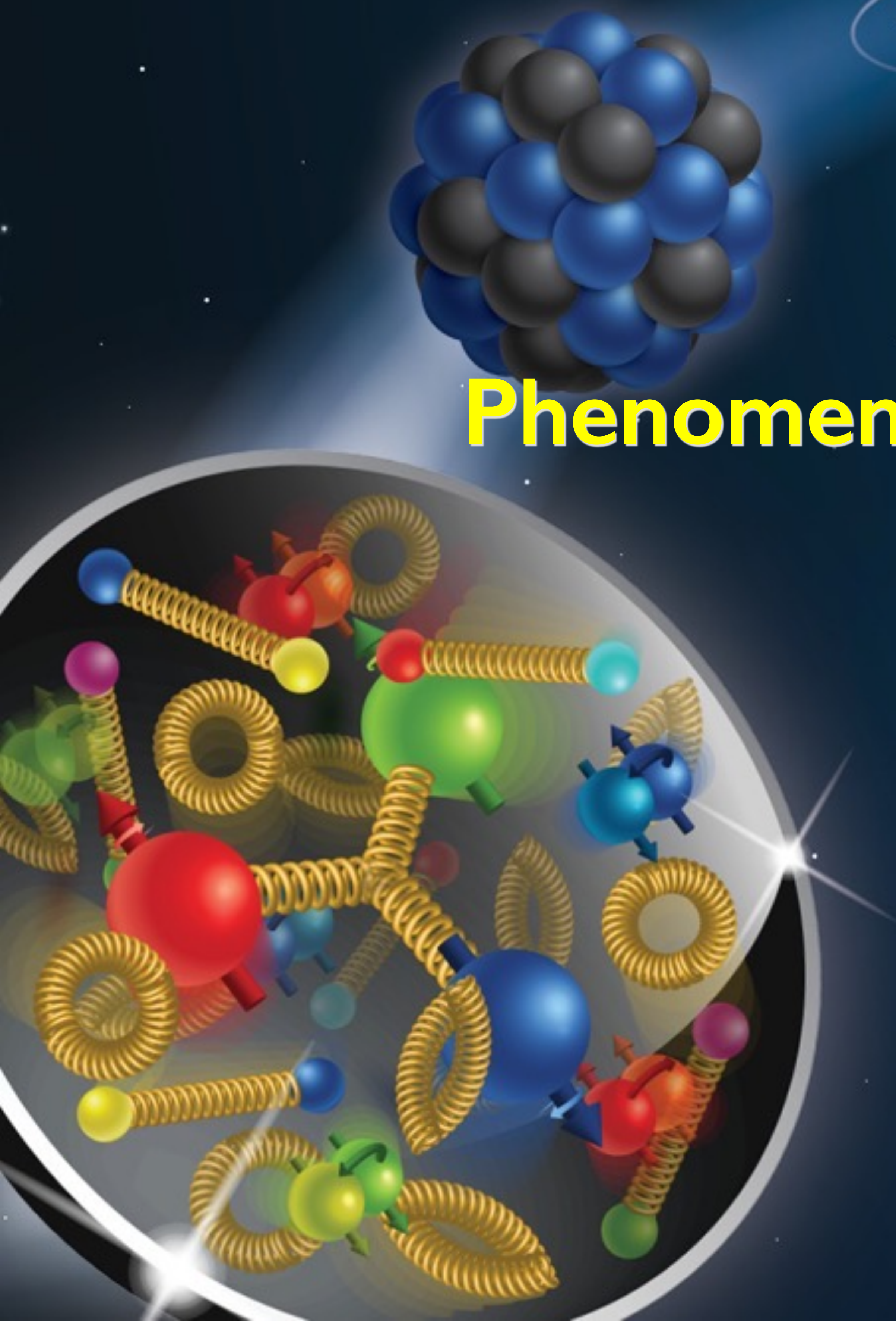
Phenomenology of TMDs: lecture II



Alexei Prokudin

The plan:

- Lecture I:

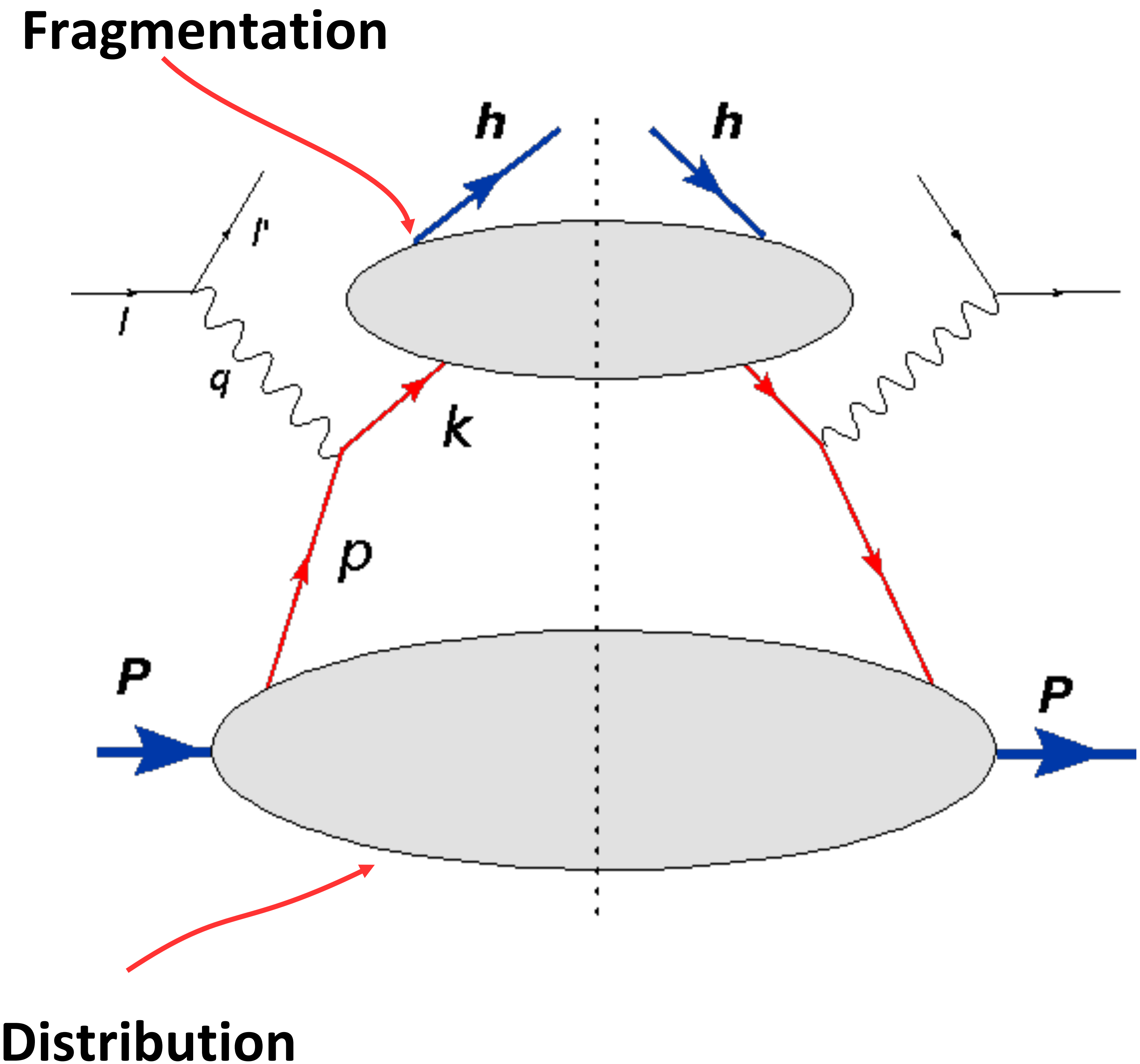
  Structure of the nucleon
  Transverse Momentum Dependent distributions (TMDs)

- Lecture II

  Semi Inclusive Deep Inelastic Scattering (SIDIS)
  Calculations of SIDIS structure functions using Mathematica

Semi Inclusive Deep Inelastic Scattering (SIDIS)

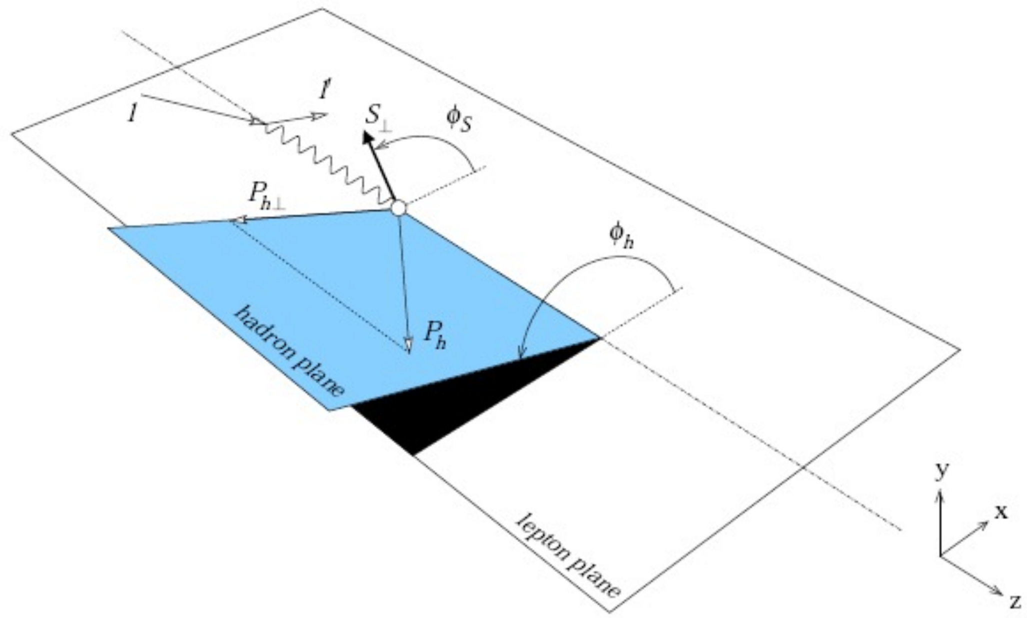
$\ell P \to \ell' \pi X$ — Factorization



$$\sigma_{\mathbf{SIDIS}} = \mathbf{D_{q/h}} \otimes \hat{\sigma}_{\mathbf{lq \to l'q'}} \otimes \mathbf{f_{q/P}}$$

Semi Inclusive Deep Inelastic scattering

$$\ell P \to \ell' \pi X$$



One can rewrite the cross-section in terms of 18 structure functions

Each structure function encodes parton dynamics via convolutions of TMDs when factorization is applicable

Mulders, Tangerman (1995),
Boer, Mulders (1998)
Bacchetta et al (2007)

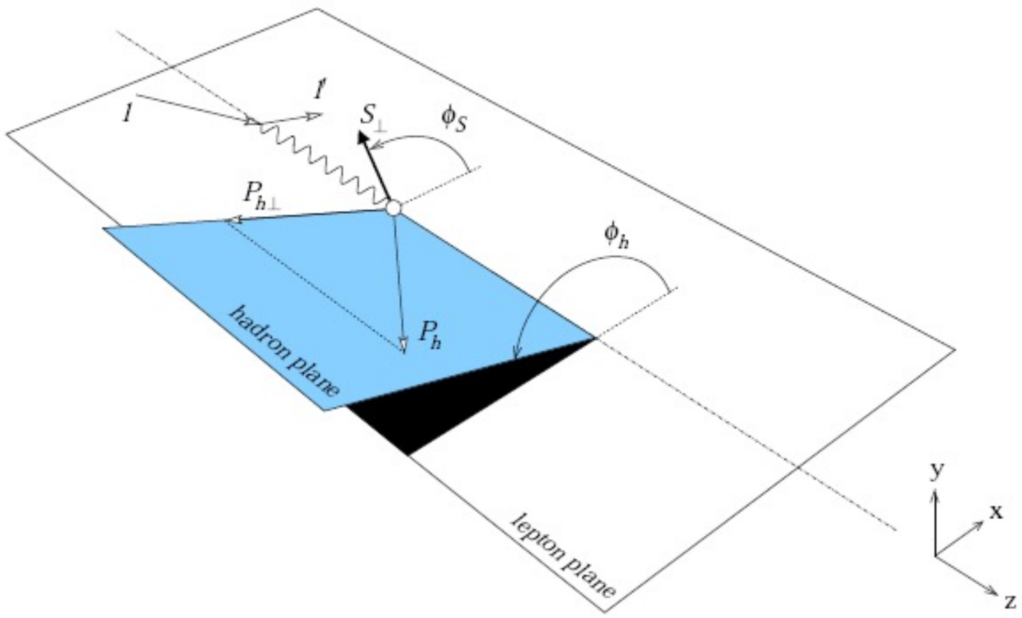
$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$

$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}\;+\ldots$$

Semi Inclusive Deep Inelastic scattering

$$\ell P \to \ell' \pi X$$



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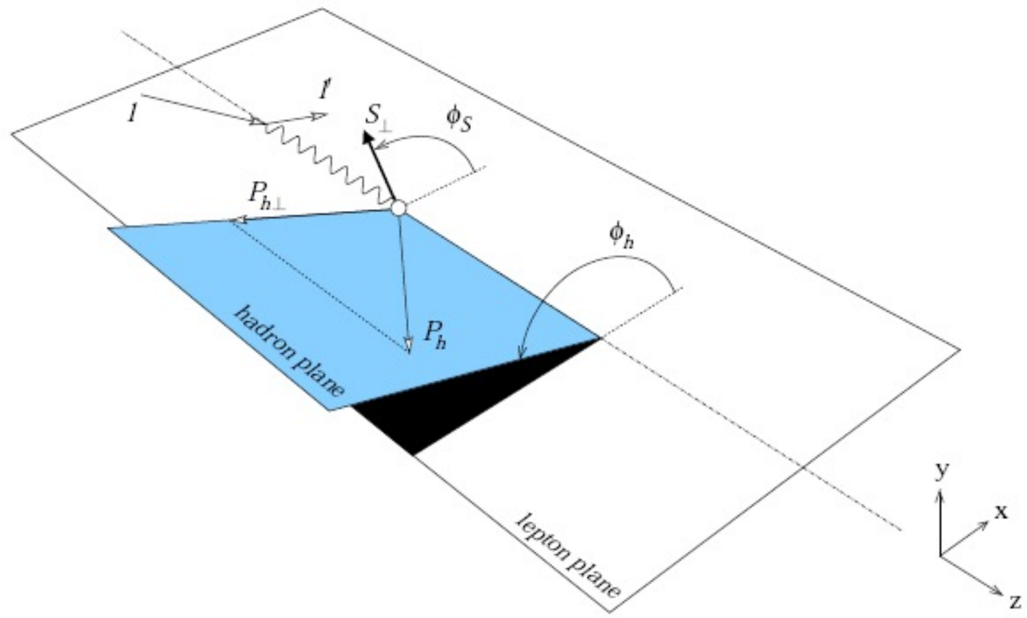
Bacchetta et al (2007)

$$x = \frac{Q^2}{2\,P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z = \frac{P \cdot P_h}{P \cdot q}, \quad Q^2 = -q^2$$

Semi Inclusive Deep Inelastic scattering

$$\ell P \to \ell' \pi X$$



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Bacchetta et al (2007)

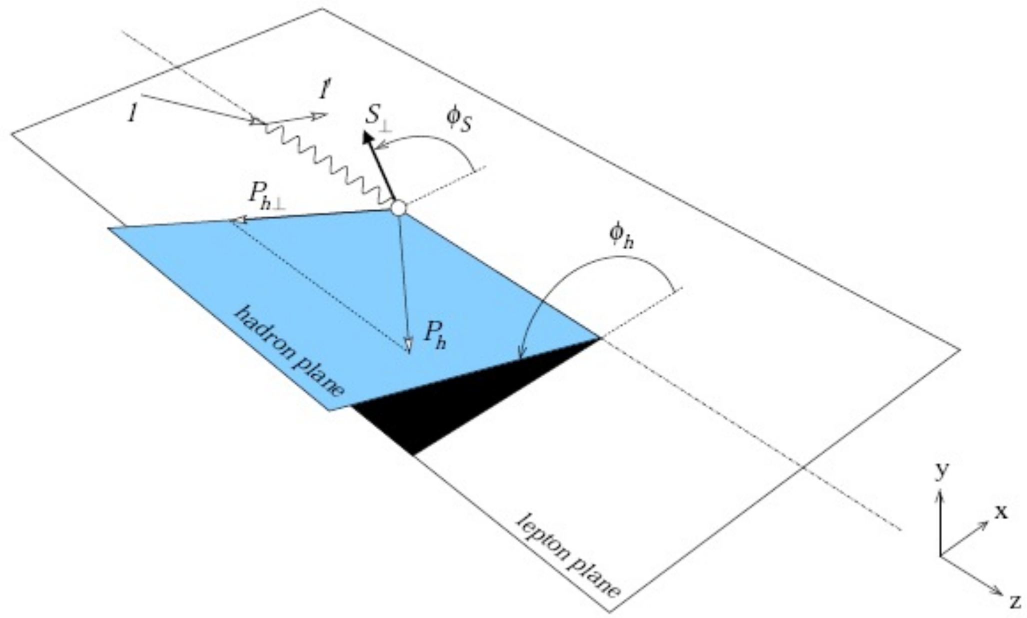
The TMD factorization is valid in the region

$$P_{hT}/z \ll Q$$

Interesting QCD regime, when recoil is happening from a low transverse momentum – important for studies of non perturbative physics.

