Introduction to Electrical Circuits
Sec: G

Finalterm
Week: 8

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Book

Introductory Circuit Analysis

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Eleventh Edition

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Instantaneous Value to Phasor or Polar form

Instantaneous Value	Phasor or Polar Form
$v(t) = V_m \sin(\omega t + \theta_v)$	$\boldsymbol{V} = \frac{V_m}{\sqrt{2}} \angle \theta_v = V_{rms} \angle \theta_v = V \angle \theta_v$ where, $V = V_{rms} = \frac{V_m}{\sqrt{2}}$
$v(t) = V_m \sin(\omega t - \theta_v)$	$\boldsymbol{V} = \frac{V_m}{\sqrt{2}} \angle -\theta_v = V_{rms} \angle -\theta_v = V \angle -\theta_v$ where, $V = V_{rms} = \frac{V_m}{\sqrt{2}}$
$i(t) = I_m \sin(\omega t + \theta_i)$	$\boldsymbol{I} = \frac{I_m}{\sqrt{2}} \angle \theta_i = I_{rms} \angle \theta_i = I \angle \theta_i$ where, $I = I_{rms} = \frac{I_m}{\sqrt{2}}$
$i(t) = I_m \sin(\omega t - \theta_i)$	$\boldsymbol{I} = \frac{I_m}{\sqrt{2}} \angle -\theta_i = I_{rms} \angle -\theta_i = I \angle -\theta_i$ where, $I = I_{rms} = \frac{I_m}{\sqrt{2}}$

Example

The following instantaneous voltage convert to (**i**) the phasor or polar form, (**ii**) the exponential form, and (**iii**) the Cartesian or rectangular form.

$$v(t) = 100\sin(\omega t + 45^\circ)$$

Solution: **RMS value is:** $V = V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.71$

Phasor or Polar form: $\vec{V} = \boldsymbol{V} = 70.71\angle 45^\circ$

Exponential form: $\vec{V} = \boldsymbol{V} = 70.71e^{j45^\circ}$

Cartesian or Rectangular form:

$$\vec{V} = \boldsymbol{V} = 70.71(\cos 45^\circ + j\sin 45^\circ) = 70.71(0.7071 + j0.7071) = 50 + j50$$

Example

The following current convert to (**i**) the exponential form, (ii) (**i**) the Cartesian or rectangular form, and (**iii**) write the instantaneous expression.

$$\boldsymbol{I} = 90\angle -40^{\circ}$$

Solution: $I = I_{rms} = \frac{I_m}{\sqrt{2}} = 90\ \text{A}$ $\qquad \theta_i = -40^{\circ}$

Exponential form: $\vec{I} = \boldsymbol{I} = 90e^{-j40^{\circ}}$

Cartesian or Rectangular form: $\vec{I} = \boldsymbol{I} = 90(\cos 40^{\circ} - j\sin 40^{\circ}) = 69 - j57.9$

Peak value: $I_m = \sqrt{2}I_{rms} = \sqrt{2} \times 90 = 127.28\ \text{A}$

Instantaneous expression: $i(t) = \sqrt{2}I\sin(\omega t - 40^{\circ}) = \sqrt{2}I_{rms}\sin(\omega t - 40^{\circ})\ \text{A}$

$$i(t) = 127.28\sin(\omega t - 40^{\circ})\ \text{A}$$

Impedance

According to ohm's Law, Impedance is the ratio of voltage to current. The unit of impedance is **ohm** [Ω].

$$\boldsymbol{Z} = \frac{v}{i} = \frac{V_m \sin(\omega t + \theta_v)}{I_m \sin(\omega t + \theta_i)} \quad \text{ohm } [\Omega]$$

$$\boldsymbol{Z} = \frac{V_{rms}\angle\theta_v}{I_{rms}\angle\theta_i} = \frac{V_{rms}}{I_{rms}}\angle(\theta_v - \theta_i) = \frac{V}{I}\angle(\theta_v - \theta_i) = |Z|\angle\theta_z \quad \Omega$$

$$\boldsymbol{Z} = \frac{V_m}{I_m}\angle(\theta_v - \theta_i) = Z\angle\theta_z = R + jX = (\textbf{Resistance}) + j(\textbf{Reactance})$$

Magnitude of impedance, $Z = \dfrac{V_m}{I_m} = \dfrac{V_{rms}}{I_{rms}} = \dfrac{V}{I} \quad \Omega$

Angle of impedance, $\theta_z = \theta_v - \theta_i$

Resistance: $R = Z\cos\theta_z$

Reactance: $X = Z\sin\theta_z$

Example

The voltage and current of a circuit are given as follows: $v(t)=100\sin(314t+60^\circ)$ V and $i(t)=10\sin(314t+30^\circ)$ A. Calculate the magnitude of impedance and angle of impedance.

Solution: Here, $\omega=314$ rad/s, $V_m = 100$ V, $I_m = 10$ A, $\theta_v = 60^\circ$, and $\theta_i = 30^\circ$.

Magnitude of impedance, $Z = \dfrac{V_m}{I_m} = \dfrac{100}{10} = 10\ \Omega$

Angle of impedance, $\theta_z = \theta_v - \theta_i = 60^\circ - 30^\circ = 30^\circ$

Impedance, $\boldsymbol{Z} = 10\angle 30^\circ = 8.66 + j5\ \Omega$

Example

The voltage and current of a circuit are given as follows: $\boldsymbol{V}=15\angle 30^\circ$ V and $\boldsymbol{I}=1.2\angle 60^\circ$ A. Calculate the magnitude of impedance and angle of impedance.

Solution:

Impedance, $\boldsymbol{Z} = \dfrac{\boldsymbol{V}}{\boldsymbol{I}} = \dfrac{15\angle 30^\circ}{1.2\angle 60^\circ} = 12.5\angle -30^\circ = 10.83 - j6.25\ \Omega$

Magnitude of impedance, $Z = 12.5\ \Omega$

Angle of impedance, $\theta_z = \theta_v - \theta_i = -30^\circ$

Power (or Average or Real or Active or True or Wattfull Power)

The average values of second and third terms of Eq. ($p5$) are zero, so the average value of total instantaneous power equals to the constant term which is called ***power (or average power or real or active or true or wattfull power)***.

The **unit** of real power is called **watt**.

The real power is measured by **wattmeter**.

The power or real power or active power or true power or wattfull power can be written as follows:

$$\boxed{P = P_r = \frac{V_m I_m}{2}\cos\theta = V_{rms} I_{rms}\cos\theta = VI\cos\theta}$$

Physical Significance of Power or real power or average power:

The ***power (or average power or real or active or true or wattfull power)*** represents the power which is consumed by load (resistor).

The real power is the power which converts from electrical energy to other form of energy.

Reactive or imaginary or Quadrature or Wattless Power (or Reactive Volt-Ampere)

The maximum value of instantaneous reactive or imaginary or quadrature or wattless power (or instantaneous reactive volt-ampere) is called the ***reactive or imaginary or quadrature or wattless power*** (or ***reactive volt-ampere***).

The **unit** of reactive power is called **var** **(reactive volt-ampere)**.

The reactive power is measured by **varmeter**.

The instantaneous reactive or imaginary or quadrature or wattless power (or instanatneous reactive volt-ampere) can be written as follows:

$$Q = P_x = \frac{V_m I_m}{2} \sin\theta = V_{rms} I_{rms} \sin\theta = VI \sin\theta$$

Since the reactive power depends of the angle of sine, it may be positive (for capacititor) or negative (for inductor).

Volt-Ampere or Apparent Power

The volt-ampere or apparent power represents the maximum possible supply power by a source.

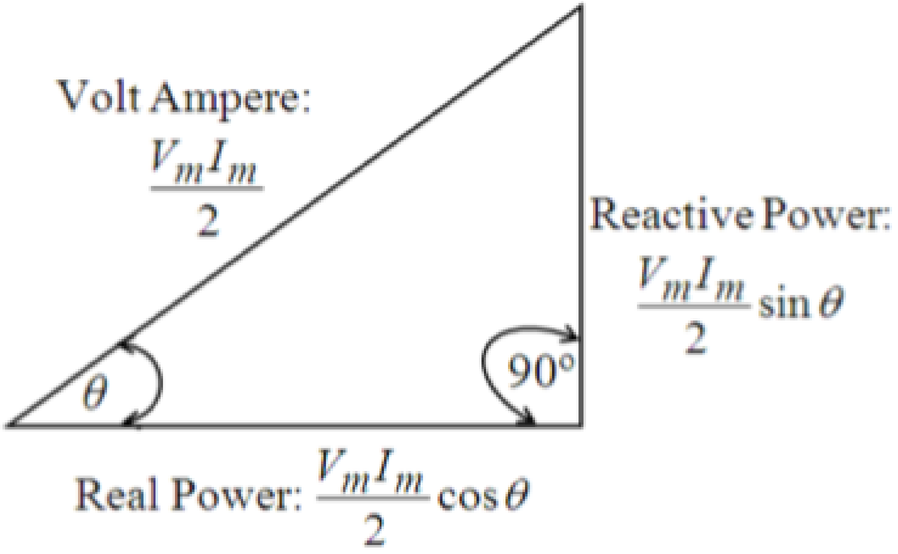
The apparent power can be obtained by combining the real and reactive power as follows:

$$S = \sqrt{P_r^2 + P_x^2} = \sqrt{P^2 + Q^2} = \frac{V_m I_m}{2} = V_{rms} I_{rms} = VI$$

The **unit** of apparent power is called **VA (volt-ampere)**.

