# Introduction to Electrical Circuits

**Final Term**

**Week: 9**

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**Room No: D0414**

**Book**

## Introductory Circuit Analysis

Robert L. Boylestad
Eleventh Edition

# Series Parallel Network Analysis

For the network in Fig. 16.1:

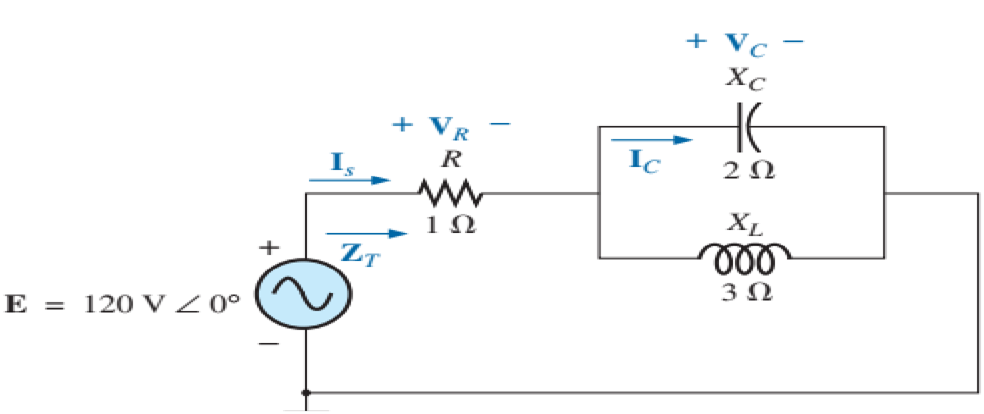
a. Calculate **Z***T*.

b. Determine **I***s*.

c. Calculate **V***R* and **V***C*   d. Find **I***C*.

e. Compute the power delivered.

f. Find *Fp* of the network.



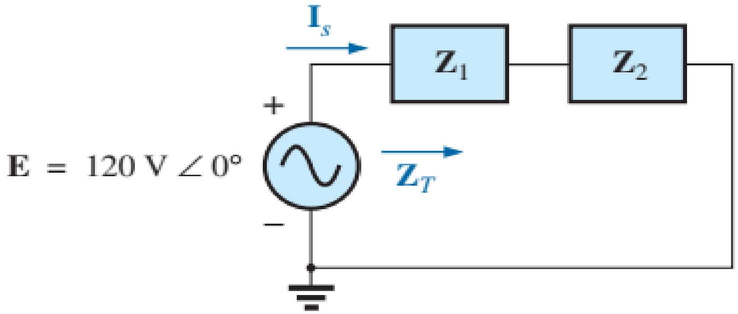




**FIG. 16.2**

*Network in Fig. 16.1 after assigning the block impedances*

## *Solutions:*

**Z***T* =**Z1** +**Z2**

with

$$\mathbf{Z}_1 = R\angle 0° = 1\ \Omega\angle 0°$$

$$\mathbf{Z}_2 = \mathbf{Z}_C \parallel \mathbf{Z}_L = \frac{(X_C\angle -90°)(X_L\angle 90°)}{-j\,X_C + j\,X_L} = \frac{(2\ \Omega\angle -90°)(3\ \Omega\angle 90°)}{-j\,2\ \Omega + j\,3\ \Omega}$$

$$= \frac{6\ \Omega\angle 0°}{j\,1} = \frac{6\ \Omega\angle 0°}{1\angle 90°} = 6\ \Omega\angle -90°$$

and

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 1\ \Omega - j\,6\ \Omega = \mathbf{6.08\ \Omega\angle -80.54°}$$

b. $$\mathbf{I}_s = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{120\ \text{V}\angle 0°}{6.08\ \Omega\angle -80.54°} = \mathbf{19.74\ A\angle 80.54°}$$

c. Referring to Fig. 16.2, we find that $\mathbf{V}_R$ and $\mathbf{V}_C$ can be found by a direct application of Ohm's law:

$$\mathbf{V}_R = \mathbf{I}_s\mathbf{Z}_1 = (19.74\ \text{A}\angle 80.54°)(1\ \Omega\angle 0°) = \mathbf{19.74\ V\angle 80.54°}$$

$$\mathbf{V}_C = \mathbf{I}_s\mathbf{Z}_2 = (19.74\ \text{A}\angle 80.54°)(6\ \Omega\angle -90°)$$

$$= \mathbf{118.44\ V\angle -9.46°}$$

d. Now that $\mathbf{V}_C$ is known, the current $\mathbf{I}_C$ can also be found using Ohm's law.

$$\mathbf{I}_C = \frac{\mathbf{V}_C}{\mathbf{Z}_C} = \frac{118.44\ \text{V}\angle -9.46°}{2\ \Omega\angle -90°} = \mathbf{59.22\ A\angle 80.54°}$$

e. $P_{\text{del}} = I_s^2R = (19.74\ \text{A})^2(1\ \Omega) = \mathbf{389.67\ W}$

f. $F_p = \cos\theta = \cos 80.54° = \mathbf{0.164\ leading}$

# Series Parallel Network Analysis

**EXAMPLE 16.3** For the network in Fig. 16.5:

a. Calculate the voltage **V***C* using the voltage divider rule.
b. Calculate the current **I***s.*

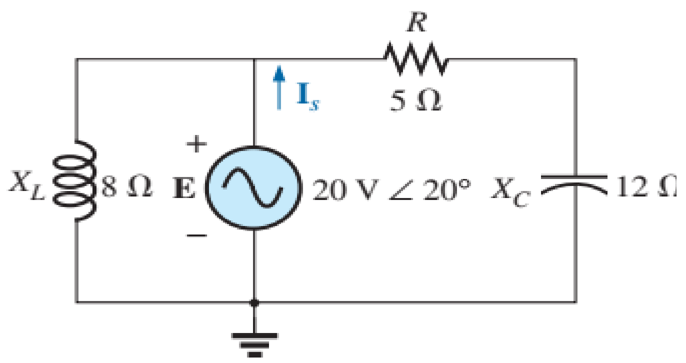



**FIG. 16.5**
*Example 16.3.*

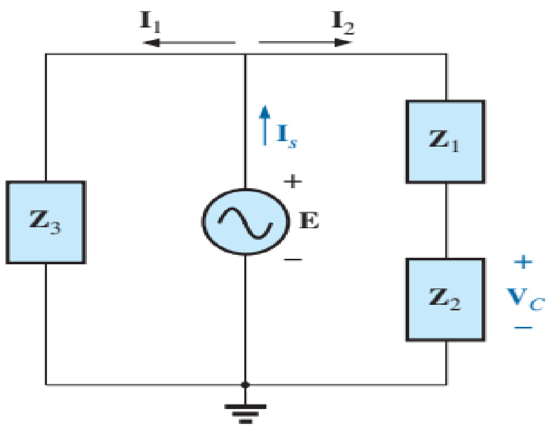



**FIG. 16.6**
*Network in Fig. 16.5 after assigning the block impedances.*

***Solutions:***

a. The network is redrawn as shown in Fig. 16.6, with

$$\begin{aligned}\mathbf{Z}_1 &= 5\ \Omega = 5\ \Omega\ \angle 0^\circ\\ \mathbf{Z}_2 &= -j\,12\ \Omega = 12\ \Omega\ \angle -90^\circ\\ \mathbf{Z}_3 &= +j\,8\ \Omega = 8\ \Omega\ \angle 90^\circ\end{aligned}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(12\ \Omega\ \angle -90^\circ)(20\ \text{V}\ \angle 20^\circ)}{5\ \Omega - j\,12\ \Omega} = \frac{240\ \text{V}\ \angle -70^\circ}{13\ \angle -67.38^\circ}$$
$$= \mathbf{18.46\ V\ \angle -2.62^\circ}$$

b. $$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_3} = \frac{20\ \text{V}\ \angle 20^\circ}{8\ \Omega\ \angle 90^\circ} = 2.5\ \text{A}\ \angle -70^\circ$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{20\ \text{V}\ \angle 20^\circ}{13\ \Omega\ \angle -67.38^\circ} = 1.54\ \text{A}\ \angle 87.38^\circ$$

and

$$\begin{aligned}\mathbf{I}_s &= \mathbf{I}_1 + \mathbf{I}_2\\ &= 2.5\ \text{A}\ \angle -70^\circ + 1.54\ \text{A}\ \angle 87.38^\circ\\ &= (0.86 - j\,2.35) + (0.07 + j\,1.54)\\ \mathbf{I}_s &= 0.93 - j\,0.81 = \mathbf{1.23\ A\ \angle -41.05^\circ}\end{aligned}$$

