$$\sum_{i=1}^{\infty} i \cdot a^{i-1}(1-a)$$

$$= (1-a)\sum_{i=1}^{\infty} i a^{i-1} = (1-a)\frac{d}{da}\left(\sum_{i=0}^{\infty} a^i\right) = (1-a)\frac{d}{da}\left(\frac{1}{1-a}\right) = (1-a)\frac{-1}{(1-a)^2}\cdot(-1)$$

$$= \frac{1}{1-a}$$

$x_t \in \{0, 1\}$

$$p(y_i | x=0) = \begin{cases} 0.7 & y_i = 10 \\ 0.3 & y_i = 20 \end{cases}$$

$$A = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$$

$$p(y_i | x=1) = \begin{cases} 0.4 & y_i = 10 \\ 0.6 & y_i = 20 \end{cases}$$

prior

$$\pi(x) = \begin{cases} 0.5 & x=1 \\ 0.5 & x=0 \end{cases}$$

$$y = \{y_i\} = [10, 10, 20, 10]$$

$$b = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$$

forward algorithm

$$\alpha_1(j) = \pi_j b_j(O_1)$$

$$\alpha_1 = \begin{bmatrix} 0.35 \\ 0.2 \end{bmatrix} \qquad \begin{matrix} 0.5 \cdot 0.7 \\ 0.5 \cdot 0.4 \end{matrix}$$

$$\alpha_{t+1} = b_j(O_{t+1}) \cdot \sum_{i=1}^{2} \alpha_t(i) a_{ij}$$

$$\alpha_2 = \begin{bmatrix} 0.2485 \\ 0.078 \end{bmatrix} \qquad \begin{matrix} 0.7 \cdot (0.35 \cdot 0.9 + 0.2 \cdot 0.2) \\ 0.4 \cdot (0.35 \cdot 0.1 + 0.2 \cdot 0.8) \end{matrix}$$

$$\alpha_3 = \begin{bmatrix} 0.071775 \\ 0.05235 \end{bmatrix} \qquad \begin{matrix} 0.3 \cdot \\ 0.6 \cdot \end{matrix} \begin{bmatrix} 0.2485 \cdot 0.9 + 0.078 \cdot 0.2 \\ 0.2485 \cdot 0.1 + 0.078 \cdot 0.8 \end{bmatrix}$$

$$\alpha_4 = \begin{bmatrix} 0.05254725 \\ 0.019623 \end{bmatrix} \qquad \begin{matrix} 0.7 \cdot \\ 0.4 \cdot \end{matrix} \begin{bmatrix} 0.071775 \cdot 0.9 + 0.05235 \cdot 0.2 \\ 0.071775 \cdot 0.1 + 0.05235 \cdot 0.8 \end{bmatrix}$$

backward algorithm

$$\beta_4 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\beta_t(i) = \sum_j a_{ij}\, \beta_{t+1}(j)\, b_j(O_{t+1})$$

$$\beta_3 = \begin{bmatrix} 0.67 \\ 0.46 \end{bmatrix}$$

$0.9 \cdot 1 \cdot 0.7 + 0.1 \cdot 1 \cdot 0.4$

$0.2 \cdot 1 \cdot 0.7 + 0.8 \cdot 1 \cdot 0.4$

$$\beta_2 = \begin{bmatrix} 0.2085 \\ 0.261 \end{bmatrix}$$

$0.9 \cdot 0.67 \cdot 0.3 + 0.1 \cdot 0.46 \cdot 0.6$

$0.2 \cdot 0.67 \cdot 0.3 + 0.8\ 0.46 \cdot 0.6$

$$\beta_1 = \begin{bmatrix} 0.141795 \\ 0.11271 \end{bmatrix}$$

$0.9 \cdot 0.2085 \cdot 0.7 + 0.1 \cdot 0.261 \cdot 0.4$

$0.2 \cdot 0.2085 \cdot 0.7 + 0.8 \cdot 0.261 \cdot 0.4$