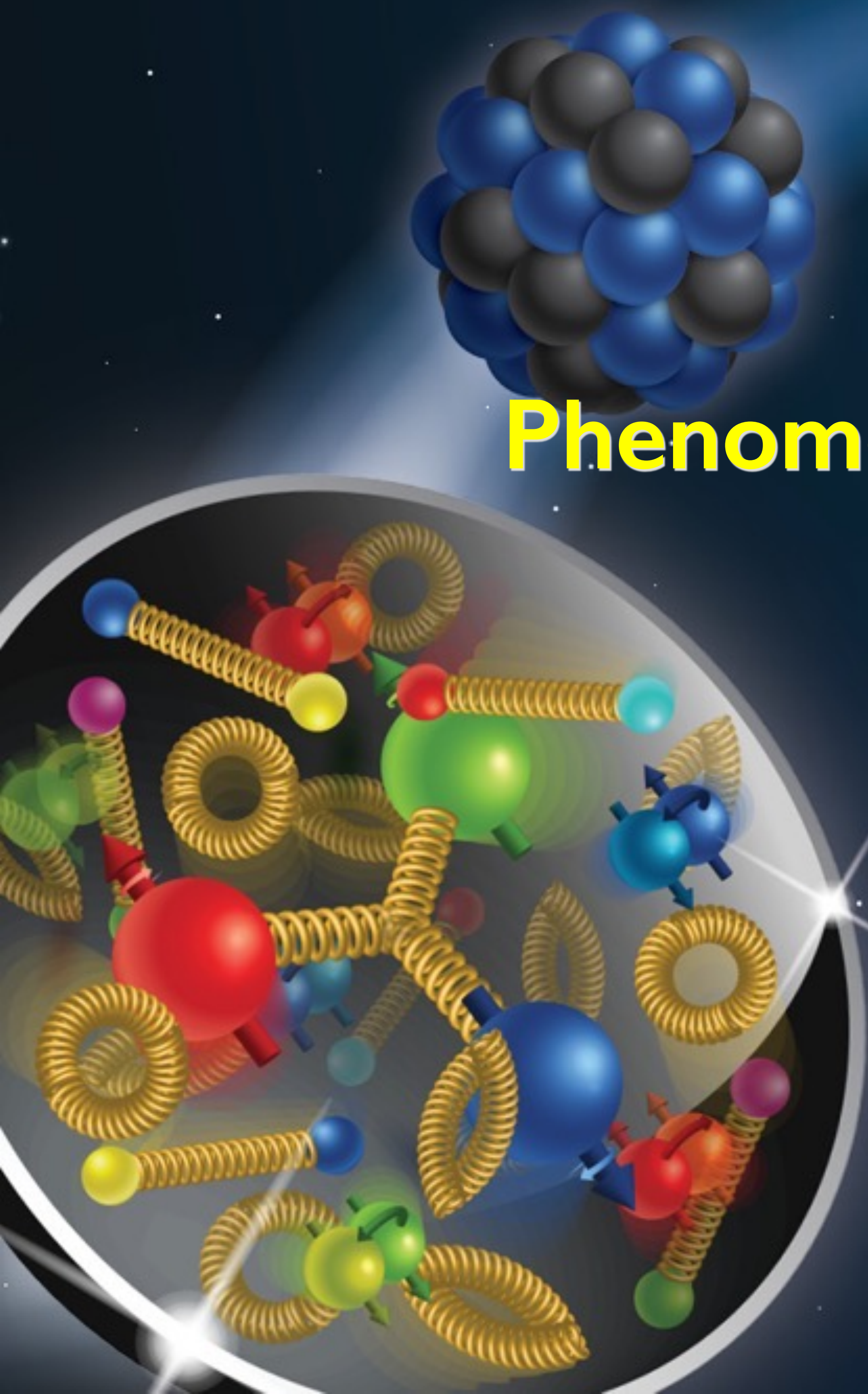


Phenomenology of TMDs: lecture I

Alexei Prokudin

The plan:

- # Lecture I:

Structure of the nucleon
Transverse Momentum Dependent distributions (TMDs)

- # Lecture II

Semi Inclusive Deep Inelastic Scattering (SIDIS)
Calculations of SIDIS structure functions using Mathematica

You will find all material in https://github.com/prokudin/tmdcollaboration22

There are 3 exercises:

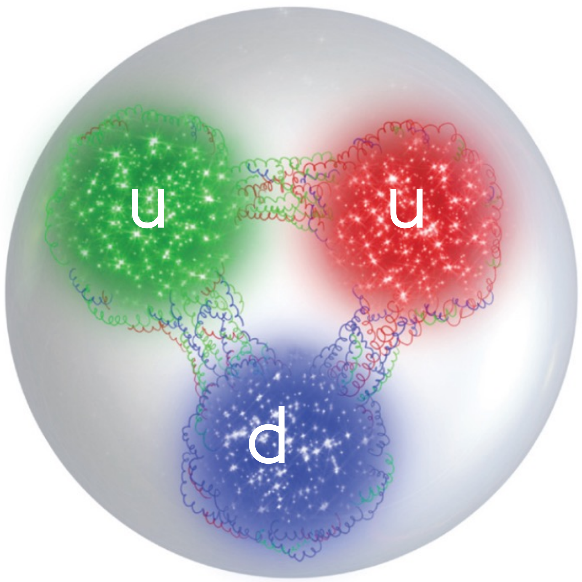
✓ Choose any file sivers.nb or collins.nb. Follow the file, calculate the structure functions and plot the asymmetries. Compare your results to plots from example.nb

✓ ** Use the definitions of Fourier transforms of TMDs from the TMD handbook Eq. (2.192), (2.193) and show that they give the same results as the ones you obtain in the previous exercise

✓ *** Rewrite the notebooks using Python, SymPy https://www.sympy.org/en/index.html to perform analytical computations

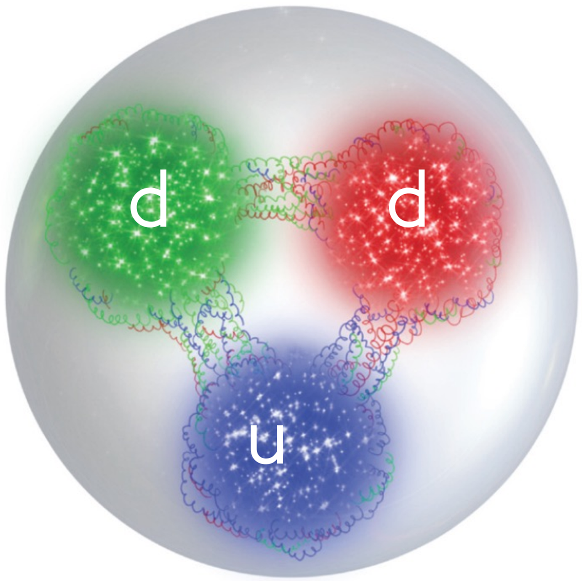
What do we study?

Proton



Two "up"
one "down"

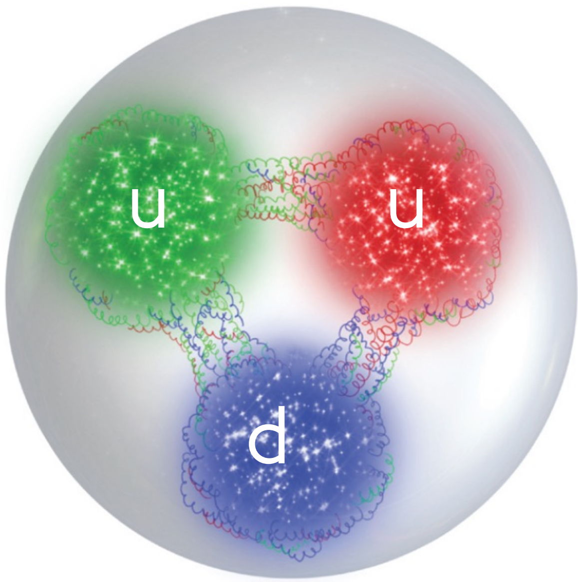
Neutron



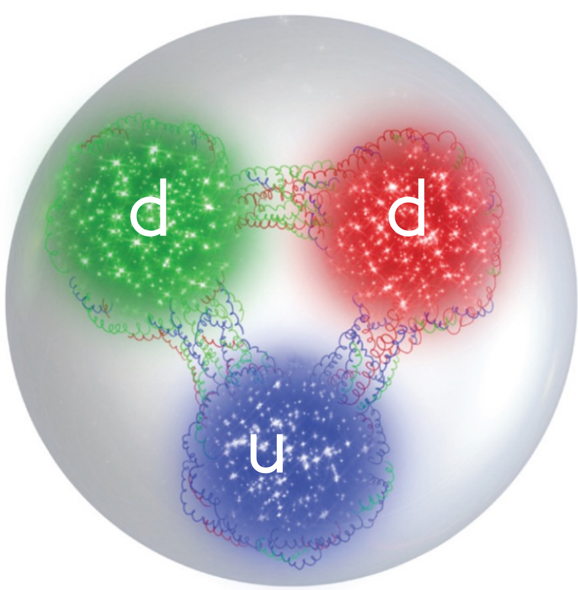
Two "down"
one "up"

Quark model

Proton



Neutron



Charge

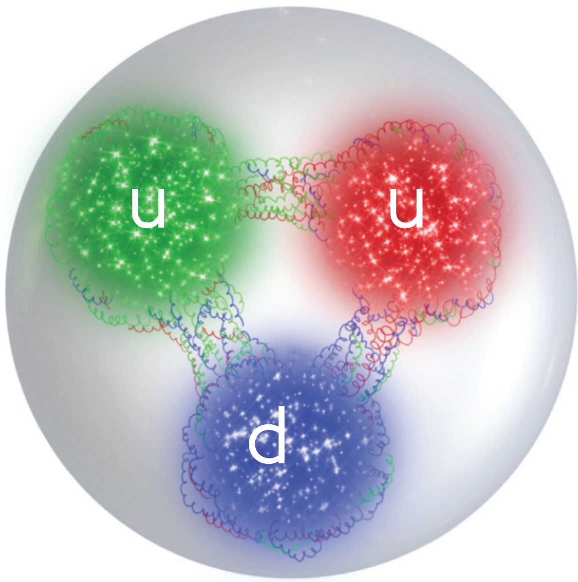
2/3 + 2/3 -1/3 = +1

Quark charges
are fractional
of positron's charge

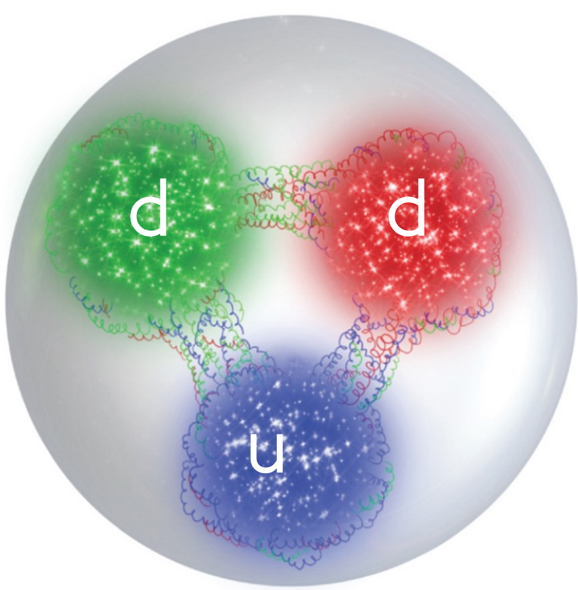
2/3 - 1/3 -1/3 = 0

Quarks are
bound (confined)
by strong interaction
(gluons)

Proton



Neutron



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Quark charges
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2/3 - 1/3 -1/3 = 0

Quarks are
bound (confined)
by strong interaction
(gluons)

No evidence of free quarks
observed directly in experiment

Long distance - quarks
are **confined** in hadrons

Short distance - quarks
behave as if they were free

Asymptotic freedom -

later in this lecture!

Quantum Electro Dynamics and Quantum Chromo Dynamics

QED

Gauge theory U(1)

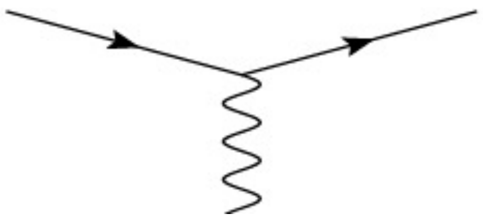
Force carrier:

 photon (electrically neutral)

 electron

 positron

Interaction $\alpha_{em} \simeq \frac{1}{137}$



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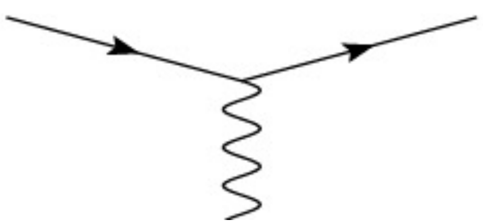

Richard Feynman, Nobel Prize 1965

"for ... fundamental work in quantum electrodynamics"

Quantum Electro Dynamics and Quantum Chromo Dynamics

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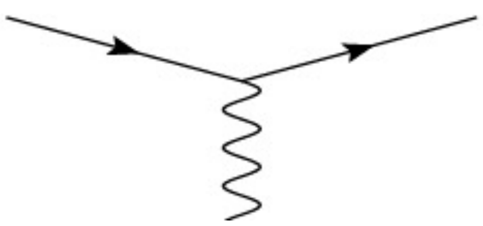


Richard Feynman, Nobel Prize 1965

"for ... fundamental work in quantum electrodynamics"

QCD

Gauge theory SU(3)

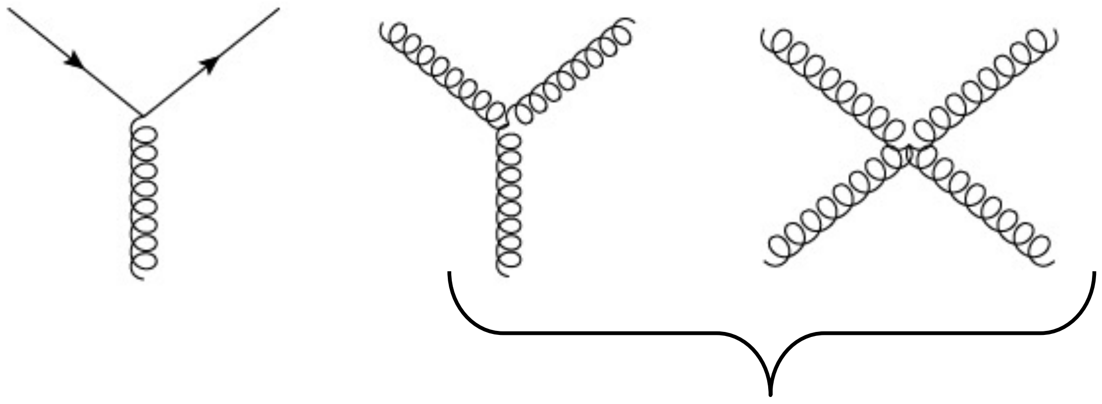
Force carriers:

 gluons (carry color)

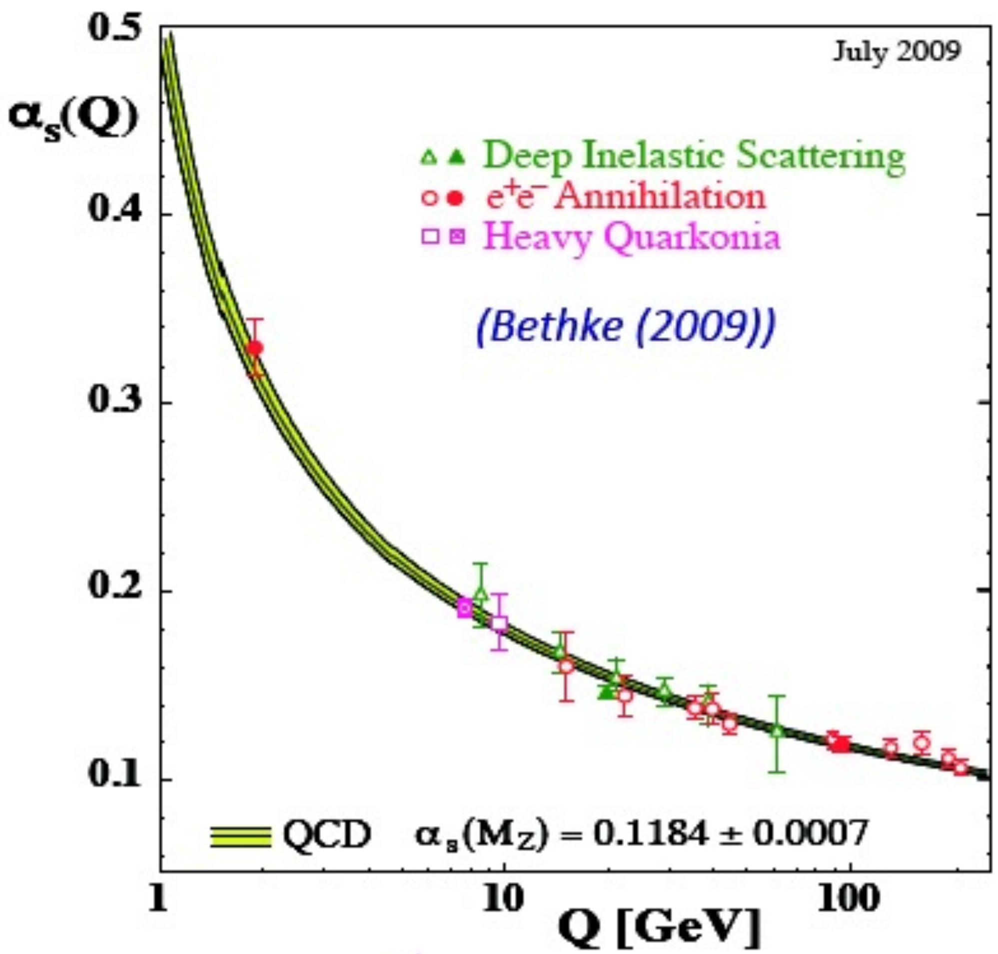
 quark

 anti-quark

Interaction $\alpha_s \sim 0.1$ strong



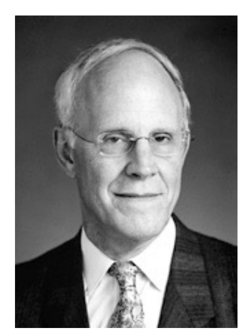
Gluons interact with each other
- non abelian theory



"long" DISTANCE "short"

The coupling depends
on the scale – consequence of
renormalizability of the theory

At short distances
quarks behave as if they were
almost free particles!




"for the discovery of asymptotic freedom in the theory of the strong interaction"

Gross, Politzer, Wilczek, Nobel Prize 2004

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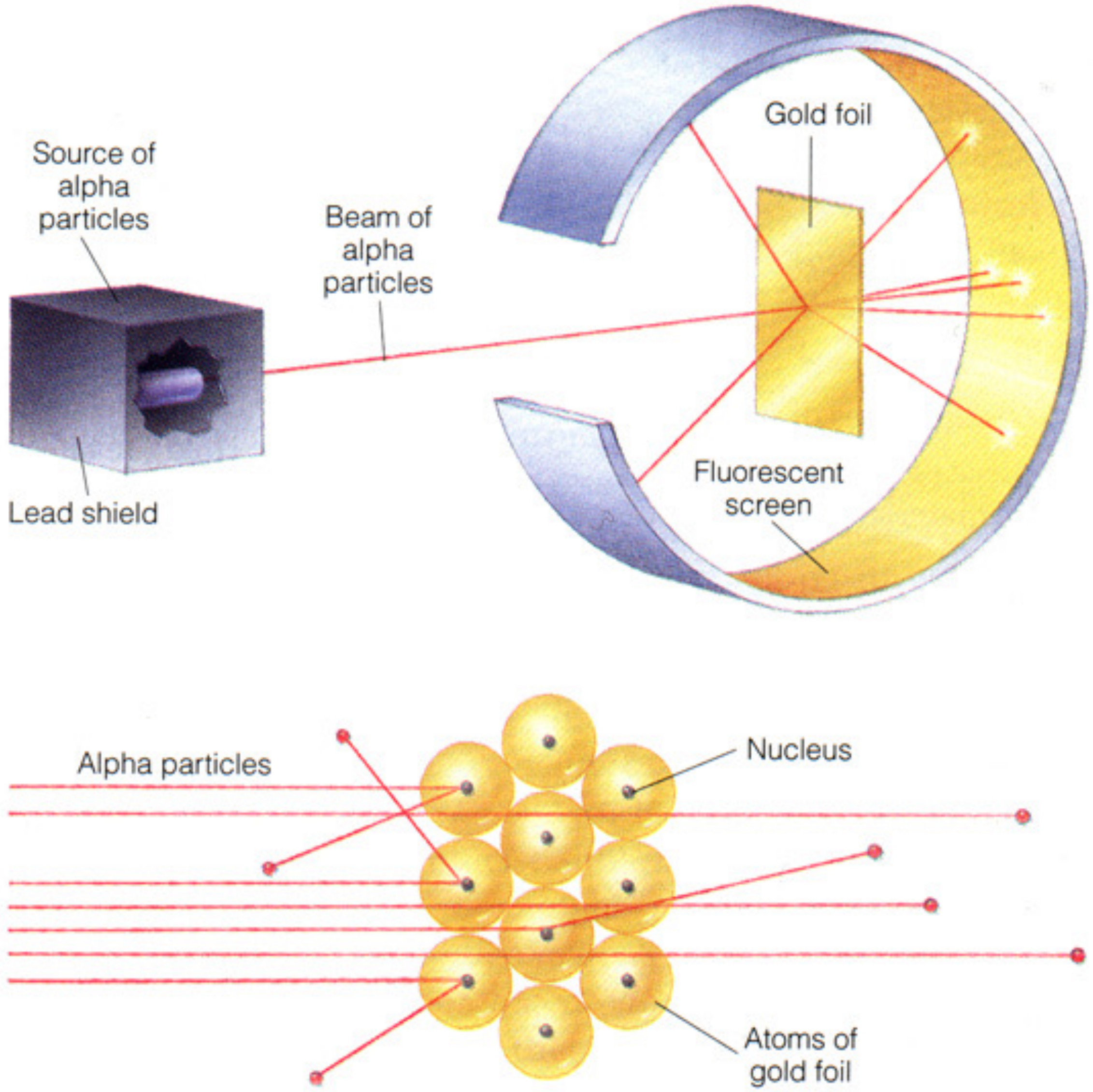
How do we study the structure?

