Hadron Structure Theory III

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The plan:

- Lecture I:

  Transverse spin structure of the nucleon

- Lecture II

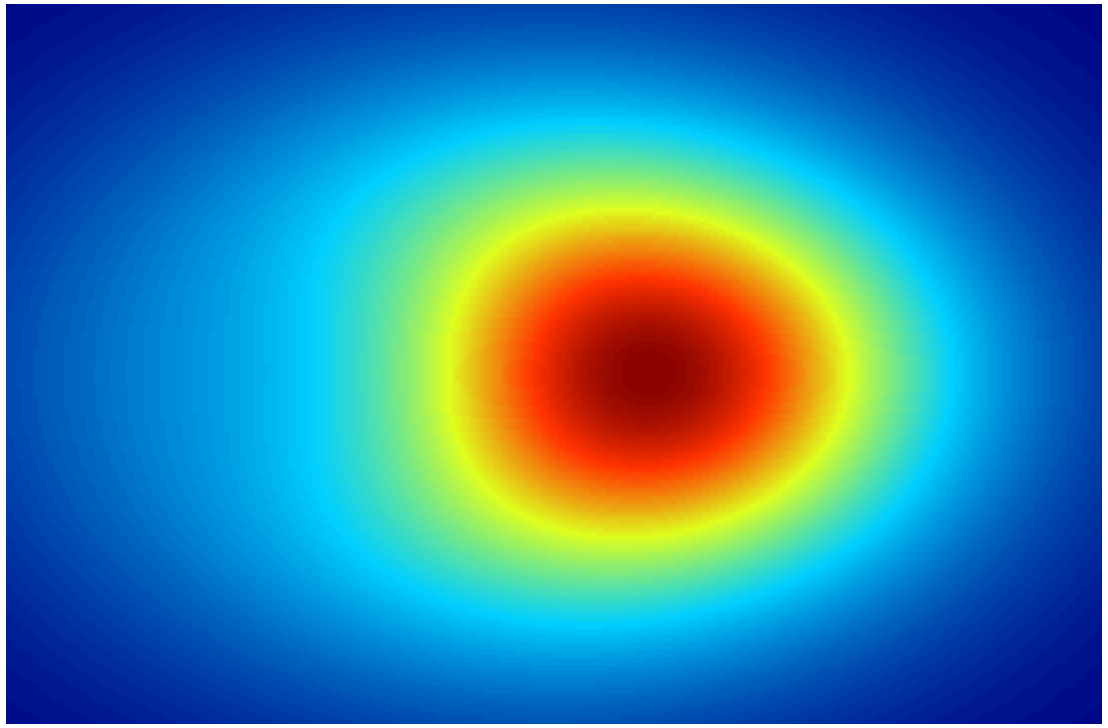
  Transverse Momentum Dependent distributions (TMDs)
  Semi Inclusive Deep Inelastic Scattering (SIDIS)

- Tutorial

  Calculations of SIDIS structure functions using Mathematica

- Lecture III

  Advanced topics. Evolution of TMDs



The polarized proton in momentum space as "seen" by the virtual photon

Factorization theorems help us to relate functions that describe the hadron structure and the experimental observables

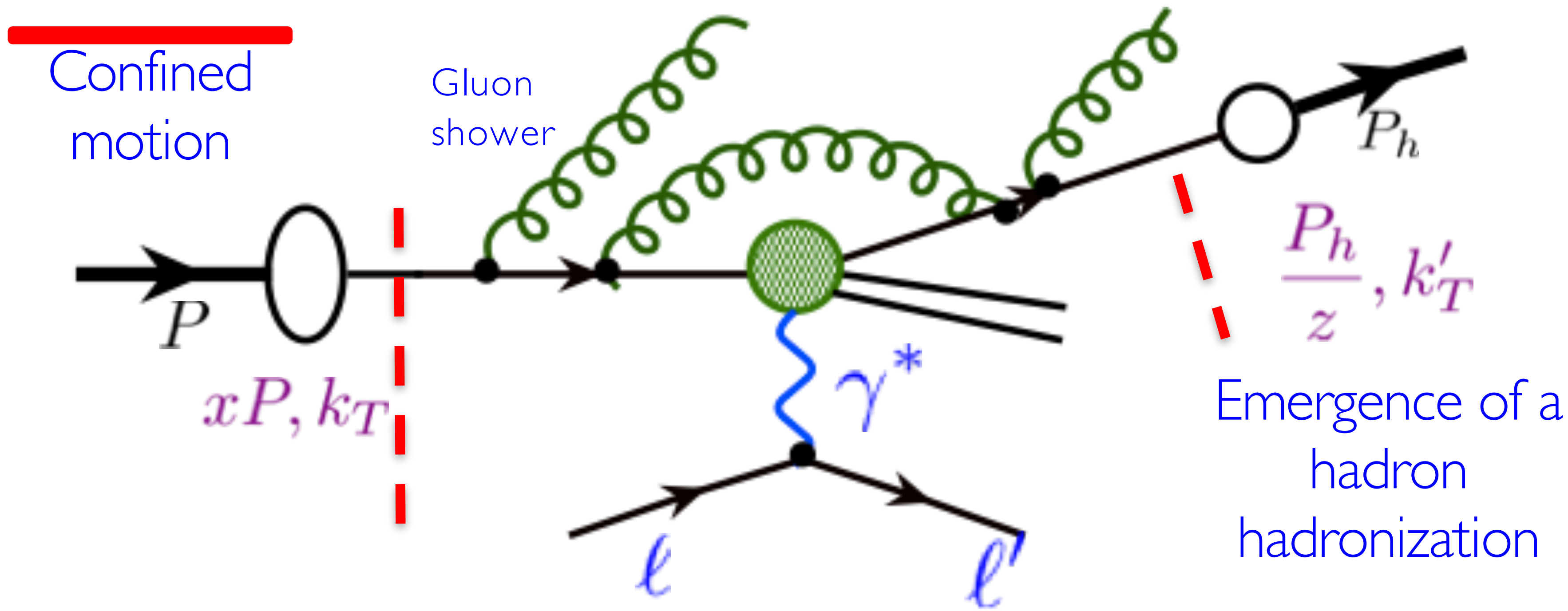
Factorization is a ***controllable approximation*** and the goal of theorists and phenomenologists is to test and improve the region of applicability of factorization and/or construct new factorization theorems

Hadron structure is the ultimate goal of measurements and phenomenology

Why TMDs, factorization, and evolution

Why QCD evolution is interesting?