Power Triangle

The power, reactive power and apparent power can be represented by graphically as shown in the following figure which is called **Power Triangle**.



Power Factor

Cosine θ ($\cos\theta$) which is a factor, by which volt-amperes are multiplied to give power, is called power factor. Power factor is always **positive**.

Power factor can be given by:

$$\text{Power Factor (pf)} = \cos\theta = \frac{P}{\frac{V_m I_m}{2}} = \frac{P}{V_{eff} I_{eff}} = \frac{P}{V_{rms} I_{rms}} = \frac{P}{S}$$

Physical Significance of Power Factor

Power factor represents how much of maximum possible supply power is utilized (or consumed by resistor).

Suppose 0.5 power factor (*i.e.* 50% pf) of a circuit means that it will utilize only 50% of the apparent power whereas 0.8 power factor means 80% utilization of apparent power. For this reason, we wish that the power factor of the circuit to be near 1 (unity) as possible.

Phase difference between voltage and current	Reactive factor	Sign in reactive power	Nature of power factor	Nature of load
$\theta = \theta_z = \theta_v - \theta_i = 0$	Zero	Zero	unity	Resistor
$\theta = \theta_z = \theta_v - \theta_i = 90^o$	1	Q = S	Zero lagging	Inductor
$\theta = \theta_z = \theta_v - \theta_i = -90^o$	-1	$Q = -S$	Zero leading	Capacitor
$\theta = \theta_z = \theta_v - \theta_i > 0$	Positive	Positive	Lagging	RL series
$\theta = \theta_z = \theta_v - \theta_i < 0$	negative	Negative	Leading	RC series

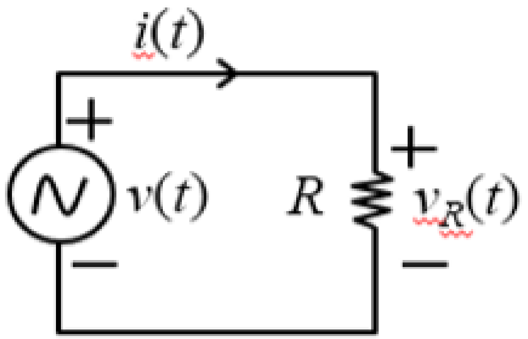
Pure Resistive Circuit

Let, the input is $v(t) = V_m \sin\omega t$ V, according to KVL, we have: $v(t) = v_R(t) = V_m \sin\omega t$

For a resistance the relation of voltage and current is: $v_R(t) = Ri(t)$

$$Ri(t) = V_m \sin\omega t$$

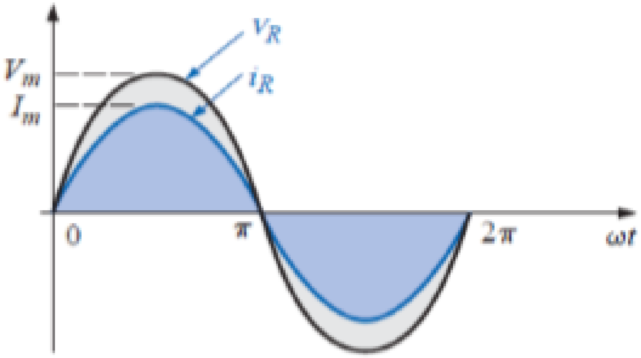
$$i(t) = \frac{V_m}{R}\sin\omega t = I_m \sin\omega t$$



Magnitude of impedance, $Z = \dfrac{V_m}{I_m} = R \quad \Omega$

Angle of impedance, $\theta_z = \theta_v - \theta_i = 0°$

Impedance of a Resistor, $\boldsymbol{Z} = \boldsymbol{Z}_R = R\angle 0° = R\ \Omega$



The phase difference between voltage across and current through a resistor is zero.

For a purely resistive element, the voltage across and the current through the element are in phase.

Power of Resistive Load

The power of a resistive load:

$$p(t) = (V_m \sin \omega t)(I_m \sin \omega t) = V_m I_m \sin^2 \omega t = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$$

$$p(t) = V_{rms} I_{rms} - V_{rms} I_{rms} \cos 2\omega t$$

Power Factor:

$$\text{pf} = \cos(\theta_z) = \cos(0) = 1$$

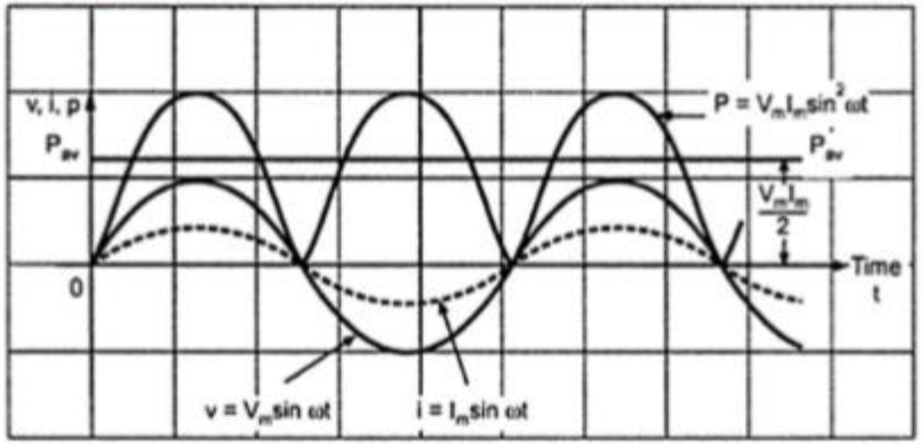
For resistive load, the power factor is 1 which called **unity power factor**.

Reactive Factor:

$$\text{rf} = \sin(\theta_z) = \sin(0) = 0$$

Average or Real Power:

$$P = P_{ave} = V_{rms} I_{rms} = \frac{V_{rms}^2}{R} = I_{rms}^2 R \quad \text{W}$$



v, i and p for purely resistive circuit

Reactive Power:

$$Q = P_x = V_{rms} I_{rms} \sin \theta_z = 0 \quad \text{VAR}$$

Apparent Power:

$$S = \sqrt{P_r^2 + P_x^2} = \sqrt{P^2 + Q^2} = P = \frac{V_m I_m}{2} = V_{rms} I_{rms} = VI \quad \text{VA}$$

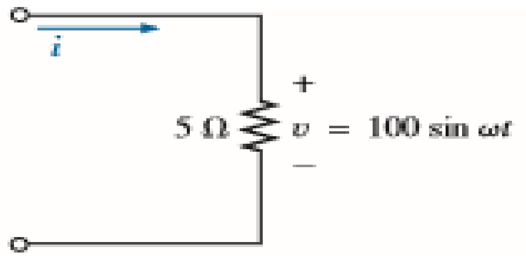


FIG. 15.2
Example 15.1.

EXAMPLE 15.1 Using complex algebra, find the current i for the circuit in Fig. 15.2. Sketch the waveforms of v and i.

Solution: Note Fig. 15.3:

$$v = 100 \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = 70.71 \text{ V} \angle 0^\circ$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_R} = \frac{V \angle \theta}{R \angle 0^\circ} = \frac{70.71 \text{ V} \angle 0^\circ}{5\ \Omega \angle 0^\circ} = 14.14 \text{ A} \angle 0^\circ$$

and $$i = \sqrt{2}(14.14) \sin \omega t = \mathbf{20 \sin \omega t}$$

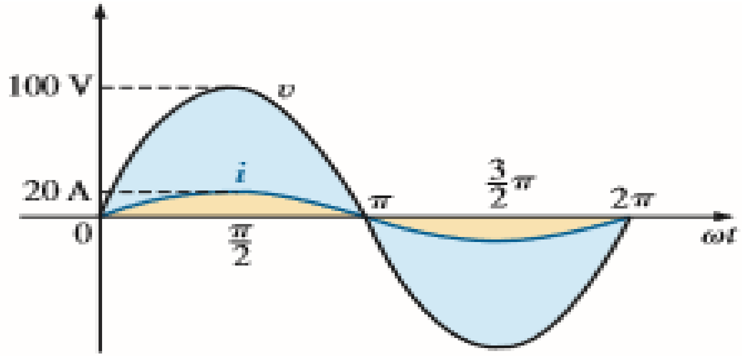


FIG. 15.3
Waveforms for Example 15.1.

EXAMPLE 15.2 Using complex algebra, find the voltage v for the circuit in Fig. 15.4. Sketch the waveforms of v and i.

Solution: Note Fig. 15.5:

$$i = 4 \sin(\omega t + 30^\circ) \Rightarrow \text{phasor form } \mathbf{I} = 2.828 \text{ A} \angle 30^\circ$$

$$\begin{aligned}\mathbf{V} &= \mathbf{I}\mathbf{Z}_R = (I \angle \theta)(R \angle 0^\circ) = (2.828 \text{ A} \angle 30^\circ)(2\ \Omega \angle 0^\circ)\\ &= 5.656 \text{ V} \angle 30^\circ\end{aligned}$$

and $$v = \sqrt{2}\,(5.656) \sin(\omega t + 30^\circ) = \mathbf{8.0 \sin(\omega t + 30^\circ)}$$

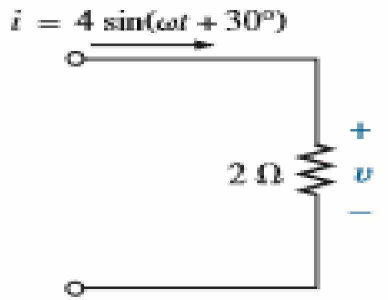


FIG. 15.4
Example 15.2.