Scattering as the method of study

"for his investigations into the disintegration of the elements, and the chemistry of radioactive substances"

Ernest Rutherthord, Nobel Prize 1908



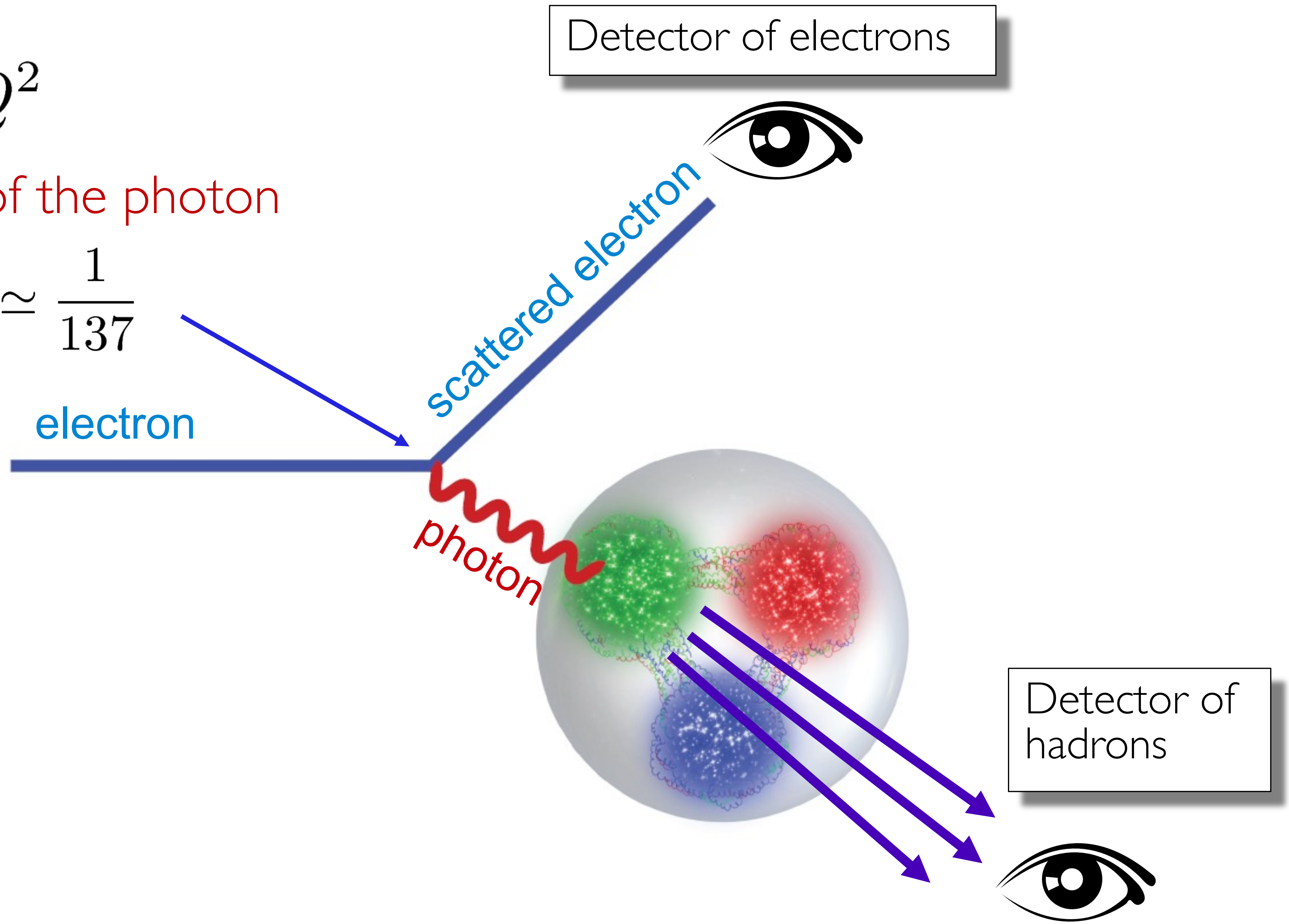
1911

Electron Scattering

$$q^2 = -Q^2$$

Virtuality of the photon

$$\alpha_{em} \simeq \frac{1}{137}$$



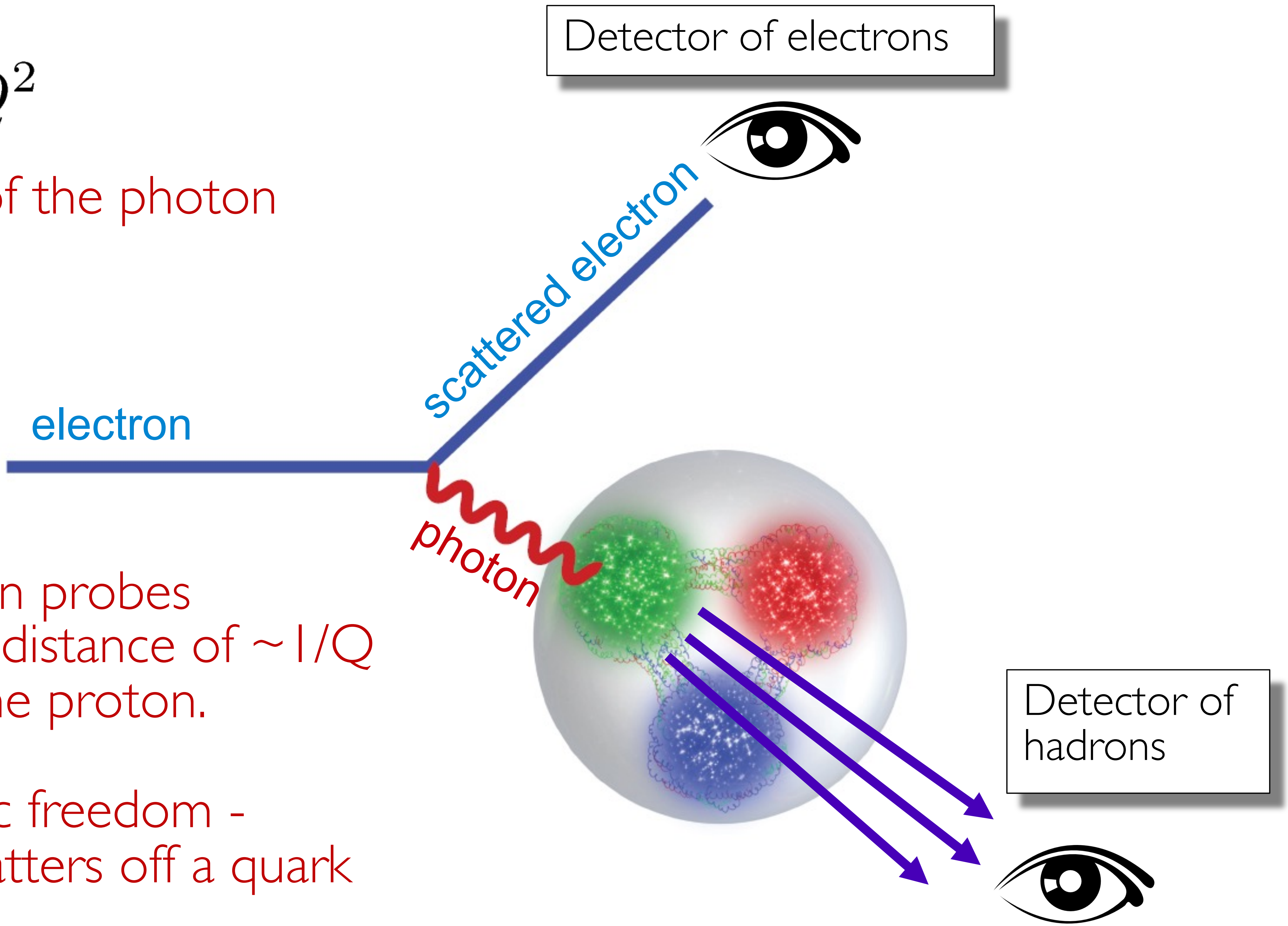
Electron Scattering

$$q^2 = -Q^2$$

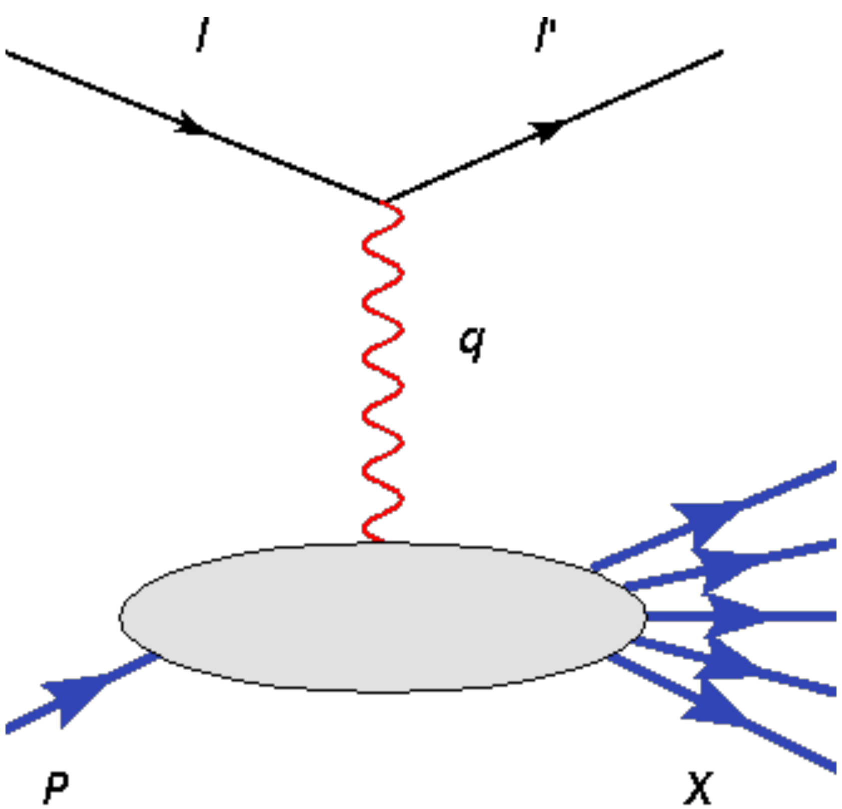
Virtuality of the photon

The photon probes transverse distance of ~1/Q inside of the proton.

Asymptotic freedom - photon scatters off a quark



Electron Scattering: interpretation

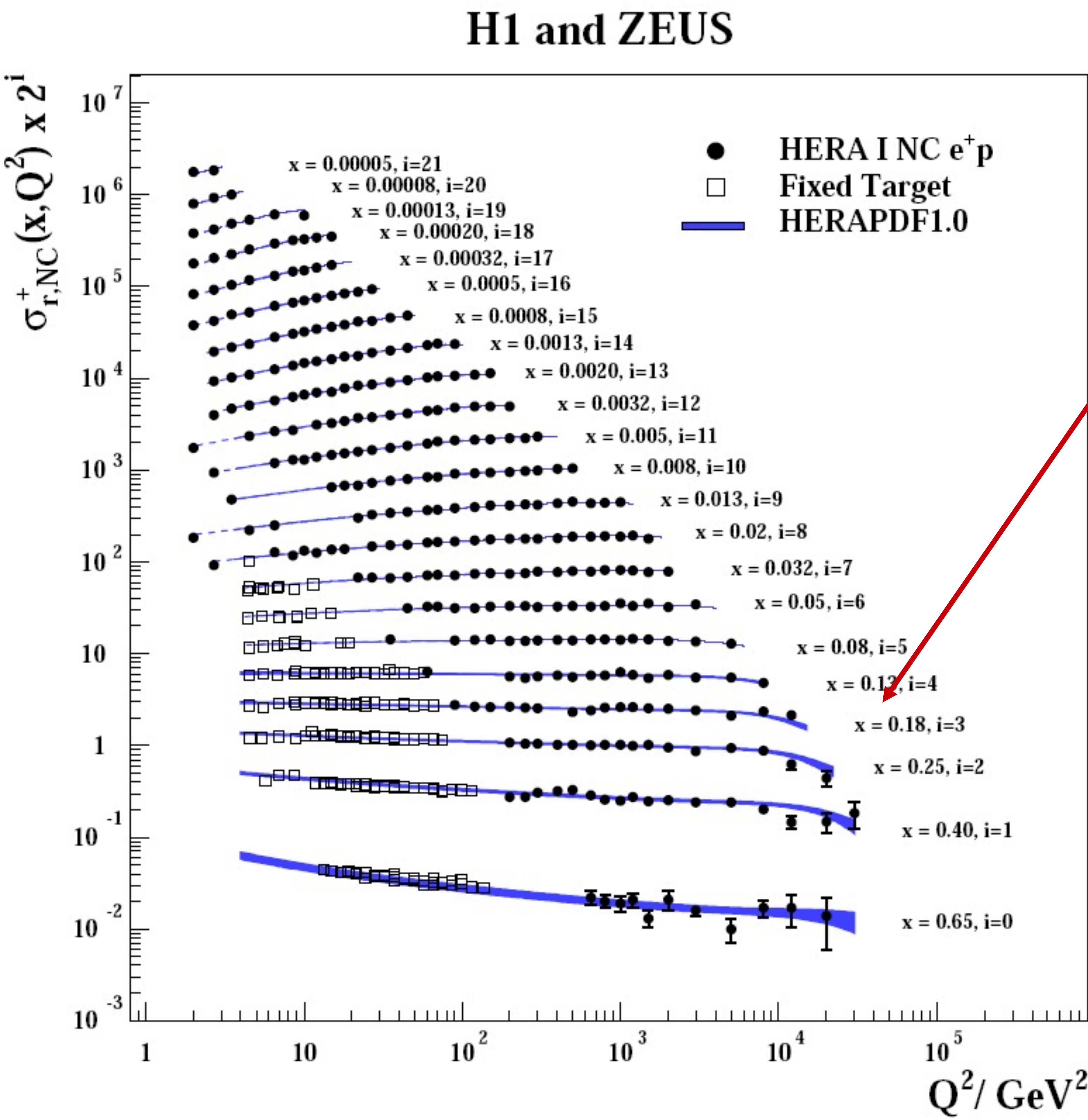


Electromagnetic probe resolves a quark or anti-quark with momentum p=xP inside of the proton of momentum P.

$$x_{Bj} = \frac{Q^2}{2P \cdot q}$$ Bjorken variable

Gives the fraction of longitudinal momentum of the proton carried by the parton

Electron Scattering: interpretation

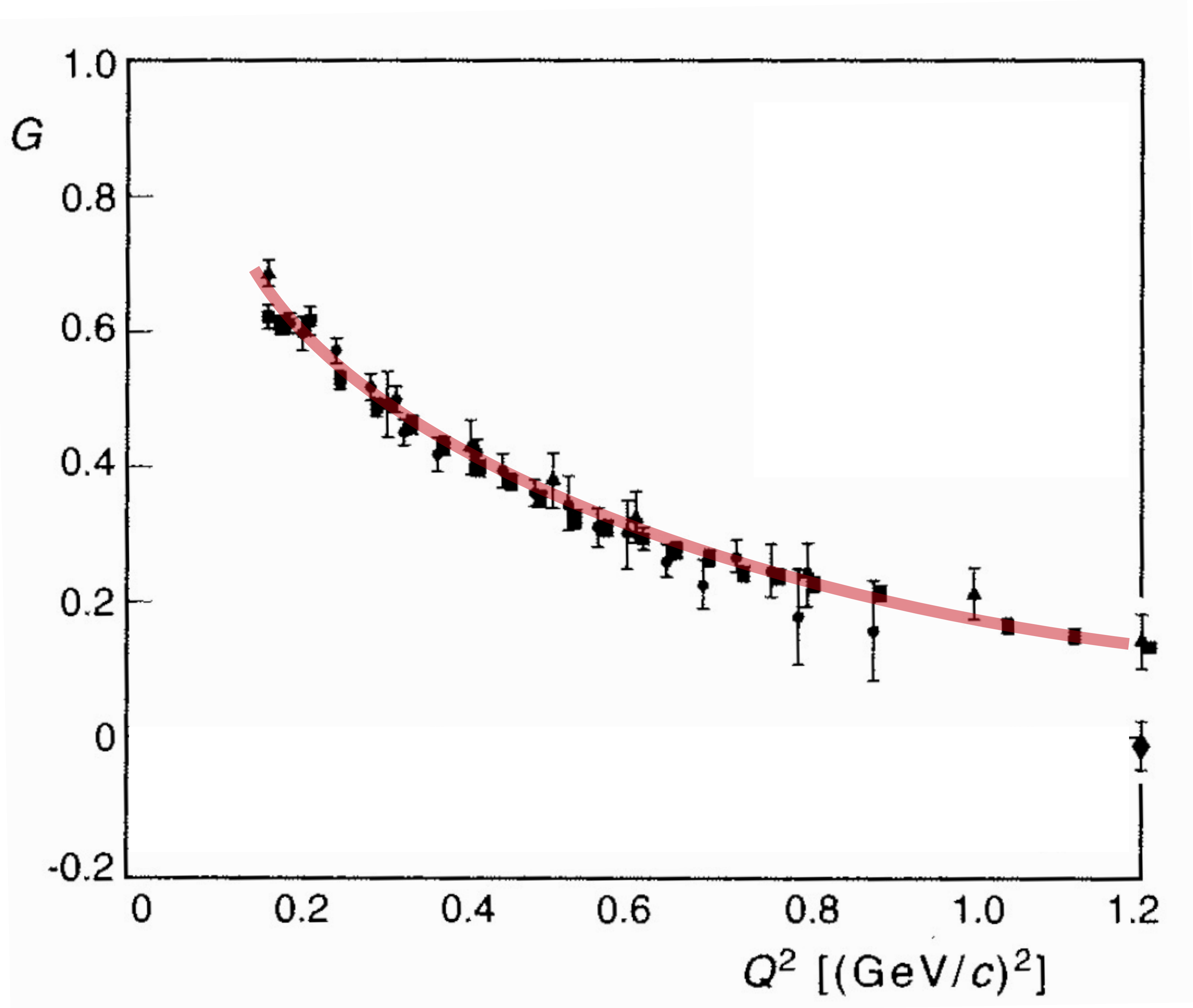


Experimental data

$$x_{Bj} = 0.18$$

Behavior is almost flat
– quarks are pointlike

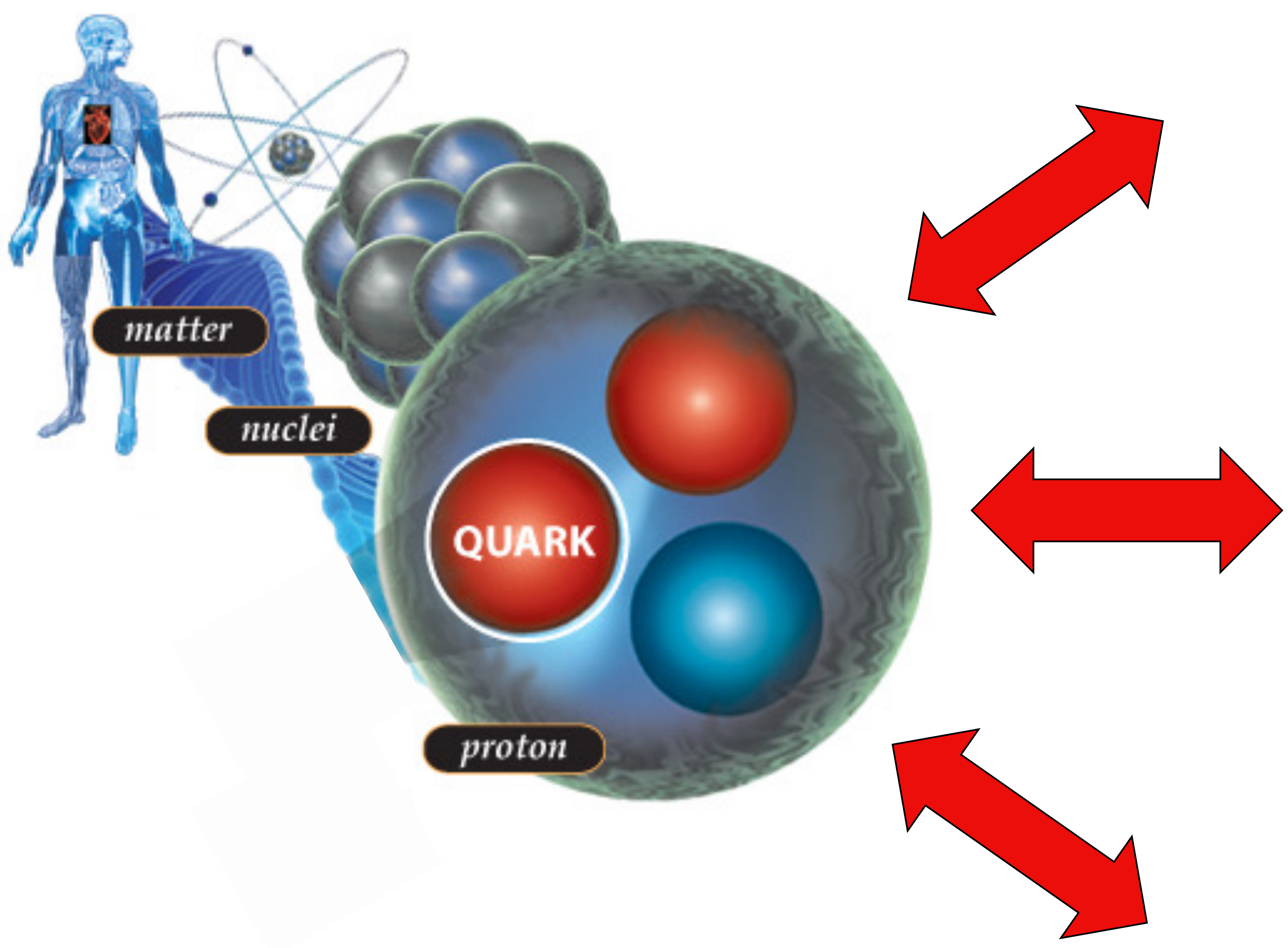
Electron Scattering: interpretation



Unlike the proton itself:

Proton size ~ 1 fm

Exploring the nucleon: a fundamental quest

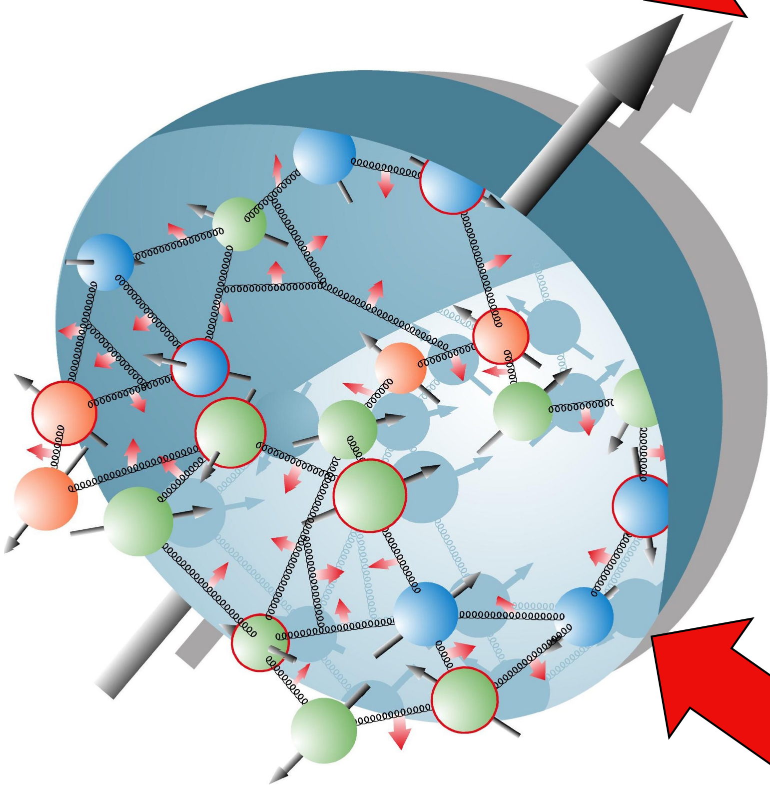


Know what we are made of !