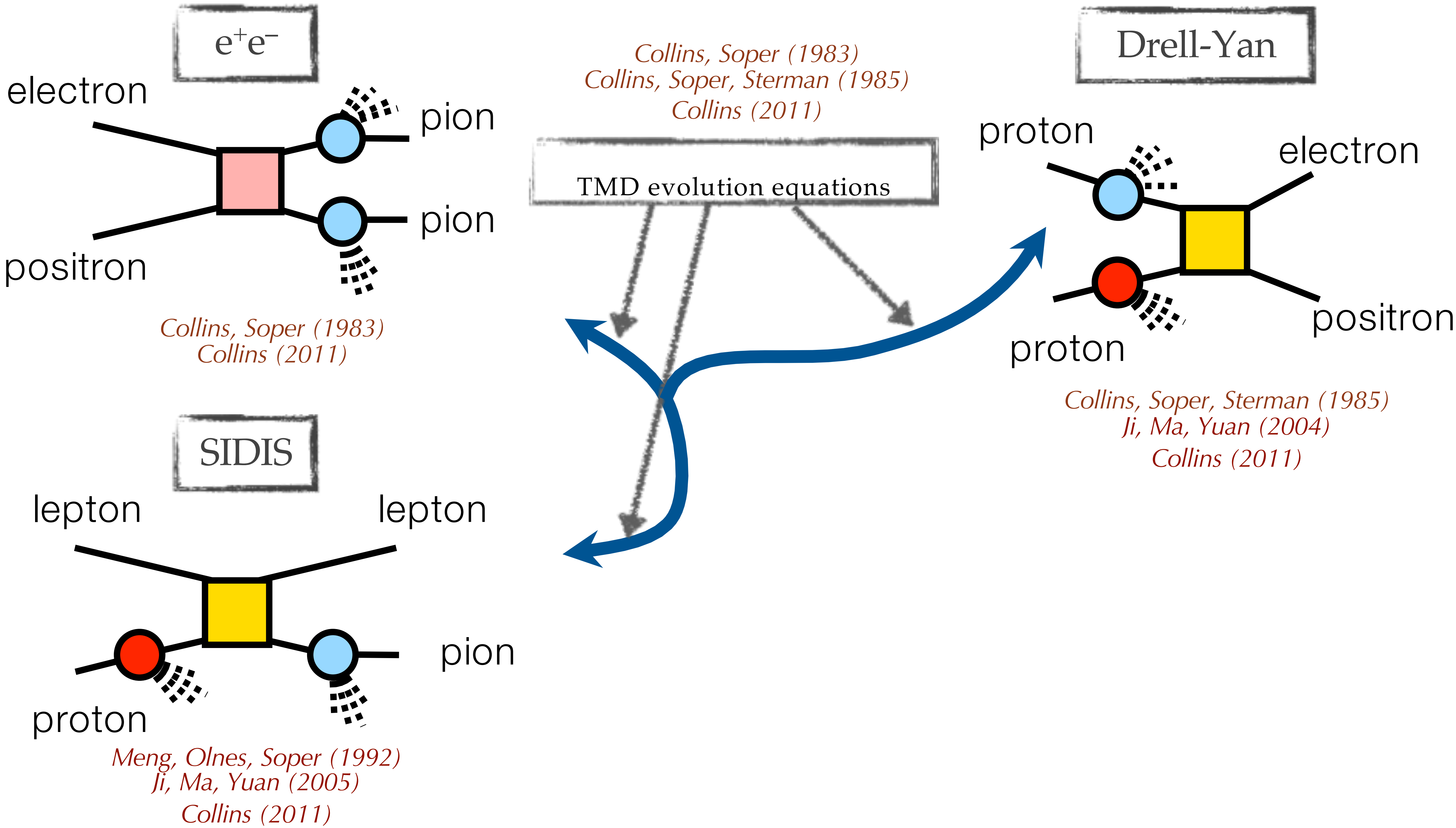
Study of evolution gives us insight on different aspects and origin of confined motion of partons, gluon radiation, parton fragmentation



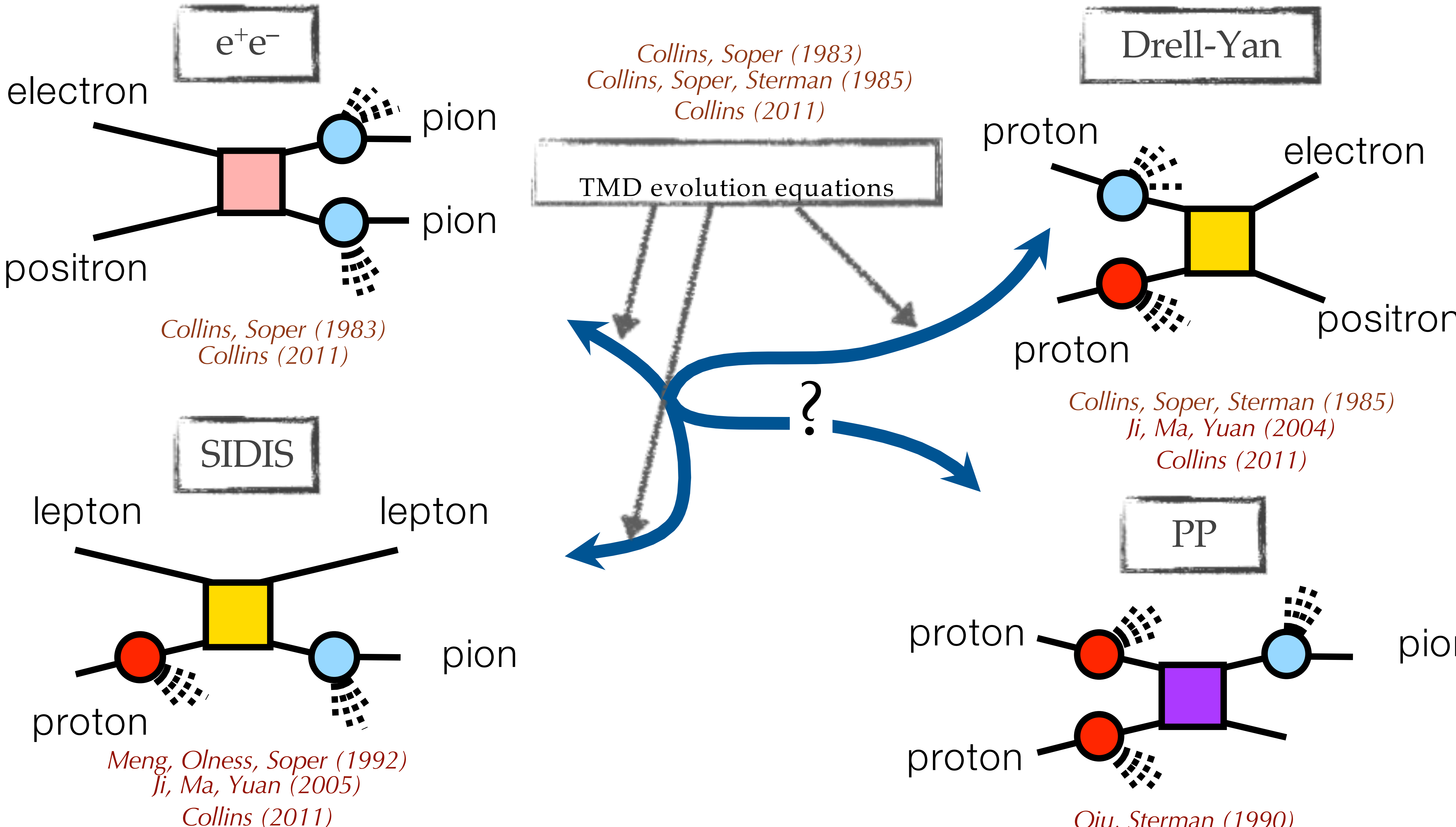
Evolution allows to connect measurements at very different scales.