# Series Parallel Network Analysis

Example: 16.7

a. Compute **I**.
b. Find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.
c. Verify Kirchhoff's current law by showing that

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

d. Find the total impedance of the circuit.

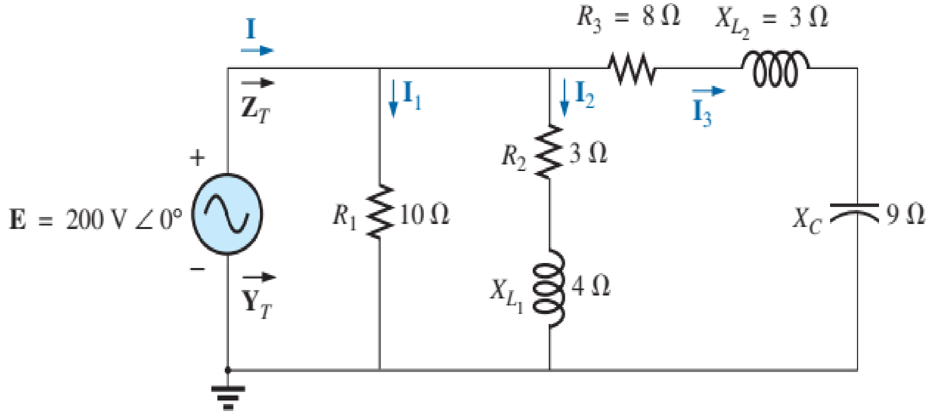



**FIG. 16.14**
*Example 16.7.*

**Solutions:**

a. Redrawing the circuit as in Fig. 16.15 reveals a strictly parallel network where

$$\mathbf{Z}_1 = R_1 = 10\ \Omega\ \angle 0°$$
$$\mathbf{Z}_2 = R_2 + j\,X_{L_1} = 3\ \Omega + j\,4\ \Omega$$
$$\mathbf{Z}_3 = R_3 + j\,X_{L_2} - j\,X_C = 8\ \Omega + j\,3\ \Omega - j\,9\ \Omega = 8\ \Omega - j\,6\ \Omega$$

The total admittance is

$$\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3$$
$$= \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} = \frac{1}{10\ \Omega} + \frac{1}{3\ \Omega + j\,4\ \Omega} + \frac{1}{8\ \Omega - j\,6\ \Omega}$$
$$= 0.1\ \mathrm{S} + \frac{1}{5\ \Omega\ \angle 53.13°} + \frac{1}{10\ \Omega\ \angle -36.87°}$$
$$= 0.1\ \mathrm{S} + 0.2\ \mathrm{S}\ \angle -53.13° + 0.1\ \mathrm{S}\ \angle 36.87°$$
$$= 0.1\ \mathrm{S} + 0.12\ \mathrm{S} - j\,0.16\ \mathrm{S} + 0.08\ \mathrm{S} + j\,0.06\ \mathrm{S}$$
$$= 0.3\ \mathrm{S} - j\,0.1\ \mathrm{S} = 0.316\ \mathrm{S}\ \angle -18.435°$$

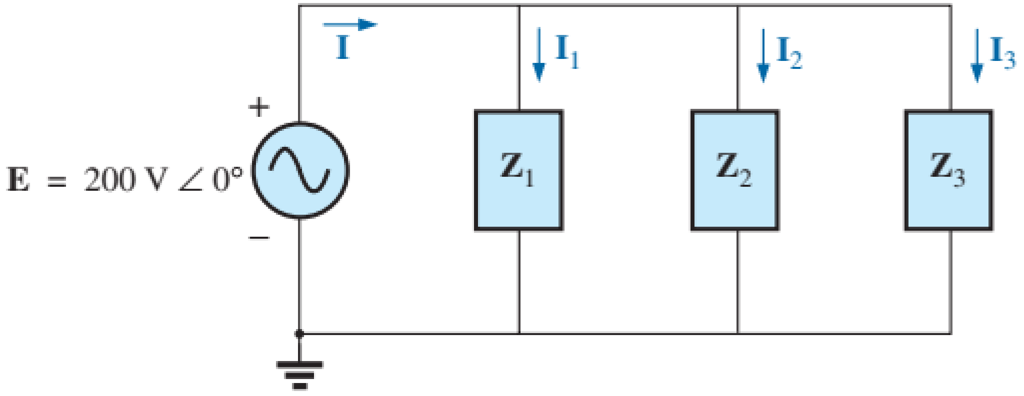



**FIG. 16.15**
*Network in Fig. 16.14 following the assignment of the subscripted impedances*

# Real Power, Reactive Power and Apparent Power Measurement

## General Equation:

For any system such as in Fig. 19.1, the power delivered to a load at any instant is defined by the product of the applied voltage and the resulting current; that is,

$$p = vi$$

In this case, since $v$ and $i$ are sinusoidal quantities, let us establish a general case where

$$v = V_m \sin(\omega t + \theta)$$

and

$$i = I_m \sin \omega t$$

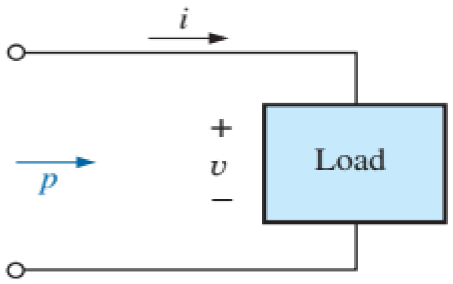



**FIG. 19.1**
*Defining the power delivered to a load.*

The chosen $v$ and $i$ include all possibilities because, if the load is purely resistive, $\theta = 0°$. If the load is purely inductive or capacitive, $\theta = 90°$ or $\theta = -90°$, respectively. For a network that is primarily inductive, $\theta$ is positive ($v$ leads $i$). For a network that is primarily capacitive, $\theta$ is negative ($i$ leads $v$).

$$p = V_m I_m \sin \omega t \sin(\omega t + \theta) \qquad \textbf{(19.1)}$$

Applying the trigonometric product-to-sum identity of: $\sin A \sin B = \dfrac{\cos(A-B) - \cos(A+B)}{2}$

$$\boxed{p = VI\cos\theta(1 - \cos 2\omega t) + VI\sin\theta(\sin 2\omega t)}$$

where $V$ and $I$ are the rms values.