Semi Inclusive Deep Inelastic scattering

$$\ell P \to \ell' \pi X$$



One can rewrite the cross-section in terms of 18 structure functions

Each structure function encodes parton dynamics via convolutions of TMDs when factorization is applicable

Mulders, Tangerman (1995),
Boer, Mulders (1998)
Bacchetta et al (2007)

The TMD factorization is valid in the region

$$F \sim \int d^2\vec{k}_\perp d^2\vec{p}_\perp \delta^{(2)}(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{hT}) \omega f(x, \vec{k}_\perp) D(z, \vec{p}_\perp)$$

Final transverse momentum is related to transverse momenta of parent and fragmenting partons

Bacchetta et al (2007)

Cross-section

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right](F_{UU,T}+\varepsilon F_{UU,L})\cdot$$

$$\Big(1+\cos\phi_h\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UU}^{\cos\phi_h}+\cos 2\phi_h\,\varepsilon\,A_{UU}^{\cos 2\phi_h}$$
$$+\lambda\,\sin\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LU}^{\sin\phi_h}$$
$$+S_L\left[\sin\phi_h\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UL}^{\sin\phi_h}+\sin 2\phi_h\,\varepsilon\,A_{UL}^{\sin 2\phi_h}\right]$$
$$+S_L\lambda\left[\sqrt{1-\varepsilon^2}\,A_{LL}+\cos\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LL}^{\cos\phi_h}\right]$$
$$+S_T\,\sin(\phi_h-\phi_S)\,A_{UT}^{\sin(\phi_h-\phi_S)}$$
$$+S_T\,\sin(\phi_h+\phi_S)\,\varepsilon\,A_{UT}^{\sin(\phi_h+\phi_S)}$$
$$+S_T\,\sin(3\phi_h-\phi_S)\,\varepsilon\,A_{UT}^{\sin(3\phi_h-\phi_S)}$$
$$+S_T\,\sin\phi_S\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UT}^{\sin\phi_S}$$
$$+S_T\,\sin(2\phi_h-\phi_S)\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UT}^{\sin(2\phi_h-\phi_S)}$$
$$+S_T\lambda\,\cos(\phi_h-\phi_S)\,\sqrt{1-\varepsilon^2}\,A_{LT}^{\cos(\phi_h-\phi_S)}$$
$$+S_T\lambda\,\cos\phi_S\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LT}^{\cos\phi_S}$$
$$+S_T\lambda\,\cos(2\phi_h-\phi_S)\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LT}^{\cos(2\phi_h-\phi_S)}\Big)$$

virtual photon polarization

λ beam polarization

Target polarization

S_L longitudinal

S_T transverse

$$S=(0,\boldsymbol{S}_T,S_L)\ \text{with}\ \boldsymbol{S}_T^2+S_L^2=1.$$

Longitudinal to transverse photon flux ratio

$$\varepsilon=\frac{1-y-\frac{1}{4}\gamma^2y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2y^2}$$

$$\gamma=\frac{2Mx}{Q}$$

Bacchetta et al (2007)

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} &= \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right]\left(F_{UU,T}+\varepsilon F_{UU,L}\right)\cdot \\
&\Big(1+\cos\phi_h\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UU}^{\cos\phi_h}+\cos 2\phi_h\ \varepsilon\ A_{UU}^{\cos 2\phi_h} \\
&\quad +\lambda\ \sin\phi_h\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LU}^{\sin\phi_h} \\
&\quad +S_L\left[\sin\phi_h\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UL}^{\sin\phi_h}+\sin 2\phi_h\ \varepsilon\ A_{UL}^{\sin 2\phi_h}\right] \\
&\quad +S_L\lambda\left[\sqrt{1-\varepsilon^2}\ A_{LL}+\cos\phi_h\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LL}^{\cos\phi_h}\right] \\
&\quad +S_T\ \sin(\phi_h-\phi_S)\ A_{UT}^{\sin(\phi_h-\phi_S)} \\
&\quad +S_T\ \sin(\phi_h+\phi_S)\ \varepsilon\ A_{UT}^{\sin(\phi_h+\phi_S)} \\
&\quad +S_T\ \sin(3\phi_h-\phi_S)\ \varepsilon\ A_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\quad +S_T\ \sin\phi_S\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UT}^{\sin\phi_S} \\
&\quad +S_T\ \sin(2\phi_h-\phi_S)\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UT}^{\sin(2\phi_h-\phi_S)} \\
&\quad +S_T\lambda\ \cos(\phi_h-\phi_S)\ \sqrt{1-\varepsilon^2}\ A_{LT}^{\cos(\phi_h-\phi_S)} \\
&\quad +S_T\lambda\ \cos\phi_S\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LT}^{\cos\phi_S} \\
&\quad +S_T\lambda\ \cos(2\phi_h-\phi_S)\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LT}^{\cos(2\phi_h-\phi_S)}\Big)
\end{aligned}
$$

Asymmetry

Angular dependence

$$A_{UU}^{\cos\phi_h}$$

Beam Polarization

Target

Bacchetta et al (2007)

$$
\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right]\left(F_{UU,T}+\varepsilon F_{UU,L}\right)\cdot
$$

$$
\begin{aligned}
\Big(&1+\cos\phi_h\;\sqrt{2\varepsilon(1+\varepsilon)}\;A_{UU}^{\cos\phi_h}+\cos 2\phi_h\;\varepsilon\;A_{UU}^{\cos 2\phi_h}\\
&\boxed{\begin{aligned}&+\lambda\;\sin\phi_h\;\sqrt{2\varepsilon(1-\varepsilon)}\;A_{LU}^{\sin\phi_h}\\
&+S_L\left[\sin\phi_h\;\sqrt{2\varepsilon(1+\varepsilon)}\;A_{UL}^{\sin\phi_h}+\sin 2\phi_h\;\varepsilon\;A_{UL}^{\sin 2\phi_h}\right]\end{aligned}}\\
&+S_L\lambda\left[\sqrt{1-\varepsilon^2}\;A_{LL}+\cos\phi_h\;\sqrt{2\varepsilon(1-\varepsilon)}\;A_{LL}^{\cos\phi_h}\right]\\
&\boxed{\begin{aligned}&+S_T\;\sin(\phi_h-\phi_S)\;A_{UT}^{\sin(\phi_h-\phi_S)}\\
&+S_T\;\sin(\phi_h+\phi_S)\;\varepsilon\;A_{UT}^{\sin(\phi_h+\phi_S)}\\
&+S_T\;\sin(3\phi_h-\phi_S)\;\varepsilon\;A_{UT}^{\sin(3\phi_h-\phi_S)}\\
&+S_T\;\sin\phi_S\;\sqrt{2\varepsilon(1+\varepsilon)}\;A_{UT}^{\sin\phi_S}\\
&+S_T\;\sin(2\phi_h-\phi_S)\;\sqrt{2\varepsilon(1+\varepsilon)}\;A_{UT}^{\sin(2\phi_h-\phi_S)}\end{aligned}}\\
&+S_T\lambda\;\cos(\phi_h-\phi_S)\;\sqrt{1-\varepsilon^2}\;A_{LT}^{\cos(\phi_h-\phi_S)}\\
&+S_T\lambda\;\cos\phi_S\;\sqrt{2\varepsilon(1-\varepsilon)}\;A_{LT}^{\cos\phi_S}\\
&+S_T\lambda\;\cos(2\phi_h-\phi_S)\;\sqrt{2\varepsilon(1-\varepsilon)}\;A_{LT}^{\cos(2\phi_h-\phi_S)}\Big)
\end{aligned}
$$

Single Spin Asymmetry

Bacchetta et al (2007)

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right]\left(F_{UU,T}+\varepsilon F_{UU,L}\right)\cdot$$

$$\Big(1+\cos\phi_h\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UU}^{\cos\phi_h}+\cos 2\phi_h\ \varepsilon\ A_{UU}^{\cos 2\phi_h}$$

$$+\lambda\ \sin\phi_h\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LU}^{\sin\phi_h}$$

$$+S_L\left[\sin\phi_h\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UL}^{\sin\phi_h}+\sin 2\phi_h\ \varepsilon\ A_{UL}^{\sin 2\phi_h}\right]$$

$$+S_L\lambda\left[\sqrt{1-\varepsilon^2}\ A_{LL}+\cos\phi_h\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LL}^{\cos\phi_h}\right]$$

$$+S_T\ \sin(\phi_h-\phi_S)\ A_{UT}^{\sin(\phi_h-\phi_S)}$$

$$+S_T\ \sin(\phi_h+\phi_S)\ \varepsilon\ A_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+S_T\ \sin(3\phi_h-\phi_S)\ \varepsilon\ A_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+S_T\ \sin\phi_S\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UT}^{\sin\phi_S}$$

$$+S_T\ \sin(2\phi_h-\phi_S)\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UT}^{\sin(2\phi_h-\phi_S)}$$

$$+S_T\lambda\ \cos(\phi_h-\phi_S)\ \sqrt{1-\varepsilon^2}\ A_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+S_T\lambda\ \cos\phi_S\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LT}^{\cos\phi_S}$$

$$+S_T\lambda\ \cos(2\phi_h-\phi_S)\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LT}^{\cos(2\phi_h-\phi_S)}\Big)$$

Single Spin Asymmetry

Double Spin Asymmetry

Cross-section

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right]\left(F_{UU,T}+\varepsilon F_{UU,L}\right)\cdot$$

$$\Big(1 + \cos\phi_h\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UU}^{\cos\phi_h} + \cos 2\phi_h\ \varepsilon\ A_{UU}^{\cos 2\phi_h}$$

$$+\lambda\ \sin\phi_h\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LU}^{\sin\phi_h}$$

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$$+S_L\lambda\left[\sqrt{1-\varepsilon^2}\ A_{LL} + \cos\phi_h\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LL}^{\cos\phi_h}\right]$$

$$+S_T\ \sin(\phi_h-\phi_S)\ A_{UT}^{\sin(\phi_h-\phi_S)}$$

$$+S_T\ \sin(\phi_h+\phi_S)\ \varepsilon\ A_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+S_T\ \sin(3\phi_h-\phi_S)\ \varepsilon\ A_{UT}^{\sin(3\phi_h-\phi_S)}$$

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$$+S_T\ \sin(2\phi_h-\phi_S)\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UT}^{\sin(2\phi_h-\phi_S)}$$

$$+S_T\lambda\ \cos(\phi_h-\phi_S)\ \sqrt{1-\varepsilon^2}\ A_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+S_T\lambda\ \cos\phi_S\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LT}^{\cos\phi_S}$$

$$+S_T\lambda\ \cos(2\phi_h-\phi_S)\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LT}^{\cos(2\phi_h-\phi_S)}\Big)$$

Unpolarized modulations

Single Spin Asymmetry

Double Spin Asymmetry

Bacchetta et al (2007)

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right]\boxed{(F_{UU,T}+\varepsilon F_{UU,L})}\cdot$$

$$\Big(\boxed{1+\cos\phi_h\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UU}^{\cos\phi_h}+\cos 2\phi_h\,\varepsilon\,A_{UU}^{\cos 2\phi_h}}$$

$$+\lambda\,\sin\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LU}^{\sin\phi_h}$$

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$$+S_L\lambda\left[\sqrt{1-\varepsilon^2}\,A_{LL}+\cos\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LL}^{\cos\phi_h}\right]$$

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$$+S_T\,\sin(\phi_h+\phi_S)\,\varepsilon\,A_{UT}^{\sin(\phi_h+\phi_S)}$$

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$$+S_T\,\sin(2\phi_h-\phi_S)\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UT}^{\sin(2\phi_h-\phi_S)}$$

$$+S_T\lambda\,\cos(\phi_h-\phi_S)\,\sqrt{1-\varepsilon^2}\,A_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+S_T\lambda\,\cos\phi_S\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LT}^{\cos\phi_S}$$

$$+S_T\lambda\,\cos(2\phi_h-\phi_S)\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LT}^{\cos(2\phi_h-\phi_S)}\Big)$$

Unpolarized cross section

Unpolarized modulations

Single Spin Asymmetry

Double Spin Asymmetry

Bacchetta et al (2007) ✓ – twist 2 ✗ – twist 3

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right](F_{UU,T}+\varepsilon F_{UU,L})\cdot$$

$$\Big(1+\cos\phi_h\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UU}^{\cos\phi_h}+\cos 2\phi_h\,\varepsilon\,A_{UU}^{\cos 2\phi_h}$$

$$\boxed{+\lambda\,\sin\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LU}^{\sin\phi_h} +S_L\left[\sin\phi_h\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UL}^{\sin\phi_h}+\sin 2\phi_h\,\varepsilon\,A_{UL}^{\sin 2\phi_h}\right]}$$

$$+S_L\lambda\left[\sqrt{1-\varepsilon^2}\,A_{LL}+\cos\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LL}^{\cos\phi_h}\right]$$

$$\boxed{\begin{aligned}&+S_T\,\sin(\phi_h-\phi_S)\,A_{UT}^{\sin(\phi_h-\phi_S)}\\&+S_T\,\sin(\phi_h+\phi_S)\,\varepsilon\,A_{UT}^{\sin(\phi_h+\phi_S)}\\&+S_T\,\sin(3\phi_h-\phi_S)\,\varepsilon\,A_{UT}^{\sin(3\phi_h-\phi_S)}\\&+S_T\,\sin\phi_S\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UT}^{\sin\phi_S}\\&+S_T\,\sin(2\phi_h-\phi_S)\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UT}^{\sin(2\phi_h-\phi_S)}\end{aligned}}$$

$$+S_T\lambda\,\cos(\phi_h-\phi_S)\,\sqrt{1-\varepsilon^2}\,A_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+S_T\lambda\,\cos\phi_S\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LT}^{\cos\phi_S}$$

$$+S_T\lambda\,\cos(2\phi_h-\phi_S)\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LT}^{\cos(2\phi_h-\phi_S)}\Big)$$

Single Spin Asymmetry

$$A_{LU}^{\sin(\phi_h)} \sim \frac{M}{Q}(xe\otimes H_1^\perp+\dots)$$ ✗

$$A_{UL}^{\sin(\phi_h)} \sim \frac{M}{Q}(h_{1L}^\perp\otimes\tilde{H}+\dots)$$ ✗

$$A_{UL}^{\sin(2\phi_h)} \sim h_{1L}^\perp\otimes H_1^\perp$$ ✓

$$A_{UT}^{\sin(\phi_h-\phi_S)} \sim f_{1T}^\perp\otimes D_1$$ ✓

$$A_{UT}^{\sin(\phi_h+\phi_S)} \sim h_1^\perp\otimes H_1^\perp$$ ✓

$$A_{UT}^{\sin(3\phi_h-\phi_S)} \sim h_{1T}^\perp\otimes H_1^\perp$$ ✓

$$A_{UT}^{\sin(\phi_h)} \sim \frac{M}{Q}(xf_T\otimes D_1+\dots)$$ ✗

$$A_{UT}^{\sin(2\phi_h-\phi_S)} \sim \frac{M}{Q}(xf_T^\perp\otimes D_1+\dots)$$ ✗

PennState Berks

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right](F_{UU,T}+\varepsilon F_{UU,L})\cdot$$

$$\Big(1+\cos\phi_h\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UU}^{\cos\phi_h}+\cos 2\phi_h\,\varepsilon\,A_{UU}^{\cos 2\phi_h}$$

Double Spin Asymmetry

$$+\lambda\,\sin\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LU}^{\sin\phi_h}$$

$$+S_L\left[\sin\phi_h\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UL}^{\sin\phi_h}+\sin 2\phi_h\,\varepsilon\,A_{UL}^{\sin 2\phi_h}\right]$$

$$\boxed{+S_L\lambda\left[\sqrt{1-\varepsilon^2}\,A_{LL}+\cos\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LL}^{\cos\phi_h}\right]}$$

$$+S_T\,\sin(\phi_h-\phi_S)\,A_{UT}^{\sin(\phi_h-\phi_S)}$$

$$+S_T\,\sin(\phi_h+\phi_S)\,\varepsilon\,A_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+S_T\,\sin(3\phi_h-\phi_S)\,\varepsilon\,A_{UT}^{\sin(3\phi_h-\phi_S)}$$

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$$\boxed{\begin{aligned}&+S_T\lambda\,\cos(\phi_h-\phi_S)\,\sqrt{1-\varepsilon^2}\,A_{LT}^{\cos(\phi_h-\phi_S)}\\&+S_T\lambda\,\cos\phi_S\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LT}^{\cos\phi_S}\\&+S_T\lambda\,\cos(2\phi_h-\phi_S)\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LT}^{\cos(2\phi_h-\phi_S)}\Big)\end{aligned}}$$

$$A_{LL}\sim g_1\otimes D_1 \quad ✓$$

$$A_{LL}^{\cos\phi_h}\sim\frac{M}{Q}(g_1\otimes\tilde{D}_1+\ldots) \quad ✗$$

$$A_{LT}^{\cos(\phi_h-\phi_S)}\sim g_{1T}\otimes D_1 \quad ✓$$

$$A_{LT}^{\cos\phi_h}\sim\frac{M}{Q}(g_{1T}\otimes\tilde{D}_1+\ldots) \quad ✗$$

$$A_{LT}^{\cos(2\phi_h-\phi_S)}\sim\frac{M}{Q}(g_{1T}\otimes\tilde{D}_1+\ldots) \quad ✗$$

Bacchetta et al (2007) – twist 2 – twist 3

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right](F_{UU,T}+\varepsilon F_{UU,L})\cdot$$

$$\Big(\boxed{1+\cos\phi_h\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UU}^{\cos\phi_h}+\cos 2\phi_h\,\varepsilon\,A_{UU}^{\cos 2\phi_h}}$$

$$+\lambda\,\sin\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LU}^{\sin\phi_h}$$

$$+S_L\left[\sin\phi_h\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UL}^{\sin\phi_h}+\sin 2\phi_h\,\varepsilon\,A_{UL}^{\sin 2\phi_h}\right]$$

$$+S_L\lambda\left[\sqrt{1-\varepsilon^2}\,A_{LL}+\cos\phi_h\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LL}^{\cos\phi_h}\right]$$

$$+S_T\,\sin(\phi_h-\phi_S)\,A_{UT}^{\sin(\phi_h-\phi_S)}$$

$$+S_T\,\sin(\phi_h+\phi_S)\,\varepsilon\,A_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+S_T\,\sin(3\phi_h-\phi_S)\,\varepsilon\,A_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+S_T\,\sin\phi_S\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UT}^{\sin\phi_S}$$

$$+S_T\,\sin(2\phi_h-\phi_S)\,\sqrt{2\varepsilon(1+\varepsilon)}\,A_{UT}^{\sin(2\phi_h-\phi_S)}$$

$$+S_T\lambda\,\cos(\phi_h-\phi_S)\,\sqrt{1-\varepsilon^2}\,A_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+S_T\lambda\,\cos\phi_S\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LT}^{\cos\phi_S}$$

$$+S_T\lambda\,\cos(2\phi_h-\phi_S)\,\sqrt{2\varepsilon(1-\varepsilon)}\,A_{LT}^{\cos(2\phi_h-\phi_S)}\Big)$$

Unpolarized modulations
Single Spin Asymmetry

$$A_{UU}^{\cos\phi_h} \sim \frac{M}{Q}(f_1\otimes\tilde{D}_1+\dots)$$

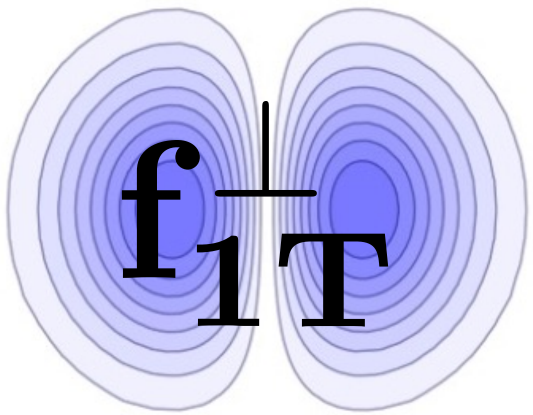
$$A_{UU}^{\cos 2\phi_h} \sim h_1^\perp\otimes\tilde{H}_1^\perp$$

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2\,d\phi_h\,d\psi} = \left[\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\right](F_{UU,T}+\varepsilon F_{UU,L})\cdot$$

$$\Big(1+\cos\phi_h\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UU}^{\cos\phi_h}+\cos 2\phi_h\ \varepsilon\ A_{UU}^{\cos 2\phi_h}$$
$$+\lambda\ \sin\phi_h\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LU}^{\sin\phi_h}$$
$$+S_L\left[\sin\phi_h\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UL}^{\sin\phi_h}+\sin 2\phi_h\ \varepsilon\ A_{UL}^{\sin 2\phi_h}\right]$$
$$+S_L\lambda\left[\sqrt{1-\varepsilon^2}\ A_{LL}+\cos\phi_h\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LL}^{\cos\phi_h}\right]$$
$$+S_T\ \sin(\phi_h-\phi_S)\ A_{UT}^{\sin(\phi_h-\phi_S)}$$
$$+S_T\ \sin(\phi_h+\phi_S)\ \varepsilon\ A_{UT}^{\sin(\phi_h+\phi_S)}$$
$$+S_T\ \sin(3\phi_h-\phi_S)\ \varepsilon\ A_{UT}^{\sin(3\phi_h-\phi_S)}$$
$$+S_T\ \sin\phi_S\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UT}^{\sin\phi_S}$$
$$+S_T\ \sin(2\phi_h-\phi_S)\ \sqrt{2\varepsilon(1+\varepsilon)}\ A_{UT}^{\sin(2\phi_h-\phi_S)}$$
$$+S_T\lambda\ \cos(\phi_h-\phi_S)\ \sqrt{1-\varepsilon^2}\ A_{LT}^{\cos(\phi_h-\phi_S)}$$
$$+S_T\lambda\ \cos\phi_S\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LT}^{\cos\phi_S}$$
$$+S_T\lambda\ \cos(2\phi_h-\phi_S)\ \sqrt{2\varepsilon(1-\varepsilon)}\ A_{LT}^{\cos(2\phi_h-\phi_S)}\Big)$$

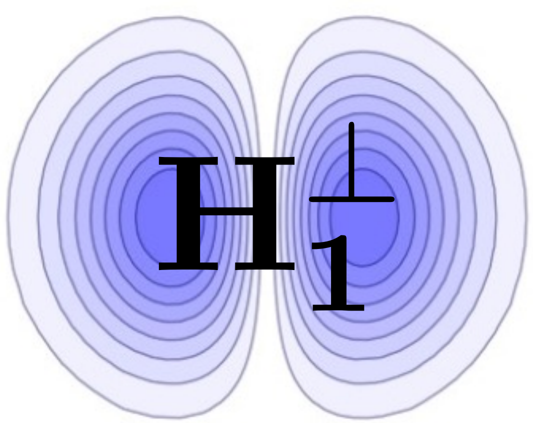
$$F_{UU,T}\sim f_1\otimes D_1$$

What do we know about structure functions in SIDIS?