TMD evolution has also a universal non-perturbative part. The result of evolution cannot be uniquely predicted using evolution equations until the non-perturbative part is reliably extracted from the data.

TMD factorization

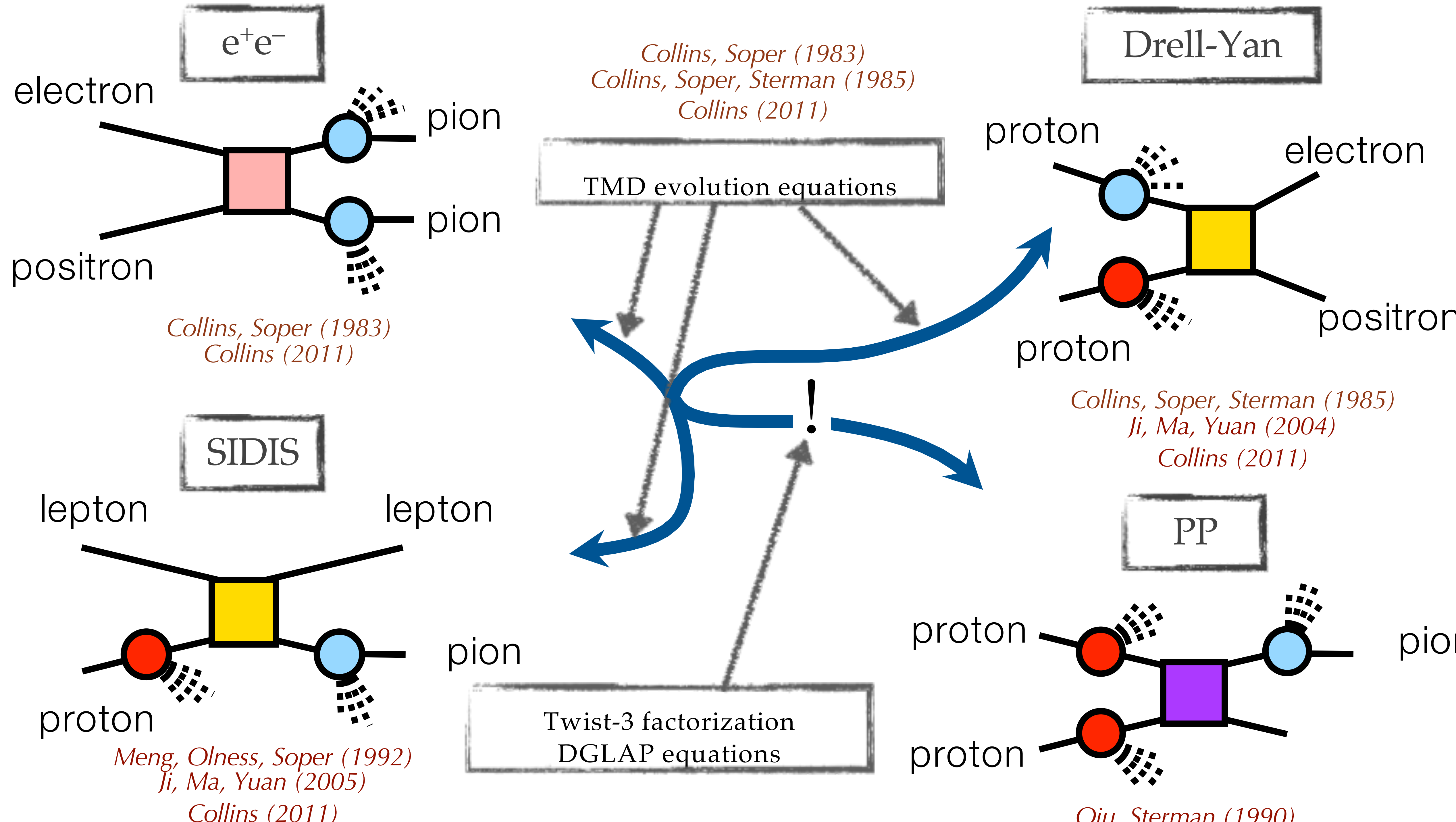


TMD factorization



Only one scale is measured in PP
TMD factorization is not applicable?

TMD factorization



- Twist-3 functions are related to TMD via OPE
- TMD and twist-3 factorizations are related in high QT region
- Global analysis of TMDs and twist-3 is possible:

All four processes can be used.