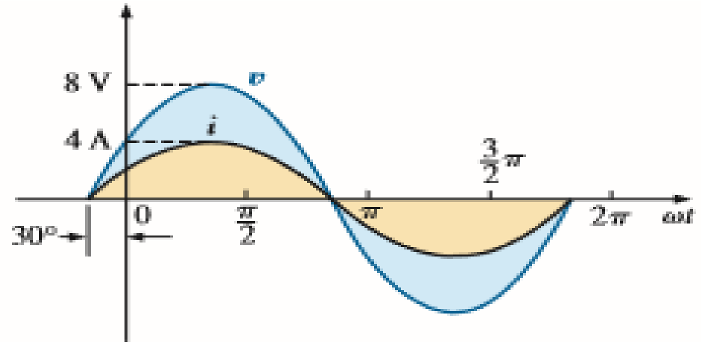


FIG. 15.5
Waveforms for Example 15.2.

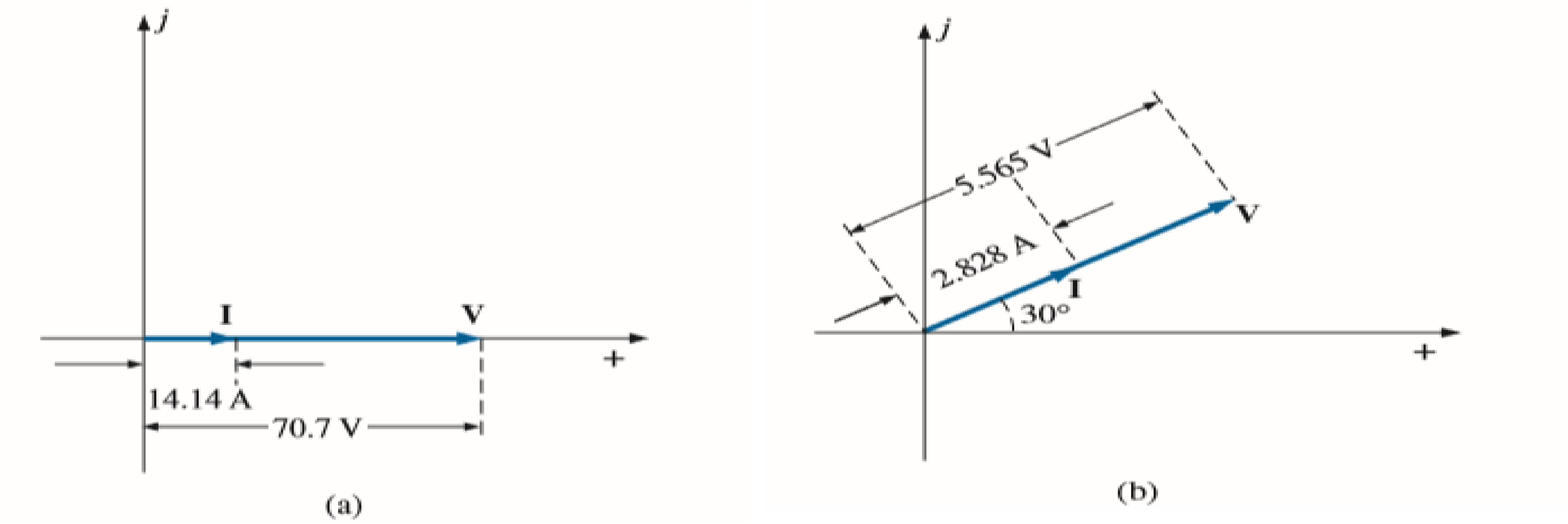


FIG. 15.6

Phasor diagrams for Examples 15.1 and 15.2.

Home Work

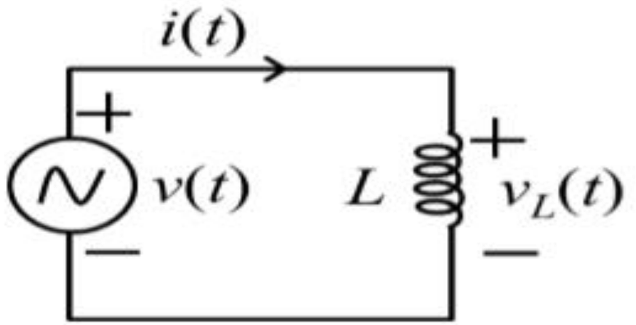
Problem 1: The current $i(t) = 4\sin(\omega t - 20°)$ A flows through a 8 Ω resistor. (***i***) What is the sinusoidal expression for the voltage? (***ii***) Calculate the real power, reactive power, power factor, reactive factor. (***iii***) write the expression of instantaneous power. (***iv***) Sketch the *v* and *i* sinusoidal waveforms on the same axis. (***v***) Draw the phasor diagram.

Problem 2: The voltage $v(t) = 20\sin(\omega t + 30°)$ V is applied to a 4 Ω resistor. (*i*) What is the sinusoidal expression for the current? (**ii**) Calculate the real power, reactive power, power factor, reactive factor. (**iii**) write the expression of instantaneous power. (**iv**) Sketch the *v* and *i* sinusoidal waveforms on the same axis. (**v**) Draw the phasor diagram.

Pure Inductive Circuit

Let, the input is $v(t) = V_m \sin\omega t$ V, according to KVL, we have: $v(t) = v_L(t) = V_m \sin\omega t$

For an inductance the relation of voltage and current is:

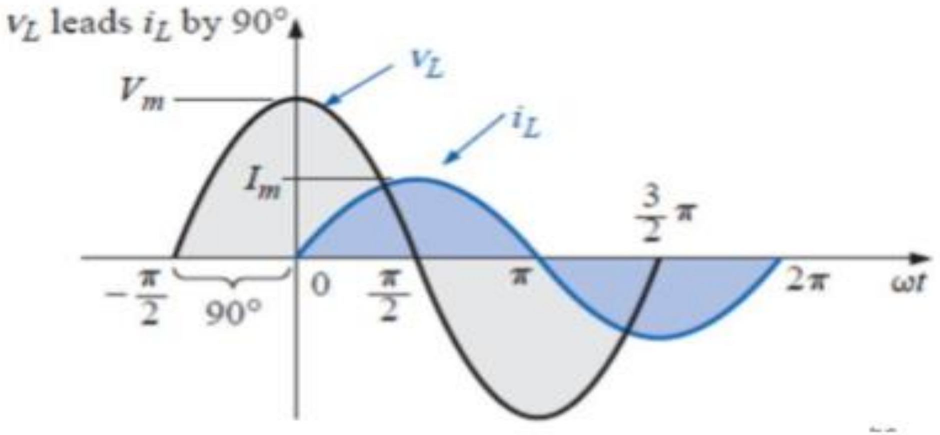


$$i_L(t) = \frac{1}{L}\int v_L(t)dt = \frac{V_m}{L}\int \sin\omega t dt = -\frac{V_m}{\omega L}\cos\omega t$$

$$i_L(t) = \frac{V_m}{\omega L}\sin(\omega t - 90°) = I_m \sin(\omega t + \theta_i)$$

Magnitude of impedance, $Z = \dfrac{V_m}{I_m} = \omega L = X_L \quad \Omega$

Inductive reactance, $X_L = \omega L = 2\pi f L \quad \Omega$



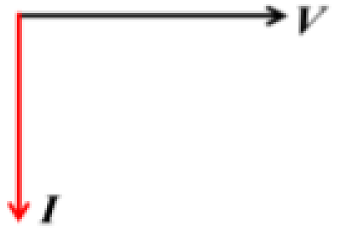
Angle of impedance, $\theta_z = \theta_v - \theta_i = 90°$

Impedance of a Inuctor, $\boldsymbol{Z} = \boldsymbol{Z}_L = X_L\angle 90°$

$$= jX_L \ \Omega$$

The phase difference between voltage across and current through an inductor is 90°.

***For a purely inductive element, the voltage leads the current through the inductive element by 90°*. Or, *the current lags the voltage in an inductive element by 90°*.**



Phasor Diagram

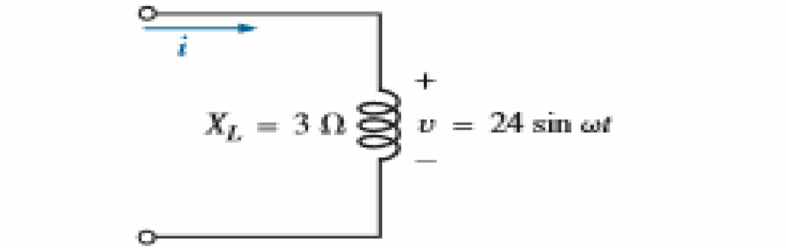


FIG. 15.8
Example 15.3.