If Eq. (19.1) is expanded to the form

$$p = \underbrace{VI\cos\theta}_{\text{Average}} - \underbrace{VI\cos\theta}_{\text{Peak}}\underbrace{\cos 2\omega t}_{2x} + \underbrace{VI\sin\theta}_{\text{Peak}}\underbrace{\sin 2\omega t}_{2x}$$

# Resistive Circuit

## Resistive Load

For a purely resistive circuit (such as that in Fig. 19.2), $v$ and $i$ are in phase, and $\theta = 0°$, as appearing in Fig. 19.3. Substituting $\theta = 0°$ into Eq. (19.1), we obtain

$$p_R = VI\cos(0°)(1 - \cos 2\omega t) + VI\sin(0°)\sin 2\omega t$$
$$= VI(1 - \cos 2\omega t) + 0$$

$$\boxed{p_R = VI - VI\cos 2\omega t}$$

where $VI$ is the average or dc term and $-VI\cos 2\omega t$ is a negative cosine wave with twice the frequency of either input quantity ($v$ or $i$) and a peak value of $VI$.

***the total power delivered to a resistor will be dissipated in the form of heat.***

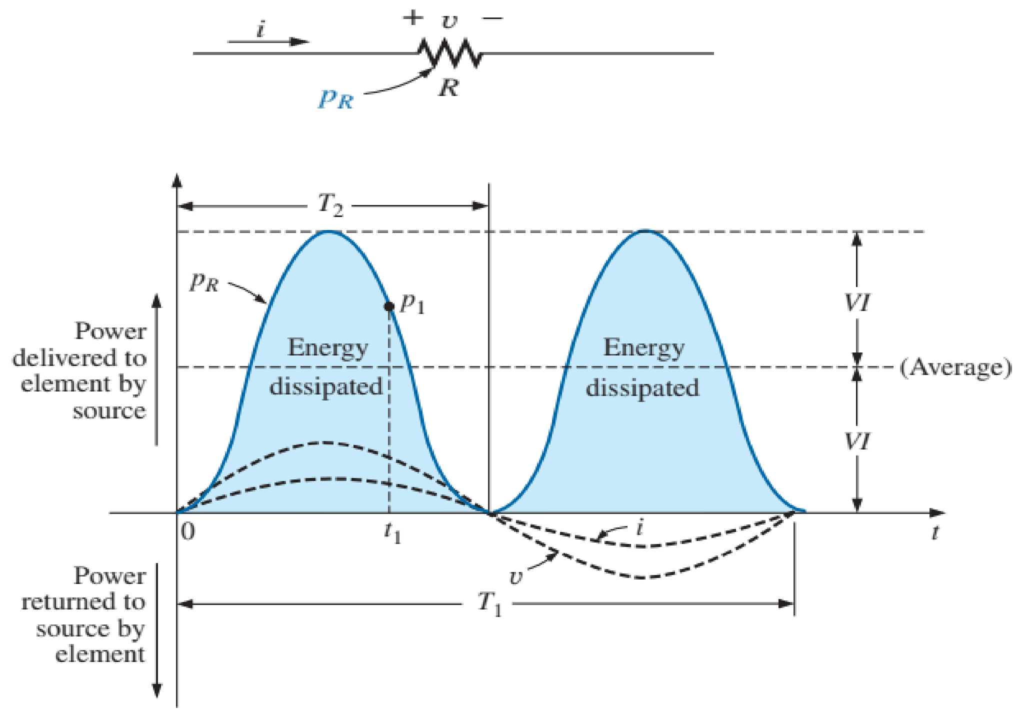



**FIG. 19.3**
*Power versus time for a purely resistive load.*

The **average** **(real) power** from Eq. (19.2), or Fig. 19.3, is $VI$; or, as a summary,

$$\boxed{P = VI = \frac{V_m I_m}{2} = I^2R = \frac{V^2}{R}} \quad \text{(watts, W)}$$

The energy dissipated by the resistor ($W_R$) over one full cycle $W = Pt$
where $P$ is the average value and $t$ is the period of the applied voltage;

$$\boxed{W_R = VIT_1} \quad \text{(joules, J)}$$

$$\boxed{W_R = \frac{VI}{f_1}} \quad \text{(joules, J)}$$

# Apparent Power

## Apparent Power

Apparent power is the product of the rms value of voltage and the rms value o current. The **unit** of apparent power is called **VA (volt-ampere)**.

$$\boxed{S = VI} \qquad \text{(volt-amperes, VA)}$$

or, since $V = IZ$ and $I = \dfrac{V}{Z}$

then $$\boxed{S = I^2 Z} \qquad \text{(VA)}$$

and $$\boxed{S = \dfrac{V^2}{Z}} \qquad \text{(VA)}$$

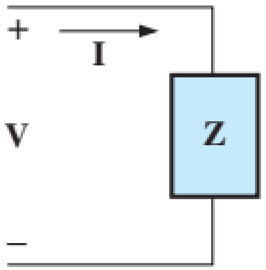
The average power to the load in Fig. 19.4 is

$$P = VI \cos\theta$$

However, $$S = VI$$

Therefore, $$\boxed{P = S\cos\theta} \qquad \text{(W)}$$

and the power factor of a system $F_p$ is




$$\boxed{F_p = \cos\theta = \dfrac{P}{S}} \qquad \text{(unitless)}$$

$$P = VI = S$$

$$F_p = \cos\theta = \dfrac{P}{S} = 1$$

# Inductive Circuit and Reactive Power

## Inductive Circuit and Reactive Power

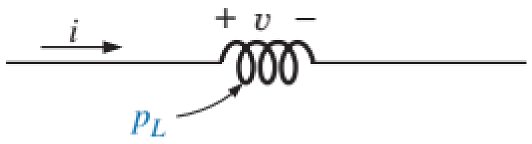
For a purely inductive circuit $v$ leads $i$ by 90°, Substituting $\theta = 90°$

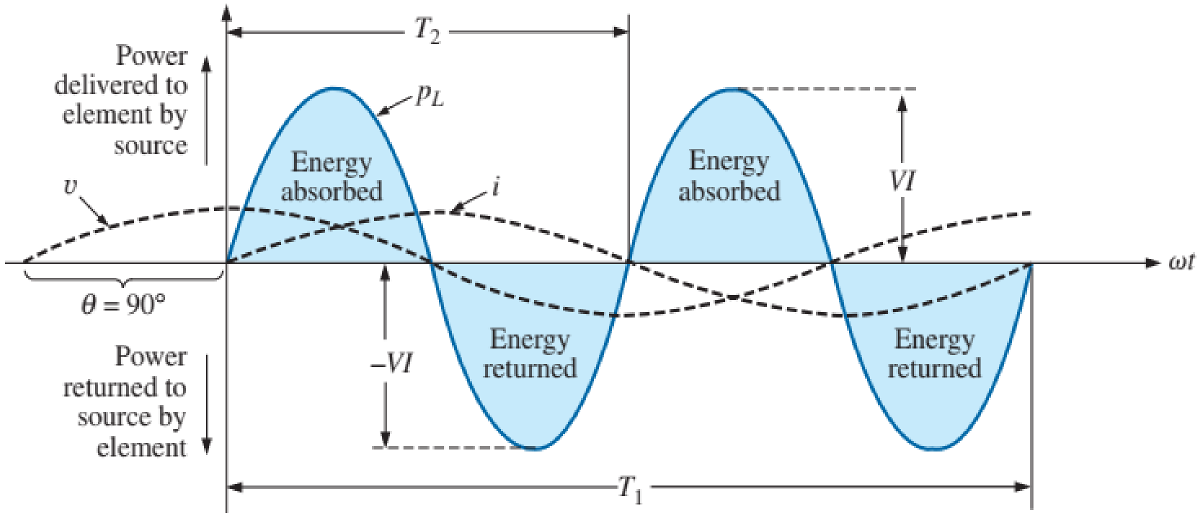
$$p_L = VI\cos(90°)(1 - \cos 2\omega t) + VI\sin(90°)(\sin 2\omega t)$$
$$= 0 + VI\sin 2\omega t$$

$$\boxed{p_L = VI\sin 2\omega t}$$

where $VI \sin 2\omega t$ is a sine wave with twice the frequency

***The net flow of power to the pure (ideal) inductor is zero over a full cycle, and no energy is lost in the transaction.***