- Data are from HERMES, COMPASS, JLab,

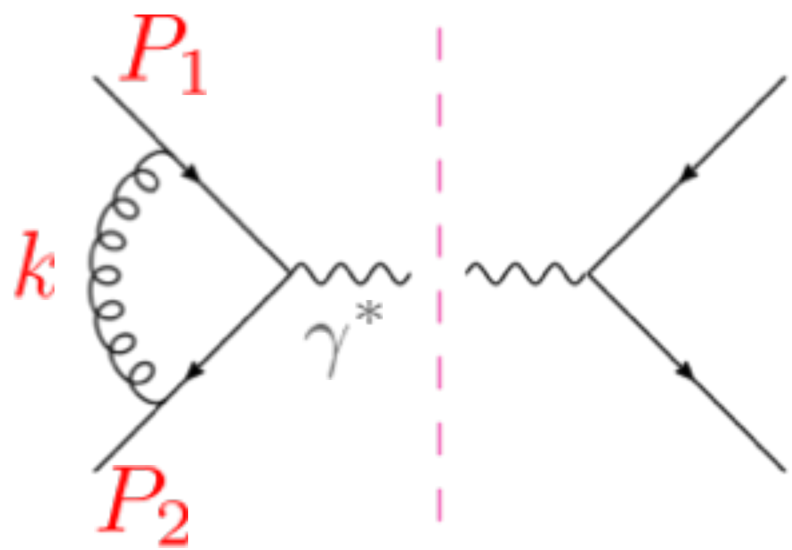
BaBar, Belle, RHIC, LHC, Fermilab

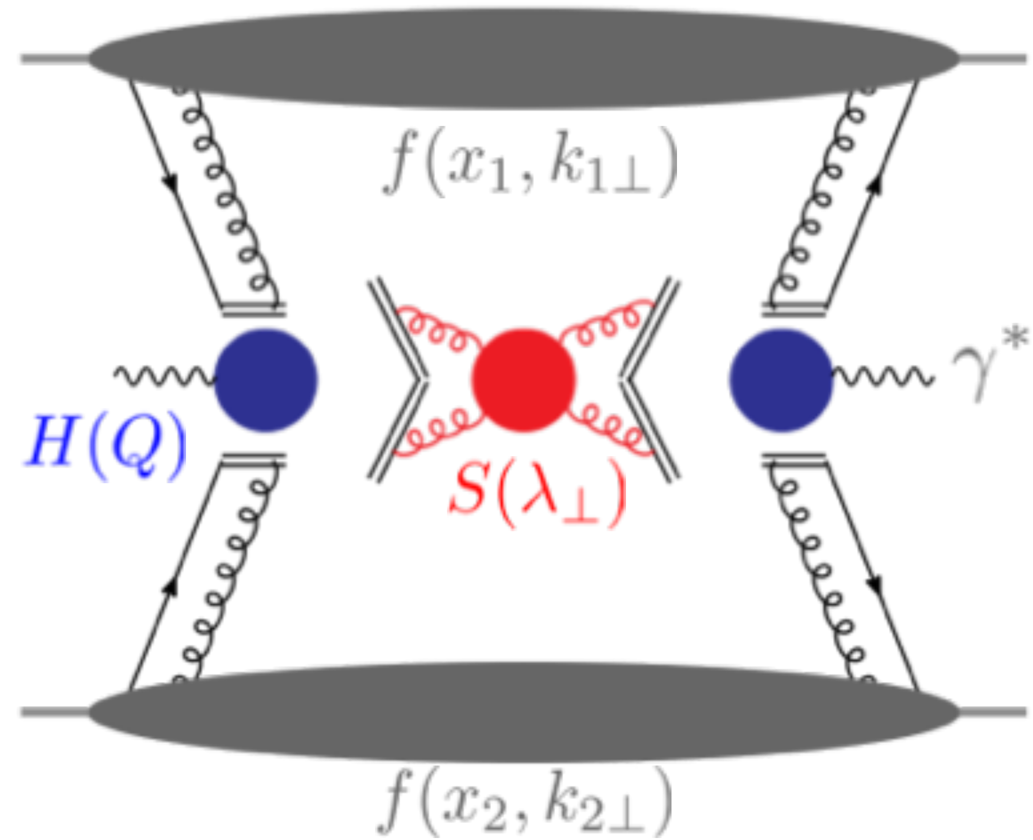
Global fit is needed.
Work in progress

Why TMD Evolution?

TMD factorization in a nut-shell

Drell-Yan: $p + p \to [\gamma^* \to \ell^+ \ell^-] + X$





Factorization of regions:
(1) k//P1, (2) k//P2, (3) k soft, (4) k hard

Factorized form and mimicking "parton model"

$$\frac{d\sigma}{dQ^2 dy d^2 q_\perp} \propto \int d^2k_{1\perp}\, d^2k_{2\perp}\, d^2\lambda_\perp H(Q) f(x_1, k_{1\perp}) f(x_2, k_{2\perp}) S(\lambda_\perp) \delta^2(k_{1\perp} + k_{2\perp} + \lambda_\perp - q_\perp)$$

$$= \int \frac{d^2b}{(2\pi)^2} e^{iq_\perp \cdot b} H(Q) f(x_1, b) f(x_2, b) S(b)$$

$$F(x, b) = f(x, b)\sqrt{S(b)}$$

$$\boxed{= \int \frac{d^2b}{(2\pi)^2} e^{iq_\perp \cdot b} H(Q) F(x_1, b) F(x_2, b)}$$

mimic "parton model"

slide courtesy of Z. Kang

TMDs evolve

Just like collinear PDFs, TMDs also depend on the scale of the probe = evolution

Collinear PDFs

$$F(x, Q)$$

- ✓ DGLAP evolution
- ✓ Resum $\left[\alpha_s \ln(Q^2/\mu^2)\right]^n$
- ✓ Kernel: purely perturbative

TMDs

$$F(x, k_\perp; Q)$$

- ✓ Collins-Soper/rapidity evolution equation
- ✓ Resum $\left[\alpha_s \ln^2(Q^2/k_\perp^2)\right]^n$
- ✓ Kernel: can be non-perturbative when $k_\perp \sim \Lambda_{\rm QCD}$

$$F(x, Q_i) \;\Downarrow\; R^{\rm coll}(x, Q_i, Q_f) \;\Downarrow\; F(x, Q_f)$$

$$F(x, k_\perp, Q_i) \;\Downarrow\; R^{\rm TMD}(x, k_\perp, Q_i, Q_f) \;\Downarrow\; F(x, k_\perp, Q_f)$$

TMD evolution and non-perturbative component

Fourier transform back to the momentum space, one needs the whole b region (large b): need some non-perturbative extrapolation

Many different methods/proposals to model this non-perturbative part

$$F(x,k_\perp;Q)=\frac{1}{(2\pi)^2}\int d^2b e^{ik_\perp\cdot b}F(x,b;Q)=\frac{1}{2\pi}\int_0^\infty db\, bJ_0(k_\perp b)F(x,b;Q)$$

Collins, Soper, Sterman 85, ResBos, Qiu, Zhang 99, Echevarria, Idilbi, Kang, Vitev, 14, Aidala, Field, Gamberg, Rogers, 14, Sun, Yuan 14, D'Alesio, Echevarria, Melis, Scimemi, 14, Rogers, Collins, 15, Vladimirov, Scimemi 17…