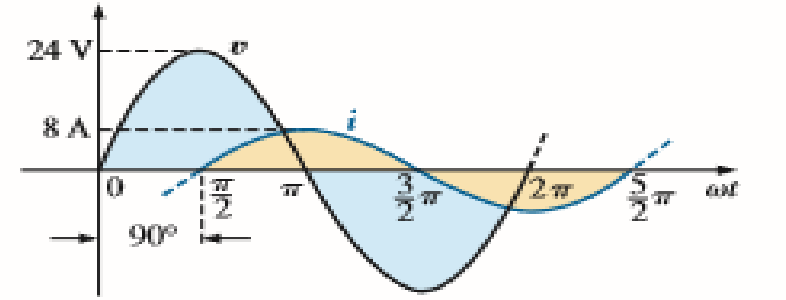


FIG. 15.9
Waveforms for Example 15.3.

EXAMPLE 15.3 Using complex algebra, find the current i for the circuit in Fig. 15.8. Sketch the v and i curves.

Solution: Note Fig. 15.9:

$$v = 24 \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = 16.968 \text{ V } \angle 0^\circ$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_L} = \frac{V \angle \theta}{X_L \angle 90^\circ} = \frac{16.968 \text{ V} \angle 0^\circ}{3 \ \Omega \angle 90^\circ} = 5.656 \text{ A} \angle -90^\circ$$

and $\quad i = \sqrt{2}(5.656) \sin(\omega t - 90^\circ) = \mathbf{8.0 \sin(\omega t - 90^\circ)}$

EXAMPLE 15.4 Using complex algebra, find the voltage v for the circuit in Fig. 15.10. Sketch the v and i curves.

Solution: Note Fig. 15.11:

$$i = 5 \sin(\omega t + 30^\circ) \Rightarrow \text{phasor form } \mathbf{I} = 3.535 \text{ A} \angle 30^\circ$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_L = (I \angle \theta)(X_L \angle 90^\circ) = (3.535 \text{ A} \angle 30^\circ)(4 \ \Omega \angle +90^\circ)$$
$$= 14.140 \text{ V} \angle 120^\circ$$

and $\quad v = \sqrt{2}(14.140) \sin(\omega t + 120^\circ) = \mathbf{20 \sin(\omega t + 120^\circ)}$

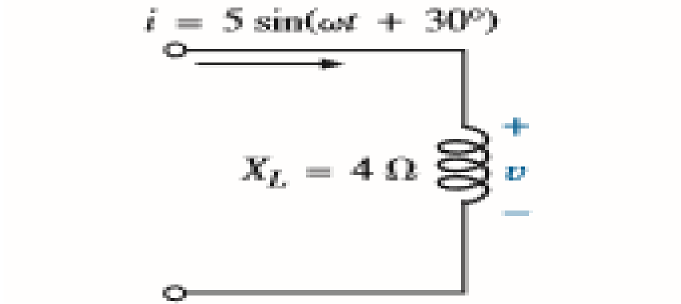


FIG. 15.10
Example 15.4.

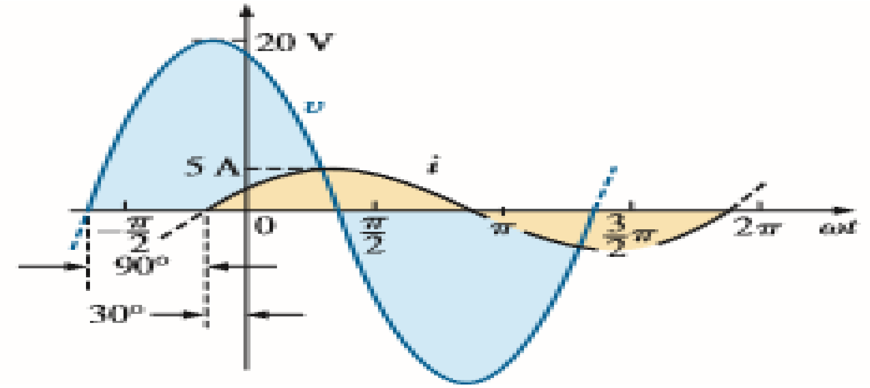


FIG. 15.11
Waveforms for Example 15.4.

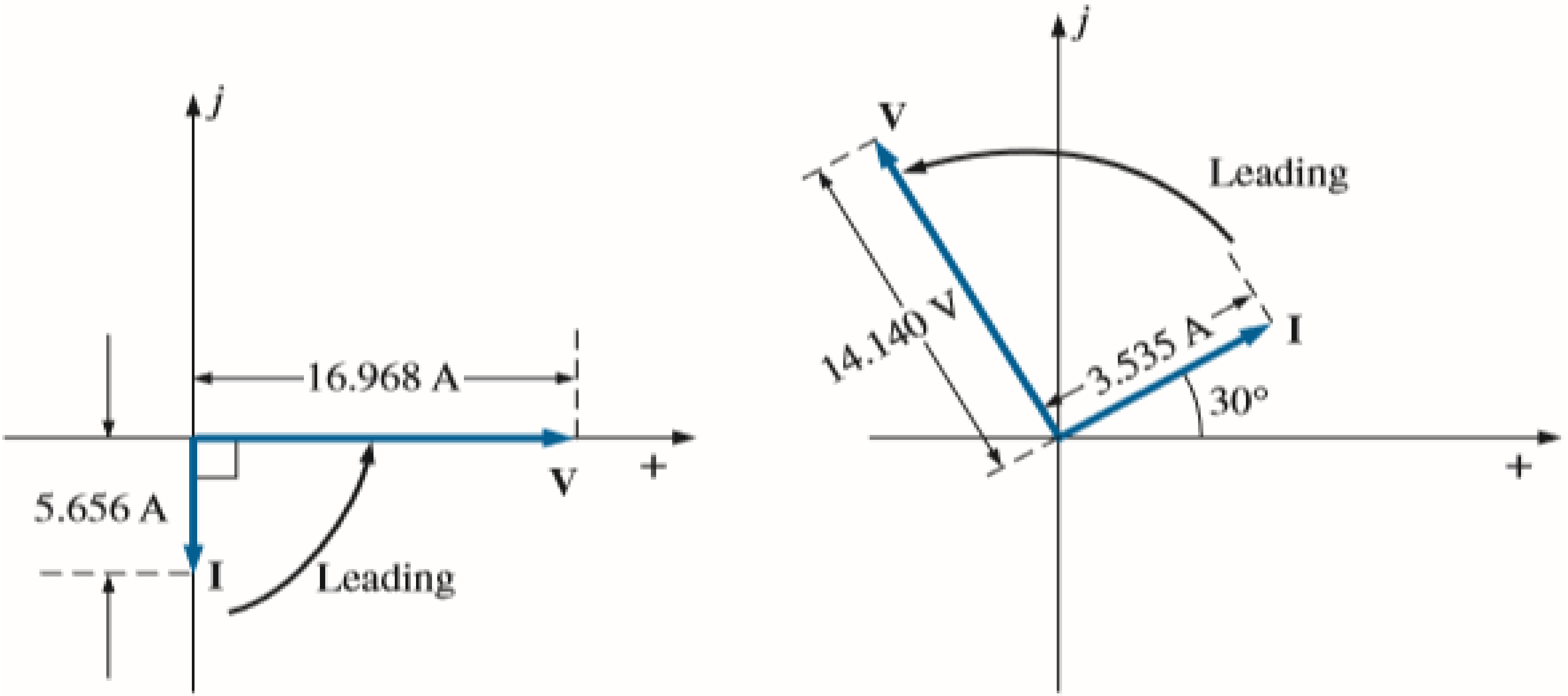


FIG. 15.12
Phasor diagrams for Examples 15.3 and 15.4.

Home Work

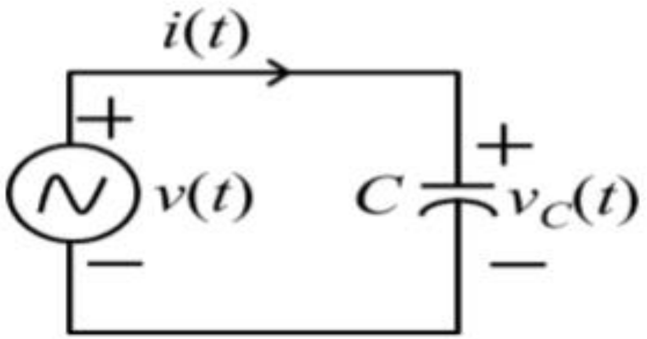
Problem 4: The current $i(t)$=5sin(ωt+30°) A flows through a 4 Ω inductive reactance. (***i***) What is the sinusoidal expression for the voltage? (***ii***) Calculate the real power, reactive power, power factor, reactive factor. (***iii***) write the expression of instantaneous power. (***iv***) Sketch the v and i sinusoidal waveforms on the same axis. (***v***) Draw the phasor diagram.

Problem 5: The voltage $v(t)$=24sinωt V is applied to a 3 Ω inductive reactance. (***i***) What is the sinusoidal expression for the current? (**ii**) Calculate the real power, reactive power, power factor, reactive factor. (**iii**) write the expression of instantaneous power. (**iv**) Sketch the v and i sinusoidal waveforms on the same axis. (**v**) Draw the phasor diagram.