Sivers function

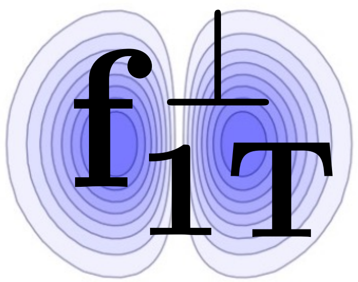
Non universal



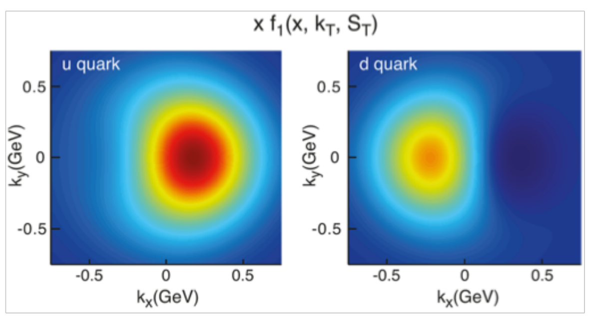
Collins function

Universal

Sivers function

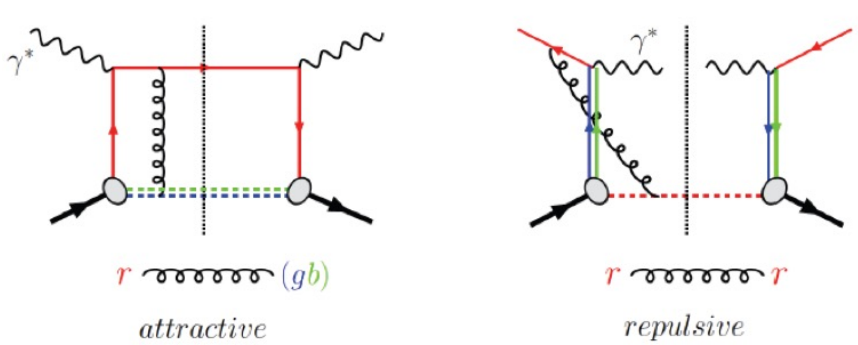


- Describes unpolarized quarks inside of transversely polarized nucleon
- Encodes the correlation of orbital motion with the spin



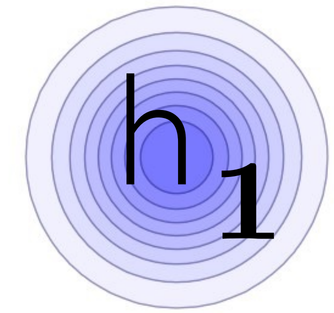
- Sign change of Sivers function is fundamental consequence of QCD

Brodsky, Hwang, Schmidt (2002), Collins (2002)



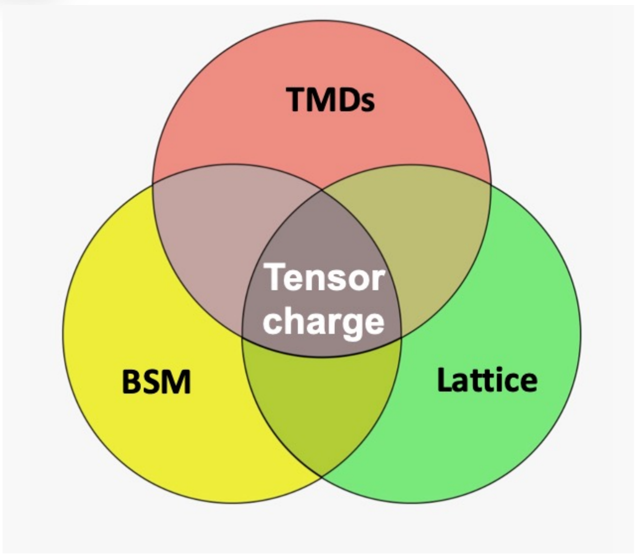
$$f_{1T}^{\perp SIDIS} = -f_{1T}^{\perp DY}$$

Transversity



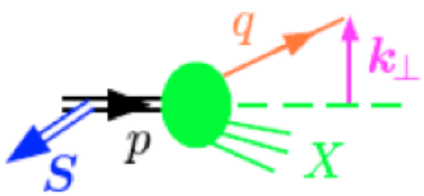
- The only source of information on tensor charge of the nucleon
- Couples to Collins fragmentation function or di-hadron interference fragmentation functions in SIDIS

$$\delta q \equiv g_T^q = \int_0^1 dx \; \left[h_1^q(x,Q^2) - h_1^{\bar{q}}(x,Q^2) \right]$$



Sivers function: unpolarized quark distribution inside a transversely polarized nucleon

Sivers 1989



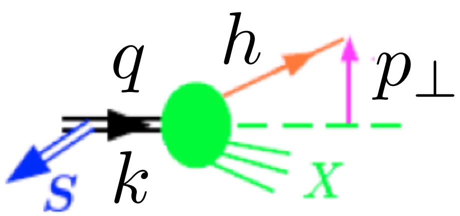
$$f_{q/h^\uparrow}(x,\vec{k}_\perp,\vec{S}) = f_{q/h}(x,k_\perp^2) - \frac{1}{M} f_{1T}^{\perp q}(x,k_\perp^2)\vec{S}\cdot(\hat{P}\times\vec{k}_\perp)$$

Spin independent

Spin dependent

Collins function: unpolarized hadron from a transversely polarized quark

Collins 1992



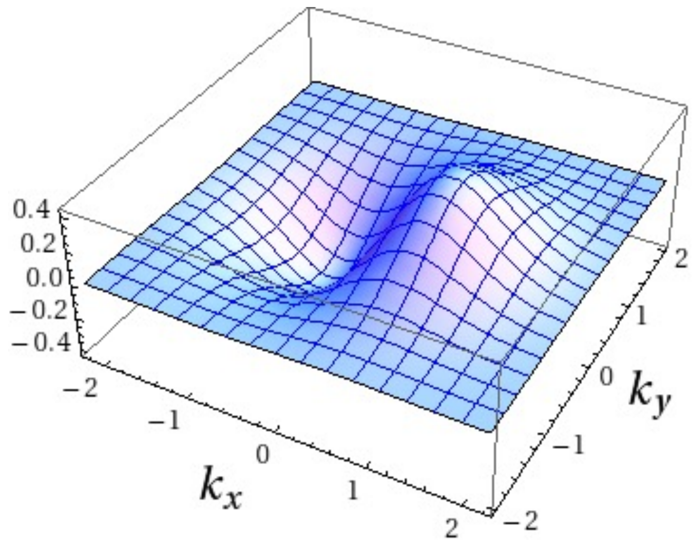
$$D_{q/h}(z,\vec{p}_\perp,\vec{s}_q) = D_{q/h}(z,p_\perp^2) + \frac{1}{zM_h} H_1^{\perp q}(z,p_\perp^2)\vec{s}_q\cdot(\hat{k}\times\vec{p}_\perp)$$

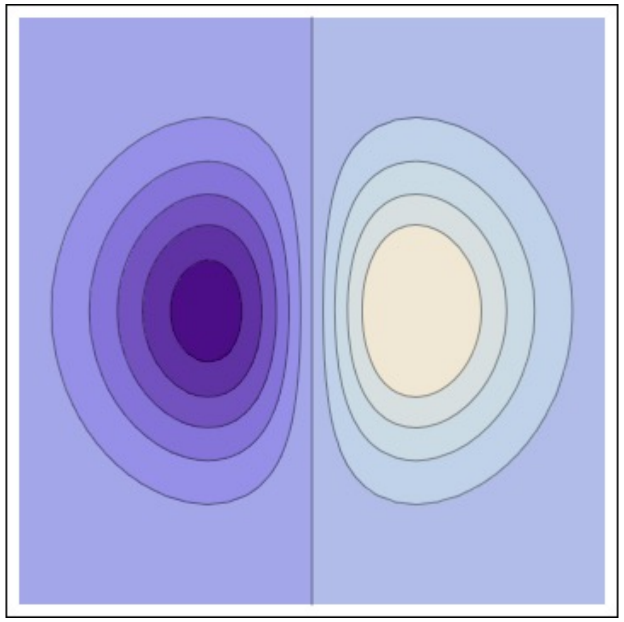
$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_x}}{M}$$

Suppose the spin is along Y direction: $S_T = (0, 1)$

Deformation in momentum space is:

This is the "dipole" deformation. $x \cdot f(x^2 + y^2)$





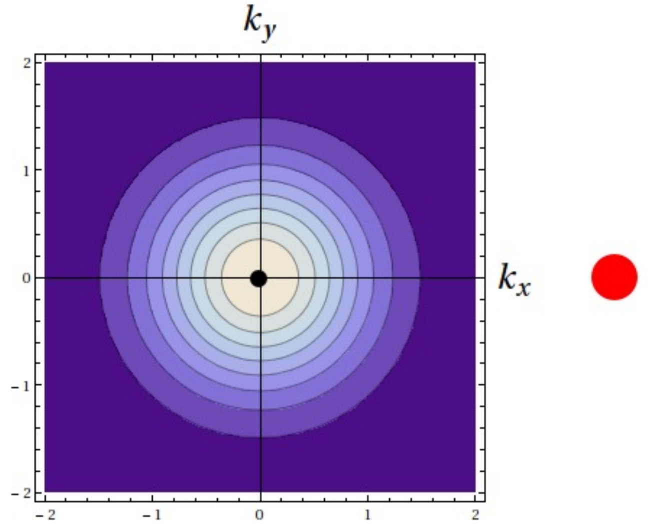
$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_x}}{M}$$

Suppose the spin is along Y direction: $S_T = (0, 1)$

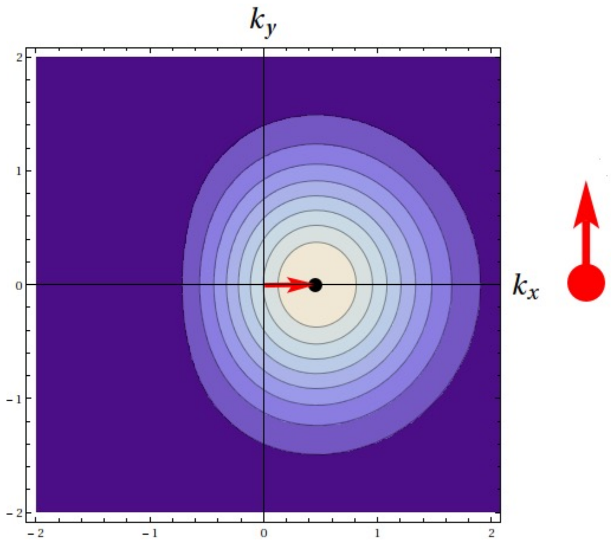
Deformation in momentum space is: $x \cdot f(x^2 + y^2)$

This is the "dipole" deformation.

No correlation:

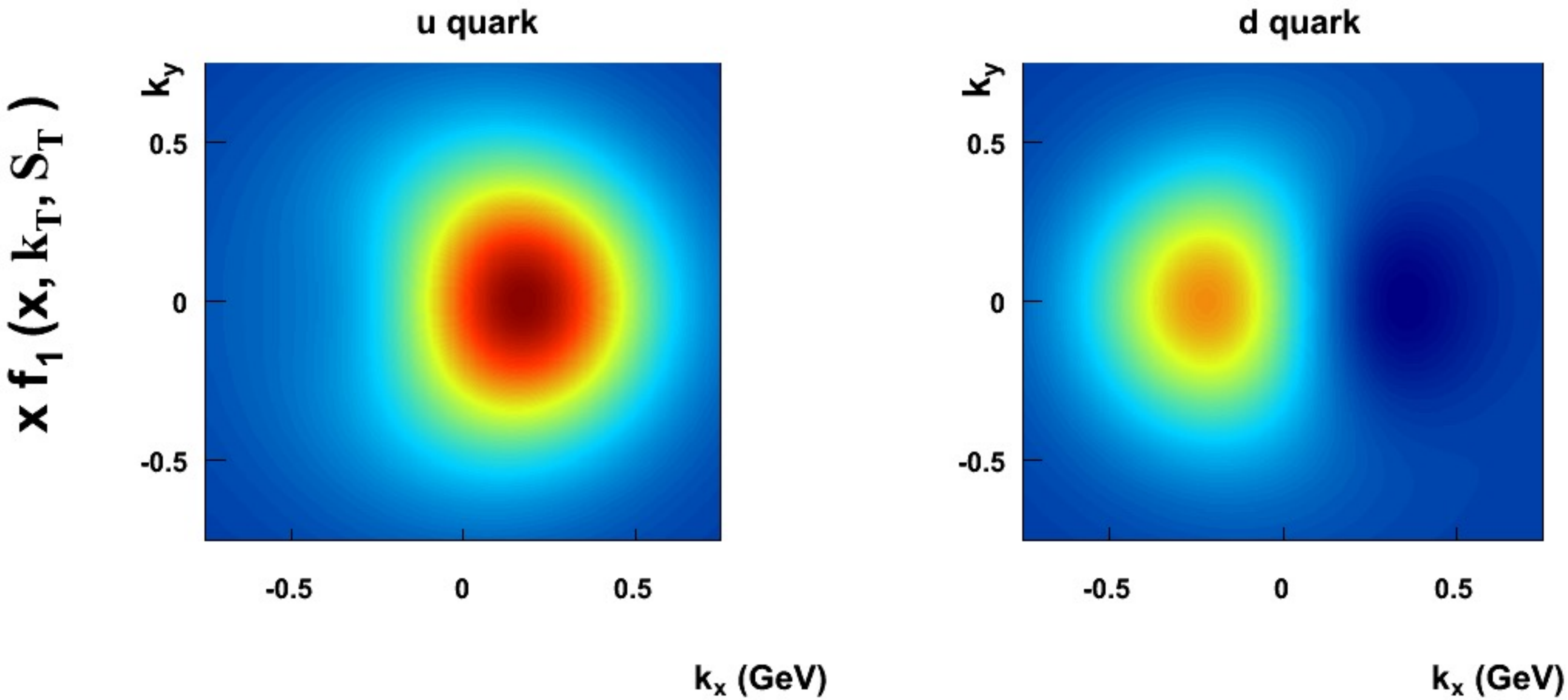


Correlation:



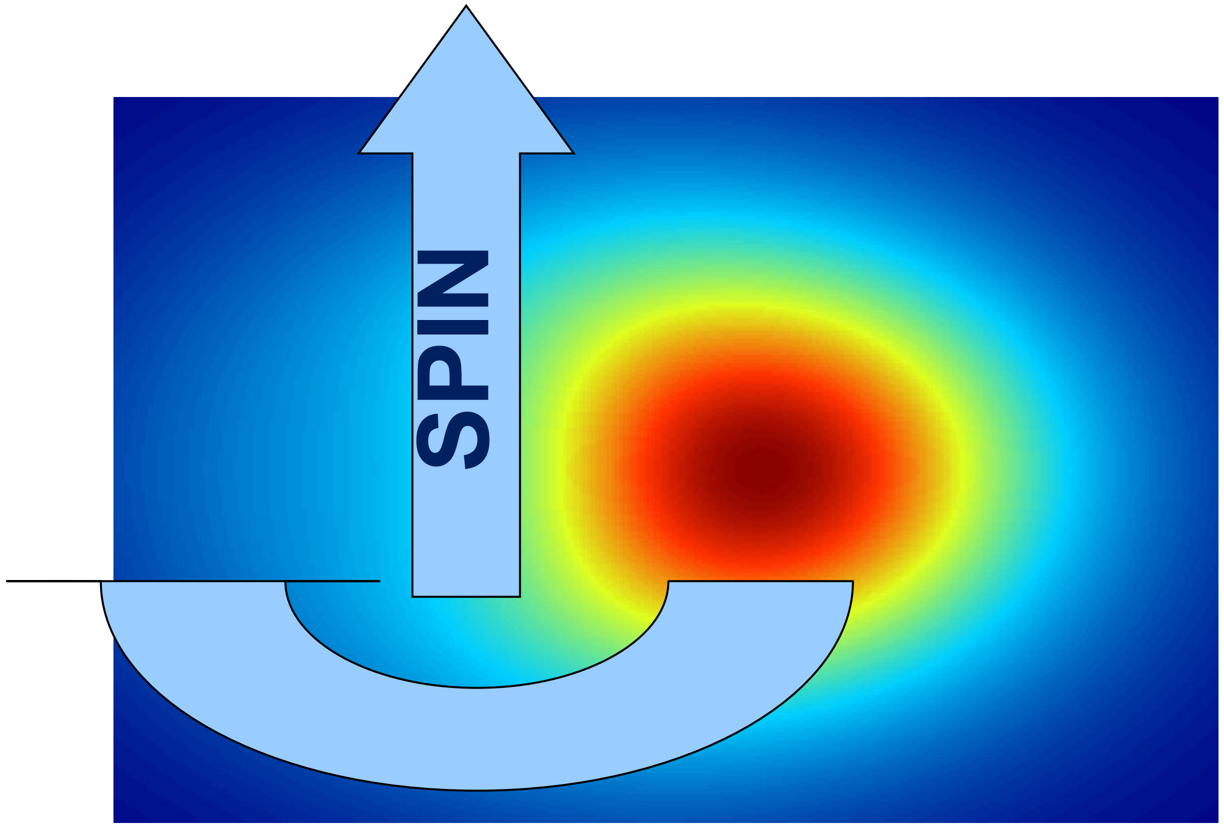
PennState Berks

Tomographic scan of the nucleon



Anselmino et al 2009

Tomographic scan of the nucleon



Internal motion of quarks is correlated with the spin of the proton!

