as (neglecting $1/Q^2$ contributions)

$$\delta^4(q - k - p) = \delta(q^+ - p^+)\,\delta(q^- - k^-)\,\delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T), \tag{4.10}$$

fixing $xP_1^+ = p^+ = q^+ = x_1 P_1^+$, i.e., $x = x_1$ and similarly $\bar{x} = x_2$, and allows up to $1/Q^2$ corrections for integration over p^- and k^+. However, the transverse momentum integrations cannot be separated, unless one integrates over the transverse momentum of the photon. In that case one arrives at correlation functions also integrated over their transverse momentum dependence, such that they only depend on the momentum fractions x, y and $\bar{x}, \bar{y}$. These partly integrated correlation functions $\Phi(x), \overline{\Phi}(\bar{x}), \Phi_A^\alpha(x,y)$ and $\overline{\Phi}_A^\alpha(\bar{x},\bar{y})$ are parametrized in terms of the distribution functions. For details see Chap. 3.

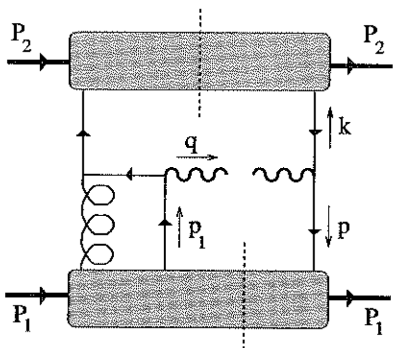


Figure 4.2: Subleading order contribution to the Drell-Yan process

Up to order $1/Q$ the five relevant diagrams (cf. Fig. 4.2 for a subleading order contribution) lead to the following expression for the hadron tensor integrated over the transverse momentum of the photon:

$$\begin{aligned}
\int d^2\boldsymbol{q}_T\, \mathcal{W}^{\mu\nu} &= \frac{1}{3}\Bigg\{\mathrm{Tr}\left(\Phi(x)\gamma^\mu\overline{\Phi}(\bar{x})\gamma^\nu\right) \\
&+ \int dy\, \mathrm{Tr}\left(\Phi_A^\alpha(y,x)\gamma^\mu\overline{\Phi}(\bar{x})\gamma_\alpha\frac{\not{n}_+}{Q\sqrt{2}}\frac{x-y}{x-y+i\epsilon}\gamma^\nu\right) \\
&+ \int dy\, \mathrm{Tr}\left(\Phi_A^\alpha(x,y)\gamma^\mu\frac{\not{n}_+}{Q\sqrt{2}}\frac{x-y}{x-y-i\epsilon}\gamma_\alpha\overline{\Phi}(\bar{x})\gamma^\nu\right) \\
&- \int d\bar{y}\, \mathrm{Tr}\left(\Phi(x)\gamma^\mu\overline{\Phi}_A^\alpha(\bar{y},\bar{x})\gamma^\nu\frac{\not{n}_-}{Q\sqrt{2}}\frac{\bar{x}-\bar{y}}{\bar{x}-\bar{y}+i\epsilon}\gamma_\alpha\right) \\
&- \int d\bar{y}\, \mathrm{Tr}\left(\Phi(x)\gamma_\alpha\frac{\not{n}_-}{Q\sqrt{2}}\frac{\bar{x}-\bar{y}}{\bar{x}-\bar{y}-i\epsilon}\gamma^\mu\overline{\Phi}_A^\alpha(\bar{x},\bar{y})\gamma^\nu\right)\Bigg\}. \qquad (4.11)
\end{aligned}$$

We will explain the above expression. The factor $1/3$ arises from the color averaging in the $q\bar{q}$ annihilation. We have omitted flavor indices and summation; furthermore, there is a contribution from diagrams with reversed fermion flow, which is similar to the above expression but with $\mu \leftrightarrow \nu$ and $q \to -q$ replacements. In the expression the terms with $\not{n}_\pm$ arise from the fermion propagators in the hard part neglecting contributions that will appear suppressed by powers of Q^2,

$$\frac{\not p_1 - \not q + m}{(p_1-q)^2 - m^2 + i\epsilon} \approx -\frac{\not n_+}{Q\sqrt{2}} \frac{x-y}{x-y+i\epsilon}, \tag{4.12}$$

$$\frac{\not q - \not k_1 + m}{(q-k_1)^2 - m^2 + i\epsilon} \approx \frac{\not n_-}{Q\sqrt{2}} \frac{\bar{x}-\bar{y}}{\bar{x}-\bar{y}+i\epsilon}, \tag{4.13}$$

where the approximations hold only when the propagators are embedded in the diagrams (see also the next chapter Eqs. (5.66) and (5.67)).

We note that in the above expression one cannot simply replace $\Phi_A^\alpha(x,y)$ by $\Phi_D^\alpha(x,y)$. One must take into account the difference proportional to $\Phi_\partial^\alpha(x) = \int d^2\boldsymbol{p}_T p_T^\alpha \Phi(x,\boldsymbol{p}_T)$ (due to $iD^\alpha = i\partial^\alpha + gA^\alpha$). This difference is only zero, in case there are no transverse polarization vectors present. Similarly for the difference between $\overline{\Phi}_A^\alpha$ and $\overline{\Phi}_D^\alpha$.

From these expressions one observes that the case $x = y$, i.e., the case of a zero-momentum gluon in the quark-gluon correlation function, corresponds to an on-shell quark (see Fig. 4.2) and hence is sensitive to the imaginary part of the quark propagator. We also note that

$$\int dy\, \Phi_A^\alpha(x,y)\, \frac{x-y}{x-y+i\epsilon} \overset{\Phi_A^\alpha(x,x)=0}{\Longrightarrow} \int dy\, \Phi_A^\alpha(x,y), \tag{4.14}$$

which is implicitly assumed in [76] and as a consequence they do not find any single spin asymmetries. Let us first investigate this case and after that study what happens if one assumes that $\Phi_A^\alpha(x,x) \neq 0$.

4.2 The Drell-Yan cross section in terms of distribution functions

We will now discuss the Drell-Yan cross section in case one integrates over the transverse momentum of the photon and assumes $\Phi_A^\alpha(x,x) = 0$. The cross section is obtained by contracting the hadron tensor with the lepton tensor:

$$\frac{d\sigma}{d\Omega\, d^4q} = \frac{\alpha^2}{2\, s\, Q^4} L_{\mu\nu} W^{\mu\nu}, \tag{4.15}$$

where the (unpolarized) lepton tensor in terms of the lepton momenta l, l' is given as

$$L_{\mu\nu} = 2 l_{\{\mu} l'_{\nu\}} - Q^2 g_{\mu\nu}. \tag{4.16}$$

By integrating over the transverse momentum of the photon one arrives at[1]:

$$\frac{d\sigma}{d\Omega\, dx_1\, dx_2} = \frac{\pi\alpha^2}{2\, Q^4} L_{\mu\nu} \int d^2\boldsymbol{q}_T\, W^{\mu\nu}. \tag{4.17}$$

[1]The lepton tensor is for every choice of its components just a function of $y = l^-/q^-$ and hence does not depend on $\boldsymbol{q}_T$.

One uses the parametrizations of the correlation functions in terms of distribution functions (cf. Eq. (3.24)) in the expression for the integrated hadron tensor as given in Eq. (4.11). The parametrizations in terms of distribution functions are defined with help of the vectors n_+, n_- and several transverse vectors. However, angles we are going to discuss with respect to another set of vectors. Depending on the choice of this set, we find different combinations of distribution functions with and without a tilde (cf. Eqs. (3.135)–(3.140)). Needless to say that the cross section itself is an observable and does not depend on the choice of vectors, even though its appearance changes.

We choose the following sets of normalized vectors:

$$\hat{t} \equiv q/Q, \tag{4.18}$$

$$\hat{z} \equiv (1-c)\frac{2x_1}{Q}\tilde{P}_1 - c\frac{2x_2}{Q}\tilde{P}_2, \tag{4.19}$$

$$\hat{x} \equiv q_T/Q_T = (q - x_1 P_1 - x_2 P_2)/Q_T, \tag{4.20}$$

characterized by a parameter c and where $\tilde{P}_i \equiv P_i - q/(2x_i)$, such that:

$$n_+^\mu = \frac{1}{\sqrt{2}}\left[\hat{t}^\mu + \hat{z}^\mu - 2c\frac{Q_T}{Q}\hat{x}^\mu\right], \tag{4.21}$$

$$n_-^\mu = \frac{1}{\sqrt{2}}\left[\hat{t}^\mu - \hat{z}^\mu - 2(1-c)\frac{Q_T}{Q}\hat{x}^\mu\right]. \tag{4.22}$$

So the parameter c distributes the transverse momentum between P_1 and P_2 in different ways (Fig. 4.3). If $c = 0$ ($c = 1$), then P_1 (P_2) has no transverse component. The symmetric case $c = 1/2$ is the one used in Ref. [82].

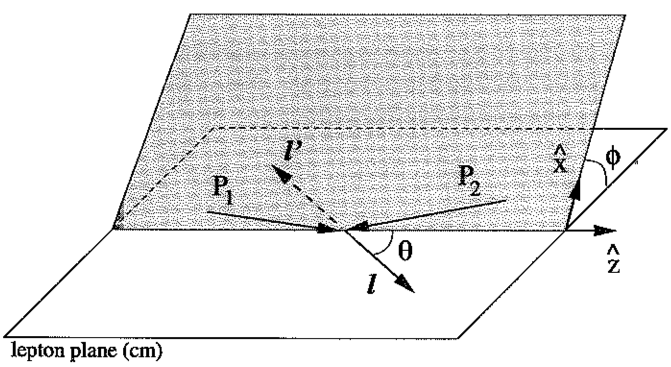


Figure 4.3: Kinematics of the Drell-Yan process in the lepton center of mass frame, for a particular value of c.

In this way we arrive at the following expression for the Drell-Yan cross section in case of unpolarized leptons:

$$
\begin{aligned}
\frac{d\sigma(h_1 h_2 \to l\bar{l}X)}{d\Omega dx_1 dx_2} &= \frac{\alpha^2}{3Q^2} \sum_{a,\bar{a}} e_a^2 \Big\{ A(y) \left(f_1 \overline{f}_1 - \lambda_1 \lambda_2 g_1 \overline{g}_1\right) \\
&+ B(y)\, |\boldsymbol{S}_{1T}|\, |\boldsymbol{S}_{2T}|\, \cos(\phi_{S_1} + \phi_{S_2})\, \left(h_1 \overline{h}_1\right) + 2\, C(y) \times \\
&\Big[\lambda_2\, |\boldsymbol{S}_{1T}|\, \cos(\phi_{S_1}) \left(\frac{M_1}{Q}\, x_1 \left((1-c) g_T + c\tilde{g}_T\right) \overline{g}_1 + \frac{M_2}{Q}\, x_2\, h_1 \left(c\overline{h}_L + (1-c)\tilde{\overline{h}}_L\right)\right) \\
&- \lambda_1\, |\boldsymbol{S}_{2T}|\, \cos(\phi_{S_2}) \left(\frac{M_2}{Q}\, x_2\, g_1 \left(c\overline{g}_T + (1-c)\tilde{\overline{g}}_T\right) + \frac{M_1}{Q}\, x_1 \left(\left((1-c) h_L + c\tilde{h}_L\right) \overline{h}_1\right)\right)\Big]\Big\},
\end{aligned}
\tag{4.23}
$$

where $d\Omega = 2dy\, d\phi^l$; ϕ^l gives the orientation of $\hat{l}_\perp^\mu \equiv \left(g^{\mu\nu} - \hat{t}^{\{\mu}\hat{t}^{\nu\}} + \hat{z}^{\{\mu}\hat{z}^{\nu\}}\right) l_\nu$, the perpendicular part of the lepton momentum l; ϕ_{S_i} is the angle between $\boldsymbol{S}_{iT}$ and $\hat{l}_\perp$ and $y = l^-/q^-$. In this result we encounter the following functions of y, which in the lepton center of mass frame equals $y = (1 + \cos\theta)/2$, where θ is the angle of $\hat{z}$ with respect to the momentum of the outgoing lepton l (cf. Fig. 4.3):

$$A(y) = \left(\frac{1}{2} - y + y^2\right) \stackrel{cm}{=} \frac{1}{4}\left(1 + \cos^2\theta\right), \tag{4.24}$$

$$B(y) = y\,(1-y) \stackrel{cm}{=} \frac{1}{4}\sin^2\theta, \tag{4.25}$$

$$C(y) = (1-2y)\sqrt{y\,(1-y)} \stackrel{cm}{=} -\frac{1}{4}\sin(2\theta). \tag{4.26}$$

Furthermore, $f_1\overline{f}_1 = f_1^a(x_1)\overline{f}_1^a(x_2)$, etc., where a is the flavor index.

For $c = 1/2$ we find agreement with the results of [76] for the cross section. Hence, we confirm the deviation of that result from the one found in [11], where a collinear expansion of the soft part is performed (cf. their Eq. (65)). The reason that this is not allowed is that the intrinsic transverse momentum in the soft part is not small compared to the soft hadronic scale M (or Λ)[2]. A collinear expansion of the soft part will result in $\delta^2(\boldsymbol{q}_T)$ and hence, in the absence of tilde functions.

There is no choice of c to eliminate the tilde functions from this expression, nor to only retain tilde functions. For $c = 1$ ($c = 0$) only tilde functions associated with hadron one (two) appear. This shows the nontrivial role of intrinsic transverse momentum of partons and one cannot discard it. This means that unlike in the case of DIS, one cannot take just $\Phi(x)$ and $\Phi_D^\alpha(x, y)$ as a basis.

In conclusion: under the assumption $\Phi_A^\alpha(x, x) = 0$ one only finds double spin asymmetries in the case of q_T-integrated DY at tree level including $1/Q$ corrections. The cross section will always depend on the difference between Φ_A^α and Φ_D^α as a consequence of nonzero intrinsic transverse momentum.

[2]Renormalization of the operators in the matrix elements might result in a dependence on a hard scale (compared to $|\boldsymbol{k}_T|$ and M), nevertheless terms proportional to $\boldsymbol{k}_T^2/M^2$ will remain present, hence expansion of the correlation function in $\boldsymbol{k}_T^2$ is not permitted.

4.3 Gluonic poles and time-reversal odd behavior

Now we are interested in the behavior of the quark-gluon correlation function Φ_A^α in case $x = y$, when the gluon has zero-momentum. For this purpose, we define (α is a transverse index)

$$\Phi^\alpha_{Fij}(x,y) \equiv \int \frac{d\lambda}{2\pi}\frac{d\eta}{2\pi} e^{i\lambda x} e^{i\eta(y-x)} \langle P,S|\overline{\psi}_j(0) F^{+\alpha}(\eta n_-)\psi_i(\lambda n_-)|P,S\rangle \tag{4.27}$$

and $F^{\rho\sigma}(z) = \frac{i}{g}\left[D^\rho(z), D^\sigma(z)\right]$. This matrix element has the same hermiticity, but the opposite time reversal behavior as Φ_D^α and Φ_A^α and we will parametrize it analogously with help of functions called $G_F, \tilde{G}_F, H_F$ and E_F (compare with Eq. (3.105)). So for the correlation function Φ_F^α we use the following parametrization

$$\begin{aligned}\Phi_F^\alpha(x,y) \;=\; & \frac{M_1}{2}\Big[G_F(x,y)\, i\epsilon_T^{\alpha\beta} S_{1T\beta} \not{P}_1 + \tilde{G}_F(x,y)\, S_{1T}^\alpha \gamma_5 \not{P}_1 \\ & + H_F(x,y)\lambda_1\gamma_5\gamma_T^\alpha \not{P}_1 + E_F(x,y)\gamma_T^\alpha \not{P}_1\Big].\end{aligned} \tag{4.28}$$

Hermiticity, parity and time reversal invariance yield the following relations:

$$[\Phi_F^\alpha(x,y)]^\dagger = \gamma_0\, \Phi_F^\alpha(y,x)\, \gamma_0 \qquad \text{[Hermiticity]} \tag{4.29}$$
$$\Phi_F^\alpha(x,y) = \gamma_0\, \Phi_{F\alpha}(x,y)\, \gamma_0 \qquad \text{[Parity]} \tag{4.30}$$
$$[\Phi_F^\alpha(x,y)]^* = -\gamma_5 C\, \Phi_{F\alpha}(x,y)\, C^\dagger \gamma_5 \qquad \text{[Time reversal]} \tag{4.31}$$

Hermiticity then gives for the two-argument functions in Eq. (4.28) the following constraints:

$$G_F(x,y) = -G_F^*(y,x), \tag{4.32}$$
$$\tilde{G}_F(x,y) = \tilde{G}_F^*(y,x), \tag{4.33}$$
$$H_F(x,y) = H_F^*(y,x), \tag{4.34}$$
$$E_F(x,y) = -E_F^*(y,x). \tag{4.35}$$

Just as for $G_A, \ldots$, the real and imaginary parts of these two-argument functions have definite symmetry properties under the interchange of the two arguments. If we would impose time reversal invariance all four functions must be imaginary and $\tilde{G}_F$ and H_F are then antisymmetric and G_F and E_F are symmetric under interchange of the two arguments, such that at $x = y$ only G_F and E_F survive (in contrast to G_D and E_D).

In the lightcone gauge $A^+ = 0$ one has $F^{+\alpha} = \partial^+ A_T^\alpha$ and by partial integration one finds

$$(x-y)\Phi_A^\alpha(x,y) = -i\Phi_F^\alpha(x,y). \tag{4.36}$$

If a specific Dirac projection of $\Phi_F^\alpha(x,x)$ is nonvanishing, then the corresponding projection of $\Phi_A^\alpha(x,x)$ has a pole, hence the name gluonic pole. An example is the function $T(x, S_T) = \pi \mathrm{Tr}\left[\Phi_F^\alpha(x,x)\, \epsilon_{T\beta\alpha} S_T^\beta \not{n}_-\right]/P^+ = 2\pi i M S_T^2 G_F(x,x)$ discussed by Qiu and Sterman [79].

In order to define Eq. (4.36) at the pole, one needs a prescription, which is related to the choice of boundary conditions on $A_T^\alpha(\eta = \pm\infty)$ inside physical matrix elements. Possible inversions of $F^{+\alpha} = \partial^+ A_T^\alpha$ are (only considering the dependence on the minus component):

$$\begin{aligned}
A_T^\alpha(y^-) &= A_T^\alpha(\infty) - \int_{-\infty}^{\infty} dz^-\ \theta(z^- - y^-)\, F^{+\alpha}(z^-) \\
&= A_T^\alpha(-\infty) + \int_{-\infty}^{\infty} dz^-\ \theta(y^- - z^-)\, F^{+\alpha}(z^-) \\
&= \frac{A_T^\alpha(\infty) + A_T^\alpha(-\infty)}{2} - \frac{1}{2}\int_{-\infty}^{\infty} dz^-\ \epsilon(z^- - y^-)\, F^{+\alpha}(z^-).
\end{aligned} \tag{4.37}$$

One can use the representations for the θ and ϵ functions,

$$\pm i\theta(\pm x) = \int \frac{dk}{2\pi}\frac{e^{ikx}}{k \mp i\epsilon}, \qquad i\epsilon(x) = \int \frac{dk}{2\pi}\mathrm{P}\frac{e^{ikx}}{k}, \tag{4.38}$$

to obtain

$$\begin{aligned}
\Phi_A^\alpha(x,y) &= \delta(x-y)\,\Phi_{A(\infty)}^\alpha(x) + \frac{-i}{x-y+i\epsilon}\,\Phi_F^\alpha(x,y) \\
&= \delta(x-y)\,\Phi_{A(-\infty)}^\alpha(x) + \frac{-i}{x-y-i\epsilon}\,\Phi_F^\alpha(x,y) \\
&= \delta(x-y)\,\frac{\Phi_{A(\infty)}^\alpha(x) + \Phi_{A(-\infty)}^\alpha(x)}{2} + \mathrm{P}\frac{-i}{x-y}\,\Phi_F^\alpha(x,y),
\end{aligned} \tag{4.39}$$

where

$$\delta(x-y)\,\Phi_{A(\pm\infty)\,ij}^\alpha(x) \equiv \int \frac{d\lambda}{2\pi}\frac{d\eta}{2\pi} e^{i\lambda x} e^{i\eta(y-x)} \langle P,S|\overline{\psi}_j(0) g A_T^\alpha(\eta = \pm\infty)\psi_i(\lambda n_-)|P,S\rangle. \tag{4.40}$$

So Eq. (4.39) shows the importance of boundary conditions in the inversion of Eq. (4.36), if matrix elements containing $A_T^\alpha(\eta = \pm\infty)$ do not vanish. When such matrix elements vanish (implicitly assumed in [76]) the pole prescription does not matter. Also one obtains

$$2\pi\,\Phi_F^\alpha(x,x) = \left[\Phi_{A(\infty)}^\alpha(x) - \Phi_{A(-\infty)}^\alpha(x)\right], \tag{4.41}$$

which shows the relation between the zero-momentum quark-gluon correlation function and the boundary conditions.

The behavior of $\Phi_{A(\pm\infty)}^\alpha(x)$ under time reversal is:

$$\Phi_{A(\pm\infty)}^{\alpha *}(x) = \gamma_5 C\, \Phi_{A(\mp\infty)\alpha}(x)\, C^\dagger \gamma_5. \tag{4.42}$$

One can show that this relation implies that time reversal invariance only allows for symmetric or antisymmetric boundary conditions (this assumes continuity of the functions). Both situations (if nonvanishing) lead to a singularity in $\Phi_A^\alpha(x,y)$ at the point $x = y$, but only the antisymmetric case will be called a gluonic pole. The symmetric case leads to a delta-function singularity.

4.4 Effective T-odd distribution functions

To study the effect of gluonic poles we will consider the (nonvanishing) antisymmetric boundary condition[3], $\Phi^{\alpha}_{A(\infty)}(x) = -\Phi^{\alpha}_{A(-\infty)}(x)$, which implies

$$\pi\,\Phi^{\alpha}_{F}(x,x) = \Phi^{\alpha}_{A(\infty)}(x), \tag{4.43}$$

$$\Phi^{\alpha *}_{A(\pm\infty)}(x) = -\gamma_5 C\,\Phi_{A(\pm\infty)\alpha}(x)\,C^{\dagger}\gamma_5. \tag{4.44}$$

In the calculation of the hadron tensor as given in Eq. (4.11) one always encounters the pole of the matrix element (in this case in the principal value prescription, cf. Eq. (4.39)) multiplied with the propagator in the hard subprocess (having a causal prescription), so we define an effective quark-gluon correlation function[4]

$$\begin{aligned}\Phi^{\alpha\,\mathrm{eff}}_{A}(y,x) &\equiv \frac{x-y}{x-y+i\epsilon}\,\Phi^{\alpha}_{A}(y,x) = \frac{-i}{x-y+i\epsilon}\,\Phi^{\alpha}_{F}(y,x)\\ &= \Phi^{\alpha}_{A}(y,x) - \pi\,\delta(x-y)\,\Phi^{\alpha}_{F}(y,x).\end{aligned} \tag{4.45}$$

Since $\Phi^{\alpha}_{F}(x,y)$ has the opposite behavior under time reversal compared to $\Phi^{\alpha}_{A}(x,y)$, the effective correlation function $\Phi^{\alpha\,\mathrm{eff}}_{A}(x,y)$ does not have definite behavior under time reversal symmetry. Specifically, the allowed T-even functions of $\Phi^{\alpha}_{F}(x,x)$, i.e., $G_F(x,x)$ and $E_F(x,x)$, can be identified with T-odd functions in the effective correlation function $\Phi^{\alpha\,\mathrm{eff}}_{A}(x,y)$. This implies that $G^{\mathrm{eff}}_{A}(x,y)$ and $E^{\mathrm{eff}}_{A}(x,y)$ will have an imaginary part and this gives rise to two "effective" T-odd distribution functions $\tilde{f}^{\mathrm{eff}}_{T}(x)$ and $\tilde{h}^{\mathrm{eff}}(x)$ via the (imaginary part of the) e.o.m.

To say it again in a different way: by partial integration we find for instance

$$(x-y)\,G_A(x,y) = -i\,G_F(x,y). \tag{4.46}$$

If one applies time reversal invariance, $G_A(x,y)$ will be a real function and $G_F(x,y)$ imaginary. So one expects the pole prescription to be the principal value. But when convoluting the pole of the matrix element (with this principal value prescription) with the propagator in the hard subprocess (with a causal prescription), it is formally possible to shift the imaginary part from the pole of the latter to the pole of the former. This will effectively give rise to a causal prescription in Eq. (4.46), instead of a principal value (but without the additional boundary term as would be required by time reversal, cf. Eq. (4.39)). This implies that $G_A(x,y)$ (and also $E_A(x,y)$) will effectively acquire an imaginary part.

Since by identification

$$i\pi\,G_F(x,x) = \int dy\,\mathrm{Im}\,G^{\mathrm{eff}}_{A}(y,x), \tag{4.47}$$

$$i\pi\,E_F(x,x) = \int dy\,\mathrm{Im}\,E^{\mathrm{eff}}_{A}(y,x), \tag{4.48}$$

[3] The consistency of antisymmetric boundary conditions with the Maxwell equations has already been shown in [37].

[4] Inversions like Eq. (4.39) are only allowed when accompanied by a regular function at the point $x=y$, which the propagator is not. However, Eq. (4.36) *can* be used safely always.

it follows from the imaginary parts of the e.o.m. Eqs. (3.136) and (3.140) that

$$x\tilde{f}_T^{\text{eff}}(x) = i\pi G_F(x,x) = \frac{1}{2MS_T^2}T(x,S_T), \tag{4.49}$$

$$x\tilde{h}^{\text{eff}}(x) = 2i\pi E_F(x,x) = \frac{-i\pi}{2MP^+}\mathrm{Tr}\left[\Phi_F^\alpha(x,x)\,\gamma_{T\alpha}\not{n}_-\right]. \tag{4.50}$$

The function $\tilde{e}_L^{\text{eff}}$ receives no gluonic pole contribution, since time reversal symmetry requires $H_F(x,x) = 0$.

Of course, the mechanism for generating finite projections of $\Phi_F^\rho(x,x)$ remains unknown. We just can conclude that if there is indeed a nonzero gluonic pole (in the case of nonzero antisymmetric boundary conditions), then at twist three there are two nonzero "effective" T-odd distribution functions, namely $\tilde{f}_T$ and $\tilde{h}$. The first one generates the twist-three single spin asymmetry found by Hammon *et al.* [77], which in their notation[5] is proportional to $T(x,x)$. The second one leads to an asymmetry with the same angular dependence (see next section). Summarizing, we find for the T-*even* parametrization of $\Phi_{A(\infty)}^\alpha(x)$ and equivalently $\pi\,\Phi_F^\alpha(x,x)$ (cf. Eq. (4.43)),

$$\Phi_{A(\infty)}^\alpha(x) \;=\; -\frac{ixM}{2}\left[\tilde{f}_T^{\text{eff}}(x)\,i\epsilon_T^{\alpha\beta}S_{T\beta}\not{P} + \frac{1}{2}\tilde{h}^{\text{eff}}(x)\gamma_T^\alpha\not{P}\right], \tag{4.51}$$

which is constrained by time reversal symmetry but behaves exactly opposite to for instance $\Phi_\partial^\alpha(x)$, which is another one-argument function with one Lorentz index, hence in their parametrizations the meaning of time-reversal even or odd functions are reversed too.

4.5 Single spin asymmetries

We will again discuss the Drell-Yan cross section in case one integrates over the transverse momentum of the photon, but now under the assumption that $\Phi_A^\alpha(x,x) \neq 0$ with the above discussed antisymmetric boundary conditions.

In this case we arrive at the following expression for the single spin asymmetries in the Drell-Yan cross section in case of unpolarized leptons (where we drop the superscript "eff"):

$$\begin{aligned}\frac{d\sigma(h_1h_2\to l\bar{l}X)}{d\Omega dx_1 dx_2} &= \frac{\alpha^2}{3Q^2}\sum_{a,\bar{a}} e_a^2\Bigg\{\ldots + 2\,C(y)\times \\ &\Bigg[\,|\boldsymbol{S}_{1T}|\,\sin(\phi_{S_1})\left(\frac{M_1}{Q}x_1\left((1-c)f_T + c\tilde{f}_T\right)\bar{f}_1 + \frac{M_2}{Q}x_2\,h_1\left(c\bar{\tilde{h}} + (1-c)\bar{\tilde{h}}\right)\right) \\ &+ |\boldsymbol{S}_{2T}|\,\sin(\phi_{S_2})\left(\frac{M_2}{Q}x_2\,f_1\left(c\bar{f}_T + (1-c)\bar{\tilde{f}}_T\right) + \frac{M_1}{Q}x_1\left((1-c)h + c\tilde{h}\right)\bar{h}_1\right)\Bigg]\Bigg\},\end{aligned} \tag{4.52}$$

where ... are the T-even contributions given in Eq. (4.23). We observe single transverse-spin asymmetries with two possible angular dependences, namely $\sin(\phi_{S_1})$ and $\sin(\phi_{S_2})$. Each of

[5]In [77] the asymmetry is in fact proportional to $T(x,x) - x\,dT(x,x)/dx$. The second terms arises due to an incorrect Fierz decomposition.

them comes with two products of an unpolarized function (f_1 or h) times a polarized one (f_T or h_1).

If we assume that the presence of T-odd distribution functions is only effective, i.e., arising from gluonic poles, and that $\Phi^{\alpha}_{A(\infty)} = \Phi^{\alpha}_{D(\infty)}$ (thereby neglecting transverse momentum of the fields at infinity), then $\tilde{f}_T^{\rm eff} = f_T^{\rm eff}$ and $\tilde{h}^{\rm eff} = h^{\rm eff}$. This implies the following single spin asymmetry (taking hadron two unpolarized), given in the lepton center of mass frame:

$$A_T = \frac{4\sin(2\theta)\ \sin(\phi_{S_1})}{1+\cos^2\theta}\frac{|\boldsymbol{S}_{1T}|}{Q}\frac{\sum_a e_a^2\left[M_1\,x_1\,f_T^a(x_1)f_1^{\bar a}(x_2) + M_2\,h_1^a(x_1)x_2\,h^{\bar a}(x_2)\right]}{\sum_a e_a^2\,f_1^a(x_1)f_1^{\bar a}(x_2)}. \quad (4.53)$$

The first term in the asymmetry (proportional to f_T) is the one discussed in [77] (in their notation it is proportional to $T(x,x)q(y)$). The second, new term is another gluonic pole contribution to the same single spin asymmetry in DY. It is not proportional to $T(x,S_T)$, but to a chiral-odd projection of Φ_F^{α} at the point $x = y$, cf. Eq. (4.50). Only the first asymmetry can be present in semi-inclusive DIS, which provides a means to eliminate one mechanism from which the T-odd function f_T can originate. If in DY it would for instance arise due to soft initial state interactions between the two incoming hadrons [65], then it would not be present in semi-inclusive DIS.

4.6 Discussion

The antisymmetric nonvanishing boundary condition for $\Phi^{\alpha}_{A(\pm\infty)}(x)$ might for instance arise from a linear A-field, giving a constant field strength (cf. e.g. [83, 84]). The constant field strength should be understood as an average value of the gluonic chromomagnetic field, which is nonzero due to a correlation with the direction of the proton spin. The large-distance origin of the asymmetries arising from such a gluonic pole is apparent.

The case of nonvanishing *symmetric* boundary conditions is less interesting, since $\Phi_F^{\alpha}(x,x) = 0$. The delta-function singularity in this case will contribute to the functions $\tilde{G}_A(x,x)$ and $\tilde{H}_A(x,x)$ and hence, by using the real part of the e.o.m., to T-even tilde functions $\tilde{g}_T$ and $\tilde{h}_L$, respectively. This would only affect the magnitude of (time-reversal even) double spin asymmetries.

We like to point out that both gluonic and fermionic poles play a role in off-forward scattering, such as prompt photon production [85, 79, 80, 81]. For instance, in that process a gluonic pole gives rise to an asymmetry proportional to $T(x,S_T)g(\bar{x})$ (see Fig. 4.4). Fermionic 'poles' in $\Phi_A^{\alpha}(x,y)$ proportional to $\delta(x)$ and $\delta(y)$ could also contribute in DY. Moreover, they will also give rise to $\delta(x)$ contributions to (and possibly violations of) the BC sum rule as discussed in the previous chapter.

The single spin asymmetries as discussed in the previous section have a different origin compared to the analogous asymmetries in inclusive hadron production in electron-positron

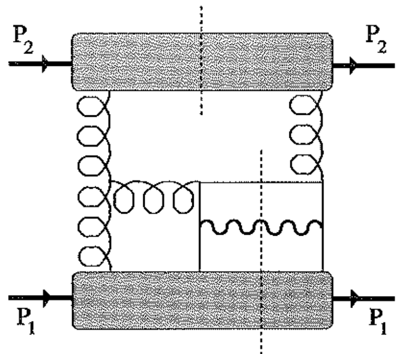


Figure 4.4: A diagram yielding a single transverse spin asymmetry in prompt photon production

annihilation [86, 26], which can also arise from final state interactions between particles within a jet, which are expected to be present always. The *fragmentation* function called D_T –the analogue of the distribution function f_T– shows up in a single spin asymmetry in hadron production in electron-positron annihilation [86, 26, 87] and is allowed because final state interactions lead to T-odd fragmentation functions, without a violation of time reversal symmetry (see next chapter). In Ref. [87] gluonic poles were taken into account and observed to contribute in the same way as final state interactions. Since the poles were given causal prescriptions (without including boundary terms), this result can be seen as an example of the relation between gluonic poles and *effective* fragmentation functions (arising possibly next to regular fragmentation functions).

Since we have chosen a different lightcone gauge for each soft part, we have to make a comment on color gauge invariance. In Chap. 2 we discussed how in DIS A^+ gluons generate the link operator necessary to make the definitions of the hadronic matrix elements color gauge invariant. In the present case of DY one must show that the A^+ gluons in $\Phi_A^+, \Phi_{AA}^{++}, \ldots, \Phi_{AA}^{\alpha+}, \ldots$, generate the appropriate links in $\Phi, \Phi_D^\alpha, \ldots$ (with straight paths along the n_- direction) and the A^- in $\overline{\Phi}_A^-, \overline{\Phi}_{AA}^{--}, \ldots, \overline{\Phi}_{AA}^{\alpha-}, \ldots$, generate the appropriate links in $\overline{\Phi}, \overline{\Phi}_D^\alpha, \ldots$ (with straight paths along the n_+ direction). In order to show this one needs to choose the correct pole prescriptions for the propagators, like given in the hadron tensor Eq. (4.11), and then use the above given representations for the θ functions. The case of DY without integration over the transverse momentum of the photon is slightly more involved as we will discuss in Chap. 7, when we investigate color gauge invariance for electron-positron annihilation into two hadrons (without integrating over $\boldsymbol{q}_T$), which is very similar to DY.

4.7 Conclusions

We have shown how the effects of so-called gluonic poles in twist-three hadronic matrix elements, which were first discussed by Qiu and Sterman [79], cannot be distinguished from that of T-odd distribution functions. We investigated this for the Drell-Yan process, which is expressed in terms of products of distribution functions. Even in the absence of T-odd distribution functions, imaginary parts of hard subprocesses together with gluonic poles would give rise to *effective* T-odd distribution functions. This would lead to single spin asymmetries for the Drell-Yan process, such as the one found by Hammon *et al.* [77]. We have found a similar asymmetry also arising from a gluonic pole, which involves chiral-odd distribution functions. We have moreover shown that the presence of gluonic poles is in accordance with time reversal invariance and requires a large distance gluonic field with antisymmetric boundary conditions. Our analysis shows also the nonnegligible role of intrinsic transverse momentum of the partons for the DY cross section at subleading order, also for the regular T-even asymmetries.

Chapter 5

Asymmetries in electron-positron annihilation

We present the results of the tree-level calculation of inclusive two-hadron production in electron-positron annihilation via one photon up to and including order $1/Q$. We consider the situation where the two hadrons belong to different, back-to-back jets. We include polarization of the produced hadrons and discuss azimuthal dependences of asymmetries. New asymmetries are found, in particular there is a *leading* $\cos(2\phi)$ asymmetry, which is even present when hadron polarization is absent, since it arises solely due to the intrinsic transverse momenta of the quarks.

5.1 Introduction

Three of the basic hard scattering processes in which the structure of hadrons is studied are (semi-)inclusive lepton-hadron scattering, the Drell-Yan process and inclusive hadron production in e^+e^- annihilation. In this chapter we will focus on the latter, where we restrict ourselves to photon exchange. The (timelike) photon momentum q sets the scale Q, where $Q^2 \equiv q^2$, which is much larger than characteristic hadronic scales.