Understand the strong force: "QCD"

Use protons as tool for discovery (e.g. LHC)

Xiangdong Ji at DIS08



Spin is a fundamental quantum degree of freedom

Spin plays a critical role in determining the basic structure of fundamental interactions

Test of a theory is not complete without a full test of spin-dependent decays and scattering

Spin provides a unique opportunity to probe the inner structure of a composite system (such as the proton)

"Experiments with spin have killed more theories than any other single physical parameter"

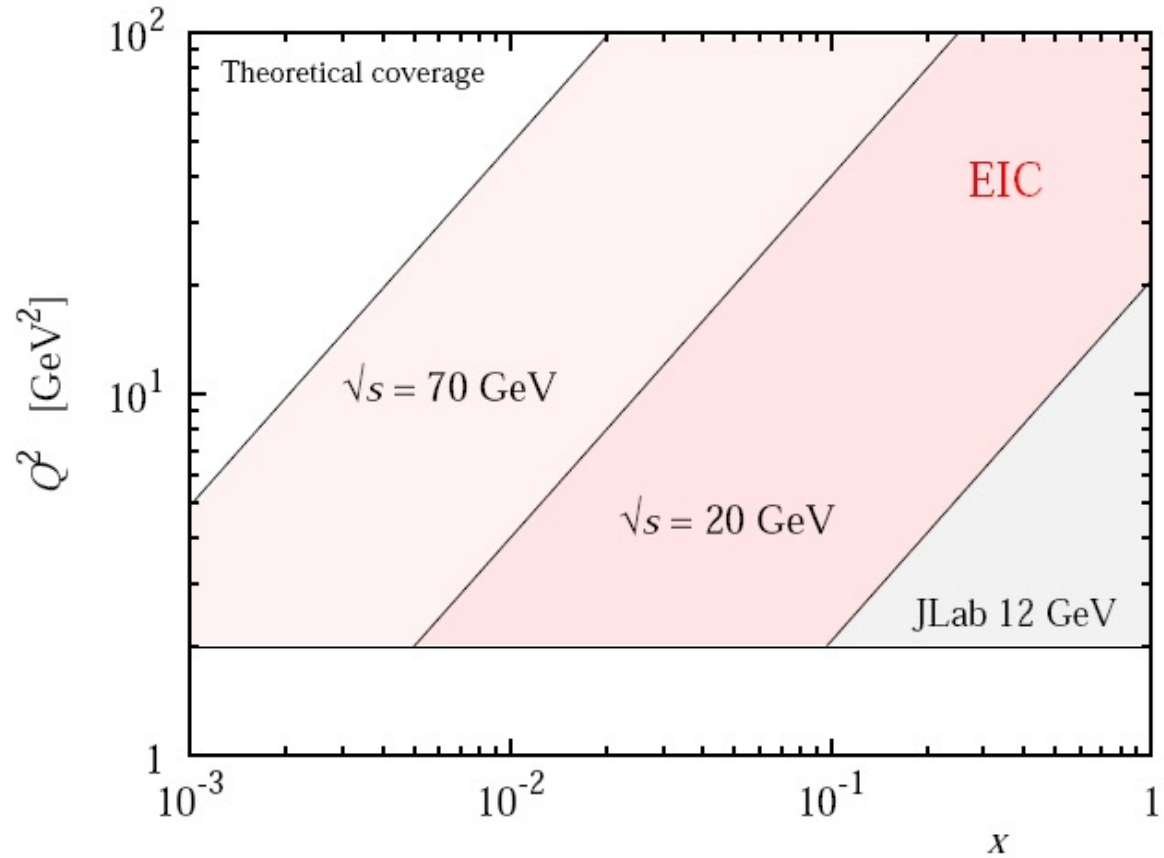
Elliot Leader, Spin in Particle Physics, Cambridge U. Press (2001)

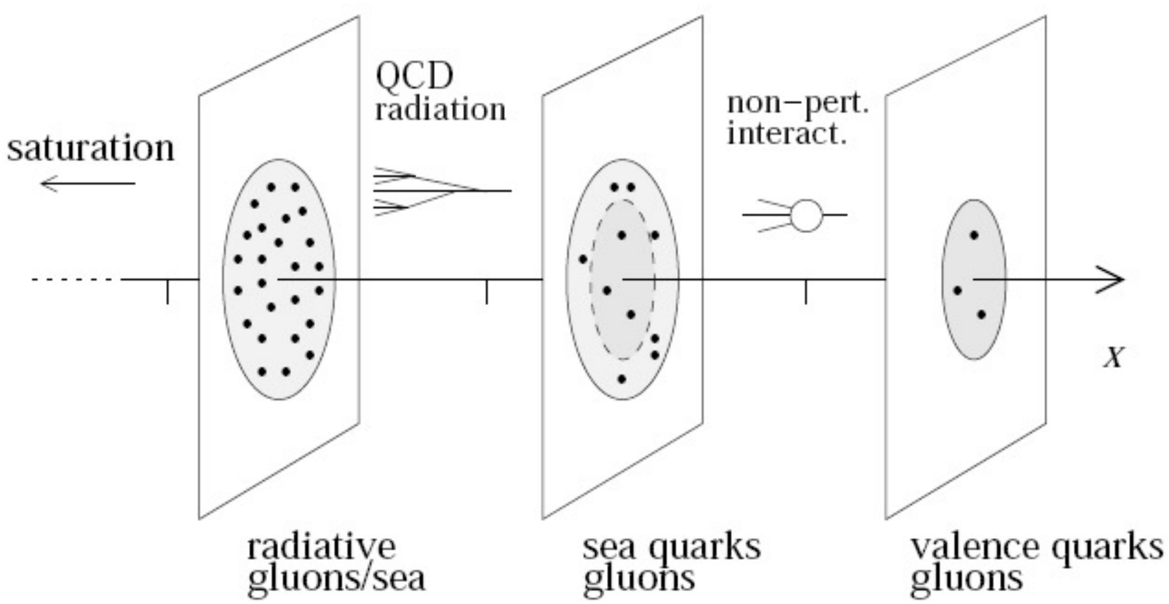
"Polarization data has often been the graveyard of fashionable theories. If theorists had their way they might well ban such measurements altogether out of self-protection."

J. D. Bjorken, Proc. Adv. Research Workshop on QCD Hadronic Processes, St. Croix, Virgin Islands (1987).

The nucleon landscape

Nucleon landscape





Nucleon is a many body dynamical system of quarks and gluons

Changing x we probe different aspects of nucleon wave function

How partons move and how they are distributed in space is one of the directions of development of nuclear physics

Technically such information is encoded into Generalised Parton Distributions (GPDs) and Transverse Momentum Dependent distributions (TMDs)

These distributions are also referred to as 3D (three-dimensional) distributions

Unified View of Nucleon Structure

5D

Wigner function

$W(x, \mathbf{k}_\perp, \mathbf{r}_\perp)$

Transverse
Momentum
Dependent
distributions

$\int d^2\mathbf{r}_\perp$

$\int d^2\mathbf{k}_\perp$

Generalized
Parton
Distributions

$f(x, \mathbf{k}_\perp)$

3D

$H(x, \xi, t)$

$\int d^2\mathbf{k}_\perp$

Parton
Distribution
Functions

Form
Factors

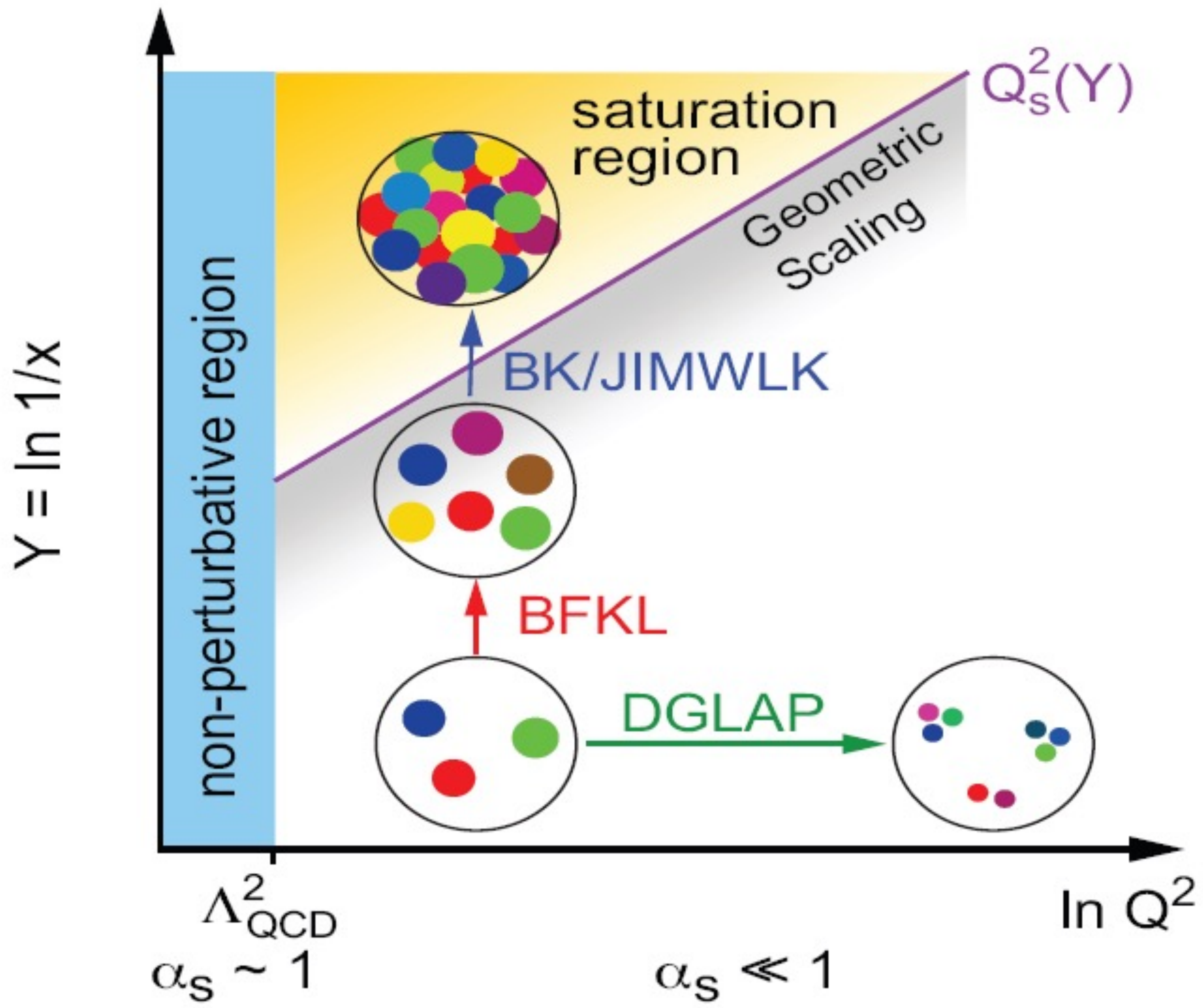
$\int dx$

$f(x)$

1D

$F(Q^2)$

Nucleon landscape

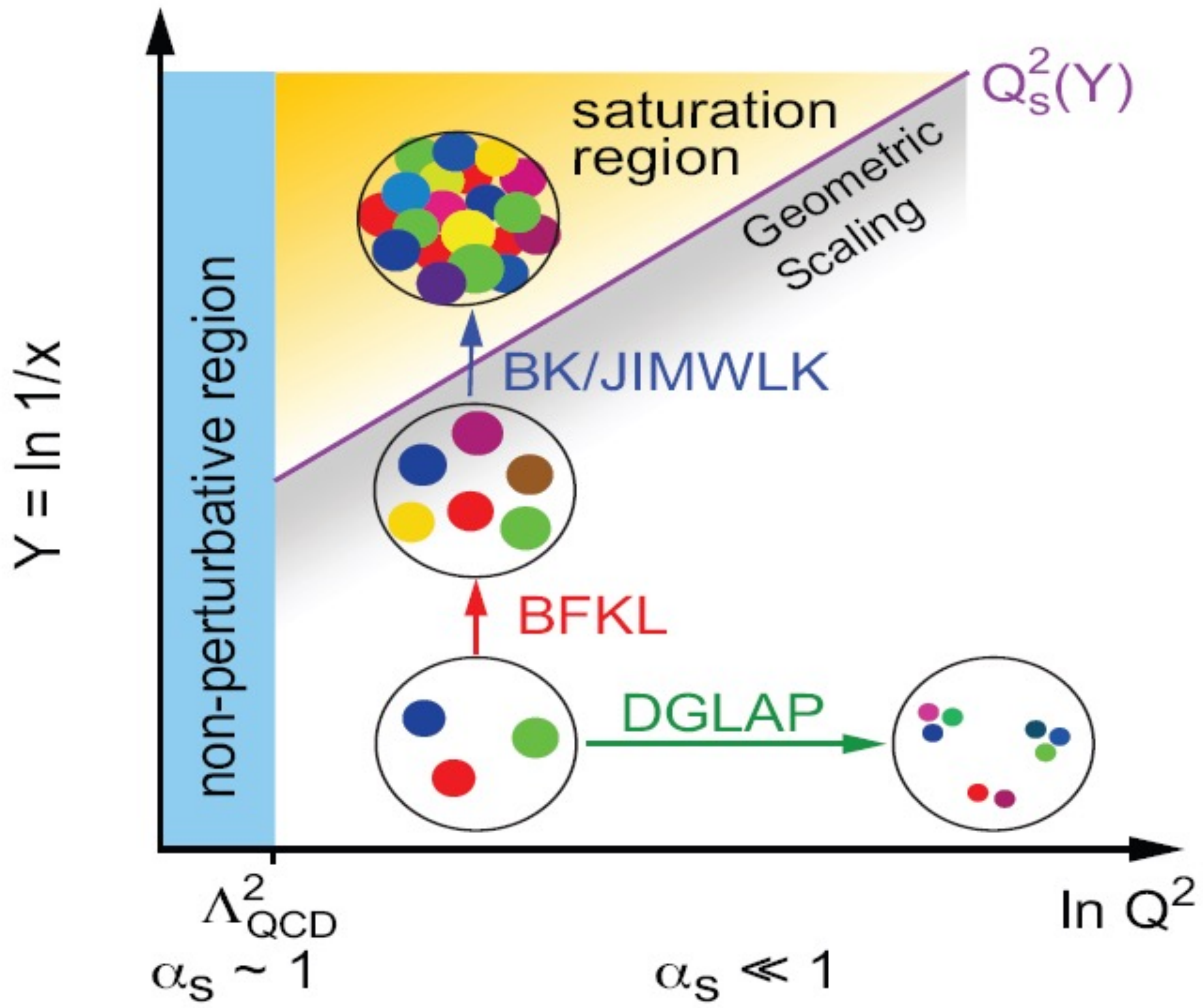


Virtual photon serves as a microscopic probe of the nucleon:

Larger Q^2 probe smaller distances – DGLAP evolution

Plot from EIC whitepaper

Nucleon landscape

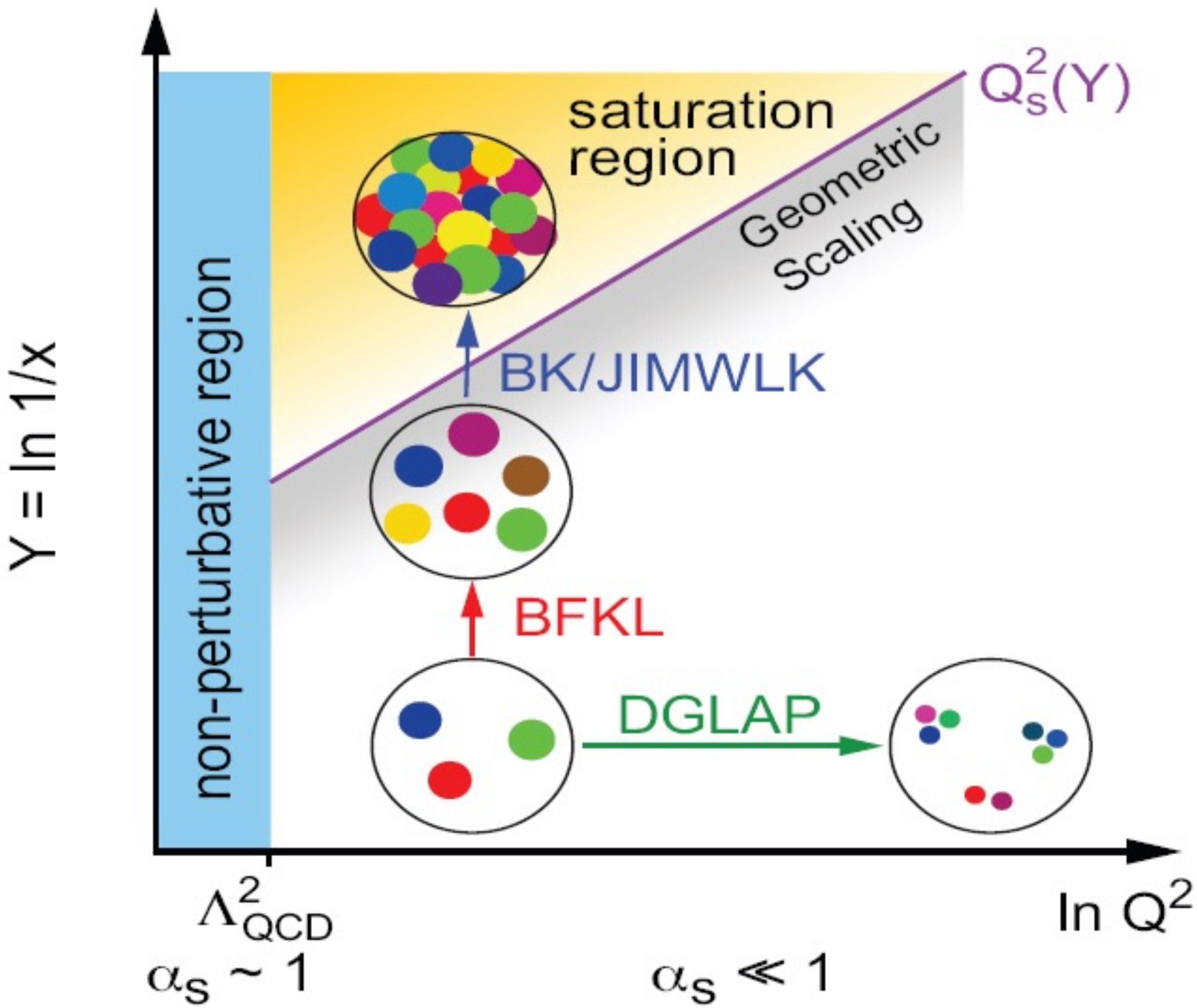


Virtual photon serves as a microscopic probe of the nucleon:

Fixing Q^2 and changing the energy we probe BFKL evolution

Plot from EIC whitepaper

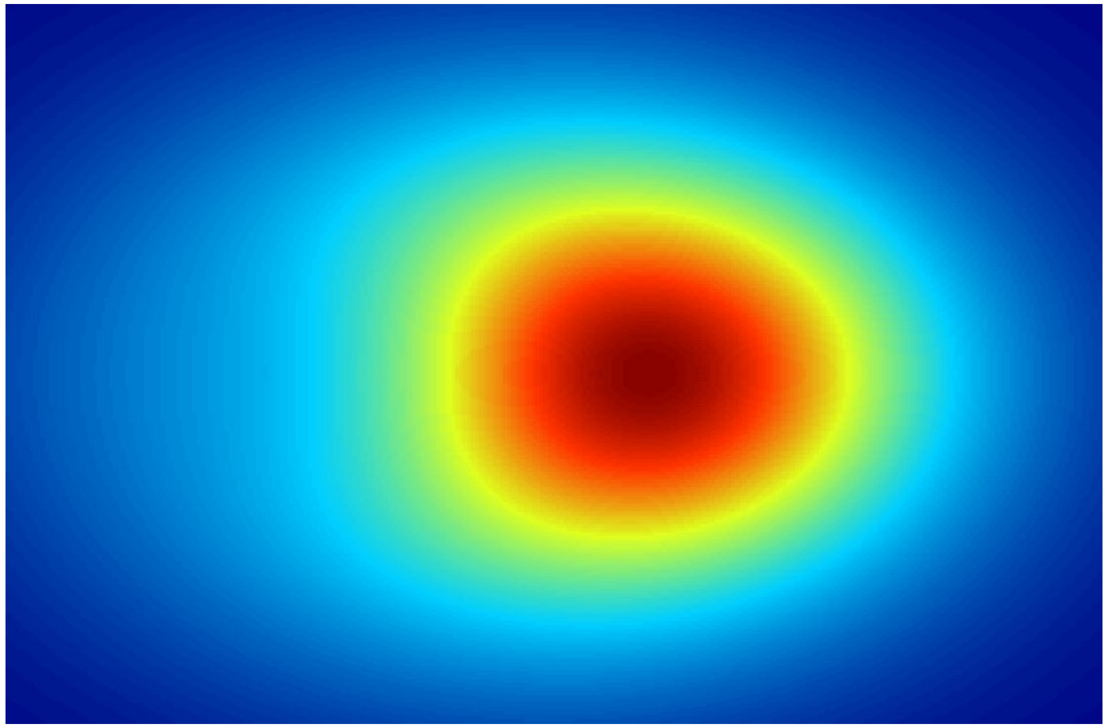
Nucleon landscape



Virtual photon serves as a microscopic probe of the nucleon:

Recombination of gluons leads to non linear effects – BK/JIMWLK evolution and phenomenon of saturation. Dilute vs dense regime of QCD.