Definitions

Sivers function: $f_{1T}^{\perp q}$ describes strength of correlation $\vec{S} \cdot (\hat{P} \times \vec{k}_\perp)$

Sivers 1989

Collins function: $H_1^{\perp q}$ describes strength of correlation $\vec{s}_q \cdot (\hat{k} \times \vec{p}_\perp)$

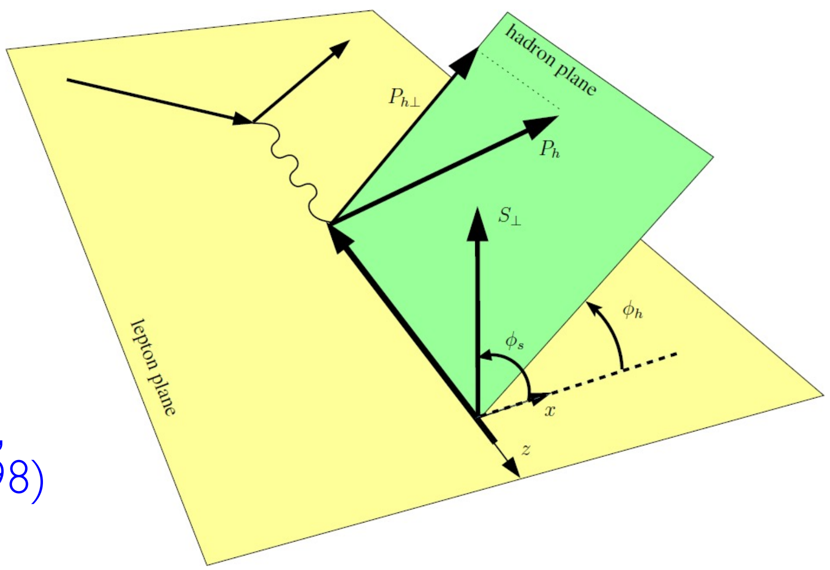
Collins 1992

Both functions extensively studied experimentally, phenomenologically, theoretically

$$\ell P \to \ell' \pi X$$

Sivers function and Collins function can give rise to Single Spin Asymmetries in scattering processes. For instance in Semi Inclusive Deep Inelastic process

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)



$$d\sigma(S) \sim \sin(\phi_h + \phi_S) h_1 \otimes H_1^\perp + \sin(\phi_h - \phi_S) f_{1T}^\perp \otimes D_1 + \ldots$$

Sivers function

Large – N_c result $$f_{1T}^{\perp u} = -f_{1T}^{\perp d}$$ Pobylitsa 2003

→ Confirmed by phenomenological extractions

→ Confirmed by experimental measurements

Relation to GPDs (E) and anomalous magnetic moment Burkardt 2002

$$f_{1T}^{\perp q} \sim \kappa^q$$

→ Predicted correct sign of Sivers asymmetry in SIDIS

→ Shown to be model-dependent Meissner, Metz, Goeke 2007

→ Used in phenomenological extractions Bacchetta, Radici 2011

Sivers function

Sum rule

Burkardt 2004

→ Conservation of transverse momentum

→ Average transverse momentum shift of a quark inside a transversely polarized nucleon

$$\langle k_T^{i,q} \rangle = \varepsilon_T^{ij} S_T^j f_{1T}^{\perp(1)q}(x)$$

$$f_{1T}^{\perp(1)q}(x) = \int d^2 k_\perp \frac{k_\perp^2}{2M^2} f_{1T}^{\perp q}(x, k_\perp^2)$$

→ Sum rule

$$\sum_{a=q,g} \int_0^1 dx \langle k_T^{i,a} \rangle = 0 \qquad \sum_{a=q,g} \int_0^1 dx\, f_{1T}^{\perp(1)a}(x) = 0$$

PennState Berks

Sivers function

Extractions

→ Many extractions without taking into account TMD evolution

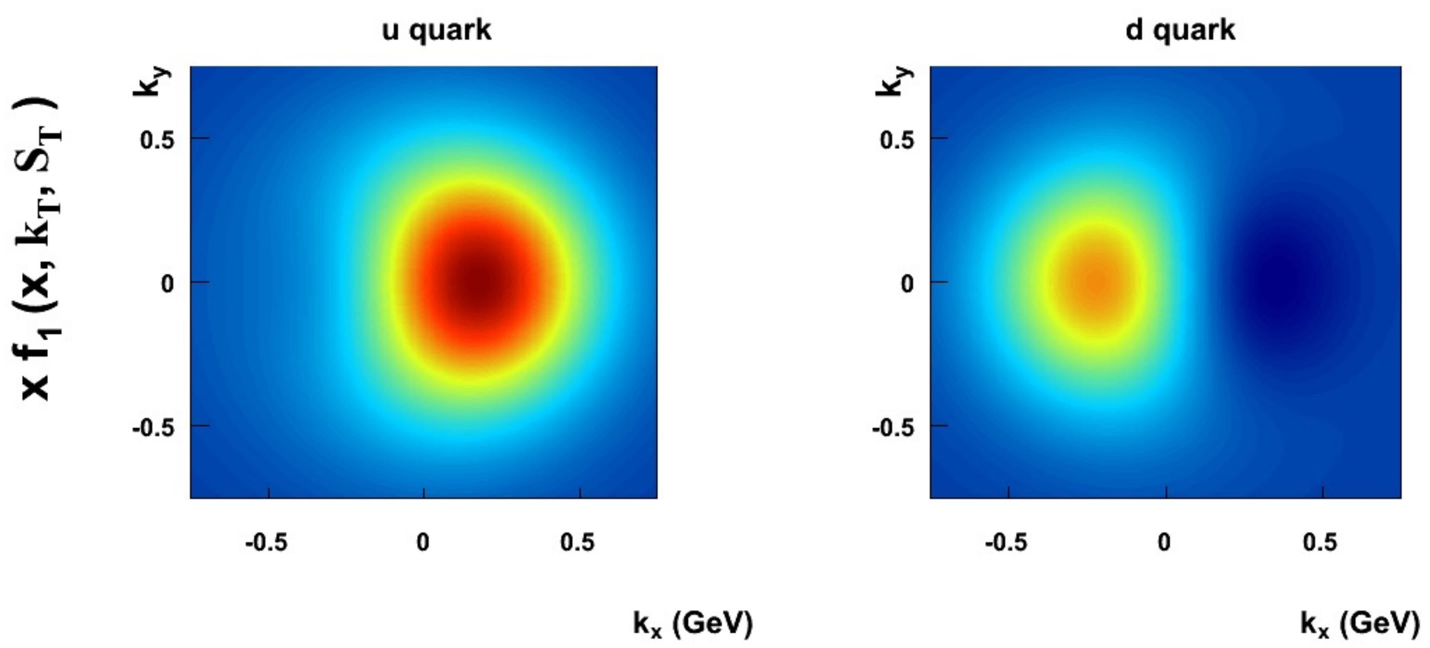
Efremov et al 2005, Vogelsang, Yuan 2005, Anselmino et al 2005,
Collins et al 2006, Anselmino et al 2009, 2011, 2016, Bacchetta Radici 2011, Cammarota et al 21

→ Extractions with TMD evolution

Echevarria et al 2014, Sun Yuan 2013,
Del Carro et al 21,
Echevarria, Kang, Terry 21, AP, Vladimirov 21

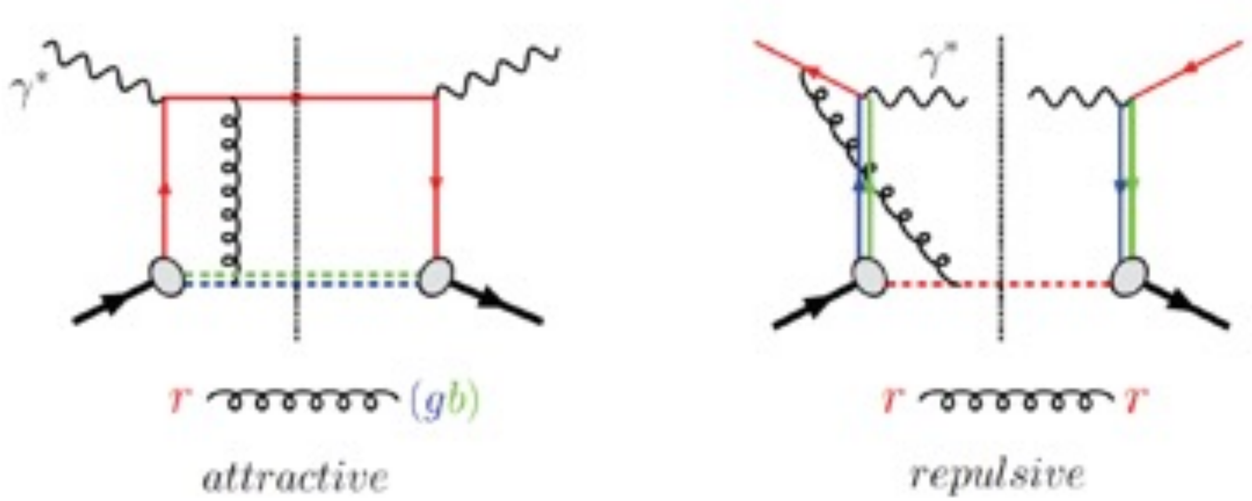
→ Relation to the tomography of the nucleon

Anselmino et al 2009

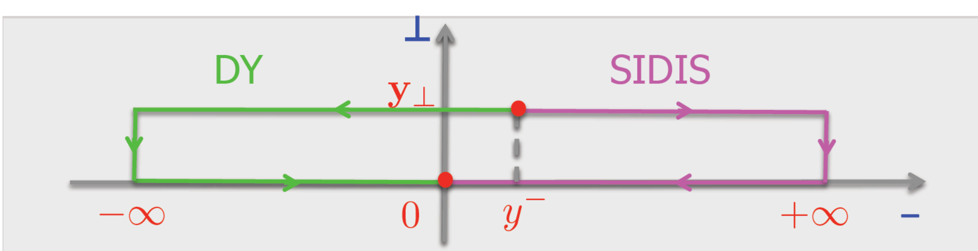


→ Agreement with the sum rule and large N_c prediction

Colored objects are surrounded by gluons, profound consequence of gauge invariance:
Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell-Yan)



$$f_{1T}^{\perp SIDIS} = -f_{1T}^{\perp DY}$$



Brodsky,Hwang,Schmidt;
Belitsky,Ji,Yuan;
Collins;
Boer,Mulders,Pijlman;
Kang, Qiu;
Kovchegov, Sievert;
etc

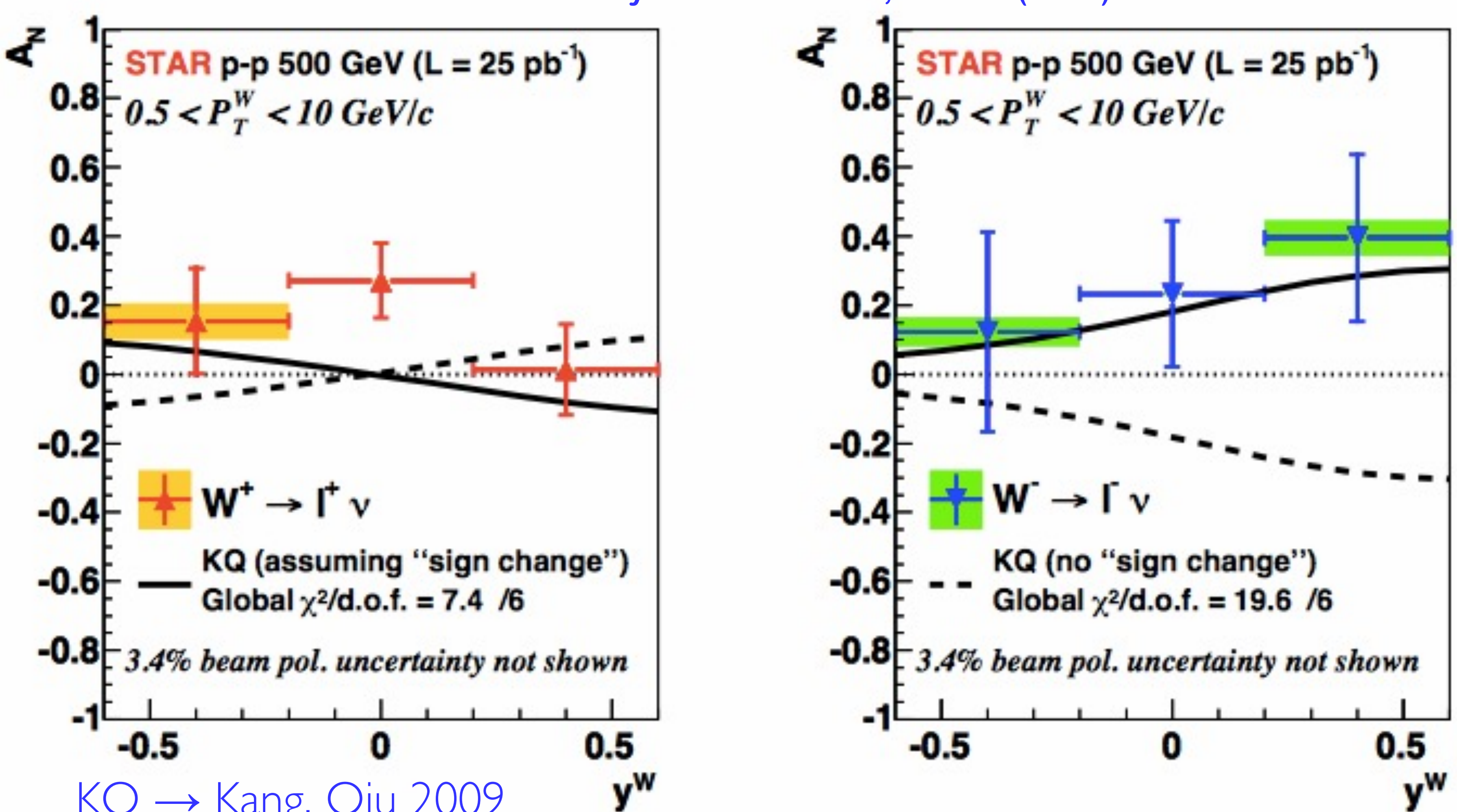
Crucial test of TMD factorization and collinear twist-3 factorization
Several labs worldwide aim at measurement of Sivers effect in Drell-Yan
BNL, CERN, GSI, IHEP, JINR, FERMILAB etc
Barone et al., Anselmino et al., Yuan,Vogelsang, Schlegel et al., Kang,Qiu, Metz,Zhou etc
The verification of the sign change is an NSAC (DOE and NSF) milestone

Process dependence of Sivers function

STAR 2016

- First experimental hint on the sign change: A_N in W and Z production

STAR Collab. Phys. Rev. Lett. 116, 132301 (2016)



KQ → Kang, Qiu 2009

$$p^\uparrow p \to W^\pm X$$
$$p^\uparrow p \to Z^0 X$$

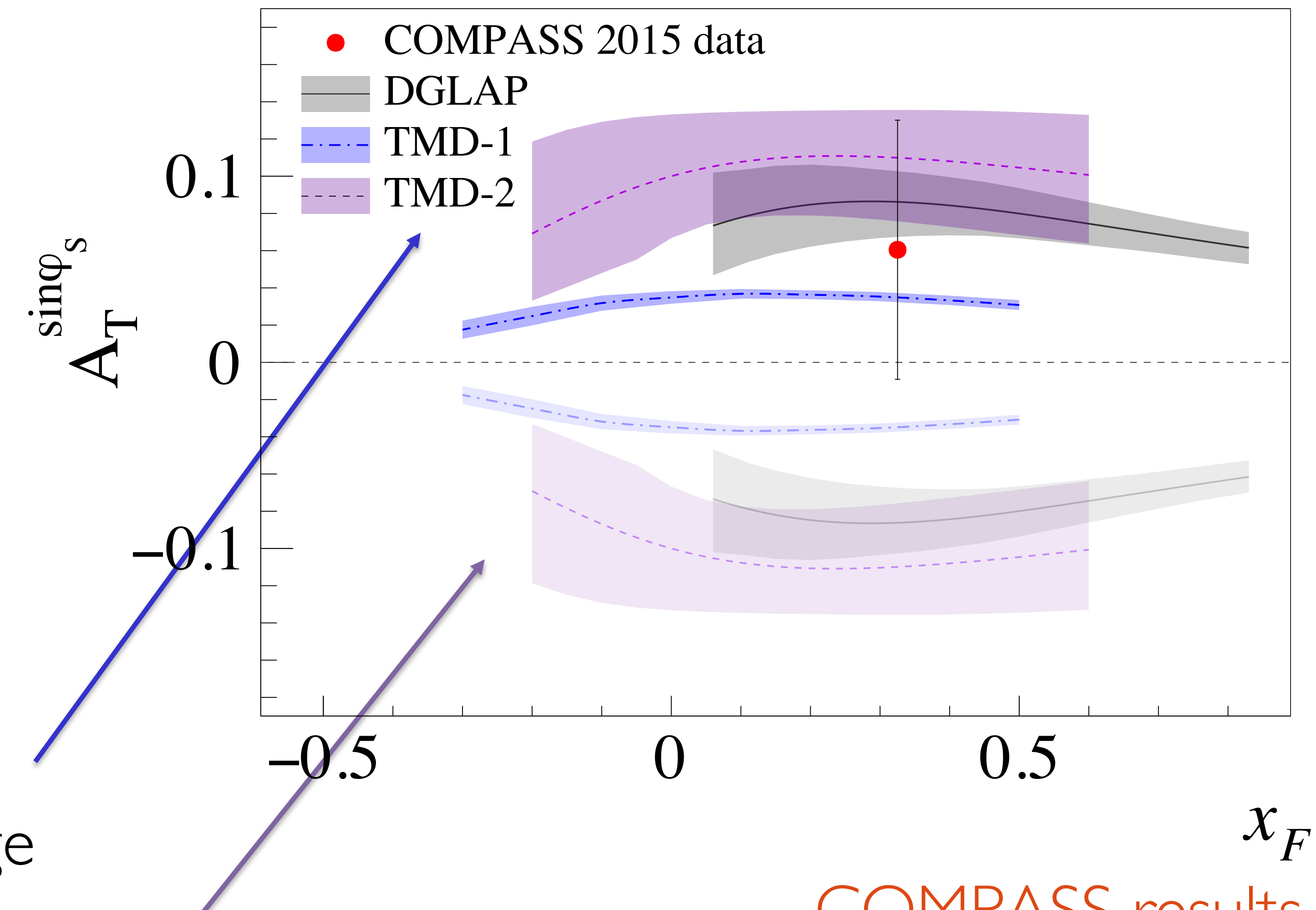
- Sign change $\chi^2/d.o.f \sim 1.2$
- No sign change $\chi^2/d.o.f \sim 3.2$

- Large uncertainties of predictions
- No antiquark Sivers functions

Process dependence of Sivers function

COMPASS 2017

→ First experimental hint on the sign change in Drell-Yan



→ Sign change

→ No sign change

→ COMPASS results hint on sign change

Collins function

Schafer-Teryaev sum rule

Schafer Teryaev 1999
Meissner, Metz, Pitonyak 2010

→ Conservation of transverse momentum

$$\langle P_T^i(z)\rangle \sim H_1^{\perp(1)}(z) \qquad H_1^{\perp(1)}(z) = \int d^2p_\perp \frac{p_\perp^2}{2z^2M_h^2} H_1^\perp(z,p_\perp^2)$$