Eventually evolved TMDs in b-space

$$F(x,b;Q)\approx \boxed{C\otimes F(x,c/b^*)}\times \boxed{\exp\left\{-\int_{c/b^*}^{Q}\frac{d\mu}{\mu}\left(A\ln\frac{Q^2}{\mu^2}+B\right)\right\}}\times \boxed{\exp\left(-S_{\text{non-pert}}(b,Q)\right)}$$

longitudinal/collinear part

transverse part

- ✓ Non-perturbative: fitted from data
- ✓ The key ingredient – ln(Q) piece is spin-independent

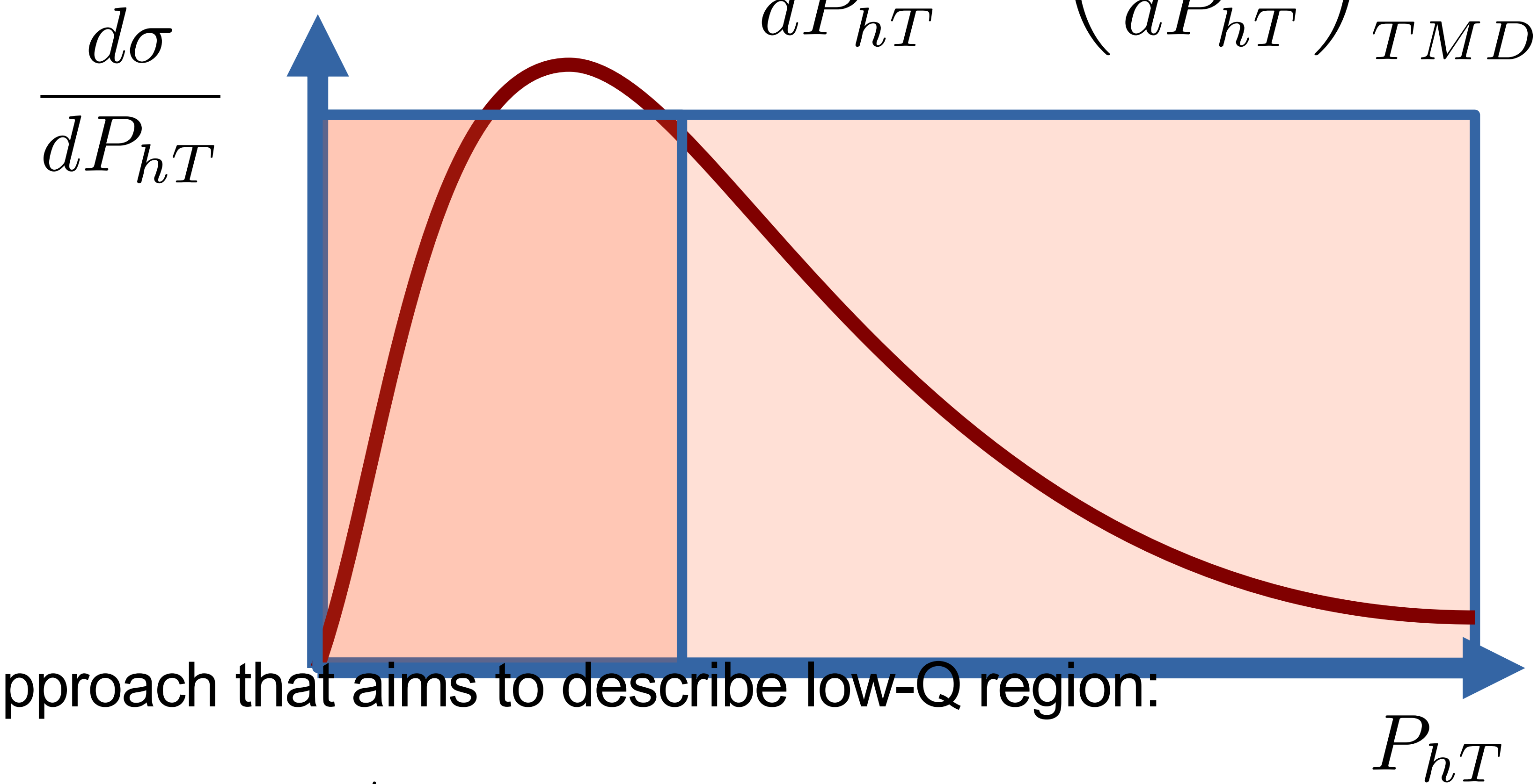
Since the polarized scattering data is still limited kinematics, we can use unpolarized data to constrain/extract key ingredients for the non-perturbative part

TMD

TMD factorization has a validity region $P_{hT}/z \ll Q$
(two scale problem)

In order to describe cross section in a wide region of transverse momentum one needs to add a Y term