The EFP approach to DIS was extended (based on analogy) to other processes in which the OPE is not applicable (for an alternative higher twist approach based on a nonlocal operator expansion see [88]). As mentioned in the previous chapter, the Drell-Yan process has been studied to leading and subleading order. Its cross section consists of a sum of products of two distribution functions. The complete tree level result up to and including order $1/Q$ for semi-inclusive polarized lepton-hadron scattering has also been obtained [42]. In this case one needs to include so-called fragmentation functions [89, 6] and the cross section is a sum of products of a distribution and a fragmentation function.

The present chapter focuses on the third process with two soft parts: inclusive two-hadron production in e^+e^- annihilation up to and including order $1/Q$, where the two hadrons belong to different, back-to-back jets. The cross section involves products of fragmentation functions, the number of which is larger than the number of distribution functions (we will assume from now on that T-odd distribution functions are absent, as are gluonic poles), due to the presence of time-reversal odd fragmentation functions, which are allowed since time reversal does not pose constraints on the functions, in the case of fragmentation, as will be explained. Interesting features like the Collins effect [90] show up, as do new asymmetries.

Some asymmetries, e.g. those due to the Collins effect, are leading effects, not suppressed by powers of $1/Q$, but one needs to include intrinsic transverse momentum in the fragmentation functions [6]. As was discussed in the previous chapter intrinsic transverse momentum plays a crucial role in processes with two soft parts, since the transverse momenta are linked by momentum conservation. Those distribution or fragmentation functions which, as functions of transverse momentum, would not contribute in processes with only one soft part, will show up in these processes with two soft parts. The idea that intrinsic transverse momentum always gives rise to suppression is incorrect, although it is the case in processes with one soft part. In the pioneering work [91, 92] on azimuthal dependences due to intrinsic transverse momentum, all effects are found to be suppressed by at least a factor of $1/Q$, as opposed to the ones we will discuss.

The outline of this chapter is as follows. In next section we present the formalism of the e^+e^- annihilation process, with emphasis on the kinematics. The third section contains the analysis of the soft parts of the process, in particular the fragmentation functions are studied. This is followed by the details of the complete tree-level calculation of the hadron tensor, at leading and subleading twist, in the fourth section, the result of which is given in the second appendix. In the three remaining sections we investigate special cases, which give more insight than the full result and are useful from a practical point of view. In the fifth section we discuss the result after integration over the transverse momentum of the photon. In the sixth section we study the differential cross section, i.e., not integrated over transverse momentum of the photon, but restricted to leading twist and the case where only one hadron is polarized (the case of two polarized hadrons is given in the third appendix). In the seventh section this is compared with the integrated cross section weighted with factors of the transverse momentum of the photon. Finally, the results are summarized in the last section.

Many aspects of the calculation are identical to the case of DY -discussed in the previous chapter-, but we will nevertheless write down the details for the present case very explicitly, since we do not necessarily integrate over the transverse momentum of the photon and this requires that some aspects concerning transverse momentum have to be specified more explicitly. Transverse momentum can for instance not just be taken into account by including the analogue of $\Phi^{\alpha}_{\partial}(x)$. On the other hand, to make the analysis not more complicated than necessary, we choose one particular frame in which to discuss angles (in the language of the previous chapter we choose a particular value for the parameter c).

5.2 Kinematics

We consider $e^- + e^+ \to$ hadrons, where the two leptons with momenta l and l' annihilate into a photon with momentum $q = l + l'$, which is timelike with $q^2 \equiv Q^2 \to \infty$. Denoting the momentum of outgoing hadrons by P_h ($h = 1, 2, \dots$) we use invariants $z_h = 2P_h \cdot q/Q^2$. The momenta can also be considered as jet momenta. We will consider the general case of polarized leptons with helicities $\pm\lambda_e$ and production of hadrons of which the spin states are characterized by a spin vector S_h ($h = 1, 2, \dots$), satisfying $S_h^2 = -1$ and $P_h \cdot S_h = 0$. In this way we can treat the case of unpolarized final states or final state hadrons with spin-0 and spin-1/2. We will work in the limit where Q^2 and $P_h \cdot q$ are large, keeping the ratios z_h finite.

The square of the amplitude can be split into a purely leptonic and a purely hadronic part,

$$|\mathcal{M}|^2 = \frac{e^4}{Q^4} L_{\mu\nu} H^{\mu\nu}, \tag{5.1}$$

with the helicity-conserving lepton tensor (neglecting the lepton masses) given by

$$L_{\mu\nu}(l, l'; \lambda_e) = 2l_\mu l'_\nu + 2l_\nu l'_\mu - Q^2 g_{\mu\nu} + 2i\lambda_e\, \epsilon_{\mu\nu\rho\sigma} l^\rho l'^\sigma. \tag{5.2}$$

For the case of two observed hadrons in the final state, the product of hadronic current matrix elements is written as

$$H_{\mu\nu}(P_X; P_1S_1; P_2S_2) = \langle 0|J_\mu(0)|P_X; P_1S_1; P_2S_2\rangle\langle P_X; P_1S_1; P_2S_2|J_\nu(0)|0\rangle, \tag{5.3}$$

where a summation over spins of the unobserved *out*-state X is understood. The cross section (including a factor 1/2 from averaging over incoming polarizations) is given by: for *two-particle inclusive* e^+e^- annihilation

$$\frac{P_1^0\, P_2^0\, d\sigma^{(e^+e^-)}}{d^3P_1\, d^3P_2} = \frac{\alpha^2}{4\,Q^6} L_{\mu\nu} \mathcal{W}^{\mu\nu}, \tag{5.4}$$

with

$$\mathcal{W}_{\mu\nu}(q; P_1S_1; P_2S_2) = \int \frac{d^3P_X}{(2\pi)^3 2P_X^0} \delta^4(q - P_X - P_1 - P_2) H_{\mu\nu}(P_X; P_1S_1; P_2S_2), \tag{5.5}$$

for *one-particle inclusive* e^+e^- annihilation

$$P_h^0 \frac{d\sigma}{d^3P_h} = \frac{\alpha^2}{2\,Q^6}\, L_{\mu\nu} W^{\mu\nu}, \tag{5.6}$$

with

$$\begin{aligned} W_{\mu\nu}(q; P_hS_h) = \frac{1}{(2\pi)} \int \frac{d^3P_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta^4(q - P_X - P_h) \\ \times\langle 0|J_\mu(0)|P_X; P_hS_h\rangle\langle P_X; P_hS_h|J_\nu(0)|0\rangle, \end{aligned} \tag{5.7}$$

and for the *totally inclusive* annihilation cross section the well-known result

$$\sigma(e^+e^- \to \text{hadrons}) = \frac{4\pi^2\,\alpha^2}{Q^6}\,L_{\mu\nu}R^{\mu\nu}, \tag{5.8}$$

with the tensor $R_{\mu\nu}$ given by

$$\begin{aligned} R_{\mu\nu}(q) &= \int \frac{d^3P_X}{(2\pi)^3 2P_X^0}(2\pi)^4\delta^4(q-P_X)\langle 0|J_\mu(0)|P_X\rangle\langle P_X|J_\nu(0)|0\rangle \\ &= \int d^4x\; e^{iq\cdot x}\,\langle 0|[J_\mu(x),J_\nu(0)]|0\rangle. \end{aligned} \tag{5.9}$$

Recall that the totally inclusive cross section is directly related to the vacuum polarization. Also note that the totally inclusive process is short-distance dominated, whereas the one-particle inclusive case is lightcone dominated, but only in the former the OPE can be applied.

In order to expand the lepton and hadron tensors in terms of independent Lorentz structures, it is convenient to work with vectors orthogonal to q. A normalized timelike vector is defined by q and a normalized spacelike vector is defined by $\tilde{P}^\mu = P^\mu - (P\cdot q/q^2)\,q^\mu$ for one of the outgoing momenta, say P_2,

$$\hat{t}^\mu \;\equiv\; \frac{q^\mu}{Q}, \tag{5.10}$$

$$\hat{z}^\mu \;\equiv\; \frac{Q}{P_2\cdot q}\,\tilde{P}_2^\mu \;=\; 2\,\frac{P_2^\mu}{z_2 Q} - \frac{q^\mu}{Q}. \tag{5.11}$$

Note that we (as usual) have neglected $1/Q^2$ corrections.

Vectors orthogonal to $\hat{z}$ and $\hat{t}$ are obtained with help of the tensors

$$g_\perp^{\mu\nu} \equiv g^{\mu\nu} - \hat{t}^\mu\hat{t}^\nu + \hat{z}^\mu\hat{z}^\nu, \tag{5.12}$$

$$\epsilon_\perp^{\mu\nu} \equiv -\epsilon^{\mu\nu\rho\sigma}\hat{t}_\rho\hat{z}_\sigma \;=\; \frac{1}{(P_2\cdot q)}\,\epsilon^{\mu\nu\rho\sigma}P_{2\,\rho}q_\sigma. \tag{5.13}$$

Since we have chosen hadron two to define the longitudinal direction, the momentum P_1 of hadron one can be used to express the directions orthogonal to $\hat{t}$ and $\hat{z}$. One obtains $P_{1\perp}^\mu = g_\perp^{\mu\nu}\,P_{1\nu}$ (see Fig. 5.1). We define the normalized vector $\hat{h}^\mu = P_{1\perp}^\mu/|\boldsymbol{P}_{1\perp}|$ and the second orthogonal direction is given by $\epsilon_\perp^{\mu\nu}\hat{h}_\nu$.

In the calculation of the hadron tensor it will be convenient to define *lightlike* directions using the hadronic (or jet) momenta. Consider two hadronic momenta P_1 and P_2 not belonging to one jet (i.e., their inner product $P_1\cdot P_2$ is of order Q^2). Like for DY, the momenta can then be parametrized using the dimensionless lightlike vectors n_+ and n_-,

$$P_1^\mu \equiv \frac{\zeta_1\tilde{Q}}{\sqrt{2}}\,n_-^\mu + \frac{M_1^2}{\zeta_1\tilde{Q}\sqrt{2}}\,n_+^\mu, \tag{5.14}$$

$$P_2^\mu \equiv \frac{M_2^2}{\zeta_2\tilde{Q}\sqrt{2}}\,n_-^\mu + \frac{\zeta_2\tilde{Q}}{\sqrt{2}}\,n_+^\mu, \tag{5.15}$$

$$q^\mu \equiv \frac{\tilde{Q}}{\sqrt{2}}\,n_-^\mu + \frac{\tilde{Q}}{\sqrt{2}}\,n_+^\mu + q_T^\mu, \tag{5.16}$$

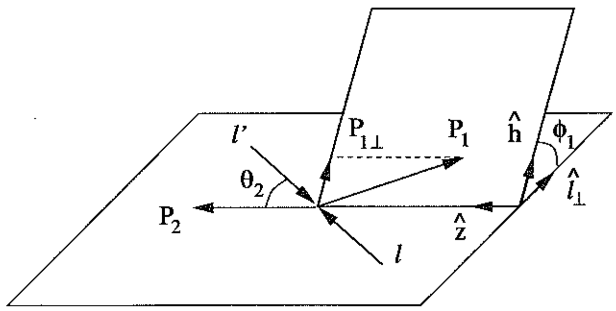


Figure 5.1: Kinematics of the annihilation process in the lepton center of mass frame for a back-to-back jet situation. P_2 is the momentum of a jet or of a fast hadron in a jet, P_1 is the momentum of a hadron belonging to the other jet.

where $\tilde{Q}^2 = Q^2 + Q_T^2$ with $q_T^2 \equiv -Q_T^2$. For the case of two back-to-back jets $Q_T^2 \ll Q^2$ and up to Q_T^2/Q^2, which we neglect, one has $\tilde{Q} = Q$, $\zeta_1 = z_1$ and $\zeta_2 = z_2$. If momentum P_2 is used to define the vector $\hat{z}^\mu$, then

$$P_{1\perp}^\mu = -z_1\, q_T^\mu = z_1\, Q_T\, \hat{h}^\mu. \tag{5.17}$$

Vectors transverse to n_+ and n_- one obtains using the tensors

$$g_T^{\mu\nu} \equiv g^{\mu\nu} - n_+^{\{\mu} n_-^{\nu\}}, \tag{5.18}$$

$$\epsilon_T^{\mu\nu} \equiv \epsilon^{\mu\nu\rho\sigma} n_{+\rho} n_{-\sigma}. \tag{5.19}$$

Note that these *transverse* tensors are not identical to the *perpendicular* ones defined above if the transverse momentum of the outgoing hadron does not vanish. The lightlike directions, however, can easily be expressed in $\hat{t}$, $\hat{z}$ and a perpendicular vector, say $\hat{h}$,

$$n_+^\mu = \frac{1}{\sqrt{2}}\left[\hat{t}^\mu + \hat{z}^\mu\right], \tag{5.20}$$

$$n_-^\mu = \frac{1}{\sqrt{2}}\left[\hat{t}^\mu - \hat{z}^\mu + 2\,\frac{Q_T}{Q}\,\hat{h}^\mu\right], \tag{5.21}$$

cf. Eqs. (4.21) and (4.22). This shows that the differences between $g_\perp^{\mu\nu}$ and $g_T^{\mu\nu}$ are of order $1/Q$. We will see however that taking transverse momentum into account does not automatically lead to suppression. Especially for the treatment of azimuthal asymmetries, it is important to keep track of these differences.

In summary, we use two sets of basis vectors, the first set constructed from the photon momentum (q) and one of the hadron momenta (P_2), the second set from the two hadron momenta (P_1 and P_2). The respective frames where the momenta q and P_2, or P_1 and P_2, are collinear are the natural ones connected to these two sets. In the first P_1 has a perpendicular component $P_{1\perp}$, in the second q has a transverse component q_T. One can get from one frame to the other via a Lorentz transformation that leaves the minus components unchanged [93].

5.3 Fragmentation correlation functions

In this section we discuss the relevant soft hadronic matrix elements that appear in the diagrammatic expansion of the process under consideration. Assuming the two hadrons to belong to two different jets we encounter two types of soft parts in the process under consideration: one describes the fragmentation of a quark into a hadron plus a remainder which is not detected and the other describes the similar fragmentation for an antiquark. Up to and including order $1/Q$ the quark fragmentation is described with help of two types

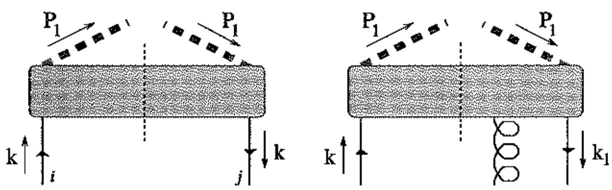


Figure 5.2: Correlation functions Δ and Δ_A.

of correlation functions: the quark correlation function $\Delta(P_1, S_1; k)$ [6] and the quark-gluon correlation function $\Delta_A^\alpha(P_1, S_1; k, k_1)$ [93] (Fig. 5.2):

$$\Delta_{ij}(P_1, S_1; k) = \sum_X \int \frac{d^4x}{(2\pi)^4}\, e^{ik\cdot x}\, \langle 0|\psi_i(x)|P_1, S_1; X\rangle\langle P_1, S_1; X|\overline{\psi}_j(0)|0\rangle, \tag{5.22}$$

$$\Delta^\alpha_{A\,ij}(P_1, S_1; k, k_1) = \sum_X \int \frac{d^4x}{(2\pi)^4}\frac{d^4y}{(2\pi)^4}\, e^{ik\cdot y + ik_1\cdot(x-y)} \times\langle 0|\psi_i(x)\, gA_T^\alpha(y)\,|P_1, S_1; X\rangle\langle P_1, S_1; X|\,\overline{\psi}_j(0)|0\rangle, \tag{5.23}$$

where k, k_1 are the quark momenta and an averaging over color indices is understood. If one chooses the gauge $A^- = 0$ only a transverse gluon is relevant. In fact, in a calculation up to subleading order, we only encounter the partly integrated correlation functions $\int dk^+\, \Delta(P_1, S_1; k)$ and $\int dk^+ d^4k_1\, \Delta_A^\alpha(P_1, S_1; k, k_1)$, which are functions of k^- and $\boldsymbol{k}_T$ only. Note that the definition of Δ_A^α includes one power of the strong coupling constant g. All this is analogous[1] to the correlation functions Φ and Φ_A^α.

The above matrix elements as functions of invariants are assumed to vanish sufficiently fast above a characteristic hadronic scale which is much smaller than Q^2. This means that in the above matrix elements $k^2, k\cdot P_1 \ll Q^2$. Hence, we make the following Sudakov decomposition for the quark momentum k:

$$k \equiv \frac{z_1 Q}{z\sqrt{2}}\, n_- + \frac{z(k^2 + \boldsymbol{k}_T^2)}{z_1 Q\sqrt{2}}\, n_+ + k_T \approx \frac{1}{z} P_1 + k_T. \tag{5.24}$$

[1]There are also differences between Φ and Δ, such as the behavior under time reversal symmetry and the normalization of the functions. Also, one cannot simply use anticommutation of spinors to derive symmetry relations between Δ and $\overline{\Delta}$.

Similarly, we decompose the spin vector S_1:

$$S_1 \equiv \frac{\lambda_1 z_1 Q}{M_1\sqrt{2}}\, n_- - \frac{\lambda_1 M_1}{z_1 Q\sqrt{2}}\, n_+ + S_{1T} \approx \frac{\lambda_1}{M_1} P_1 + S_{1T}, \tag{5.25}$$

with for a pure state $\lambda_1^2 + \boldsymbol{S}_{1T}^2 = 1$.

The Dirac structure of the quark correlation function can be expanded in a number of amplitudes, i.e., functions of invariants built up from the quark and hadron momenta, constrained by hermiticity and parity (cf. Eq. (3.10)). Here we directly integrate the correlation function over k^+, which up to and including order $1/Q$ can be parametrized as follows [42]

$$\begin{aligned}
\frac{1}{4z}\int dk^+\, \Delta(P_1,S_1;k)\Big|_{k^-=P_1^-/z,\ \boldsymbol{k}_T} &= \frac{M_1}{4P_1^-}\Big\{ E\,\mathbf{1} + D_1 \frac{\not{P}_1}{M_1} + D_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P_1^\nu k_T^\rho S_{1T}^\sigma}{M_1^2} \\
&+ D^\perp \frac{\not{k}_T}{M_1} + D_T\, \epsilon_{\mu\nu\rho\sigma} n_+^\mu n_-^\nu \gamma^\rho S_{1T}^\sigma + \lambda\, D_L^\perp \frac{\epsilon_{\mu\nu\rho\sigma} n_+^\mu n_-^\nu \gamma^\rho k_T^\sigma}{M_1} - E_s\, i\gamma_5 - G_{1s} \frac{\not{P}_1\gamma_5}{M_1} \\
&- G_T'\, \not{S}_{1T}\gamma_5 - G_s^\perp \frac{\not{k}_T\gamma_5}{M_1} - H_{1T} \frac{i\sigma_{\mu\nu}\gamma_5\, S_{1T}^\mu P_1^\nu}{M_1} - H_{1s}^\perp \frac{i\sigma_{\mu\nu}\gamma_5\, k_T^\mu P_1^\nu}{M_1^2} \\
&- H_T^\perp \frac{i\sigma_{\mu\nu}\gamma_5\, S_{1T}^\mu k_T^\nu}{M_1} - H_s\, i\sigma_{\mu\nu}\gamma_5\, n_-^\mu n_+^\nu + H_1^\perp \frac{\sigma_{\mu\nu} k_T^\mu P_1^\nu}{M_1^2} + H\, \sigma_{\mu\nu} n_-^\mu n_+^\nu \Big\},
\end{aligned} \tag{5.26}$$

where the shorthand notation G_{1s} stands for the combination

$$G_{1s}(z,\boldsymbol{k}_T) = \lambda_1\, G_{1L} + G_{1T} \frac{(\boldsymbol{k}_T\cdot \boldsymbol{S}_{1T})}{M_1}, \tag{5.27}$$

etc. The *real* functions $E, D_1, \ldots$ in Eq. (5.26) and $G_{1L}, G_{1T}, \ldots$ in $G_{1s}, \ldots$ are the fragmentation functions. One wants to express the fragmentation functions in terms of the hadron momentum, hence, the arguments of the fragmentation functions are chosen to be the lightcone (momentum) fraction $z = P_1^-/k^-$ of the produced hadron with respect to the fragmenting quark and $\boldsymbol{k}_T' \equiv -z\boldsymbol{k}_T$, which is the transverse momentum of the hadron in a frame where the quark has no transverse momentum. In order to switch from quark to hadron transverse momentum a Lorentz transformation leaving k^- and P_1^- unchanged needs to be performed. The fragmentation functions depend on z and $\boldsymbol{k}_T'^2$ only.

Inverting the above expression, the fragmentation functions appear in specific Dirac projections of the correlation functions, integrated over k^+:

$$\begin{aligned}
\Delta^{[\Gamma]}(z,\boldsymbol{k}_T) &\equiv \frac{1}{4z}\int dk^+\, \mathrm{Tr}(\Delta\,\Gamma)\Big|_{k^-=P_1^-/z,\ \boldsymbol{k}_T} \\
&= \sum_X \int \frac{dx^+ d^2\boldsymbol{x}_T}{4z\,(2\pi)^3}\, e^{ik\cdot x}\, \mathrm{Tr}\langle 0|\psi(x)|P_1,S_1;X\rangle\langle P_1,S_1;X|\overline{\psi}(0)\Gamma|0\rangle\Big|_{x^-=0},
\end{aligned} \tag{5.28}$$

for which we can distinguish the leading fragmentation functions:

$$\Delta^{[\gamma^-]}(z,\boldsymbol{k}_T) = D_1(z,\boldsymbol{k}_T'^2) + \frac{\epsilon_T^{ij}k_{Ti}S_{1Tj}}{M_1}\,D_{1T}^\perp(z,\boldsymbol{k}_T'^2), \tag{5.29}$$

$$\Delta^{[\gamma^-\gamma_5]}(z,\boldsymbol{k}_T) = G_{1s}(z,\boldsymbol{k}_T), \tag{5.30}$$

$$\Delta^{[i\sigma^{i-}\gamma_5]}(z,\boldsymbol{k}_T) = S_{1T}^i\,H_{1T}(z,\boldsymbol{k}_T'^2) + \frac{k_T^i}{M_1}\,H_{1s}^\perp(z,\boldsymbol{k}_T) + \frac{\epsilon_T^{ij}k_{Tj}}{M_1}\,H_1^\perp(z,\boldsymbol{k}_T'^2); \tag{5.31}$$

furthermore we obtain subleading projections (i,j are transverse indices):

$$\Delta^{[1]}(z,\boldsymbol{k}_T) = \frac{M_1}{P_1^-}\,E(z,\boldsymbol{k}_T'^2), \tag{5.32}$$

$$\Delta^{[\gamma^i]}(z,\boldsymbol{k}_T) = \frac{k_T^i}{P_1^-}\,D^\perp(z,\boldsymbol{k}_T'^2) + \frac{\lambda_1\,\epsilon_T^{ij}k_{Tj}}{P_1^-}\,D_L^\perp(z,\boldsymbol{k}_T'^2) + \frac{M_1\,\epsilon_T^{ij}S_{1Tj}}{P_1^-}\,D_T(z,\boldsymbol{k}_T'^2), \tag{5.33}$$

$$\Delta^{[i\gamma_5]}(z,\boldsymbol{k}_T) = \frac{M_1}{P_1^-}\,E_s(z,\boldsymbol{k}_T), \tag{5.34}$$

$$\Delta^{[\gamma^i\gamma_5]}(z,\boldsymbol{k}_T) = \frac{M_1\,S_{1T}^i}{P_1^-}\,G_T'(z,\boldsymbol{k}_T'^2) + \frac{k_T^i}{P_1^-}\,G_s^\perp(z,\boldsymbol{k}_T), \tag{5.35}$$

$$\Delta^{[i\sigma^{ij}\gamma_5]}(z,\boldsymbol{k}_T) = \frac{S_{1T}^i k_T^j - k_T^i S_{1T}^j}{P_1^-}\,H_T^\perp(z,\boldsymbol{k}_T'^2) + \frac{M_1\,\epsilon_T^{ij}}{P_1^-}\,H(z,\boldsymbol{k}_T'^2), \tag{5.36}$$

$$\Delta^{[i\sigma^{-+}\gamma_5]}(z,\boldsymbol{k}_T) = \frac{M_1}{P_1^-}\,H_s(z,\boldsymbol{k}_T). \tag{5.37}$$

The order at which a function first can contribute depends on the power of M_1/P_1^- in front of the function as it appears in the projections. Each factor M_1/P_1^- leads to a suppression with a power of M_1/Q in cross sections. We will refer to the function multiplying a power $(M_1/P_1^-)^{t-2}$ as being of 'twist' t.

The naming scheme is similar to that of the distribution functions. All functions obtained after tracing with a scalar ($\mathbf{1}$) or pseudoscalar ($i\gamma_5$) Dirac matrix are given the name $E_{..}$, those traced with a vector matrix (γ^μ) are given the name $D_{..}$ (to connect to the common notation $D_{a/H}$), those traced with an axial vector matrix ($\gamma^\mu\gamma_5$) are given the name $G_{..}$ and, finally, those traced with the second rank tensor $i\sigma^{\mu\nu}\gamma_5$ are given the name $H_{..}$. A subscript 1 is given to the leading functions, subscripts L or T refer to the connection with the hadron spin being longitudinal or transverse and a superscript $\perp$ signals the explicit presence of transverse momenta with a noncontracted index. In the literature sometimes the fragmentation functions are denoted by lower-case names, but supplemented by a hat ($\hat{e},\hat{g},\hat{h}$), with the one exception that D is named $\hat{f}$. We note that after integration over $\boldsymbol{k}_T$ several functions disappear. In the case of $\Delta^{[i\sigma^{i-}\gamma_5]}$ and $\Delta^{[\gamma^i\gamma_5]}$ specific combinations remain, namely $H_1 \equiv H_{1T} + (\boldsymbol{k}_T^2/2M_1^2)\,H_{1T}^\perp$ and $G_T \equiv G_T' + (\boldsymbol{k}_T^2/2M_1^2)\,G_T^\perp$, respectively.

The choice of factors in the definition of fragmentation functions is such that

$$\int dz\, d^2\boldsymbol{k}_T'\, D_1(z,\boldsymbol{k}_T') = N_h, \tag{5.38}$$

where N_h is the number of produced hadrons. The twist-two fragmentation functions have natural interpretations as decay functions. The projection $\Delta^{[\gamma^-]}$ is (after proper normalizing) the probability of a quark to produce a spin-1/2 hadron in a specific spin state, $\Delta^{[\gamma^-\gamma_5]}$ is the difference of the probabilities for a chirally right and chirally left quark to produce such a hadron, while $\Delta^{[i\sigma^{i-}\gamma_5]}$ is the difference of opposite transverse spin states (along direction i) of a quark to produce such a hadron.

Note that the decay probability for an unpolarized quark with nonzero transverse momentum can lead to a transverse polarization in the production of spin-1/2 particles. This polarization is orthogonal to the quark transverse momentum and the probability is given by the function $D_{1T}^{\perp}$. In the same way, oppositely transversely polarized quarks with nonzero transverse momentum can produce unpolarized hadrons or spinless particles, with different probabilities. In other words: there can be a preference for one or the other transverse polarization direction of the quark to fragment into an unpolarized hadron. This difference is described by the function $H_1^{\perp}$. It is the one appearing in the so-called Collins or sheared-jet effect [90], which predicts a single transverse-spin asymmetry in for instance semi-inclusive DIS, and arises due to intrinsic transverse momentum (in Collins' notation: $\Delta\hat{D}_{H/a} \sim \epsilon_T^{ij} S_{1Ti} k_{Tj}\, H_1^{\perp}$).

The functions $D_{1T}^{\perp}$ and $H_1^{\perp}$ are examples of what are generally called 'time-reversal odd' fragmentation functions, the analogues of $f_{1T}^{\perp}$ and $h_1^{\perp}$, cf. Fig. 3.2. In the case of fragmentation functions this is a misleading terminology, which refers to the behavior of the functions under the so-called *naive* time-reversal operation T_N [94], which acts as follows on the correlation functions:

$$\Delta(P_1, S_1; k) \xrightarrow{T_N} (\gamma_5 C\, \Delta(\bar{P}_1, \bar{S}_1; \bar{k})\, C^{\dagger}\gamma_5)^*, \tag{5.39}$$

where $\bar{k} = (k^0, -\boldsymbol{k})$, etc. If T_N invariance would apply, the functions $D_{1T}^{\perp}$, $H_1^{\perp}$, $D_L^{\perp}$, D_T, E_L, E_T and H would be purely imaginary. On the other hand, hermiticity requires the functions to be real, so these functions should then vanish.

The operation T_N differs from the actual time-reversal operation T in that the former does not transforms *in* into *out*-states and vice versa. Due to final state interactions, the *out*-state $|P_1, S_1; X\rangle$ in $\Delta(P_1, S_1; k)$ is not a plane wave state and thus, is not simply related to an *in*-state. Therefore, one has $T_N \neq T$ and since T itself does not pose any constraints on the functions, they need not vanish.

In the analogous case of distribution functions, which *are* derived from matrix elements with plane wave states, $T = T_N$ and therefore there are no time-reversal odd distribution functions, unless they arise from some nonstandard mechanism (like factorization breaking mechanisms or gluonic poles, as discussed in the previous chapter). But the final state interactions between particles in one jet, which could lead to nonzero fragmentation functions, do not require any such mechanism.

The quark-gluon correlation functions can be expressed in terms of the quark correlation functions with help of the classical equations of motion (e.o.m.). If we again define Dirac projections:

$$\Delta_A^{\alpha[\Gamma]}(z,\boldsymbol{k}_T) = \frac{1}{4z}\int dk^+\, d^4k_1 \mathrm{Tr}\,(\Delta_A^\alpha \Gamma)\Big|_{k^-=P_1^-/z,\ \boldsymbol{k}_T}$$
$$= \sum_X \int \frac{dx^+ d^2x_\perp}{4z\,(2\pi)^3}\, e^{i\,k\cdot x}\,\mathrm{Tr}\,\langle 0|\psi(x)\, gA_T^\alpha(x)\,|P_1,S_1;X\rangle\langle P_1,S_1;X|\,\overline{\psi}(0)\Gamma|0\rangle\Big|_{x^-=0}, \quad (5.40)$$

we find as a consequence of the e.o.m.:

$$\Delta_{A\alpha}^{[\sigma^{\alpha -}]} = -\epsilon_T^{\alpha\beta}\,\Delta_A^{\alpha[i\sigma^{\beta -}\gamma_5]} = \frac{M_1}{z}\left(\tilde{H} + i\,\tilde{E}\right) - \epsilon_T^{ij}\,k_{Ti}S_{1Tj}\left(\frac{1}{z}\tilde{H}_T^\perp + i\,\frac{m}{M_1}\,D_{1T}^\perp\right), \quad (5.41)$$

$$\Delta_{A\alpha}^{[i\sigma^{\alpha -}\gamma_5]} = \frac{M_1}{z}\left(\tilde{H}_s + i\,\tilde{E}_s\right), \quad (5.42)$$

$$\Delta_A^{\alpha[\gamma^-]} + i\epsilon_T^{\alpha\beta}\,\Delta_{A\beta}^{[\gamma^-\gamma_5]} = k_T^\alpha\left(\frac{1}{z}\tilde{D}^\perp + i\,\frac{m}{M_1}\,H_1^\perp\right) - \frac{\left(k_T^\alpha k_T^i + \frac{1}{2}k_T^2 g_T^{\alpha i}\right)}{M_1}\,\epsilon_T^{ij}S_{1Tj}\,D_{1T}^\perp$$
$$+ i\epsilon_T^{\alpha\beta}k_{T\beta}\,\frac{1}{z}\left(\tilde{G}_s^\perp - i\,\lambda_1\tilde{D}_L^\perp\right) + i\epsilon_T^{\alpha\beta}S_{1T\beta}\,\frac{M_1}{z}\left(\tilde{G}_T' - i\,\tilde{D}_T\right), \quad (5.43)$$

where the functions indicated with a tilde ($\tilde{H}$, $\tilde{E}$, ...) differ from the corresponding twist-3 functions (H, E, ...) by a twist-2 part, namely

$$E = \frac{m}{M_1}zD_1 + \tilde{E}, \quad (5.44)$$
$$D^\perp = zD_1 + \tilde{D}^\perp, \quad (5.45)$$
$$D_L^\perp = \tilde{D}_L^\perp, \quad (5.46)$$
$$D_T = -\frac{\boldsymbol{k}_T^2}{2M_1^2}zD_{1T}^\perp + \tilde{D}_T, \quad (5.47)$$
$$E_s = \tilde{E}_s, \quad (5.48)$$
$$G_T' = \frac{m}{M_1}zH_{1T} + \tilde{G}_T', \quad (5.49)$$
$$G_s^\perp = zG_{1s} + \frac{m}{M_1}zH_{1s}^\perp + \tilde{G}_s^\perp, \quad (5.50)$$
$$G_T = \frac{\boldsymbol{k}_T^2}{2M_1^2}zG_{1T} + \frac{m}{M_1}zH_1 + \tilde{G}_T, \quad (5.51)$$
$$H_T^\perp = zH_{1T} + \tilde{H}_T^\perp, \quad (5.52)$$
$$H = -\frac{\boldsymbol{k}_T^2}{M_1^2}zH_1^\perp + \tilde{H}, \quad (5.53)$$
$$H_s = \frac{m}{M_1}zG_{1s} - \frac{\boldsymbol{k}_T\cdot\boldsymbol{S}_{1T}}{M_1}zH_{1T} - \frac{\boldsymbol{k}_T^2}{M_1^2}zH_{1s}^\perp + \tilde{H}_s. \quad (5.54)$$

We have included G_T in this list since it is relevant for $\boldsymbol{k}_T'$-integrated functions and note that $\tilde{G}_T = \tilde{G}_T' + (\boldsymbol{k}_T^2/2M_1^2)\,\tilde{G}_T^\perp$. The functions in Eqs. (5.41) to (5.43) are interaction dependent and vanish for the case of a quark fragmenting in a quark (as can be checked with the help of App. 5.A). Note that the time-reversal odd twist-two functions[2] $D_{1T}^\perp$ and $H_1^\perp$ are in fact interaction dependent, which agrees with the fact that their presence is due to final state interactions of the produced hadrons. The separation of twist-three functions in this way is analogous to the case of distribution functions and the twist-two parts are again called Wandzura-Wilczek parts.

For the fragmentation of an antiquark most things are analogous to the quark fragmentation. The major difference in our case is that the role of the + and − direction is reversed. We will denote the antiquark correlation functions by $\overline{\Delta}(P_2, S_2; p)$ and $\overline{\Delta}_A^\alpha(P_2, S_2; p, p_1)$. The antiquark fragmentation functions are obtained from

$$\overline{\Delta}_{ij}(P_2, S_2; p) = \sum_X \int \frac{d^4x}{(2\pi)^4}\, e^{-ip\cdot x}\, \langle 0|\overline{\psi}_j(0)|P_2, S_2; X\rangle\langle P_2, S_2; X|\psi_i(x)|0\rangle. \tag{5.55}$$

They should be defined consistently with the replacement $\psi \to \psi^c = C\overline{\psi}^T$, or $\overline{\Delta}^{[\Gamma]} = \Delta^{c[\Gamma]}$ for $\Gamma = \gamma_\mu, i\sigma_{\mu\nu}\gamma_5, i\gamma_5$ and $\overline{\Delta}^{[\Gamma]} = -\Delta^{c[\Gamma]}$ for $\Gamma = \mathbf{1}, \gamma_\mu\gamma_5$, where we have defined the projections as:

$$\overline{\Delta}^{[\Gamma]}(\bar{z}, \boldsymbol{p}_T) \;=\; \frac{1}{4\bar{z}} \int dp^-\ \mathrm{Tr}(\overline{\Delta}\,\Gamma)\Big|_{p^+ = P_2^+/\bar{z},\ \boldsymbol{p}_T}. \tag{5.56}$$

Here we made the following Sudakov decomposition for the antiquark momentum p:

$$p \equiv \frac{z_2 Q}{\bar{z}\sqrt{2}}\, n_+ + \frac{\bar{z}(p^2 + \boldsymbol{p}_T^2)}{z_2 Q\sqrt{2}}\, n_- + p_T \approx \frac{1}{\bar{z}} P_2 + p_T. \tag{5.57}$$

Similarly, we decompose the spin vector S_2:

$$S_2 \equiv \frac{\lambda_2 z_2 Q}{M_2\sqrt{2}}\, n_+ - \frac{\lambda_2 M_2}{z_2 Q\sqrt{2}}\, n_- + S_{2T} \approx \frac{\lambda_2}{M_2} P_2 + S_{2T}, \tag{5.58}$$

with for a pure state $\lambda_2^2 + \boldsymbol{S}_{2T}^2 = 1$. The antiquark fragmentation functions are denoted by $\overline{D}_1(\bar{z}, \boldsymbol{p}_T'^2), \ldots$, with $\boldsymbol{p}_T' = -\bar{z}\boldsymbol{p}_T$.