→ Sum rule

$$\sum_h \int_0^1 dz \langle P_T^i(z)\rangle = 0$$

→ If only pions are considered $H_1^{\perp fav}(z) \sim -H_1^{\perp unf}(z)$

Universality of TMD fragmentation functions

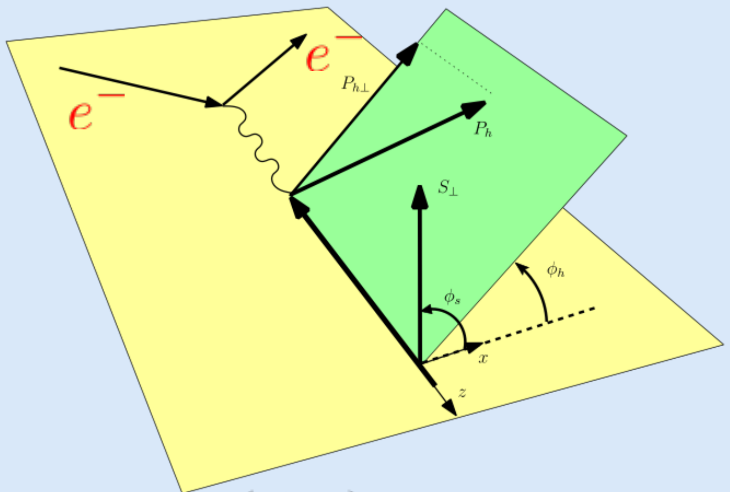
Metz 2002, Metz, Collins 2004, Yuan 2008
Gamberg, Mukherjee, Mulders 2011
Boer, Kang, Vogelsang, Yuan 2010

$$H_1^\perp(z)|_{SIDIS} = H_1^\perp(z)|_{e^+e^-} = H_1^\perp(z)|_{pp}$$

→ Very non trivial results

→ Agrees with phenomenology, allows global fits

SIDIS and e+e-: combined global analysis

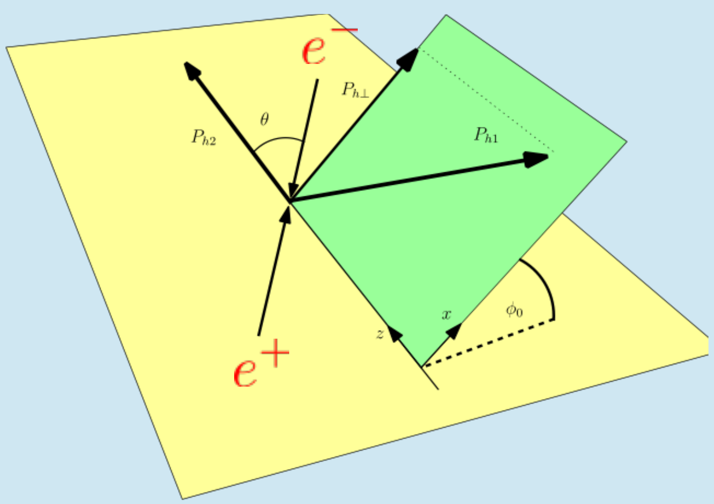


$$F_{UT}^{\sin(\phi_h+\phi_s)} \sim h_1(x_B,k_\perp)H_1^\perp(z_h,p_\perp)$$

transversity

Collins function

$$\frac{d\sigma(S_\perp)}{dx_B\,dy\,dz_h\,d^2P_{h\perp}} = \sigma_0(x_B,y,Q^2)\left[F_{UU} + \sin(\phi_h+\phi_s)\,\frac{2(1-y)}{1+(1-y)^2}\,F_{UT}^{\sin(\phi_h+\phi_s)} + \ldots\right]$$



$$Z_{\text{collins}}^{h_1h_2} \sim H_1^\perp(z_1,p_{1\perp})H_1^\perp(z_2,p_{2\perp})$$

Collins function

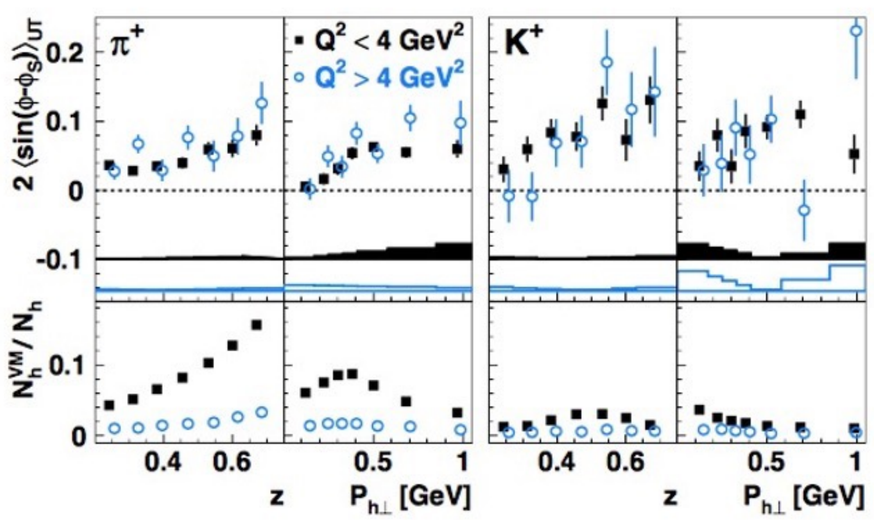
Collins function

$$\frac{d\sigma^{e^+e^-\to h_1h_2+X}}{dz_{h1}\,dz_{h2}\,d^2P_{h\perp}\,d\cos\theta} = \frac{N_c\pi\alpha_{\text{em}}^2}{2Q^2}\left[\left(1+\cos^2\theta\right)Z_{uu}^{h_1h_2} + \sin^2\theta\cos(2\phi_0)Z_{\text{collins}}^{h_1h_2}\right]$$

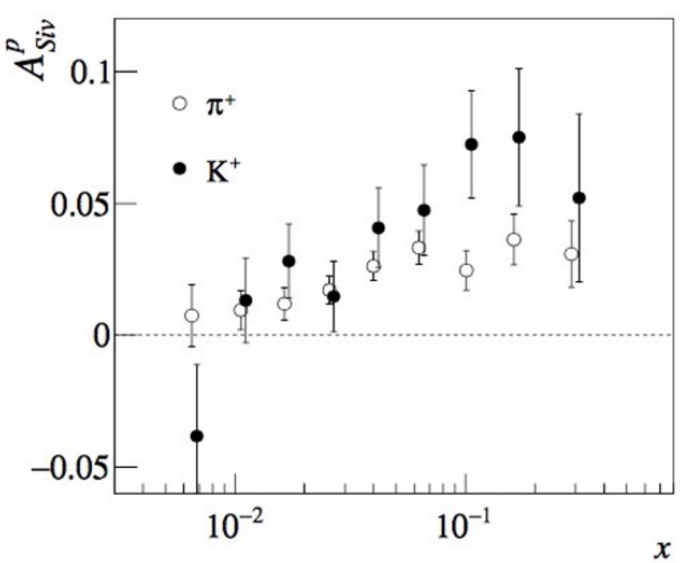
TRANSVERSE SPIN ASYMMETRIES

Transverse Single Spin Asymmetries (SSAs) have been observed in a variety of processes

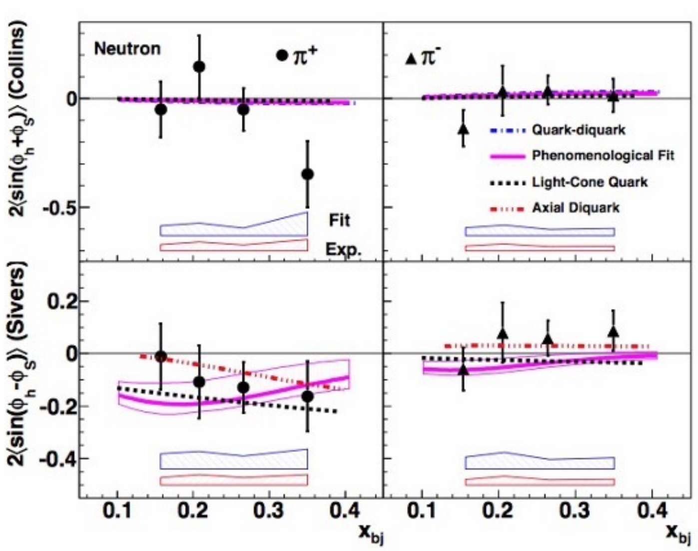
Sivers asymmetry in SIDIS



HERMES (09)



COMPASS (15)



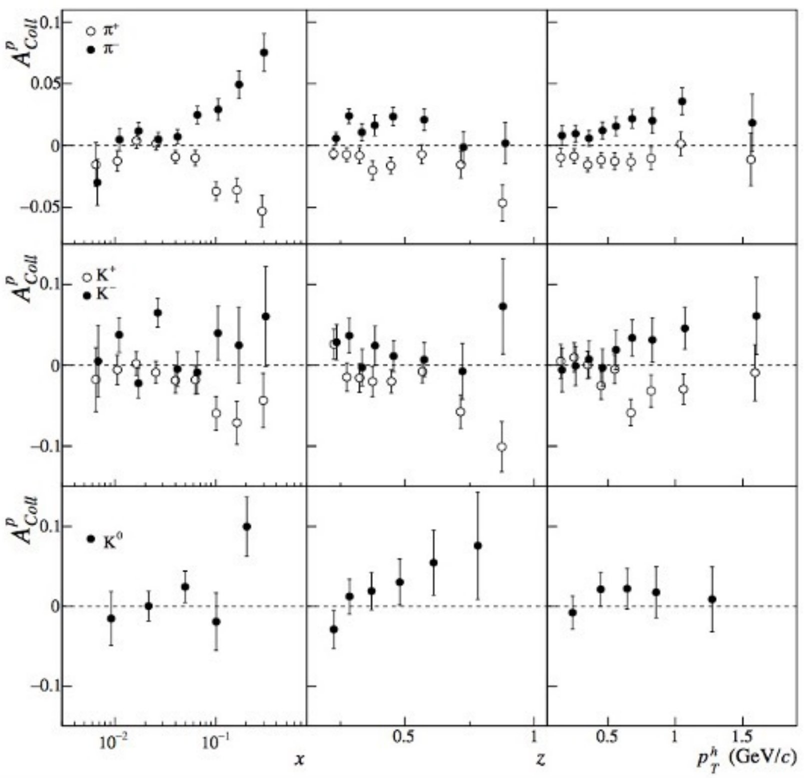
JLAB (11)

$$F_{UT}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{k}_T}{M}\boldsymbol{f_{1T}^{\perp}}D_1\right]$$

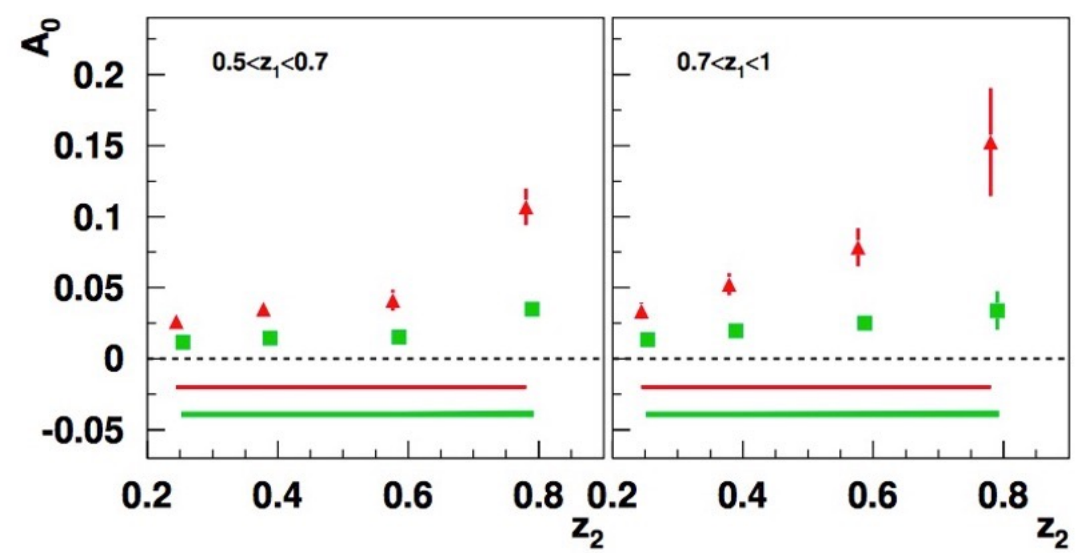
TRANSVERSE SPIN ASYMMETRIES

Transverse Single Spin Asymmetries (SSAs) have been observed in a variety of processes

Collins asymmetry in SIDIS and e⁺e⁻



COMPASS (15),
also HERMES (05,10, 20), JLab (11,14)



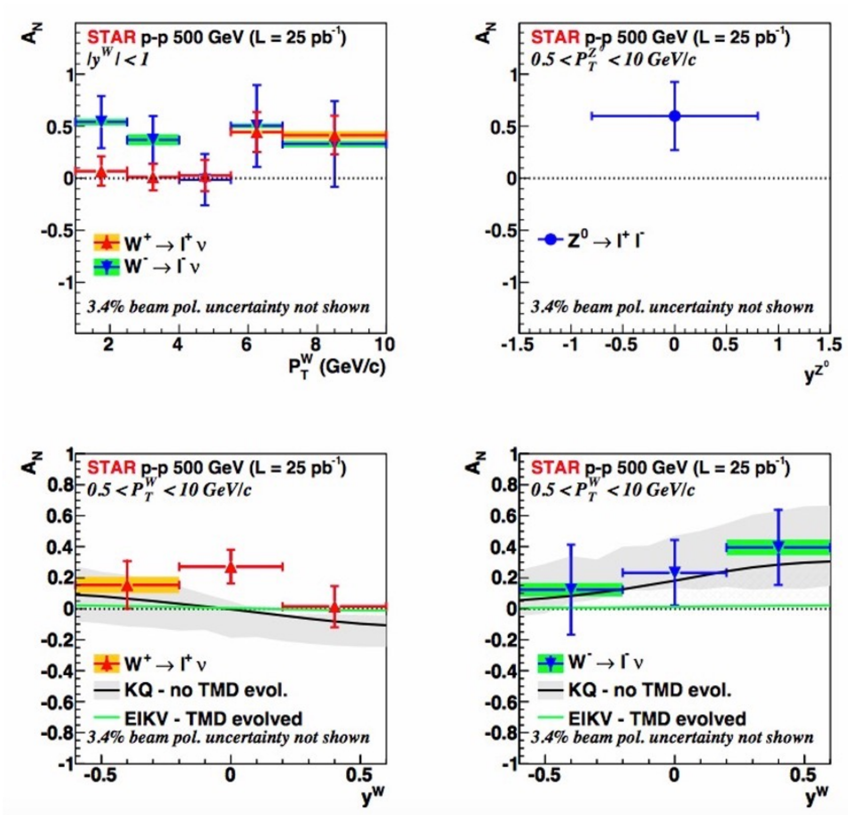
BELLE (08),
also BaBar (14), BESIII (16)

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{p}_\perp}{M_h}\boldsymbol{h_1}\boldsymbol{H_1^\perp}\right] \qquad F_{UU}^{\cos(2\phi_0)} = \mathcal{C}\left[\frac{2\hat{h}\cdot\vec{p}_{a\perp}\,\hat{h}\cdot\vec{p}_{b\perp}-\vec{p}_{a\perp}\cdot\vec{p}_{b\perp}}{M_aM_b}\boldsymbol{H_1^\perp}\,\boldsymbol{\bar{H}_1^\perp}\right]$$

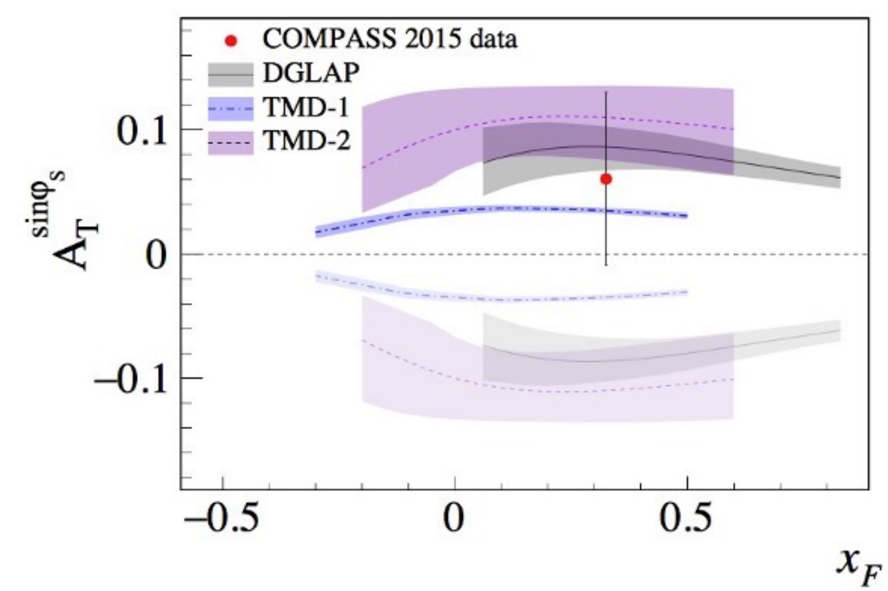
TRANSVERSE SPIN ASYMMETRIES

Transverse Single Spin Asymmetries (SSAs) have been observed in a variety of processes

Sivers effect in Drell-Yan



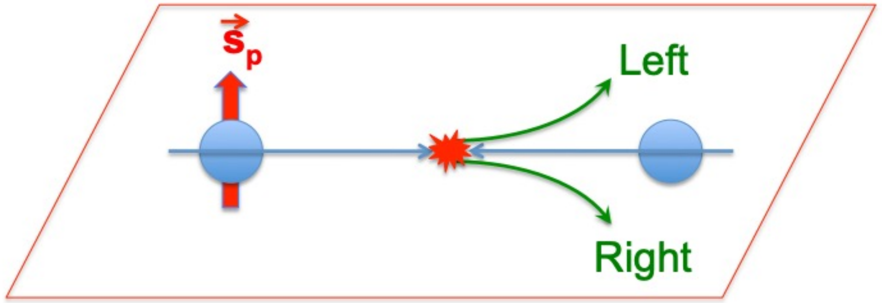
STAR (15)



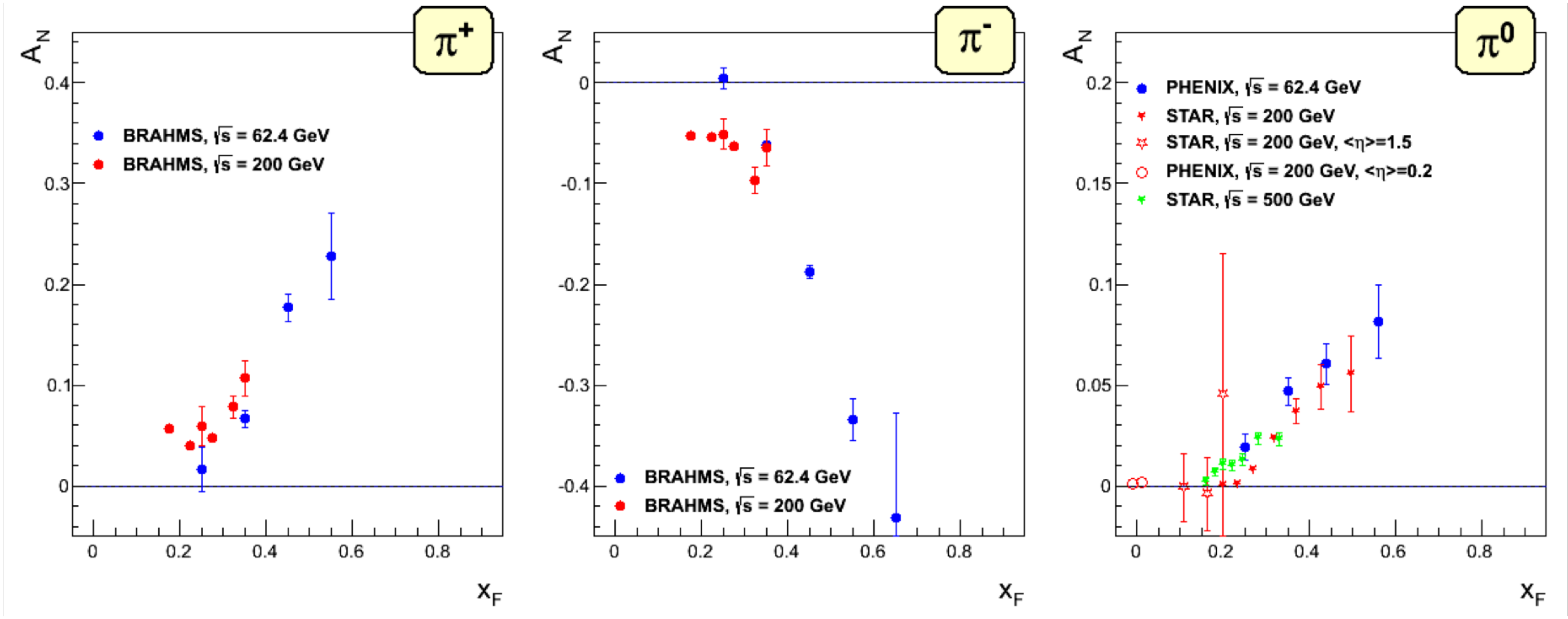
COMPASS (17)

$$F_{TU}^{1} = \mathcal{C}\left[-\frac{\vec{h}\cdot\vec{k}_{aT}}{M_a}\, f_{1T}^{\perp}\, \bar{f}_1\right]$$