Plot from EIC whitepaper



The polarized proton in momentum space as "seen" by the virtual photon

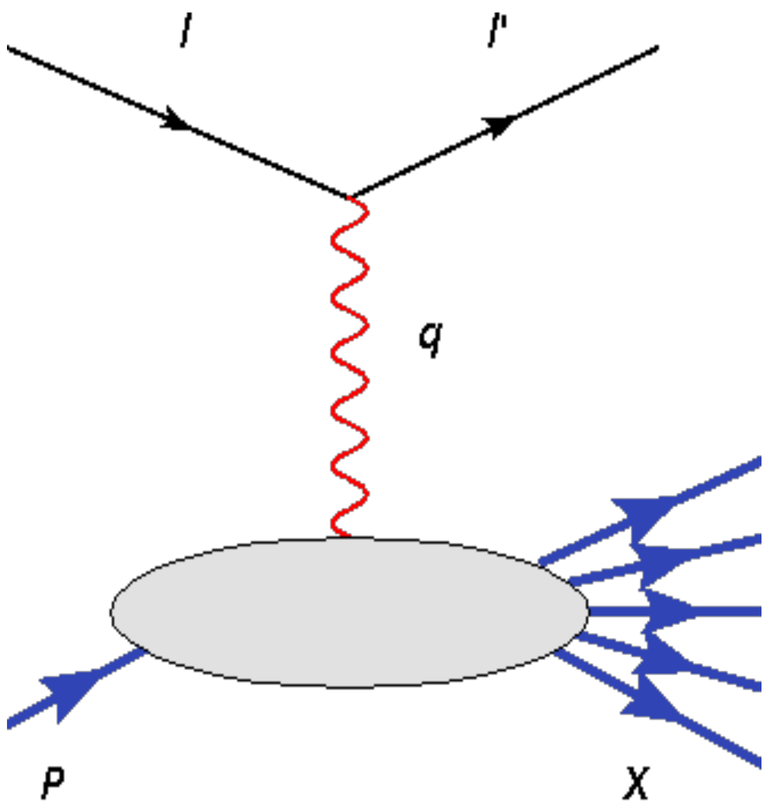
Factorization theorems help us to relate functions that describe the hadron structure and the experimental observables

Factorization is a ***controllable approximation*** and the goal of theorists and phenomenologists is to test and improve the region of applicability of factorization and/or construct new factorization theorems

Hadron structure is the ultimate goal of measurements and phenomenology

How do we study the structure of the nucleon?

In order to access **distributions** we could use
deep inelastic scattering



The energy is big enough to
transform the proton in
a lot of final states

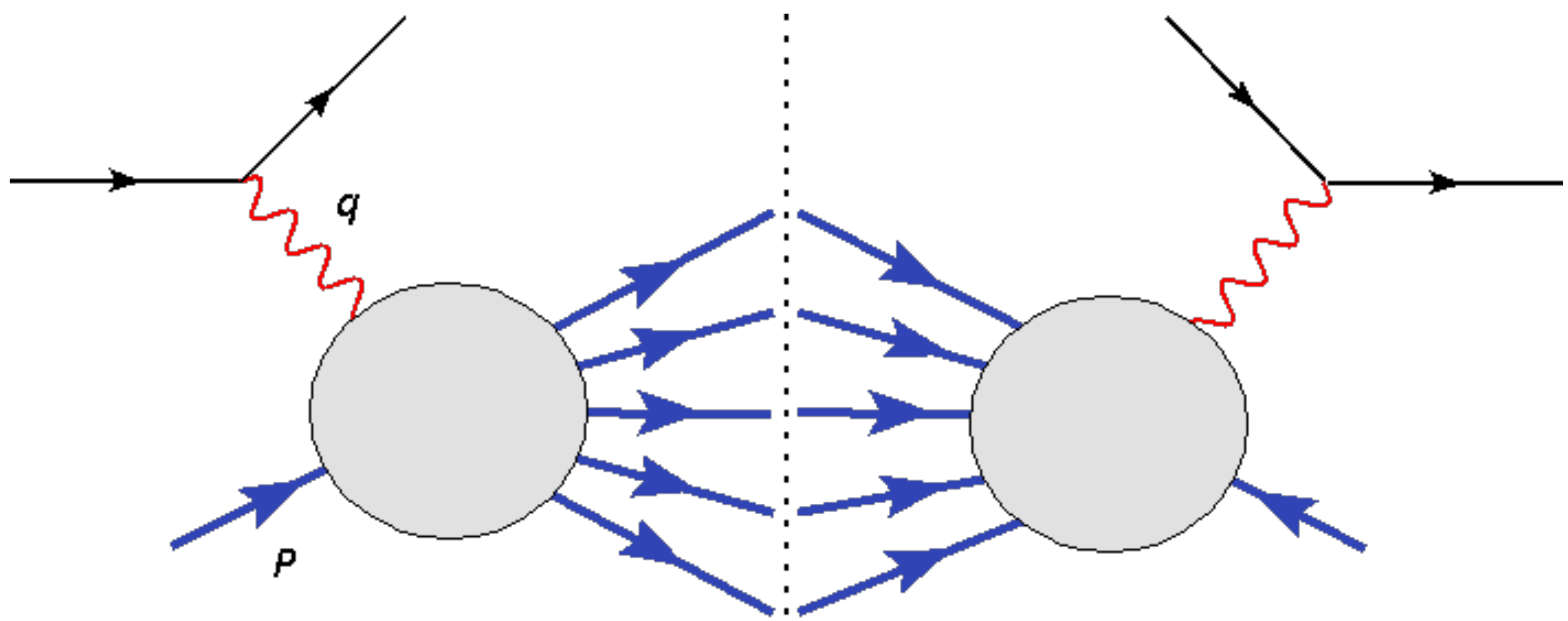
Bjorken limit is

$$\mathbf{Q^2 \to \infty}$$

$$\mathbf{P \cdot q \to \infty}$$

$$\mathbf{x_{Bj} \equiv \frac{Q^2}{2P \cdot q} \to const}$$

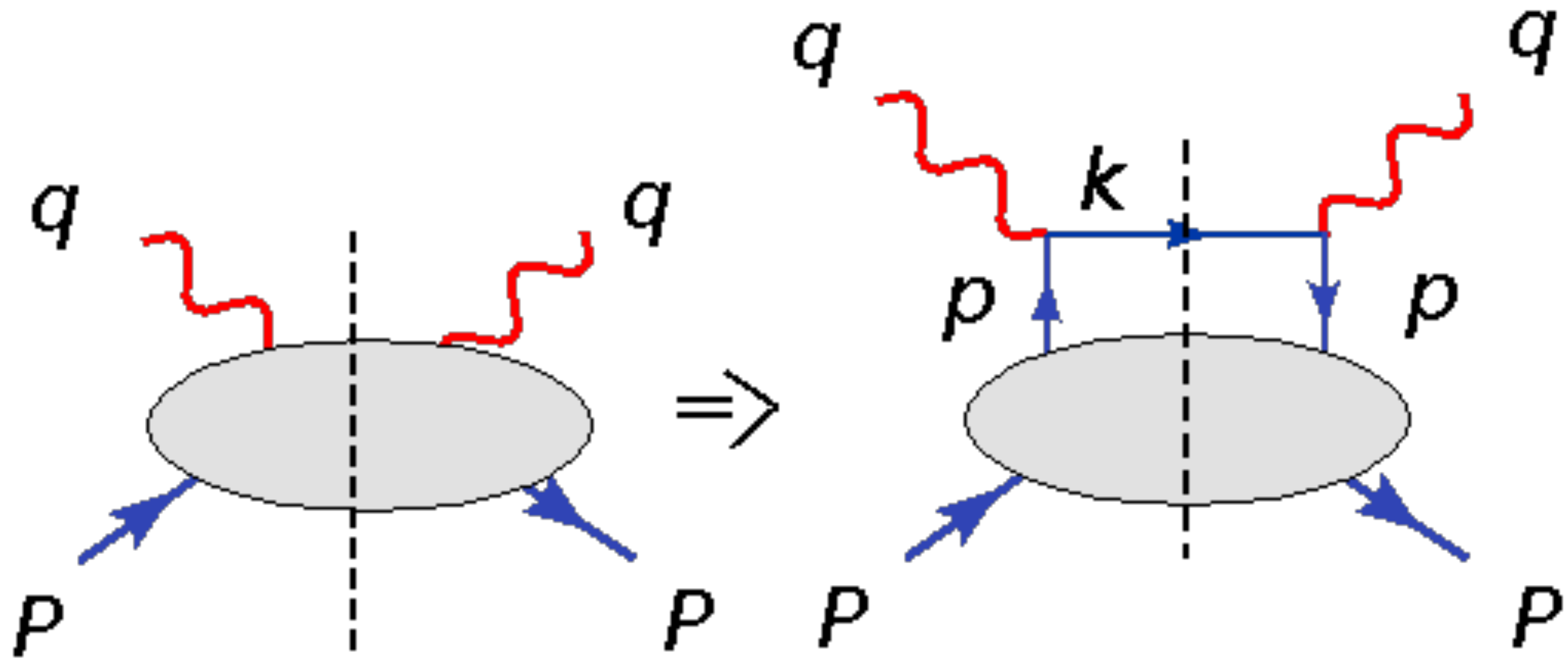
Distributions measured in deep inelastic scattering



$$\sum_{\mathbf{X}} |\mathbf{X}\rangle\langle\mathbf{X}|$$

This sum makes it sensitive to parton structure!

Parton model is a logical step, partons are pointlike and dilute, so the photon interacts with them incoherently



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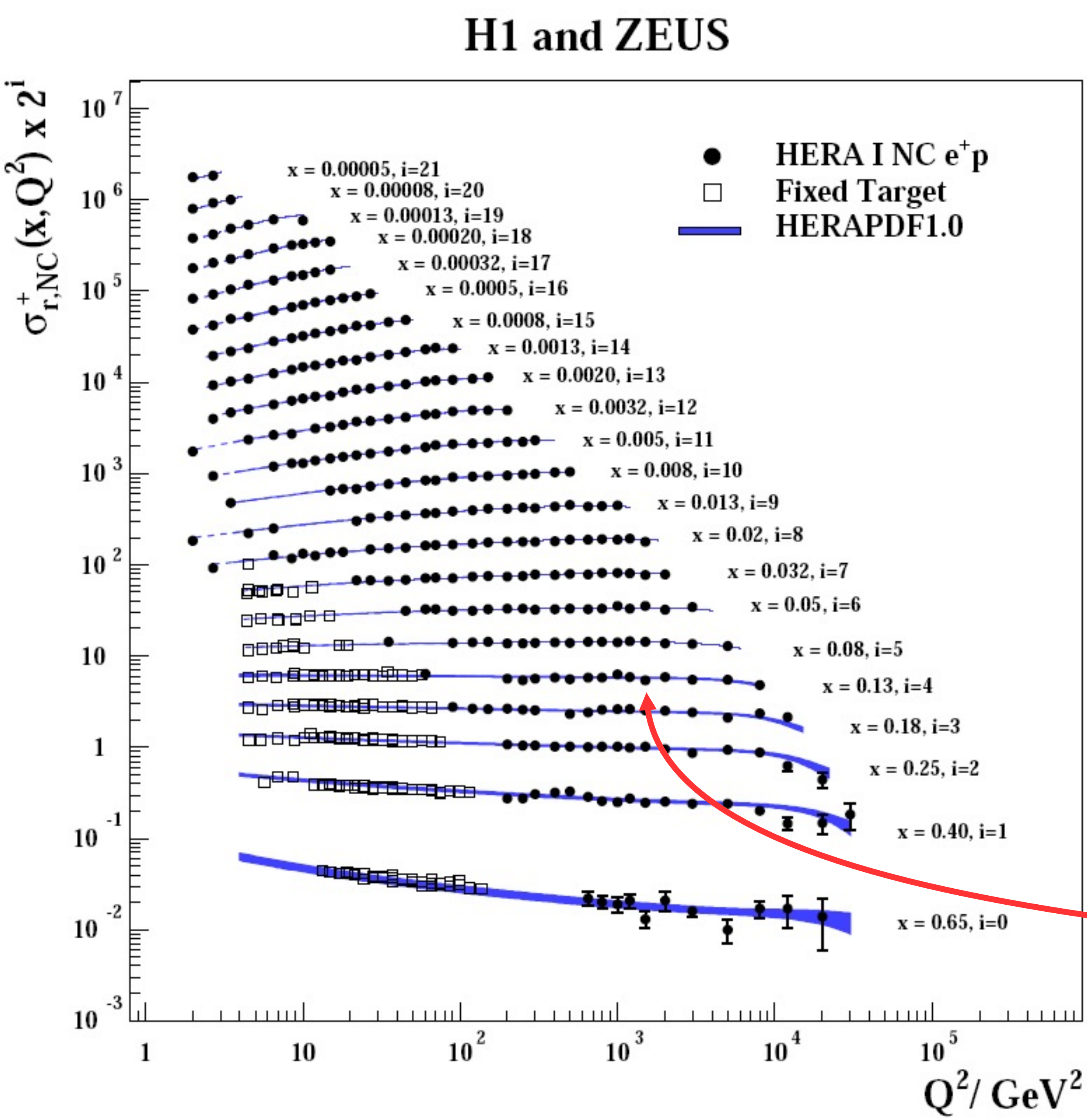
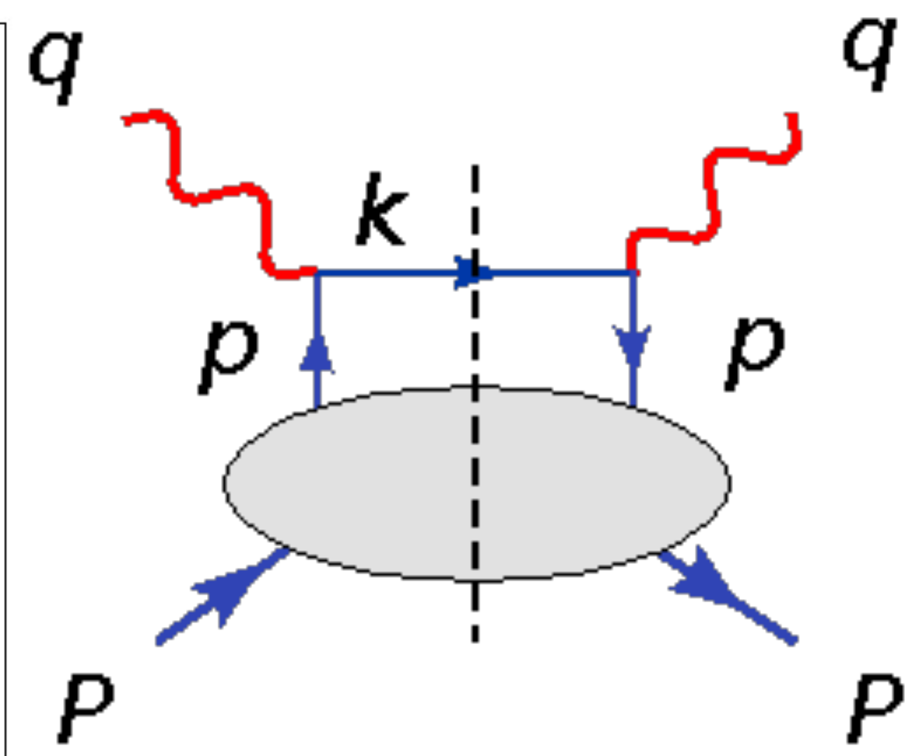
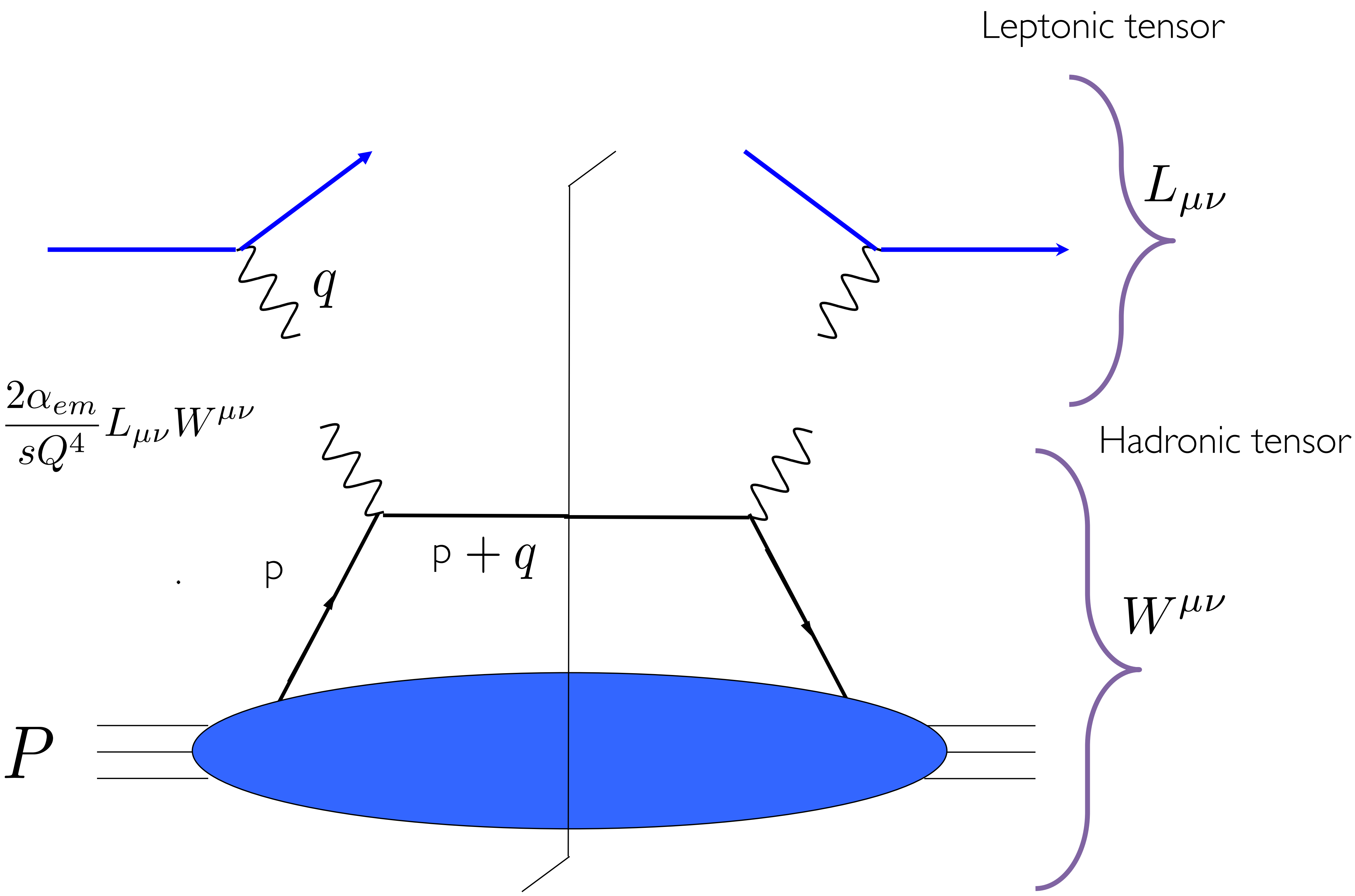


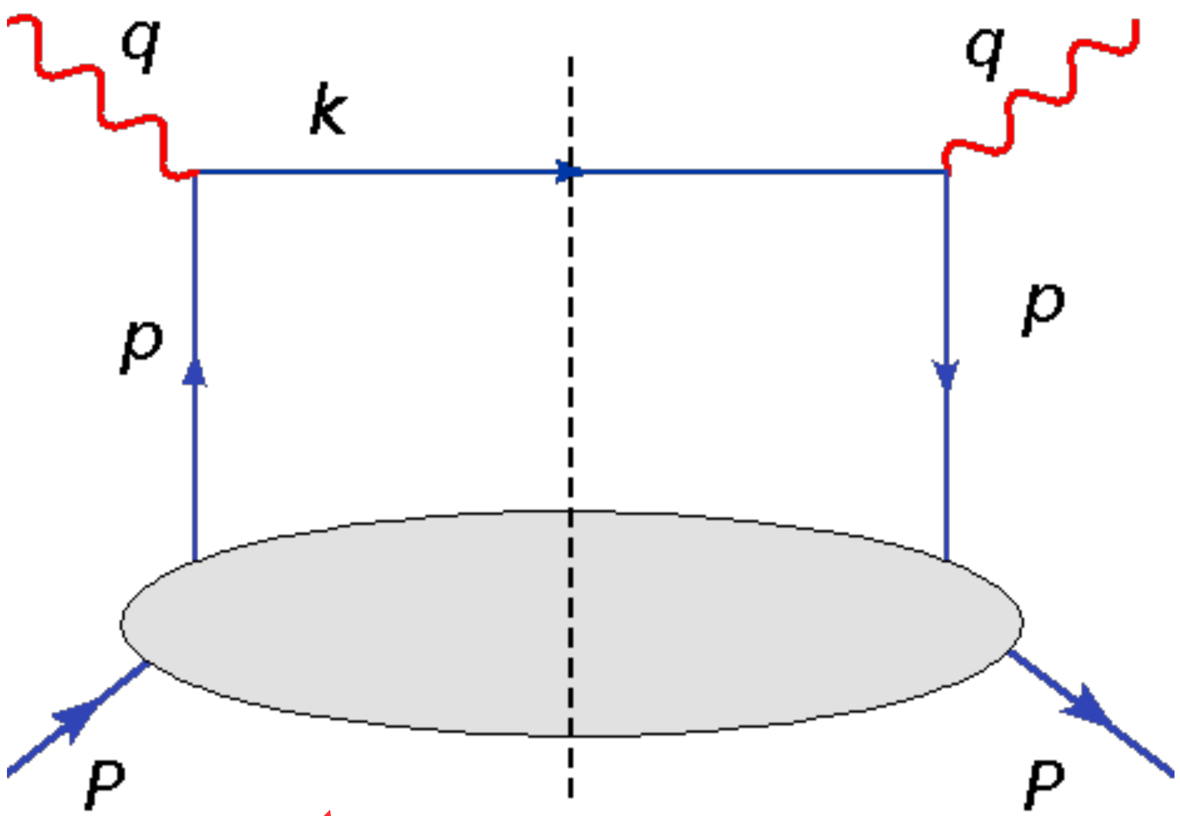


CONSTANT!

$$E'\frac{d\sigma}{d^3\mathbf{l}'} = \frac{2\alpha_{em}}{sQ^4} L_{\mu\nu} W^{\mu\nu}$$



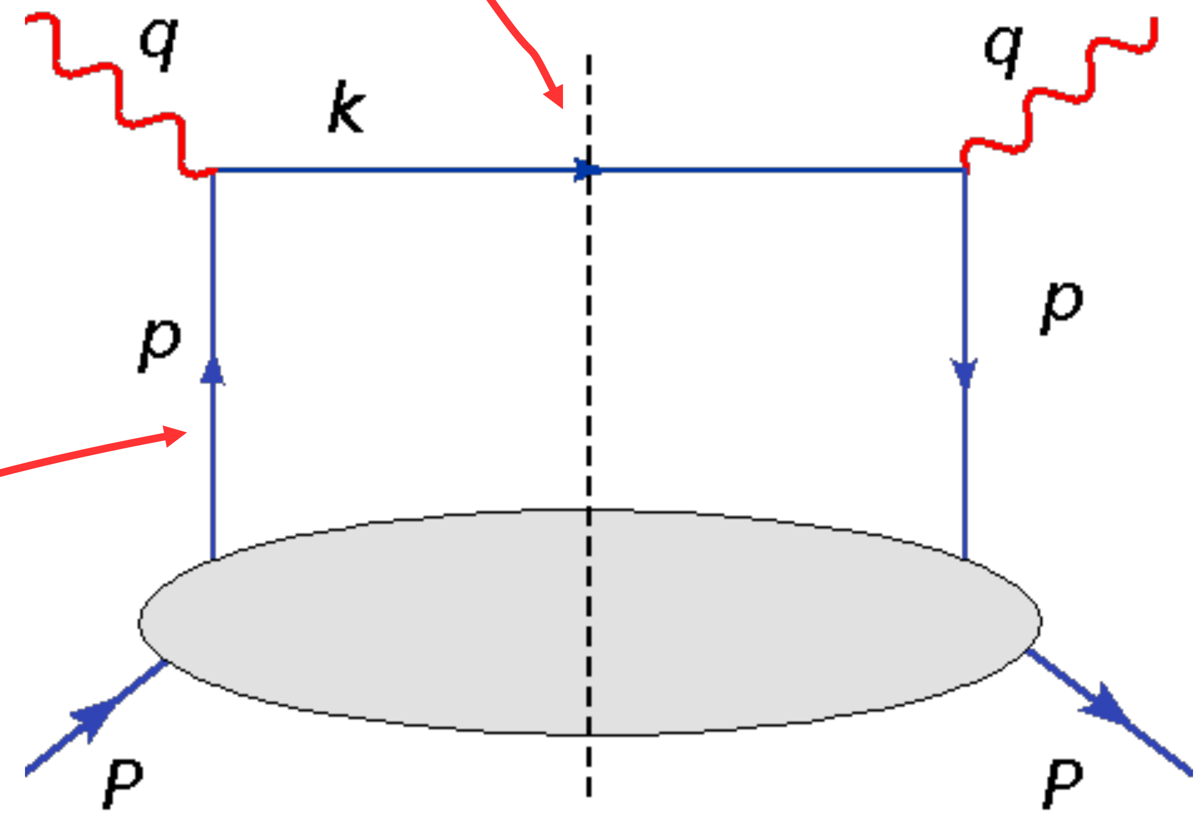
This diagram is called "handbag diagram"



$$\mathbf{\Phi(p, P)}$$ - parton correlator

Why quarks are on mass-shell?