Although the antiquark-gluon correlation function is straightforwardly defined, we still give here the relations which follow from the e.o.m., since these differ nontrivially from those for the quark-gluon correlation functions. We will not use tilde functions here, since in the nonsymmetric frame in which we will express the hadron tensor (nonsymmetric between quark and antiquark fragmentation part), they do not show up in a natural way (recall the

[2]The arbitrariness in the definition of $\tilde{D}_T$ and $\tilde{H}$ in Eqs. (5.47) and (5.53) is fixed by the requirement that the functions $D_{1T}^\perp$ and $H_1^\perp$ do not appear in the integrated versions of Eqs. (5.41) to (5.43).

$c = 0$ versus $c = 1$ case in DY, cf. the discussion below Eq. (4.22)). We find

$$\overline{\Delta}_{A\alpha}^{[\sigma^{\alpha +}]} = \epsilon_T^{\alpha\beta}\,\overline{\Delta}_A^{\alpha[i\sigma^{\beta +}\gamma_5]} = i\left(\frac{M_2}{z}\overline{E} - m\,\overline{D}_1 + i\,\frac{M_2}{z}\,\overline{H} - i\,\frac{p_T^2}{M_2}\,\overline{H}_1^\perp\right)$$
$$+\epsilon_T^{ij}\,p_{Ti}S_{2Tj}\left(\overline{H}_{1T} - \frac{1}{z}\overline{H}_T^\perp + i\,\frac{m}{M_2}\,\overline{D}_{1T}^\perp\right), \tag{5.59}$$

$$\overline{\Delta}_{A\alpha}^{[i\sigma^{\alpha +}\gamma_5]} = -\frac{M_2}{z}\overline{H}_s + m\,\overline{G}_{1s} + i\,\frac{M_2}{z}\overline{E}_s + (p_T\cdot S_{2T})\overline{H}_{1T} + \frac{p_T^2}{M_2}\,\overline{H}_{1s}^\perp, \tag{5.60}$$

$$\overline{\Delta}_A^{\alpha[\gamma^+]} - i\epsilon_T^{\alpha\beta}\,\overline{\Delta}_{A\beta}^{[\gamma^+\gamma_5]} = -p_T^\alpha\left(\frac{1}{z}\overline{D}^\perp - \overline{D}_1 - i\,\frac{m}{M_2}\,\overline{H}_1^\perp\right) - \frac{p_T^\alpha}{M_2}\,\epsilon_T^{ij}p_{Ti}S_{2Tj}\,\overline{D}_{1T}^\perp$$
$$-i\epsilon_T^{\alpha\beta}p_{T\beta}\left(\frac{1}{z}\overline{G}_s^\perp - \overline{G}_{1s} - \frac{m}{M_2}\,\overline{H}_{1s}^\perp + i\,\frac{\lambda_2}{z}\overline{D}_L^\perp\right)$$
$$-i\epsilon_T^{\alpha\beta}S_{2T\beta}\left(\frac{M_2}{z}\,\overline{G}_T' - m\,\overline{H}_{1T} + i\,\frac{M_2}{z}\,\overline{D}_T\right). \tag{5.61}$$

We have not commented on color gauge invariance of the correlation functions. This will be done extensively in Chap. 7. As given above the correlation functions are gauge-invariant quantities displayed in a specific gauge. In general, one has to include link operators. At this point we assume that matrix elements with multiple A^--gluon fields in $\Delta_A^-, \Delta_{AA}^{--}, \ldots$ (multiple A^+-gluon fields in $\overline{\Delta}_A^+, \overline{\Delta}_{AA}^{++}, \ldots$) will combine into an appropriate link operator with path along the $+$ direction ($-$ direction) in Δ ($\overline{\Delta}$). For the $\boldsymbol{k}_T$-dependent functions which involve transverse separations, the path from the point 0 to x in Δ will run along the $+$ direction via $x^+ = \infty$. The transverse part of the path, which is at ∞, does not contribute, since matrix elements are assumed to vanish there.

We will furthermore use that the relation

$$\Delta_A^{\alpha[\Gamma]}(z,\boldsymbol{k}_T) = \Delta_D^{\alpha[\Gamma]}(z,\boldsymbol{k}_T) - k^\alpha\,\Delta^{[\Gamma]}(z,\boldsymbol{k}_T), \tag{5.62}$$

where

$$\sum_X\int\frac{d^4x}{(2\pi)^4}\,e^{ik\cdot x}\,\langle 0|\psi_i(x)\,i\partial^\mu\,|P_1,S_1;X\rangle\langle P_1,S_1;X|\,\overline{\psi}_j(0)|0\rangle = k^\mu\,\Delta_{ij}(P_1,S_1;k), \tag{5.63}$$

which holds in the $A^- = 0$ gauge.

5.4 The leading and subleading twist calculation

Up to and including order $1/Q$ there are five tree-level diagrams to consider. The simplest diagram (Fig. 5.3) involving only quarks contributes at order 1 and $1/Q$, the other four (Fig. 5.4) involve one gluon which connects to one of the two soft parts. Note that one power of

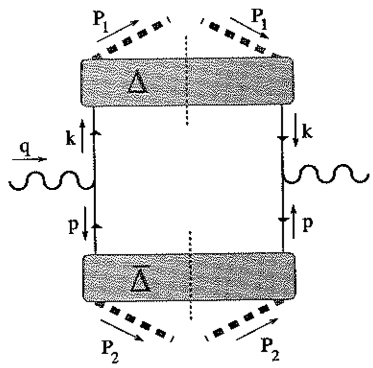


Figure 5.3: Quark diagram contributing to e^+e^- annihilation in leading order. There is a similar diagram with reversed fermion flow.

the coupling constant is included in the definition of the soft part, such that the diagrams are of order $(\alpha_s)^0$.

The four-momentum conservation delta-function at the photon vertex is written as (neglecting $1/Q^2$ contributions)

$$\delta^4(q-k-p) = \delta(q^+ - p^+)\,\delta(q^- - k^-)\,\delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T), \tag{5.64}$$

fixing $P_2^+/\bar{z} = p^+ = q^+ = P_2^+/z_2$ and $P_1^-/z = k^- = q^- = P_1^-/z_1$. Eq. (5.64) shows why only the k^+ and p^--integrated correlation functions are relevant. Note that the quark transverse momentum integrations are linked. The five diagrams lead to the following expression for the full result up to and including order $1/Q$:

$$\begin{aligned}
\mathcal{W}^{\mu\nu} = 3\int dp^- dk^+ d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T)\Bigg\{ &\mathrm{Tr}\left(\overline{\Delta}(p)\gamma^\mu \Delta(k)\gamma^\nu\right) \\
-\mathrm{Tr}\left(\overline{\Delta}_A^\alpha(p)\gamma^\mu\Delta(k)\gamma_\alpha\frac{\not{n}_+}{Q\sqrt{2}}\gamma^\nu\right) &- \mathrm{Tr}\left(\left(\gamma_0\overline{\Delta}_A^{\alpha\dagger}(p)\gamma_0\right)\gamma^\mu\frac{\not{n}_+}{Q\sqrt{2}}\gamma_\alpha\Delta(k)\gamma^\nu\right) \\
+\,\mathrm{Tr}\left(\overline{\Delta}(p)\gamma^\mu\left(\gamma_0\Delta_A^{\alpha\dagger}(k)\gamma_0\right)\gamma^\nu\frac{\not{n}_-}{Q\sqrt{2}}\gamma_\alpha\right) &+ \mathrm{Tr}\left(\overline{\Delta}(p)\gamma_\alpha\frac{\not{n}_-}{Q\sqrt{2}}\gamma^\mu\Delta_A^\alpha(k)\gamma^\nu\right)\Bigg\}\Bigg|_{p^+ k^-}.
\end{aligned} \tag{5.65}$$

The factor 3 originates from the color summation. We have omitted the flavor indices and summation; furthermore, there is a contribution from diagrams with reversed fermion flow, which results from the above expression by replacing $\mu \leftrightarrow \nu$ and $q \to -q$ (and in the end in a summation over flavors and antiflavors).

In the expression the terms with $\not{n}_\pm$ arise from the fermion propagators in the hard part neglecting contributions that will appear suppressed by powers of Q^2,

$$\frac{\not{q} - \not{p}_1 + m}{(q-p_1)^2 - m^2} \approx \frac{(q^+ - p_1{}^+)\gamma^-}{2(q^+ - p_1{}^+)q^-} = \frac{\gamma^-}{2q^-} = \frac{\not{n}_+}{Q\sqrt{2}}, \tag{5.66}$$

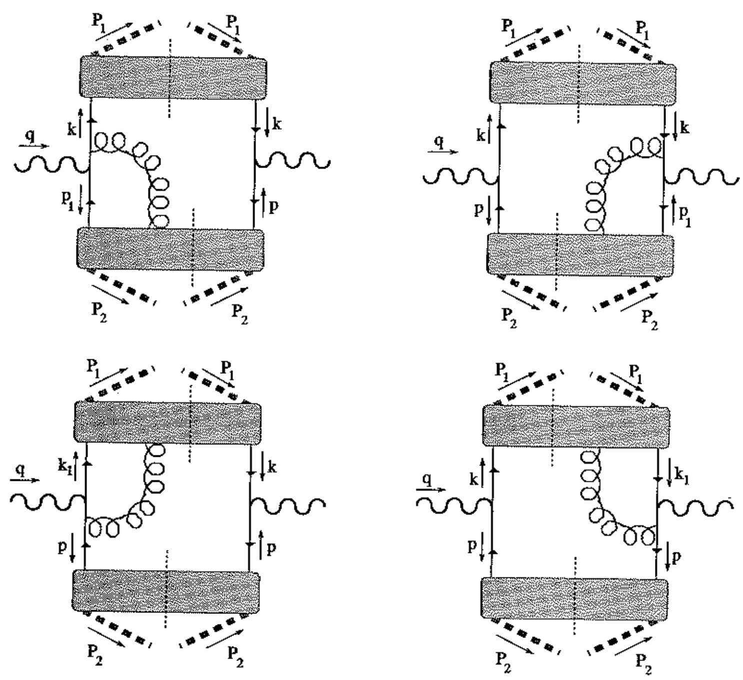


Figure 5.4: Diagrams contributing to e^+e^- annihilation at order $1/Q$.

$$\frac{\not{k}_1 - \not{q} + m}{(k_1 - q)^2 - m^2} \approx \frac{(k_1{}^- - q^-)\gamma^+}{-2(k_1{}^- - q^-)q^+} = \frac{\gamma^+}{-2q^+} = -\frac{\not{n}_-}{Q\sqrt{2}}, \tag{5.67}$$

where the approximations hold only when the propagators are embedded in the diagrams. We have assumed that gluonic poles are absent (cf. Eqs. (4.12) and (4.13)), such that one can always integrate out one of the momenta of $\Delta_A^\alpha(k, k_1)$ or $\overline{\Delta}_A^\alpha(p, p_1)$ and apply the e.o.m. immediately. The quantity $\Delta_A^\alpha(k)$ arises from integrating out the second argument of $\Delta_A^\alpha(k, k_1)$ instead of the first which yields the combination $\gamma_0 \Delta_A^{\alpha\dagger}(k)\gamma_0$:

$$\begin{aligned}\int d^4k_1 \Delta^\alpha_{A\,ij}(k, k_1) &= \sum_X \int \frac{d^4x}{(2\pi)^4}\, e^{ik\cdot x} \langle 0|\psi_i(x)\, gA_T^\alpha(x)\, |P_1, S_1; X\rangle\langle P_1, S_1; X|\overline{\psi}_j(0)|0\rangle \\ &= \Delta^\alpha_{A\,ij}(k),\end{aligned} \tag{5.68}$$

$$\begin{aligned}\int d^4k_1 \Delta^\alpha_{A\,ij}(k_1, k) &= \sum_X \int \frac{d^4x}{(2\pi)^4}\, e^{ik\cdot x} \langle 0|\psi_i(x)\, |P_1, S_1; X\rangle\langle P_1, S_1; X|\, gA_T^\alpha(0)\, \overline{\psi}_j(0)|0\rangle \\ &= (\gamma_0 \Delta_A^{\alpha\dagger}\gamma_0)_{ij}(k)\end{aligned} \tag{5.69}$$

and similarly for $\overline{\Delta}_A^\alpha(p)$ and $\gamma_0 \overline{\Delta}_A^{\alpha\dagger}(p)\gamma_0$. To deal with these combinations one can use the relation:

$$\left(\gamma_0 \Delta_A^{\alpha\dagger} \gamma_0\right)^{[\Gamma]} = \left(\Delta_A^{\alpha[\Gamma]}\right)^* \tag{5.70}$$

and a similar one for $\gamma_0 \overline{\Delta}_A^{\alpha\dagger}(p)\gamma_0$.

To obtain the expressions for the symmetric and antisymmetric parts of the hadron tensor we expand all vectors in $\Delta, \overline{\Delta}, \Delta_A^\alpha$ and $\overline{\Delta}_A^\alpha$ in the perpendicular basis ($\hat{t}$, $\hat{z}$ and $\perp$ directions). In particular, we reexpress the transverse vectors k_T, p_T, S_{1T} and S_{2T} in terms of their perpendicular parts and a part along $\hat{t}$ and $\hat{z}$. For this we need

$$g_T^{\mu\nu} = g_\perp^{\mu\rho} g_{T\rho}{}^{\nu} - \frac{Q_T}{Q}\,(\hat{t}^\mu + \hat{z}^\mu)\hat{h}^\nu. \tag{5.71}$$

We will refer to these perpendicular projections as $k_\perp$, etc. (instead of $k_{T\perp}$). Thus

$$k_\perp^\mu \equiv g_\perp^{\mu\nu} k_{T\nu} = k_T^\mu + \frac{\boldsymbol{q}_T \cdot \boldsymbol{k}_T}{Q}\,(\hat{t}^\mu + \hat{z}^\mu), \tag{5.72}$$

and similarly for $p_\perp$, $S_{1\perp}$ and $S_{2\perp}$. We note that for these four vectors with this definition the two-component perpendicular parts are the same as the two-component transverse parts, i.e., $\boldsymbol{k}_\perp = \boldsymbol{k}_T$, $\boldsymbol{S}_{1\perp} = \boldsymbol{S}_{1T}$, etc.

The full expressions for the symmetric and antisymmetric parts of the hadron tensor (expressed in the perpendicular frame defined in section 2) are given in App. 5.B. We note that the expressions are not symmetric in the interchange of the hadrons one and two, because the choice of perpendicular direction ($P_{2\perp} \equiv 0$) is nonsymmetric.

The cross sections are obtained from the hadron tensor after contraction with the lepton tensor

$$\begin{aligned} L^{\mu\nu} \;=\;& Q^2\Big[-\left(1-2y+2y^2\right) g_\perp^{\mu\nu} + 4y(1-y)\hat{z}^\mu\hat{z}^\nu - 4y(1-y)\left(\hat{l}_\perp^\mu \hat{l}_\perp^\nu + \frac{1}{2} g_\perp^{\mu\nu}\right) \\ & - 2(1-2y)\sqrt{y(1-y)}\;\hat{z}^{\{\mu}\hat{l}_\perp^{\nu\}} + i\lambda_e\,(1-2y)\,\epsilon_\perp^{\mu\nu} - 2i\lambda_e\sqrt{y(1-y)}\;\hat{l}_{\perp\rho}\epsilon_\perp^{\rho\,[\mu}\hat{z}^{\nu]}\Big]. \end{aligned} \tag{5.73}$$

The fraction y is defined to be $y = P_2 \cdot l / P_2 \cdot q \approx l^-/q^-$, which in the lepton center of mass frame equals $y = (1+\cos\theta_2)/2$, where θ_2 is the angle of hadron 2 with respect to the momentum of the incoming leptons. The contractions of specific tensor structures in the hadron tensor, given in Table 5.1, contain azimuthal angles inside the perpendicular plane defined with respect to $\hat{l}_\perp^\mu$, defined to be the normalized perpendicular part of the lepton momentum l, $\hat{l}_\perp^\mu = l_\perp^\mu/(Q\sqrt{y(1-y)})$:

$$\hat{l}_\perp \cdot a_\perp = -|\boldsymbol{a}_\perp| \cos\phi_a, \tag{5.74}$$

$$\epsilon_\perp^{\mu\nu}\hat{l}_{\perp\mu} a_{\perp\nu} = |\boldsymbol{a}_\perp| \sin\phi_a, \tag{5.75}$$

for a generic vector a.

Table 5.1: Contractions of the lepton tensor $L_{\mu\nu}$ with tensor structures appearing in the hadron tensor.

$w^{\mu\nu}$	$L_{\mu\nu}w^{\mu\nu}/(4Q^2)$
$-g_\perp^{\mu\nu}$	$\left(\frac{1}{2} - y + y^2\right)$
$a_\perp^{\{\mu}b_\perp^{\nu\}} - (a_\perp \cdot b_\perp)\, g_\perp^{\mu\nu}$	$-y\,(1-y)\,\vert\boldsymbol{a}_\perp\vert\,\vert\boldsymbol{b}_\perp\vert\,\cos(\phi_a + \phi_b)$
$\frac{1}{2}\left(a_\perp^{\{\mu}\,\epsilon_\perp^{\nu\}\rho} b_{\perp\rho} + b_\perp^{\{\mu}\,\epsilon_\perp^{\nu\}\rho} a_{\perp\rho}\right)$ $= a_\perp^{\{\mu}\,\epsilon_\perp^{\nu\}\rho} b_{\perp\rho} - (\epsilon_\perp^{\rho\sigma} a_{\perp\rho} b_{\perp\sigma})\, g_\perp^{\mu\nu}$	$y\,(1-y)\,\vert\boldsymbol{a}_\perp\vert\,\vert\boldsymbol{b}_\perp\vert\,\sin(\phi_a + \phi_b)$
$\hat{z}^{\{\mu}a_\perp^{\nu\}}$	$-(1-2y)\sqrt{y(1-y)}\,\vert\boldsymbol{a}_\perp\vert\,\cos\phi_a$
$\hat{z}^{\{\mu}\,\epsilon_\perp^{\nu\}\rho}\,a_{\perp\rho}$	$(1-2y)\sqrt{y(1-y)}\,\vert\boldsymbol{a}_\perp\vert\,\sin\phi_a$
$i\,\epsilon_\perp^{\mu\nu}$	$-\lambda_e\,\left(\frac{1}{2} - y\right)$
$i\,a_\perp^{[\mu}b_\perp^{\nu]}$	$-\lambda_e\,\left(\frac{1}{2} - y\right)\,\vert\boldsymbol{a}_\perp\vert\,\vert\boldsymbol{b}_\perp\vert\,\sin(\phi_b - \phi_a)$
$i\,\hat{z}^{[\mu}a_\perp^{\nu]}$	$\lambda_e\,\sqrt{y(1-y)}\,\vert\boldsymbol{a}_\perp\vert\,\sin\phi_a$
$i\,\hat{z}^{[\mu}\,\epsilon_\perp^{\nu]\rho}\,a_{\perp\rho}$	$\lambda_e\,\sqrt{y(1-y)}\,\vert\boldsymbol{a}_\perp\vert\,\cos\phi_a$

5.5 Integration over transverse photon momentum

In the next sections we will discuss explicit expressions for cross sections. Instead of giving the complete cross section, which can be obtained from the hadron tensor (App. 5.B), we treat a number of special cases. In this section we consider cross sections integrated over all transverse momenta.

After integration over the transverse momentum of the photon (or equivalently over the perpendicular momentum of hadron one $\boldsymbol{P}_{1\perp} = -z_1\boldsymbol{q}_T$), the integrations over $\boldsymbol{k}_T$ and $\boldsymbol{p}_T$ in the hadron tensor (Eqs. (5.B2) and (5.B2)) can be performed leading to

$$
\begin{aligned}
\int d^2\boldsymbol{q}_T\; \mathcal{W}_S^{\mu\nu} &= 12 z_1 z_2 \sum_{a,\bar{a}} e_a^{\,2} \\
&\times\Bigg\{ - g_\perp^{\mu\nu}\Big[D_1\overline{D}_1 - \lambda_1\lambda_2 G_1\overline{G}_1\Big] - \Big(S_{1\perp}^{\{\mu}S_{2\perp}^{\nu\}} + g_\perp^{\mu\nu}\,\boldsymbol{S}_{1\perp}\!\cdot\!\boldsymbol{S}_{2\perp}\Big)\Big[H_1\overline{H}_1\Big] \\
&- 2\frac{\hat{z}^{\{\mu}S_{1\perp}^{\nu\}}}{Q}\lambda_2\left[M_1\frac{\tilde{G}_T}{z_1}\overline{G}_1 + M_2 H_1\frac{\overline{H}_L}{z_2}\right] + 2\frac{\hat{z}^{\{\mu}S_{2\perp}^{\nu\}}}{Q}\lambda_1\left[M_2 G_1\frac{\overline{G}_T}{z_2} + M_1\frac{\tilde{H}_L}{z_1}\overline{H}_1\right] \\
&+ 2\frac{\hat{z}^{\{\mu}\epsilon_\perp^{\nu\}\rho}S_{1\perp\rho}}{Q}\left[M_1\frac{\tilde{D}_T}{z_1}\overline{D}_1 + M_2 H_1\frac{\overline{H}}{z_2}\right] + 2\frac{\hat{z}^{\{\mu}\epsilon_\perp^{\nu\}\rho}S_{2\perp\rho}}{Q}\left[M_2 D_1\frac{\overline{D}_T}{z_2} + M_1\frac{\tilde{H}}{z_1}\overline{H}_1\right]\Bigg\}
\end{aligned}
\tag{5.76}
$$

and

$$
\begin{aligned}
\int d^2\boldsymbol{q}_T\, \mathcal{W}_A^{\mu\nu} &= 12 z_1 z_2 \sum_{a,\bar{a}} {e_a}^2 \Bigg\{ i\epsilon_\perp^{\mu\nu} \Big[\lambda_1 G_1 \overline{D}_1 - \lambda_2 D_1 \overline{G}_1 \Big] \\
&+ 2i \frac{\hat{z}^{[\mu} S_{1\perp}^{\nu]}}{Q} \lambda_2 \Big[M_1 \frac{\tilde{D}_T}{z_1} \overline{G}_1 - M_2 H_1 \frac{\overline{E}_L}{z_2} \Big] + 2i \frac{\hat{z}^{[\mu} S_{2\perp}^{\nu]}}{Q} \lambda_1 \Big[- M_2 G_1 \frac{\overline{D}_T}{z_2} + M_1 \frac{\tilde{E}_L}{z_1} \overline{H}_1 \Big] \\
&+ 2i \frac{\hat{z}^{[\mu} \epsilon_\perp^{\nu]\rho} S_{1\perp\rho}}{Q} \Big[M_1 \frac{\tilde{G}_T}{z_1} \overline{D}_1 + M_2 H_1 \frac{\overline{E}}{z_2} \Big] + 2i \frac{\hat{z}^{[\mu} \epsilon_\perp^{\nu]\rho} S_{2\perp\rho}}{Q} \Big[M_2 D_1 \frac{\overline{G}_T}{z_2} + M_1 \frac{\tilde{E}}{z_1} \overline{H}_1 \Big] \Bigg\}.
\end{aligned}
\tag{5.77}
$$

We have now included the summation over flavor indices and e_a is the quark charge in units of e. The fragmentation functions are flavor dependent and only depend on the longitudinal momentum fractions, e.g. $D_1\overline{D}_1 = D_1^a(z_1)\,\overline{D}_1^a(z_2)$. The result is expressed in terms of the fragmentation functions which survive the $\boldsymbol{k}_T$-integration of the Dirac projections of the correlation functions (cf. Eqs. (5.29) to (5.37)): $D_1, G_1 = G_{1L}, H_1, E, E_L, G_T, H, H_L, D_T$ [95, 96].

Note that the tilde functions arise naturally in the hadron-one sector. The reason that this does not occur for the hadron-two sector is due to the nonsymmetric choice of frame. This nonsymmetric feature only shows up at subleading order. The leading order is symmetric (note that $\epsilon_\perp^{\mu\nu}$ acquires a minus sign, due to the interchange of the vectors n_+ and n_-).

From the hadron tensors we easily arrive at the following expressions for the cross sections, which we separate into parts for unpolarized (O) and polarized (L) leptons:

$$
\begin{aligned}
\frac{d\sigma^O(e^+e^- \to h_1 h_2 X)}{d\Omega dz_1 dz_2} &= \frac{3\alpha^2}{Q^2} \sum_{a,\bar{a}} {e_a}^2 \Bigg\{ A(y) \Big(D_1\overline{D}_1 - \lambda_1\lambda_2 G_1 \overline{G}_1 \Big) \\
&+ B(y)\, |\boldsymbol{S}_{1T}|\, |\boldsymbol{S}_{2T}|\, \cos(\phi_{S_1} + \phi_{S_2}) \Big(H_1 \overline{H}_1 \Big) \\
&+ C(y)\, D(y)\, |\boldsymbol{S}_{1T}|\, \sin(\phi_{S_1}) \left(\frac{2M_1}{Q} \frac{\tilde{D}_T}{z_1} \overline{D}_1 + \frac{2M_2}{Q} H_1 \frac{\overline{H}}{z_2} \right) \\
&+ C(y)\, D(y)\, |\boldsymbol{S}_{2T}|\, \sin(\phi_{S_2}) \left(\frac{2M_2}{Q} D_1 \frac{\overline{D}_T}{z_2} + \frac{2M_1}{Q} \frac{\tilde{H}}{z_1} \overline{H}_1 \right) \\
&+ C(y)\, D(y)\, \lambda_2\, |\boldsymbol{S}_{1T}|\, \cos(\phi_{S_1}) \left(\frac{2M_1}{Q} \frac{\tilde{G}_T}{z_1} \overline{G}_1 + \frac{2M_2}{Q} H_1 \frac{\overline{H}_L}{z_2} \right) \\
&- C(y)\, D(y)\, \lambda_1\, |\boldsymbol{S}_{2T}|\, \cos(\phi_{S_2}) \left(\frac{2M_2}{Q} G_1 \frac{\overline{G}_T}{z_2} + \frac{2M_1}{Q} \frac{\tilde{H}_L}{z_1} \overline{H}_1 \right) \Bigg\}
\end{aligned}
\tag{5.78}
$$

and

$$
\begin{aligned}
\frac{d\sigma^L(e^+e^- \to h_1 h_2 X)}{d\Omega dz_1 dz_2} &= \frac{3\alpha^2}{Q^2} \lambda_e \sum_{a,\bar{a}} {e_a}^2 \Bigg\{ \frac{C(y)}{2} \Big(\lambda_2 D_1 \overline{G}_1 - \lambda_1 G_1 \overline{D}_1 \Big) \\
&+ D(y)\, |\boldsymbol{S}_{2T}|\, \cos(\phi_{S_2}) \left(\frac{2M_2}{Q} D_1 \frac{\overline{G}_T}{z_2} + \frac{2M_1}{Q} \frac{\tilde{E}}{z_1} \overline{H}_1 \right)
\end{aligned}
$$

$$+ D(y)\,|\boldsymbol{S}_{1T}|\,\cos(\phi_{S_1})\left(\frac{2M_1}{Q}\frac{\tilde{G}_T}{z_1}\overline{D}_1 + \frac{2M_2}{Q}H_1\frac{\overline{E}}{z_2}\right)$$
$$- D(y)\,\lambda_1\,|\boldsymbol{S}_{2T}|\,\sin(\phi_{S_2})\left(\frac{2M_2}{Q}G_1\frac{\overline{D}_T}{z_2} - \frac{2M_1}{Q}\frac{\tilde{E}_L}{z_1}\overline{H}_1\right)$$
$$+ D(y)\,\lambda_2\,|\boldsymbol{S}_{1T}|\,\sin(\phi_{S_1})\left(\frac{2M_1}{Q}\frac{\tilde{D}_T}{z_1}\overline{G}_1 - \frac{2M_2}{Q}H_1\frac{\overline{E}_L}{z_2}\right)\Bigg\}, \tag{5.79}$$

where $d\Omega = 2dy\,d\phi^l$, with ϕ^l giving the orientation of $\hat{l}_\perp^\mu$, see Fig. 5.1. Note that on the r.h.s. of the above equations the dependence on ϕ^l enters in the azimuthal angles, which are defined with respect to $\hat{l}_\perp^\mu$, cf. Eqs. (5.74) and (5.75). We use the following factors:

$$A(y) = \left(\frac{1}{2} - y + y^2\right) \stackrel{cm}{=} \frac{1}{4}\left(1 + \cos^2\theta_2\right), \tag{5.80}$$
$$B(y) = y\,(1-y) \stackrel{cm}{=} \frac{1}{4}\sin^2\theta_2, \tag{5.81}$$
$$C(y) = (1-2y) \stackrel{cm}{=} -\cos\theta_2, \tag{5.82}$$
$$D(y) = \sqrt{y\,(1-y)} \stackrel{cm}{=} \frac{1}{2}\sin\theta_2. \tag{5.83}$$

The first three terms in Eq. (5.78) coincide with the ones found in [97], if one neglects the contributions associated to Z exchange in their Eq. (45) (see also next chapter). One observes that besides these three leading contributions, one finds subleading single and double spin azimuthal asymmetries.

To reduce the expression to the one-particle inclusive cross section, one must take the fragmentation functions for a quark fragmenting into a quark (see App. 5.A) and sum over spins. Only $D_1^a(z_1)$ survives and after summation over spins becomes a delta-function. We find for the one-hadron inclusive integrated cross sections (using h as index instead of 2 and realizing that $\overline{D}_1^a = D_1^{\bar{a}}$):

$$\frac{d\sigma^O(e^+e^- \to hX)}{d\Omega dz_h} = \frac{3\alpha^2}{Q^2}\sum_{a,\bar{a}} e_a{}^2\left\{A(y)D_1^a(z_h)\right.$$
$$\left. + C(y)\,D(y)\,|\boldsymbol{S}_{hT}|\,\sin(\phi_{S_h})\,\frac{2M_h}{Q}\frac{D_T^a(z_h)}{z_h}\right\} \tag{5.84}$$

and

$$\frac{d\sigma^L(e^+e^- \to hX)}{d\Omega dz_h} = \frac{3\alpha^2}{Q^2}\,\lambda_e\sum_{a,\bar{a}} e_a{}^2\left\{-\frac{C(y)}{2}\,\lambda_h G_1^a(z_h)\right.$$
$$\left. + D(y)\,|\boldsymbol{S}_{hT}|\,\cos(\phi_{S_h})\,\frac{2M_h}{Q}\frac{G_T^a(z_h)}{z_h}\right\}. \tag{5.85}$$

If the hadrons are unpolarized we find:

$$\frac{d\sigma^O(e^+e^- \to hX)}{d\Omega dz_h} = \frac{3\alpha^2}{Q^2}\,A(y)\sum_{a,\bar{a}} e_a{}^2\,D_1^a(z_h), \tag{5.86}$$

$$\frac{d\sigma^O(e^+e^- \to h_1h_2X)}{d\Omega dz_1 dz_2} = \frac{3\alpha^2}{Q^2} A(y) \sum_{a,\bar{a}} e_a{}^2\, D_1^a(z_1)\overline{D}_1^a(z_2), \tag{5.87}$$

and $d\sigma^L = 0$ in both cases. Hence we find for the number of produced particles

$$N_h(z_h) = \sum_{a,\bar{a}} e_a{}^2\, D_1^a(z_h) \Big/ \sum_{a,\bar{a}} e_a{}^2, \tag{5.88}$$

$$N_{h_1h_2}(z_1,z_2) = \sum_{a,\bar{a}} e_a{}^2\, D_1^a(z_1)\overline{D}_1^a(z_2) \Big/ \sum_{a,\bar{a}} e_a{}^2. \tag{5.89}$$

The case of $S_h = 0$ (summation over spins) gives the number of particles produced per spin degree of freedom, while the part proportional to λ_h gives the contributions of produced hadrons with $\lambda_h = \pm 1$. Thus the ratio of the part multiplying λ_h and the $S_h = 0$ result gives the longitudinal polarization of the produced hadrons, which must lie between -1 and $+1$. Similarly, the ratio of the part multiplying $\boldsymbol{S}_{hT}$ and the $S_h = 0$ result gives the transverse polarization, again a number between -1 and $+1$. In many cases the final state hadron will not be a stable particle, e.g. a Λ. In that case the final state ($N\pi$ for the case of a Λ) is used to determine the spin vector S_h [97].

For the one-particle inclusive cross section we see one leading polarizing effect, namely for polarized leptons the longitudinal polarization of produced spin-1/2 particles is given by

$$\Big(\text{longitudinal polarization}\Big) = -\lambda_e \frac{C(y)}{2A(y)} \frac{\sum_{a,\bar{a}} e_a{}^2 G_1^a(z_h)}{\sum_{a,\bar{a}} e_a{}^2 D_1^a(z_h)}. \tag{5.90}$$

At subleading order transverse polarization in the final state is induced given by

$$\begin{pmatrix}\text{transverse polarization}\\ \text{in lepton plane}\end{pmatrix} = \lambda_e \frac{D(y)}{A(y)} \frac{2M_h}{z_h\,Q} \frac{\sum_{a,\bar{a}} e_a{}^2 G_T^a(z_h)}{\sum_{a,\bar{a}} e_a{}^2 D_1^a(z_h)}, \tag{5.91}$$

$$\begin{pmatrix}\text{transverse polarization}\\ \text{transverse to lepton plane}\end{pmatrix} = \frac{C(y)\,D(y)}{A(y)} \frac{2M_h}{z_h\,Q} \frac{\sum_{a,\bar{a}} e_a{}^2 D_T^a(z_h)}{\sum_{a,\bar{a}} e_a{}^2 D_1^a(z_h)}, \tag{5.92}$$

where the lepton plane is spanned by l and P_2. The in-plane polarization is proportional to the lepton polarization and is determined by the fragmentation function G_T. This function is the equivalent of the distribution function g_T. An out-of-plane polarization is found for unpolarized leptons, determined by the time-reversal odd fragmentation function D_T. The asymmetry (5.92) was first discussed by Lu [86]. Experimental studies of transverse polarization of produced Λ's in Z decays show a result consistent with zero [98, 99].

For the 2-particle inclusive cross section in which one hadron has spin 1/2, e.g. $e^+e^- \to \Lambda\pi X$, a longitudinal Λ polarization is induced,

$$\Big(\text{longitudinal polarization}\Big) = -\lambda_e \frac{C(y)}{2A(y)} \frac{\sum_{a,\bar{a}} e_a{}^2 G_1^{a\to\Lambda}(z_1)\,\overline{D}_1^{a\to\pi}(z_2)}{\sum_{a,\bar{a}} e_a{}^2 D_1^{a\to\Lambda}(z_1)\overline{D}_1^{a\to\pi}(z_2)}, \tag{5.93}$$

involving one polarized fragmentation function, namely $G_1^{a\to\Lambda}$. If both hadrons have spin-1/2 a correlation between the polarizations of the two hadrons exist. The correlated longitudinal polarization involves $-\lambda_1\lambda_2\, A(y) \sum_{a,\bar{a}} e_a{}^2 G_1\overline{G}_1$; The correlated transverse polarization involves $B(y)\ |\boldsymbol{S}_{1T}|\ |\boldsymbol{S}_{2T}|\ \cos(\phi_{S_1}+\phi_{S_2})\ \sum_{a,\bar{a}} e_a{}^2 H_1 \overline{H}_1$ and provides a possibility to measure the transverse spin fragmentation function $H_1(z)$, the equivalent of the transverse spin distribution function h_1 [97]. For the two-particle inclusive cross section there are several (subleading) single spin asymmetries in unpolarized and polarized scattering.

5.6 Leading order asymmetries

Instead of integrating out the $\boldsymbol{q}_T$-dependence, we will now focus on the fully differential cross section, i.e., not integrated over transverse momentum ($\boldsymbol{P}_{1\perp} \doteq -z_1 \boldsymbol{q}_T$). We will see that the transverse momentum dependent cross sections contain asymmetries, which would vanish upon integration. Some of those asymmetries appear at leading order, to which we restrict in this section. The subleading results can be obtained from the hadron tensor in App. 5.B in a similar way.