Pure Capacitive Circuit

Let, the input is $v(t) = V_m \sin\omega t$ V, according to KVL, we have: $v(t) = v_C(t) = V_m \sin\omega t$

For a capacitance the relation of voltage and current is:



$$i(t) = i_C(t) = C\frac{dv_C(t)}{dt} = CV_m\frac{d(\sin\omega t dt)}{dt} = \omega LV_m \cos\omega t$$

$$i(t) = \omega CV_m \sin(\omega t + 90°) = I_m \sin(\omega t + \theta_i)$$

Magnitude of impedance, $Z = \dfrac{V_m}{I_m} = \dfrac{1}{\omega C} = X_C \quad \Omega$

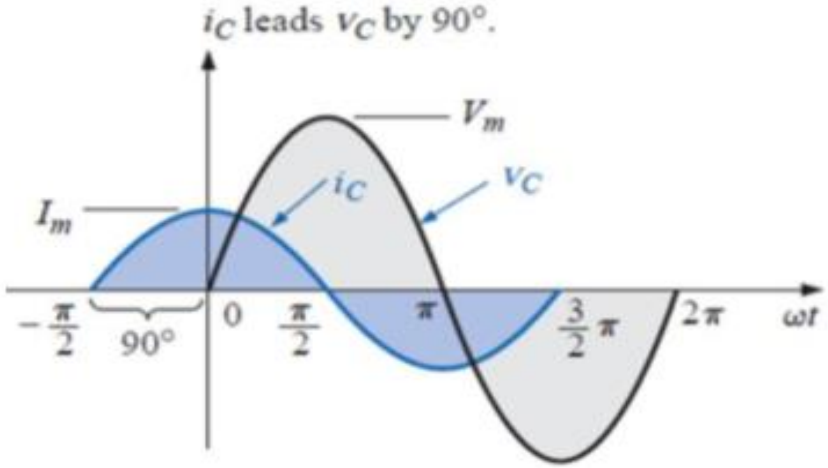
Capacitive reactance, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC} \quad \Omega$

Angle of current, $\theta_i = 90°$

Angle of impedance, $\theta_z = \theta_v - \theta_i = -90°$

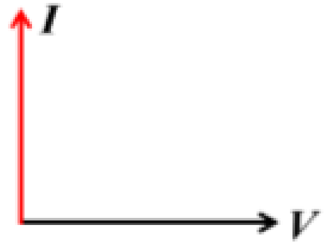
Impedance of a Capacitor, $\boldsymbol{Z} = \boldsymbol{Z}_C = X_C\angle -90°$

$$= -jX_C \ \Omega$$



The phase difference between voltage across and current through a capacitor is 90°.

***For a purely capacitive element, the voltage lags the current through the capacitive element by 90°*.** **Or, *the current leads the voltage in an capacitive element by 90°*.**



Phasor Diagram

EXAMPLE 15.5 Using complex algebra, find the current i for the circuit in Fig. 15.14. Sketch the v and i curves.

Solution: Note Fig. 15.15:

$$v = 15 \sin \omega t \Rightarrow \text{phasor notation } \mathbf{V} = 10.605 \text{ V} \angle 0^\circ$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_C} = \frac{V \angle \theta}{X_C \angle -90^\circ} = \frac{10.605 \text{ V} \angle 0^\circ}{2 \, \Omega \angle -90^\circ} = 5.303 \text{ A} \angle 90^\circ$$

and $$i = \sqrt{2}(5.303) \sin(\omega t + 90^\circ) = \mathbf{7.5 \sin(\omega t + 90^\circ)}$$

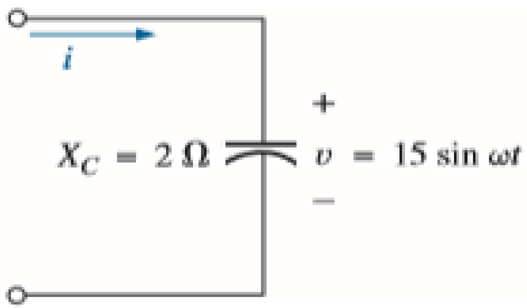
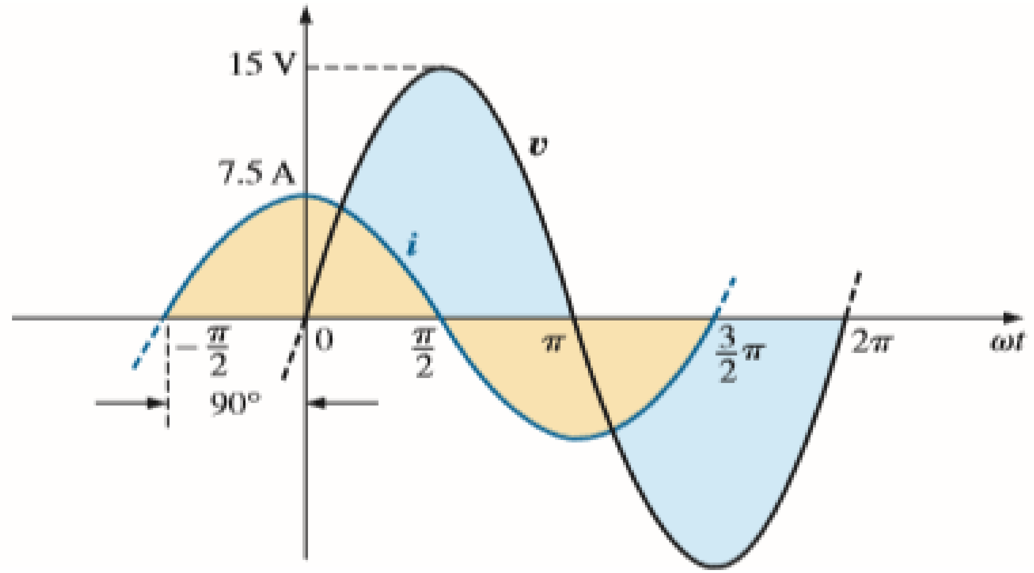


FIG. 15.14
Example 15.5.



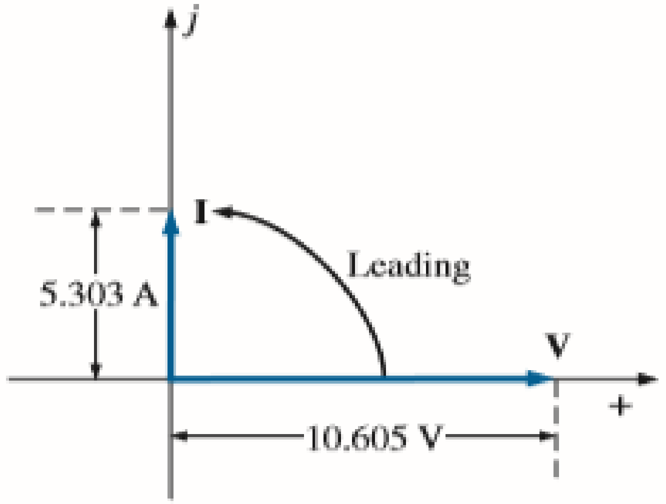


FIG. 15.15
Waveforms for Example 15.5.

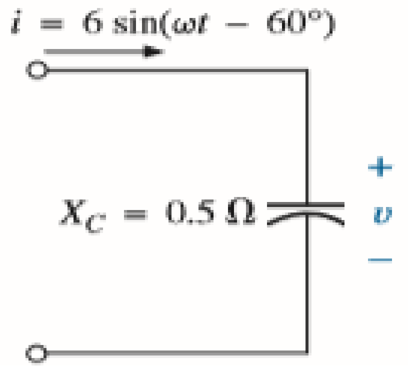
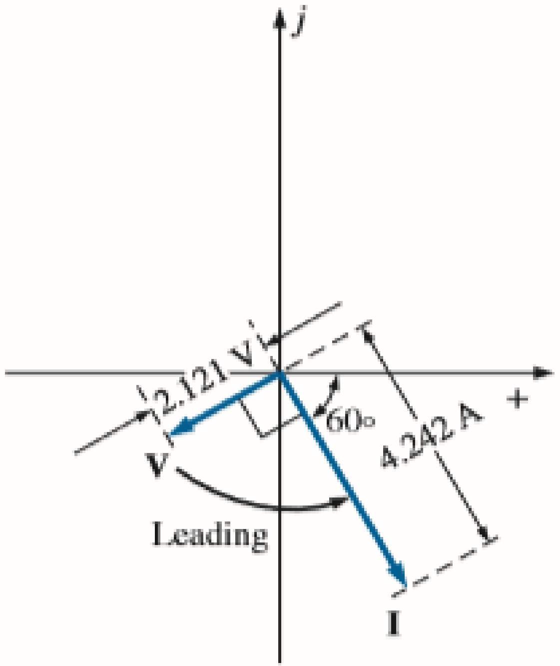


FIG. 15.16
Example 15.6.



EXAMPLE 15.6 Using complex algebra, find the voltage v for the circuit in Fig. 15.16. Sketch the v and i curves.