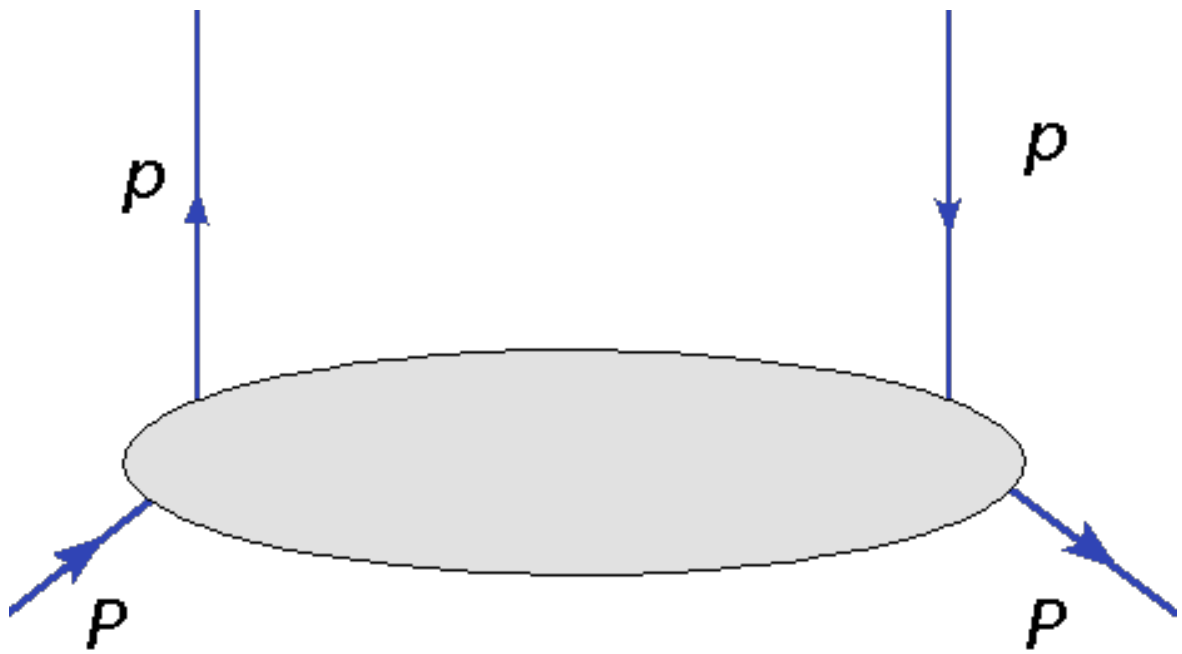
$$Im\left(\frac{1}{k^2+i\epsilon}\right)=\pi\delta(k^2) \qquad \Rightarrow \quad k^2\approx 0$$



This one is virtual! However the main contribution comes from

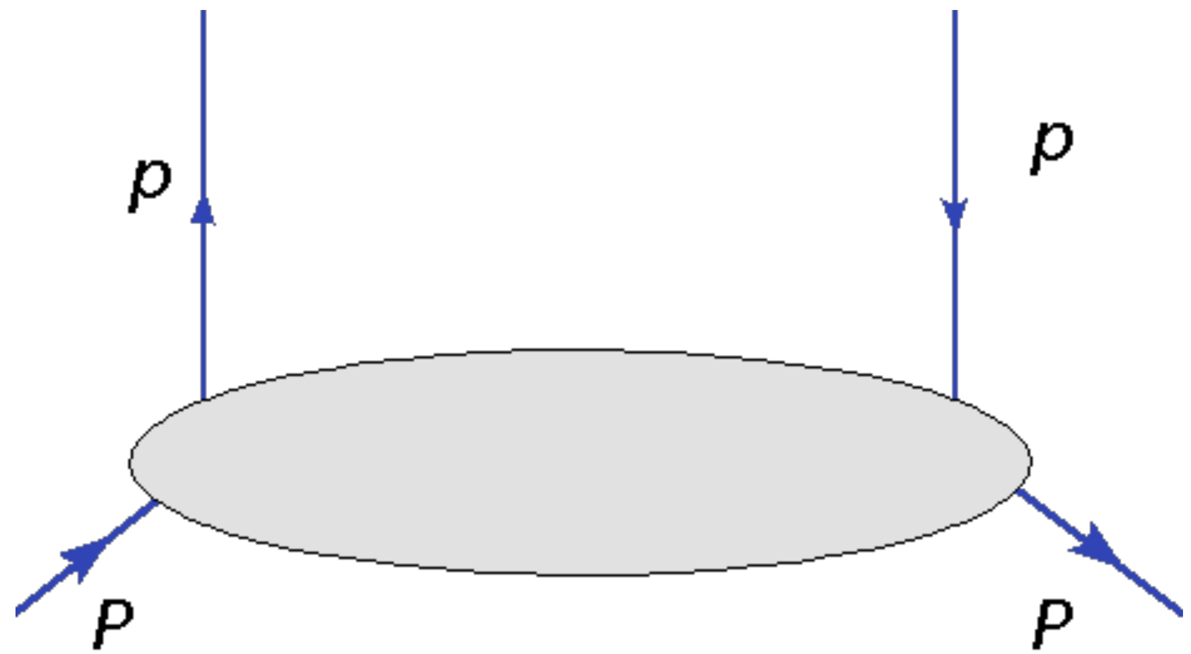
$$\int d^4p\left(\frac{1}{p^2+i\epsilon}\right)\left(\frac{1}{p^2-i\epsilon}\right) \quad \Rightarrow \quad p^2\approx 0$$

Definition of parton correlator



$$\Phi_{ij}(p,P) = \int \frac{d\xi^+ d\xi^- d^2\xi_T}{(2\pi)^4}\, e^{ip\cdot\xi}\, \langle P, S_P|\bar{\psi}_j(0)\psi_i(\xi)|P,S_P\rangle$$

Definition of parton distribution



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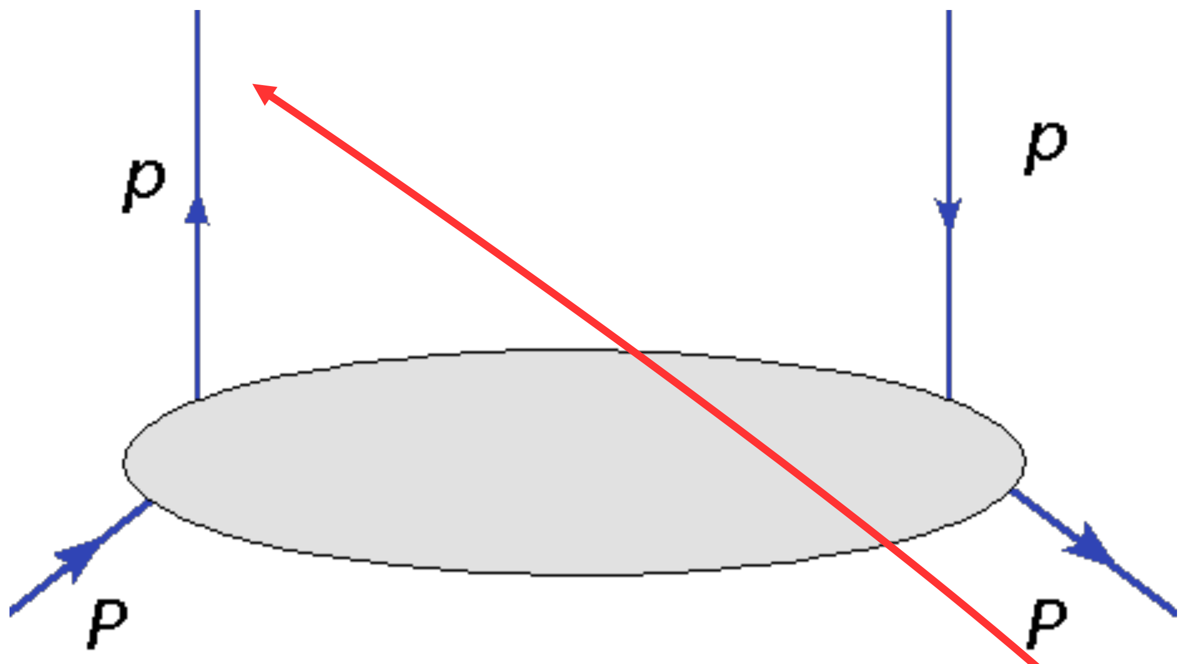
Fourier transform from coordinate to momentum space

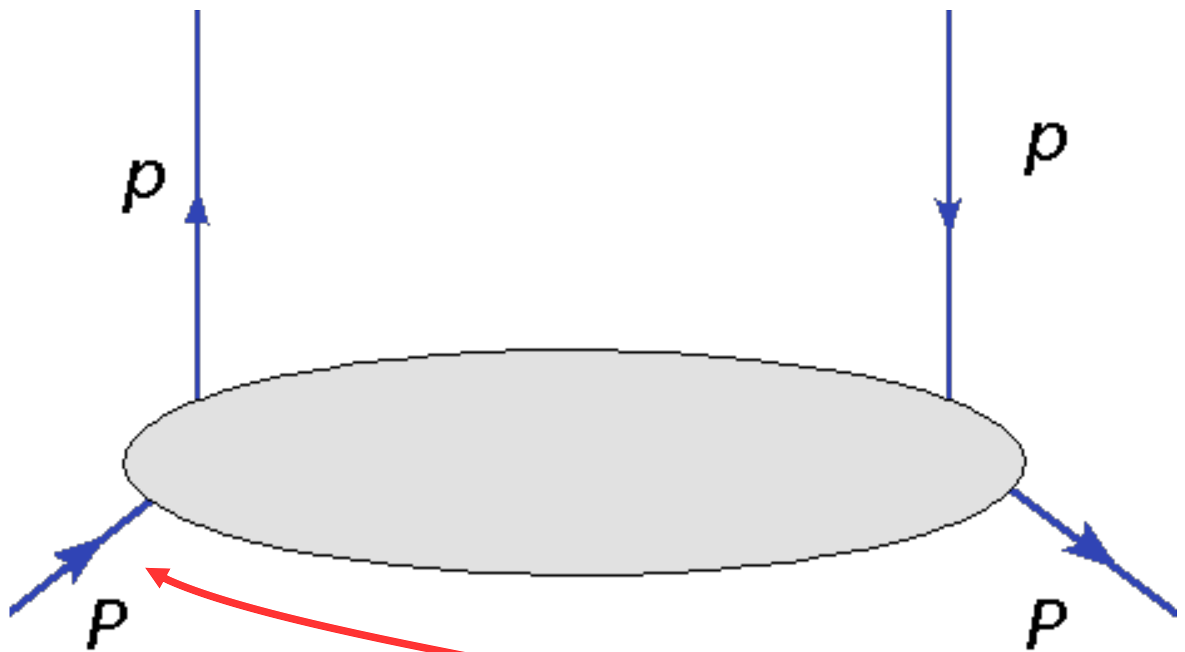
Definition of parton distribution



$$\Phi_{ij}(p,P) = \int \frac{d\xi^+ d\xi^- d^2\xi_T}{(2\pi)^4}\, e^{ip\cdot\xi}\, \langle P, S_P | \bar{\psi}_j(0)\psi_i(\xi) | P, S_P\rangle$$

Quark field operator

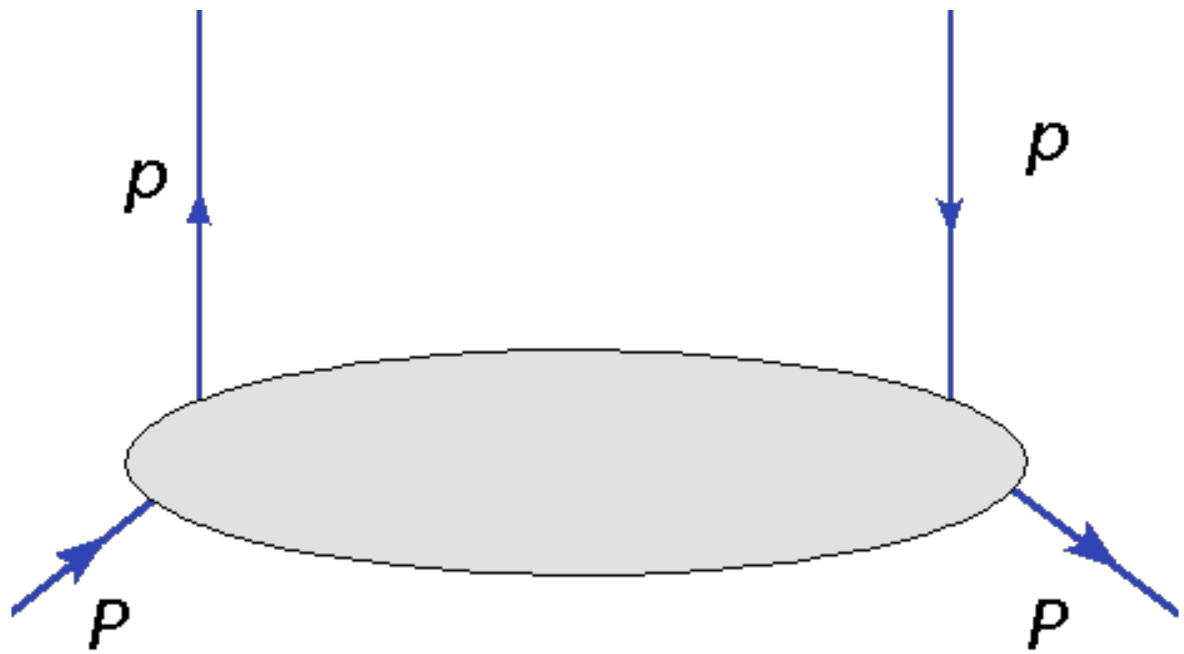
Definition of parton distribution



$$\Phi_{ij}(p,P) = \int \frac{d\xi^+ d\xi^- d^2\xi_T}{(2\pi)^4}\, e^{ip\cdot\xi}\, \langle P, S_P | \bar{\psi}_j(0)\psi_i(\xi) | P, S_P \rangle$$

The proton state vector

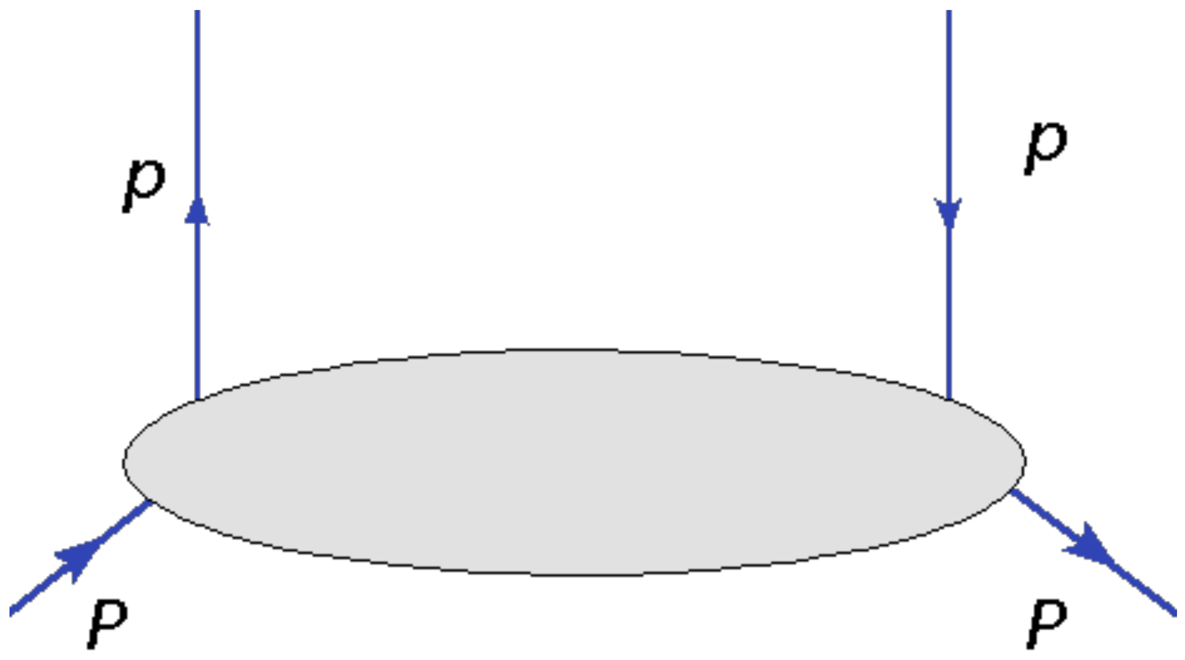
Definition of parton distribution



$$\Phi_{ij}(p,P) = \int \frac{d\xi^+ d\xi^- d^2\xi_T}{(2\pi)^4}\, e^{ip\cdot\xi}\, \langle P, S_P | \bar{\psi}_j(0)\psi_i(\xi) | P, S_P\rangle$$

Position of the field in coordinate space

Definition of parton distribution



$$\Phi_{ij}(p,P) = \int \frac{d\xi^+ d\xi^- d^2\xi_T}{(2\pi)^4}\, e^{ip\cdot\xi}\, \underline{\langle P, S_P|\bar{\psi}_j(0)\psi_i(\xi)|P,S_P\rangle}$$

This matrix element is called "bilocal"

What do we know about quark momentum? Suppose that proton is moving along Z direction with a high momentum, then

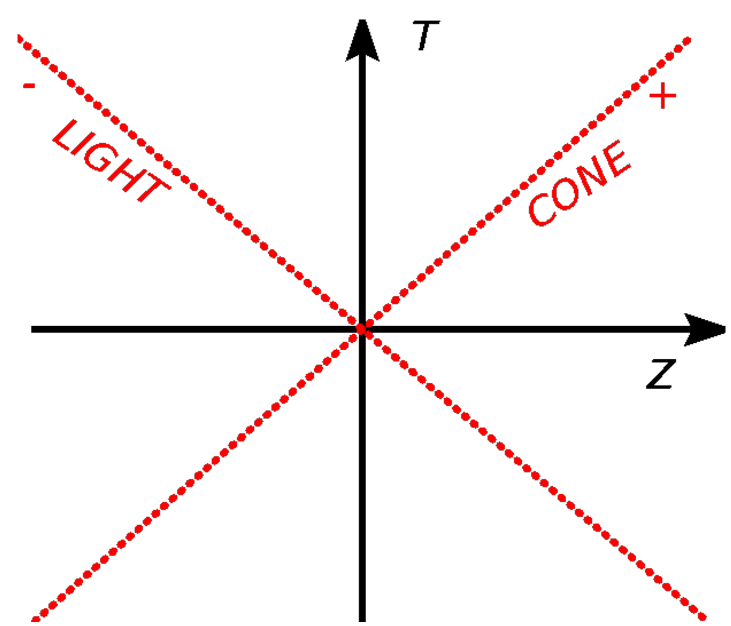
$$p^\mu = xP^+ n_+^\mu + \frac{p^2 + \mathbf{p}_\perp{}^2}{2xP^+} n_-^\mu + p_\perp^\mu$$

"Big"component $\sim Q$

$x = p^+/P^+$ is a new variable called lightcone momentum fraction

$$P^+ = \frac{1}{\sqrt{2}}\left(P^0 + P^z\right)$$

$$P^- = \frac{1}{\sqrt{2}}\left(P^0 - P^z\right)$$



What do we know about quark momentum?

$$p^\mu = xP^+ n_+^\mu + \frac{p^2 + \mathbf{p}_\perp{}^2}{2xP^+} n_-^\mu + p_\perp^\mu$$

"Big" component $\sim Q$

"Small" component $\sim 1/Q$

PennState Berks

What do we know about quark momentum?

$$p^\mu = xP^+ n_+^\mu + \frac{p^2 + \mathbf{p}_\perp{}^2}{2xP^+} n_-^\mu + p_\perp^\mu$$

"Big"component $\sim Q$

"Small" component $\sim 1/Q$

"Transverse" component $\sim \Lambda_{QCD}$

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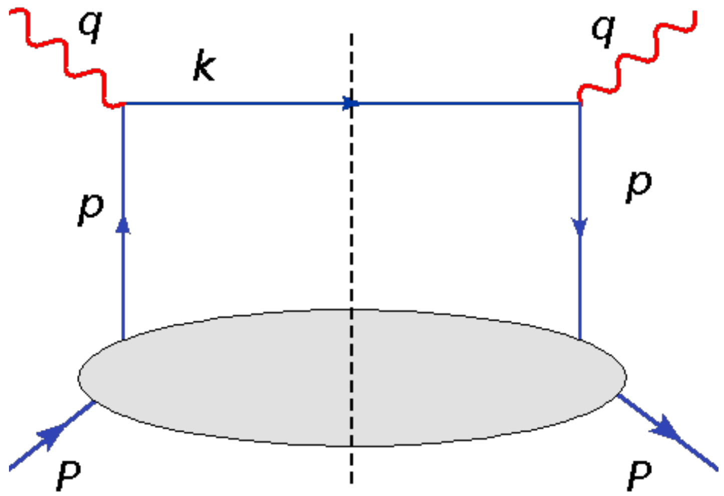
What do we know about hadronic tensor?