TRANSVERSE SPIN ASYMMETRIES



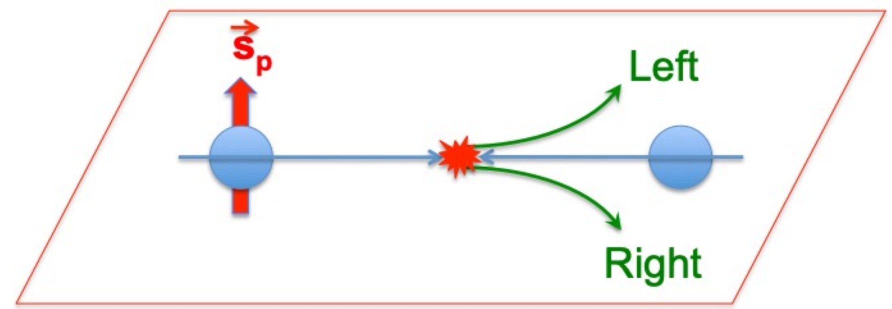
A_N in pp scattering
RHIC: STAR, BRAHMS, PHENIX



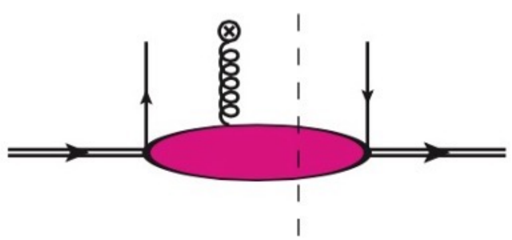
"The RHIC SPIN Program: Achievements and Future Opportunities", Aschenauer et al (15)

TRANSVERSE SPIN ASYMMETRIES

A_N in pp scattering is related to collinear twist-3 (CT3) factorization



$$d\Delta\sigma(S_T) \sim \underbrace{H_{QS} \otimes f_1 \otimes \boldsymbol{F_{FT}} \otimes D_1}_{\text{Qiu-Sterman term}} + H_F \otimes f_1 \otimes \boldsymbol{h_1} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}\right)$$

$\boldsymbol{F_{FT}} \sim$ 

quark-gluon-quark correlator

Qiu, Sterman (99), Kouvaris, et al (06)

$$\pi \boldsymbol{F_{FT}(x,x)} = \int d^2\vec{k}_T \, \frac{k_T^2}{2M^2} \boldsymbol{f_{1T}^{\perp}(x, k_T^2)} \equiv f_{1T}^{\perp(1)}(x)$$ the first moment of Sivers function

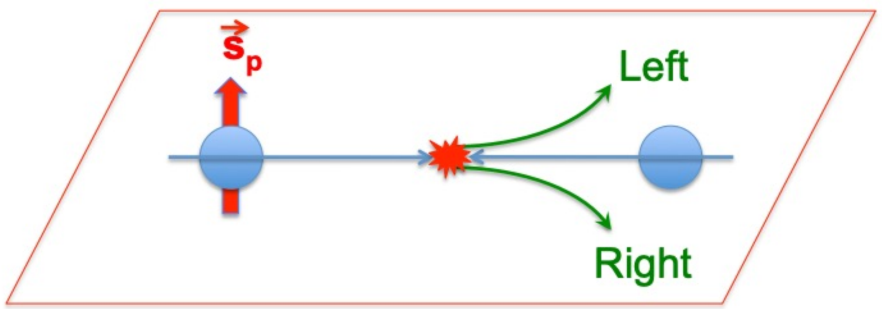
Boer, et al (03)

TMD and CT3 factorization agree in their overlapping region of applicability

Ji, et al (06); Koike, et a. (08); Zhou, et al (08, 10); Yuan and Zhou (09)

TRANSVERSE SPIN ASYMMETRIES

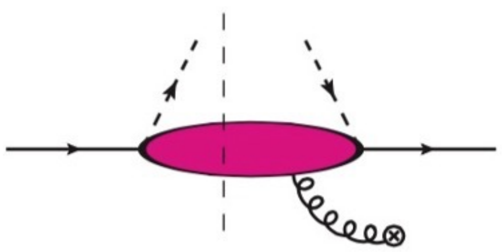
A_N in pp scattering is related to collinear twist-3 (CT3) factorization



$$d\Delta\sigma(S_T) \sim H_{QS} \otimes f_1 \otimes \boldsymbol{F_{FT}} \otimes D_1 \;+\; H_F \otimes f_1 \otimes \boldsymbol{h_1} \otimes \underbrace{\left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}\right)}_{\text{Fragmentation term}}$$

$\boldsymbol{h_1}$ collinear transversity

Kanazawa, Koike, Metz, Pitonyak, Schlegel, (16)

$\boldsymbol{H_1^{\perp(1)}}$ $\boldsymbol{\tilde{H}}$ $\sim$  quark-gluon-quark fragmentation functions

$$\boldsymbol{H_1^{\perp(1)}(z)} \equiv z^2 \int d^2\vec{p}_\perp \frac{p_\perp^2}{2M_h^2} \boldsymbol{H_1^\perp(z, z^2 p_\perp^2)}$$ the first moment of Collins FF

$$F_{UT}^{\sin\phi_S} \sim \sum_a e_a^2 \frac{2M_h}{Q} \boldsymbol{h_1^a(x)} \frac{\boldsymbol{\tilde{H}(z)}}{z}$$

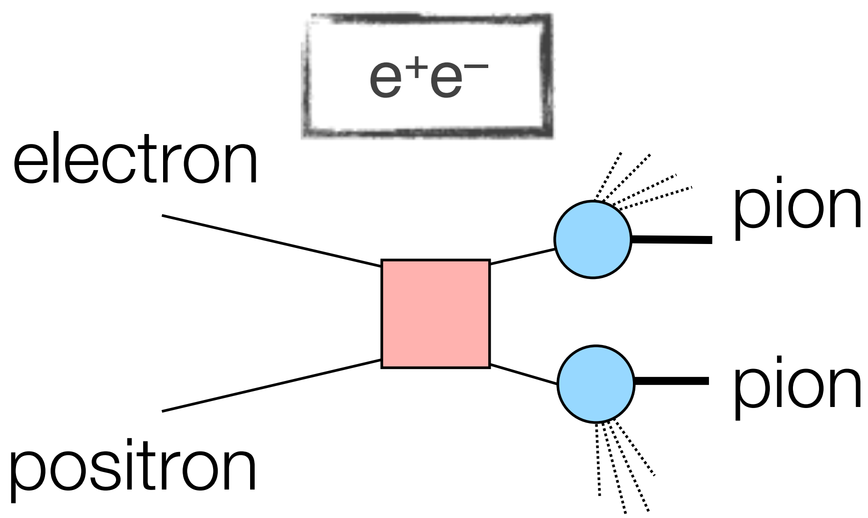
Mulders, Tangerman (96); Bacchetta, et al (07)

Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato (2020)

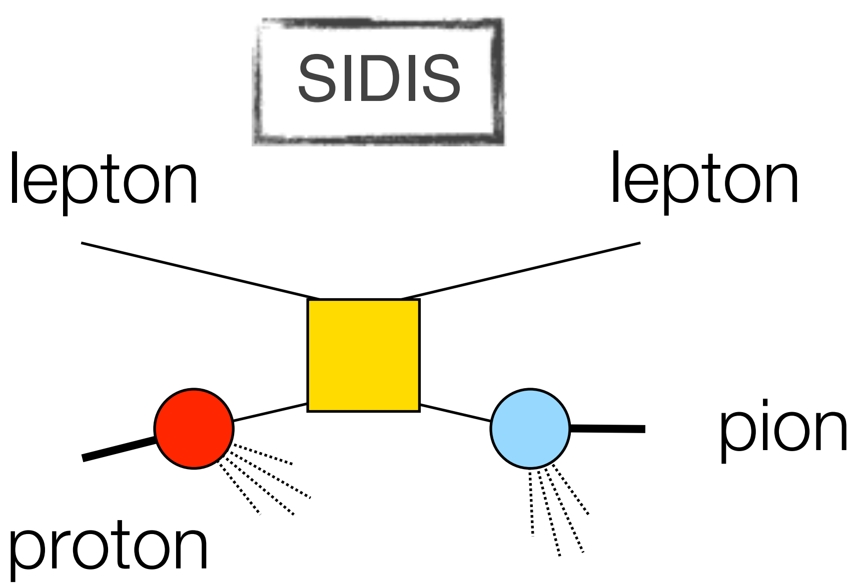
JAM20 ANALYSIS

UNIVERSAL GLOBAL FIT 2020

Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato (2020)

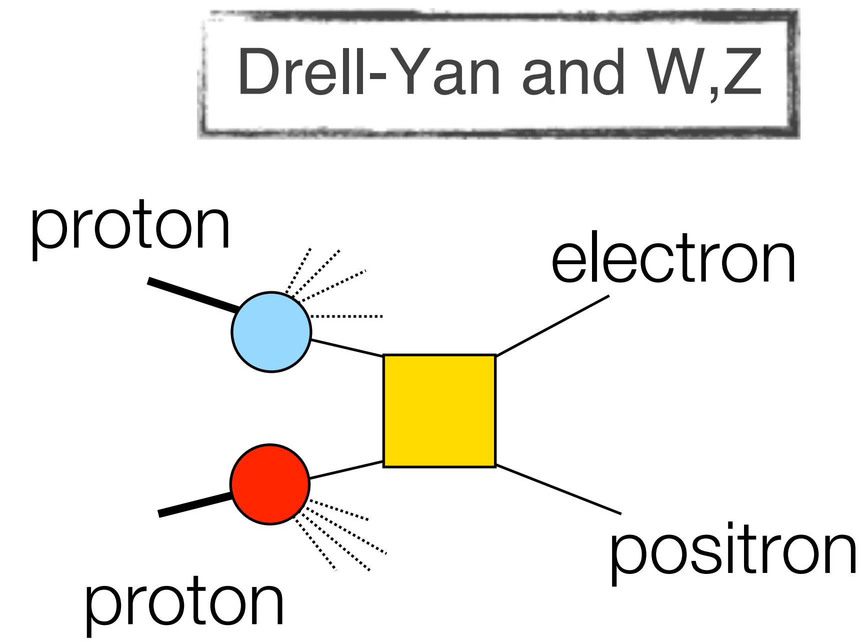


Collins asymmetries
BELLE, BaBar, BESIII data

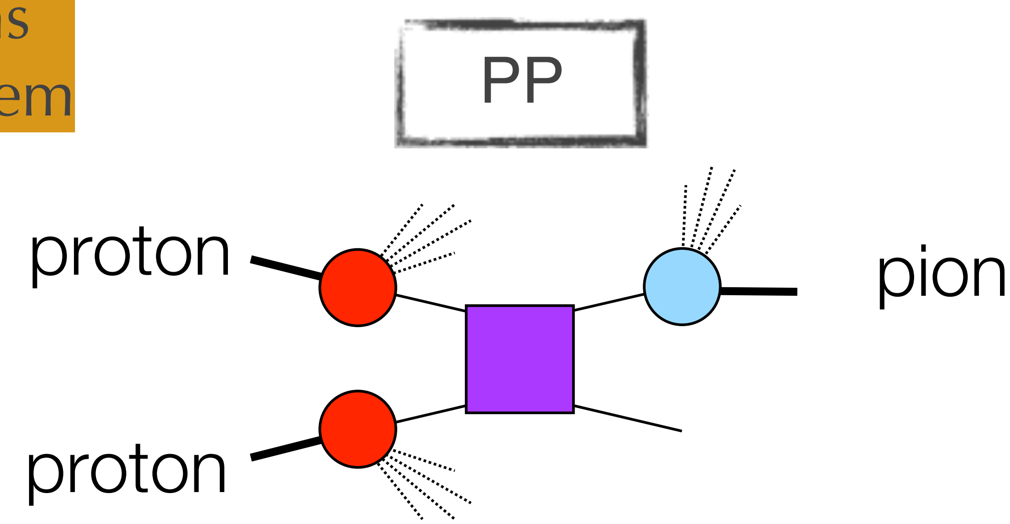


Sivers, Collins asymmetries
COMPASS, HERMES, JLab data

To demonstrate the common origin of SSAs in various processes, we combined all available data and extracted a universal set of non perturbative functions that describes all of them



Sivers asymmetries
COMPASS, STAR data



A_N asymmetry
STAR, PHENIX, BRAHMS data

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Jefferson Lab Angular Momentum Collaboration

https://www.jlab.org/theory/jam

Observable	**Reactions**	**Non-Perturbative Function(s)**	$\chi^2/N_{\rm pts.}$
$A_{\rm SIDIS}^{\rm Siv}$	$e+(p,d)^{\uparrow}\to e+(\pi^+,\pi^-,\pi^0)+X$	$f_{1T}^{\perp}(x,k_T^2)$	$150.0/126=1.19$
$A_{\rm SIDIS}^{\rm Col}$	$e+(p,d)^{\uparrow}\to e+(\pi^+,\pi^-,\pi^0)+X$	$h_1(x,k_T^2),\,H_1^{\perp}(z,z^2p_\perp^2)$	$111.3/126=0.88$
$A_{\rm SIA}^{\rm Col}$	$e^+ + e^- \to \pi^+\pi^-(UC,UL)+X$	$H_1^{\perp}(z,z^2p_\perp^2)$	$154.5/176=0.88$
$A_{\rm DY}^{\rm Siv}$	$\pi^- + p^{\uparrow}\to \mu^+\mu^- + X$	$f_{1T}^{\perp}(x,k_T^2)$	$5.96/12=0.50$
$A_{\rm DY}^{\rm Siv}$	$p^{\uparrow}+p\to (W^+,W^-,Z)+X$	$f_{1T}^{\perp}(x,k_T^2)$	$31.8/17=1.87$
A_N^h	$p^{\uparrow}+p\to(\pi^+,\pi^-,\pi^0)+X$	$h_1(x),\,F_{FT}(x,x)=\frac{1}{\pi}f_{1T}^{\perp(1)}(x),\,H_1^{\perp(1)}(z)$	$66.5/60=1.11$

Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato (2020)

- 18 observables and 6 non-perturbative functions (Sivers up/down; transversity up/down; Collins favored/unfavored)

$$h_1(x),\ F_{FT}(x,x),\ H_1^{\perp(1)}(z),\ \cancel{\tilde{H}}(z)$$

- Broad kinematical coverage to test universality
- The analysis is performed at parton level leading order, gaussian model is used for TMDs, and DGLAP-type evolution is implemented

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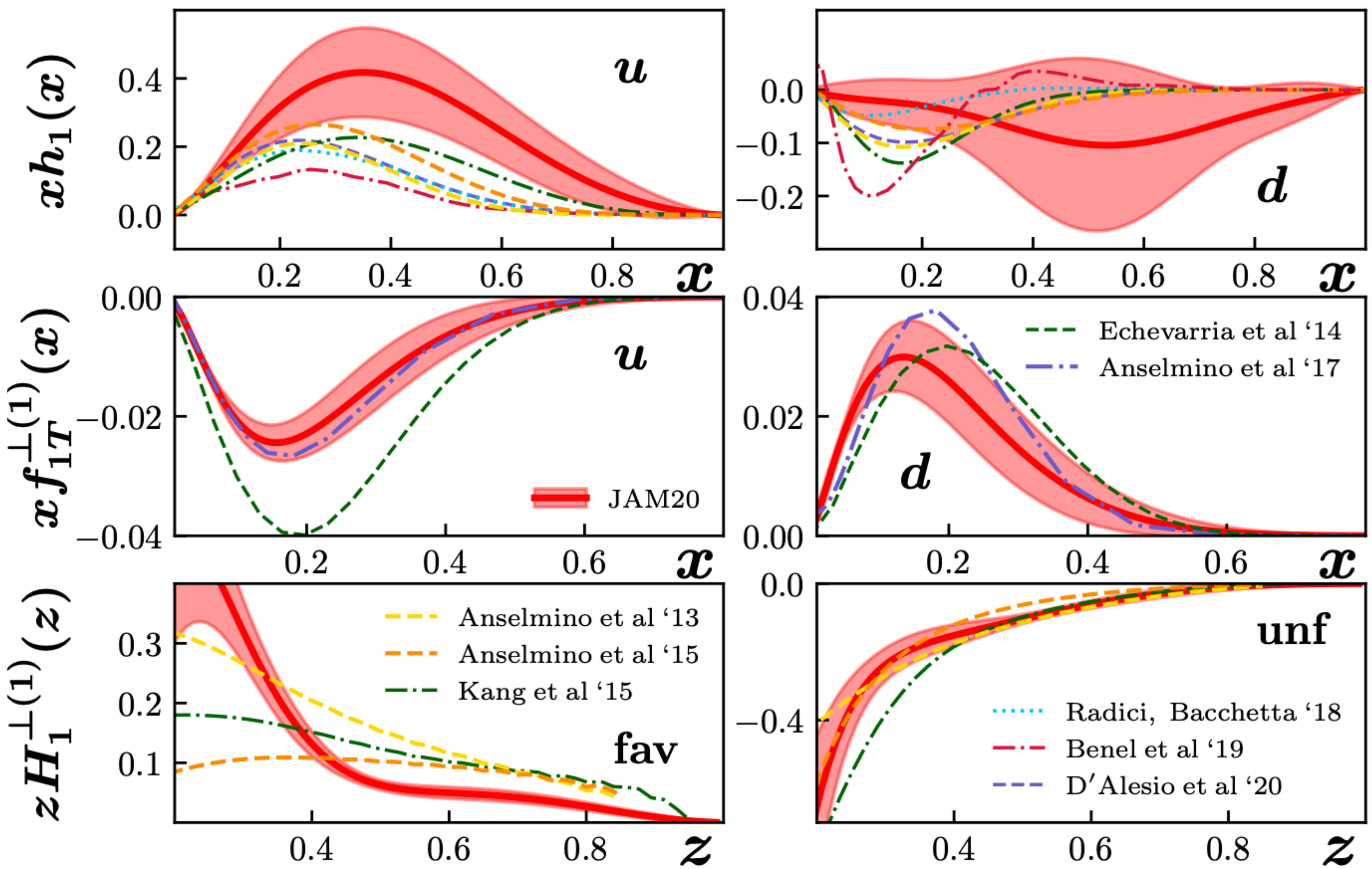
Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato (2020)