First we consider the expression for both hadrons unpolarized:

$$\frac{d\sigma^O(e^+e^- \to h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2\boldsymbol{q}_T} = \frac{3\alpha^2}{Q^2}\, z_1^2 z_2^2 \left\{ A(y)\ \mathcal{F}\left[D_1 \overline{D}_1\right] \right.$$
$$\left. +B(y)\ \cos(2\phi_1)\ \mathcal{F}\left[\left(2\,\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\ \hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T - \boldsymbol{k}_T\cdot\boldsymbol{p}_T\right)\frac{H_1^\perp \overline{H}_1^\perp}{M_1 M_2}\right]\right\}, \tag{5.94}$$

where we use the convolution notation (Ralston and Soper [9] use $I[...]$)

$$\mathcal{F}\left[D\overline{D}\right] \equiv \sum_{a,\bar{a}} e_a{}^2 \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{p}_T\, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T) D^a(z_1, z_1^2\boldsymbol{k}_T^2)\overline{D}^a(z_2, z_2^2\boldsymbol{p}_T^2), \tag{5.95}$$

and $d\sigma^L = 0$ in this case. The angle ϕ_1 is the azimuthal angle of $\hat{h}$, see Fig. 5.1. So we find that the number of produced hadrons has an azimuthal dependence:

$$N_{h_1 h_2}(z_1, z_2, \boldsymbol{q}_T, y) = z_1^2 z_2^2 \left\{ \mathcal{F}\left[D_1 \overline{D}_1\right] \right.$$
$$\left. + \frac{B(y)}{A(y)}\ \cos(2\phi_1)\ \mathcal{F}\left[\left(2\,\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\ \hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T - \boldsymbol{k}_T\cdot\boldsymbol{p}_T\right)\frac{H_1^\perp \overline{H}_1^\perp}{M_1 M_2}\right]\right\} \Big/ \sum_{a,\bar{a}} e_a{}^2. \tag{5.96}$$

This asymmetry (the second term) has no regular analogue in the Drell-Yan process or semi-inclusive lepton-hadron scattering, since it involves a product of two T-odd functions and nonzero T-odd distribution functions would require some nonstandard mechanism, which we assume to be absent when we use the term "regular". This new asymmetry goes with the same function $H_1^\perp$ as appears in the Collins effect, multiplied with the similar time-reversal odd function $\overline{H}_1^\perp$. We emphasize that this is a measurement in which no polarization of the produced hadrons (nor of the incoming leptons) is needed and the result is not suppressed by factors of $1/Q$. This in contrast to the $\cos(2\phi)$ asymmetry found by Berger [92], which does not arise from time-reversal odd functions. It is $1/Q^2$-suppressed and also arises in other processes. We will discuss this asymmetry at energies around the Z peak extensively in the next chapter.

Assuming for instance a Gaussian $\boldsymbol{k}_T$-dependence of the functions, the convolutions can be evaluated. The number of produced hadrons would then be:

$$N_{h_1h_2}(z_1,z_2,Q_T,y) = \mathcal{G}(Q_T;R)\sum_{a,\bar{a}} e_a{}^2 \Big\{ D_1^a(z_1)\overline{D}_1^a(z_2) - \frac{B(y)}{A(y)}\cos(2\phi_1)\,\frac{Q_T R^4}{M_1 M_2 R_1^2 R_2^2} H_1^{\perp a}(z_1)\overline{H}_1^{\perp a}(z_2)\Big\} \Big/ \sum_{a,\bar{a}} e_a{}^2, \qquad (5.97)$$

where $R^2 = R_1^2 R_2^2/(R_1^2 + R_2^2)$ and

$$D_1(z_1, \boldsymbol{k}_T'^2) = D_1(z_1) R_1^2 \exp(-R_1^2 \boldsymbol{k}_T^2)/\pi z_1^2 \equiv D_1(z_1)\mathcal{G}(|\boldsymbol{k}_T|;R_1)/z_1^2, \qquad (5.98)$$

etc. For details see Ref. [42].

In case we consider the expression for hadron one polarized and hadron two unpolarized, we find the following additional terms:

$$\begin{aligned}\frac{d\sigma^O(e^+e^-\to h_1h_2X)}{d\Omega dz_1 dz_2 d^2\boldsymbol{q}_T} &= \frac{3\alpha^2}{Q^2}\,z_1^2 z_2^2 \Big\{ \ldots + B(y)\,\lambda_1\,\sin(2\phi_1) \qquad (5.99)\\
&\times\mathcal{F}\left[\left(2\,\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{k}_T\,\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{p}_T - \boldsymbol{k}_T\!\cdot\!\boldsymbol{p}_T\right)\frac{H_{1L}^\perp \overline{H}_1^\perp}{M_1M_2}\right] - A(y)\,|\boldsymbol{S}_{1T}|\,\sin(\phi_1-\phi_{S_1})\\
&\times\mathcal{F}\left[\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{k}_T\,\frac{D_{1T}^\perp \overline{D}_1}{M_1}\right] + B(y)\,|\boldsymbol{S}_{1T}|\,\sin(\phi_1+\phi_{S_1})\mathcal{F}\left[\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{p}_T\,\frac{H_1\overline{H}_1^\perp}{M_2}\right] + B(y)\,|\boldsymbol{S}_{1T}|\\
&\times\sin(3\phi_1-\phi_{S_1})\mathcal{F}\left[\left(4\,\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{p}_T\,(\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{k}_T)^2 - 2\,\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{k}_T\,\boldsymbol{k}_T\!\cdot\!\boldsymbol{p}_T - \hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{p}_T\,\boldsymbol{k}_T^2\right)\frac{H_{1T}^\perp \overline{H}_1^\perp}{2M_1{}^2M_2}\right]\Big\},\end{aligned}$$

$$\begin{aligned}\frac{d\sigma^L(e^+e^-\to h_1h_2X)}{d\Omega dz_1 dz_2 d^2\boldsymbol{q}_T} &= \frac{3\alpha^2}{Q^2}\,z_1^2 z_2^2 \Big\{ -\lambda_e\,\frac{C(y)}{2}\,\lambda_1\,\mathcal{F}\left[G_1\overline{D}_1\right]\\
&-\lambda_e\,\frac{C(y)}{2}\,|\boldsymbol{S}_{1T}|\,\cos(\phi_1-\phi_{S_1})\,\mathcal{F}\left[\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{k}_T\,\frac{G_{1T}\overline{D}_1}{M_1}\right]\Big\}. \qquad (5.100)\end{aligned}$$

Again there is a term which has no analogue in regular semi-inclusive lepton-hadron scattering, namely the term with $D_{1T}^\perp$ (the analogue of the T-odd distribution function $f_{1T}^\perp$), which can be seen by comparison with the result obtained in Ref. [100]. The term with $H_1\overline{H}_1^\perp$ is the analogue of the single (transverse)-spin Collins asymmetry [90]. Note that it appears together with other single transverse-spin asymmetries (like in semi-inclusive DIS [100]).

Conversely, one can consider hadron two polarized and hadron one unpolarized, which may be simpler from the experimental point of view, because one does not need to measure the transverse momentum *and* the polarization of the same hadron. For this case we find similar expressions in which all single spin terms have, besides the obvious replacements, a sign change.

The leading order double spin asymmetries can be found in App. 5.C and are useful in for instance the case of $e^+e^- \to \Lambda\overline{\Lambda}X$.

We like to point out that the transverse momentum dependence of some of the functions can be directly probed in the situation where hadron two is taken to be a jet, which in this back-to-back jet situation is equivalent to analyzing the azimuthal structure of hadrons inside a jet. Only $\overline{D}_1$ remains and equals a delta-function, so the convolutions can be evaluated exactly. In that case Eqs. (5.94), (5.99) and (5.100) taken together yield:

$$\frac{d\sigma(e^+e^- \to h \text{ jet } X)}{d\Omega dz_h d^2\boldsymbol{q}_T} = \frac{3\alpha^2}{Q^2}\, z_h^2 \sum_{a,\bar{a}} e_a{}^2 \Bigg\{ A(y) \left[D_1^a(z_h, z_h^2 Q_T^2) + |\boldsymbol{S}_{hT}| \, \sin(\phi_h - \phi_{S_1}) \frac{Q_T}{M_h} D_{1T}^{\perp a}(z_h, z_h^2 Q_T^2) \right] - \lambda_e \, \frac{C(y)}{2} \left[\lambda_h \, G_1^a(z_h, z_h^2 Q_T^2) + |\boldsymbol{S}_{hT}| \, \cos(\phi_h - \phi_{S_1}) \, \frac{Q_T}{M_h} G_{1T}^a(z_h, z_h^2 Q_T^2) \right] \Bigg\}. \quad (5.101)$$

This result means that there is a transverse polarization transverse to the hadron plane, which is proportional to the function $D_{1T}^{\perp}$, and a transverse polarization in the hadron plane, proportional to G_{1T}. So one sees that by measuring $\boldsymbol{q}_T$ one can learn about the transverse momentum dependence of four functions in this particular case. There are no chiral-odd functions in this result, because in this case they must be accompanied by a quark mass, which gives a result proportional to m/Q, so they are present in the subleading result (not discussed here).

5.7 Weighted cross sections

The expressions in the previous section contain convolutions, which are not the objects of interest, rather one wants to learn about the (universal) fragmentation functions depending on z and $\boldsymbol{k}_T^2$. At the end of the previous section we discussed a situation in which the transverse momentum dependence of some of the functions could be extracted from the analysis of one jet. Below we will outline a way to obtain instead of the full transverse momentum dependence, the $\boldsymbol{k}_T^2$-moments of the functions, defined as:

$$F^{(n)}(z_1) = \int d^2\boldsymbol{k}_T' \left(\frac{\boldsymbol{k}_T^2}{2M_1^2}\right)^n F(z_1, \boldsymbol{k}_T'^2), \quad (5.102)$$

for a generic fragmentation function F. The lowest moment is the familiar $\boldsymbol{k}_T$-integrated fragmentation function. By constructing appropriately weighted cross sections the convolutions result in products of such $\boldsymbol{k}_T^2$-moments. The same $\boldsymbol{k}_T^2$-moments for instance show up in semi-inclusive lepton-hadron scattering, in that case multiplied by $\boldsymbol{k}_T^2$-moments of distribution functions [42]. In Sect. 5.5 we have presented the hadron tensor and cross section integrated over transverse momentum of the photon. A number of structures averaged out to zero, which are retained when the integration is weighted with an appropriate number of factors of $\boldsymbol{q}_T$.

We find for the once-weighted cross sections, where we again show only the leading results, for the case that hadron two is unpolarized:

$$\int d^2\boldsymbol{q}_T\ (\boldsymbol{q}_T\cdot\boldsymbol{a})\ \frac{d\sigma(e^+e^-\to h_1h_2X)}{d\Omega dz_1dz_2d^2\boldsymbol{q}_T} = \frac{3\alpha^2}{Q^2}\sum_{a,\bar{a}} e_a{}^2\ |\boldsymbol{a}| \Big\{ -A(y)\ |\boldsymbol{S}_{1T}|\ \sin(\phi_{S_1}-\phi_a)\left(M_1 D_{1T}^{\perp(1)}\overline{D}_1\right) - B(y)\ |\boldsymbol{S}_{1T}|\ \sin(\phi_a+\phi_{S_1})\left(M_2 H_1 \overline{H}_1^{\perp(1)}\right) -\lambda_e\frac{C(y)}{2}\ |\boldsymbol{S}_{1T}|\ \cos(\phi_{S_1}-\phi_a)\left(M_1 G_{1T}^{(1)}\overline{D}_1\right)\Big\}. \tag{5.103}$$

Constructing from this cross section the weighted one-particle inclusive cross section, by replacing $\overline{D}_1$ by a delta-function, and considering the specific case[3] $\boldsymbol{a} = \hat{\boldsymbol{l}}_\perp$, one finds:

$$\int d^2\boldsymbol{q}_T\ \left(\boldsymbol{q}_T\cdot\hat{\boldsymbol{l}}_\perp\right)\ \frac{d\sigma(e^+e^-\to hX)}{d\Omega dz_h d^2\boldsymbol{q}_T} = \frac{3\alpha^2}{Q^2}\sum_{a,\bar{a}} e_a{}^2 \Big\{ -A(y)\ |\boldsymbol{S}_{hT}|\ \sin(\phi_{S_h})\ M_h D_{1T}^{\perp(1)} - \lambda_e\frac{C(y)}{2}\ |\boldsymbol{S}_{hT}|\ \cos(\phi_{S_h})\ M_h G_{1T}^{(1)}\Big\}. \tag{5.104}$$

These $\boldsymbol{k}_T^2$-moments $D_{1T}^{\perp(1)}$ and $G_{1T}^{(1)}$ are related to the twist-three functions D_T and G_T via

$$D_T(z) = z^3\frac{d}{dz}\left[\frac{D_{1T}^{\perp(1)}(z)}{z}\right], \tag{5.105}$$

$$G_T(z) = G_1(z) - z^3\frac{d}{dz}\left[\frac{G_{1T}^{(1)}(z)}{z}\right], \tag{5.106}$$

respectively [42]. These relations are analogous to the ones discussed in Chap. 3. An experimental verification of these relations by comparing the above cross section to the one-particle inclusive results Eqs. (5.84) and (5.85) would be very interesting.

The twice-weighted cross section is:

$$\int d^2\boldsymbol{q}_T\ (\boldsymbol{q}_T\cdot\boldsymbol{a})\ (\boldsymbol{q}_T\cdot\boldsymbol{b})\ \frac{d\sigma(e^+e^-\to h_1h_2X)}{d\Omega dz_1dz_2d^2\boldsymbol{q}_T} = \frac{3\alpha^2}{Q^2}\sum_{a,\bar{a}} e_a{}^2\ |\boldsymbol{a}|\ |\boldsymbol{b}| \Big\{ -A(y)\ \cos(\phi_b-\phi_a)\left(M_2{}^2 D_1\overline{D}_1^{(1)} + M_1{}^2 D_1^{(1)}\overline{D}_1\right) + 2B(y)\ M_1M_2\left[\cos(\phi_b+\phi_a)\left(H_1^{\perp(1)}\overline{H}_1^{\perp(1)}\right) + \sin(\phi_b+\phi_a)\left(\lambda_1\ H_{1L}^{\perp(1)}\overline{H}_1^{\perp(1)}\right)\right] + \lambda_e\frac{C(y)}{2}\ \cos(\phi_b-\phi_a)\left(+\lambda_1\ M_2{}^2 G_1\overline{D}_1^{(1)} + \lambda_1\ M_1{}^2 G_1^{(1)}\overline{D}_1\right)\Big\}. \tag{5.107}$$

[3]Note that one is not allowed to take $\boldsymbol{a} = \boldsymbol{q}_T$ in Eq. (5.103), since that is the momentum one has integrated over. If one wants to weight with $\boldsymbol{q}_T^2$ one has to look at the twice-weighted cross section.

In particular, one can use $(\boldsymbol{q}_T \cdot \hat{\boldsymbol{l}}_\perp)^2$, so one puts $\boldsymbol{a} = \boldsymbol{b} = \hat{\boldsymbol{l}}_\perp$ in the above equation ($\phi_a = \phi_b = 0$), such that in case both hadrons are unpolarized

$$\int d^2\boldsymbol{q}_T \left(\boldsymbol{q}_T \cdot \hat{\boldsymbol{l}}_\perp\right)^2 \frac{d\sigma(e^+e^- \to h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2\boldsymbol{q}_T} = \frac{3\alpha^2}{Q^2} \sum_{a,\bar{a}} e_a{}^2 \Big\{ - A(y) \left(M_2{}^2 D_1 \overline{D}_1^{(1)} + M_1{}^2 D_1^{(1)} \overline{D}_1\right) + 2B(y)\, M_1 M_2\, H_1^{\perp(1)} \overline{H}_1^{\perp(1)} \Big\}. \qquad (5.108)$$

Going back to the result for $N_{h_1 h_2}$ in Eq. (5.96), one sees that weighting that result only with $\cos(2\phi_1)$ would produce a convolution of $H_1^\perp$ and $\overline{H}_1^\perp$, while the result above shows that including appropriate factors of $|\boldsymbol{q}_T|$ produces a product of $\boldsymbol{k}_T^2$-moments of fragmentation functions, in this case $H_1^{\perp(1)} \overline{H}_1^{\perp(1)}$. The $\boldsymbol{k}_T^2$-moments can be used in other processes where they also occur. The above is an illustration of the general procedure.

In App. 5.D and 5.E we give the integrated once- and twice-weighted hadron tensors, respectively, in case both hadrons are polarized. The twice-weighted result is only given to leading order.

5.8 Summary and conclusions

We have presented the complete tree-level result up to and including order $1/Q$ for inclusive two-hadron production in electron-positron annihilation. We consider the situation where the two hadrons belong to different, back-to-back jets. Polarization in the initial and final states is included for the case of spin-1/2 hadrons. In case of spinless hadrons one will focus on the ones that are produced most abundantly, like π's and K's, which also serve to study flavor dependence of fragmentation functions (see for instance [101]). For the case of spin-1/2 hadrons Λ's seem most appropriate due to their self-analyzing decays (see for instance [97, 98]). Hadrons with higher spin, like ρ's, are not considered, because in that case a spin vector is not sufficient to describe the spin states (for a treatment of spin-one hadrons, see for instance [96, 102]).

We have restricted ourselves to the case of photon exchange, since we are interested in power corrections which are, most likely, negligible in regions of Q^2 where the annihilation into Z bosons becomes important, i.e., at LEP energies. In the next chapter we will investigate the inclusion of Z's in the leading part of our result taking into account transverse momentum.

Our results include among others the following:

- We have considered the cross sections integrated over transverse momenta of the produced hadrons for both, polarized and unpolarized beams up to subleading twist. Our result agree with the terms which have been found previously in a leading order analysis for unpolarized e^+e^- annihilation [97].
- In particular, we have focussed on the information obtainable by observing transverse momentum of one of the produced hadrons (defined either relative to a jet-axis or

relative to the momentum of a hadron in the second jet). Although cross sections differential in transverse momentum are not easy to measure, they are of particular interest, since they contain leading order asymmetries, which would vanish upon integration.

We have found a number of new unpolarized, single and double spin asymmetries. Often they have no analogues in regular (semi-inclusive) DIS or the Drell-Yan process, since they involve products – or to be more specific, convolutions – of two time-reversal odd fragmentation functions. In particular, the $\cos(2\phi)$ dependence discussed in Sect. 6 is most likely measurable, since it is not suppressed by powers of $1/Q$ and does not involve polarization, neither of the beams nor of the final states (see also next chapter).

One-hadron inclusive measurements supplied with the additional determination of the jet axis gives direct access to the transverse momentum dependence of some of the fragmentation functions.

- We have discussed how convolutions of fragmentation functions can be converted into products of their $\boldsymbol{k}_T^2$-moments. This is achieved by appropriate weighting of the integration over the transverse momentum dependence, in the spirit of a Fourier analysis. This is another way of retaining asymmetries, which would vanish upon (non-weighted) integration.

5.A The fragmentation functions for a quark fragmenting into a quark

We consider the correlation function for the case of a quark (with momentum k) fragmenting into a quark (with momentum p and spin s), given by

$$\delta_{ij}(p,s;k) = u_i(k,s)\overline{u}_j(k,s)\delta^4(k-p) = \frac{1}{2}((\not{k}+m)(1+\gamma_5\not{s}))_{ij}\delta^4(k-p), \tag{5.A1}$$

where the momentum and spin of the quark are parametrized as (using the notation $k = [k^-, k^+, \boldsymbol{k}_T]$)

$$k = \left[k^-,\ \frac{\boldsymbol{k}_T^2+m^2}{2k^-},\ \boldsymbol{k}_T\right], \tag{5.A2}$$

$$s = \left[\frac{\lambda_q\, k^-}{m},\ -\frac{m\,\lambda_q}{2k^-}+\frac{\boldsymbol{k}_T\cdot\boldsymbol{s}_{qT}}{k^-}+\frac{\lambda_q\,\boldsymbol{k}_T^2}{2m\,k^-},\ \boldsymbol{s}_{qT}+\frac{\lambda_q}{m}\boldsymbol{k}_T\right] \tag{5.A3}$$

in terms of a quark lightcone helicity λ_q and a quark lightcone transverse polarization $\boldsymbol{s}_{qT}$. The twist-two projections become

$$\delta^{[\gamma^-]}(k) = \frac{1}{2}\delta\,(z-1)\,\delta^2(\boldsymbol{k}_T-\boldsymbol{p}_T), \tag{5.A4}$$

$$\delta^{[\gamma^-\gamma_5]}(k) = \frac{1}{2}\lambda_q\,\delta\,(z-1)\,\delta^2(\boldsymbol{k}_T - \boldsymbol{p}_T), \tag{5.A5}$$

$$\delta^{[i\sigma^{i-}\gamma_5]}(k) = \frac{1}{2}\boldsymbol{s}_{qT}^i\,\delta\,(z-1)\,\delta^2(\boldsymbol{k}_T - \boldsymbol{p}_T), \tag{5.A6}$$

where $z = p^-/k^-$. For twist three we obtain

$$\delta^{[1]}(k) = \frac{m}{2k^-}\,\delta\,(z-1)\,\delta^2(\boldsymbol{k}_T - \boldsymbol{p}_T), \tag{5.A7}$$

$$\delta^{[\gamma^i]}(k) = \frac{\boldsymbol{k}_T^i}{2k^-}\,\delta\,(z-1)\,\delta^2(\boldsymbol{k}_T - \boldsymbol{p}_T), \tag{5.A8}$$

$$\delta^{[\gamma^i\gamma_5]}(k) = \frac{(m\,\boldsymbol{s}_{qT}^i + \lambda_q\,\boldsymbol{k}_T^i)}{2k^-}\,\delta\,(z-1)\,\delta^2(\boldsymbol{k}_T - \boldsymbol{p}_T), \tag{5.A9}$$

$$\delta^{[i\sigma^{ij}\gamma_5]}(k) = \frac{\boldsymbol{s}_{qT}^i\boldsymbol{k}_T^j - \boldsymbol{k}_T^i\boldsymbol{s}_{qT}^j}{2k^-}\,\delta\,(z-1)\,\delta^2(\boldsymbol{k}_T - \boldsymbol{p}_T), \tag{5.A10}$$

$$\delta^{[i\sigma^{-+}\gamma_5]}(k) = \frac{m\,\lambda_q - \boldsymbol{k}_T\cdot\boldsymbol{s}_{qT}}{2k^-}\,\delta\,(z-1)\,\delta^2(\boldsymbol{k}_T - \boldsymbol{p}_T). \tag{5.A11}$$

5.B The complete expression for the hadron tensor

The full expressions for the symmetric and antisymmetric parts of the hadron tensor are (expressed in the perpendicular frame defined in the second section)

$$\begin{aligned}
\mathcal{W}_S^{\mu\nu} = 12 z_1 z_2 \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{p}_T\, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T)\Bigg\{ \\
-g_\perp^{\mu\nu}\Bigg[D_1\overline{D}_1 - G_{1s}\overline{G}_{1s} + \frac{\epsilon_\perp^{\rho\sigma}k_{\perp\rho}S_{1\perp\sigma}}{M_1}D_{1T}^\perp\overline{D}_1 - \frac{\epsilon_\perp^{\rho\sigma}p_{\perp\rho}S_{2\perp\sigma}}{M_2}D_1\overline{D}_{1T}^\perp \\
- \frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{p}_\perp\ \boldsymbol{S}_{1\perp}\!\cdot\!\boldsymbol{S}_{2\perp} - \boldsymbol{p}_\perp\!\cdot\!\boldsymbol{S}_{1\perp}\ \boldsymbol{k}_\perp\!\cdot\!\boldsymbol{S}_{2\perp}}{M_1M_2}D_{1T}^\perp\overline{D}_{1T}^\perp\Bigg] \\
-\left(S_{1\perp}^{\{\mu}S_{2\perp}^{\nu\}} + g_\perp^{\mu\nu}\,\boldsymbol{S}_{1\perp}\!\cdot\!\boldsymbol{S}_{2\perp}\right)H_{1T}\overline{H}_{1T} - \frac{k_\perp^{\{\mu}p_\perp^{\nu\}} + g_\perp^{\mu\nu}\,\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{p}_\perp}{M_1M_2}\left(H_{1s}^\perp\overline{H}_{1s}^\perp + H_1^\perp\overline{H}_1^\perp\right) \\
-\frac{k_\perp^{\{\mu}S_{2\perp}^{\nu\}} + g_\perp^{\mu\nu}\,\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{S}_{2\perp}}{M_1}H_{1s}^\perp\overline{H}_{1T} - \frac{p_\perp^{\{\mu}S_{1\perp}^{\nu\}} + g_\perp^{\mu\nu}\,\boldsymbol{p}_\perp\!\cdot\!\boldsymbol{S}_{1\perp}}{M_2}H_{1T}\overline{H}_{1s}^\perp \\
+\frac{k_\perp^{\{\mu}\epsilon_\perp^{\nu\}\rho}p_{\perp\rho} + p_\perp^{\{\mu}\epsilon_\perp^{\nu\}\rho}k_{\perp\rho}}{2M_1M_2}\left(H_{1s}^\perp\overline{H}_1^\perp - H_1^\perp\overline{H}_{1s}^\perp\right) \\
-\frac{k_\perp^{\{\mu}\epsilon_\perp^{\nu\}\rho}S_{2\perp\rho} + S_{2\perp}^{\{\mu}\epsilon_\perp^{\nu\}\rho}k_{\perp\rho}}{2M_1}H_1^\perp\overline{H}_{1T} + \frac{p_\perp^{\{\mu}\epsilon_\perp^{\nu\}\rho}S_{1\perp\rho} + S_{1\perp}^{\{\mu}\epsilon_\perp^{\nu\}\rho}p_{\perp\rho}}{2M_2}H_{1T}\overline{H}_1^\perp \\
+2\frac{\hat{z}^{\{\mu}k_\perp^{\nu\}}}{Q}\Bigg[+\frac{\tilde{D}^\perp}{z_1}\overline{D}_1 - \frac{\boldsymbol{S}_{1\perp}\!\cdot\!\boldsymbol{S}_{2\perp}}{M_1}M_2D_{1T}^\perp\frac{\overline{D}_T}{z_2} - \lambda_2\frac{\boldsymbol{p}_\perp\!\cdot\!\boldsymbol{S}_{1\perp}}{M_1}D_{1T}^\perp\frac{\overline{D}_L^\perp}{z_2} \\
+\frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{p}_\perp\ \boldsymbol{S}_{1\perp}\!\cdot\!\boldsymbol{S}_{2\perp} - \boldsymbol{p}_\perp\!\cdot\!\boldsymbol{S}_{1\perp}\ \boldsymbol{k}_\perp\!\cdot\!\boldsymbol{S}_{2\perp}}{M_1M_2}D_{1T}^\perp\overline{D}_{1T}^\perp - \frac{\tilde{G}_s^\perp}{z_1}\overline{G}_{1s}
\end{aligned}$$

$$
\begin{aligned}
&\qquad + \frac{\boldsymbol{p}_\perp\!\cdot\!\boldsymbol{S}_{1\perp}}{M_2}\,\frac{\tilde{H}_T^\perp}{z_1}\overline{H}_{1s}^\perp + \boldsymbol{S}_{1\perp}\!\cdot\!\boldsymbol{S}_{2\perp}\,\frac{\tilde{H}_T^\perp}{z_1}\overline{H}_{1T} - \frac{M_2}{M_1}\left(H_{1s}^\perp\frac{\overline{H}_s}{z_2} + H_1^\perp\frac{\overline{H}}{z_2}\right)\Bigg] \\
&+ 2\frac{\hat{z}^{\{\mu}p_\perp^{\nu\}}}{Q}\Bigg[- D_1\frac{\overline{D}^\perp}{z_2} - \frac{\boldsymbol{k}_\perp^2\;\boldsymbol{S}_{1\perp}\!\cdot\!\boldsymbol{S}_{2\perp}}{2M_1M_2}D_{1T}^\perp\overline{D}_{1T}^\perp + \lambda_1\frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{S}_{2\perp}}{M_2}\,\frac{\tilde{D}_L^\perp}{z_1}\overline{D}_{1T}^\perp \\
&\qquad + \frac{M_1}{M_2}\,\boldsymbol{S}_{1\perp}\!\cdot\!\boldsymbol{S}_{2\perp}\,\frac{\tilde{D}_T}{z_1}\overline{D}_{1T}^\perp + G_{1s}\frac{\overline{G}_s^\perp}{z_2} - \frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{S}_{2\perp}}{M_1}H_{1s}^\perp\frac{\overline{H}_T^\perp}{z_2} \\
&\qquad - \boldsymbol{S}_{1\perp}\!\cdot\!\boldsymbol{S}_{2\perp}\,H_{1T}\frac{\overline{H}_T^\perp}{z_2} + \frac{M_1}{M_2}\left(\frac{\tilde{H}}{z_1}\overline{H}_1^\perp + \frac{\tilde{H}_s}{z_1}\overline{H}_{1s}^\perp\right)\Bigg] \\
&+ 2\frac{\hat{z}^{\{\mu}S_{1\perp}^{\nu\}}}{Q}\Bigg[+ \lambda_2\frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{p}_\perp}{M_1}D_{1T}^\perp\frac{\overline{D}_L^\perp}{z_2} + \frac{M_2}{M_1}\,\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{S}_{2\perp}\,D_{1T}^\perp\frac{\overline{D}_T}{z_2} - M_1\frac{\tilde{G}_T'}{z_1}\overline{G}_{1s} \\
&\qquad - \frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{p}_\perp}{M_2}\,\frac{\tilde{H}_T^\perp}{z_1}\overline{H}_{1s}^\perp - M_2H_{1T}\frac{\overline{H}_s}{z_2} - \boldsymbol{k}_\perp\!\cdot\!\boldsymbol{S}_{2\perp}\,\frac{\tilde{H}_T^\perp}{z_1}\overline{H}_{1T}\Bigg] \\
&+ 2\frac{\hat{z}^{\{\mu}S_{2\perp}^{\nu\}}}{Q}\Bigg[- \lambda_1\frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{p}_\perp}{M_2}\,\frac{\tilde{D}_L^\perp}{z_1}\overline{D}_{1T}^\perp + \frac{\boldsymbol{p}_\perp\!\cdot\!\boldsymbol{S}_{1\perp}\;\boldsymbol{k}_\perp^2}{2M_1M_2}D_{1T}^\perp\overline{D}_{1T}^\perp - \frac{M_1}{M_2}\,\boldsymbol{p}_\perp\!\cdot\!\boldsymbol{S}_{1\perp}\,\frac{\tilde{D}_T}{z_1}\overline{D}_{1T}^\perp \\
&\qquad + M_2G_{1s}\frac{\overline{G}_T'}{z_2} + \frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{p}_\perp}{M_1}H_{1s}^\perp\frac{\overline{H}_T^\perp}{z_2} + \boldsymbol{p}_\perp\!\cdot\!\boldsymbol{S}_{1\perp}\,H_{1T}\frac{\overline{H}_T^\perp}{z_2} + M_1\frac{\tilde{H}_s}{z_1}\overline{H}_{1T}\Bigg] \\
&+ 2\frac{\hat{z}^{\{\mu}\epsilon_\perp^{\nu\}\rho}k_{\perp\rho}}{Q}\Bigg[\lambda_1\frac{\tilde{D}_L^\perp}{z_1}\overline{D}_1 - \frac{\boldsymbol{p}_\perp\!\cdot\!\boldsymbol{S}_{1\perp}}{M_1}D_{1T}^\perp\frac{\overline{D}^\perp}{z_2} - \frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{S}_{1\perp}}{M_1}D_{1T}^\perp\overline{D}_1 \\
&\qquad + \frac{m}{M_1}H_1^\perp\overline{G}_{1s} - \frac{\boldsymbol{p}_\perp\!\cdot\!\boldsymbol{S}_{1\perp}}{M_2}\,\frac{\tilde{H}_T^\perp}{z_1}\overline{H}_1^\perp + \frac{M_2}{M_1}\left(H_{1s}^\perp\frac{\overline{H}}{z_2} - H_1^\perp\frac{\overline{H}_s}{z_2}\right)\Bigg] \\
&+ 2\frac{\hat{z}^{\{\mu}\epsilon_\perp^{\nu\}\rho}p_{\perp\rho}}{Q}\Bigg[\lambda_2D_1\frac{\overline{D}_L^\perp}{z_2} - \frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{S}_{2\perp}}{M_2}\,\frac{\tilde{D}^\perp}{z_1}\overline{D}_{1T}^\perp - \frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{S}_{2\perp}}{M_1}H_1^\perp\frac{\overline{H}_T^\perp}{z_2} \\
&\qquad + \frac{M_1}{M_2}\left(\frac{\tilde{H}}{z_1}\overline{H}_{1s}^\perp - \frac{\tilde{H}_s}{z_1}\overline{H}_1^\perp\right)\Bigg] \\
&+ 2\frac{\hat{z}^{\{\mu}\epsilon_\perp^{\nu\}\rho}S_{1\perp\rho}}{Q}\Bigg[M_1\frac{\tilde{D}_T}{z_1}\overline{D}_1 + \frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{p}_\perp}{M_2}\,\frac{\tilde{H}_T^\perp}{z_1}\overline{H}_1^\perp + \frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{p}_\perp}{M_1}D_{1T}^\perp\frac{\overline{D}^\perp}{z_2} \\
&\qquad + \frac{\boldsymbol{k}_\perp^2}{2M_1}D_{1T}^\perp\overline{D}_1 + M_2H_{1T}\frac{\overline{H}}{z_2}\Bigg] \qquad\qquad (5.B1) \\
&+ 2\frac{\hat{z}^{\{\mu}\epsilon_\perp^{\nu\}\rho}S_{2\perp\rho}}{Q}\Bigg[M_2D_1\frac{\overline{D}_T}{z_2} + M_1\frac{\tilde{H}}{z_1}\overline{H}_{1T} + \frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{p}_\perp}{M_2}\,\frac{\tilde{D}^\perp}{z_1}\overline{D}_{1T}^\perp + \frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{p}_\perp}{M_1}H_1^\perp\frac{\overline{H}_T^\perp}{z_2}\Bigg]\Bigg\}
\end{aligned}
$$

and

$$
\begin{aligned}
\mathcal{W}_A^{\mu\nu} = 12 z_1 z_2 \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{p}_T\, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T)\Bigg\{ & \\
&+ i\epsilon_\perp^{\mu\nu}\Big[G_{1s}\overline{D}_1 - D_1\overline{G}_{1s}\Big] - i p_\perp^{[\mu} S_{2\perp}^{\nu]}\frac{1}{M_2}G_{1s}\overline{D}_{1T}^\perp - i k_\perp^{[\mu} S_{1\perp}^{\nu]}\frac{1}{M_1}D_{1T}^\perp\overline{G}_{1s} \\
&+ 2i\frac{\hat{z}^{[\mu}k_\perp^{\nu]}}{Q}\Bigg[+ \lambda_1\frac{\tilde{D}_L^\perp}{z_1}\overline{G}_{1s} - \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_{1\perp}}{M_1}D_{1T}^\perp\overline{G}_{1s} - \frac{\boldsymbol{S}_{1\perp}\cdot\boldsymbol{S}_{2\perp}}{M_1}\Bigg(M_2 D_{1T}^\perp\frac{\overline{G}_T'}{z_2} \\
&\qquad - m D_{1T}^\perp\overline{H}_{1T}\Bigg) - \frac{\boldsymbol{p}_\perp\cdot\boldsymbol{S}_{1\perp}}{M_1}\left(D_{1T}^\perp\frac{\overline{G}_s^\perp}{z_2} - \frac{m}{M_2}D_{1T}^\perp\overline{H}_{1s}^\perp\right) + \frac{m}{M_1}H_1^\perp\overline{D}_1 \\
&\qquad - \frac{M_2}{M_1}\left(H_{1s}^\perp\frac{\overline{E}_s}{z_2} + H_1^\perp\frac{\overline{E}}{z_2}\right)\Bigg] \\
&+ 2i\frac{\hat{z}^{[\mu}p_\perp^{\nu]}}{Q}\Bigg[- \lambda_2 G_{1s}\frac{\overline{D}_L^\perp}{z_2} + \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_{2\perp}}{M_2}\frac{\tilde{G}_s^\perp}{z_1}\overline{D}_{1T}^\perp + \frac{\boldsymbol{S}_{1\perp}\cdot\boldsymbol{S}_{2\perp}}{M_2}M_1\frac{\tilde{G}_T'}{z_1}\overline{D}_{1T}^\perp \\
&\qquad + \frac{M_1}{M_2}\left(\frac{\tilde{E}_s}{z_1}\overline{H}_{1s}^\perp + \frac{\tilde{E}}{z_1}\overline{H}_1^\perp\right)\Bigg] \\
&+ 2i\frac{\hat{z}^{[\mu}S_{1\perp}^{\nu]}}{Q}\Bigg[+ M_1\frac{\tilde{D}_T}{z_1}\overline{G}_{1s} + \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_{2\perp}}{M_1}\left(M_2 D_{1T}^\perp\frac{\overline{G}_T'}{z_2} - m D_{1T}^\perp\overline{H}_{1T}\right) + \frac{\boldsymbol{k}_\perp^2}{2M_1}D_{1T}^\perp\overline{G}_{1s} \\
&\qquad + \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{p}_\perp}{M_1 M_2}\left(M_2 D_{1T}^\perp\frac{\overline{G}_s^\perp}{z_2} - m D_{1T}^\perp\overline{H}_{1s}^\perp\right) - M_2 H_{1T}\frac{\overline{E}_s}{z_2}\Bigg] \\
&+ 2i\frac{\hat{z}^{[\mu}S_{2\perp}^{\nu]}}{Q}\Bigg[- M_2 G_{1s}\frac{\overline{D}_T}{z_2} - \frac{\boldsymbol{p}_\perp\cdot\boldsymbol{S}_{1\perp}}{M_2}M_1\frac{\tilde{G}_T'}{z_1}\overline{D}_{1T}^\perp - \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{p}_\perp}{M_2}\frac{\tilde{G}_s^\perp}{z_1}\overline{D}_{1T}^\perp + M_1\frac{\tilde{E}_s}{z_1}\overline{H}_{1T}\Bigg] \\
&+ 2i\frac{\hat{z}^{[\mu}\epsilon_\perp^{\nu]\rho}k_{\perp\rho}}{Q}\Bigg[\frac{\tilde{G}_s^\perp}{z_1}\overline{D}_1 - \frac{\tilde{D}^\perp}{z_1}\overline{G}_{1s} - \frac{\boldsymbol{p}_\perp\cdot\boldsymbol{S}_{1\perp}}{M_1 M_2}m D_{1T}^\perp\overline{H}_1^\perp + \frac{M_2}{M_1}\left(H_{1s}^\perp\frac{\overline{E}}{z_2} - H_1^\perp\frac{\overline{E}_s}{z_2}\right)\Bigg] \\
&+ 2i\frac{\hat{z}^{[\mu}\epsilon_\perp^{\nu]\rho}p_{\perp\rho}}{Q}\Bigg[D_1\frac{\overline{G}_s^\perp}{z_2} - G_{1s}\frac{\overline{D}^\perp}{z_2} - \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_{2\perp}}{M_1 M_2}m H_1^\perp\overline{D}_{1T}^\perp + \frac{M_1}{M_2}\left(\frac{\tilde{E}}{z_1}\overline{H}_{1s}^\perp - \frac{\tilde{E}_s}{z_1}\overline{H}_1^\perp\right)\Bigg] \\
&+ 2i\frac{\hat{z}^{[\mu}\epsilon_\perp^{\nu]\rho}S_{1\perp\rho}}{Q}\Bigg[M_1\frac{\tilde{G}_T'}{z_1}\overline{D}_1 + \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{p}_\perp}{M_1 M_2}m D_{1T}^\perp\overline{H}_1^\perp + M_2 H_{1T}\frac{\overline{E}}{z_2}\Bigg] \\
&+ 2i\frac{\hat{z}^{[\mu}\epsilon_\perp^{\nu]\rho}S_{2\perp\rho}}{Q}\Bigg[M_2 D_1\frac{\overline{G}_T'}{z_2} + \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{p}_\perp}{M_1 M_2}m H_1^\perp\overline{D}_{1T}^\perp + M_1\frac{\tilde{E}}{z_1}\overline{H}_{1T}\Bigg]\Bigg\}.
\end{aligned}
\qquad (5.B2)
$$