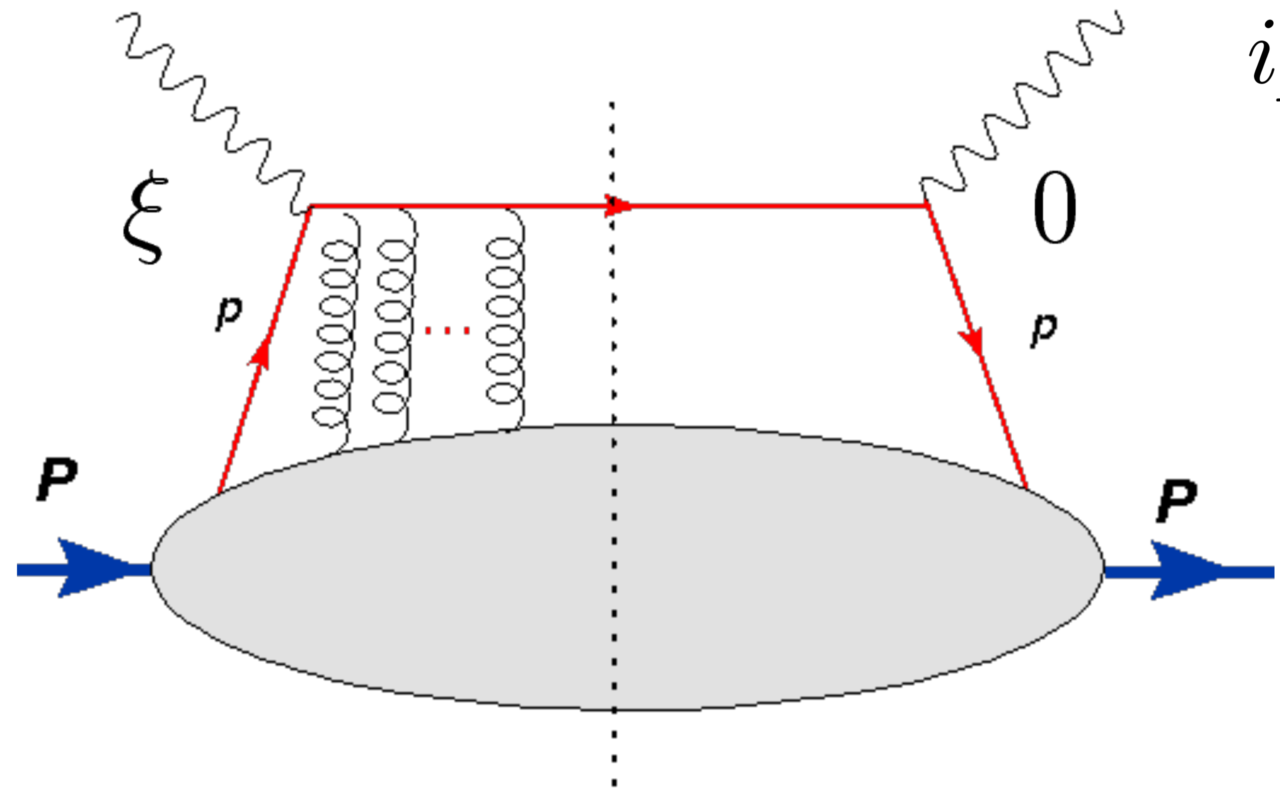


$$W^{\mu\nu} = \sum_q e_q^2 \int \frac{d^4p}{(2\pi)^4} Tr(\gamma^\mu(\not{p} + \not{q})\gamma^\nu \Phi(P,p))\delta((p+q)^2)$$

$$\delta((p+q)^2) \approx \delta(-Q^2 + 2xP\cdot q) = \frac{1}{2P\cdot q}\delta(x_{Bj} - x) \ ,$$

Quarks are "probed" at value of x_{Bj}

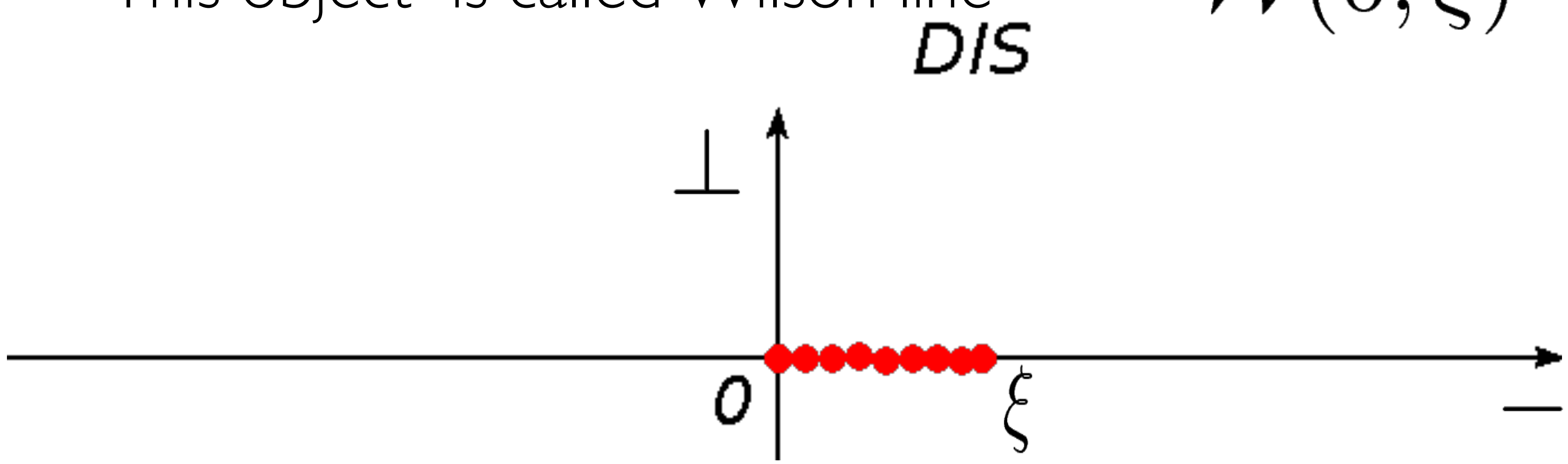
The quark and the remnant are colored thus they interact via gluon exchanges! If "–" and perpendicular component of parton momentum are neglected, then in configuration space only "–" component survives,

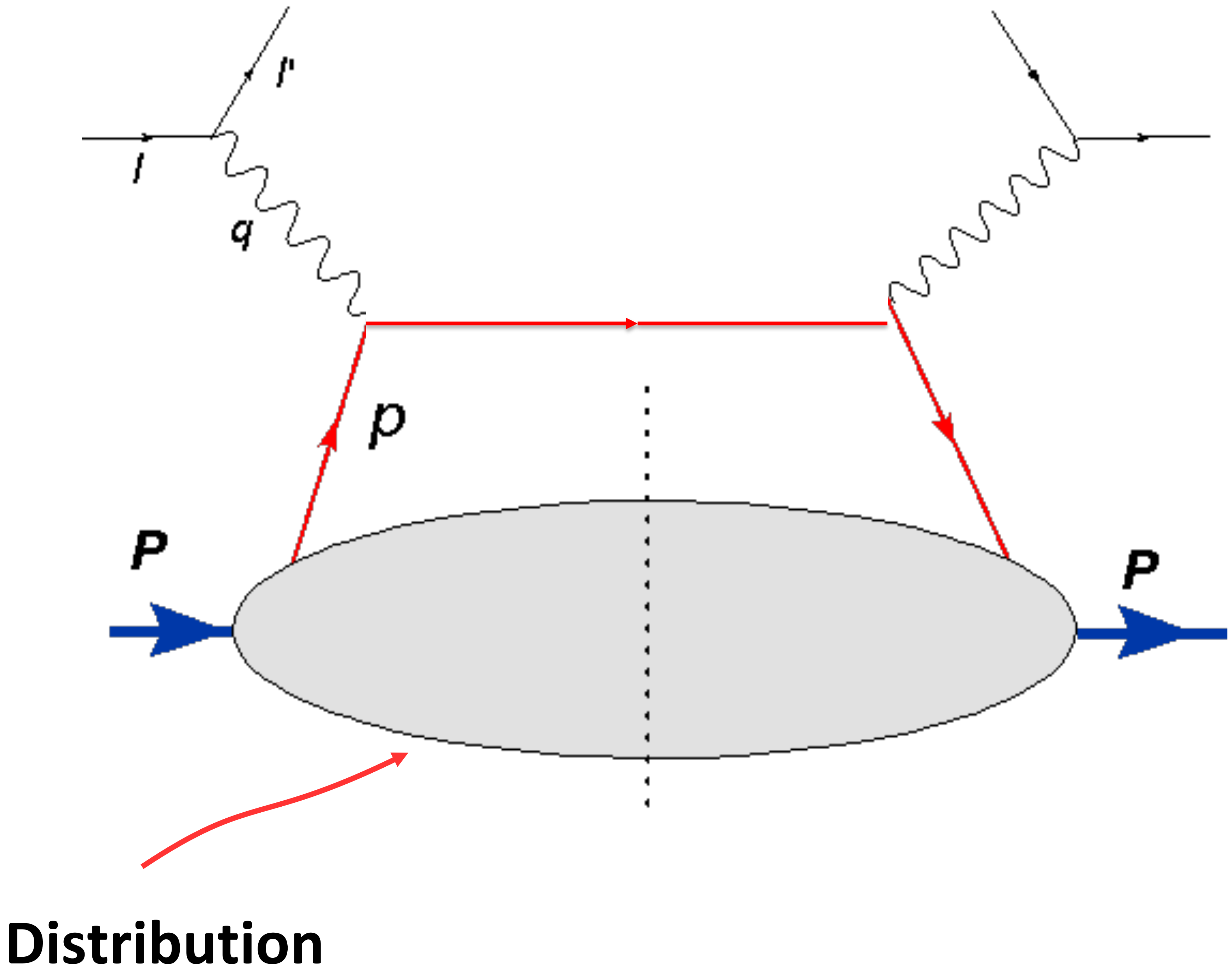


$$ip \cdot \xi = ixP^+\xi^-$$

This object is called Wilson line $\mathcal{W}(0,\xi)$

For DIS:





σ **DIS**

$||$

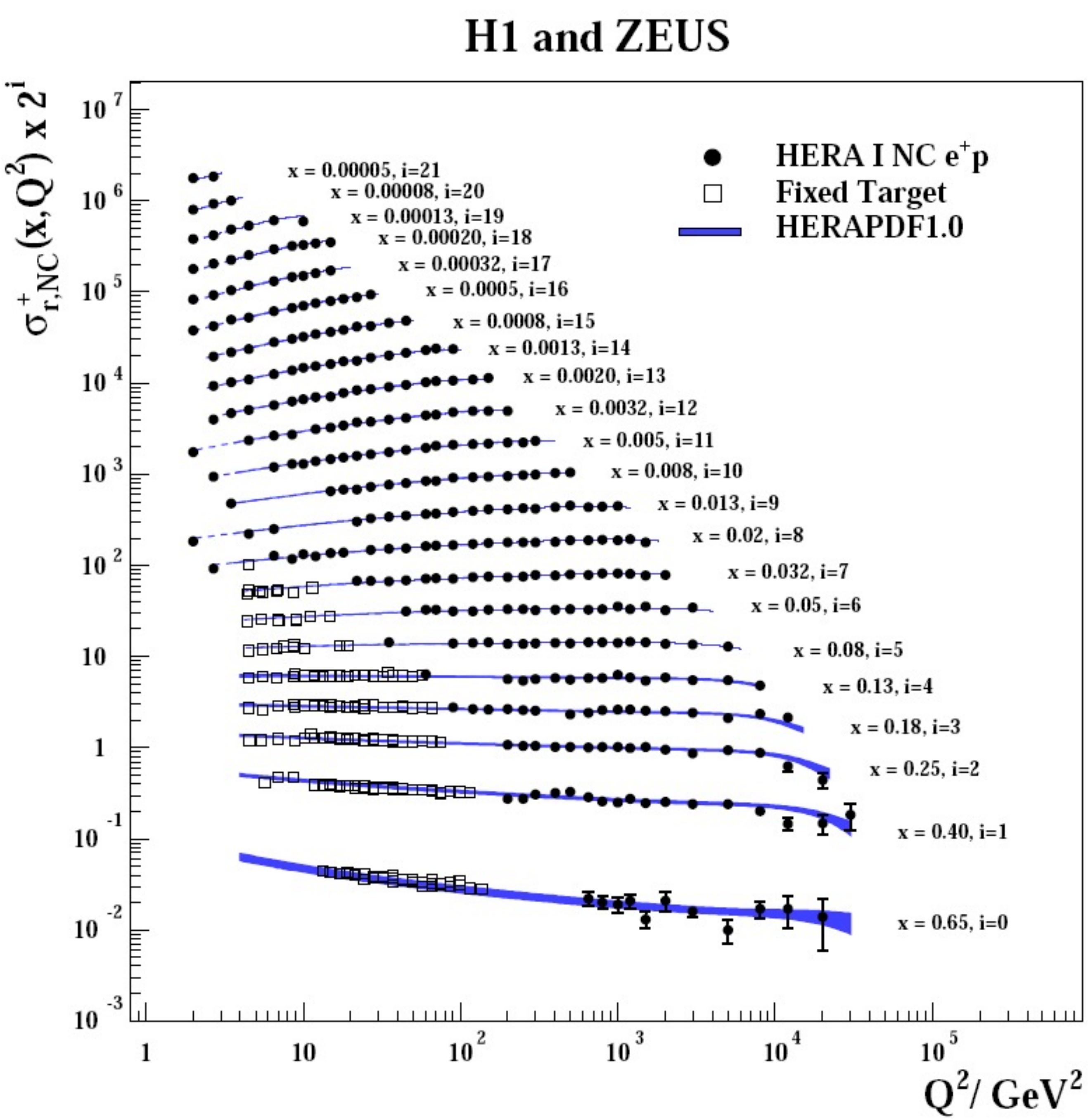
$\hat{\sigma}_{\mathbf{lq \to l'q'}}$

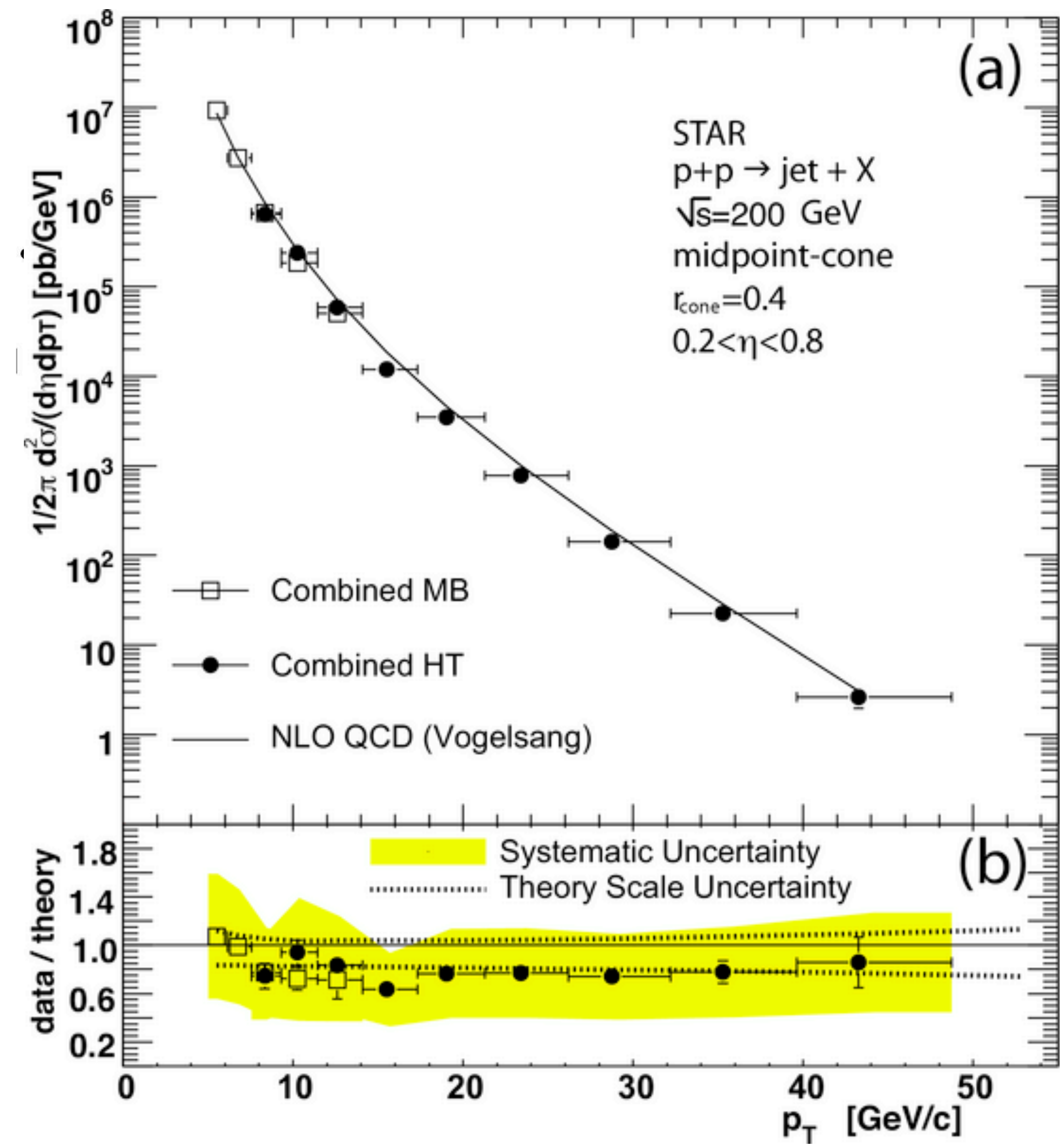
$\otimes$

$\mathbf{f_{q/P}}$

Success of QCD factorization

- Universality of PDFs: mapped in one process (say DIS), used in other processes





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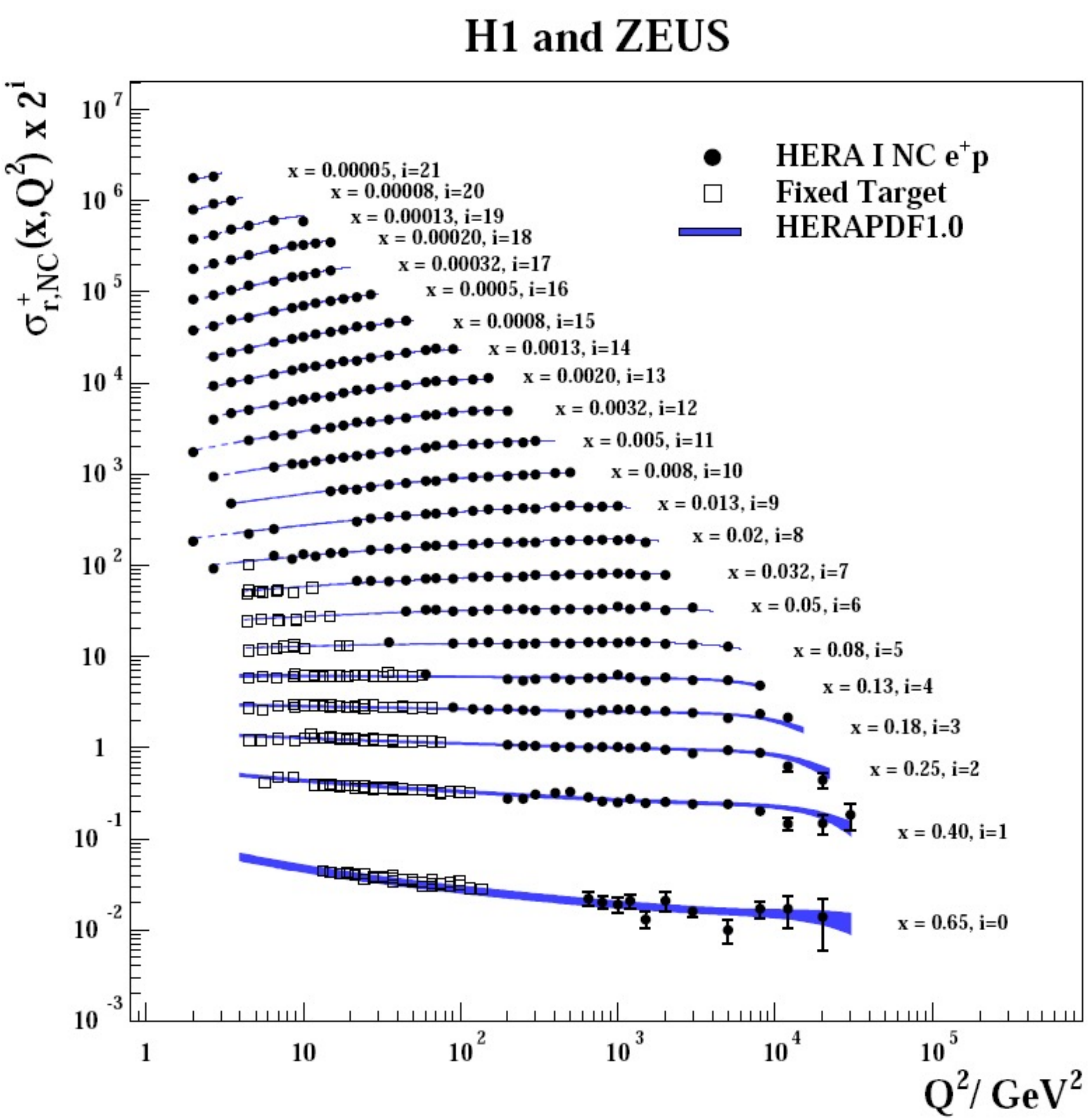