Transversity

$h_1(x)$

Sivers

$f_{1T}^{\perp(1)}(x)$

Collins FF

$H_1^{\perp(1)}(z)$

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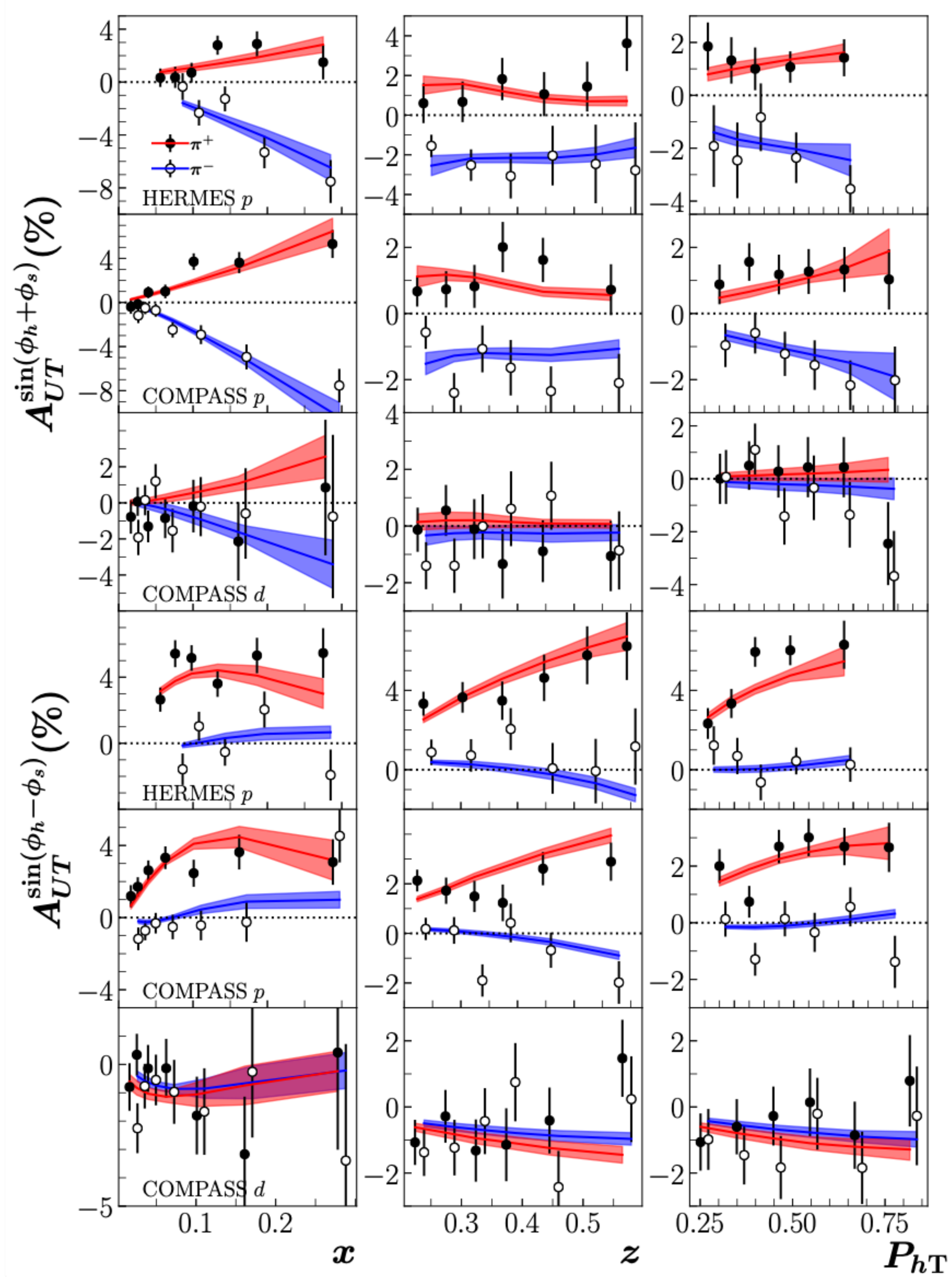
Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato (2020)

SIDIS



Collins asymmetry

$$\frac{\chi^2}{npoints} = \frac{107.1}{126} = 0.85$$

Sivers asymmetry

$$\frac{\chi^2}{npoints} = \frac{85.4}{88} = 0.97$$

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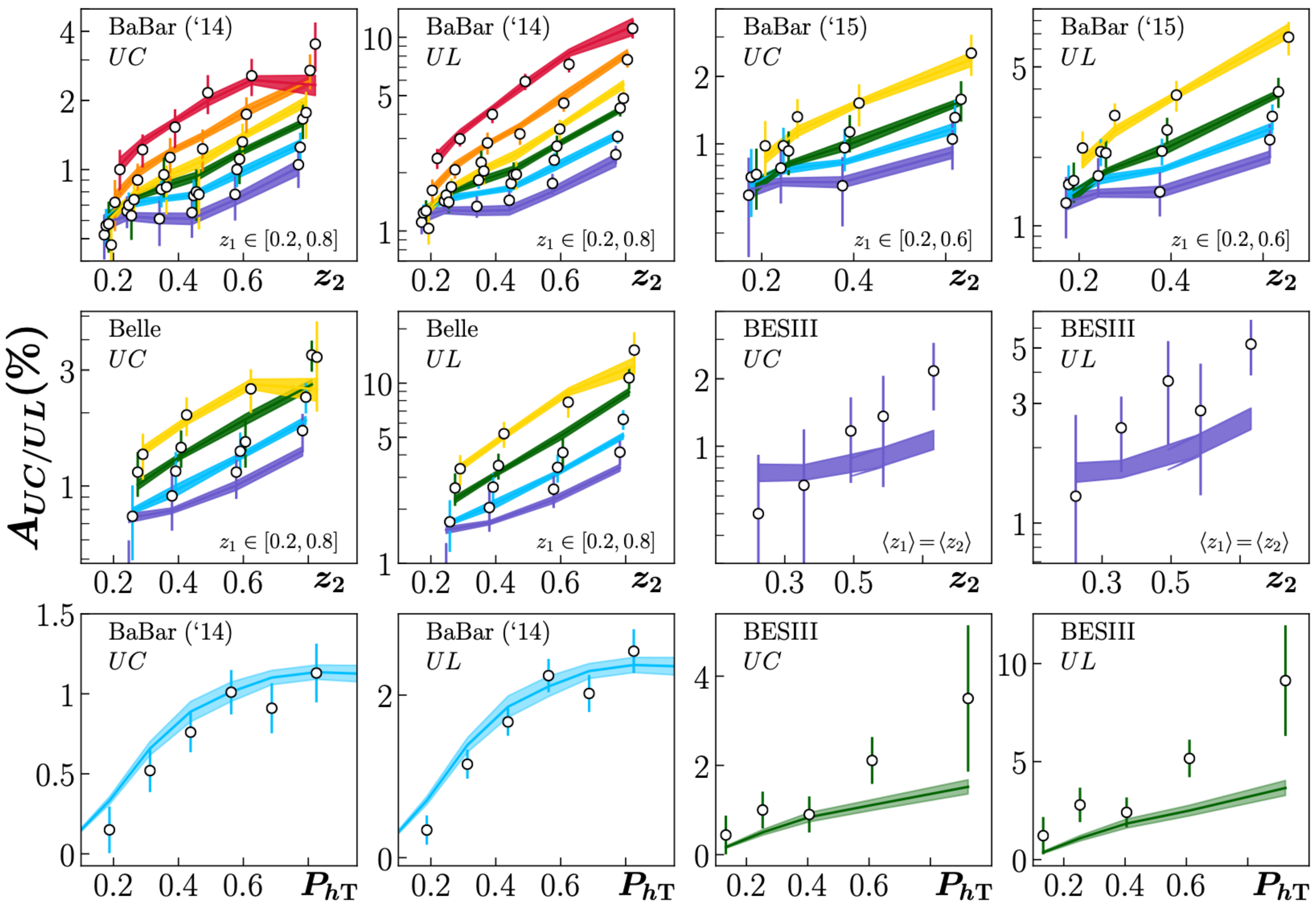
Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato (2020)

e^+e^-



$$\frac{\chi^2}{npoints} = \frac{149.6}{176} = 0.85$$

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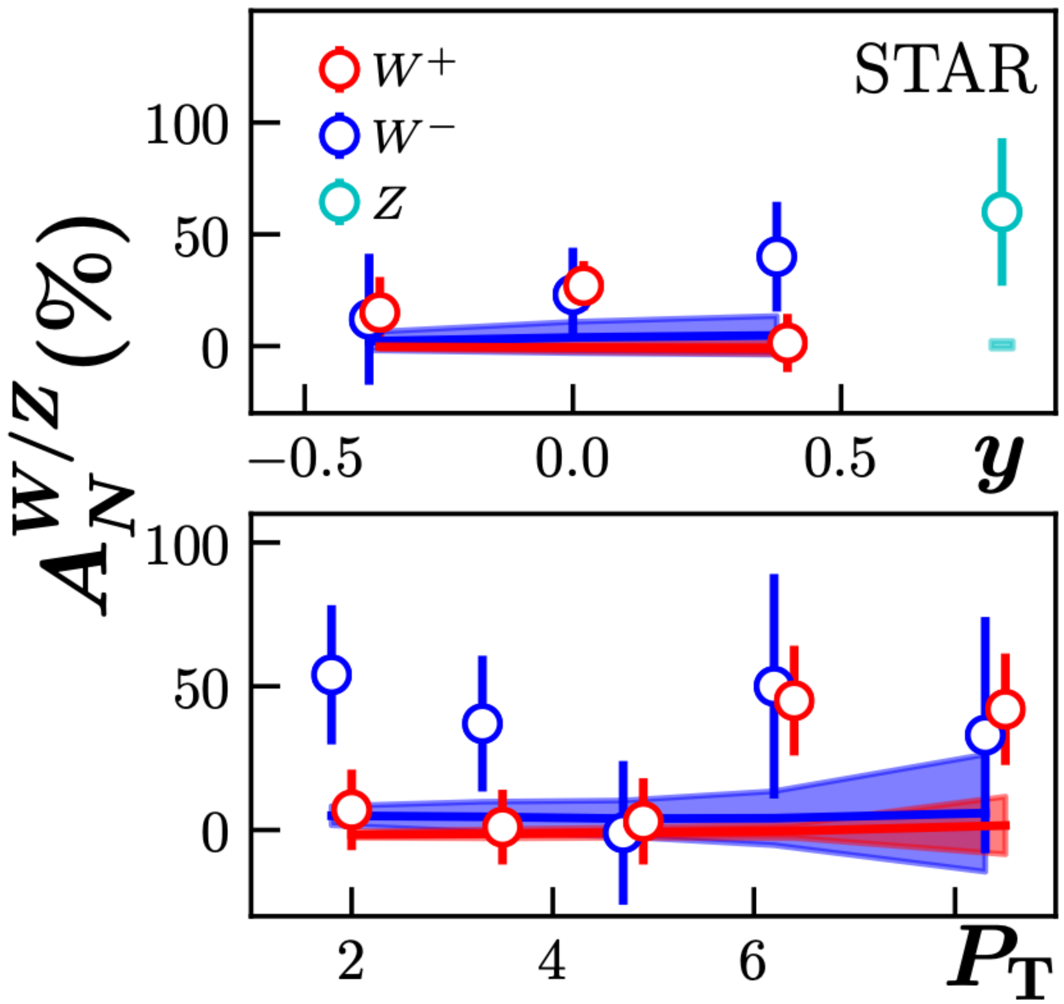
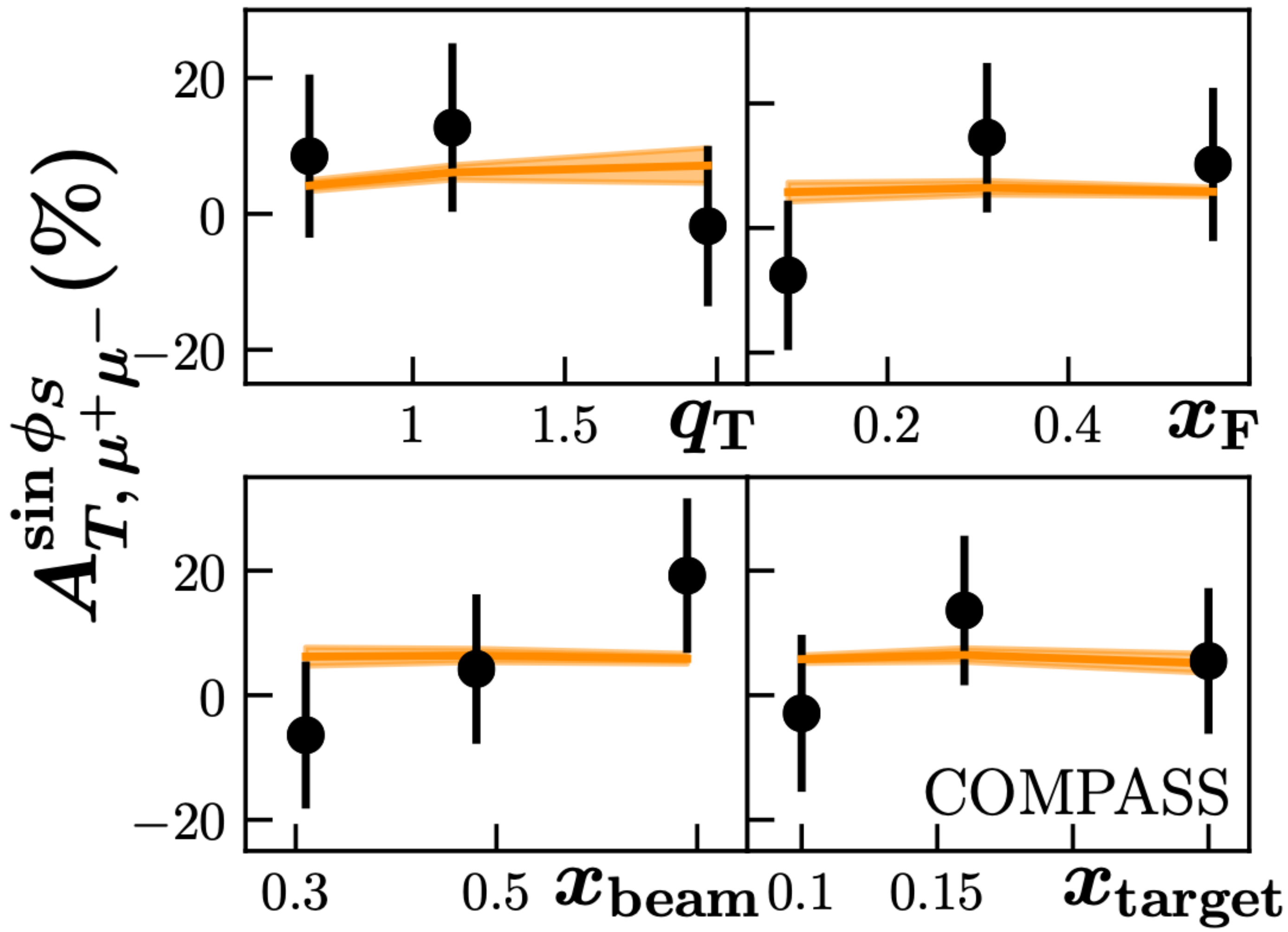
Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato (2020)

Drell-Yan





$$\frac{\chi^2}{npoints} = \frac{29.8}{17} = 1.75$$

STAR

$$\frac{\chi^2}{npoints} = \frac{7.6}{12} = 0.63$$

COMPASS DY

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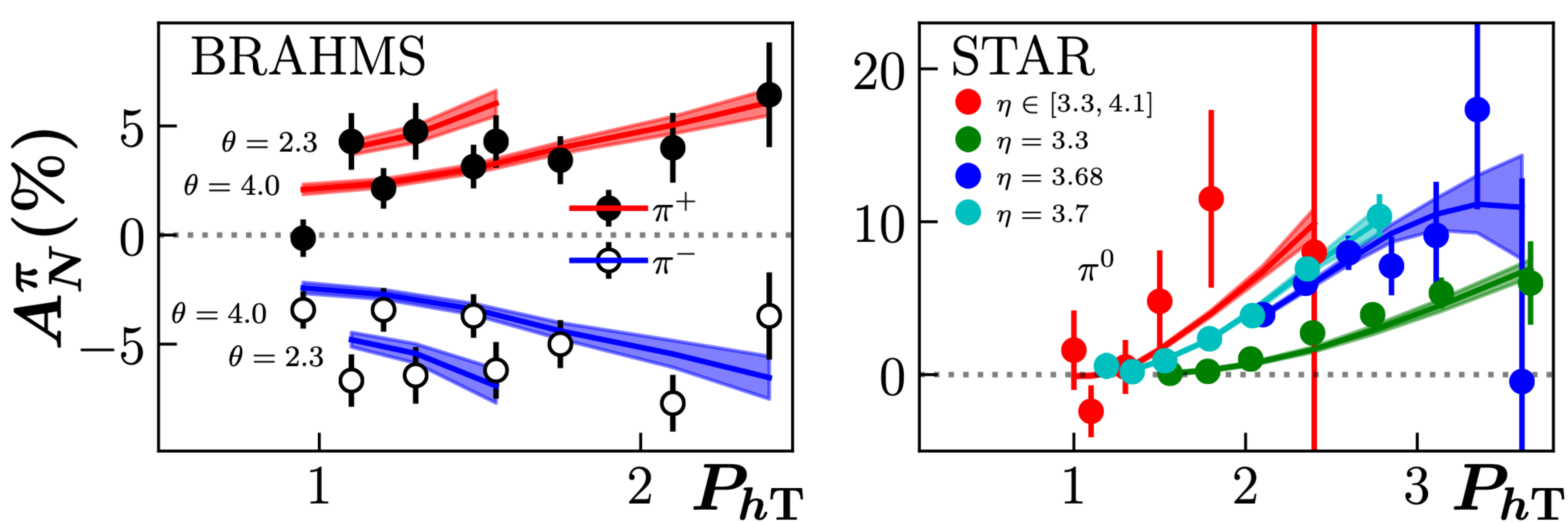
Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato (2020)