5.C Double spin asymmetries

In this appendix we give the azimuthal dependences of double spin asymmetries, as can be observed, for instance, in $\Lambda\overline{\Lambda}$ production by determination of the polarizations of both observed hadrons. The spin-independent and single-spin dependent parts of the cross section are given in Eqs. (5.94), (5.99) and (5.100).

$$
\begin{aligned}
\frac{d\sigma^{(2)}(e^+e^-\to h_1h_2X)}{d\Omega dz_1 dz_2 d^2\boldsymbol{q}_T} = \frac{3\alpha^2}{Q^2}\,z_1^2z_2^2\,\Bigg\{ &-\frac{A(y)}{2}\,\lambda_1\,\lambda_2\,\mathcal{F}\left[G_1\overline{G}_1\right] \\
&-A(y)\,\lambda_1\,|\boldsymbol{S}_{2T}|\,\cos(\phi_1-\phi_{S_2})\,\mathcal{F}\left[\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{p}_T\,\frac{G_1\overline{G}_{1T}}{M_2}\right] \\
&+\frac{A(y)}{2}\,|\boldsymbol{S}_{1T}|\,|\boldsymbol{S}_{2T}|\,\cos(2\phi_1-\phi_{S_1}-\phi_{S_2})\,\mathcal{F}\left[\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{k}_T\,\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{p}_T\,\frac{D_{1T}^\perp\overline{D}_{1T}^\perp-G_{1T}\overline{G}_{1T}}{M_1M_2}\right] \\
&-\frac{A(y)}{2}\,|\boldsymbol{S}_{1T}|\,|\boldsymbol{S}_{2T}|\,\cos(\phi_1-\phi_{S_1})\,\cos(\phi_1-\phi_{S_2})\,\mathcal{F}\left[\boldsymbol{k}_T\!\cdot\!\boldsymbol{p}_T\,\frac{D_{1T}^\perp\overline{D}_{1T}^\perp}{M_1M_2}\right] \\
&-\frac{A(y)}{2}\,|\boldsymbol{S}_{1T}|\,|\boldsymbol{S}_{2T}|\,\sin(\phi_1-\phi_{S_1})\,\sin(\phi_1-\phi_{S_2})\,\mathcal{F}\left[\boldsymbol{k}_T\!\cdot\!\boldsymbol{p}_T\,\frac{G_{1T}\overline{G}_{1T}}{M_1M_2}\right] \\
&+\frac{B(y)}{2}\,|\boldsymbol{S}_{1T}|\,|\boldsymbol{S}_{2T}|\,\cos(\phi_{S_1}+\phi_{S_2})\,\mathcal{F}\left[H_1\overline{H}_1\right] \\
&+B(y)\,\lambda_1\,|\boldsymbol{S}_{2T}|\,\cos(\phi_1+\phi_{S_2})\,\mathcal{F}\left[\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{k}_T\,\frac{H_{1L}^\perp\overline{H}_1}{M_1}\right] \\
&+\frac{B(y)}{2}\,\lambda_1\,\lambda_2\,\cos(2\phi_1)\,\mathcal{F}\left[\left(2\,\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{k}_T\,\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{p}_T\,-\,\boldsymbol{k}_T\!\cdot\!\boldsymbol{p}_T\right)\frac{H_{1L}^\perp\overline{H}_{1L}^\perp}{M_1M_2}\right] \\
&+\frac{B(y)}{2}\,|\boldsymbol{S}_{1T}|\,|\boldsymbol{S}_{2T}|\,\cos(2\phi_1-\phi_{S_1}+\phi_{S_2})\,\mathcal{F}\left[\left(2(\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{k}_T)^2-\boldsymbol{k}_T^2\right)\frac{H_{1T}^\perp\overline{H}_1}{M_1}\right] \\
&+\frac{B(y)}{2}\,\lambda_2\,|\boldsymbol{S}_{1T}|\,\cos(3\phi_1-\phi_{S_1})\,\mathcal{F}\Bigg[\Big(4\,\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{p}_T\,(\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{k}_T)^2-2\,\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{k}_T\,\boldsymbol{k}_T\!\cdot\!\boldsymbol{p}_T \\
&\qquad\qquad -\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{p}_T\,\boldsymbol{k}_T^2\Big)\frac{H_{1T}^\perp\overline{H}_{1L}^\perp}{M_1^2M_2}\Bigg] \\
&+\frac{B(y)}{8}\,|\boldsymbol{S}_{1T}|\,|\boldsymbol{S}_{2T}|\,\cos(4\phi_1-\phi_{S_1}-\phi_{S_2})\,\mathcal{F}\Bigg[\Bigg(8(\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{k}_T)^2(\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{p}_T)^2 \\
&-4\,\boldsymbol{k}_T\!\cdot\!\boldsymbol{p}_T\,\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{k}_T\,\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{p}_T\,-2(\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{k}_T)^2\,\boldsymbol{p}_T^2\,-2(\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{p}_T)^2\,\boldsymbol{k}_T^2\,+\boldsymbol{k}_T^2\,\boldsymbol{p}_T^2\Bigg)\frac{H_{1T}^\perp\overline{H}_{1T}^\perp}{M_1^2M_2^2}\Bigg] \\
&-\lambda_e\,\frac{C(y)}{2}\,\lambda_1\,|\boldsymbol{S}_{2T}|\,\sin(\phi_1-\phi_{S_2})\,\mathcal{F}\left[\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{p}_T\,\frac{G_1\overline{D}_{1T}^\perp}{M_2}\right]
\end{aligned}
$$

$$
\begin{aligned}
&-\lambda_e\,\frac{C(y)}{4}\,|\boldsymbol{S}_{1T}|\,|\boldsymbol{S}_{2T}|\,\sin(2\phi_1-\phi_{S_1}-\phi_{S_2})\,\mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\;\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\,\frac{D_{1T}^{\perp}\overline{G}_{1T}+G_{1T}\overline{D}_{1T}^{\perp}}{M_1M_2}\right]\\
&-\lambda_e\,\frac{C(y)}{2}\,|\boldsymbol{S}_{1T}|\,|\boldsymbol{S}_{2T}|\,\sin(\phi_1-\phi_{S_2})\cos(\phi_1-\phi_{S_1})\,\mathcal{F}\left[\boldsymbol{k}_T\cdot\boldsymbol{p}_T\,\frac{D_{1T}^{\perp}\overline{G}_{1T}}{M_1M_2}\right]\\
&+\;\begin{pmatrix}1\longleftrightarrow 2\\ \boldsymbol{p}\longleftrightarrow\boldsymbol{k}\end{pmatrix}\;\Bigg\}
\end{aligned}
\tag{5.C1}
$$

5.D Integrated once-weighted hadron tensor

We display the hadron tensor weighted with the factor $(\boldsymbol{q}_T\cdot\boldsymbol{a})$ when integrated over the transverse momentum of the photon. The vector $\boldsymbol{a}$ is an arbitrary vector like e.g. $\hat{\boldsymbol{l}}_\perp$.

$$
\begin{aligned}
&\int d^2\boldsymbol{q}_T\;(\boldsymbol{q}_T\cdot\boldsymbol{a})\;\mathcal{W}_S^{\mu\nu}=12z_1z_2\times\Bigg\{\\
&-g_\perp^{\mu\nu}\Big[-\lambda_1\,\boldsymbol{a}\cdot\boldsymbol{S}_{2\perp}\,M_2G_1\overline{G}_{1T}^{(1)}-\lambda_2\,\boldsymbol{a}\cdot\boldsymbol{S}_{1\perp}\,M_1G_{1T}^{(1)}\overline{G}_1\\
&\qquad+\epsilon_\perp^{\rho\sigma}a_\rho S_{1\perp\sigma}M_1D_{1T}^{\perp(1)}\overline{D}_1-\epsilon_\perp^{\rho\sigma}a_\rho S_{2\perp\sigma}M_2D_1\overline{D}_{1T}^{\perp(1)}\Big]\\
&-\left(S_{1\perp}^{\{\mu}a^{\nu\}}+g_\perp^{\mu\nu}\,\boldsymbol{a}\cdot\boldsymbol{S}_{1\perp}\right)\lambda_2M_2H_1\overline{H}_{1L}^{\perp(1)}-\left(S_{2\perp}^{\{\mu}a^{\nu\}}+g_\perp^{\mu\nu}\,\boldsymbol{a}\cdot\boldsymbol{S}_{2\perp}\right)\lambda_1M_1H_{1L}^{\perp(1)}\overline{H}_1\\
&+\left(a^{\{\mu}\epsilon_\perp^{\nu\}\rho}S_{1\perp\rho}+S_{1\perp}^{\{\mu}\epsilon_\perp^{\nu\}\rho}a_\rho\right)\frac{M_2}{2}H_1\overline{H}_1^{\perp(1)}-\left(a^{\{\mu}\epsilon_\perp^{\nu\}\rho}S_{2\perp\rho}+S_{2\perp}^{\{\mu}\epsilon_\perp^{\nu\}\rho}a_\rho\right)\frac{M_1}{2}H_1^{\perp(1)}\overline{H}_1\\
&+2\frac{\hat{z}^{\{\mu}S_{1\perp}^{\nu\}}}{Q}\,\boldsymbol{a}\cdot\boldsymbol{S}_{2\perp}\left[+M_1M_2D_{1T}^{\perp(1)}\frac{\overline{D}_T}{z_2}-M_1M_2\frac{\tilde{G}_T}{z_1}\overline{G}_{1T}^{(1)}-M_2{}^2H_1\frac{\overline{H}_T^{(1)}}{z_2}-M_1{}^2\frac{\tilde{H}_T^{\perp(1)}}{z_1}\overline{H}_1\right]\\
&+2\frac{\hat{z}^{\{\mu}S_{2\perp}^{\nu\}}}{Q}\,\boldsymbol{a}\cdot\boldsymbol{S}_{1\perp}\left[-M_1M_2\frac{\tilde{D}_T}{z_1}\overline{D}_{1T}^{\perp(1)}+M_1M_2G_{1T}^{(1)}\frac{\overline{G}_T}{z_2}+M_2{}^2H_1\frac{\overline{H}_T^{\perp(1)}}{z_2}+M_1{}^2\frac{\tilde{H}_T^{(1)}}{z_1}\overline{H}_1\right]\\
&-2\frac{\hat{z}^{\{\mu}a^{\nu\}}}{Q}\Bigg[M_2{}^2D_1\frac{\overline{D}^{\perp(1)}}{z_2}-M_1{}^2\frac{\tilde{D}^{\perp(1)}}{z_1}\overline{D}_1-M_1M_2\,\boldsymbol{S}_{1\perp}\cdot\boldsymbol{S}_{2\perp}\left(\frac{\tilde{D}_T}{z_1}\overline{D}_{1T}^{\perp(1)}-D_{1T}^{\perp(1)}\frac{\overline{D}_T}{z_2}\right)\\
&\qquad+\lambda_1\lambda_2\left(M_1{}^2\frac{\tilde{G}_L^{\perp(1)}}{z_1}\overline{G}_1-M_2{}^2G_1\frac{\overline{G}_L^{\perp(1)}}{z_2}\right)+\lambda_1\lambda_2M_1M_2\left(H_{1L}^{\perp(1)}\frac{\overline{H}_L}{z_2}-\frac{\tilde{H}_L}{z_1}\overline{H}_{1L}^{\perp(1)}\right)\\
&\qquad+M_1M_2\left(H_1^{\perp(1)}\frac{\overline{H}}{z_2}-\frac{\tilde{H}}{z_1}\overline{H}_1^{\perp(1)}\right)-\boldsymbol{S}_{1\perp}\cdot\boldsymbol{S}_{2\perp}\left(M_1{}^2\frac{\tilde{H}_T^{\perp(1)}}{z_1}\overline{H}_1-M_2{}^2H_1\frac{\overline{H}_T^{\perp(1)}}{z_2}\right)\Bigg]\\
&-2\frac{\hat{z}^{\{\mu}\epsilon_\perp^{\nu\}\rho}a_\rho}{Q}\Bigg[-\lambda_1M_1{}^2\frac{\tilde{D}_L^{\perp(1)}}{z_1}\overline{D}_1-\lambda_2M_2{}^2D_1\frac{\overline{D}_L^{\perp(1)}}{z_2}-\lambda_2mM_1H_1^{\perp(1)}\overline{G}_1\\
&\qquad+\lambda_1M_1M_2\left(\frac{\tilde{H}_L}{z_1}\overline{H}_1^{\perp(1)}-H_{1L}^{\perp(1)}\frac{\overline{H}}{z_2}\right)+\lambda_2M_1M_2\left(H_1^{\perp(1)}\frac{\overline{H}_L}{z_2}-\frac{\tilde{H}}{z_1}\overline{H}_{1L}^{\perp(1)}\right)\Bigg]\Bigg\}
\end{aligned}
\tag{5.D1}
$$

and

$$
\begin{aligned}
&\int d^2\boldsymbol{q}_T\ (\boldsymbol{q}_T\cdot\boldsymbol{a})\ \mathcal{W}_A^{\mu\nu} = 12 z_1 z_2 \times \Bigg\{ \\
&+ i\epsilon_\perp^{\mu\nu}\Big[M_1\,\boldsymbol{a}\cdot\boldsymbol{S}_{1\perp}\,G_{1T}^{(1)}\overline{D}_1 - M_2\,\boldsymbol{a}\cdot\boldsymbol{S}_{2\perp}\,D_1\overline{G}_{1T}^{(1)}\Big] \\
&+ iS_{1\perp}^{[\mu}a^{\nu]}\Big[\lambda_2 M_1 D_{1T}^{\perp(1)}\overline{G}_1\Big] + iS_{2\perp}^{[\mu}a^{\nu]}\Big[\lambda_1 M_2 G_1\overline{D}_{1T}^{\perp(1)}\Big] \\
&+ i2\frac{\hat{z}^{[\mu}S_{1\perp}^{\nu]}}{Q}\,\boldsymbol{a}\cdot\boldsymbol{S}_{2\perp}\left[M_1M_2D_{1T}^{\perp(1)}\frac{\overline{G}_T}{z_2} + M_1M_2\frac{\tilde{D}_T}{z_1}\overline{G}_{1T}^{(1)} - mM_1D_{1T}^{\perp(1)}\overline{H}_1 - {M_2}^2H_1\frac{\overline{E}_T^{(1)}}{z_2}\right] \\
&+ i2\frac{\hat{z}^{[\mu}S_{2\perp}^{\nu]}}{Q}\,\boldsymbol{a}\cdot\boldsymbol{S}_{1\perp}\left[-M_1M_2G_{1T}^{(1)}\frac{\overline{D}_T}{z_2} - M_1M_2\frac{\tilde{G}_T}{z_1}\overline{D}_{1T}^{\perp(1)} + {M_1}^2\frac{\tilde{E}_T}{z_1}(1)\overline{H}_1\right] \\
&- i2\frac{\hat{z}^{[\mu}a^{\nu]}}{Q}\Bigg[\lambda_1\lambda_2\left({M_2}^2G_1\frac{\overline{D}_L^{\perp(1)}}{z_2} - {M_1}^2\frac{\tilde{D}_L^{\perp(1)}}{z_1}\overline{G}_1\right) \\
&\qquad -M_1M_2\,\boldsymbol{S}_{1\perp}\cdot\boldsymbol{S}_{2\perp}\left(\frac{\tilde{G}_T}{z_1}\overline{D}_{1T}^{\perp(1)} - D_{1T}^{\perp}(1)\frac{\overline{G}_T}{z_2}\right) \\
&\qquad -mM_1\left(\boldsymbol{S}_{1\perp}\cdot\boldsymbol{S}_{2\perp}\,D_{1T}^{\perp(1)}\overline{H}_1 + H_1^{\perp(1)}\overline{D}_1\right) + M_1M_2\left(H_1^{\perp(1)}\frac{\overline{E}}{z_2} - \frac{\tilde{E}}{z_1}\overline{H}_1^{\perp(1)}\right) \\
&\qquad +\lambda_1\lambda_2M_1M_2\left(H_{1L}^{\perp(1)}\frac{\overline{E}_L}{z_2} - \frac{\tilde{E}_L}{z_1}\overline{H}_{1L}^{\perp(1)}\right)\Bigg] \qquad\qquad (5.D2) \\
&- i2\frac{\hat{z}^{[\mu}\epsilon_\perp^{\nu]\rho}a_\rho}{Q}\Bigg[\lambda_1{M_2}^2G_1\frac{\overline{D}^{\perp(1)}}{z_2} + \lambda_2{M_1}^2\frac{\tilde{D}^{\perp(1)}}{z_1}\overline{G}_1 - \lambda_1{M_1}^2\frac{\tilde{G}_L^{\perp(1)}}{z_1}\overline{D}_1 - \lambda_2{M_2}^2D_1\frac{\overline{G}_L^{\perp(1)}}{z_2} \\
&\qquad +M_1M_2\left(\lambda_1\frac{\tilde{E}_L}{z_1}\overline{H}_1^{\perp(1)} + \lambda_2H_1^{\perp(1)}\frac{\overline{E}_L}{z_2} - \lambda_1H_{1L}^{\perp(1)}\frac{\overline{E}}{z_2} - \lambda_2\frac{\tilde{E}}{z_1}\overline{H}_{1L}^{\perp(1)}\right)\Bigg]\Bigg\}.
\end{aligned}
$$

5.E Integrated twice-weighted hadron tensor

We display the leading terms of the hadron tensor weighted with a factor $(\boldsymbol{q}_T\cdot\boldsymbol{a})\ (\boldsymbol{q}_T\cdot\boldsymbol{b})$ when integrated over $\boldsymbol{q}_T$.

$$
\begin{aligned}
&\int d^2\boldsymbol{q}_T\ (\boldsymbol{q}_T\cdot\boldsymbol{a})\ (\boldsymbol{q}_T\cdot\boldsymbol{b})\ \mathcal{W}_S^{\mu\nu} = 12 z_2 z_1 \times \Bigg\{ \\
&\quad +g_\perp^{\mu\nu}\Bigg[-\,\boldsymbol{a}\cdot\boldsymbol{b}\left({M_1}^2D_1^{(1)}\overline{D}_1 + {M_2}^2D_1\overline{D}_1^{(1)} - \lambda_1\lambda_2{M_1}^2G_1^{(1)}\overline{G}_1 - \lambda_1\lambda_2{M_2}^2G_1\overline{G}_1^{(1)}\right) \\
&\qquad +2\,\boldsymbol{S}_{1\perp}\cdot\boldsymbol{S}_{2\perp}\ \boldsymbol{a}\cdot\boldsymbol{b}\,M_1M_2D_{1T}^{\perp(1)}\overline{D}_{1T}^{\perp(1)} \\
&\qquad +(\boldsymbol{a}\cdot\boldsymbol{S}_{1\perp}\ \boldsymbol{b}\cdot\boldsymbol{S}_{2\perp} + \boldsymbol{a}\cdot\boldsymbol{S}_{2\perp}\ \boldsymbol{b}\cdot\boldsymbol{S}_{1\perp})\,M_1M_2\left(G_{1T}^{(1)}\overline{G}_{1T}^{(1)} - D_{1T}^{\perp(1)}\overline{D}_{1T}^{\perp(1)}\right)\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
&-\boldsymbol{a}\cdot\boldsymbol{b}\left(S_{1\perp}^{\{\mu}S_{2\perp}^{\nu\}}+g_\perp^{\mu\nu}\,\boldsymbol{S}_{1\perp}\cdot\boldsymbol{S}_{2\perp}\right)\left(M_1{}^2H_1^{(1)}\overline{H}_1+M_2{}^2H_1\overline{H}_1^{(1)}-\frac{M_1{}^2}{2}H_{1T}^{\perp(2)}\overline{H}_1\right.\\
&\qquad\qquad\qquad\left.-\frac{M_2{}^2}{2}H_1\overline{H}_{1T}^{\perp(2)}\right)\\
&-\left[\boldsymbol{a}\cdot\boldsymbol{S}_{1\perp}\left(S_{2\perp}^{\{\mu}b^{\nu\}}+g_\perp^{\mu\nu}\,\boldsymbol{b}\cdot\boldsymbol{S}_{2\perp}\right)+\boldsymbol{b}\cdot\boldsymbol{S}_{1\perp}\left(S_{2\perp}^{\{\mu}a^{\nu\}}+g_\perp^{\mu\nu}\,\boldsymbol{a}\cdot\boldsymbol{S}_{2\perp}\right)\right]\frac{M_1{}^2}{2}H_{1T}^{\perp(2)}\overline{H}_1\\
&-\left[\boldsymbol{a}\cdot\boldsymbol{S}_{2\perp}\left(S_{1\perp}^{\{\mu}b^{\nu\}}+g_\perp^{\mu\nu}\,\boldsymbol{b}\cdot\boldsymbol{S}_{1\perp}\right)+\boldsymbol{b}\cdot\boldsymbol{S}_{2\perp}\left(S_{1\perp}^{\{\mu}a^{\nu\}}+g_\perp^{\mu\nu}\,\boldsymbol{a}\cdot\boldsymbol{S}_{1\perp}\right)\right]\frac{M_2{}^2}{2}H_1\overline{H}_{1T}^{\perp(2)}\\
&-2M_1M_2\left(a^{\{\mu}b^{\nu\}}+g_\perp^{\mu\nu}\,\boldsymbol{a}\cdot\boldsymbol{b}\right)\left(H_1^{\perp(1)}\overline{H}_1^{\perp(1)}+\lambda_2\lambda_1H_{1L}^{\perp(1)}\overline{H}_{1L}^{\perp(1)}\right)\\
&+M_1M_2\left(a^{\{\mu}\epsilon_\perp^{\nu\}\rho}b_\rho+b^{\{\mu}\epsilon_\perp^{\nu\}\rho}a_\rho\right)\left(\lambda_1H_{1L}^{\perp(1)}\overline{H}_1^{\perp(1)}-\lambda_2H_1^{\perp(1)}\overline{H}_{1L}^{\perp(1)}\right)\Big\}
\end{aligned}\tag{5.E1}
$$

and

$$
\begin{aligned}
&\int d^2\boldsymbol{q}_T\ (\boldsymbol{q}_T\cdot\boldsymbol{a})\ (\boldsymbol{q}_T\cdot\boldsymbol{b})\ \mathcal{W}_A^{\mu\nu}=12z_1z_2\times\Big\{\\
&\quad-i\epsilon_\perp^{\mu\nu}\,\boldsymbol{a}\cdot\boldsymbol{b}\left[\lambda_2M_2{}^2D_1\overline{G}_1^{(1)}+\lambda_2M_1{}^2D_1^{(1)}\overline{G}_1-\lambda_1M_2{}^2G_1\overline{D}_1^{(1)}-\lambda_1M_1{}^2G_1^{(1)}\overline{D}_1\right]\\
&\quad+i\left(S_{1\perp}^{[\mu}a^{\nu]}\,\boldsymbol{b}\cdot\boldsymbol{S}_{2\perp}+S_{1\perp}^{[\mu}b^{\nu]}\,\boldsymbol{a}\cdot\boldsymbol{S}_{2\perp}\right)M_1M_2D_{1T}^{\perp(1)}\overline{G}_{1T}^{(1)}\\
&\quad+i\left(S_{2\perp}^{[\mu}a^{\nu]}\,\boldsymbol{b}\cdot\boldsymbol{S}_{1\perp}+S_{2\perp}^{[\mu}b^{\nu]}\,\boldsymbol{a}\cdot\boldsymbol{S}_{1\perp}\right)M_1M_2G_{1T}^{(1)}\overline{D}_{1T}^{\perp(1)}\Big\}.
\end{aligned}\tag{5.E2}
$$

Chapter 6

Leading asymmetries in electron-positron annihilation at the Z peak

We present the leading unpolarized and single spin asymmetries in inclusive two-hadron production in electron-positron annihilation at the Z peak. The azimuthal dependence in the unpolarized differential cross section of almost back-to-back hadrons is a leading $\cos(2\phi)$ asymmetry, which arises solely due to the intrinsic transverse momenta of the quarks. An extensive discussion on how to measure this asymmetry and the accompanying time-reversal odd fragmentation functions is given. A simple estimate indicates that the asymmetry could be of the order of a percent. A comparison to a preliminary experimental study [103] is made. Also we discuss recent investigations of two-meson production (both mesons belong to the same jet) in leptoproduction (semi-inclusive DIS), which are also relevant for $e^+e^- \to$ hadrons processes.

6.1 Introduction

In the previous chapter we have presented the results of the complete tree-level calculation of inclusive two-hadron production in electron-positron annihilation via one photon up to and including order $1/Q$, where the scale Q is defined by the (timelike) photon momentum q (with $Q^2 \equiv q^2$) and given by $Q = \sqrt{s}$. The quantity Q had to be much larger than characteristic hadronic scales, but – being interested in effects at subleading order – we considered energies only well below the threshold for the production of Z bosons.

In this chapter we extend those results to electron-positron annihilation into a Z boson, such that the results can be used to analyze LEP-I data. We will neglect contributions from photon exchange and γ-Z interference terms, which are known to be numerically irrelevant at the Z peak. Only leading order $(1/Q)^0$ effects are discussed, since for $Q \gtrsim M_Z$ the power corrections of order $1/Q$ are expected to be completely negligible. Again we will only focus on tree level, i.e., order $(\alpha_s)^0$. A rich structure nevertheless arises when taking into account the intrinsic transverse momentum of the quarks and polarization of the detected hadrons in the final state. By accounting for intrinsic transverse momentum effects we extend the results of the analysis of Chen *et al.* [97], where no azimuthal asymmetries arising from transverse momenta have been considered.

6.2 The unpolarized cross section

For details of the calculation and the notation we refer to the previous chapter. Here we only present changes. We consider the same process $e^- + e^+ \to h_1 + h_2 + X$, where the two leptons (with momentum l for the e^- and l' for the e^+, respectively) now annihilate into a Z boson with momentum $q = l + l'$, which is timelike with $q^2 \equiv Q^2$. We will again consider the case where the two hadronic momenta P_1 and P_2 do not belong to the same jet (i.e., $P_1 \cdot P_2$ is of order Q^2). In principle, the momenta can also be considered to be the jet momenta themselves, but then effects due to intrinsic transverse momentum will be absent.

In this chapter (except in the appendix) we will disregard the polarization of hadron two (summation over spins). The final states which have to be identified and analyzed for the effects we discuss are simpler than the ones investigated by Artru and Collins [104], who proposed to measure azimuthal correlations of two particles in each of the two jets.

The cross section (including a factor 1/2 from averaging over incoming polarizations) for two-particle inclusive e^+e^- annihilation is given by

$$\frac{P_1^0\, P_2^0\; d\sigma^{(e^+e^-)}}{d^3P_1\, d^3P_2} = \frac{\alpha_w^2}{4\left((Q^2-M_Z^2)^2+\Gamma_Z^2 M_Z^2\right) Q^2} L_{\mu\nu} \mathcal{W}^{\mu\nu}, \tag{6.1}$$

with $\alpha_w = e^2/(16\pi \sin^2\theta_W \cos^2\theta_W)$ and the helicity-conserving lepton tensor (neglecting the lepton masses and polarization) is given by

$$L_{\mu\nu}(l,l') = ({g_V^l}^2 + {g_A^l}^2)\left[2l_\mu l'_\nu + 2l_\nu l'_\mu - Q^2 g_{\mu\nu}\right] - (2g_V^l g_A^l) 2i\,\epsilon_{\mu\nu\rho\sigma} l^\rho l'^\sigma, \tag{6.2}$$

where g_V^l, g_A^l denote the vector and axial-vector couplings of the Z boson to the leptons, respectively.

To leading order the expression for the hadron tensor, including quarks and antiquarks, is

$$\mathcal{W}^{\mu\nu} = 3\int dp^- dk^+ d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T)\ \mathrm{Tr}\left(\overline{\Delta}(p) V^\mu \Delta(k) V^\nu\right)\Big|_{p^+\,k^-} + \left(\begin{array}{c} q \leftrightarrow -q \\ \mu \leftrightarrow \nu \end{array}\right), \tag{6.3}$$

where $V^\mu = g_V\gamma^\mu + g_A\gamma_5\gamma^\mu$ is the Z-boson–quark vertex. We have omitted flavor indices and summation. The (partly integrated) correlation function Δ is parametrized as:

$$\frac{1}{z}\int dk^+ \,\Delta(P_1,S_1;k)\bigg|_{k^-=P_1^-/z,\,\boldsymbol{k}_T} = \frac{M_1}{P_1^-}\bigg\{D_1\,\frac{\not{P}_1}{M_1} + D_{1T}^\perp\,\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P_1^\nu k_T^\rho S_{1T}^\sigma}{M_1^2} - G_{1s}\,\frac{\not{P}_1\gamma_5}{M_1}$$

$$-H_{1T}\,\frac{i\sigma_{\mu\nu}\gamma_5\,S_{1T}^\mu P_1^\nu}{M_1} - H_{1s}^\perp\,\frac{i\sigma_{\mu\nu}\gamma_5\,k_T^\mu P_1^\nu}{M_1^2} + H_1^\perp\,\frac{\sigma_{\mu\nu}k_T^\mu P_1^\nu}{M_1^2}\bigg\}. \tag{6.4}$$

We parametrize the antiquark correlation function $\overline{\Delta}$ in the same way, cf. previous chapter.

The cross section is obtained from the hadron tensor after contraction with the lepton tensor, here given in the perpendicular basis

$$\begin{aligned} L^{\mu\nu} \;=\; & ({g_V^l}^2+{g_A^l}^2)\,Q^2\bigg[-\left(1-2y+2y^2\right)g_\perp^{\mu\nu} + 4y(1-y)\hat{z}^\mu\hat{z}^\nu \\ & -4y(1-y)\left(\hat{l}_\perp^\mu\hat{l}_\perp^\nu + \frac{1}{2}\,g_\perp^{\mu\nu}\right) - 2(1-2y)\sqrt{y(1-y)}\;\hat{z}^{\{\mu}\hat{l}_\perp^{\nu\}}\bigg] \\ & -(2g_V^l g_A^l)\,Q^2\left[+i\,(1-2y)\,\epsilon_\perp^{\mu\nu} - 2i\,\sqrt{y(1-y)}\;\hat{l}_{\perp\rho}\epsilon_\perp^{\rho[\mu}\hat{z}^{\nu]}\right]. \end{aligned} \tag{6.5}$$

The vector and axial-vector couplings to the Z boson are given by:

$$g_V^j \;=\; T_3^j - 2\,Q^j\,\sin^2\theta_W, \tag{6.6}$$

$$g_A^j \;=\; T_3^j, \tag{6.7}$$

where Q^j denotes the charge and T_3^j the weak isospin of particle j (i.e., $T_3^j = +1/2$ for $j = u$ and $T_3^j = -1/2$ for $j = e^-, d, s$). Combinations of the couplings occurring frequently in the formulas are

$$\begin{aligned} c_1^j &= \left({g_V^j}^2+{g_A^j}^2\right), \\ c_2^j &= \left({g_V^j}^2-{g_A^j}^2\right), \qquad\qquad j = \ell \text{ or } u,d,s \\ c_3^j &= 2g_V^j g_A^j. \end{aligned} \tag{6.8}$$

We obtain in leading order in $1/Q$ and α_s the following expression for the cross section in case of unpolarized (or spinless) final state hadrons:

$$\begin{aligned} \frac{d\sigma(e^+e^-\to h_1h_2X)}{d\Omega dz_1dz_2d^2\boldsymbol{q}_T} =& \sum_{a,\bar{a}} \frac{3\,\alpha_w^2\,Q^2}{(Q^2-M_Z^2)^2+\Gamma_Z^2M_Z^2}\;z_1^2z_2^2\;\bigg\{\left(c_1^\ell\,c_1^a\,A(y) - \tfrac{1}{2}\,c_3^\ell\,c_3^a C(y)\right) \\ &\times\mathcal{F}\left[D_1^a\overline{D}_1^a\right] + \cos(2\phi_1)\,c_1^\ell\,c_2^a\,B(y)\;\mathcal{F}\left[\left(2\,\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{k}_T\;\hat{\boldsymbol{h}}\!\cdot\!\boldsymbol{p}_T - \boldsymbol{k}_T\!\cdot\!\boldsymbol{p}_T\right)\frac{H_1^{\perp a}\overline{H}_1^{\perp a}}{M_1M_2}\right]\bigg\}. \end{aligned} \tag{6.9}$$

Recall that the angle ϕ_1 is the azimuthal angle of $\hat{h}$ (see Fig. 5.1). Compare this expression with Eq. (5.94).

6.3 Unpolarized correlation

In order to deconvolute the expression Eq. (6.9) we can define weighted cross sections

$$\langle W\rangle_A = \int \frac{d\phi^\ell}{2\pi}\, d^2\boldsymbol{q}_T\, W\, \frac{d\sigma(e^+e^- \to h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2\boldsymbol{q}_T}, \tag{6.10}$$

where $W = W(Q_T, \phi_1, \phi_2, \phi_{S_1}, \phi_{S_2})$. The subscript A denotes the polarization in the final state for hadron one, with as possibilities unpolarized (O, including the case of summation over spin), longitudinally polarized (L) or transversely polarized (T). We postpone the discussion of the additional structures and information accessible by measuring the polarization of one of the final state hadrons to the end of this chapter.