Solution: Note Fig. 15.17:

$$i = 6\sin(\omega t - 60°) \Rightarrow \text{phasor notation } \mathbf{I} = 4.242\text{ A} \angle -60°$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_C = (I \angle \theta)(X_C \angle -90°) = (4.242\text{ A} \angle -60°)(0.5\ \Omega \angle -90°)$$
$$= 2.121\text{ V} \angle -150°$$

and

$$v = \sqrt{2}(2.121)\sin(\omega t - 150°) = \mathbf{3.0\sin(\omega t - 150°)}$$

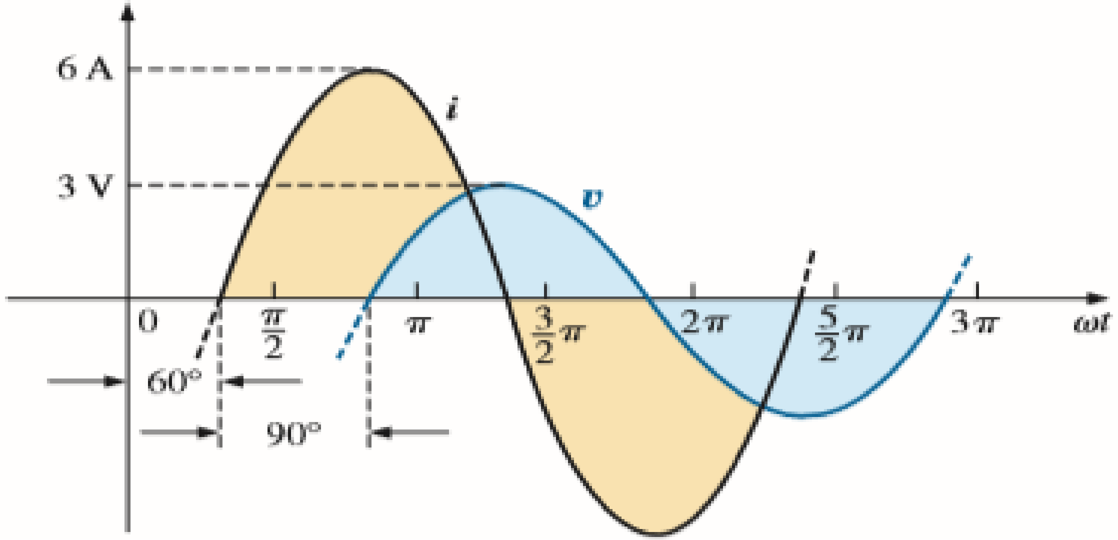


FIG. 15.17
Waveforms for Example 15.6.

	Resistance	**Inductance**	**Capacitance**
Magnitude of impedance(Z) [Ω]	R	X_L	X_C
Angle of impedance(θ=θ_z)	0°	90°	-90°
Impedance ($\boldsymbol{Z}$) [Ω]	$\boldsymbol{Z}_R=R\angle 0°=R+j0$	$\boldsymbol{Z}_L= X_L\angle 90°=0+jX_L$	$\boldsymbol{Z}_C= X_C\angle -90°=0-jX_L$
Phase difference between voltage and current	0°	90°	-90°
Relation between voltage and current	Voltage and current are in phase	Voltage leads current Current **lags** voltage	Voltage lags current Currents **leads** voltage
Power factor (pf=cosθ)	Unity (1)	Zero **lagging** power factor (pf=0)	Zero **leading** power factor (pf=0)
Reactive factor (rf=sinθ)	0	1	-1
Power (P) [W]	$V_{rms}I_{rms}=I_{rms}^2R=V_{rms}^2/R$	0	0
Reactive power (Q=P_x) [Var]	0	$V_{rms}I_{rms}=I_{rms}^2X_L$ $=V_{rms}^2/ X_L$	$- V_{rms}I_{rms}=-I_{rms}^2X_C$ $=-V_{rms}^2/ X_C$
Apparent power (S) [VA]	$S= V_{rms}I_{rms}$	$S= V_{rms}I_{rms}$	$S= V_{rms}I_{rms}$

Impedance Diagram

Now that an angle is associated with resistance, inductive reactance, and capacitive reactance, each can be placed on a complex plane diagram, as shown in Fig. 15.19. For any network, the resistance will *always* appear on the positive real axis, the inductive reactance on the positive imaginary axis, and the capacitive reactance on the negative imaginary axis.

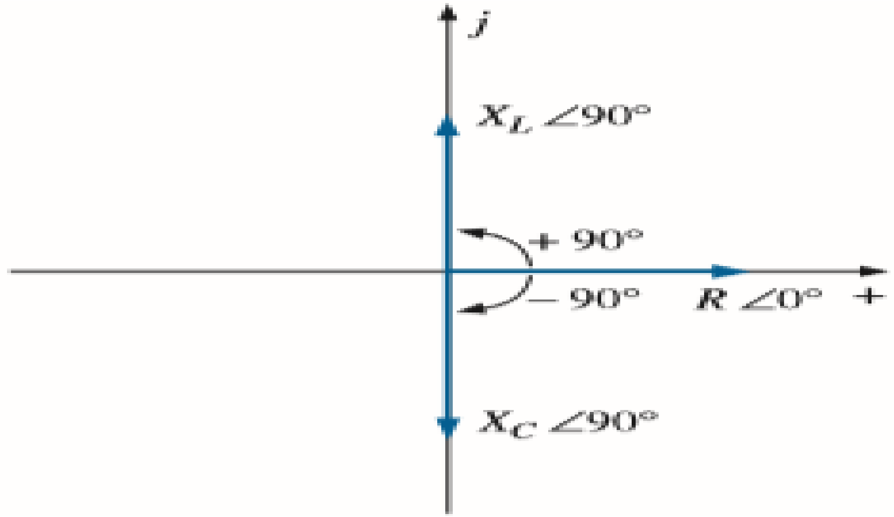


FIG. 15.19
Impedance diagram.

For any configuration (series, parallel, series-parallel, and so on), the angle associated with the total impedance is the angle by which the applied voltage leads the source current. For inductive networks, θ_T will be positive, whereas for capacitive networks, θ_T will be negative.

15.3 SERIES CONFIGURATION

The overall properties of series ac circuits (Fig. 15.20) are the same as those for dc circuits. For instance, the total impedance of a system is the sum of the individual impedances:

$$\boxed{\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \cdots + \mathbf{Z}_N} \tag{15.4}$$

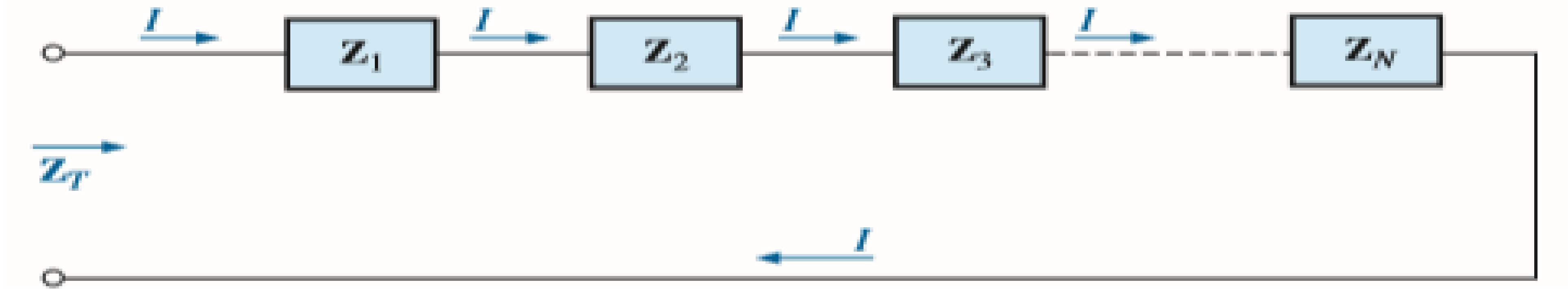


FIG. 15.20
Series impedances.

EXAMPLE 15.7 Draw the impedance diagram for the circuit in Fig. 15.21, and find the total impedance.

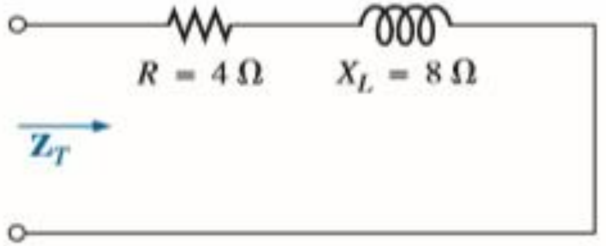


FIG. 15.21
Example 15.7.

Solution: As indicated by Fig. 15.22, the input impedance can be found graphically from the impedance diagram by properly scaling the real and imaginary axes and finding the length of the resultant vector Z_T and angle θ_T. Or, by using vector algebra, we obtain

$$
\begin{aligned}
\mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 \\
&= R \angle 0^\circ + X_L \angle 90^\circ \\
&= R + jX_L = 4\ \Omega + j\,8\ \Omega \\
\mathbf{Z}_T &= \mathbf{8.94\ \Omega \angle 63.43^\circ}
\end{aligned}
$$

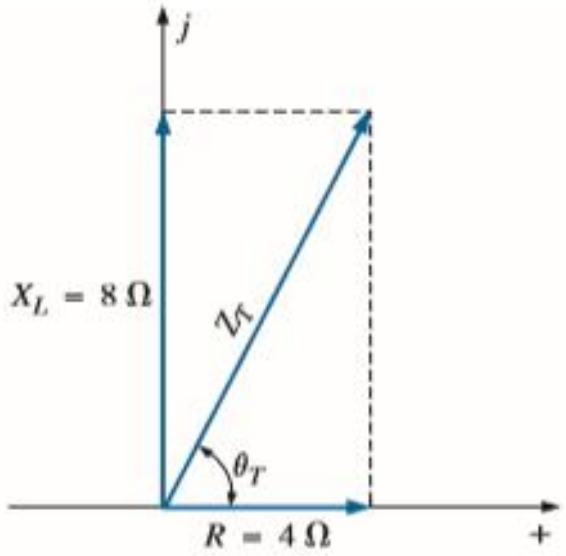


FIG. 15.22
Impedance diagram for Example 15.7.