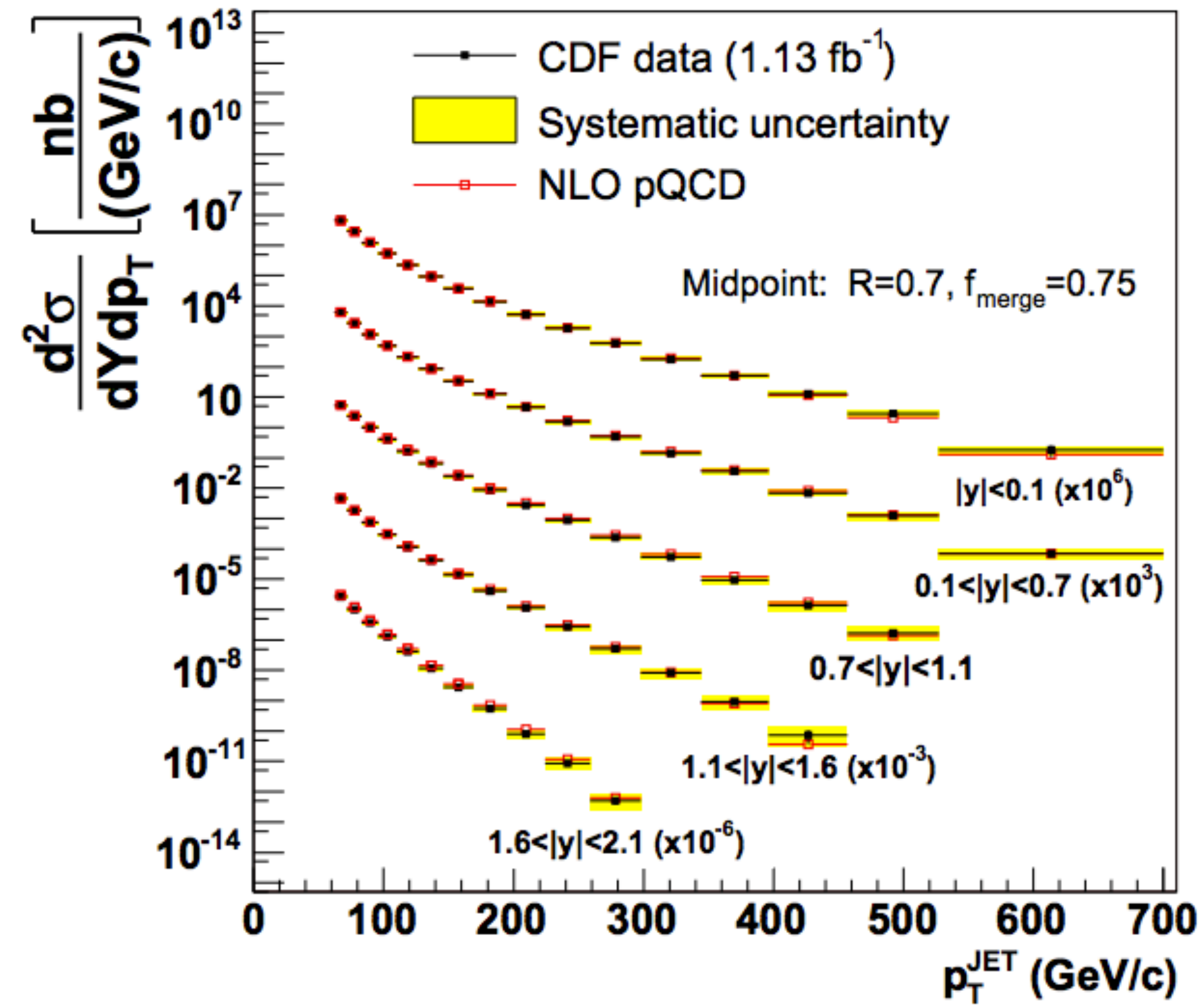

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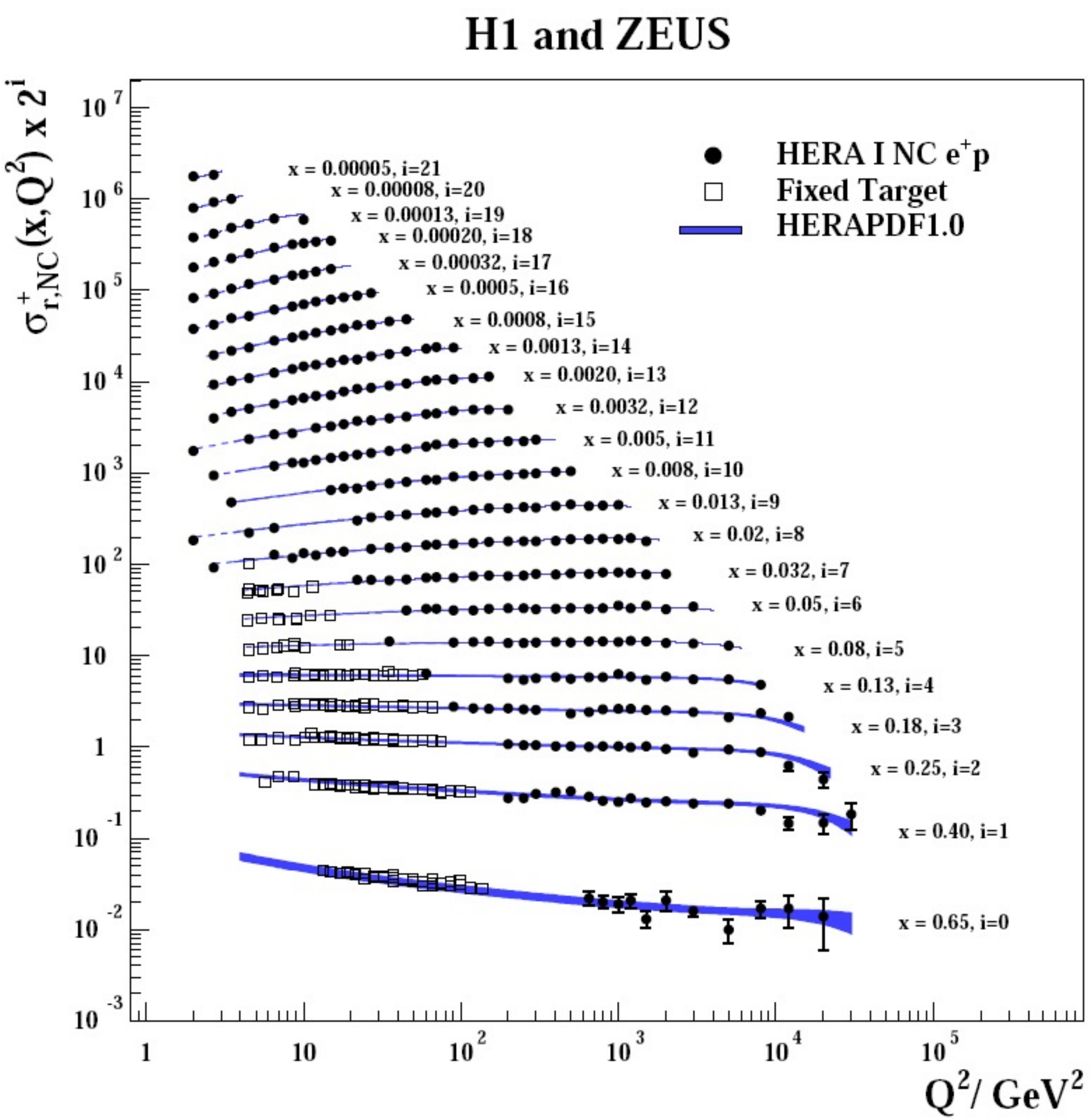

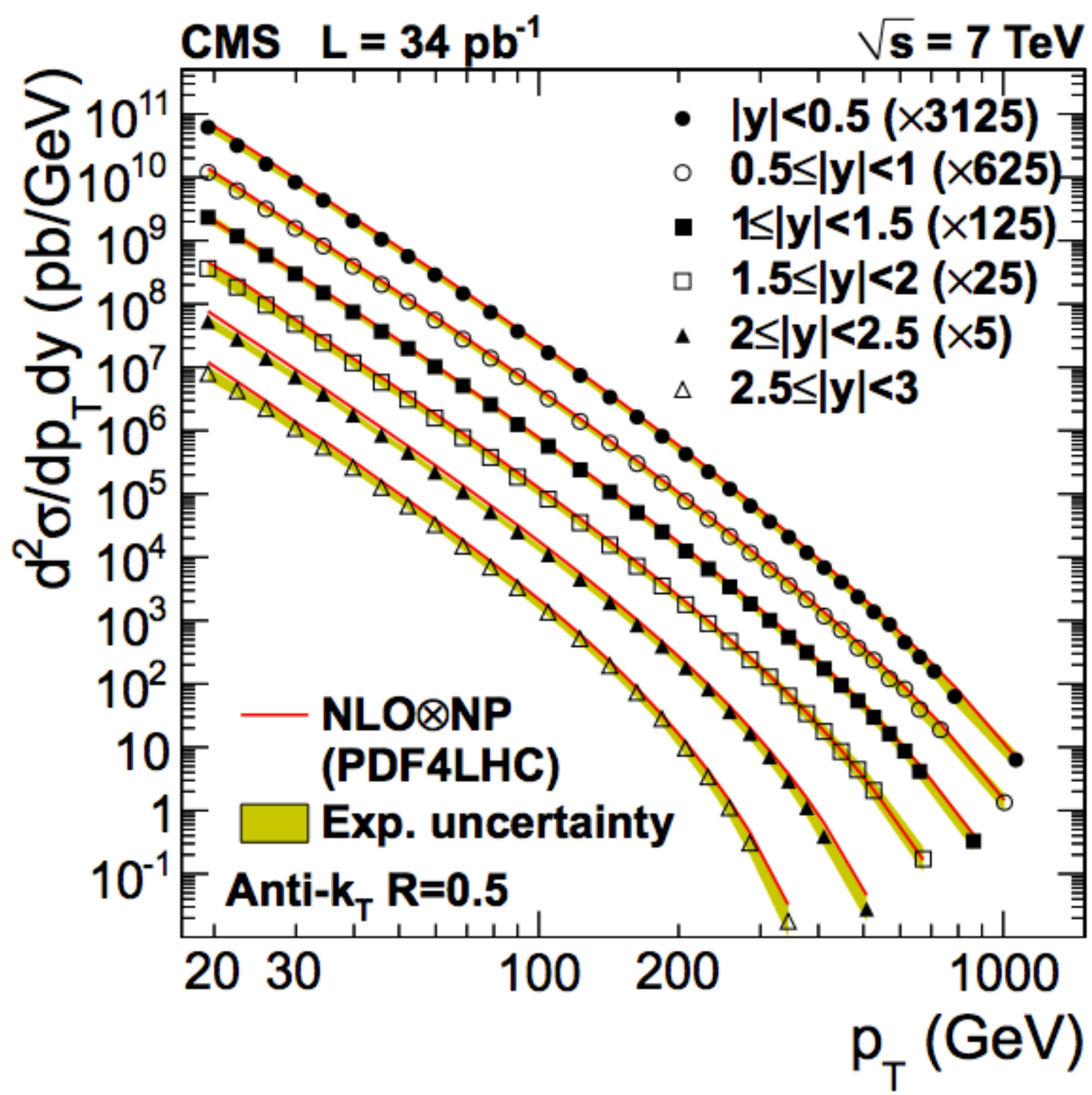

Success of QCD factorization

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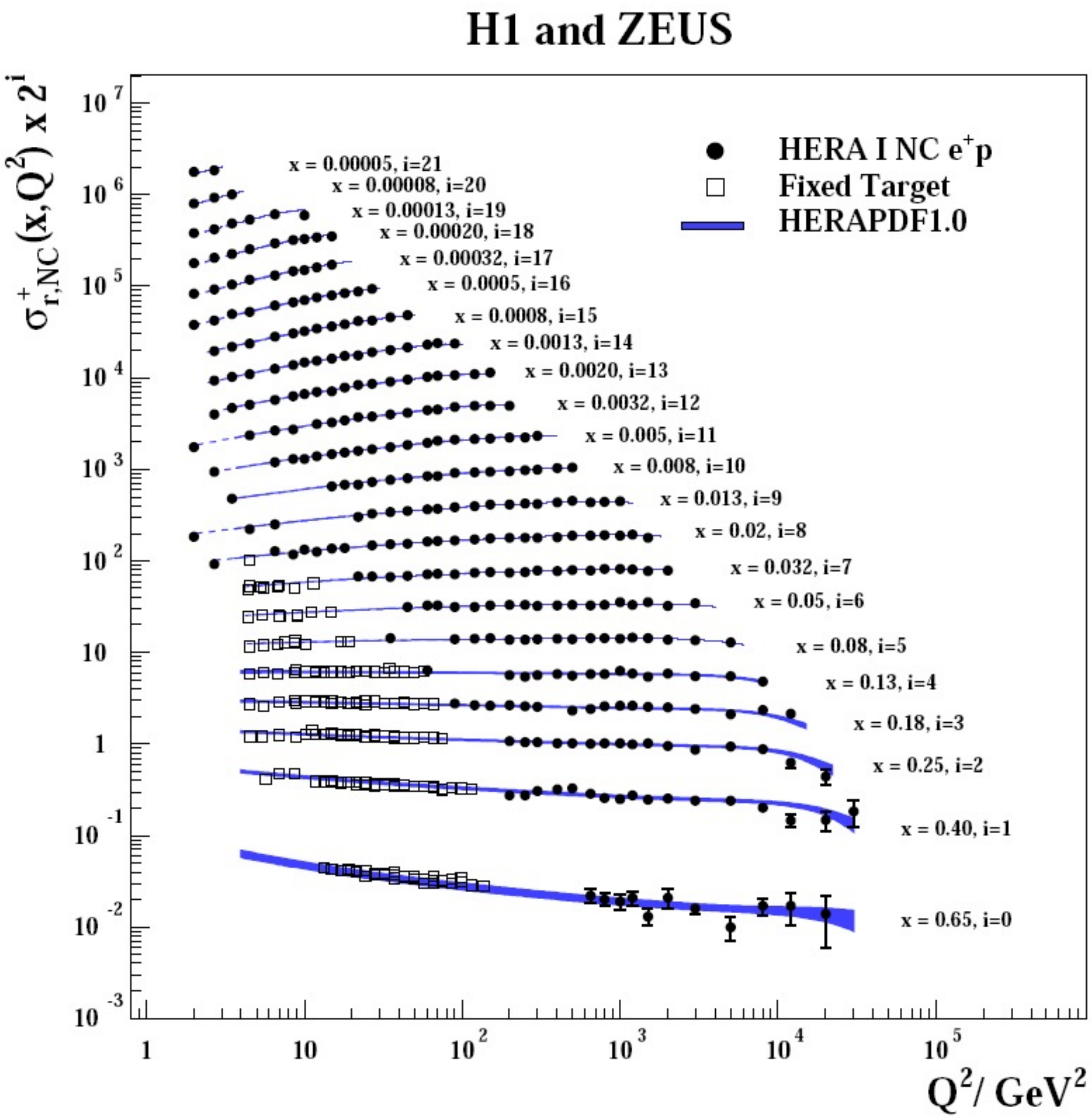
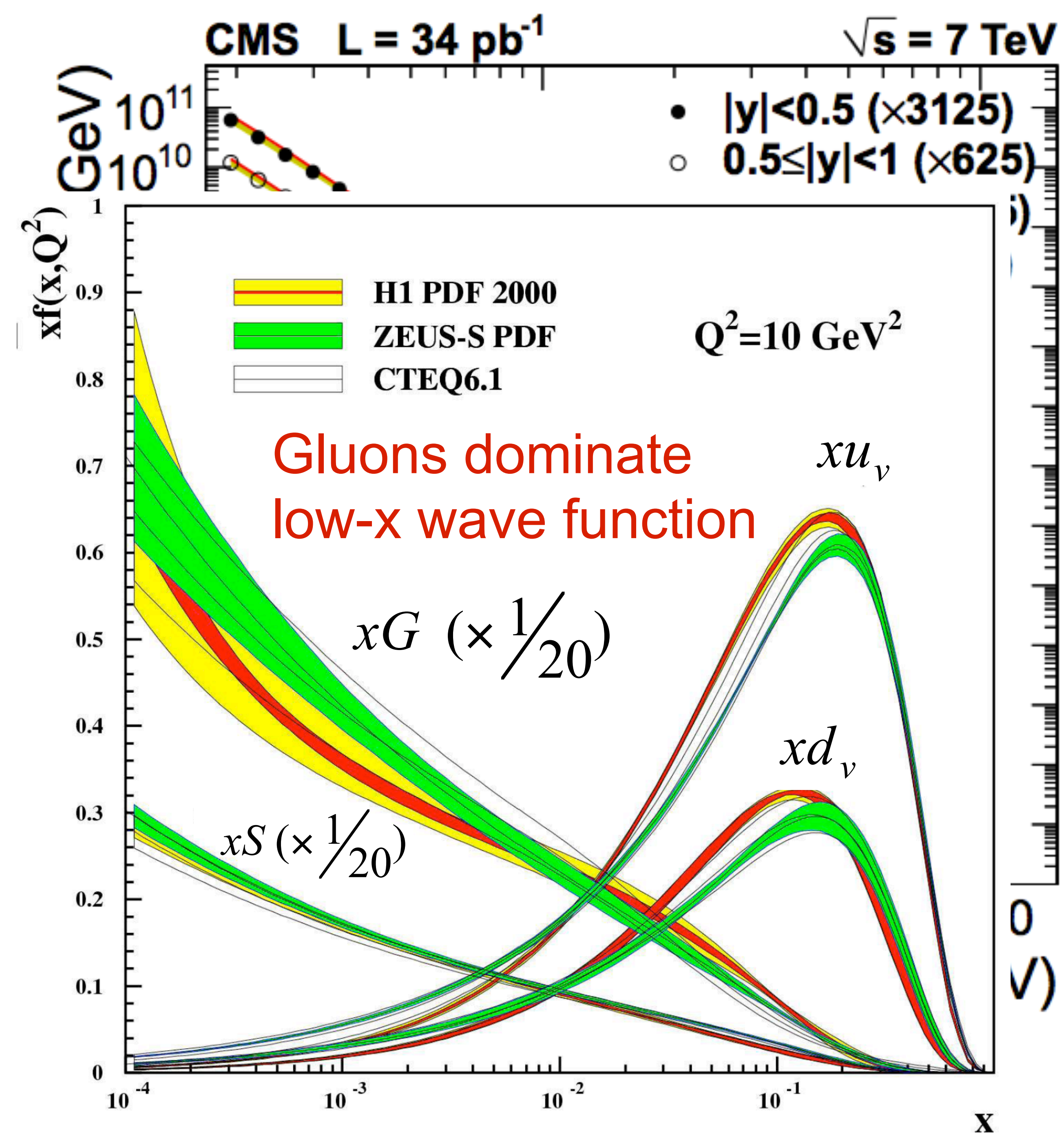




Let us go beyond collinear picture

Transverse structure: Momentum vs Position

Variables are related by 2-dimensional Fourier transform

$$\bar{\tilde{\psi}}(k_\perp, z^-) = \int d^2 z_\perp e^{-iz_\perp k_\perp} \bar{\psi}(z_\perp, z^-)$$

At the level of squared amplitudes, one has

$$\bar{\tilde{\psi}}(k_\perp)\tilde{\psi}(l_\perp) = \int d^2 z_\perp d^2 y_\perp e^{-i(z_\perp k_\perp - y_\perp l_\perp)} \bar{\psi}(z_\perp)\psi(y_\perp)$$

$$z_\perp k_\perp - y_\perp l_\perp = \frac{1}{2}(z_\perp - y_\perp)(k_\perp + l_\perp) + \frac{1}{2}(z_\perp + y_\perp)(k_\perp - l_\perp)$$

The ‘average’ transverse momentum is Fourier conjugate to position difference (TMD)

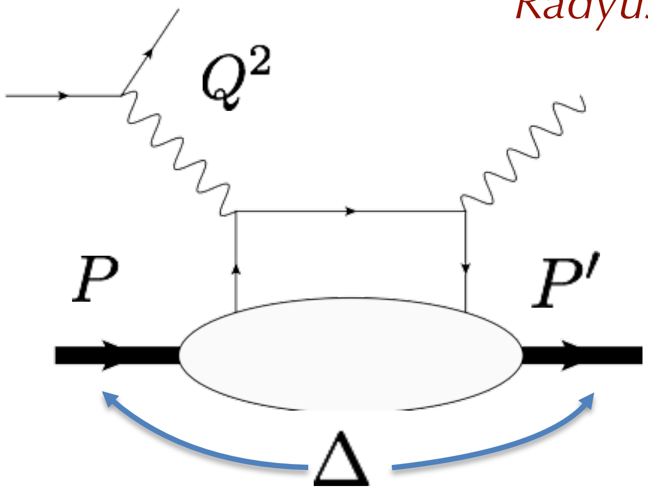
The momentum transfer is Fourier conjugate to ‘average’ position (GPD)

GPDs TMDs

DVCS

Ji (1997)

Radyushkin (1997)



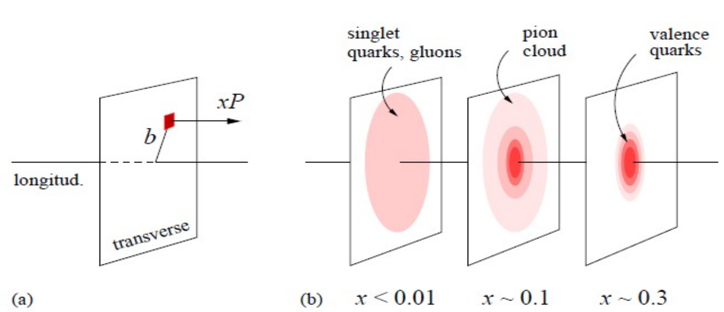
Q^2 ensures hard scale, pointlike interaction

$\Delta = P' - P$ momentum transfer can be varied independently

Connection to 3D structure Burkardt (2000) Burkardt (2003)

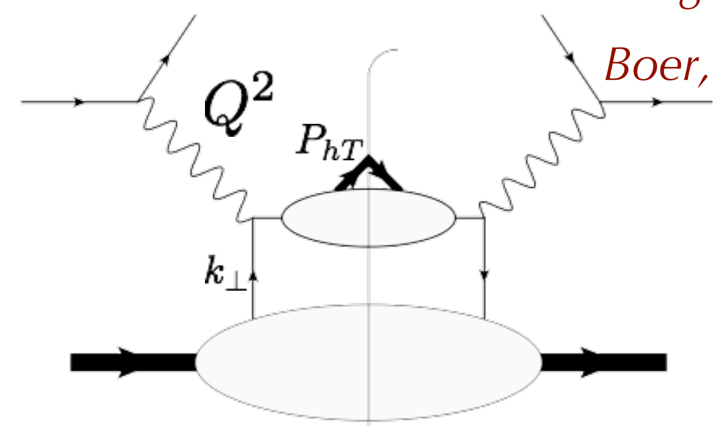
$$\rho(x, \vec{r}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{r}_\perp} H_q(x, \xi = 0, t = -\vec{\Delta}_\perp^2)$$

Drell-Yan frame $\Delta^+ = 0$ Weiss (2009)



SIDIS

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)



Imaginary part, momentum transfer is zero

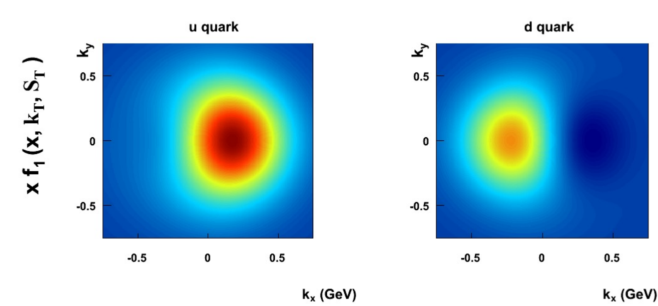
Q^2 ensures hard scale, pointlike interaction

P_{hT} final hadron transverse momentum can be varied independently

Connection to 3D structure Ji, Ma, Yuan (2004) Collins (2011)

$$\tilde{f}(x, \vec{b}) = \int d^2k_\perp e^{-i\vec{b}\cdot\vec{k}_\perp} f(x, \vec{k}_\perp)$$