Even without determining polarization of a final state hadron a subtle test of our understanding of spin transfer mechanisms in perturbative QCD can be done. The information on the production of a transversely polarized quark-antiquark pair, which subsequently fragment into unpolarized (or spinless) hadrons with probabilities depending on the orientation of the (anti)quark's spin vector relative to its transverse momentum, is contained in the $\cos(2\phi_1)$ azimuthal asymmetry. To access this information we utilize the weighted cross sections

$$\langle 1\rangle_O = \frac{3\,\alpha_w^2\, Q^2}{(Q^2-M_Z^2)^2+\Gamma_Z^2 M_Z^2} \sum_{a,\bar a} \left(c_1^\ell c_1^a\, A(y) - \tfrac{1}{2} c_3^\ell c_3^a C(y)\right) D_1^a(z_1)\, \overline{D}_1^a(z_2), \tag{6.11}$$

$$\left\langle \frac{Q_T^2}{4M_1M_2}\cos(2\phi_1)\right\rangle_O = \frac{3\,\alpha_w^2\, Q^2}{(Q^2-M_Z^2)^2+\Gamma_Z^2 M_Z^2} \sum_{a,\bar a} c_1^\ell c_2^a\, B(y)\, H_1^{\perp(1)a}(z_1)\, \overline{H}_1^{\perp(1)a}(z_2). \tag{6.12}$$

We now like to focus on the weighted cross section defined in Eq. (6.12) and discuss its possible measurement. In order to be able to observe the $\cos(2\phi_1)$ dependence one must look at two jet events in unpolarized electron-positron scattering. In each jet one identifies a fast hadron with momentum fractions z_1 and z_2 respectively. One of the hadrons (say two) together with the leptons determines the lepton scattering plane as is indicated in Fig. 5.1. In the lepton center of mass system hadron two determines the $\hat{z}$-direction with respect to which the azimuthal angles are measured. One needs in particular the azimuthal angle ϕ_1 of the other hadron (one) as well as its transverse momentum $\boldsymbol{P}_{1\perp}$, which determines $Q_T = |\boldsymbol{P}_{1\perp}|/z_1$. The $\cos(2\phi_1)$ angular dependence then can be analyzed with help of the weighted cross section of Eq. (6.12).

For an order of magnitude estimate, we consider the situation of the produced hadrons being a π^+ and a π^-. Furthermore, we assume $D_1^{u\to\pi^+}(z) = D_1^{\bar d\to\pi^+}(z)$, $D_1^{d\to\pi^-}(z) = D_1^{\bar u\to\pi^-}(z)$ and neglect unfavored fragmentation functions like $D^{d\to\pi^+}(z)$ etc.; and similar for the time-reversal odd functions. The equalities for the D_1 functions seem quite safe on grounds of isospin and charge conjugation, the same assumptions might be non-trivial for the $H_1^\perp$ functions. As a consequence of these assumptions the fragmentation functions can be taken outside the flavor summation, and we obtain

$$\left\langle \frac{Q_T^2}{4M_1M_2} \cos(2\phi_1) \right\rangle_O = F(y) \, \frac{H_1^{\perp(1)}(z_1)}{D_1(z_1)} \, \frac{H_1^{\perp(1)}(z_2)}{D_1(z_2)} \, \langle 1 \rangle_O \, , \tag{6.13}$$

where

$$F(y) = \frac{\sum_{a=u,\bar{d}} c_1^e \, c_2^a \, B(y)}{\sum_{a=u,\bar{d}} \left(c_1^e \, c_1^a \, A(y) - \frac{1}{2} c_3^e \, c_3^a \, C(y) \right)}. \tag{6.14}$$

This factor is shown in Fig. 6.1 as a function of the center of mass angle θ_2 (we use $\sin^2\theta_W = 0.2315$ [105]). At an angle close to 90° we observe the largest effect, since the nonsymmetric term proportional to $C(y)$ is small. In order to get an estimate of the true asymmetry at the level of count rates, one should compare Eq. (6.12) with the weighted cross section $\langle Q_T^2/4M_1M_2\rangle_O$. To estimate the ratio of those two quantities, we use as argued in Ref. [106], for the ratio of the (weighted) fragmentation functions $H_1^{\perp(1)}(z_1)/D_1(z_1) = \mathcal{O}(1)$, although this is likely to be an optimistic estimate. From the average transverse momentum squared of produced pions in one jet, for which we take 0.5 (GeV/c)^2 [107], one obtains an estimate for the average transverse momentum of pions in jet one with respect to a given pion in jet two. This leads at $z_1 = z_2 = 1/2$ to $\langle Q_T^2/4M_\pi^2\rangle_O \approx 50 \, \langle 1\rangle_O$ and consequently to an estimate at the percent level for the ratio $\langle (Q_T^2/4M_\pi^2) \cos(2\phi_1)\rangle_O \, / \, \langle Q_T^2/4M_\pi^2\rangle_O$. Such an azimuthal dependence in the unpolarized cross section, however, may be detectable in electron-positron scattering experiments at presently attainable energies.

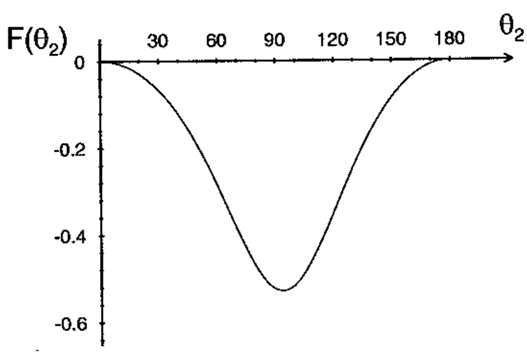


Figure 6.1: Factor defined in Eq. (6.14) depending on the center of mass angle θ_2.

6.4 Experimental study

In a preliminary study [103] a similar correlation in back-to-back jets was already experimentally investigated. We find that it involves moments of the functions $H_1^\perp$ and $\overline{H}_1^\perp$, different from the ones in the correlation Eq. (6.12) that we consider. In this study no significant result was found using the 1991 to 1994 LEP data. The main difference is that in that analysis *three* momenta in the final state need to be determined, namely besides two hadron momenta also the jet axis, and hence there are two azimuthal angles, ϕ and ϕ', yielding a $\cos(\phi+\phi')$ asymmetry. So their analyzing power (S) is a different expression of the functions $H_1^\perp$ and $\overline{H}_1^\perp$, than in the expressions for the weighted cross section we discuss and estimate (the magnitudes will be different).

Let us investigate the differences more extensively. In Ref. [103] they study the following angular dependence of the differential cross section for correlated hadron production in opposite jets:

$$\frac{d\sigma}{d\cos\theta\, d\phi\, d\phi'} \propto 1+\cos^2\theta + c_{TT} S^2 \sin^2\theta \cos(\phi+\phi'), \tag{6.15}$$

where S is the analyzing power of transversely polarized quark fragmentation (the ratio of spin-dependent to spin-independent parts of fragmentation functions), ϕ (and similarly ϕ') is the azimuthal angle of the direction of the transverse momentum of the leading particle with respect to the $q\bar{q}$ axis (the angle is defined with respect to the projection of the incoming lepton axis in the plane perpendicular to the $q\bar{q}$ axis). Since the correlation arises due to the Collins effect, the direction of the transverse momentum of the leading particle with respect to the $q\bar{q}$ axis, is equal to that of the transverse momentum of the quark with respect to the leading particle and hence, is perpendicular to the transverse spin of the quark. Hence, the name 'transverse spin correlation'. Furthermore, $c_{TT} = (|v_q|^2 - |a_q|^2)/(|v_q|^2 + |a_q|^2)$, where v_q and a_q are the vector and axial-vector couplings of quarks to the Z boson. So in our notation $c_{TT} = c_2^a/c_1^a$.

Now let us derive an expression for the analyzing power S. We start with the hadron tensor expressed in the transverse basis

$$\mathcal{W}_{(ZZ)}^{\mu\nu} = 12\, z_1 z_2 \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{p}_T\, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T)\Bigg\{ -\Big[g_T^{\mu\nu}\, c_1^a + i\epsilon_T^{\mu\nu}\, c_3^a\Big] D_1 \overline{D}_1 \\ - \frac{k_T^{\{\mu} p_T^{\nu\}} + g_T^{\mu\nu}\, \boldsymbol{k}_T\!\cdot\!\boldsymbol{p}_T}{M_1 M_2} c_2^a\, H_1^\perp \overline{H}_1^\perp \Bigg\}, \tag{6.16}$$

This expression is the one we have used, since it includes integrals over the unobserved transverse momentum of the quarks. If one would perform the integration over $\boldsymbol{q}_T$, then the last term would average to zero. This is the reason we performed a weighted integration.

Another option is the one used in Ref. [103]. If one would determine the jet-axis, which is identified with the $q\bar{q}$ axis, then a measurement of the transverse momenta of the leading particles in the two jets compared to the jet momentum is a determination of the transverse momenta of the quarks compared to the leading hadrons they fragment into. One can then keep the cross section differential in the azimuthal angles of the transverse momentum of the quarks, after which the $\boldsymbol{q}_T$ integration can be safely done and it will not average to zero unless one integrates over the azimuthal angles. In this way one will arrive at an expression involving

$$F^{[n]}(z_i) \equiv \int d|\boldsymbol{k}_T| \left[\frac{|\boldsymbol{k}_T|}{M_i}\right]^n F(z_i, |\boldsymbol{k}_T|^2), \tag{6.17}$$

which is a one-dimensional $|\boldsymbol{k}_T|$-moment, whereas our expressions contain two-dimensional $\boldsymbol{k}_T^2$-moments.

To make the transformation from the transverse basis (in which P_1 and P_2 are collinear) into the one where the $q\bar{q}$ axis defines the $\hat{z}$ axis, we must define a different perpendicular basis than defined in our analysis, where $\hat{z}$ is defined by P_2 (the new perpendicular directions we will also indicate by $\perp$). We will choose:

$$\hat{t}^\mu \equiv \frac{q^\mu}{Q}, \tag{6.18}$$

$$\hat{z}^\mu \equiv \frac{k^\mu - p^\mu}{Q}. \tag{6.19}$$

We find that

$$g_\perp^{\mu\nu} = g_T^{\mu\nu} - \frac{\sqrt{2}}{Q}\left(p_T^{\{\mu}n_+^{\nu\}} + k_T^{\{\mu}n_-^{\nu\}}\right) - \frac{2}{Q^2}p_T^{\{\mu}k_T^{\nu\}}. \tag{6.20}$$

Hence, $P_{1\perp} = -z_1\,k_T$ and $P_{2\perp} = -z_2\,p_T$.

Even though the $\hat{z}$ axis is defined differently, the contraction with the lepton tensor gives similar expressions. We give the relevant part of the contractions in Table 6.1. The functions A, B and C are the same functions of $y = P_2 \cdot l/P_2 \cdot q$ as before. If one expresses the lepton momentum l in terms of $\hat{t}$ and $\hat{z}$, i.e., $l = Q\,\hat{t}/2 + \sqrt{Q^2/4 - \boldsymbol{l}_\perp^2}\,\hat{z} + l_\perp$, one finds that $A(y) = 1/2 - B(y)$ and $C(y) = -\sqrt{1 - 4\,B(y)}$, with $B(y) = \boldsymbol{l}_\perp^2/Q^2$.

Table 6.1: Contractions of the lepton tensor $L_{\mu\nu}$ with tensor structures appearing in the hadron tensor.

$w^{\mu\nu}$	$L_{\mu\nu}w^{\mu\nu}/(4Q^2)$
$-g_T^{\mu\nu}$	$c_1^l\,A(y)$
$k_T^{\{\mu}p_T^{\nu\}} + (\boldsymbol{k}_T \cdot \boldsymbol{p}_T)\,g_T^{\mu\nu}$	$-c_1^l\,B(y)\,\lvert\boldsymbol{P}_{1\perp}\rvert\,\lvert\boldsymbol{P}_{2\perp}\rvert\,\cos(\phi + \phi')/(z_1 z_2)$
$i\,\epsilon_T^{\mu\nu}$	$c_3^l\,C(y)/2$

We obtain in leading order in $1/Q$ and α_s the following expression for the cross section differential in ϕ and ϕ', in case of unpolarized (or spinless) final state hadrons:

$$\frac{d\sigma(e^+e^- \to h_1 h_2 X)}{d\Omega dz_1 dz_2 d\phi\, d\phi'} = \sum_{a,\bar{a}} \frac{3\,\alpha_w^2\, Q^2}{(Q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}\, z_1^2 z_2^2 \left\{ \left(c_1^\ell\, c_1^a\, A(y) - \tfrac{1}{2}\, c_3^\ell\, c_3^a C(y)\right)\right.$$

$$\left. \times D_1^{[0]a}(z_1)\overline{D}_1^{[0]a}(z_2) + c_1^\ell\, c_2^a\, B(y)\, \cos(\phi + \phi')\, H_1^{\perp[1]a}(z_1)\overline{H}_1^{\perp[1]a}(z_2)\right\}. \tag{6.21}$$

This is to be compared with Eq. (6.9). We have included the forward-backward asymmetry term (proportional to $C(y)$) which was neglected in [103] since it is small. Hence, we find for the analyzing power of Ref. [103] (for one flavor)

$$S = \frac{H_1^{\perp[1]}(z_1)\overline{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1)\overline{D}_1^{[0]}(z_2)}. \tag{6.22}$$

If one simply assumes a Gaussian $\boldsymbol{k}_T$-dependence of the functions, i.e.,

$$H_1^\perp(z, \boldsymbol{k}_T'^2) = H_1^\perp(z)\, R^2\, \exp(-R^2\, \boldsymbol{k}_T^2)/(\pi\, z^2) \tag{6.23}$$

and similarly for $D_1(z, \boldsymbol{k}_T'^2)$, then one finds that

$$S = \frac{1}{\pi\, R^2\, M^2}\, \frac{H_1^\perp(z_1)\overline{H}_1^\perp(z_2)}{D_1(z_1)\overline{D}_1(z_2)}. \tag{6.24}$$

Hence,

$$S' \equiv \frac{H_1^{\perp(1)}(z_1)\overline{H}_1^{\perp(1)}(z_2)}{D_1(z_1)\overline{D}_1(z_2)} = \frac{1}{4\, R^4\, M^4}\, \frac{H_1^\perp(z_1)\overline{H}_1^\perp(z_2)}{D_1(z_1)\overline{D}_1(z_2)} = \frac{\pi}{4\, R^2\, M^2} S. \tag{6.25}$$

Let us first state the experimental result of Ref. [103] for S. With 0.3 GeV $< |\boldsymbol{k}_T| <$ 1 GeV and $\min(z, z') > 0.25$, they obtain $S < 0.32$ at the 90% confidence level. For a pion $R\,M \approx 0.5$, such that we conclude that one then finds $S' \lesssim 1$. For our optimistic estimate of about one percent we used $S' = 1$, so if one would include the 1995 data as well, one might still be able to observe the effect.

6.5 Interference fragmentation functions

In Ref. [108] it is argued that the final state interactions between P and X in $|P; X\rangle$ (cf. Eq. (5.22)), characterized by some phase shift, will average out by summation over X. Therefore they propose (along similar lines as discussed in [96, 19, 109]) to study $|P_1; P_2; X\rangle$, where P_1 and P_2 both belong to particles in the same jet. By the same reasoning this will be on average $|(P_1 P_2)_{\rm out}; X\rangle$, with only the phase shift of the $(P_1 P_2)$ system remaining after summation over X. In order to arrive at a dependence on the phase shift in the square of the amplitude, the invariant mass of the $(P_1 P_2)$ system should be at nonresonant values, hence the name interference fragmentation functions (IFF). We would like to emphasize that the IFF are leading twist functions. The $|(\pi\pi); X\rangle$ IFF have been studied already in Ref. [96] and the idea of IFF has been discussed and modeled in Refs. [19, 109]. The new idea of [108] is to use the interference of s and p waves and their well-known phase shifts to model the IFF. They show how the transversity distribution function h_1 can be probed in such inclusive two-meson production in DIS off a transversely polarized proton and also how the valence helicity distribution function can be probed by DIS with a longitudinally polarized lepton and nucleon beam.

A similar analysis can be done for $e^+e^- \to (\pi\pi)X$ or $e^+e^- \to (\pi\pi)_1(\pi\pi)_2X$, where the two pairs of pions belong to opposite jets. The last case would be similar to the situation studied in [104], which has also been studied experimentally in Ref. [103], but no significant nonzero result was observed.

An interesting fact is that due to parity no helicity IFF (which would be the analogue of G_1) can be measured in semi-inclusive two-meson production in DIS, because no pseudoscalar can be constructed from the available vectors in $ep \to \pi\pi X$ [109]. This is also the reason why longitudinal handedness [110], which is proportional to the longitudinal quark polarization, is defined by three momenta.

6.6 Polarized correlations

Let us focus again on the process that is the subject of this chapter, namely $e^+e^- \to h_1h_2X$, where the two hadrons belong to opposite jets. The experimental determination of the polarization of (one of) the final state hadrons offers further opportunities to reveal hadronic properties in terms of spin-dependent fragmentation functions. We assume in the following that the spin vector S_1 of hadron one is known (reconstructed), having in mind the example of a produced Λ and its self-analyzing properties. We observe a rich structure of angular dependences due to polarization.

Again, weighting cross sections is an appropriate means to separate out specific functions. For instance, the weighted cross section

$$\left\langle \frac{Q_T}{M_2}\sin(\phi_1+\phi_{S_1})\right\rangle_T = |\boldsymbol{S}_{1T}|\,\frac{3\,\alpha_w^2\,Q^2}{(Q^2-M_Z^2)^2+\Gamma_Z^2M_Z^2}\sum_{a,\bar{a}} c_1^\ell\,c_2^a\,B(y)\,H_1^a(z_1)\,\overline{H}_1^{\perp(1)a}(z_2) \tag{6.26}$$

picks out the term that is the closest analogue of the original Collins asymmetry [90] in semi-inclusive lepton-hadron scattering [42, 111]. We note that a confirmation of the $\cos(2\phi)$ asymmetry discussed before, also implies a confirmation of the Collins effect, *without* the need for polarization measurements on either incoming or outgoing particles.

A complete list of leading order, weighted cross sections for $S_1 \neq 0, S_2 = 0$ is given in Table 6.2. The full hadron tensor for both hadrons polarized is given in App. 6.A for completeness.

Table 6.2: Weighted cross sections for $S_1 \neq 0, S_2 = 0$

W	A	$\langle W \rangle_A \cdot [3\,\alpha_w^2\,Q^2/\,((Q^2-M_Z^2)^2+\Gamma_Z^2 M_Z^2)]^{-1}$
1	L	$-\lambda_1 \sum_{a,\bar{a}} \left(c_1^\ell\,c_3^a\,A(y) - \frac{1}{2}\,c_3^\ell\,c_1^a\,C(y)\right)\; G_{1L}^a(z_1)\,\overline{D}_1^a(z_2)$
$(Q_T^2/4M_1M_2)\,\sin(2\phi_1)$	L	$\lambda_1 \sum_{a,\bar{a}}\; c_1^\ell\,c_2^a\,B(y)\; H_{1L}^{\perp(1)a}(z_1)\,\overline{H}_1^{\perp(1)a}(z_2)$
$(Q_T/M_1)\,\sin(\phi_1-\phi_{S_1})$	T	$\lvert \boldsymbol{S}_{1T}\rvert \sum_{a,\bar{a}} \left(c_1^\ell\,c_1^a\,A(y) - \frac{1}{2}\,c_3^\ell\,c_3^a\,C(y)\right)\; D_{1T}^{\perp(1)a}(z_1)\,\overline{D}_1^a(z_2)$
$(Q_T/M_2)\,\sin(\phi_1+\phi_{S_1})$	T	$-\lvert \boldsymbol{S}_{1T}\rvert \sum_{a,\bar{a}}\; c_1^\ell\,c_2^a\,B(y)\; H_1^a(z_1)\,\overline{H}_1^{\perp(1)a}(z_2)$
$(Q_T^3/6M_1^2M_2)\,\sin(3\phi_1-\phi_{S_1})$	T	$-\lvert \boldsymbol{S}_{1T}\rvert \sum_{a,\bar{a}}\; c_1^\ell\,c_2^a\,B(y)\; H_{1T}^{\perp(2)a}(z_1)\,\overline{H}_1^{\perp(1)a}(z_2)$
$(Q_T/M_1)\,\cos(\phi_1-\phi_{S_1})$	T	$\lvert \boldsymbol{S}_{1T}\rvert \sum_{a,\bar{a}} \left(c_1^\ell\,c_3^a\,A(y) - \frac{1}{2}\,c_3^\ell\,c_1^a\,C(y)\right)\; G_{1T}^{(1)a}(z_1)\,\overline{D}_1^a(z_2)$

6.7 Conclusions

We have presented the leading asymmetries in inclusive two-hadron production in electron-positron annihilation at the Z peak. We have investigated unpolarized and single spin asymmetries. We included the effects of intrinsic transverse momentum and in this respect our results are an extension of those of Ref. [97]. The azimuthal dependence in the unpolarized differential cross section is a $\cos(2\phi)$ asymmetry, which arises solely due to the intrinsic transverse momenta of the quarks. An extensive discussion on how to measure this asymmetry and the accompanying time-reversal odd fragmentation functions is given. A simple estimate indicates that the asymmetry could be at the percent level, hence it can perhaps be observed in electron-positron scattering experiments at presently attainable energies. A comparison to a preliminary experimental study [103] is made. Our estimate is still compatible with their upper bound on the analyzing power of the two-hadron correlation and suggests that only a small improvement of the accuracy of the data might be required (in the most optimistic scenario). In confirming the existence of this asymmetry one would also confirm the Collins effect, without the need of a polarization measurement. Also we briefly discussed recent investigations of two-meson production (both mesons belong to the same jet) in leptoproduction, concerning so-called interference fragmentation functions, which are also relevant for $e^+e^- \to$ hadrons processes.

6.A The complete expression for the hadron tensor

The full expressions for the leading order symmetric and antisymmetric parts of the hadron tensor for the case of $e^+e^- \to Z \to \vec{h}_1\vec{h}_2X$ are (expressed in the perpendicular frame defined in Chap. 5)

$$
\begin{aligned}
\mathcal{W}_S^{\mu\nu} = 12\, z_1 z_2 \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{p}_T\, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T)\Bigg\{ \\
- g_\perp^{\mu\nu}\Bigg[+ (g_V^2 + g_A^2)\Bigg(D_1\overline{D}_1 - G_{1s}\overline{G}_{1s} + \frac{\epsilon_\perp^{\rho\sigma} k_{\perp\rho} S_{1\perp\sigma}}{M_1} D_{1T}^\perp \overline{D}_1 - \frac{\epsilon_\perp^{\rho\sigma} p_{\perp\rho} S_{2\perp\sigma}}{M_2} D_1 \overline{D}_{1T}^\perp \\
- \frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{p}_\perp\ \boldsymbol{S}_{1\perp}\!\cdot\!\boldsymbol{S}_{2\perp} - \boldsymbol{p}_\perp\!\cdot\!\boldsymbol{S}_{1\perp}\ \boldsymbol{k}_\perp\!\cdot\!\boldsymbol{S}_{2\perp}}{M_1 M_2} D_{1T}^\perp \overline{D}_{1T}^\perp \Bigg) \\
+ 2 g_V g_A \Bigg(D_1\overline{G}_{1s} - G_{1s}\overline{D}_1 + \frac{\epsilon_\perp^{\rho\sigma} k_{\perp\rho} S_{1\perp\sigma}}{M_1} D_{1T}^\perp \overline{G}_{1s} + \frac{\epsilon_\perp^{\rho\sigma} p_{\perp\rho} S_{2\perp\sigma}}{M_2} G_{1s} \overline{D}_{1T}^\perp \Bigg)\Bigg] \\
+ (g_V^2 - g_A^2)\Bigg[- \left(S_{1\perp}^{\{\mu} S_{2\perp}^{\nu\}} + g_\perp^{\mu\nu}\, \boldsymbol{S}_{1\perp}\!\cdot\!\boldsymbol{S}_{2\perp} \right) H_{1T}\overline{H}_{1T} \\
- \frac{k_\perp^{\{\mu} p_\perp^{\nu\}} + g_\perp^{\mu\nu}\, \boldsymbol{k}_\perp\!\cdot\!\boldsymbol{p}_\perp}{M_1 M_2} \left(H_{1s}^\perp \overline{H}_{1s}^\perp + H_1^\perp \overline{H}_1^\perp \right) \\
- \frac{k_\perp^{\{\mu} S_{2\perp}^{\nu\}} + g_\perp^{\mu\nu}\, \boldsymbol{k}_\perp\!\cdot\!\boldsymbol{S}_{2\perp}}{M_1} H_{1s}^\perp \overline{H}_{1T} - \frac{p_\perp^{\{\mu} S_{1\perp}^{\nu\}} + g_\perp^{\mu\nu}\, \boldsymbol{p}_\perp\!\cdot\!\boldsymbol{S}_{1\perp}}{M_2} H_{1T} \overline{H}_{1s}^\perp \\
+ \frac{k_\perp^{\{\mu} \epsilon_\perp^{\nu\}\rho} p_{\perp\rho} + p_\perp^{\{\mu} \epsilon_\perp^{\nu\}\rho} k_{\perp\rho}}{2 M_1 M_2} \left(H_{1s}^\perp \overline{H}_1^\perp - H_1^\perp \overline{H}_{1s}^\perp \right) \qquad (6.\mathrm{A1}) \\
- \frac{k_\perp^{\{\mu} \epsilon_\perp^{\nu\}\rho} S_{2\perp\rho} + S_{2\perp}^{\{\mu} \epsilon_\perp^{\nu\}\rho} k_{\perp\rho}}{2 M_1} H_1^\perp \overline{H}_{1T} + \frac{p_\perp^{\{\mu} \epsilon_\perp^{\nu\}\rho} S_{1\perp\rho} + S_{1\perp}^{\{\mu} \epsilon_\perp^{\nu\}\rho} p_{\perp\rho}}{2 M_2} H_{1T} \overline{H}_1^\perp \Bigg]\Bigg\}
\end{aligned}
$$

and

$$
\begin{aligned}
\mathcal{W}_A^{\mu\nu} = 12\, z_1 z_2 \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{p}_T\, \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T)\Bigg\{ \\
+ i\epsilon_\perp^{\mu\nu}\Bigg[- 2 g_V g_A \Bigg(D_1\overline{D}_1 - G_{1s}\overline{G}_{1s} - \frac{\boldsymbol{k}_\perp\!\cdot\!\boldsymbol{p}_\perp\ \boldsymbol{S}_{1\perp}\!\cdot\!\boldsymbol{S}_{2\perp} - \boldsymbol{p}_\perp\!\cdot\!\boldsymbol{S}_{1\perp}\ \boldsymbol{k}_\perp\!\cdot\!\boldsymbol{S}_{2\perp}}{M_1 M_2} D_{1T}^\perp \overline{D}_{1T}^\perp \Bigg) \\
+ (g_V^2 + g_A^2)\left(-D_1\overline{G}_{1s} + G_{1s}\overline{D}_1 \right)\Bigg] \\
+ i\frac{k_\perp^{[\mu} S_{1\perp}^{\nu]}}{M_1}\left[- 2 g_V g_A D_{1T}^\perp \overline{D}_1 - (g_V^2 + g_A^2) D_{1T}^\perp \overline{G}_{1s} \right] \\
+ i\frac{p_\perp^{[\mu} S_{2\perp}^{\nu]}}{M_2}\left[+ 2 g_V g_A D_1 \overline{D}_{1T}^\perp - (g_V^2 + g_A^2) G_{1s} \overline{D}_{1T}^\perp \right]\Bigg\}. \qquad (6.\mathrm{A2})
\end{aligned}
$$

Chapter 7

Technical aspects of hard scattering processes

7.1 Factorization

In previous chapters hard scattering processes with one or two soft parts are considered. Assuming factorization EFP's approach was used for processes for which the OPE cannot be applied. Soft parts were parametrized by distribution and fragmentation functions, which are assumed to be universal, which means that the functions that appear in DIS also appear in for instance the DY process. This universality depends on the validity of factorization, which means that the processes can be separated into hard and soft parts. One has to prove factorization in each process and show that the soft parts are identical.

Proofs of factorization have been given for some processes for the leading twist, like for instance, for the case of back-to-back jets in electron-positron annihilation [112]. Some of the proofs include effects of intrinsic transverse momentum and/or polarization (for discussions on this cf. [113, 90]). For the DY process, strong arguments have been given why factorization also holds for the first subleading twist [13, 114]. For the processes investigated in this thesis, proofs of factorization remain to be given, since we consider both leading and next-to-leading twist and include intrinsic transverse momentum and polarization. Nevertheless, the above mentioned studies suggest that it is not unreasonable to assume that a proof for the factorized form we have used can indeed be given. We will not attempt to give such a proof, but in this section we will discuss in a bit more detail what is the current status of the relevant factorization proofs.

Let us shortly repeat the essentials of discussions in Ref. [113, 90] on factorization. The factorization theorem for the case of leading twist unpolarized Drell-Yan scattering without intrinsic transverse momentum states [115, 116, 113]:

$$\frac{d\sigma}{d\Omega dx_1 dx_2} = \sum_{a,b} \int_{x_1}^{1} dx \int_{x_2}^{1} d\bar{x}\, f_1^a(x)\, \frac{d\hat{\sigma}}{d\Omega dx d\bar{x}}\, \overline{f}_1^b(\bar{x}), \tag{7.1}$$

where $d\hat{\sigma}$ is the hard scattering (differential) cross section for the elementary process $q+\bar{q} \rightarrow$

$l+\bar{l}$. In fact, for the case of polarized scattering a similar form holds, but with f_1 replaced by g_1 or h_1 (similarly for $\overline{f}_1$). Also, it is not necessary that $\boldsymbol{q}_T$ is integrated over, one could also keep the ratio Q_T/Q fixed (recall that $Q_T^2 = -\boldsymbol{q}_T^2$) as $Q^2 \to \infty$, such that the energy conservation delta-function can be approximated as $\delta^2(\boldsymbol{q}_T - \boldsymbol{p}_T - \boldsymbol{k}_T) \approx \delta^2(\boldsymbol{q}_T)$.

The above theorem is not valid for $Q_T \ll Q$, where the cross section is in fact largest. In that case, one has the following factorized form [117]

$$\begin{aligned}\frac{d\sigma}{d\Omega dx_1 dx_2 d^2\boldsymbol{q}_T} &= \sum_{a,b}\int_{x_1}^1 dx \int_{x_2}^1 d\bar{x} \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{p}_T\, f_1^a(x,\boldsymbol{k}_T)\,\frac{d\hat{\sigma}}{d\Omega dx d\bar{x} d^2\boldsymbol{q}_T}\,\overline{f}_1^b(\bar{x},\boldsymbol{p}_T)\\ &+ Y(x_1,x_2,Q,Q_T),\end{aligned} \tag{7.2}$$

where the first term dominates at small Q_T and the second term is necessary to reproduce Eq. (7.1) after integration over $\boldsymbol{q}_T$ or for fixed $Q_T \sim Q$ values. The first term now contains $\delta^2(\boldsymbol{q}_T - \boldsymbol{p}_T - \boldsymbol{k}_T)$. For nongauge theories the above factorized cross section is a theorem [118]. In gauge theories, the gauge bosons will lead to so-called Sudakov form factors [119], which broaden the transverse momentum distribution as Q increases (for a discussion of this for the DY process see [117], for e^+e^- annihilation see [112] and for semi-inclusive DIS see [90, 120]). Again, polarization can be taken into account, without changing the form of the equation.

The first term in Eq. (7.2) is of the form we have used (cf. the first term in Eq. (5.65)), since we focus on the low Q_T region. However, in some cases we have considered $\boldsymbol{q}_T$-weighted cross sections, hence it seems that we have integrated over all values of Q_T and not only over the small values. However, due to the delta-function $\delta^2(\boldsymbol{q}_T - \boldsymbol{p}_T - \boldsymbol{k}_T)$, large values of $|\boldsymbol{q}_T|$ implies large values of $|\boldsymbol{p}_T|$ and/or $|\boldsymbol{k}_T|$, for which the functions $f_1^a(x,\boldsymbol{k}_T^2)$ and $\overline{f}_1^b(\bar{x},\boldsymbol{p}_T^2)$ fall off fast. In other words, the distributions force $\boldsymbol{q}_T$ to be small via the delta-function. Large values of Q_T are connected to the additional function Y, which we have not included.

We conclude that for the leading twist, including transverse momentum and polarization, a factorized form as we have used for the first term of Eq. (5.65) in e^+e^- annihilation is justifiable. For the case of first subleading twist a proof will be along the lines of Refs. [13, 114]. They extended the factorization program to order $1/Q^2$ corrections for unpolarized DY [13] and to order $1/Q$ corrections for the polarized case [114]. The general form of the factorized cross sections is written as [121]

$$\sigma(Q) = H^0 \otimes f_2 \otimes f_2 + \frac{1}{Q^n} H^1 \otimes f_2 \otimes f_{2+n} + \mathcal{O}\left(\frac{1}{Q^{n+1}}\right), \tag{7.3}$$

where $n = 1,2$ for the polarized and unpolarized case respectively. The hard parts are denoted by $H^{\cdot\cdot}$ and are convoluted ($\otimes$) with the soft parts f_t, where t denotes the twist. For instance, f_2 could be f_1, g_1, h_1 and f_3 could be e, g_T, h_L. The first term equals Eq. (7.1). Eq. (7.3) coincides with the form of the $\boldsymbol{q}_T$-integrated DY hadron tensor we have used, Eq. (4.11). The extrapolation that we made is to use the form of the first term of Eq. (7.2) not only for the leading twist term, but also for the subleading twist term, cf. Eq. (5.65).

The general form as given in Eq. (7.3) is consistent with the known failure of factorization at order $1/Q^4$ in the unpolarized case [122]. For processes with two soft parts factorization will eventually break down, but the above discussion and also explicit pQCD correction calculations suggest that factorization indeed occurs at first subleading order. Our investigations assumed this and our neglect of order $1/Q^2$ contributions for polarized scattering, i.e., next-to-next-to-leading twist, is reasonable from this point of view. Moreover, the experimental knowledge of twist-three distribution and fragmentation functions is very limited (if existent at all), so to go beyond the first subleading twist is not relevant for present-day phenomenology. Experimentally, the most well-known functions are the leading order unpolarized and polarized distribution functions f_1 and g_1, respectively, [123] and the fragmentation function D_1 [124]. The leading order transversity distribution h_1 will be measured in the near future at RHIC, COMPASS and possibly HERA-$\vec{N}$. Also, the twist-three distribution function g_2 has been measured [17], but the data are still consistent with its twist-two (Wandzura-Wilczek) part.

7.2 Evolution

As said, factorization means that a process factorizes into soft and hard parts. This must be such that soft and collinear divergences arising in calculations of perturbative corrections either cancel or can be absorbed in redefinitions of the soft parts to all orders in the coupling constant. For the absorption of (collinear) divergences into distribution (or fragmentation) functions, one needs to introduce a scale, the factorization scale, which is the separation scale between soft and hard energy scales. Changes of this factorization scale determine how a distribution function changes with energy, i.e., how it evolves.

Hence, factorization allows to make two kinds of predictions. One follows from the universality discussed in the previous section: if a distribution function is measured in DIS, then it can be used to predict cross sections of other processes involving the same function. The other kind of prediction follows from evolution: if one knows the cross section at a certain energy scale one can predict its magnitude at other energies.

Here we give the example of the leading order (LO) evolution equation for the nonsinglet, unpolarized quark distribution function $q(x,Q^2)(= f_1(x,Q^2))$:

$$\frac{dq(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y} q(y,Q^2) P_{qq}(\frac{x}{y}), \tag{7.4}$$

where the Altarelli-Parisi (AP) [125] splitting function P_{qq} is given by

$$P_{qq}(x) = C_F\left[\frac{(1+x^2)}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]. \tag{7.5}$$

This equation gives the leading order QCD evolution of the distribution function as a function of Q^2. Also, the LO and NLO (α_s^2) evolution kernels of the other leading twist distribution functions, g_1 [49] and h_1 [50, 51, 52], have been calculated.

Evolution calculations of twist-three distribution and fragmentation functions have been performed and are conceptually not different from the leading twist ones and lead to factorization to the order that is considered. But the evolution equations are more complicated than Eq. (7.4) due to mixing of functions, which is already the case for polarized distribution functions at the leading twist level. Many LO evolution calculations for the twist-three function g_2 have been performed [38, 126, 127], both for moments as well as for the x-dependent function as a whole. Also for the other twist-three distribution functions e and h_L, the LO evolution kernels have been obtained [128, 129, 130] and similarly for the analogous fragmentation functions [131, 127]. These results can be used to evolve the tree-level results for the $\boldsymbol{q}_T$-integrated cross sections.