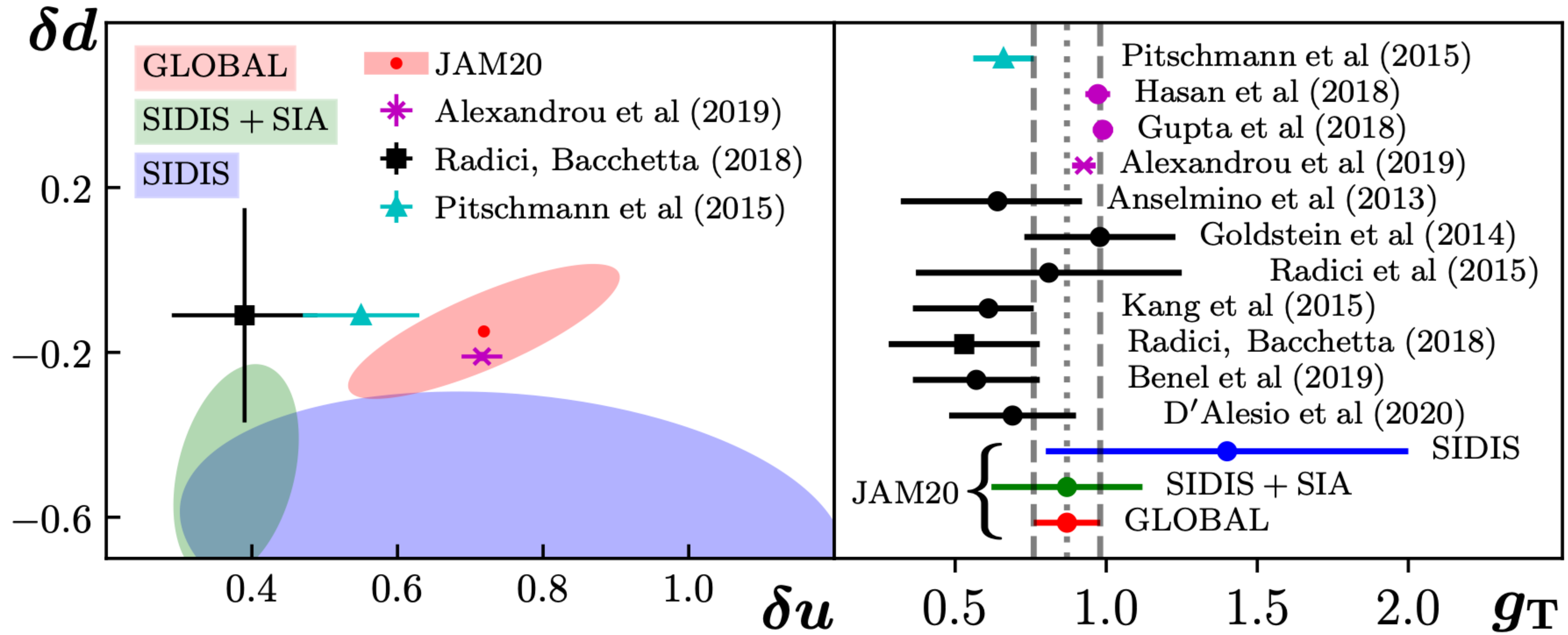
proton-proton A_N



$$\frac{\chi^2}{npoints} = \frac{72.0}{60} = 1.2$$

UNIVERSAL GLOBAL FIT 2020

Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato (2020)



- **Tensor charge** from up and down quarks is constrained and compatible with lattice results
- Isovector tensor charge $g_T = \delta u - \delta d$

$g_T = 0.89 \pm 0.12$ compatible with lattice results

δu and δd Q^2=4 GeV^2

$\delta u = 0.65 \pm 0.22$

$\delta d = -0.24 \pm 0.2$

Complementarity of SIDIS, e+e- and Drell-Yan, and hadron-hadron

Various processes allow study and test of evolution, universality and extractions of distribution and fragmentation functions. We need information from all of them

$$f(x) \otimes D(z)$$

Semi Inclusive DIS – convolution of distribution functions and fragmentation functions

$$\ell + P \to \ell' + h + X$$

$$f(x_1) \otimes f(x_2)$$

Drell-Yan – convolution of distribution functions

$$P_1 + P_2 \to \bar{\ell}\ell + X$$

$$D(z_1) \otimes D(z_2)$$

e+ e- annihilation – convolution of fragmentation functions

$$\bar{\ell} + \ell \to h_1 + h_2 + X$$

$$f(x_1) \otimes f(x_2) \otimes D(z)$$

Hadron-hadron – convolutions of PDF and fragmentation functions

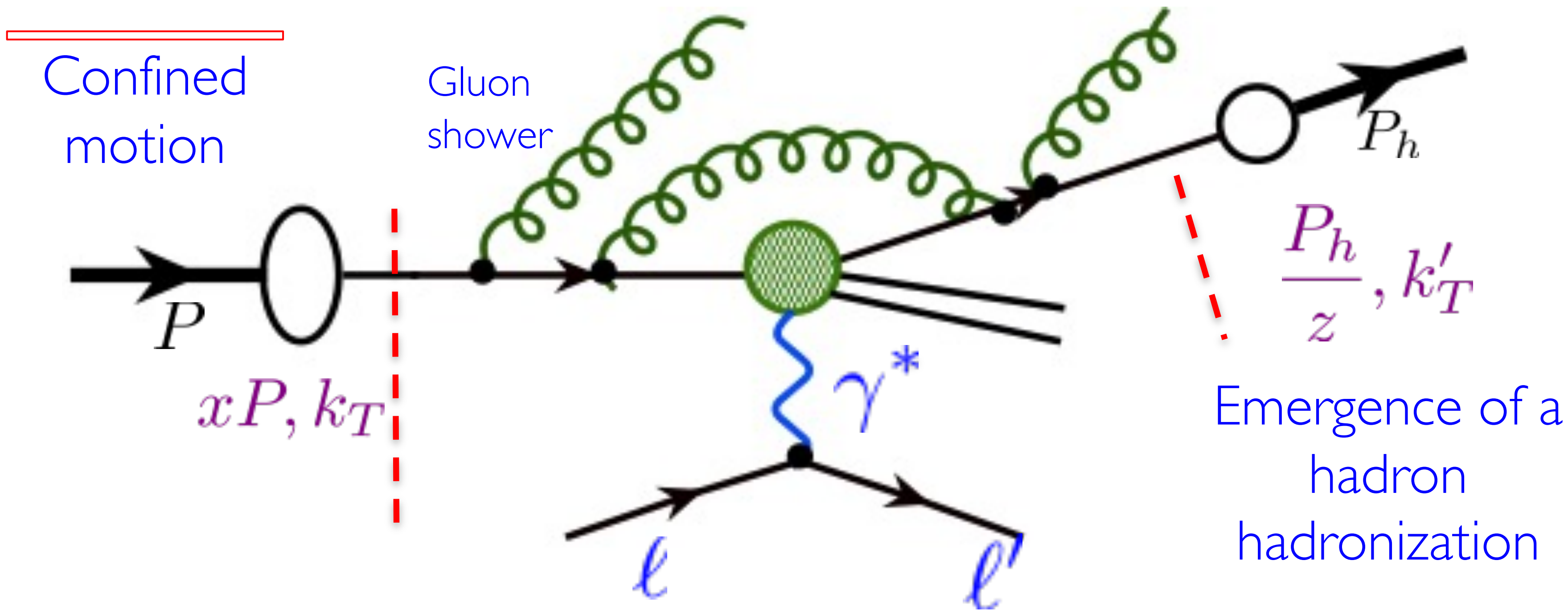
$$h_1 + h_2 \to h_3(\gamma, jet, W, ...) + X$$

Combining measurements from all above is important

Why TMDs, factorization, and evolution

PennState Berks

Study of evolution gives us insight on different aspects and origin of confined motion of partons, gluon radiation, parton fragmentation

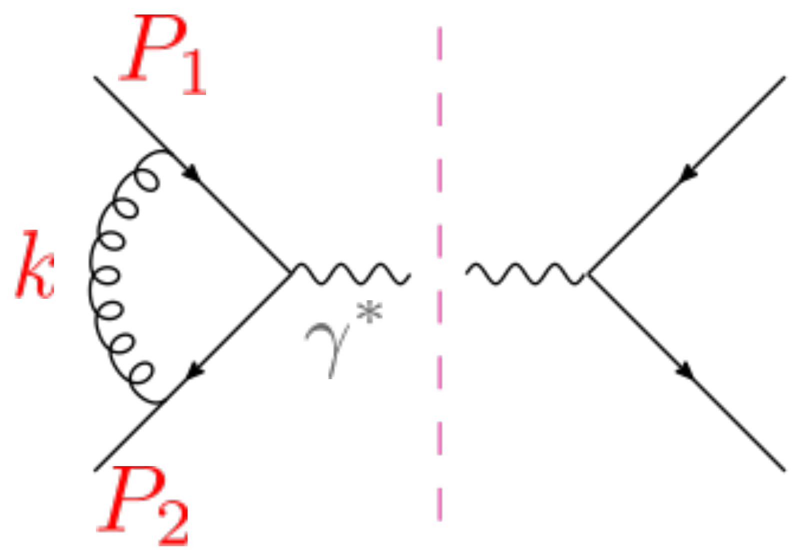


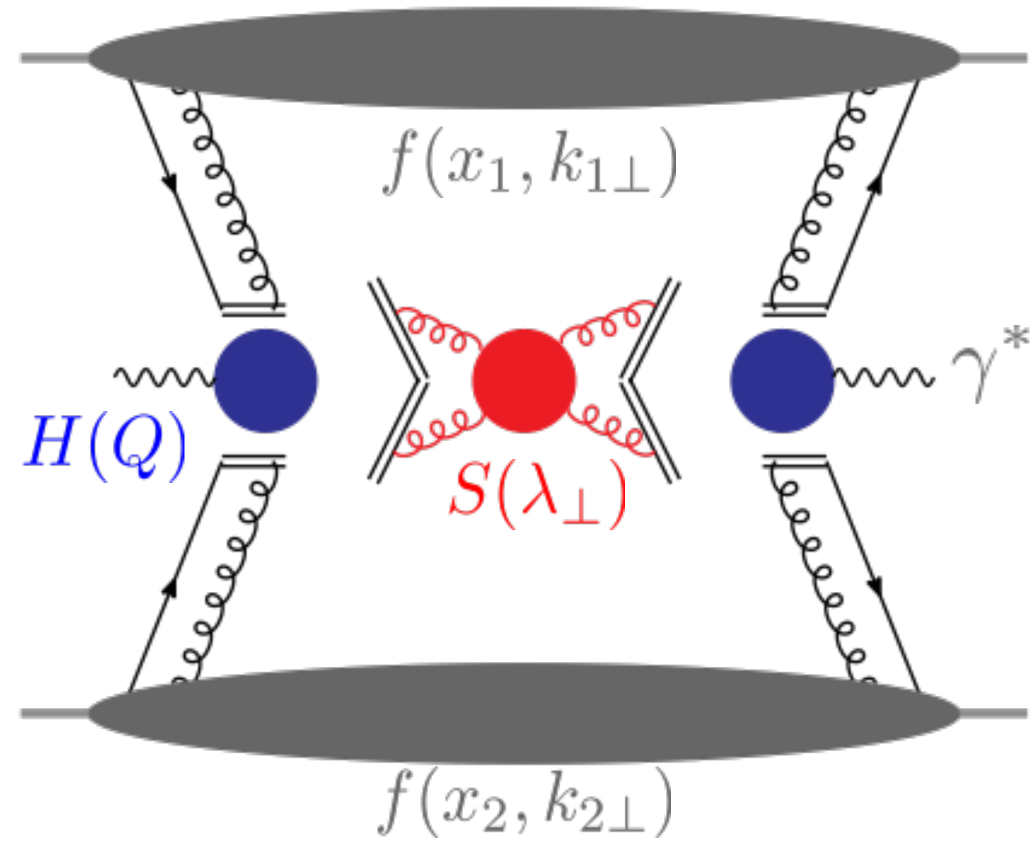
Evolution allows to connect measurements at very different scales.

TMD evolution has also a universal non-perturbative part. The result of evolution cannot be uniquely predicted using evolution equations until the non-perturbative part is reliably extracted from the data.

TMD factorization in a nut-shell

Drell-Yan: $p + p \to [\gamma^* \to \ell^+ \ell^-] + X$





Factorization of regions:
(1) $k//P_1$, (2) $k//P_2$, (3) k soft, (4) k hard

Factorized form and mimicking "parton model"

$$\frac{d\sigma}{dQ^2 dy d^2 q_\perp} \propto \int d^2k_{1\perp}\, d^2k_{2\perp}\, d^2\lambda_\perp H(Q) f(x_1, k_{1\perp}) f(x_2, k_{2\perp}) S(\lambda_\perp) \delta^2(k_{1\perp} + k_{2\perp} + \lambda_\perp - q_\perp)$$

$$= \int \frac{d^2b}{(2\pi)^2} e^{iq_\perp \cdot b} H(Q) f(x_1, b) f(x_2, b) S(b)$$

$$F(x, b) = f(x, b)\sqrt{S(b)}$$

$$\boxed{= \int \frac{d^2b}{(2\pi)^2} e^{iq_\perp \cdot b} H(Q) F(x_1, b) F(x_2, b)}$$

mimic "parton model"

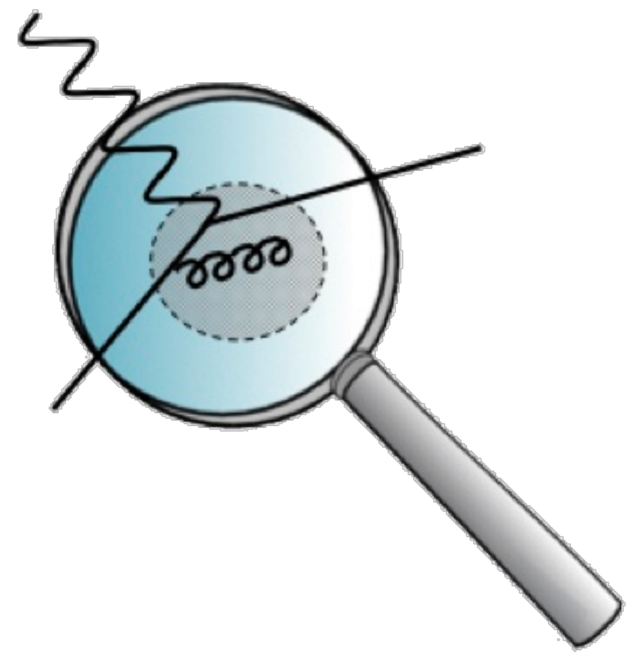
TMDs evolve

Just like collinear PDFs, TMDs also depend on the scale of the probe = evolution

Collinear PDFs

$$F(x, Q)$$

- ✓ DGLAP evolution
- ✓ Resum $\left[\alpha_s \ln(Q^2/\mu^2)\right]^n$
- ✓ Kernel: purely perturbative



TMDs

$$F(x, k_\perp; Q)$$

- ✓ Collins-Soper/rapidity evolution equation
- ✓ Resum $\left[\alpha_s \ln^2(Q^2/k_\perp^2)\right]^n$
- ✓ Kernel: can be non-perturbative when $k_\perp \sim \Lambda_{\rm QCD}$

$$F(x, Q_i) \;\downarrow\; R^{\rm coll}(x, Q_i, Q_f) \;\downarrow\; F(x, Q_f)$$

$$F(x, k_\perp, Q_i) \;\downarrow\; R^{\rm TMD}(x, k_\perp, Q_i, Q_f) \;\downarrow\; F(x, k_\perp, Q_f)$$

TMD evolution and non-perturbative component

Fourier transform back to the momentum space, one needs the whole b region (large b): need some non-perturbative extrapolation

Many different methods/proposals to model this non-perturbative part

$$F(x,k_\perp;Q) = \frac{1}{(2\pi)^2}\int d^2b e^{ik_\perp\cdot b}F(x,b;Q) = \frac{1}{2\pi}\int_0^\infty db\, bJ_0(k_\perp b)F(x,b;Q)$$

Collins, Soper, Sterman 85, ResBos, Qiu, Zhang 99, Echevarria, Idilbi, Kang, Vitev, 14, Aidala, Field, Gamberg, Rogers, 14, Sun, Yuan 14, D'Alesio, Echevarria, Melis, Scimemi, 14, Rogers, Collins, 15, Vladimirov, Scimemi 17…

Eventually evolved TMDs in b-space

$$F(x,b;Q) \approx C\otimes F(x,c/b^*) \times \exp\left\{-\int_{c/b^*}^{Q}\frac{d\mu}{\mu}\left(A\ln\frac{Q^2}{\mu^2}+B\right)\right\} \times \exp\left(-S_{\text{non-pert}}(b,Q)\right)$$

longitudinal/collinear part

transverse part

- ✓ Non-perturbative: fitted from data
- ✓ The key ingredient – ln(Q) piece is spin-independent

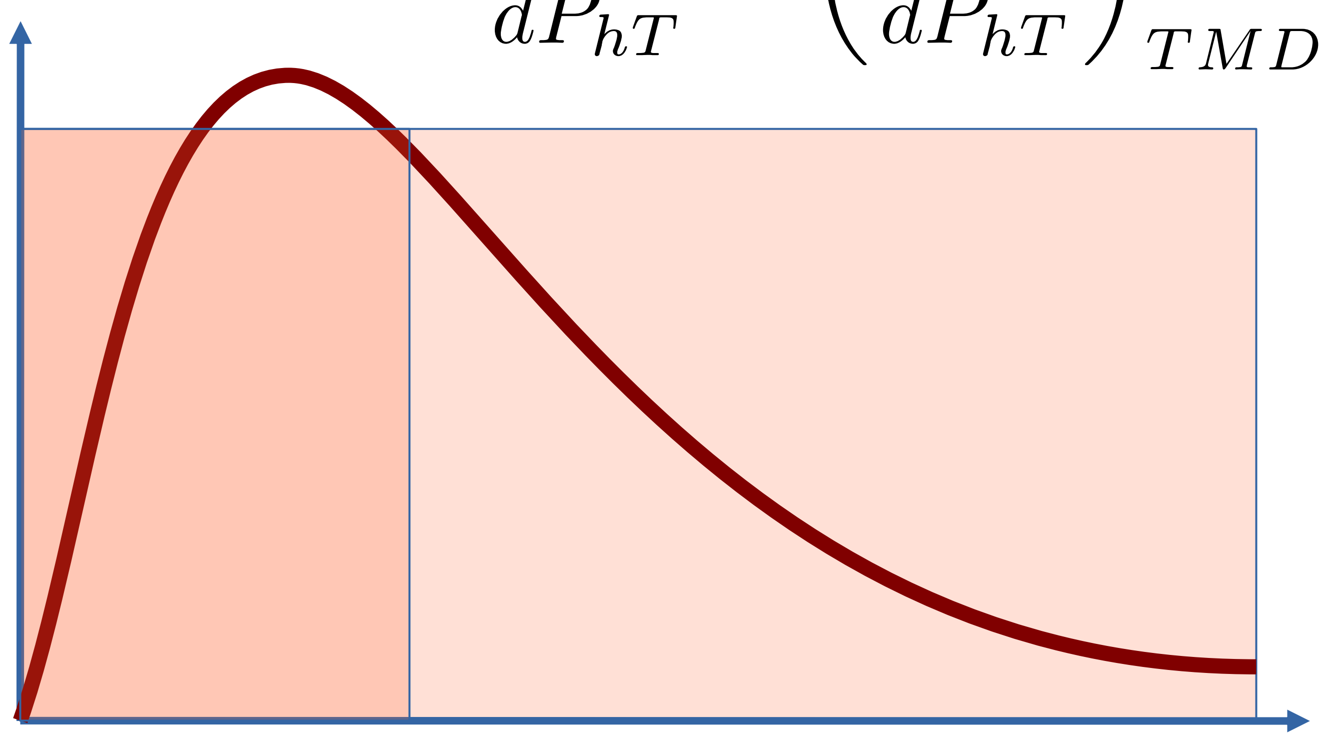
Since the polarized scattering data is still limited kinematics, we can use unpolarized data to constrain/extract key ingredients for the non-perturbative part

TMD

TMD factorization has a validity region $P_{hT}/z \ll Q$
(two scale problem)

In order to describe cross section in a wide region of transverse momentum one needs to add a Y term

$$\frac{d\sigma}{dP_{hT}} = \left(\frac{d\sigma}{dP_{hT}}\right)_{TMD} + Y$$



Improved approach that aims to describe low-Q region:

$$P_{hT}/z \ll Q$$

Collins, Gamberg, AP, Rogers, Sato, Wang arXiv:1605.00671

TMD related studies have been extremely active in the past few years, lots of progress have been made

We look forward to the future experimental results from COMPASS, RHIC, Jefferson Lab, LHC, Fermilab, future Electron Ion Collider

Many TMD related groups are created throughout the world:

Italy, Netherlands, Belgium, Germany, Japan, China, Russia, and the USA