$x_t \in \{0, 1\}$

$$A = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$$

$$p(y_i | x=0) = \begin{cases} 0.7 & y_i = 10 \\ 0.3 & y_i = 20 \end{cases}$$

$$p(y_i | x=1) = \begin{cases} 0.4 & y_i = 10 \\ 0.6 & y_i = 20 \end{cases}$$

prior

$$\pi(x_1) = \begin{cases} 0.5 & x=1 \\ 0.5 & x=0 \end{cases}$$

$y = \{y_i\} = [10, 10, 20, 10]$

$$b = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$$

forward algorithm

$$\alpha_1(j) = \pi_j b_j(O_1)$$

$$\alpha_1 = \begin{bmatrix} 0.35 \\ 0.2 \end{bmatrix}$$

$$\begin{matrix} 0.5 \cdot 0.7 \\ 0.5 \cdot 0.4 \end{matrix}$$

$$\alpha_2 = \begin{bmatrix} 0.2485 \\ 0.078 \end{bmatrix}$$

$$\alpha_{t+1} = b_j(O_{t+1}) \cdot \sum_{i=1}^{2} \alpha_t(i)\, a_{ij}$$

$$\begin{matrix} 0.7 \cdot (0.35 \cdot 0.9 + 0.2 \cdot 0.2) \\ 0.4 \cdot (0.35 \cdot 0.1 + 0.2 \cdot 0.8) \end{matrix}$$

$$\alpha_3 = \begin{bmatrix} 0.071775 \\ 0.05235 \end{bmatrix}$$

$$\begin{matrix} 0.3 \cdot [0.2485 \cdot 0.9 + 0.078 \cdot 0.2] \\ 0.6 \cdot [0.2485 \cdot 0.1 + 0.078 \cdot 0.8] \end{matrix}$$

$$\alpha_4 = \begin{bmatrix} 0.05254725 \\ 0.019623 \end{bmatrix}$$

$$\begin{matrix} 0.7 \cdot [0.071775 \cdot 0.9 + 0.05235 \cdot 0.2 \\ 0.4 \cdot [0.071775 \cdot 0.1 + 0.05235 \cdot 0.8 \end{matrix}$$