It would be very interesting to obtain evolution equations for functions which still have a dependence on transverse momentum. In [132, 133] evolution equations are discussed for the explicitly $\boldsymbol{k}_T$-dependent (unpolarized) functions, where the transverse momentum dependence is scaled by factors of the lightcone momentum fraction x. In the notation of Chap. 2 the parton momentum is decomposed as $k = x(p + \alpha\, n + \rho_T)$ and $\rho^2 \equiv -\rho_T^\mu \rho_{T\mu}$. In [133] an evolution equation for the unpolarized distribution of partons of flavor a *at fixed* ρ is given:

$$\frac{dq_a(x,\rho,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \sum_b \int_x^1 \frac{dy}{y} q_b(y,\rho,Q^2) P_{ab}(\frac{x}{y}). \tag{7.6}$$

From [132] it can be seen that this assumes that the splitting functions are independent of ρ_T, i.e., that the parton differential cross section $d\hat{\sigma}_a(k)$ factorizes in the following way:

$$d\hat{\sigma}_a(x,\rho_T) = \sum_b \int_x^1 \frac{dy}{y} d\hat{\sigma}_b(x,\rho_T) P_{ab}(\frac{x}{y}). \tag{7.7}$$

This is correct, since the transverse momentum in the hard scattering part leads to suppression. However, the hard scattering part is a matrix in the spinor indices of the partons and therefore will contain more information than just the part which is projected out by the unpolarized distribution function $q_a(x,\rho)\, (= f_1^a(x,\rho))$. This is already the case for $\boldsymbol{k}_T$-integrated functions; $h_1(x)$ evolves with a different splitting function than $f_1(x)$. In the same vein, the evolution kernels for $h_{1L}^\perp(x,\boldsymbol{k}_T^2)$ and $h_{1T}^\perp(x,\boldsymbol{k}_T^2)$ are expected to be different. At next-to-leading twist the transverse momentum in the hard scattering part *will* play a role for the evolution of the relevant distribution functions and it is not obvious that simple evolution equations, like Eq. (7.6), will result.

It is also possible that the $\boldsymbol{k}_T^2$-weighted functions satisfy AP-like evolution equations, but this is a nontrivial issue, since it seems that weighting the integrals with factors of $\boldsymbol{k}_T^2$ might cause them to diverge. For the first $\boldsymbol{k}_T^2$-moment, for instance $g_{1T}^{(1)}(x)$, one could check if the relations to twist-three functions as derived in Chap. 3 remain valid under evolution. Although higher $\boldsymbol{k}_T^2$-moments will most likely diverge, this need not be a problem, since the weighted cross section in which they appear will probably also diverge. This subject needs to be investigated.

We would also like to point out that pQCD corrections affect the tree level results in two ways. We will illustrate this by referring to the leading order unpolarized e^+e^- cross section differential in $\boldsymbol{q}_T$, Eq. (5.94). The first way is that pQCD corrections affect the magnitude of asymmetries, again in possibly two ways, namely due to collinear divergences and due to soft gluons. The former case, which just gives the Q^2 evolution of the distribution and fragmentation functions, is irrelevant from our point of view, since the magnitude of the functions was unknown anyway. It gives the Q^2 dependence of the functions $D_1(z^2, \boldsymbol{k}_T^2), \dots$ in Eq. (5.94). In the second case, the soft gluons give rise to Sudakov form factors, diluting the asymmetries we have considered, at higher energies, even for the ones that were not suppressed by $1/Q$ (those ones would be overtaken by α_s corrections in any case). So one should keep in mind the restrictions of $Q_T \ll Q$ and Q not too high, such that dilution due to Sudakov factors does not play a dominant rôle. In the example of Eq. (5.94) the Sudakov factor will modify the $\boldsymbol{q}_T$ behavior of the hadron tensor.

The second way in which pQCD corrections affect the tree level results is by introducing new structures, that were not present at tree level. An example is the longitudinal structure function F_L, which is zero at tree level. This might affect the conclusions drawn about the tree-level azimuthal asymmetries. For instance, we studied certain asymmetries in order to project out from all possible structures the ones we are interested in. In this way one can conclude that at tree level a certain asymmetry is sensitive to a particular twist-three function. But it could be possible that pQCD corrections cause a twist-two function to appear in the same asymmetry. This might even be a larger contribution, such that the effect of the twist-three function is in fact negligible. However, for the azimuthal asymmetries that arise from intrinsic transverse momentum one can argue that the asymmetries we have found are indeed governed by the tree-level expressions. In order to generate for instance the $\cos(2\phi)$ asymmetry of the previous chapter, one needs to have a particular transverse momentum dependence multiplying the functions and in case this has to arise from the hard scattering part it will automatically be suppressed. This is precisely the reason why the $\cos(2\phi)$ asymmetry of [92] appears with a factor of $1/Q^2$. Hence it would contribute to the $\cos(2\phi)$ asymmetry in Eq. (5.94) proportional to $D_1\overline{D}_1/Q^2$, which is expected to be much smaller than $H_1^\perp \overline{H}_1^\perp/M_1M_2$.

For the previous argument to hold, one assumes that all functions are roughly of the same order of magnitude and that suppression only occurs due to factors of $1/Q$. So for instance, h_L is expected to be of the same size as h_1, but the former appears in the cross section suppressed by a factor of $1/Q$. This assumption is somewhat justified by the fact that (most) twist-three functions have a twist-two (Wandzura-Wilczek) part. But there is *a priori* no particular reason why the T-odd function $H_1^\perp$ should be of the same order of magnitude as D_1, but neither why it should be much smaller than $D_1 M/Q$. Hence, it is *a priori* not obvious that in the region of $Q_T \ll Q$ and moderate values of Q the azimuthal asymmetries as considered in this thesis are dominated by pQCD correction terms. In fact, there are clear indications that the pQCD corrections fall short of explaining asymmetries in processes involving transverse quark polarization. The LO pQCD correction to such processes, like for instance $p + p^\uparrow \to \pi + X$, will be proportional to $\alpha_s m_q/\sqrt{s}$ [134] and hence will be negligible at higher energies. The data [135] cannot be explained by assuming that the asymmetries arise solely due to elementary scattering processes (see [136] for a comparison of different mechanisms that might be responsible for single spin asymmetries in several processes, including the Drell-Yan process). Another striking observation is made in [78], namely that pQCD correction predict the wrong sign (compared to the data) for an observable related to an azimuthal asymmetry in $h_1 + h_2 \to V + X$ (where V is a vector boson) and hence, in Drell-Yan scattering.

In doing pQCD correction calculations the choice of gauge is important. As we have discussed before and will discuss very extensively in the next section, in EFP-like approaches it is not necessary to choose a lightcone gauge (or even several lightcone gauges or general axial gauges), since the factorization should be such that color gauge invariant soft parts arise. But in the calculation of perturbative corrections to leading logarithmic approximation, a physical gauge is often convenient. However, a drawback is that spurious poles will be present and choosing a proper prescription to deal with them will make things more complicated again. The objections against using a physical gauge in the proof of factorization [113], for instance, that it obstructs contour deformations, can be overcome by choosing the Mandelstam-Leibbrandt prescription, however only at the cost of other complications. In App. 7.A we discuss some of the issues involved.

7.3 Color gauge invariance

The issue of color gauge invariance in processes with two soft parts is another theoretical topic not yet fully addressed in the literature. One has to show that a process with two soft parts factorizes into color gauge invariant objects, which upon choosing a lightcone gauge reduce to the correlation functions that we have used. The fact that we used different gauges for different correlation functions implicitly assumed that they are separately color gauge invariant.

For DIS, a process with one soft part, it is well-known that Ward identities applied to correlation functions with arbitrary numbers of A^+-gluons, yield the desired path-ordered exponentials, called link operators, that render the correlation functions color gauge invariant (cf. Chap. 2). In [39, 12] this issue was considered for the DY process, but the intrinsic transverse momentum of the quarks is not included in the way we need it. Unlike [39, 12] we focussed on the case of small transverse momentum of the photon.

We will now show that a process with two soft parts, taking into account the intrinsic transverse momentum of the quarks (polarization is irrelevant), as is necessary at small values of Q_T, and including $1/Q$ contributions, actually contains color gauge invariant correlation functions containing link operators with straight paths along a lightlike direction going via lightcone infinity. Such a link operator will be of the form:

$$\mathcal{L}(0,x) = \mathcal{P}\exp\left(-ig\int_0^\infty dy^-\, A^+(y^-,0^+,0_T) + ig\int_{x^-}^\infty dy^-\, A^+(y^-,x^+,\boldsymbol{x}_T)\right). \tag{7.8}$$

The only assumption we will make is that the gluon fields vanish at infinity inside *physical* matrix elements, thereby neglecting possible gluonic poles. Again we will study the tree level situation and note that the coupling constants appearing in the link operators are part of the color gauge invariant definition of correlation functions, hence, count as tree level objects. We will consider the case of electron-positron annihilation, but the case of DY or semi-inclusive DIS will be completely similar, since whether the photon is timelike or spacelike is irrelevant.

Since the cross section as a whole is gauge invariant, we will choose *one* lightcone gauge $A^- = 0$ and show that inclusion of A^+ gluons leads to appropriate link operators in one set of correlation functions (the antiquark fragmentation correlation functions $\overline{\Delta}, \overline{\Delta}_D^\alpha$), thereby demonstrating that choosing a lightcone gauge in correlation functions is inessential and moreover, that upon choosing such a gauge they reduce to the functions we have used.

We will show that matrix elements with multiple A^+-gluon fields in $\overline{\Delta}_A^+, \overline{\Delta}_{AA}^{++}, \ldots$ and $\overline{\Delta}_{AA}^{\alpha+}, \overline{\Delta}_{AAA}^{\alpha++}, \ldots$ will combine into link operators in $\overline{\Delta}$ and $\overline{\Delta}_D^\alpha$, with paths as given above (running along the $-$ direction via $y^- = \infty$). Note that the correlation function $\overline{\Delta}_D^\alpha$ appears in the end, since the explicit A^α in $\overline{\Delta}_A^\alpha$ is not gauge invariant. Note that one has to be careful in writing

$$\overline{\Delta}_A^\alpha(z, \boldsymbol{k}_T) = \overline{\Delta}_D^\alpha(z, \boldsymbol{k}_T) - k^\alpha\, \overline{\Delta}(z, \boldsymbol{k}_T) \tag{7.9}$$

for color gauge invariant correlation functions, since ∂_T also acts on the link operator. A proper color gauge invariant definition of $\overline{\Delta}_A^\alpha$ must be given, since $\overline{\Delta}_A^\alpha$ and $\overline{\Delta}_D^\alpha$ do not just differ by the replacement $gA \to iD$ as is the case in the $A^+ = 0$ gauge.

In the previous chapter we depicted the relevant diagrams for the tree level calculations including $1/Q$ corrections, Figs. 5.3 and 5.4. The gluons appearing in the latter figure all had a transverse index. In the $A^- = 0$ gauge one has to include A^+ gluons in all possible ways. There are four types of diagrams, namely those where all gluons emanating from Δ and/or $\overline{\Delta}$ connect to the right or left side of the diagram. We will restrict to the case where all gluons will connect to the left side of the diagram. The other cases are similar (all connecting to the right) or less complicated (gluons from Δ to the left/right and from $\overline{\Delta}$ to the right/left).

Next we observe that A^+ gluons emanating from Δ will give rise to $1/Q^2$ suppression, so we only need to consider the cases where the A^+ gluons emanate from $\overline{\Delta}$. Also, no diagram with a triple gluon vertex appears, since these will either be suppressed or are not 1PI in the hanging legs (cf. Chap. 2). The remaining diagrams with *zero or one* A^+ gluon we need to consider are (schematically) displayed in Fig. 7.1. We have only drawn the relevant left part of the diagrams and left out the blobs denoting the correlation functions (gluons going to the top/bottom emanate from $\Delta/\overline{\Delta}$). The index α in Fig. 7.1b can be either transverse (T) or $+$ and it turns out that α and β in Fig. 7.1c will be T and $+$ respectively, otherwise the diagram will be suppressed; on the other diagrams we indicated the appropriate index).

To discuss the expressions for the diagrams as given in the figure, we will introduce the shorthand notation:

$$[\ldots]' = \int d^4k\, d^4p\, \delta^4(q - p - k)\, \mathrm{Tr}\,[\ldots] \tag{7.10}$$

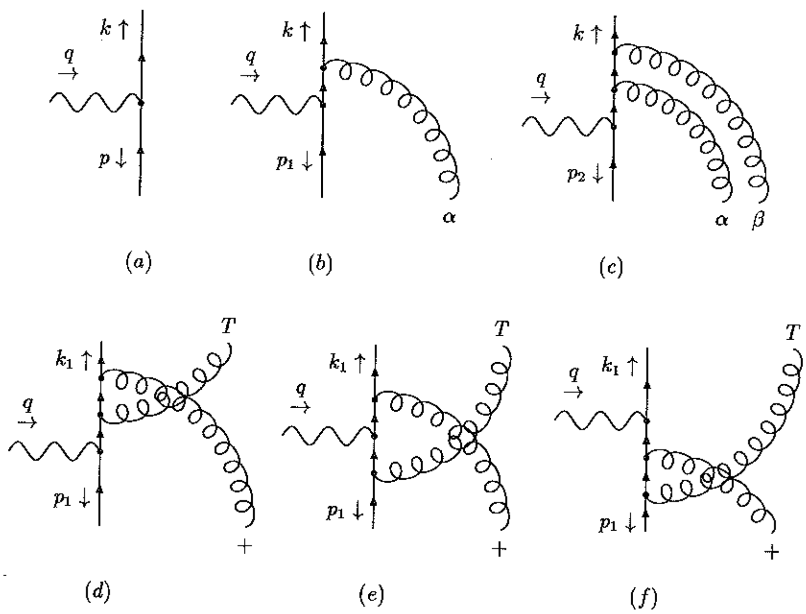


Figure 7.1: A subset of diagrams to be considered upon inclusion of one A^+ gluon.

Also, we need the precise definitions of $\overline{\Delta}_A^\alpha(p_1, p)$ and $\overline{\Delta}_{AA}^{\alpha\beta}(p_2, p_1, p)$, not given in the previous chapters (α, β not restricted to be transverse):

$$\overline{\Delta}_A^\alpha(p_1, p) = \sum_X \int \frac{d^4x}{(2\pi)^4} \frac{d^4y}{(2\pi)^4}\, e^{-ip\cdot y}\, e^{-ip_1\cdot(x-y)} \times \langle 0|\overline{\psi}(0)|P_2, S_2; X\rangle\langle P_2, S_2; X|\, gA^\alpha(y)\, \psi(x)|0\rangle, \tag{7.11}$$

$$\overline{\Delta}_{AA}^{\alpha\beta}(p_2, p_1, p) = \sum_X \int \frac{d^4x}{(2\pi)^4} \frac{d^4y}{(2\pi)^4} \frac{d^4z}{(2\pi)^4}\, e^{-ip\cdot z}\, e^{-ip_1\cdot(y-z)}\, e^{-ip_2\cdot(x-y)} \times \langle 0|\overline{\psi}(0)|P_2, S_2; X\rangle\langle P_2, S_2; X|\, gA^\beta(z)\, gA^\alpha(y)\, \psi(x)|0\rangle. \tag{7.12}$$

Furthermore, we decompose the parton momenta p_i as follows

$$p_i \equiv \frac{\xi_i Q}{\sqrt{2}}\, n_+ + \frac{(p_i^2 + \boldsymbol{p}_{iT}^2)}{\xi_i Q\sqrt{2}}\, n_- + p_{iT}. \tag{7.13}$$

We find for diagrams (a)-(d):

$$(a) = \left[\overline{\Delta}(p)\gamma^\mu \Delta(k)\gamma^\nu\right]', \tag{7.14}$$

$$(b) = -\left[\overline{\Delta}_A^\alpha(p)\gamma^\mu\Delta(k)\gamma_{\alpha T}\frac{\not{n}_+}{\sqrt{2}Q}\gamma^\nu\right]' - \left[\int d^4p_1\,\overline{\Delta}_A^+(p_1,p)\,\frac{\sqrt{2}}{Q(1-\xi_1+i\epsilon)}\,\gamma^\mu\Delta(k)\gamma^\nu\right]'$$

$$-\left[\int d^4p_1\,\overline{\Delta}_A^+(p_1,p)\gamma^\mu\Delta(k)\frac{\sqrt{2}}{Q(1-\xi_1+i\epsilon)}\left\{\frac{\not{n}_+}{\sqrt{2}Q}\left(\not{k}_T+\not{p}_T-\not{p}_{1T}\right)-P_+\right\}\gamma^\nu\right]', \tag{7.15}$$

$$(c) = \left[\int d^4p_2\,d^4p_1\,\overline{\Delta}_{AA}^{\alpha\,+}(p_2,p_1,p)\,\frac{\sqrt{2}}{Q(1-\xi_1+i\epsilon)}\,\gamma^\mu\Delta(k)\gamma_{\alpha T}\frac{\not{n}_+}{\sqrt{2}Q}\gamma^\nu\right]', \tag{7.16}$$

$$(d) = \left[\int d^4p_1\,\overline{\Delta}_A^+(p_1,p)\gamma^\mu\left(\int d^4k_1\,\Delta_D^\beta(k_1,k)\gamma_{\beta T}-\Delta(k)\not{k}_T\right)\frac{\sqrt{2}}{Q(1-\xi_1+i\epsilon)}\frac{\not{n}_+}{\sqrt{2}Q}\gamma^\nu\right]'. \tag{7.17}$$

In the last equation we have used the fact that in the $A^-=0$ gauge no link will be present in Δ_A^β. We can then apply the e.o.m. on $\int d^4k_1\,\Delta_D^\beta(k_1,k)\gamma_{\beta T}$, which results in a cancellation of the term with the P_+-projector in the expression for diagram (b). Next we observe that (to order g in the link operator)

$$\overline{\Delta}(p) - \int d^4p_1\,\overline{\Delta}_A^+(p_1,p)\frac{\sqrt{2}}{Q(1-\xi_1+i\epsilon)} = \overline{\Delta}[\infty,x^-](p), \tag{7.18}$$

$$\overline{\Delta}_A^\alpha(p) - \int d^4p_2\,d^4p_1\,\overline{\Delta}_{AA}^{\alpha\,+}(p_2,p_1,p)\frac{\sqrt{2}}{Q(1-\xi_1+i\epsilon)} = \overline{\Delta}_A^\alpha[\infty,x^-](p), \tag{7.19}$$

where

$$\overline{\Delta}[\infty,x^-](p) = \sum_X\int\frac{d^4x}{(2\pi)^4}\,e^{-ip\cdot x}\,\langle 0|\overline{\psi}(0)|P_2,S_2;X\rangle$$
$$\times\langle P_2,S_2;X|\mathcal{P}\exp\left(ig\int_{x^-}^\infty dy^-\,A^+(y^-,x^+,\boldsymbol{x}_T)\right)\,\psi(x)|0\rangle, \tag{7.20}$$

$$\overline{\Delta}_A^\alpha[\infty,x^-](p) = \sum_X\int\frac{d^4x}{(2\pi)^4}\,e^{-ip\cdot x}\,\langle 0|\overline{\psi}(0)|P_2,S_2;X\rangle$$
$$\times\langle P_2,S_2;X|\mathcal{P}\exp\left(ig\int_{x^-}^\infty dy^-\,A^+(y^-,x^+,\boldsymbol{x}_T)\right)\,gA^\alpha(x)\,\psi(x)|0\rangle. \tag{7.21}$$

The sum of the four diagram (a)-(d) then results in (to order g in the link operator)

$$\left[\overline{\Delta}[\infty,x^-](p)\gamma^\mu\Delta(k)\gamma^\nu\right]' - \left[\overline{\Delta}_A^\alpha[\infty,x^-](p)\gamma^\mu\Delta(k)\gamma_{\alpha T}\frac{\not{n}_+}{\sqrt{2}Q}\gamma^\nu\right]'$$
$$-\left[\int d^4p_1\,\overline{\Delta}_A^+(p_1,p)\gamma^\mu\Delta(k)\frac{\sqrt{2}}{Q(1-\xi_1+i\epsilon)}\frac{\not{n}_+}{\sqrt{2}Q}\left(\not{p}_T-\not{p}_{1T}\right)\gamma^\nu\right]'. \tag{7.22}$$

By taking account of the derivative of the link operator, the last two terms nicely combine into the following expression containing exclusively the color gauge invariant functions

$\overline{\Delta}[\infty,x^-](p)$ and $\overline{\Delta}_D^\alpha[\infty,x^-](p)$ (with the latter obtained from Eq. (7.21) by replacing gA^α by iD^α):

$$(a)+(b)+(c)+(d) = \left[\overline{\Delta}[\infty,x^-](p)\gamma^\mu\Delta(k)\gamma^\nu\right]' \tag{7.23}$$

$$-\left[\left\{\overline{\Delta}_D^\alpha[\infty,x^-](p)+p_T^\alpha\,\overline{\Delta}[\infty,x^-](p)\right\}\gamma^\mu\Delta(k)\gamma_{\alpha T}\frac{\not{n}_+}{\sqrt{2}Q}\gamma^\nu\right]'. \tag{7.24}$$

The diagrams (e) and (f) also turn out to combine, yielding the expected expression:

$$(e)+(f) = -\left[\overline{\Delta}[\infty,x^-](p)\gamma^\mu\left\{\gamma_0\Delta_D^{\beta\dagger}(k)\gamma_0 - k_T^\beta\Delta(k)\right\}\gamma^\nu\frac{\not{n}_-}{\sqrt{2}Q}\gamma_{\beta T}\right]'. \tag{7.25}$$

Inclusion of *two* A^+ gluons is done in a similar fashion, where we note that from diagram (c) with $\alpha=\beta=+$ one can see that a product of two theta-functions in the $-$ component appears, which gives rise to the path-ordering. Knowing this, it is a matter of iteratively treating each order like the first order we have discussed just now, by treating $\left\{\overline{\Delta}_D^\alpha[\infty,x^-](p)+p_T^\alpha\,\overline{\Delta}[\infty,x^-](p)\right\}$ as an effective $\overline{\Delta}_A^\alpha(p)$.

When the A^+ gluons emanating from $\overline{\Delta}$ connect to the right-side of the diagrams, then one arrives at functions $\overline{\Delta}[0^-,\infty](p)$ and $\overline{\Delta}_D^\alpha[0^-,\infty](p)$, which contain the link operator

$$[0^-,\infty] = \mathcal{P}\exp\left(-ig\int_{0^-}^{\infty}dy^-\,A^+(y^-,0^+,\mathbf{0}_T)\right). \tag{7.26}$$

At tree level it is now not difficult to convince oneself that the complete set of diagrams with arbitrary numbers of A^+ gluons emanating from $\overline{\Delta}$ will result in the appearance of color gauge invariant correlation functions (containing link operators and covariant derivatives) which upon choosing the $A^+=0$ gauge reduce to the ones we have been using.

A proof to all orders in α_s will be much more complicated and is part of the general (and still lacking) factorization proof given in a gauge independent manner. But we can already conclude that our tree level analysis for processes with two soft parts at leading and next-to-leading twist, including transverse momentum and polarization is manifestly color gauge invariant[1].

7.A Axial gauges

In this appendix we discuss some aspects of axial gauges in general. They are not important for tree level investigations, but they are important for calculations of pQCD corrections to the distribution or fragmentation functions.

[1]We would like to point out that we have not obtained color gauge invariance at the cost of introducing a path dependence.

Axial gauges are of the form $n \cdot A = 0$, where n is an arbitrary, constant vector, with $n^2 \neq 0$. When $n^2 = 0$ it is the so-called lightcone gauge. In order to obtain the propagator in an axial gauge, one can add two types of gauge fixing terms in the Lagrangian, namely:

$$\mathcal{L}_1 = -\frac{\lambda}{2}(n \cdot A)^2, \tag{7.A1}$$

with $\lambda \to \infty$, or

$$\mathcal{L}_2 = \alpha(n \cdot A), \tag{7.A2}$$

where α is a Lagrange-multiplier field. The propagator one finds for this lightcone gauge is:

$$G^{\mu\nu} = \frac{1}{k^2}\left[-g^{\mu\nu} + \frac{(k^\mu n^\nu + k^\nu n^\mu)}{k \cdot n}\right]. \tag{7.A3}$$

The propagator one finds with $\mathcal{L}_1$ for arbitrary λ and $n^2 \neq 0$ is:

$$G^{\mu\nu} = \frac{1}{k^2}\left[-g^{\mu\nu} + \frac{(k^\mu n^\nu + k^\nu n^\mu)}{k \cdot n} - n^2(1 + \frac{k^2}{\lambda n^2})\frac{k^\mu k^\nu}{(k \cdot n)^2}\right]. \tag{7.A4}$$

Note that when λ in $\mathcal{L}_1$ is not infinite, one does not have the condition $n \cdot A = 0$, in which case one speaks of an inhomogeneous axial gauge. Only in the limit $\lambda \to \infty$, the propagator is equal to the one one arrives at by using $\mathcal{L}_2$, because the Euler-Lagrange equation for α is $n \cdot A = 0$.

Axial gauges are called physical gauges, because contraction of the propagator with k_μ cancels the physical pole at $k^2 = 0$:

$$\lim_{k^2 \to 0} k^2 k_\mu G^{\mu\nu} = 0, \tag{7.A5}$$

which means that the degrees of freedom are transverse. The Landau gauge ($\mathcal{L}_{gf} = -\frac{\lambda}{2}(\partial \cdot A)^2$, with $\lambda \to \infty$) also has this property, but does not have the other properties of axial gauges.

Advantages of axial gauges:

- Link operators in color gauge invariant correlation functions can be made unity.
- Absence of Faddeev-Popov ghosts.
- In leading logarithmic approximation only ladder diagrams contribute to evolution [137].

Elaboration on the second point: In order to investigate the ghost sector, one has to know how the gauge fixing term $F = n \cdot A$ transforms under gauge transformations $A_\mu \to A_\mu + D_\mu \Lambda$:

$$\frac{\delta F(y)}{\delta \Lambda(z)} = n \cdot D\, \delta^4(y - z) = n \cdot \partial\, \delta^4(y - z), \tag{7.A6}$$

i.e., it is independent of A. There is no ghost–gauge-boson interaction and one can integrate out the (free) ghost fields, to arrive at a constant which one can absorb in the normalization of the path-integral.

Disadvantages of axial gauges:

• Introduction of a Lorentz covariance breaking vector (one cannot address this preferred direction in a frame independent way).
• The occurrence of spurious poles.

As can be seen from the propagator, the axial gauges introduce at least a simple pole of the form $(n \cdot k)^{-1}$ and sometimes even double poles $(n \cdot k)^{-2}$. To deal with these spurious poles one needs a prescription. We will look at the Cauchy Principal Value (CPV) and the Mandelstam-Leibbrandt (ML) prescriptions. The CPV prescription,

$$\frac{1}{n \cdot k} \equiv \lim_{\epsilon \to 0} \frac{1}{2} \left[\frac{1}{n \cdot k + i\epsilon} + \frac{1}{n \cdot k - i\epsilon} \right], \tag{7.A7}$$

is unfortunately inconsistent in the lightcone gauge [138]. For space-like axial gauges this prescription *can* be used, but still it has some unwanted features. One of the problems here is that the poles always lie in adjacent quadrants and therefore do not allow for Wick rotation without picking up extra terms. Another problem is that powers of this CPV $(n \cdot k)^{-1}$ distribution are not well-defined, i.e., they are not distributions.

The ML prescription has been devised as an analogue of the causal prescription for the normal covariant pole. A causal prescription normally arises by requiring physical boundary conditions, however this does not apply to the unphysical spurious poles. Leibbrandt [138] posed it as:

$$\frac{1}{k^+} \equiv \lim_{\epsilon \to 0} \frac{k^-}{k^+ k^- + i\epsilon}. \tag{7.A8}$$

One can easily verify that the positions of the poles in the complex k_0-plane are in the same quadrants as those of the covariant poles (like $k_0^2 - |\vec{k}|^2 + i\epsilon$ one now has $k_0^2 - k_3^2 + i\epsilon$).

The CPV and the ML prescription do not simply differ by boundary conditions, i.e. one cannot arrive at CPV from ML via residual gauge transformations (cf. [139]).

Advantages of the ML prescription:

• Allows for Wick rotations without picking up additional terms.
• Products of ML distributions are already regularized, i.e., they are distributions (except for the one originally posed by Mandelstam [140]).
• It leads to a free propagator which is a tempered distribution, i.e., it admits a Fourier transform, contrary to the CPV prescription, cf. [141].

Disadvantages of the ML prescription:

• Requires the introduction of *two* Lorentz covariance breaking vectors (one has to specify two directions).
• The possibility of nonlocal counterterms arises (however in [142] this is shown to be harmless).

Chapter 8

Summary, conclusions and outlook

In this thesis we investigated azimuthal asymmetries in (semi-)inclusive hard scattering processes, in particular, the deep inelastic scattering (DIS) process, the Drell-Yan (DY) process and inclusive two-particle production in electron-positron annihilation. The hard scale Q present in these hard scattering processes served to separate the perturbative – hard – from the nonperturbative –soft– part of the process. In Chap. 2 we focussed on this separation for DIS, a process with one soft part. We analyzed this separation for power corrections, following the work of Ellis, Furmanski and Petronzio [4] and Qiu [36]. This factorization was established at each order in $1/Q$ separately (albeit at tree level). The resulting hard parts were manifestly electromagnetically gauge invariant by themselves. The soft parts were described by nonlocal operator matrix elements and the connection to the operator product expansion was elaborated upon. The polarized twist-three and unpolarized twist-four dynamical power corrections were investigated in much detail, also taking into account quark masses. Our main conclusions were that Qiu's factorization procedure can be applied in case quark masses are present and that the nonlocal operators consist of good fields only.

The soft parts are represented by correlation functions, which are Fourier transforms of hadronic matrix elements of nonlocal operators. In Chap. 3 we studied their properties by first using the symmetries of QCD to restrict the parametrization in terms of independent functions called distribution functions. Since these distribution functions entail information on the structure of a hadron, they often satisfy certain restrictions themselves, like integral relations, called sum rules. Apart from well-known sum rules like the Burkhardt-Cottingham, the Burkardt (h_2) and the Efremov-Leader-Teryaev sum rules, we also derived some new sum rules. We discussed justifications and assumptions, like on interchanging of integrals, choosing boundary conditions, neglecting higher order corrections, etc. This is very important in order to understand what conclusions can be drawn in case a certain sum rule is found to be violated. Time reversal symmetry and its implications were discussed extensively because of its prominent role in the ensuing chapters.

After the determination of the complete set of leading and subleading twist distribution functions, one can arrive at expressions for cross-sections and asymmetries in terms of these functions. Next one can deduce how to measure and separate them. Universality of the functions means that they can be measured in one process and then yield predictions for another. For instance, the DY process is described in terms of two soft parts and one of them equals the soft part of the DIS process and the other one is related to it (the antiquark correlation function).

Another reason to study processes with two soft parts is that one can be sensitive to the transverse momentum of the partons inside a hadron. Hence, we studied distribution functions not only as functions of the lightcone momentum fraction x, but also as functions of intrinsic transverse momentum of the quarks and gluons inside a hadron. The consequences can be seen in certain asymmetries, which can even appear at leading order in $1/Q$ (as was seen in Chap. 5). This is one of the main conclusions of our investigations concerning intrinsic transverse momentum, namely that it is not allowed to make a collinear expansion of the soft part and consequently, the azimuthal asymmetries arising due to intrinsic transverse momentum are not necessarily suppressed by inverse powers of the hard scale.

In Chap. 4 we studied DY at subleading twist, integrated over the transverse momentum of the photon, such that many structures averaged out, but still a number of interesting azimuthal spin asymmetries remained. Following Hammon *et al.* [77] we discussed single spin asymmetries in DY, which might arise due to so-called gluonic poles in twist-three hadronic matrix elements, which were first discussed by Qiu and Sterman [79]. We have shown how the single spin asymmetries possibly arising due to such gluonic poles cannot be distinguished from those due to T-odd distribution functions. The gluonic poles together with imaginary parts of hard subprocesses lead to *effective* T-odd distribution functions. However, time reversal symmetry does not force the latter functions to vanish. We also observed that gluonic poles require large distance gluonic fields with antisymmetric boundary conditions. Our analysis moreover showed the nonnegligible role of intrinsic transverse momentum of the partons for the DY cross section at subleading order, also for the regular T-even asymmetries. That role became even more apparent in the two ensuing chapters in which we discussed inclusive two-hadron production in electron-positron annihilation at measured transverse momentum of the photon.

We looked at this process at energies far below the Z mass –Chap. 5– and at the Z mass peak –Chap. 6–. In both chapters the focus was on azimuthal asymmetries arising as spin asymmetries and/or as manifestations of intrinsic transverse momentum of the quarks. In Chap. 5 we studied the complete tree level result for leading and subleading twist and investigated several types of observables, like for instance, weighted cross sections. Around the Z peak subleading asymmetries are expected to be negligible, hence in Chap. 6 we have restricted to leading twist. We extended results of an earlier theoretical study [97] of this process, where no azimuthal asymmetries arising from transverse momenta have been considered, but only the leading twist cross section for unpolarized leptons, integrated over the transverse momentum of the photon and Z-boson.

In order to describe the fragmentation of a quark into a hadron plus other, undetected particles one needs fragmentation functions, which are very similar to distribution functions except for the T-odd part. While T-odd distribution functions only arise via subtle mechanisms, T-odd fragmentation functions are expected to be nonzero simply due to final state interactions.

In this thesis we focussed on the leading and next-to-leading powers in $1/Q$ and restricted to tree level. In Chap. 7 we discussed in a qualitative way the effects of including perturbative QCD corrections and concluded that it is not expected that such corrections necessarily dominate over the tree level observables that we have discussed (at least for intermediate Q^2, small $|\boldsymbol{q}_T|$ and large x). In fact, certain experimental results suggest the contrary. We discussed some of the more technical issues concerning factorization, evolution and color gauge invariance, which partly remain to be investigated or extended. The main result was that the color gauge invariant definition of $\boldsymbol{k}_T$-dependent distribution and fragmentation functions contains a path-ordered exponential with straight paths along a lightlike direction going via lightcone infinity where the transverse part of the path is located.

Polarization in the initial and final states was included for the case of spin-1/2 hadrons. Hadrons with higher spin, like ρ's, were not considered, because in that case a spin vector is not sufficient to describe the spin states (for a treatment of spin-one hadrons, see for instance [96, 102]), but there is no conceptual obstacle to extend the analysis. The same holds for the so-called interference fragmentation functions [96, 19, 109, 108], which describe a quark fragmenting into two hadrons plus other, undetected hadrons, cf. section 6.5.

Other obvious extensions are to study the cases of γ and Z interference effects, charged currents, gluon correlation functions, singlet distribution and fragmentation functions and extensions to other processes. Also, modeling of in particular T-odd fragmentation functions needs to be done and estimates of the various asymmetries have to be made. On the technical side the questions whether the $\boldsymbol{k}_T^2$-weighted functions satisfy simple Altarelli-Parisi evolution equations and whether or not the relations between first $\boldsymbol{k}_T^2$-moments of twist-two functions and twist-three functions, as given in Chap. 3, remain valid under evolution, are very interesting and deserve investigation.

Many of the observables in this thesis contain unknown functions, which are not just parametrizations of ignorance, but constitute essential information about the structure of hadrons as can be probed in hard scattering processes. The problem is simply that one at present is not able to calculate these nonperturbative objects from first principles, but estimates have been made using lattice, QCD sum rule or model calculations. Most of the attempts to estimate the magnitude and shape of distribution and fragmentation functions restricted to the ones that are known experimentally, namely f_1, g_1 and D_1 (g_2 hardly can be called to be "known" yet), all as functions of x and Q^2. Also a number of asymmetries have been measured of which it is not clear-cut which combination of functions one is in fact measuring. The latter was actually the focus of our investigations, rather than trying to model functions and make predictions, which would be an obvious next step.

Apart from the simple observations that many functions are unknown and that many of the asymmetries discussed in this thesis are beyond the reach of experimental investigation at present, we can nevertheless point out the most promising lines of experimental investigation.

First of all, we would like to emphasize the importance of discovering the origin of single spin asymmetries by doing several, different hard scattering experiments. Several mechanisms that might be responsible for single spin asymmetries have been proposed and a study on how to distinguish them in experiment has been performed [136]. We can add to those results, that the gluonic pole mechanism can in principle be distinguished from that of the factorization breaking initial state interactions mechanism; the latter will not show up in semi-inclusive DIS.

Besides this example we studied an interesting asymmetry involving a product of two T-odd fragmentation functions, which was discussed in Chap. 5 and was the main focus of Chap. 6. It is a leading $\cos(2\phi)$ asymmetry, which is even present when hadron polarization is absent. It arises solely due to the intrinsic transverse momenta of transversely polarized quarks. A measurement of this asymmetry would implicitly confirm the Collins effect *without* the need for polarization measurements on incoming or outgoing particles. We made a simple estimate which indicated that the asymmetry could be of the order of a percent. A comparison was made to a preliminary experimental study [103], which provided an upper bound on the asymmetry. Improvement of statistics is necessary and require the inclusion of data obtained afterwards.

So the other main point is to focus on the opportunities offered by the Collins effect in general and the unpolarized inclusive two-hadron production in electron-positron annihilation in particular. For instance, the measurement of the function $H_1^\perp$ in the latter process could provide a means to measure h_1 in semi-inclusive DIS [111].

As a final remark, we would like to point out the important role of polarization in many of the asymmetries, hence experiments involving Λ production with the possibility to reconstruct its polarization are amongst the most promising ones.

Bibliography

[1] D.J. Gross and F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343;
H.D. Politzer, Phys. Rev. Lett. 30 (1973) 1346.