*The power curve for a purely inductive load.*

We find that the instantaneous power can be written as:

$$p(t) = \underbrace{[V_{rms}I_{rms}\cos\theta](1-\cos 2\omega t)}_{\text{Active Power}} + \underbrace{[V_{rms}I_{rms}\sin\theta]\sin 2\omega t}_{\text{Reactive Power}}$$

The peak or maximum value of instantaneous reactive power (or instantaneous reactive volt ampere) is called the ***reactive or imaginary or quadrature or wattless power*** (or ***reactive volt ampere***).

The symbol for reactive power is $Q$, and its unit is the *volt-ampere reactive* (VAR).

$$\boxed{Q_L = VI\sin\theta} \qquad \text{(volt-ampere reactive, VAR)}$$

where $\theta$ is the phase angle between $V$ and $I$.

# Inductive Circuit and Reactive Power

Inductive Circuit

$$\boxed{Q_L = I^2 X_L}$$

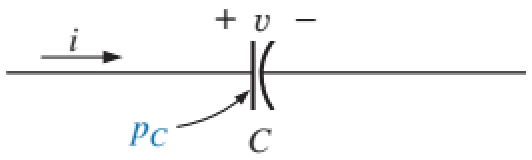
$$\boxed{Q_L = \frac{V^2}{X_L}}$$

The apparent power associated with an inductor is $S = VI$, and the average power is $P = 0$

$$F_p = \cos\theta = \frac{P}{S} = \frac{0}{VI} = 0$$

# Capacitive Circuit and Reactive Power

## Capacitive Circuit and Reactive Power




For a purely capacitive circuit $i$ leads $v$ by $90°$,

$\theta = -90°$

$$p_C = VI\cos(-90°)(1 - \cos 2\omega t) + VI\sin(-90°)(\sin 2\omega t)$$
$$= 0 - VI\sin 2\omega t$$

$$\boxed{p_C = -VI\sin 2\omega t}$$

where $-VI\sin 2\omega t$ is a negative sine wave with twice the frequency

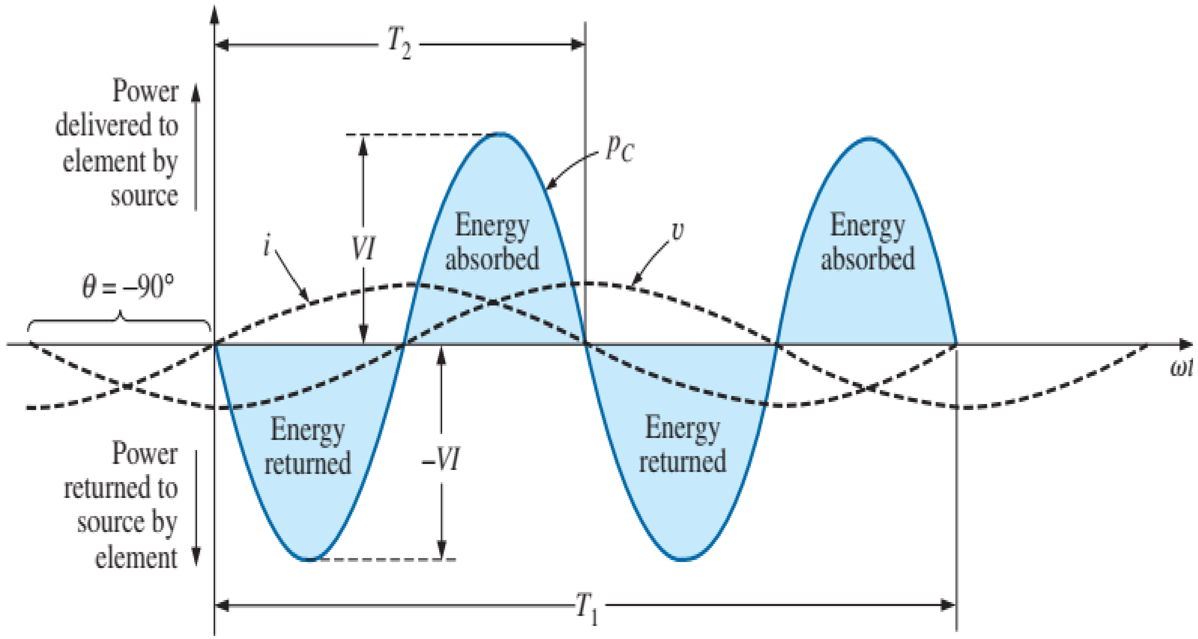
***The net flow of power to the pure (ideal) capacitor is zero over a full cycle,***

The reactive power associated with the capacitor is equal to the peak value of the $p_C$ curve, as follows:

since $V = IX_C$ and $I = V/X_C$,

$$\boxed{Q_C = I^2X_C}$$

$$\boxed{Q_C = \frac{V^2}{X_C}}$$




*The power curve for a purely capacitive load.*

The apparent power associated with the capacitor is

$$\boxed{S = VI} \qquad \text{(VA)}$$

and the average power is $P = 0$.

# Capacitive Circuit and Reactive Power

For Capacitor

The apparent power associated with the capacitor is

$$\boxed{S = VI} \qquad \text{(VA)}$$

and the average power is $P = 0$.

$$F_p = \cos\theta = \frac{P}{S} = \frac{0}{VI} = 0$$

# Instantaneous Equation of Power for R, L andC

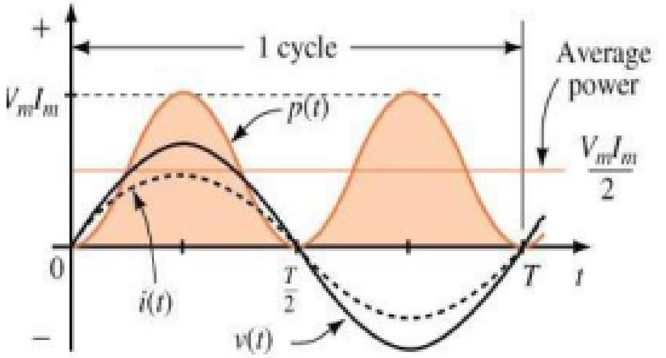
Energy Stored:

| **Resistor** | **Inductor** or **Choke** | **Capacitor** or **Condenser** |
|---|---|---|