$$\frac{d\sigma}{dP_{hT}} = \left(\frac{d\sigma}{dP_{hT}}\right)_{TMD} + Y$$



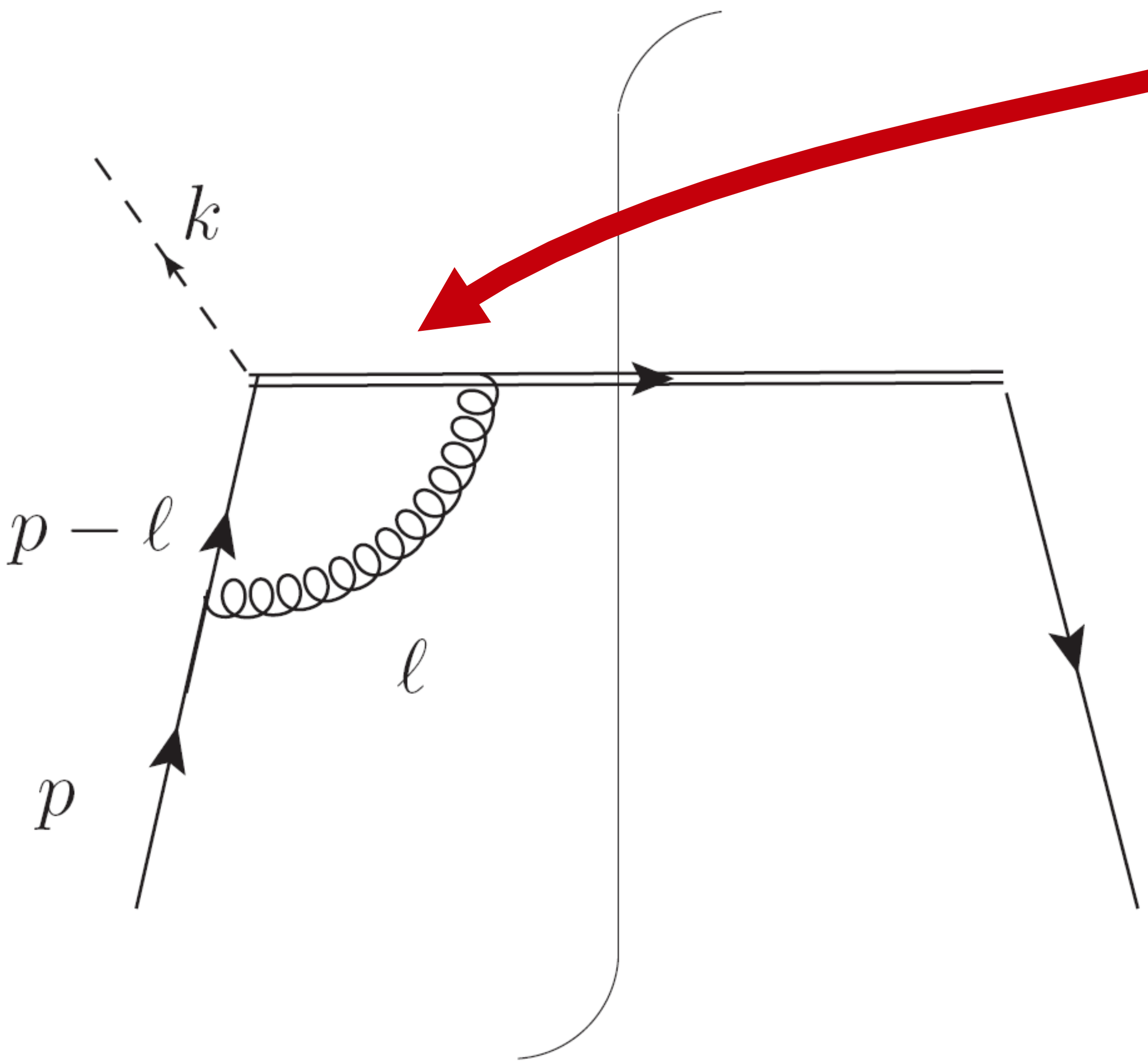
Improved approach that aims to describe low-Q region:

$$P_{hT}/z \ll Q$$

Collins, Gamberg, AP, Rogers, Sato, Wang   arXiv:1605.00671

It seems too easy...