[2] EMC Collaboration, J. Ashman *et al.*, Phys. Lett. B 202 (1988) 603; Nucl. Phys. B 328 (1989) 1.

[3] R.P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, MA, 1972).

[4] R.K. Ellis, W. Furmanski and R. Petronzio, Nucl. Phys. B 212 (1983) 29; Nucl. Phys. B 207 (1982) 1.

[5] D.E. Soper, Phys. Rev. D 15 (1977) 1141; Phys. Rev. Lett. 43 (1979) 1847.

[6] J.C. Collins and D.E. Soper, Nucl. Phys. B 194 (1982) 445.

[7] H.D. Politzer, Nucl. Phys. B 172 (1980) 349.

[8] A.V. Efremov and O.V. Teryaev, Sov. J. Nucl. Phys. 39 (1984) 962.

[9] J.P. Ralston and D.E. Soper, Nucl. Phys. B 152 (1979) 109.

[10] R.L. Jaffe and X. Ji, Phys. Rev. Lett. 67 (1991) 552.

[11] R.L. Jaffe and X. Ji, Nucl. Phys. B 375 (1992) 527.

[12] J. Qiu and G. Sterman, Nucl. Phys. B 353 (1991) 105.

[13] J. Qiu and G. Sterman, Nucl. Phys. B 353 (1991) 137.

[14] J. Bjorken, Phys. Rev. 179 (1969) 1547.

[15] E.D. Bloom *et al.*, Phys. Rev. Lett. 23 (1969) 930;
M. Breidenbach *et al.*, Phys. Rev. Lett. 23 (1969) 935.

[16] M. Virchaux and A. Milsztajn, Phys. Lett. B 274 (1992) 221.

[17] E143 Collaboration, K. Abe *et al.*, Phys. Rev. Lett. 76 (1996) 587; preprint SLAC-PUB-7753, hep-ph/9802357;
SMC Collaboration, D. Adams *et al.*, Phys. Rev. D 56 (1997) 5330.

[18] X. Ji, Z.-K. Zhu, preprint MIT-CTP-2259, hep-ph/9402303;
J. Rodrigues, A. Henneman and P.J. Mulders, in *Confinement Physics*, Proceedings of the First ELFE Summer School, Eds. S.D. Bass and P.A.M. Guichon, Cambridge UK (1995), p. 373 (nucl-th/9510036);
M. Nzar and P. Hoodbhoy, Phys. Rev. D 51 (1995) 32.

[19] J.C. Collins and G.A. Ladinsky, preprint PSU-TH-114, hep-ph/9411444.

[20] R. Jakob, P.J. Mulders and J. Rodrigues, Nucl. Phys. A 626 (1997) 937.

[21] J. Levelt, *Deep Inelastic Semi-Inclusive Processes*, Ph.D. thesis (Free University, Amsterdam, 1993).

[22] R.D. Tangerman, *Higher-Twist Correlations in Polarized Hadrons*, Ph.D. thesis (Free University, Amsterdam, 1996).

[23] M. Anselmino, A. Efremov and E. Leader, Phys. Rep. 261 (1995) 1.

[24] D. Boer and R.D. Tangerman, Phys. Lett. B 381 (1996) 305;
Proceedings of the 12th International Symposium on High-Energy Spin Physics, Eds. C.W. de Jager *et al.*, (World Scientific, 1997), p. 225 (hep-ph/9610289).

[25] D. Boer, P.J. Mulders and O.V. Teryaev, Phys. Rev. D 57 (1998) 3057;
Proceedings of the VII Workshop on High Energy Spin Physics (SPIN 97), Eds. A.V. Efremov, O.V. Selyugin, E2-97-413, Dubna (1997), p. 139;
Proceedings of the Workshop "Deep Inelastic Scattering off Polarized Targets: Theory meets Experiment", Eds. J. Blümlein *et al.*, DESY 97-200, Zeuthen (1997), p. 373 (hep-ph/9710525).

[26] D. Boer, R. Jakob and P.J. Mulders, Nucl. Phys. B 504 (1997) 345.

[27] D. Boer, R. Jakob and P.J. Mulders, Phys. Lett. B 424 (1998) 143.

[28] R.G. Roberts, *The Structure of the Proton* (Cambridge University Press, 1990).

[29] R.L. Jaffe, *Spin, Twist and Hadron Structure in Deep Inelastic Processes*, Lecture notes, Erice, August 3-10, 1995, hep-ph/9602236.

[30] T-P. Cheng and L-F. Li, *Gauge Theories of Elementary Particle Physics* (Oxford University Press, 1984).

[31] T. Muta, *Foundations of Quantum Chromodynamics* (World Scientific, 1987).

[32] O. Nachtmann, Nucl. Phys. B 63 (1973) 237;
H. Georgi and H.D. Politzer, Phys. Rev. Lett. 36 (1976) 1281; Phys. Rev. D 14 (1976) 1829.

[33] J. Kodaira, Nucl. Phys. B 165 (1980) 129.

[34] B. Ehrnsperger, A. Schäfer and L. Mankiewicz, Phys. Lett. B 323 (1994) 439.

[35] M. Maul, B. Ehrnsperger, E. Stein and A. Schäfer, Z. Phys. A 356 (1997) 443.

[36] J. Qiu, Phys. Rev. D 42 (1990) 30.

[37] J.B. Kogut and D.E. Soper, Phys. Rev. D 1 (1970) 2901.

[38] A.P. Bukhvostov, E.A. Kuraev and L.N. Lipatov, Sov. Phys. JETP 60 (1984) 22; Sov. J. Nucl. Phys. 39 (1984) 121; Sov. J. Nucl. Phys. 38 (1983) 263; JETP Letters 37 (1984) 483.

[39] A.V. Efremov and A.V. Radyushkin, Theor. Math. Phys. 44 (1981) 774.

[40] R.L. Jaffe and X. Ji, Phys. Rev. D 43 (1991) 724.

[41] C.H. Llewellyn Smith and J.P. de Vries, Nucl. Phys. B 296 (1988) 991.

[42] P.J. Mulders and R.D. Tangerman, Nucl. Phys. B 461 (1996) 197; Nucl. Phys. B 484 (1997) 538 (E).

[43] W. Lu, X. Li and H. Hu, Phys. Rev. D 53 (1996) 131.

[44] S.H. Lee, Phys. Rev. D 49 (1994) 2242.

[45] C. Itzykson and J-B. Zuber, *Quantum Field Theory* (McGraw-Hill, 1980).

[46] R.L. Jaffe, Proceedings of the Workshop "Deep Inelastic Scattering off Polarized Targets: Theory meets Experiment", Eds. J. Blümlein *et al.*, DESY 97-200, Zeuthen (1997), p. 167 (hep-ph/9710465).

[47] J.T. Donohue and S. Gottlieb, Phys. Rev. D 23 (1981) 2577; Phys. Rev. D 23 (1981) 2581.

[48] A.M. Kotzinian and P.J. Mulders, Phys. Rev. D 54 (1996) 1229.

[49] R. Mertig and W. van Neerven, Z. Phys. C 70 (1996) 637;
W. Vogelsang, Phys. Rev. D 54 (1996) 2023.

[50] X. Artru and M. Mekhfi, Z. Phys. C 45 (1990) 669.

[51] A. Hayashigaki, Y. Kanazawa and Y. Koike, Phys. Rev. D 56 (1997) 7350;
S. Kumano and M. Miyama, Phys. Rev. D 56 (1997) 2504.

[52] W. Vogelsang, Phys. Rev. D 57 (1998) 1886.

[53] H. Burkhardt and W.N. Cottingham, Ann. Phys. (N.Y.) 56 (1976) 453.

[54] L. Mankiewicz and A. Schäfer, Phys. Lett. B 265 (1991) 167.

[55] S. Wandzura and F. Wilczek, Phys. Lett. B 72 (1977) 195.

[56] M. Burkardt, Phys. Rev. D 52 (1995) 3841.

[57] R.L. Heimann, Nucl. Phys. B 64 (1973) 429.

[58] M. Burkardt, in Proceedings of 10th International Symposium on High Energy Spin Physics, Eds. Hasegawa *et al.* (Universal Academy Press, Tokyo, 1993).

[59] A.V. Efremov, O.V. Teryaev, and E. Leader, Phys. Rev. D 55 (1997) 4307.

[60] J. Blümlein and N. Kochelev, Nucl. Phys. B 498 (1997) 285.

[61] C. Bourrely, J. Soffer and O.V. Teryaev, Phys. Lett. B 420 (1998) 375.

[62] M.G. Doncel and E. de Rafael, Nuovo Cimento A 4 (1971) 363;
W. Lu and J-J. Yang, Z. Phys. C 73 (1997) 689;
J-J. Yang and W. Lu, J. Phys. G 23 (1997) 1085;
C-G. Duan and W. Lu, Z. Phys. C 74 (1997) 531.

[63] D. Boer and P.J. Mulders, Phys. Rev. D 57 (1998) 5780;
J. Levelt and P.J. Mulders, Phys. Lett. B 338 (1994) 357.

[64] D. Sivers, Phys. Rev. D 41 (1990) 83; Phys. Rev. D 43 (1991) 261.

[65] M. Anselmino, M. Boglione and F. Murgia, Phys. Lett. B 362 (1995) 164.

[66] M. Anselmino, A. Drago and F. Murgia, hep-ph/9703303.

[67] O.V. Teryaev, Proceedings of the 12th International Symposium on High-Energy Spin Physics, Eds. C.W. de Jager *et al.*, (World Scientific, 1997), p. 594.

[68] M. Diehl and T. Gousset, hep-ph/9801233.

[69] R.L. Jaffe, Nucl. Phys. B 229 (1983) 205.

[70] C.H. Llewellyn Smith, Proceedings, *Symmetry Violations in Subatomic Physics*, Kingston (1988), p. 139.

[71] E. Merzbacher, *Quantum Mechanics*, second edition (Wiley International Edition, 1970);
B.H. Marsden and C.J. Joachain, *Introduction to Quantum Mechanics* (Longman Scientific & Technical, 1989).

[72] S. Gasiorowicz, *Elementary Particle physics* (Wiley, New York, 1966).

[73] S.D. Drell and T.-M. Yan, Phys. Rev. Lett. 25 (1970) 316; Ann. Phys. (NY) 66 (1971) 578.

[74] F.E. Close and D. Sivers, Phys. Rev. Lett. 39 (1977) 1116;
C.S. Lam and W-K. Tung, Phys. Rev. D 7 (1978) 2447;
J.L. Cortes, B. Pire, and J.P. Ralston, Z. Phys. C 55 (1992) 409.

[75] F. Baldracchini *et al.*, Fortsch. Phys. 30 (1981) 505;
X. Artru, Proceedings of the *Polarized Collider Workshop*, Eds. J.C. Collins, S.F. Heppelman and R.W. Robinett, University Park, PA (1990), AIP Conf. Proc. No. 223, (AI, New York, 1990), p. 176;
X. Artru and M. Mekhfi, Nucl. Phys. A 532 (1991) 351c.

[76] R.D. Tangerman and P.J. Mulders, Phys. Rev. D 51 (1995) 3357; NIKHEF preprint NIKHEF-94-P7, hep-ph/9408305.

[77] N. Hammon, O. Teryaev and A. Schäfer, Phys. Lett. B 390 (1997) 409.

[78] A. Brandenburg, O. Nachtmann, and E. Mirkes, Z. Phys. C 60 (1993) 697.

[79] J. Qiu and G. Sterman, Phys. Rev. Lett. 67 (1991) 2264; Nucl. Phys. B 378 (1992) 52.

[80] V.M. Korotkiyan and O.V. Teryaev, Dubna preprint E2-94-200; Phys. Rev. D 52 (1995) R4775.

[81] A.V. Efremov, V.M. Korotkiyan and O.V. Teryaev, Phys. Lett. B 384 (1995) 577.

[82] R. Meng, F.I. Olness, and D.E. Soper, Nucl. Phys. B 371 (1992) 79.

[83] A. Schäfer, L. Mankiewicz, P. Gornicki and S. Güllenstern, Phys. Rev. D 47 (1993) R1.

[84] B. Ehrnsperger, A. Schäfer, W. Greiner and L. Mankiewicz, Phys. Lett. B 321 (1994) 121.

[85] A.V. Efremov and O.V. Teryaev, Phys. Lett. B 150 (1985) 383.

[86] W. Lu, Phys. Rev. D 51 (1995) 5305.

[87] W. Lu, X. Li, H. Hu, Phys. Lett. B 368 (1996) 281.

[88] I.I. Balitsky and V.M. Braun, Nucl. Phys. B 361 (1991) 93.

[89] A.H. Mueller, Phys. Rev. D 18 (1978) 3705; Phys. Rep. 73 (1981) 238.

[90] J.C. Collins, Nucl. Phys. B 396 (1993) 161.

[91] R.N. Cahn, Phys. Lett. B 78 (1978) 269.

[92] E.L. Berger, Z. Phys. C 4 (1980) 289; Phys. Lett. B 89 (1980) 241.

[93] J. Levelt and P.J. Mulders, Phys. Rev. D 49 (1994) 96.

[94] A. de Rujula, J.M. Kaplan, and E. de Rafael, Nucl. Phys. B 35 (1971) 365;
K. Hagiwara, K. Hikasa and N. Kai, Phys. Rev. D 27 (1983) 84;
D. Atwood, G. Eilam and A. Soni, Phys. Rev. Lett. 71 (1993) 492.

[95] R.L. Jaffe and X. Ji, Phys. Rev. Lett. 71 (1993) 2547.

[96] X. Ji, Phys. Rev. D 49 (1994) 114.

[97] K. Chen, G.R. Goldstein, R.L. Jaffe and X. Ji, Nucl. Phys. B 445 (1995) 380.

[98] ALEPH Collaboration, D. Buskulic *et al.*, Phys. Lett. B 374 (1996) 319.

[99] OPAL Collaboration, K. Ackerstaff *et al.*, Eur. Phys. J. C 2 (1998) 49.

[100] R.D. Tangerman and P.J. Mulders, Phys. Lett. B 352 (1995) 129.

[101] SLD Collaboration, J. Schwiening, Proceedings of '97 QCD and High Energy Hadronic Interactions, Les Arcs, France (1997), p. 293 (hep-ph/9705454).

[102] P. Hoodbhoy, R.L. Jaffe and A. Manohar, Nucl. Phys. B 312 (1989) 571;
R.L. Jaffe and A. Manohar, Nucl. Phys. B 321 (1989) 343;
A. Anselm, M. Anselmino, F. Murgia and M.G. Ryskin, JETP Lett. 60 (1994) 496;
P. Ball, V.M. Braun, Y. Koike and K. Tanaka, hep-ph/9802299.

[103] DELPHI Collaboration, W. Bonivento *et al.*, Internal Note DELPHI-95-81 PHYS 516, Contribution eps0549 to the EPS-HEP 95 conference, Brussels (1995), unpublished.

[104] X. Artru and J.C. Collins, Z. Phys. C 69 (1996) 277.

[105] Particle Data Group, Phys. Rev. D 54 (1996) 1.

[106] A.M. Kotzinian and P.J. Mulders, Phys. Lett. B 406 (1997) 373.

[107] EMC Collaboration, M. Arneodo *et al.*, Z. Phys. C 34 (1987) 277;
EMC Collaboration, J. Ashman *et al.*, Z. Phys. C 52 (1991) 361;
E665 Collaboration, M.R. Adams *et al.*, Phys. Rev. D 48 (1993) 5057.

[108] R.L. Jaffe, X. Jin and J. Tang, Phys. Rev. Lett. 80 (1998) 1166; Phys. Rev. D 57 (1998) 5920.

[109] J.C. Collins, S.F. Heppelmann and G.A. Ladinsky, Nucl. Phys. B 420 (1994) 565.

[110] O. Nachtmann, Nucl. Phys. B 127 (1977) 314;
A.V. Efremov, Sov. J. Nucl. Phys. 28 (1978) 83;
A.V. Efremov, L. Mankiewicz and N. Törnqvist, Phys. Lett. B 284 (1992) 394.

[111] R. Jakob, D. Boer and P.J. Mulders, hep-ph/9805410.

[112] J.C. Collins and D.E. Soper, Nucl. Phys. B 193 (1981) 381.

[113] J.C. Collins, Nucl. Phys. B 394 (1993) 169.

[114] J. Qiu and G. Sterman, Proceedings of the *Polarized Collider Workshop*, Eds. J.C. Collins, S.F. Heppelman and R.W. Robinett, University Park, PA (1990), AIP Conf. Proc. No. 223, (AI, New York, 1990), p. 249.

[115] J.C. Collins, D.E. Soper, and G. Sterman, Nucl. Phys. B 261 (1985) 104.

[116] J.C. Collins, D.E. Soper, and G. Sterman, in *Perturbative Quantum Chromodynamics*, Ed. A.H. Mueller (World Scientific, Singapore, 1989), p. 1.

[117] J.C. Collins, D.E. Soper, and G. Sterman, Nucl. Phys. B 250 (1985) 199.

[118] J.C. Collins, Phys. Rev. D 21 (1980) 2962.

[119] J.C. Collins, in *Perturbative Quantum Chromodynamics*, Ed. A.H. Mueller (World Scientific, Singapore, 1989), p. 573.

[120] R. Meng, F.I. Olness, and D.E. Soper, Phys. Rev. D 54 (1996) 1919.

[121] M. Luo, J. Qiu and G. Sterman, Proceedings of the Meeting of the APS Division of Particles and Fields, Vancouver (1991), p. 633.

[122] R. Doria, J. Frenkel and J.C. Taylor, Nucl. Phys. B 168 (1980) 93;
C. Di'Lieto, S. Gendron, I.G. Halliday and C.T. Sachrajda, Nucl. Phys. B 183 (1981) 223;
R. Basu, A.J. Ramalho and G. Sterman, Nucl. Phys. B 244 (1984) 221.

[123] H. Plothow-Besch, Int. J. Mod. Phys. A 10 (1995) 2901, and references therein;
G.A. Ladinsky, preprint MSU-51120, hep-ph/9601287, and references therein.

[124] European Muon Collaboration, M. Arneodo *et al.*, Nucl. Phys. B 321 (1989) 541;
J. Binnewies, B.A. Kniehl and G. Kramer, Phys. Rev. D 52 (1995) 4947.

[125] G. Altarelli and G. Parisi, Nucl. Phys. B 126 (1977) 298.

[126] X. Ji and C. Chou, Phys. Rev. D 42 (1990) 3637;
E.V. Shuryak and A.I. Vainshtein, Nucl. Phys. B 201 (1982) 141;
P.G. Ratcliffe, Nucl. Phys. B 264 (1986) 493;
I.I. Balitsky and V.M. Braun, Nucl. Phys. B 311 (1989) 541;
A. Ali, V.M. Braun and G. Hiller, Phys. Lett. B 266 (1991) 117;
D. Müller, Phys. Lett. B 407 (1997) 314.

[127] A.V. Belitsky, Lectures given at the XXXI PNPI Winter School on Nuclear and Particle Physics, St. Petersburg, Repino, February, 1997, hep-ph/9703432.

[128] Y. Koike and K. Tanaka, Phys. Rev. D 51 (1995) 6125; Prog. Theor. Phys. Suppl. 120 (1995) 247.

[129] I.I. Balitskii, V.M. Braun, Y. Koike and K. Tanaka, Phys. Rev. Lett. 77 (1996) 3078.

[130] A.V. Belitsky and D. Müller, Nucl. Phys. B 503 (1997) 279.

[131] M. Stratmann and W. Vogelsang, Nucl. Phys. B 496 (1997) 41.

[132] H. Georgi, Phys. Rev. Lett. 42 (1979) 294;
H. Georgi and J. Sheiman, Phys. Rev. D 20 (1979) 111.

[133] A. König, Z. Phys. C 18 (1983) 63;
A. König and P. Kroll, Z. Phys. C 16 (1982) 89.

[134] G.L. Kane, J. Pumplin and W. Repko, Phys. Rev. Lett. 41 (1978) 1689;
A.V. Efremov and O.V. Teryaev, Sov. J. Nucl. Phys. 36 (1982) 140.

[135] FNAL E704 Collab., D.L. Adams *et al.*, Phys. Lett. B 261 (1991) 201; Phys. Lett. B 264 (1991) 462; Phys. Lett. B 276 (1992) 531; Z. Phys. C 56 (1992) 181.

[136] C. Boros, Z. Liang and R. Rittel, J. Phys. G 24 (1998) 75.

[137] Yu.L. Dokshitzer, D.I. Dyakonov and S.I. Troyan, Phys. Rep. 58 (1980) 269; G. Curci, W. Furmanski and R. Petronzio, Nucl. Phys. B 175 (1980) 27.

[138] G. Leibbrandt, Phys. Rev. D 29 (1984) 1699.

[139] A. Bassetto, G. Nardelli and R. Soldati, *Yang-Mills Theories in Algebraic Non-Covariant Gauges* (World Scientific, 1991).

[140] S. Mandelstam, Nucl. Phys. B 213 (1983) 149.

[141] A. Bassetto, Phys. Rev. D 46 (1992) 3676.

[142] C. Acerbi and A. Bassetto, Phys. Rev. D 49 (1994) 1067.

[143] J.A.M. Vermaseren, *Symbolic Manipulation with FORM, version 2* (CAN, Amsterdam, 1991).

Samenvatting

Azimutale asymmetrieën in harde verstrooiingsprocessen

Quantumchromodynamica (QCD) is de theorie van de sterke interacties. Het is de theorie over quarks en gluonen, die de gebonden toestanden vormen die we hadronen noemen, zoals bijvoorbeeld protonen, neutronen en pionen. Bij lage energieën bestaan alleen gebonden toestanden, een eigenschap die we opsluiting of kluistering (*confinement*) noemen. De interactie tussen quarks ontstaat door uitwisseling van gluonen en de resulterende kracht neemt af in sterkte met toenemende energie. Alleen in de limiet van oneindig hoge energie gaat de koppeling van de gluonen aan de quarks naar nul en de quarks worden dan vrije deeltjes, vandaar de naam asymptotische vrijheid. Als de energie daarentegen afneemt, dan blijft de koppeling groeien, zodat alleen boven een bepaalde energieschaal de interactie als een storing op de theorie van vrije deeltjes beschouwd kan worden; een machtreeksontwikkeling in de koppelingsconstante is dan geoorloofd om de effecten van de storing te benaderen. Bij lagere energieën, waar het massaspectrum van de hadronen zich bevindt, zit men echter buiten het toepassingsgebied van storingsrekening.

Hadronen zijn intrinsiek niet-storingstheoretische objecten en dat vormt tot op heden het voornaamste obstakel om een hadron volledig te beschrijven met QCD. Een mogelijkheid om meer inzicht in de structuur van hadronen te krijgen is door middel van het doen van hoog-energetische verstrooiingsexperimenten. Een voorbeeld hiervan is het proces genaamd diep-inelastische verstrooiing of inclusieve lepton-hadron verstrooiing, waarbij 'inclusief' betekent dat de geproduceerde deeltjes niet geobserveerd worden uitgezonderd het verstrooide lepton. Zo'n hoog-energetisch verstrooiingsexperiment is in ieder geval voor een deel storingstheoretisch te beschrijven en de resterende onwetendheid kan in kaart worden gebracht met behulp van een beperkte set objecten, die als parameters fungeren. Deze parameters kunnen gemeten worden en de voorspellende waarde ligt in het feit dat er meer observabelen zijn dan parameters.

In dit proefschrift werden deze parameters, de zogenaamde distributie- en fragmentatiefuncties, onderzocht. Aangezien ze kennis van de structuur van hadronen vertegenwoordigen, bezitten ze, naast voorspellende waarde, ook een intrinsieke, universele betekenis. Verder bestudeerden we verscheidene observabelen, zoals werkzame doorsneden en asymmetrieën, in termen van deze functies.

In meer detail, we onderzochten voornamelijk de spinstructuur van hadronen door middel van harde verstrooiingsprocessen. De aanwezige, harde energieschaal Q leidt tot de scheiding van de beschrijving in berekenbare "harde" stukken en de bovengenoemde "zachte" parameters. De harde stukken kunnen erg nauwkeurig worden benaderd door het uitrekenen van storingscorrecties. Er is ook nog een ander type correctie aan de leidende term in de beschrijving van de observabelen, namelijk de zogenaamde machtscorrecties. De storingscorrecties gedragen zich als logaritmische functies van de harde energieschaal, terwijl de machtscorrecties als negatieve machten van de harde schaal gaan. De machtscorrecties vormden voornamelijk het onderwerp van deze studie, maar ook de leidende term zelf bevat al heel interessante structuren, die ook uitgebreid aan de orde zijn gekomen.

In hoofdstuk 2 hebben we de bovengenoemde scheiding onderzocht voor die machtscorrecties in het diep-inelastische verstrooiingsproces (DIS). Deze analyse was voornamelijk gebaseerd op werk van Ellis, Furmanski en Petronzio [4] en Qiu [36]. De hard-zacht scheiding kan apart worden bewerkstelligd voor iedere twist, i.e., de orde t van de machtscorrectie, waarbij de werkzame doorsnede zich gedraagt als Q^{2-t}. De resulterende harde stukken hebben de gunstige eigenschap dat ze electromagnetisch ijkinvariant zijn en de zachte stukken worden beschreven in termen van hadronische matrixelementen van niet-lokale operatoren. De relatie van deze laatste operatoren met de lokale operatoren uit de operatorprodukt-ontwikkeling werd uit de doeken gedaan. De subleidende, d.w.z. de gepolariseerde twist-drie (orde $1/Q$) en ongepolariseerde twist-vier (orde $1/Q^2$), dynamische machtscorrecties zijn uitgebreid bestudeerd, met in achtneming van quarkmassa's. Een van de voornaamste conclusies was dat de scheidings- of factorisatieprocedure, zoals voorgesteld door Qiu, ook toegepast kan worden in het geval dat quarkmassa's in de berekening worden meegenomen.

De zachte stukken worden beschreven door correlatiefuncties, die Fouriergetransformeerden zijn van de bovengenoemde hadronische matrixelementen van niet-lokale operatoren. In hoofdstuk 3 hebben we hun eigenschappen bestudeerd. De symmetrieën van QCD beperken de parametrisatie in termen van onafhankelijke functies, de bovengenoemde parameters, in dit geval de distributiefuncties. Zoals gezegd vertegenwoordigen deze distributiefuncties informatie over de structuur en eigenschappen van hadronen, daardoor zijn er vaak ook restricties op deze functies, bijvoorbeeld in de vorm van somregels (integraalrelaties). Afgezien van bekende somregels, zoals de Burkhardt-Cottingham en de Efremov-Leader-Teryaev somregels, hebben we ook nieuwe somregels afgeleid. We bespraken onderbouwingen en aannames, zoals over het verwisselen van integraties, het kiezen van randvoorwaarden, het verwaarlozen van hogere orde correcties, enzovoort. Dit onderzoeken is vooral van belang om te kunnen begrijpen wat het betekent als een bepaalde somregel geschonden blijkt te zijn. De gevolgen van tijdomkeersymmetrie zijn ook uitgebreid besproken vanwege de prominente rol in latere hoofdstukken.

Met behulp van de volledige set distributiefuncties van leidende en subleidende twist, kan men uitdrukkingen voor werkzame doorsneden en asymmetrieën vinden in termen van deze functies. Vervolgens kan men afleiden hoe ze te meten en van elkaar te scheiden zijn. Universaliteit van deze functies betekent dat ze in het ene proces gemeten kunnen worden en voorspellingen kunnen genereren voor een ander. Het Drell-Yan (DY) proces, of ook wel leptonpaarproduktie in hadron-hadron verstrooiing genaamd, wordt bijvoorbeeld beschreven in termen van twee afzonderlijke zachte stukken, waarvan de ene identiek is aan die van DIS en de ander is er aan gerelateerd (de antiquark correlatiefunctie).

Een andere reden om processen met twee zachte stukken te onderzoeken is dat in dat geval de transversale impuls van de deeltjes in een hadron een rol speelt. Daarom hebben we distributiefuncties niet alleen bestudeerd als functie van de lichtkegel-impulsfractie x van een quark of gluon ten opzichte van het hadron, maar ook als functie van de intrinsieke transversale impuls van de quarks en gluonen in dat hadron.

Een van de belangrijkste conclusies volgend uit onze onderzoekingen met betrekking tot intrinsieke transversale impuls is dat het niet toegestaan is een (collineaire) ontwikkeling te maken van de zachte stukken in deze transversale impuls in verhouding tot de harde schaal. Bijgevolg zijn de azimutale asymmetrieën die ontstaan ten gevolge van intrinsieke transversale impuls niet noodzakelijk onderdrukt als inverse machten van de harde schaal.

In hoofdstuk 4 bestudeerden we het DY proces tot op subleidende twist geïntegreerd over de transversale impuls van het leptonpaar, zodat veel structuren wegvallen, maar er blijven desalniettemin een aantal interessante azimutale spin asymmetrieën over. In navolging van Hammon *et al.* [77] bediscussieerden we enkel-spin asymmetrieën (d.w.z. het verschil in werkzame doorsnede van een proces met een spinvector in een bepaalde en de daaraan tegengestelde richting), die mogelijk ontstaan door zogenaamde gluonische polen in de twist-drie hadronische matrixelementen, d.w.z. singulier gedrag van een matrixelement als een gluon-impuls naar nul gaat. We hebben aangetoond dat deze asymmetrieën niet onderscheiden kunnen worden van die tengevolge van tijdomkeer (T) oneven distributiefuncties. Deze laatste functies zijn normaal gesproken afwezig tengevolge van tijdomkeersymmetrie. De gluonische polen leiden samen met imaginaire delen van harde subprocessen tot *effectieve* T-oneven distributiefuncties. Opmerkelijk is dat tijdomkeersymmetrie deze laatste functies wel toestaat. We observeerden tevens dat gluonische polen lange afstand gluonvelden met antisymmetrische randvoorwaarden vereisen. Onze analyse toonde bovendien de niet-verwaarloosbare rol van de intrinsieke transversale impuls voor de werkzame doorsnede in het DY proces op subleidende twist. Dit gold eveneens voor de (normale) T-even asymmetrieën. De rol van transversale impuls kwam nog duidelijker naar voren in de twee volgende hoofdstukken waarin we inclusieve twee-hadron produktie in elektron-positron annihilatie beschouwden.

We hebben dit proces beschouwd in eerste instantie bij energieën ver beneden de Z-boson produktiedrempel –hoofdstuk 5– en in het daaropvolgende hoofdstuk bij energieën rond de Z-massawaarde. In beide hoofdstukken ging het om azimutale asymmetrieën die ontstaan als spin asymmetrieën en/of als manifestaties van intrinsieke transversale impuls van de quarks. In hoofdstuk 5 hebben we het complete resultaat voor leidende en subleidende twist bestudeerd en onderzochten we verscheidene soorten observabelen, zoals bijvoorbeeld gewogen werkzame doorsneden. De subleidende asymmetrieën zijn naar verwachting verwaarloosbaar rond de Z-massawaarde, dus beperkten we ons in hoofdstuk 6 tot de leidende twist. We hebben resultaten uitgebreid van een eerdere theoretische studie [97] van dit proces, waar geen azimutale asymmetrieën tengevolge van transversale impulsen waren beschouwd.

Om fragmentatie van een quark in een hadron plus andere, niet-waargenomen deeltjes te beschrijven, zijn fragmentatiefuncties nodig, die erg lijken op de distributiefuncties behalve wat betreft het T-oneven deel. Terwijl T-oneven distributiefuncties alleen zouden kunnen ontstaan door subtiele mechanismen (zoals gluonische polen of factorisatiebreking), kunnen T-oneven fragmentatiefuncties ongelijk aan nul zijn eenvoudigweg door de altijd aanwezige eindtoestandsinteracties.

De T-oneven fragmentatiefuncties traden op in een interessante asymmetrie, die het hoofdonderwerp van hoofdstuk 6 vormde. Het is een leidende $\cos(2\phi)$ asymmetrie, die geen polarisatiemetingen aan inkomende of uitgaande deeltjes vereist. Experimentele bevestiging van deze asymmetrie zou tegelijkertijd het zogenaamde Collins' effect aantonen. Dat is het effect dat transversaal gepolariseerde quarks in ongepolariseerde hadronen kunnen fragmenteren, waarbij het aantal hadronen afhangt van de richting van de transversale impuls ten opzichte van de polarisatie. Een experimentele (voor-)studie [103] van de bovengenoemde asymmetrie heeft het bestaan vooralsnog niet aangetoond en slechts een bovengrens opgeleverd. In hoofdstuk 6 hebben een grove schatting gemaakt van de grootte van de asymmetrie, die in het meest optimistische geval in de buurt van die bovengrens komt. Een verbetering van de statistiek lijkt dus noodzakelijk en is bovendien mogelijk door het opnemen van later verkregen data.

In dit proefschrift hebben we ons gericht op de leidende en subleidende machten in $1/Q$ en beperkten we ons tot laagste orde in de sterke koppelingsconstante. In hoofdstuk 7 bediscussieerden we kwalitatief de effecten van het opnemen van storingstheoretische correcties en daaruit concludeerden we dat het niet te verwachten is dat zulke correcties de observabelen die wij bespraken en die van laagste orde in de koppelingsconstante zijn, altijd zullen overschaduwen (althans niet voor middelgrote waarden van Q^2, kleine transversale impuls van het foton of Z-boson en grote waarden van x). Sterker nog, bepaalde experimentele resultaten wijzen op het tegendeel. We bediscussieerden enkele meer technische aspecten, zoals factorisatie, evolutie en kleur-ijkinvariantie, die deels nog moeten worden onderzocht of uitgebreid. Een hoofdconclusie was dat de kleur-ijkinvariante definities van transversale-impuls-afhankelijke distributie- en fragmentatiefuncties, zogenaamde padgeordende exponenten bevatten, met rechte paden langs een lichtachtige richting, die lopen via lichtkegel-oneindig waar het transversale deel van het pad zich bevindt.

Tenslotte concluderen we dat de resultaten van dit proefschrift onder andere tonen dat de spinstructuur van hadronen nog relatief onontgonnen en onbekend terrein vormen en dat de studie van azimutale asymmetrieën in harde verstrooiingsprocessen één van de mogelijkheden is om interessante en belangrijke kennis van de hadronstructuur op te doen.

Dankwoord

Natuurlijk was dit proefschrift niet tot stand gekomen zonder de hulp van vele anderen en die wil ik nu graag bedanken. Allereerst mijn promotor Piet Mulders: Beste Piet, ik ben zeer dankbaar voor de vrijheid en de kansen die je me geboden hebt. Je nooit aflatende enthousiasme en werklust zal altijd een inspiratie voor me blijven. Ik ben ook erg blij dat je me in contact bracht met een aantal wetenschappers in het buitenland; de beste stimulans voor een beginnend wetenschapper is immers dat hij zich serieus genomen voelt.

Een paar jaar lang heb ik nauw samengewerkt met Rainer Jakob: Dear Rainer, thank you for the very pleasant and fruitful collaboration. Without you this thesis would not look the same. I think we were nicely complementary and I strongly hope you will stay in physics.

Verder wil ik de andere twee mensen waarmee ik artikelen heb geschreven bedanken, Rik Tangerman en Oleg Teryaev. Beste Rik, door jou kwam voor mij de boel eigenlijk aan het rollen en ik heb veel geleerd van je kritische houding. Dear Oleg, it was a pleasure to work with you. Thank you for the many conversations and emails; you are a reservoir of knowledge and ideas. We met thanks to Rik and it resulted in an interesting piece of physics (and a pleasant week in Dubna).

Dan dank ik natuurlijk mijn (ex-)kamergenoten, Léonard, João, Kasper (de reddende engel op computergebied) en Peter, en de andere mede-Nikheffers en VU-mensen, die voor een prettige sfeer hebben gezorgd, die nodig is om constructief bezig te kunnen zijn. Daarbij wil ik Daniël van Dierendonck apart bedanken voor zijn aanmoedigingen wat betreft de $\cos(2\phi)$ asymmetrie.

Furthermore, I would like to thank 'my' readingcommittee, consisting of Mauro Anselmino, Ben Bakker, Bernard de Wit, Jos Engelen en nogmaals Oleg. Dan de mensen die een aanbeveling voor me schreven: Justus Koch en nogmaals Bernard, Oleg en Piet. Mijn vervolgbaan heb ik voor een belangrijk deel aan jullie te danken. Eric Laenen dank ik voor al zijn VS-hulp en Jos Vermaseren in het algemeen voor zijn prachtige programma FORM [143].

I also want to thank the following *selection* of people for valuable discussions: J.C. Collins, R.L. Jaffe, X. Ji, L. Mankiewicz, W. van Neerven, J. Smith (especially for the QCD lectures given in Utrecht a long time ago) and G. Sterman.

Tot slot, dank ik natuurlijk mijn vrienden en familie. In het bijzonder dank ik mijn ouders voor hun onvoorwaardelijke steun.