**Instantaneous Power Equation**: $p(t)=P(1\text{-}\cos 2\omega t)+Q\sin 2\omega t$

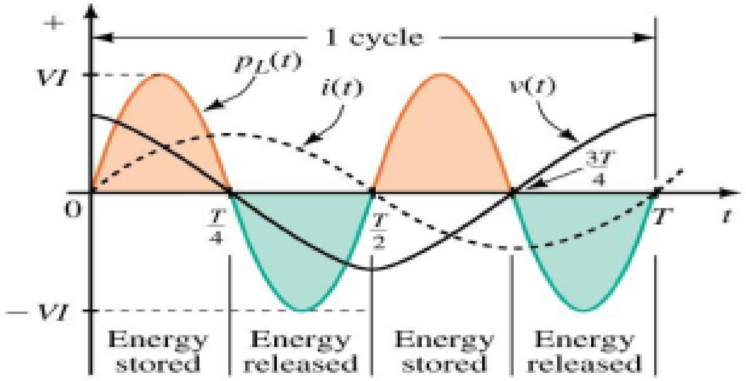
**Resistor**

$$p(t)=P(1-\cos 2\omega t)$$
$$=V_{rms}I_{rms}(1-\cos 2\omega t)$$



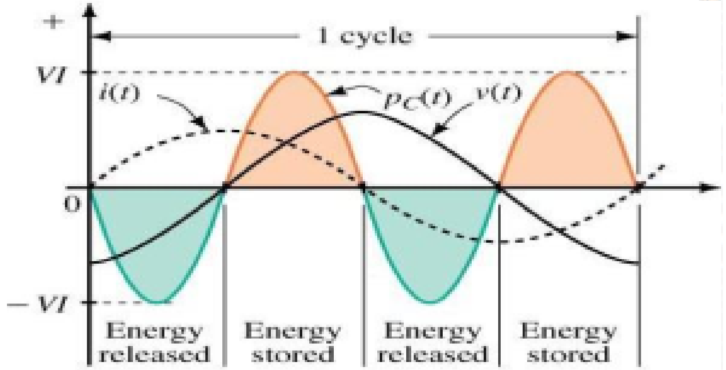

**Inductor or Choke**

$$p(t)=Q\sin 2\omega t$$
$$=V_{rms}I_{rms}\sin 2\omega t$$




**Capacitor or Condenser**

$$p(t)=Q\sin 2\omega t$$
$$=-V_{rms}I_{rms}\sin 2\omega t$$




**Maximum Energy Stored by an Inductor:**

$$W_L=\int_0^{T/4}-\frac{V_mI_m}{2}\sin 2\omega t\,dt=\frac{V_mI_m}{4\omega}[\cos 2\omega t]_0^{T/4}$$
$$=\frac{V_mI_m}{2\omega}=\frac{\omega LI_m^2}{2\omega}=\frac{1}{2}LI_m^2 \quad \text{J}$$

**Maximum Energy Stored by a Capacitor:**

$$W_C=\int_0^{T/4}-\frac{V_mI_m}{2}\sin 2\omega t\,dt=\frac{V_mI_m}{4\omega}[\cos 2\omega t]_0^{T/4}$$
$$=\frac{V_mI_m}{2\omega}=\frac{\omega CV_m^2}{2\omega}=\frac{1}{2}CV_m^2 \quad \text{J}$$

# Power or Average Power

Real Power:

**Power (or Average or Real or Active or True or Wattfull Power)**

Since the average value of sine and cosine terms of instantaneous power are zero in one cycle, so the average power of instantaneous power can be written by:

$$P = P_{ave} = V_{rms} I_{rms} \cos\theta \quad [\mathrm{W}]$$

$$P = I_{Rrms}^2 R = \frac{V_{Rrms}^2}{R} \quad [\mathrm{W}]$$

The average power is also called ***real or active or true or wattfull power or*** **simply** ***Power***.
The **unit** of real power is called **watt**. The real power is measured by **wattmeter**.

# Reactive Power or Reactive Volt-Amp

Reactive Power:

**Reactive or imaginary or Quadrature or Wattless Power (or Reactive Volt-Ampere)**

The peak or maximum value of instantaneous reactive power (or instantaneous reactive volt-ampere) is called the ***reactive or imaginary or quadrature or wattless power*** (or ***reactive volt-ampere***).

The **unit** of reactive power is called **var** **(reactive volt-ampere)**. The reactive power is measured by **varmeter**.

The reactive power or imaginary power or quadrature power or wattless power can be written as follows:

$$\boxed{Q = P_x = V_{rms} I_{rms} \sin\theta = VI \sin\theta \quad [\text{var}]}$$

$Q = 0$ for resistive load; $Q > 0$ for inductive load; $Q < 0$ for capacitive load

$$\boxed{Q_L = I_{Lrms}^2 X_L = \frac{V_{Lrms}^2}{X_L} \quad [\text{var}]} \qquad \boxed{Q_C = -I_{Crms}^2 X_C = -\frac{V_{Crms}^2}{X_C} \quad [\text{var}]}$$

# Volt-Amp or Apparent Power

Volt-Amp.:

**Volt-Ampere or Apparent Power**

The apparent power can be obtained by combining the real and reactive power as follows:

$$S = \sqrt{P_r^2 + P_x^2} = \sqrt{P^2 + Q^2} = \frac{V_m I_m}{2} = V_{rms} I_{rms} = VI \;\; [\text{VA}]$$

Apparent power is the product of the rms value of voltage and the rms value of current.

The **unit** of apparent power is called **VA (volt-ampere)**.

$$S = I_{rms}^2 Z_m = \frac{V_{rms}^2}{Z_m} \;\; [\text{VA}]$$

# Power Factor and Power Factor Angle

Power Factor Leading or Lagging:

**Power Factor and Power Factor Angle**

Cosine $\theta$ ($\cos\theta$) which is a factor, by which volt-amperes are multiplied to give power, is called power factor. Power factor is always **positive**. Power factor can be given by:

$$\textbf{Power Factor},\ pf = \cos\theta = \frac{P}{V_{rms} I_{rms}} = \frac{P}{S}$$

**Power Factor Angle** is the phase difference between voltage and current.

$$\textbf{Power Factor Angle},\ \theta = \theta_v - \theta_i$$

If voltage $v(t)$ and current $i(t)$ are **in phase**, power factor is **unity**.

Power factor **unity** means voltage $v(t)$ and current $i(t)$ are **in phase**.

If current $i(t)$ **lags** voltage $v(t)$, power factor is **lagging**.

Power factor **lagging** means current $i(t)$ **lags** voltage $v(t)$.

If current $i(t)$ **leads** voltage $v(t)$, power factor is **leading**.

Power factor **leading** means current $i(t)$ **leads** voltage $v(t)$.

# Reactive Factor

Reactive Factor:

**Reactive Factor**

Sine $\theta$ ($\sin\theta$) which is a factor, by which volt-amperes are multiplied to give reactive power, is called reactive factor. Reactive factor may be **positive** or **negative**.

Reactive factor can be given by:

$$\text{Reactive Factor (rf)} = \sin\theta = \frac{Q}{V_{rms} I_{rms}} = \frac{Q}{S}$$

If voltage $v(t)$ and current $i(t)$ are **in phase**, reactive factor is **zero**.

Reactive factor **zero** means voltage $v(t)$ and current $i(t)$ are **in phase**.

If current $i(t)$ **lags** voltage $v(t)$, reactive factor is **positive**.

Reactive factor **positive** means current $i(t)$ **lags** voltage $v(t)$.

If current $i(t)$ **leads** voltage $v(t)$, reactive factor is **negative**.

Reactive factor is **negative** means current $i(t)$ **leads** voltage $v(t)$.

# Power Triangle

Power Triangle:

## 19.7 THE POWER TRIANGLE

The three quantities **average power, apparent power,** and **reactive power** can be related in the vector domain by

$$\boxed{\mathbf{S} = \mathbf{P} + \mathbf{Q}} \tag{19.27}$$

with

$$\mathbf{P} = P \angle 0^\circ \quad \mathbf{Q}_L = Q_L \angle 90^\circ \quad \mathbf{Q}_C = Q_C \angle -90^\circ$$

For an inductive load, the *phasor power* **S**, as it is often called, is defined by

$$\mathbf{S} = P + j\,Q_L$$

as shown in Fig. 19.14.