In fact it is not easy…

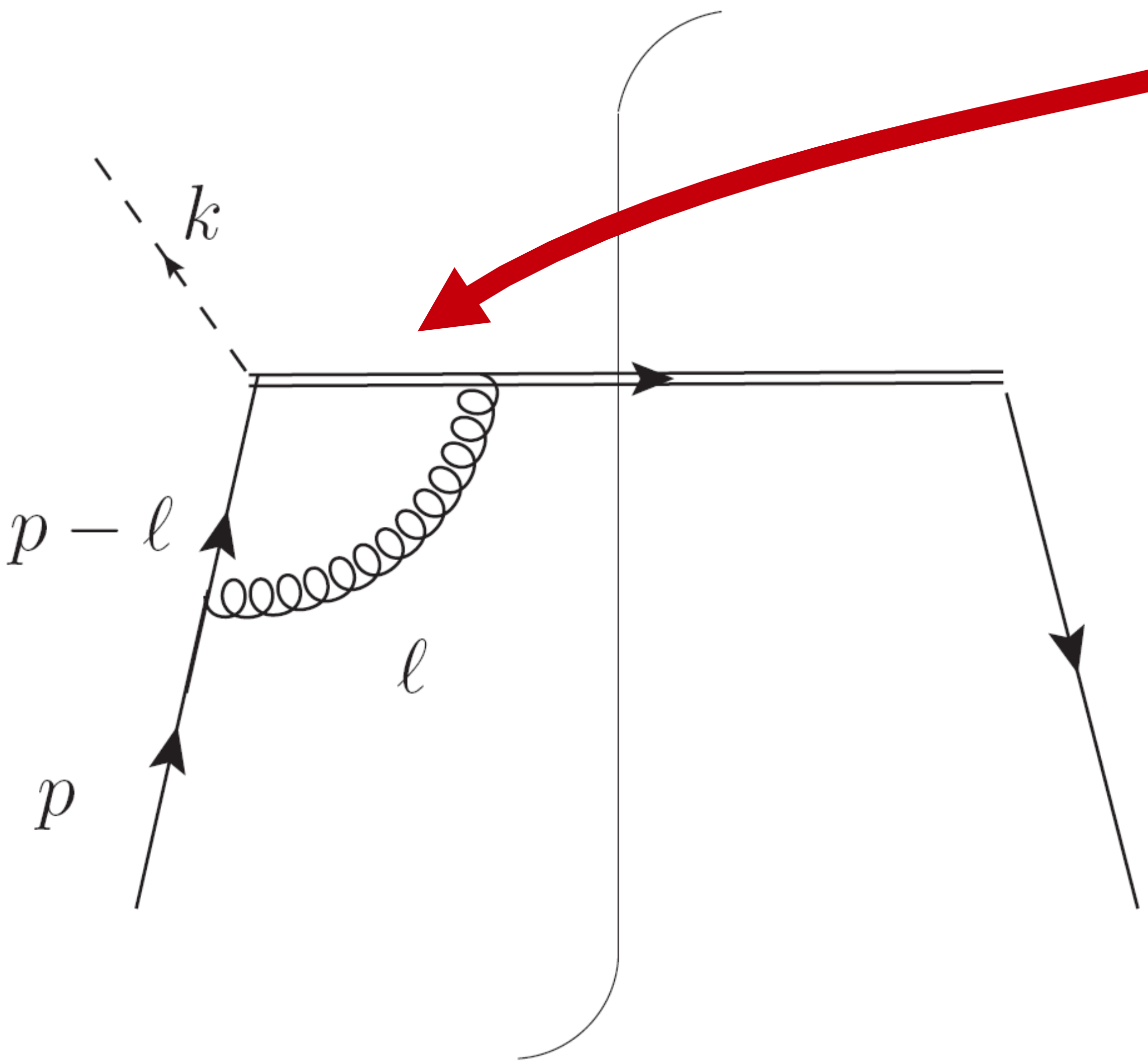


If we consider NLO corrections this diagram diverges

$$\ell = (1-\alpha)p$$

$$\propto \int_0^1 d\alpha \frac{\alpha}{1-\alpha}$$

In fact it is not easy…



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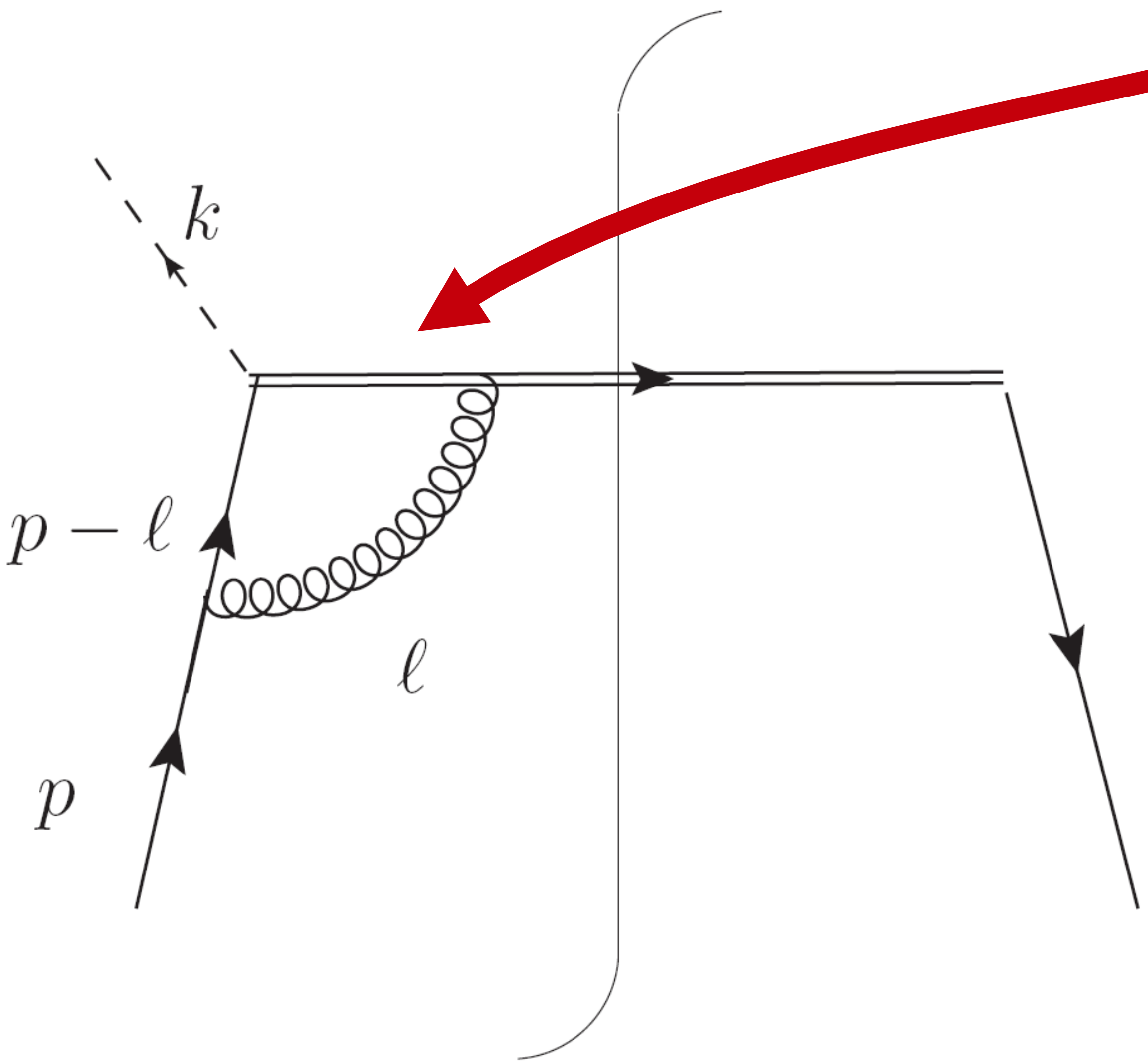

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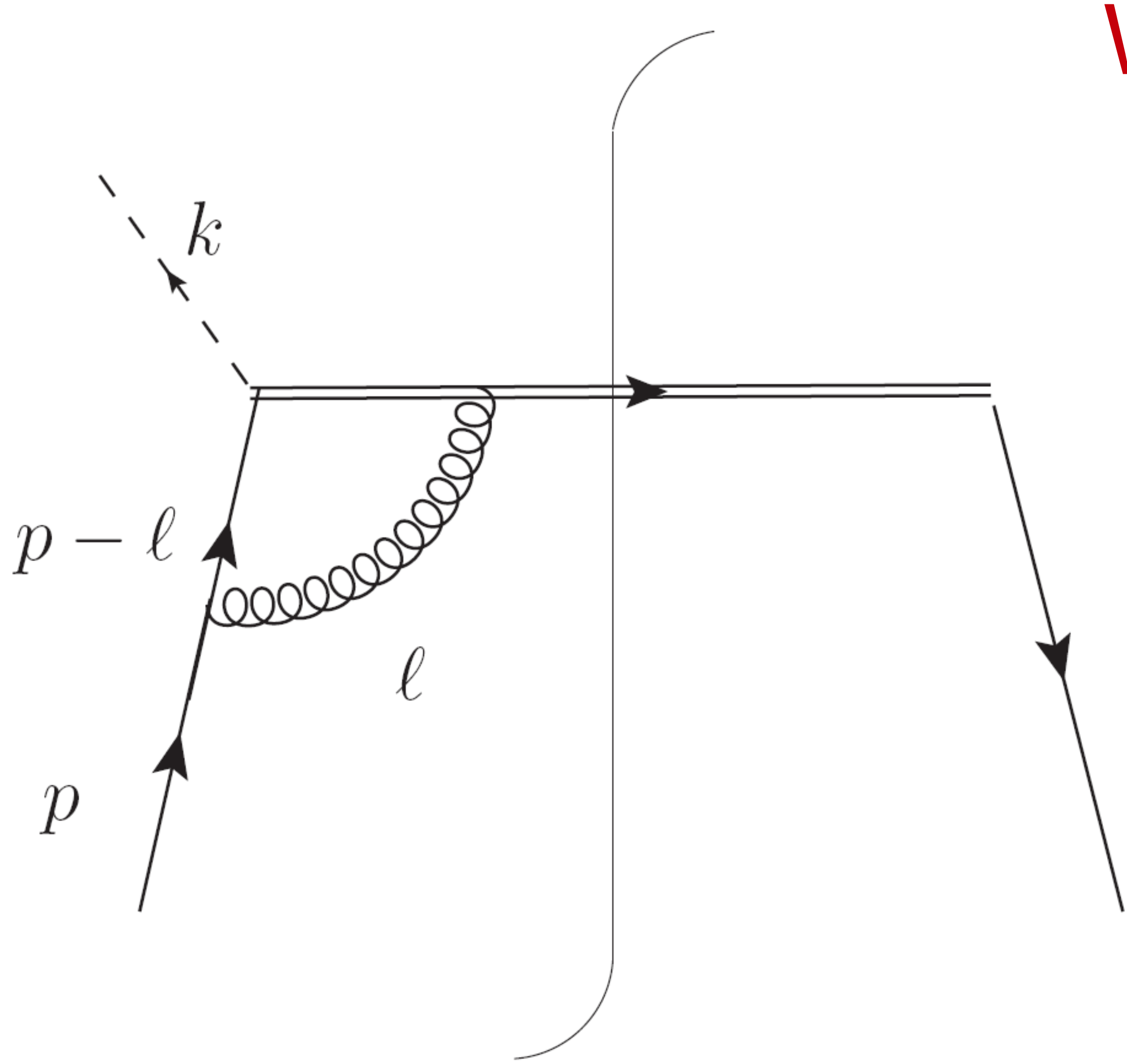
$$\ell = (1-\alpha)p$$