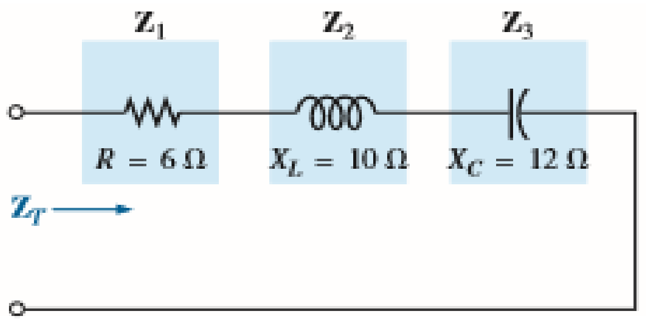


FIG. 15.23
Example 15.8

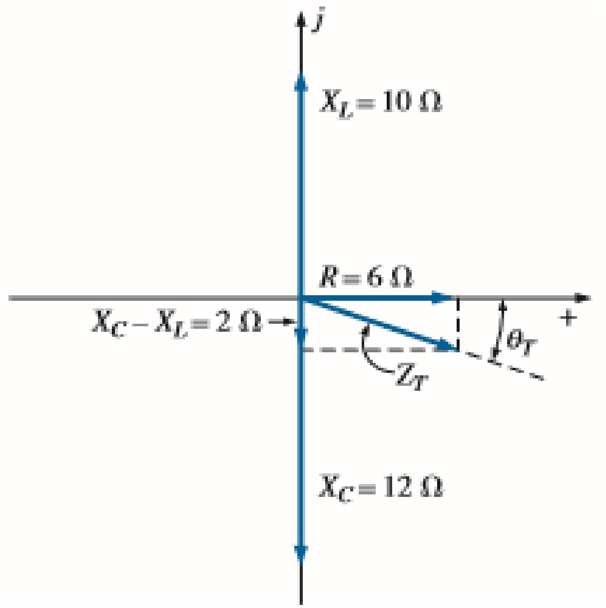


FIG. 15.24
Impedance diagram for Example 15.8.

EXAMPLE 15.8 Determine the input impedance to the series network in Fig. 15.23. Draw the impedance diagram.

Solution:

$$
\begin{aligned}
\mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 \\
&= R \angle 0^\circ + X_L \angle 90^\circ + X_C \angle -90^\circ \\
&= R + jX_L - jX_C \\
&= R + j(X_L - X_C) = 6\ \Omega + j(10\ \Omega - 12\ \Omega) = 6\ \Omega - j2\ \Omega \\
\mathbf{Z}_T &= \mathbf{6.32\ \Omega \angle -18.43^\circ}
\end{aligned}
$$

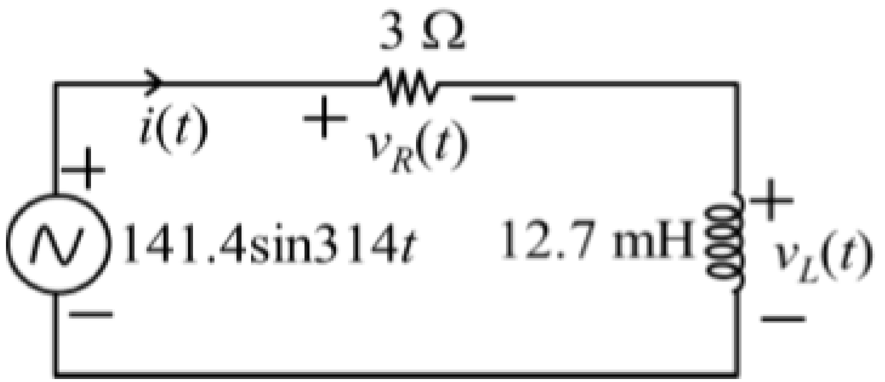
Resistance and Inductance Series Circuit

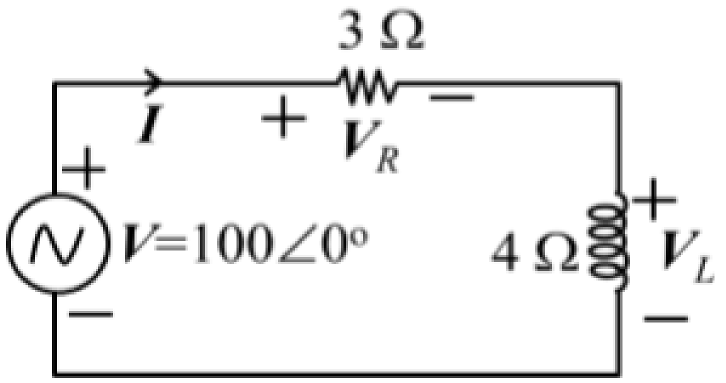
Example

A voltage 141.4sin314t V is applied to a *RL* series circuit which consists *R*=3 ohms, *L* = 12.7 mH. (**a**) Draw the circuit diagram. (**b**) Calculate (**i**) the inductive reactance, (**ii**) the impedance, (**iii**) the current, (**iv**) the voltage drop across the resistance and inductance, (**v**) the power factor and reactive factor, (vi) the power, reactive power, apparent power. (**vi**) Verify the KVL. (**c**) Write the instantaneous expression of current, voltage drop across the resistance, voltage drop across the inductance. (**d**) Draw the impedance diagram, phasor diagram, and power triangle.

Solution: $X_L = \omega L = 314 \times 0.0127 = 4 \;\; \Omega$ $\qquad \boldsymbol{V} = \dfrac{V_m}{\sqrt{2}} \angle \theta_v = \dfrac{141.4}{\sqrt{2}} \angle 0° = 100\angle 0° \;\; \text{V}$

$$\boldsymbol{Z}_L = jX_L = j4 = 4\angle 90° \;\; \Omega \qquad \boldsymbol{Z}_R = 3 = 3\angle 0° \;\; \Omega$$





$$\boldsymbol{Z} = \boldsymbol{Z}_{RL} = 3 + j4 = 5\angle 53.13^\circ \quad \Omega \qquad V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 100 \ \text{V} \quad \boldsymbol{V} = 100\angle 0^\circ \ \text{V}$$

Current: $$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}} = \frac{100\angle 0^\circ}{5\angle 53.13^\circ} = 20\angle -53.13^\circ \ \text{A}$$

Voltage drop across the resistance: $\boldsymbol{V}_R = \boldsymbol{I}\boldsymbol{Z}_R = (20\angle -53.13^\circ)(3\angle 0^\circ) = 60\angle -53.13^\circ \ \text{V}$

Voltage drop across the inductance: $\boldsymbol{V}_L = \boldsymbol{I}\boldsymbol{Z}_L = (20\angle -53.13^\circ)(4\angle 90^\circ) = 80\angle 36.87^\circ \ \text{V}$

Verification of KVL: $\boldsymbol{V} = \boldsymbol{V}_R + \boldsymbol{V}_L = 60\angle -53.13^\circ + 80\angle 36.87^\circ \ \text{V}$

$\boldsymbol{V} = 36 - j48 + 64 + j48 = 100 \ \text{V}$ (equal to supply voltage)

Angle of impedance, $\theta = \theta_z = \theta_v - \theta_i = 53.13^\circ$

Power Factor $:\text{pf} = \cos(\theta_z) = \cos[53.13^\circ] = 0.6$ lagging

Reactive Factor $: \text{rf} = \sin(\theta_z) = \sin[53.13^\circ] = 0.8$

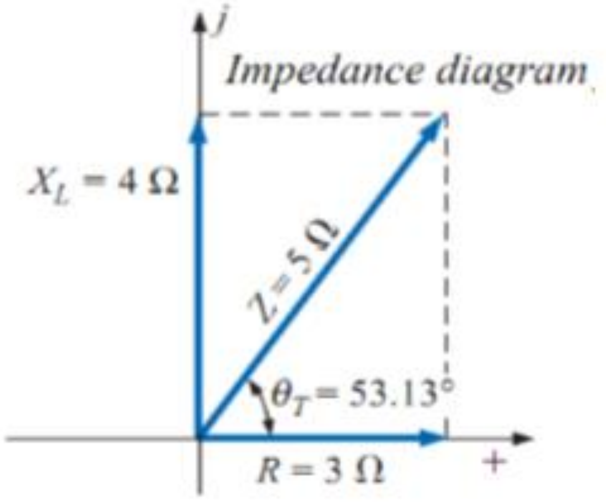
Power or **Active Power** : $P = VI\cos\theta_Z = 100 \times 20 \times 0.6 = 1200$ W