The 90° shift in $Q_L$ from $P$ is the source of another term for reactive power: *quadrature power.*

For a capacitive load, the phasor power **S** is defined by

$$\mathbf{S} = P - j\,Q_C$$

as shown in Fig. 19.15.

If a network has both capacitive and inductive elements, the reactive component of the power triangle will be determined by the *difference* between the reactive power delivered to each. If $Q_L > Q_C$, the resultant

power triangle will be similar to Fig. 19.14. If $Q_C > Q_L$, the resultant power triangle will be similar to Fig. 19.15.

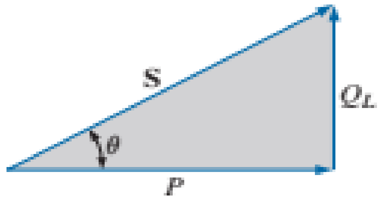



**FIG. 19.14**
*Power diagram for inductive loads.*

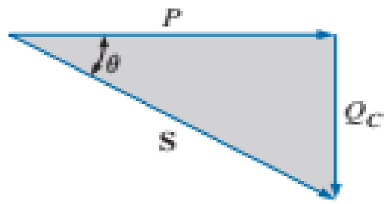



**FIG. 19.15**
*Power diagram for capacitive loads.*

# Example on Power Triangle

Example:

Consider, for example, the simple *R-L* circuit in Fig. 19.19, where

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{10\text{ V}\angle 0^\circ}{3\ \Omega + j\,4\ \Omega} = \frac{10\text{ V}\angle 0^\circ}{5\ \Omega\angle 53.13^\circ} = 2\text{ A}\angle -53.13^\circ$$

The real power (the term *real* being derived from the positive real axis of the complex plane) is

$$P = I^2R = (2\text{ A})^2(3\ \Omega) = 12\text{ W}$$

and the reactive power is

$$Q_L = I^2X_L = (2\text{ A})^2(4\ \Omega) = 16\text{ VAR }(L)$$

with $\mathbf{S} = P + j\,Q_L = 12\text{ W} + j\,16\text{ VAR }(L) = 20\text{ VA}\angle 53.13^\circ$

as shown in Fig. 19.20. Applying Eq. (19.29) yields

$$\mathbf{S} = \mathbf{VI}^* = (10\text{ V}\angle 0^\circ)(2\text{A}\angle +53.13^\circ) = 20\text{ VA}\angle 53.13^\circ$$

as obtained above.

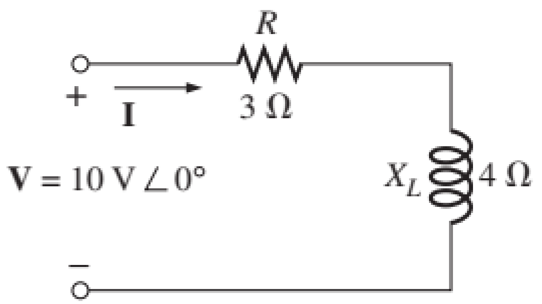



FIG. 19.19

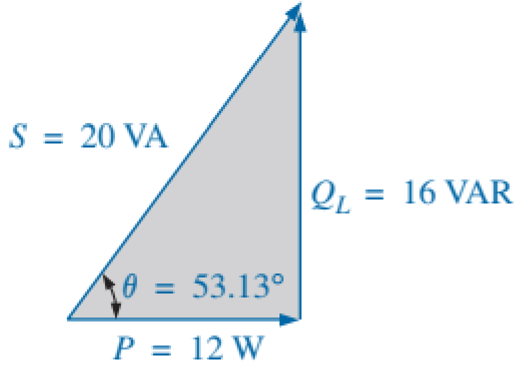



FIG. 19.20
*The power triangle for the circuit in Fig. 19.19.*

# Example on Power Calculation

Example:

**Example**: The voltage and current of a circuit are given as follows: $v(t)$=100sin(314$t$+60°) V and $i(t)$=10sin(314$t$+30°) A.
(i) Calculate the apparent power, the power or active power, the reactive power, the power factor, the reactive factor. (ii) Write the expression of instantaneous power. (iii) Draw the power triangle.

**Solution**: Here, $\omega$=314 rad/s, $V_m$ = 100 V, $I_m$ = 10 A, $\theta_v$ = 60°, and $\theta_i$ = 30°.
Thus $\theta = \theta_z = \theta_v - \theta_i$ = 60° - 30° = 30°.

$$(i)\ S = V_{rms} I_{rms} = \frac{V_m I_m}{2} = \frac{100\times 10}{2} = 500 \text{ VA} \qquad \text{pf} = \cos\theta = \cos(30^\circ) = 0.866$$

$$\text{rf} = \sin\theta = \sin(30^\circ) = 0.5 \qquad P = \frac{V_m I_m}{2}\cos\theta = \frac{100\times 10}{2}\cos(30^\circ) = 433 \text{ W}$$

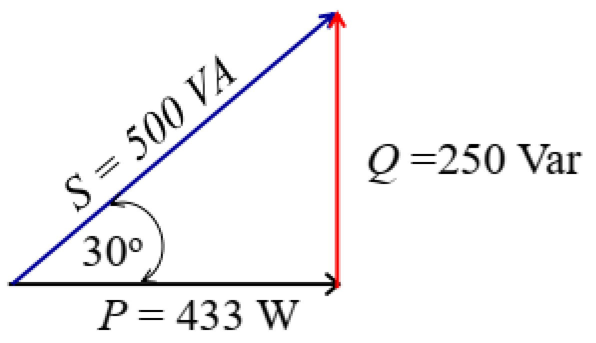
$$Q = \frac{V_m I_m}{2}\sin\theta = \frac{100\times 10}{2}\sin(30^\circ) = 250 \text{ VAR}$$

# Example on Power Calculation

Power Calculation:

(***ii***) The instantaneous real or active or true or wattfull power can be given by:

$$p(t) = P(1-\cos 2\omega t) + Q\sin 2\omega t = 433(1-\cos 628t) + 250\cos 628t$$




**Power Triangle**

# Example of Power Calculation

Example:

**Example**: The voltage of $v(t)$=100sin(314$t$-60°) V applied to an impedance of $\boldsymbol{Z}$=8.66 + $j$ 5 ohm. (*i*) Calculate the apparent power, the power or active power, the reactive power, the power factor, the reactive factor. (*ii*) Write the expression of instantaneous power. (*iii*) Draw the power triangle.