$$\propto \int_0^1 d\alpha \frac{\alpha}{1-\alpha}$$

Physics: The gluon becomes collinear to the Wilson line (struck quark) and its rapidity goes to $-\infty$

"Rapidity divergence"

In fact it is not easy...



We know how to deal with it:

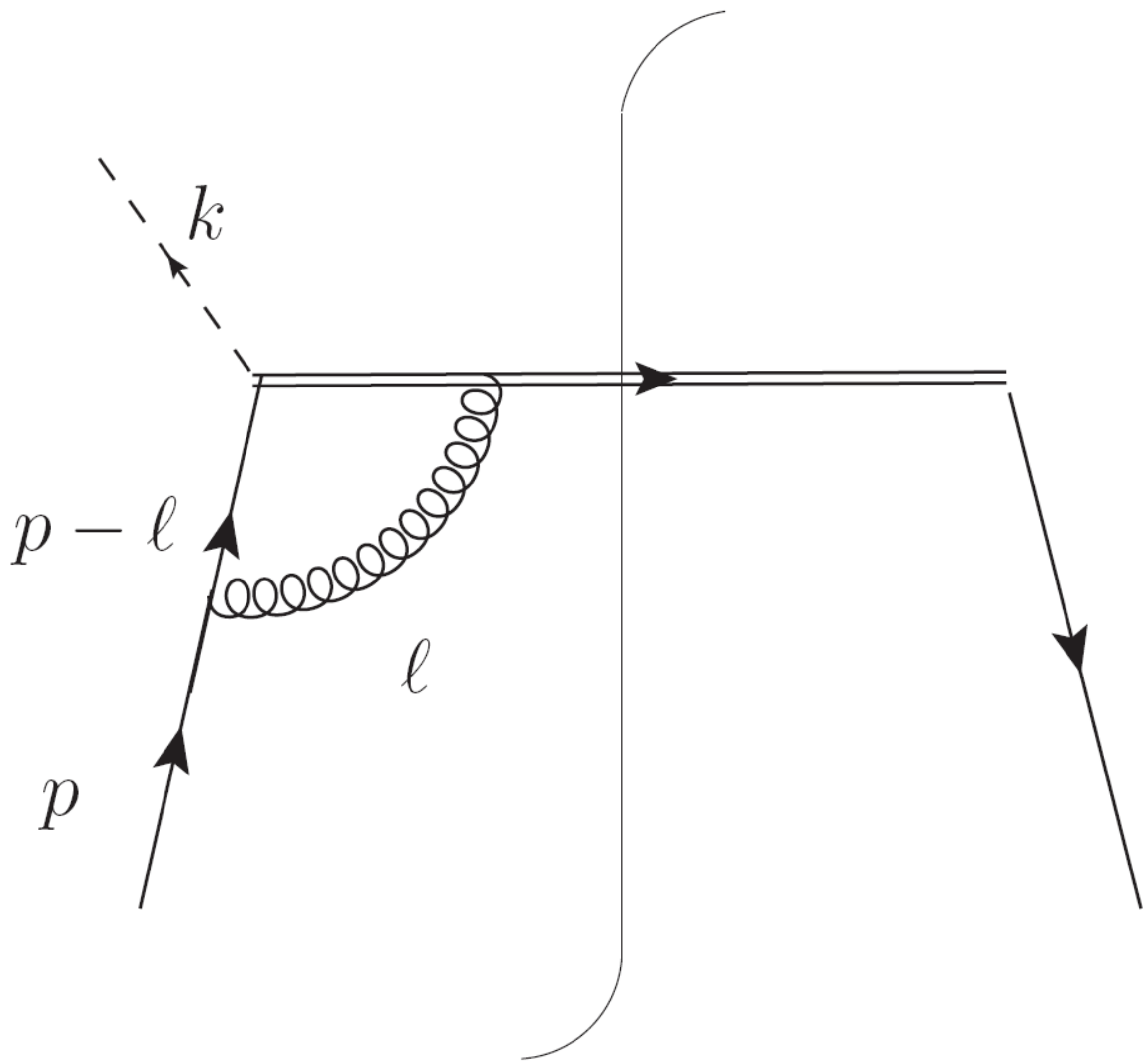
$$\propto \int_0^1 d\alpha \frac{\alpha}{(1-\alpha)_+} T(\alpha) =$$

$$= \int_0^1 d\alpha \frac{\alpha T(\alpha) - T(1)}{(1-\alpha)}$$

"+ prescription"

$$T(\alpha = 1) - T(1) = 0$$

In fact it is not easy...



Not working for TMDs:

$$\propto \int_0^1 d\alpha \frac{\alpha}{(1-\alpha)_+} T(\alpha, k_\perp) =$$

$$= \int_0^1 d\alpha \frac{\alpha T(\alpha, k_\perp) - T(1, 0_\perp)}{(1-\alpha)}$$

"+ prescription"

$$T(\alpha = 1, k_\perp) - T(1, 0_\perp) \neq 0$$

John Collins, Acta Phys.Polon. B34 (2003) 3103

TMD related studies have been extremely active in the past few years, lots of progress have been made

We look forward to the future experimental results from COMPASS, RHIC, Jefferson Lab, LHC, Fermilab, future Electron Ion Collider

Many TMD related groups are created throughout the world:

Italy, Netherlands, Belgium, Germany, Japan, China, Russia, and the USA