$\vec{b}$ is the transverse separation of parton fields in configuration space

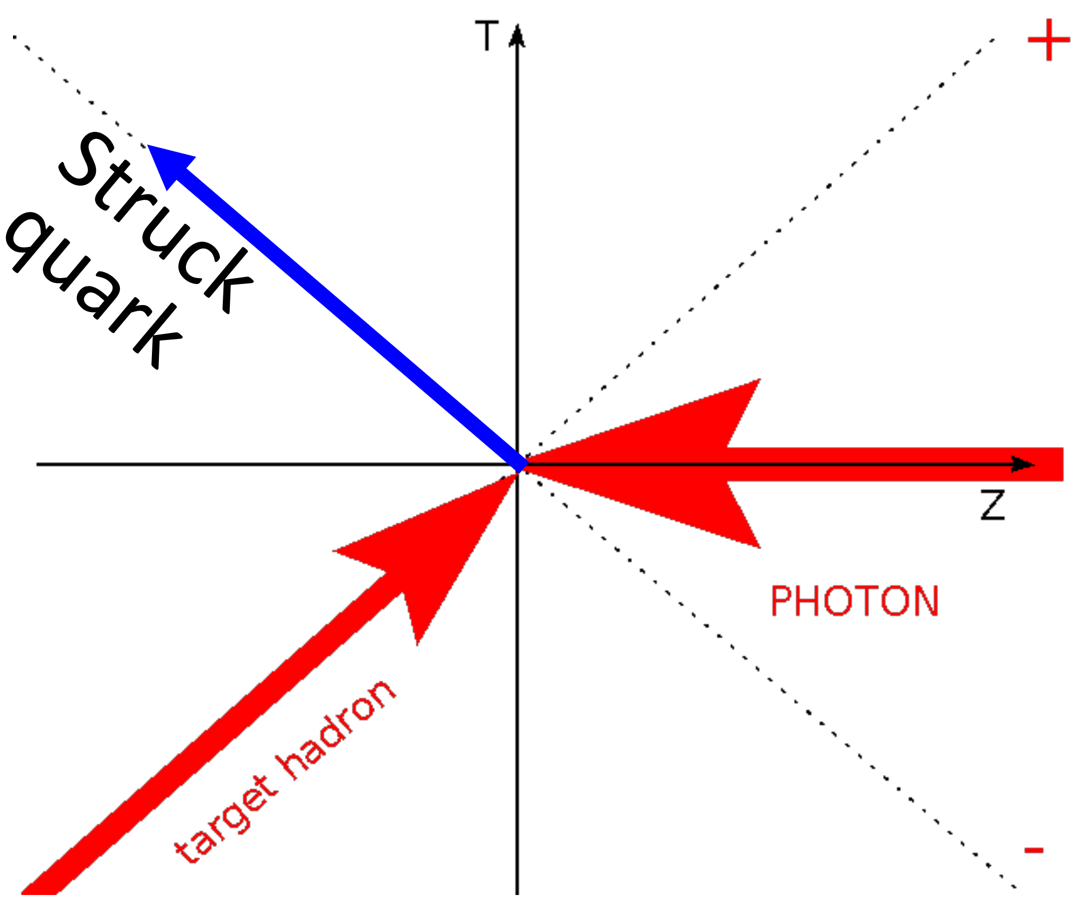


AP (2012)

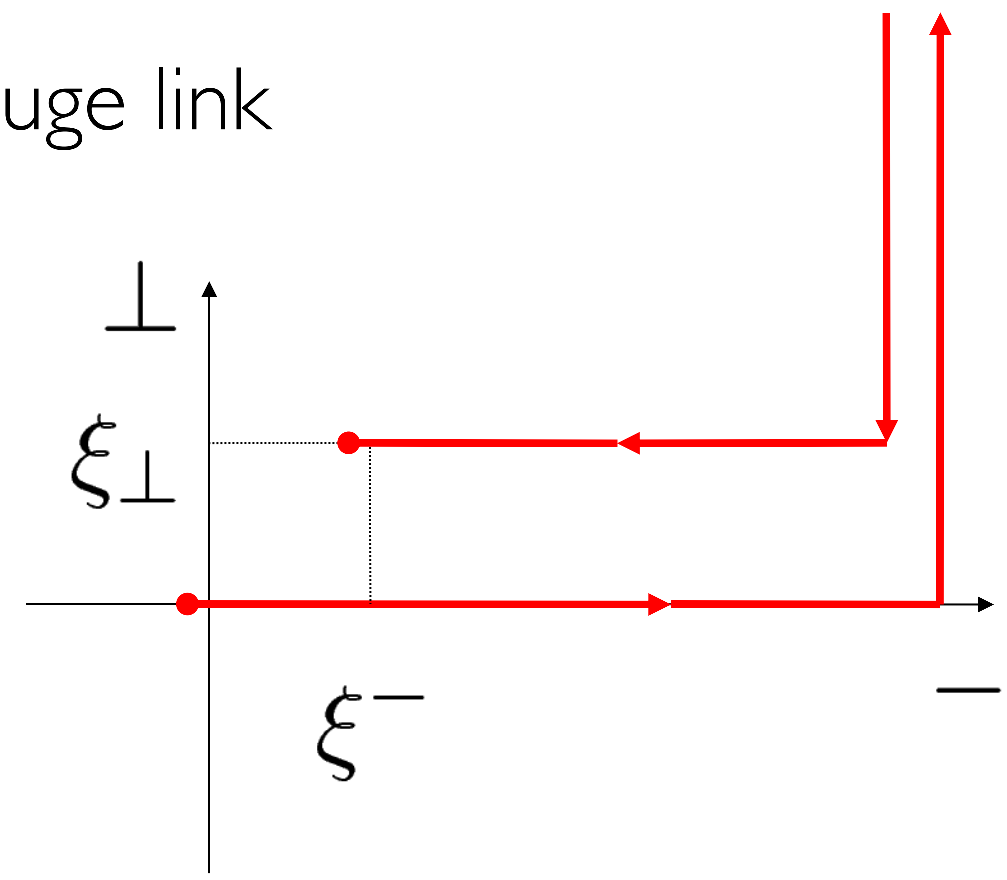
Transverse Momentum Dependent distributions

$$\Phi_{ij}(x,\mathbf{k}_\perp) = \int \frac{d\xi^-}{(2\pi)}\frac{d^2\xi_\perp}{(2\pi)^2}\, e^{ixP^+\xi^- - i\mathbf{k}_\perp\xi_\perp} \langle P, S_P|\bar\psi_j(0)\mathcal{U}(\mathbf{0},\xi)\psi_i(\xi)|P,S_P\rangle|_{\xi^+=0}$$

SIDIS in IMF:



Gauge link



$$\mathcal{U}(a,b;n) = e^{-ig\int_a^b d\lambda n\cdot A_\alpha(\lambda n)t_\alpha}$$

Ensures gauge invariance of the distribution, cannot be canceled by gauge choice

Transverse Momentum Dependent distributions

Individual TMDs can be projected out of the correlator

Unpolarized quarks

$$\frac{1}{2}\,\mathrm{Tr}\left[\gamma^+\,\Phi(x,k_\perp)\right] = f_1 - \frac{\varepsilon^{jk}k_\perp^j S_T^k}{M_N}\,f_{1T}^\perp$$

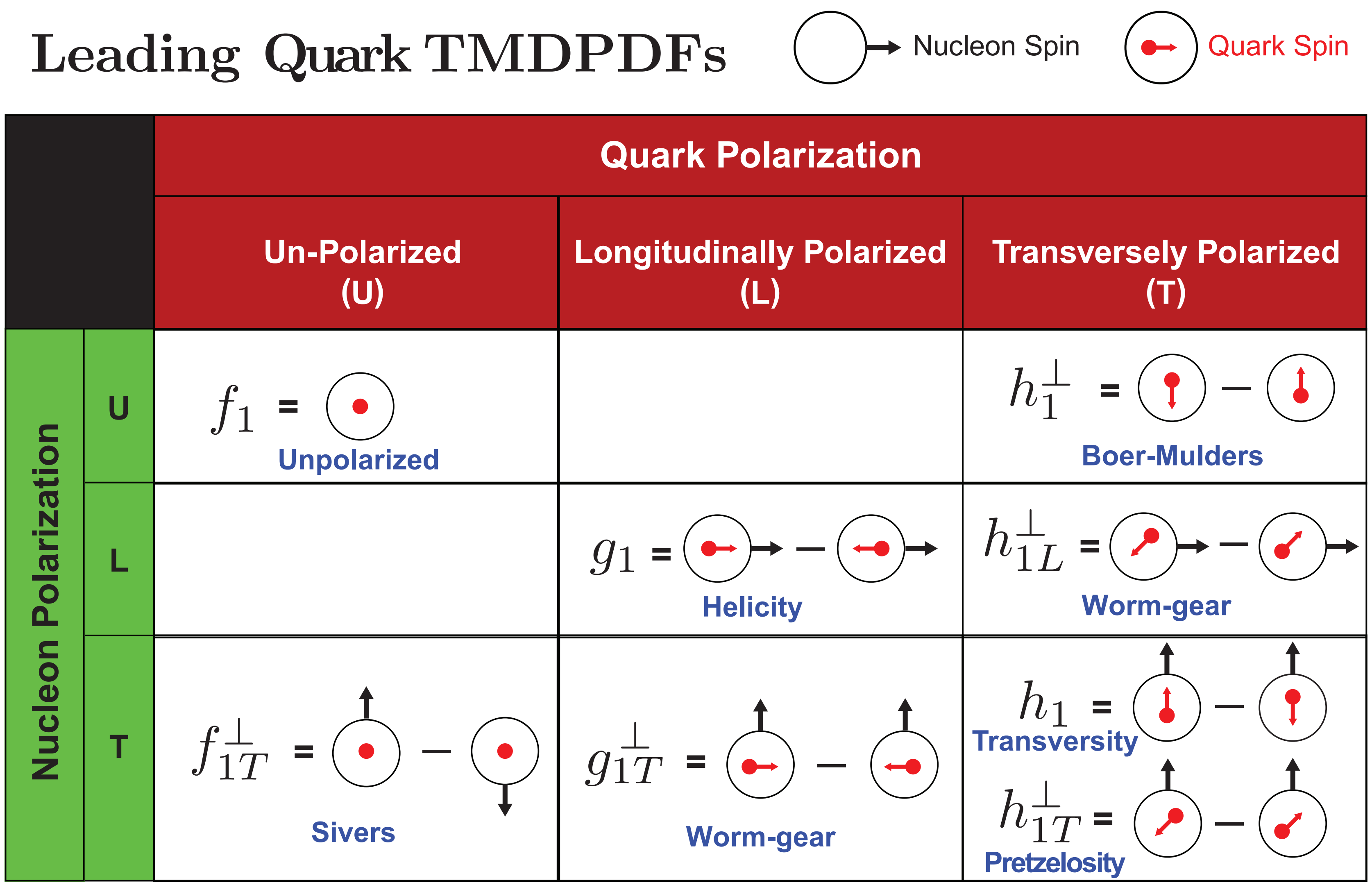
Longitudinally polarized quarks

$$\frac{1}{2}\,\mathrm{Tr}\left[\gamma^+\gamma_5\,\Phi(x,k_\perp)\right] = S_L\,g_1 + \frac{k_\perp\cdot S_T}{M_N}\,g_{1T}^\perp$$

Transversely polarized quarks

$$\frac{1}{2}\,\mathrm{Tr}\left[i\sigma^{j+}\gamma^+\,\Phi(x,k_\perp)\right] = S_T^j\,h_1 + S_L\,\frac{k_\perp^j}{M_N}\,h_{1L}^\perp + \frac{\kappa^{jk}S_T^k}{M_N^2}\,h_{1T}^\perp + \frac{\varepsilon^{jk}k_\perp^k}{M_N}\,h_1^\perp$$

$$\kappa^{jk} \equiv (k_\perp^j k_\perp^k - \frac{1}{2}\,k_\perp^2\delta^{jk})$$



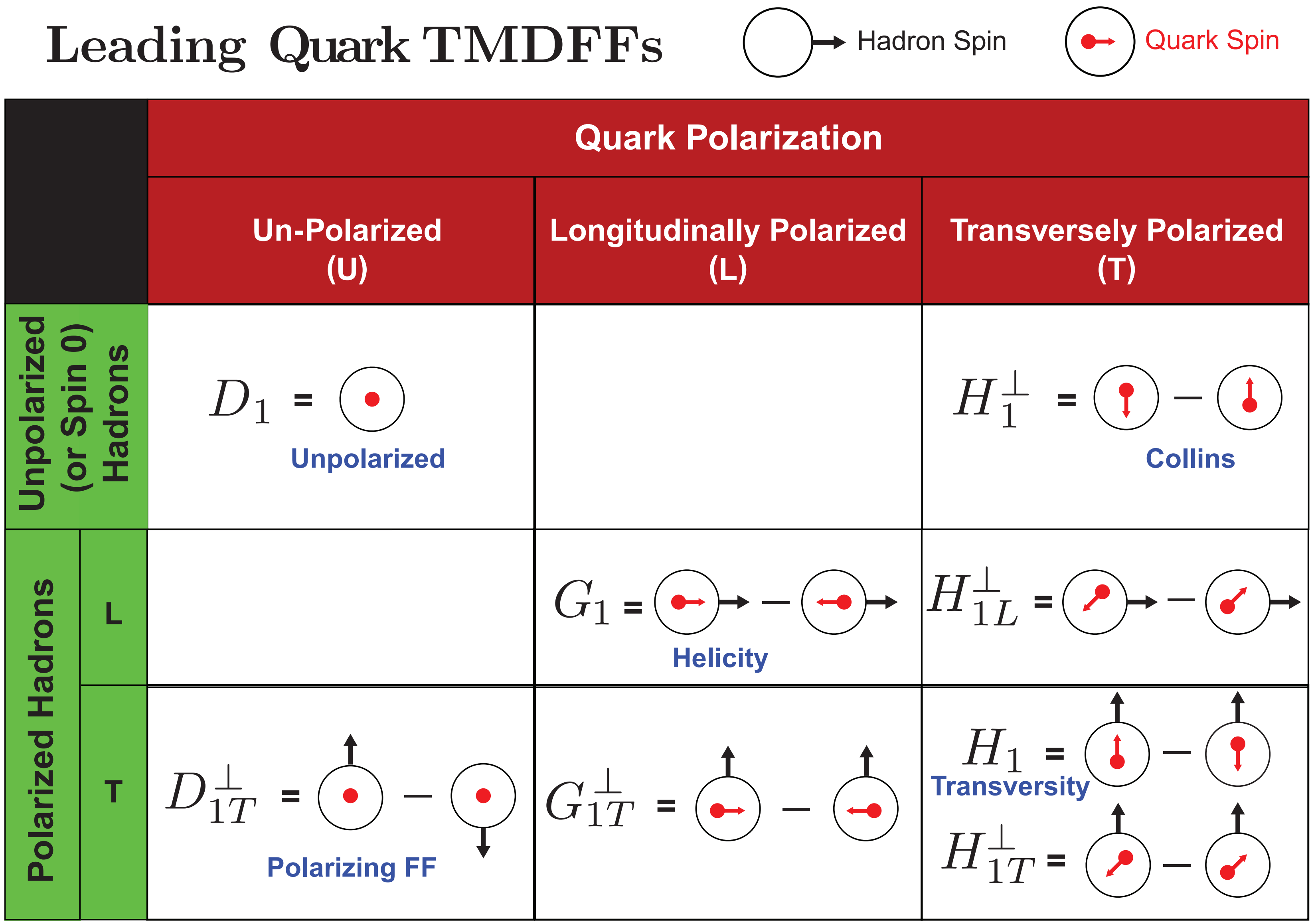
8 functions in total (at leading twist)

Each represents different aspects of partonic structure

Each depends on Bjorken-x, transverse momentum, the scale

Each function is to be studied

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)



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More at higher twist!

$$\frac{1}{2}\mathrm{Tr}\Big[1\;\Phi(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\left[\quad e - \frac{\varepsilon^{jk}k_\perp^j S_T^k}{M_N}\,e_T^\perp\right],$$

$$\frac{1}{2}\mathrm{Tr}\Big[i\gamma_5\Phi(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\left[S_L e_L + \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_T}{M_N}\,e_T\right],$$

$$\frac{1}{2}\mathrm{Tr}\Big[\gamma^j\,\Phi(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\left[\frac{k_\perp^j}{M_N}f^\perp + \varepsilon^{jk}S_T^k f_T + S_L\frac{\varepsilon^{jk}k_\perp^k}{M_N}f_L^\perp - \frac{\kappa^{jk}\varepsilon^{kl}S_T^l}{M_N^2}f_T^\perp\right],$$

$$\frac{1}{2}\mathrm{Tr}\Big[\gamma^j\gamma_5\Phi(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\left[S_T^j\,g_T + S_L\,\frac{k_\perp^j}{M_N}g_L^\perp + \frac{\kappa^{jk}S_T^k}{M_N^2}\,g_T^\perp + \frac{\varepsilon^{jk}k_\perp^k}{M_N}\,g^\perp\right],$$

$$\frac{1}{2}\mathrm{Tr}\Big[i\,\sigma^{jk}\gamma_5\Phi(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\left[\frac{S_T^j k_\perp^k - S_T^k k_\perp^j}{M_N}\,h_T^\perp - \varepsilon^{jk}\,h\right],$$

$$\frac{1}{2}\mathrm{Tr}\Big[i\,\sigma^{+-}\,\gamma_5\,\Phi(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\left[S_L\,h_L + \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_T}{M_N}\,h_T\right].$$

More at higher twist!

STAMP COLLECTING?

$$\frac{1}{2}\mathrm{Tr}\Big[1\,\Phi(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\Big[\quad e - \frac{\varepsilon^{jk}k^j \ldots}{\ldots}\ldots\Big]$$

$$\frac{1}{2}\mathrm{Tr}\Big[i\gamma_5\Phi(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\Big[S_L \ldots \frac{\ldots}{\ldots M_N}\, e_T\Big],$$

$$\frac{1}{2}\mathrm{Tr}\Big[\gamma^j\,\Phi(x,\boldsymbol{k}_\perp)\Big] = \frac{M_{\ldots}}{\ldots}\Big[\ldots + \varepsilon^{jk}S_T^k f_T + S_L\frac{\varepsilon^{jk}k_\perp^k}{M_N}f_L^\perp - \frac{\kappa^{jk}\varepsilon^{kl}S_T^l}{M_N^2}f_T^\perp\Big],$$

$$\frac{1}{2}\mathrm{Tr}\Big[\gamma^j\gamma_5\Phi(x,\ldots\Big] = \frac{\ldots}{\ldots^+}\Big[S_T^j\,g_T + S_L\,\frac{k_\perp^j}{M_N}g_L^\perp + \frac{\kappa^{jk}S_T^k}{M_N^2}\,g_T^\perp + \frac{\varepsilon^{jk}k_\perp^k}{M_N}\,g^\perp\Big],$$

$$\frac{1}{2}\mathrm{Tr}\Big[i\,\sigma^{jk}\ldots)\Big] = \frac{M_N}{P^+}\Big[\frac{S_T^j k_\perp^k - S_T^k k_\perp^j}{M_N}\,h_T^\perp - \varepsilon^{jk}\,h\Big],$$

$$\frac{1}{2}\mathrm{Tr}\Big[\ldots(x,\boldsymbol{k}_\perp)\Big] = \frac{M_N}{P^+}\Big[S_L\,h_L + \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_T}{M_N}\,h_T\Big].$$

PennState Berks

We are in a good company

Group

Metal
Metalloid
Nonmetal

Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H Hydrogen 1.0079																	2 He Helium 4.003
2	3 Li Lithium 6.941	4 Be Beryllium 9.012											5 B Boron 10.811	6 C Carbon 12.011	7 N Nitrogen 14.007	8 O Oxygen 15.999	9 F Fluorine 18.998	10 Ne Neon 20.180
3	11 Na Sodium 22.990	12 Mg Magnesium 24.305											13 Al Aluminum 26.982	14 Si Silicon 28.086	15 P Phosphorus 30.974	16 S Sulfur 32.066	17 Cl Chlorine 35.453	18 Ar Argon 39.948
4	19 K Potassium 39.098	20 Ca Calcium 40.078	21 Sc Scadium 44.956	22 Ti Titanium 47.88	23 V Vanadium 50.942	24 Cr Chromium 51.996	25 Mn Manganese 54.938	26 Fe Iron 55.845	27 Co Cobalt 58.933	28 Ni Nickel 58.69	29 Cu Copper 63.546	30 Zn Zinc 65.39	31 Ga Gallium 69.723	32 Ge Germanium 72.61	33 As Arsenic 74.922	34 Se Selenium 78.96	35 Br Bromine 79.904	36 Kr Krypton 83.8
5	37 Rb Rubidium 85.468	38 Sr Strontium 87.62	39 Y Yttrium 88.906	40 Zr Zirconium 91.224	41 Nb Niobium 92.906	42 Mo Molybdenum 95.94	43 Tc Technetium (98)	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.906	46 Pd Palladium 106.42	47 Ag Silver 107.868	48 Cd Cadmium 112.411	49 In Indium 114.82	50 Sn Tin 118.71	51 Sb Antimony 121.76	52 Te Tellurium 127.60	53 I Iodine 126.905	54 Xe Xenon 131.29
6	55 Cs Cesium 132.905	56 Ba Barium 137.327	57 La Lanthanum 138.906	72 Hf Hafnium 178.49	73 Ta Tantalum 180.948	74 W Tungsten 183.84	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.22	78 Pt Platinum 195.08	79 Au Gold 196.967	80 Hg Mercury 200.59	81 Tl Thallium 204.383	82 Pb Lead 207.2	83 Bi Bismuth 208.980	84 Po Polonium (209)	85 At Astatine (210)	86 Rn Radon (222)
7	87 Fr Francium (223)	88 Ra Radium 226.025	89 Ac Actinium 227.028	104 Rf Rutherfordium (261)	105 Db Dubnium (262)	106 Sg Seaborgium (266)	107 Bh Bohrium (264)	108 Hs Hassium (269)	109 Mt Meitnerium (268)	110 Ds Darmstadtium (271)	111 Rg Roentgenium (272)	112 Cn Copernicum (285)	113 Uut	114 Fl Flerovium 289	115 Uup	116 Lv Livermorium 293	117 Uus	118 Uuo

Lanthanides	58 Ce Cerium 140.115	59 Pr Praseodymium 140.908	60 Nd Neodymium 144.24	61 Pm Promethium (145)	62 Sm Samarium 150.36	63 Eu Europium 151.964	64 Gd Gadolinium 157.25	65 Tb Terbium 158.925	66 Dy Dyspsrosium 162.5	67 Ho Holmium 164.93	68 Er 68 Erbium 167.26	69 Tm Thulium 168.934	70 Yb Ytterbium 173.04	71 Lu Lutetium 174.967
Actinides	90 Th Thorium 232.038	91 Pa Protactinium 231.036	92 U Uranium 238.029	93 Np Neptunium 237.05	94 Pu Plutonium (244)	95 Am Americium (243)	96 Cm Curium (247)	97 Bk Berkelium (247)	98 Cf Californium (251)	99 Es Einsteinium (252)	100 Fm Fermium (257)	101 Md Mendelevium (258)	102 No Nobelium (259)	103 Lr Lawrencium (262)