**Solution**: Here, $\omega$=314 rad/s, $V_m$ = 100 V, $\theta_v$ = 60°

$$\boldsymbol{V} = \frac{100}{\sqrt{2}}\angle 60° = 70.7\angle 60° \ \text{V}$$

$$\boldsymbol{Z} = 8.66 + j5 = 10\angle 30° \ \Omega$$

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}} = \frac{70.7\angle -60°}{10\angle 30°} = 7.07\angle -30° \ \text{A}$$

$\theta = \theta_z = \theta_v - \theta_i$ = -60° + 30° = -30°.

$$(i)\ S = V_{rms} I_{rms} = 70.7 \times 7.07 = 500 \ \text{VA}$$

$$\text{pf} = \cos\theta = \cos(-30°) = 0.866$$

$$\text{rf} = \sin\theta = \sin(-30°) = -0.5$$

$$P = S\cos\theta = 500\cos(-30°) = 433 \ \text{W}$$

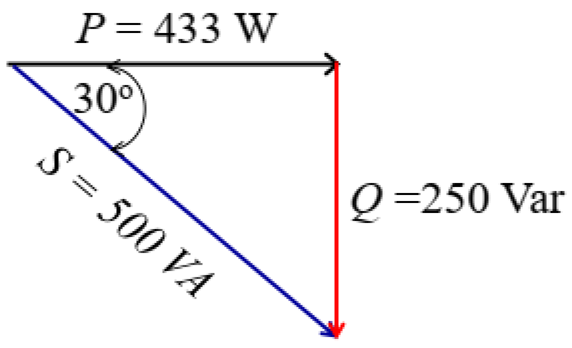
$$Q = S\sin\theta = 500\sin(-30°) = -250 \ \text{VAR}$$

# Example on Power Calculation

Power Triangle:

(***ii***) The instantaneous real or active or true or wattfull power can be given by

$$p(t) = P(1-\cos 2\omega t) + Q\sin 2\omega t = 433(1-\cos 628t) + 250\cos 628t$$




**Power Triangle**

# Complex Power

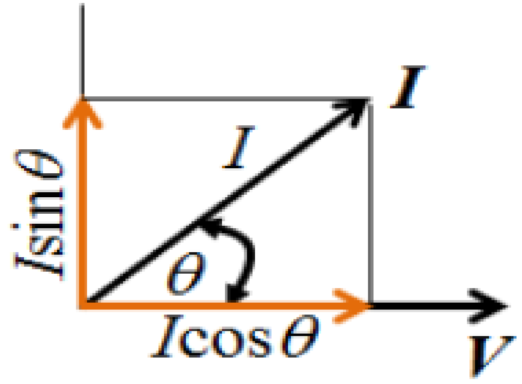
Complex Power:

## Complex Power

Let, voltage as a reference then the phasor diagram of voltage and current is as shown in the following figure.

Active Power : $P = V(I\cos\theta)$

Reactive Power : $Q = V(I\sin\theta)$




Since $I\cos\theta$ is multiplied with $V$ for active power, the component of current ($I$) $I\cos\theta$ is called ***active or real or true or wattfull component of current***.

Similarly, since $I\sin\theta$ is multiplied with $V$ for reactive power, the component of current ($I$) $I\sin\theta$ is called ***reactive or imaginary or quadrature or wattless component of current***.

# Voltage Current in Cartesian/Rectangular Form

Real/Reactive Power:

## Voltage and Current in Cartesian or Rectangular Form

**Voltage:**

$$\boldsymbol{V} = V\angle\theta_v = V\cos\theta_v + jV\sin\theta_v = V_r + jV_i$$

$$V_r = V\cos\theta_v; \qquad V_i = V\sin\theta_v$$

**Current:**

$$\boldsymbol{I} = I\angle\theta_i = I\cos\theta_i + jI\sin\theta_i = I_r + jI_i$$

$$I_r = I\cos\theta_i; \qquad I_i = I\sin\theta_i$$

## Real or Active Power

$$P = VI\cos\theta = VI\cos(\theta_v - \theta_i) = VI\cos\theta_v\cos\theta_i + VI\sin\theta_v\sin\theta_i$$

$$P = (V\cos\theta_v)(I\cos\theta_i) + (V\sin\theta_v)(I\sin\theta_i) = V_rI_r + V_iI_i$$

## Reactive Power

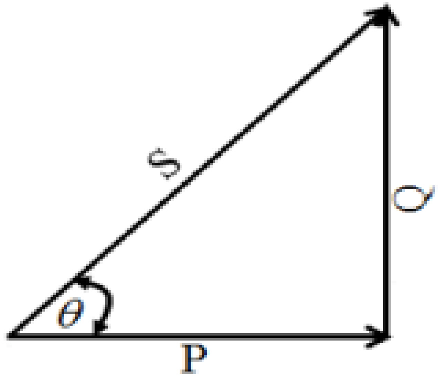
$$Q = VI\sin\theta = VI\sin(\theta_v - \theta_i) = VI\sin\theta_v\cos\theta_i - VI\cos\theta_v\sin\theta_i$$

$$Q = (V\sin\theta_v)(I\cos\theta_i) - (V\cos\theta_v)(I\sin\theta_i) = V_iI_r - V_rI_i$$

# Complex Power by Conjugate Method

Conjugate Method:

**Complex Power by Conjugate Current**




According to the power triangle the complex power can be written as follows:

$$\boldsymbol{S} = P + jQ = (V_r I_r + V_i I_i) + j(V_i I_r - V_r I_i)$$

$$\boldsymbol{S} = P + jQ = (V_r + jV_i)(I_r - jI_i) = \boldsymbol{V}\boldsymbol{I}^*$$

$$P = \mathrm{Re}[\boldsymbol{S}] = \mathrm{Re}[\boldsymbol{V}\boldsymbol{I}^*]; \qquad Q = \mathrm{Im}[\boldsymbol{S}] = \mathrm{Im}[\boldsymbol{V}\boldsymbol{I}^*]$$

**Complex Power by Conjugate Voltage**

$$\boldsymbol{S} = P - jQ = (V_r I_r + V_i I_i) - j(V_i I_r - V_r I_i)$$

$$\boldsymbol{S} = P - jQ = (V_r - jV_i)(I_r + jI_i) = \boldsymbol{V}^*\boldsymbol{I}$$

$$P = \mathrm{Re}[\boldsymbol{S}] = \mathrm{Re}[\boldsymbol{V}^*\boldsymbol{I}]; \qquad Q = -\mathrm{Im}[\boldsymbol{S}] = \mathrm{Im}[\boldsymbol{V}^*\boldsymbol{I}]$$

# Complex Power by Conjugate Method

Example:

**Example**: The voltage and current of a circuit are given as follows: $v(t)=100\sin(314t+60°)$ V and $i(t)=10\sin(314t+30°)$ A.
Calculate the power or active power, the reactive power, the apparent power by using complex conjugate method by conjugation current.

**Solution**: Here, $V_m = 100$ V, $I_m = 10$ A, $\theta_v = 60°$, and $\theta_i = 30°$. Thus,

$$V = V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.7 \quad \text{V}$$

$$I = I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07 \quad \text{A}$$

$$\boldsymbol{V} = 70.7\angle 60° = 35.35 + j61.24 \quad \text{V}$$

$$\boldsymbol{I} = 7.07\angle 30° = 6.122 + j3.535 \quad \text{A}$$

$$\boldsymbol{S} = P + jQ = \boldsymbol{VI}^* = (35.35 + j61.24)(6.122 - j3.535) = 433 + j250$$

$$P = \text{Re}[\boldsymbol{S}] = \text{Re}[\boldsymbol{VI}^*] = 433 \text{ W}; \qquad Q = \text{Im}[\boldsymbol{S}] = \text{Im}[\boldsymbol{VI}^*] = 250 \text{ VAR}$$

$$S = \sqrt{P^2 + Q^2} = \sqrt{433^2 + 250^2} = 500 \text{ VA}$$

# Complex Power by Conjugate Method

Example:

**Example**: The voltage and current of a circuit are given as follows: $\boldsymbol{V} = 50\angle 30^\circ$ V and $\boldsymbol{I} = 5\angle 60^\circ$ A. Calculate the power or active power, the reactive power, the apparent power by using complex conjugate method by conjugation voltage.

**Solution**: Here, $V_{rms} = 50$ V, $I_{rms} = 5$ A, $\theta_v = 30^\circ$, and $\theta_i = 60^\circ$.
Thus $\theta = \theta_z = \theta_v - \theta_i = 30^\circ - 60^\circ = -30^\circ$.

$$\boldsymbol{V} = 50\angle 30^\circ = 43.3 + j25 \ \ \text{V} \qquad \boldsymbol{I} = 5\angle 60^\circ = 2.5 + j4.33 \ \ \text{A}$$

$$\boldsymbol{S} = P - jQ = \boldsymbol{V}^*\boldsymbol{I} = (43.3 - j25)(2.5 + j4.33) = 216.5 + j125$$

$$P = \text{Re}[\boldsymbol{S}] = \text{Re}[\boldsymbol{V}^*\boldsymbol{I}] = 216.5 \text{ W}; \qquad Q = -\text{Im}[\boldsymbol{S}] = -\text{Im}[\boldsymbol{V}^*\boldsymbol{I}] = -125 \text{ VAR}$$

$$S = \sqrt{P^2 + Q^2} = \sqrt{216.5^2 + 125^2} = 250 \text{ VA}$$

# Power calculation RLC Series Circuit

Problem:

**EXAMPLE 19.4**

a. Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor $F_p$ for the network in Fig. 19.23.
b. Sketch the power triangle.
c. Find the energy dissipated by the resistor over one full cycle of the input voltage if the frequency of the input quantities is 60 Hz.
d. Find the energy stored in, or returned by, the capacitor or inductor over one half-cycle of the power curve for each if the frequency of the input quantities is 60 Hz.

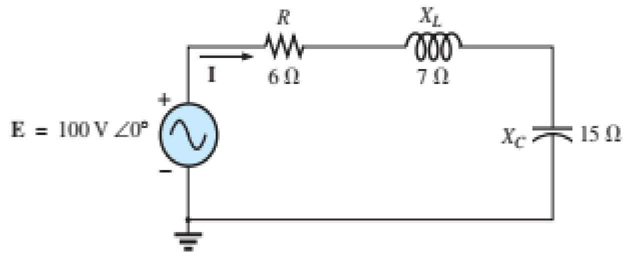



**FIG. 19.23**
*Example 19.4.*

***Solutions:***

a. $$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\text{ V}\angle 0^\circ}{6\ \Omega + j\,7\ \Omega - j\,15\ \Omega} = \frac{100\text{ V}\angle 0^\circ}{10\ \Omega\angle -53.13^\circ} = 10\text{ A}\angle 53.13^\circ$$

$$\mathbf{V}_R = (10\text{ A}\angle 53.13^\circ)(6\ \Omega\angle 0^\circ) = 60\text{ V}\angle 53.13^\circ$$
$$\mathbf{V}_L = (10\text{ A}\angle 53.13^\circ)(7\ \Omega\angle 90^\circ) = 70\text{ V}\angle 143.13^\circ$$
$$\mathbf{V}_C = (10\text{ A}\angle 53.13^\circ)(15\ \Omega\angle -90^\circ) = 150\text{ V}\angle -36.87^\circ$$

$$P_T = EI\cos\theta = (100\text{ V})(10\text{ A})\cos 53.13^\circ = \mathbf{600\ W}$$
$$= I^2R = (10\text{ A})^2(6\ \Omega) = \mathbf{600\ W}$$
$$= \frac{V_R^2}{R} = \frac{(60\text{ V})^2}{6} = \mathbf{600\ W}$$

$$S_T = EI = (100\text{ V})(10\text{ A}) = \mathbf{1000\ VA}$$
$$= I^2Z_T = (10\text{ A})^2(10\ \Omega) = \mathbf{1000\ VA}$$
$$= \frac{E^2}{Z_T} = \frac{(100\text{ V})^2}{10\ \Omega} = \mathbf{1000\ VA}$$

$$Q_T = EI\sin\theta = (100\text{ V})(10\text{ A})\sin 53.13^\circ = \mathbf{800\ VAR}$$
$$= Q_C - Q_L$$
$$= I^2(X_C - X_L) = (10\text{ A})^2(15\ \Omega - 7\ \Omega) = \mathbf{800\ VAR}$$

$$Q_T = \frac{V_C^2}{X_C} - \frac{V_L^2}{X_L} = \frac{(150\text{ V})^2}{15\ \Omega} - \frac{(70\text{ V})^2}{7\ \Omega}$$
$$= 1500\text{ VAR} - 700\text{ VAR} = \mathbf{800\ VAR}$$

$$F_p = \frac{P_T}{S_T} = \frac{600\text{ W}}{1000\text{ VA}} = \mathbf{0.6\ leading}\ (C)$$

b. The power triangle is as shown in Fig. 19.24.

c. $$W_R = \frac{V_R I}{f_1} = \frac{(60\text{ V})(10\text{ A})}{60\text{ Hz}} = \mathbf{10\ J}$$

d. $$W_L = \frac{V_L I}{\omega_1} = \frac{(70\text{ V})(10\text{ A})}{(2\pi)(60\text{ Hz})} = \frac{700\text{ J}}{377} = \mathbf{1.86\ J}$$

$$W_C = \frac{V_C I}{\omega_1} = \frac{(150\text{ V})(10\text{ A})}{377\text{ rad/s}} = \frac{1500\text{ J}}{377} = \mathbf{3.98\ J}$$

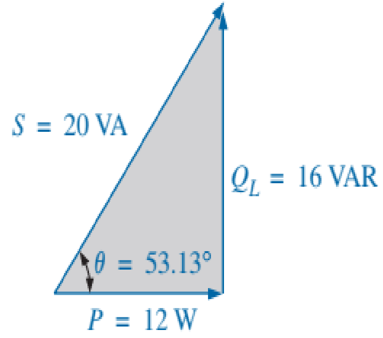



**FIG. 19.20**
*The power triangle for the circuit in Fig. 19.19.*

# Example Problem

Example:

**EXAMPLE 19.6** An electrical device is rated 5 kVA, 100 V at a 0.6 power-factor lag. What is the impedance of the device in rectangular coordinates?

***Solution:***

$$S = EI = 5000\ \text{VA}$$

Therefore,
$$I = \frac{5000\ \text{VA}}{100\ \text{V}} = 50\ \text{A}$$

For $F_p = 0.6$, we have

$$\theta = \cos^{-1} 0.6 = 53.13°$$

Since the power factor is lagging, the circuit is predominantly inductive, and **I** lags **E**. Or, for $\mathbf{E} = 100\ \text{V} \angle 0°$,

$$\mathbf{I} = 50\ \text{A} \angle -53.13°$$

However,

$$\mathbf{Z}_T = \frac{\mathbf{E}}{\mathbf{I}} = \frac{100\ \text{V} \angle 0°}{50\ \text{A} \angle -53.13°} = 2\ \Omega \angle 53.13° = \mathbf{1.2\ \Omega + \mathit{j}\ 1.6\ \Omega}$$

which is the impedance of the circuit in Fig. 19.27.

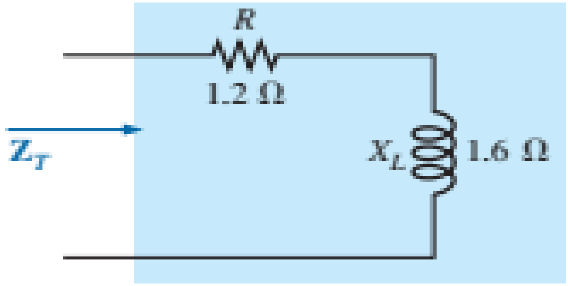



**FIG. 19.27**
*Example 19.6.*