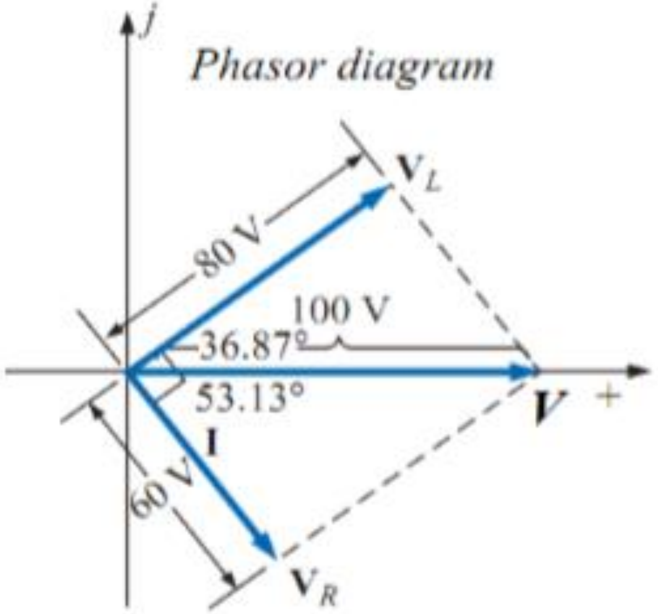
$$P = I^2 R = 20^2 \times 3 = 1200\,\text{W} \qquad P = \frac{V_R^2}{R} = \frac{60^2}{3} = 1200\,\text{W}$$

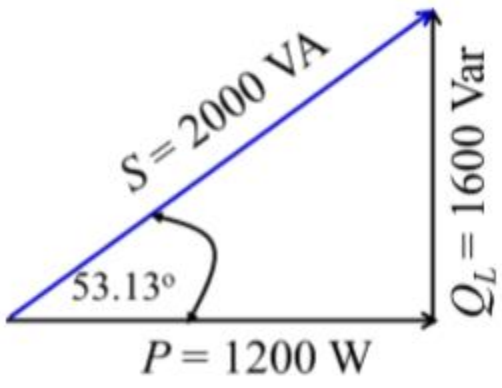
Reactive Power : $Q_L = VI\sin\theta_Z = 100 \times 20 \times 0.8 = 1600$ Var

$$Q_L = I^2 X_L = 20^2 \times 4 = 1600 \text{ Var} \qquad Q_L = \frac{V_L^2}{X_L} = \frac{80^2}{4} = 1600 \text{ Var}$$

Apparent Power : $S = VI = 100 \times 20 = 2000$ VA







Power Triangle

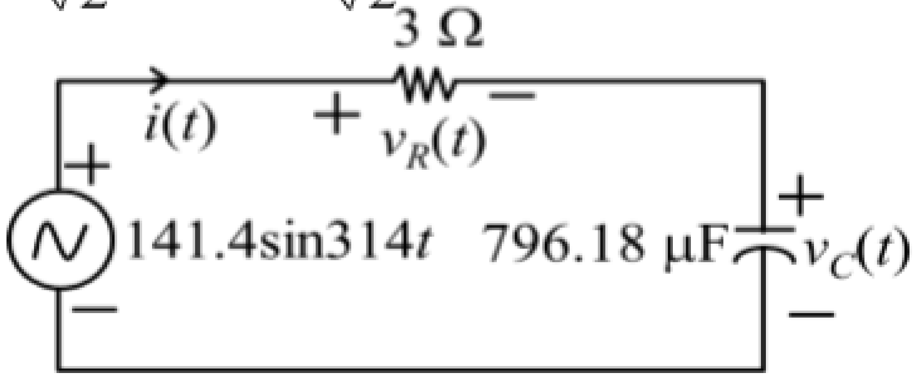
Resistance and Capacitance Series Circuit

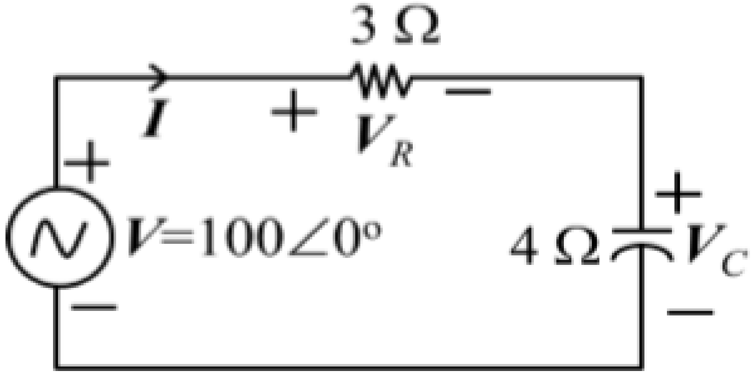
Example

A voltage 141.4sin314t V is applied to a *RC* series circuit which consists *R*=3 ohms, *C*=796.18 μF. (**a**) Draw the circuit diagram. (**b**) Calculate (**i**) the capacitive reactance, (**ii**) the impedance, (**iii**) the current, (**iv**) the voltage drop across the resistance and capacitance, (**v**) the power factor and reactive factor, (vi) the power, reactive power, apparent power. (**vi**) Verify the KVL. (**c**) Write the instantaneous expression of current, voltage drop across the resistance, voltage drop across the capacitance. (**d**) Draw the impedance diagram, phasor diagram, and power triangle.

Solution: **Capacitive reactance**: $X_C = \dfrac{1}{\omega C} = \dfrac{1}{314 \times 796.18 \times 10^{-6}} = 21.1 \ \ \Omega$

$$\boldsymbol{V} = \frac{V_m}{\sqrt{2}} \angle \theta_v = \frac{141.4}{\sqrt{2}} \angle 0° = 100 \angle 0° \ \ \text{V}$$





$$Z_C = -jX_C = -j4 = 4\angle -90^\circ \ \ \Omega \quad Z_R = 3 = 3\angle 0^\circ \ \ \Omega$$

$$Z = Z_{RC} = 3 - j4 = 5\angle -53.13^\circ \quad \Omega$$

Current: $$I = \frac{V}{Z} = \frac{100\angle 0^\circ}{5\angle -53.13^\circ} = 20\angle 53.13^\circ \ \ \text{A}$$

Voltage drop across the resistance: $$V_R = IZ_R = (20\angle 53.13^\circ)(3\angle 0^\circ) = 60\angle 53.13^\circ \ \ \text{V}$$

Voltage drop across the inductance: $$V_C = IZ_C = (20\angle 53.13^\circ)(4\angle -90^\circ) = 80\angle -36.87^\circ \ \ \text{V}$$

Verification of KVL: $$V = V_R + V_C = 60\angle 53.13^\circ + 80\angle -36.87^\circ \ \ \text{V}$$

$$V = 36 + j48 + 64 - j48 = 100 \ \ \text{V (equal to supply voltage)}$$

Angle of impedance, $\theta = \theta_z = \theta_v - \theta_i = -53.13^\circ$

Power Factor : $\text{pf} = \cos(\theta_z) = \cos[-53.13^\circ] = 0.6$ leading

Reactive Factor : $\text{rf} = \sin(\theta_z) = \sin[-53.13^\circ] = -0.8$

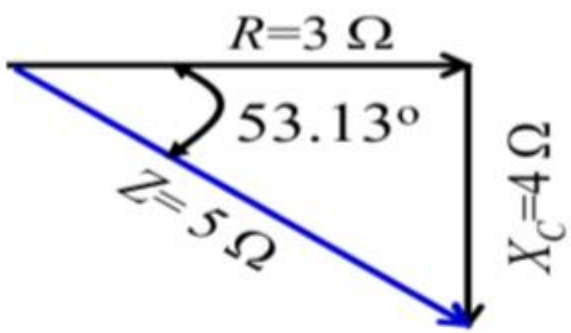
Power or **Active Power** : $P = VI\cos\theta_Z = 100\times 20\times 0.6 = 1200$ W

$$P = I^2R = 20^2\times 3 = 1200\,\text{W} \qquad P = \frac{V_R^2}{R} = \frac{60^2}{3} = 1200\,\text{W}$$

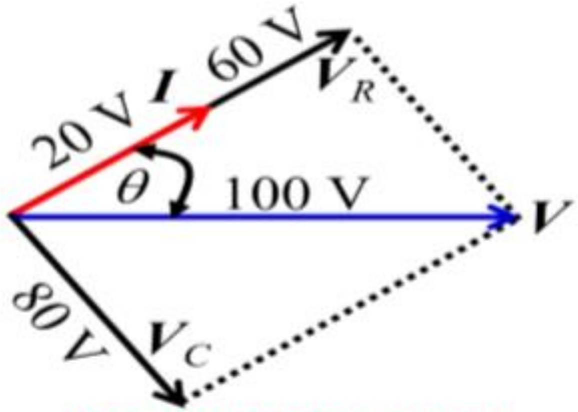
Reactive Power : $Q_L = VI\sin\theta_Z = 100\times 20\times -0.8 = -1600$ Var

$$Q_L = -I^2X_C = -20^2\times 4 = -1600\ \text{Var} \qquad Q_C = -\frac{V_C^2}{X_C} = -\frac{80^2}{4} = -1600\ \text{Var}$$

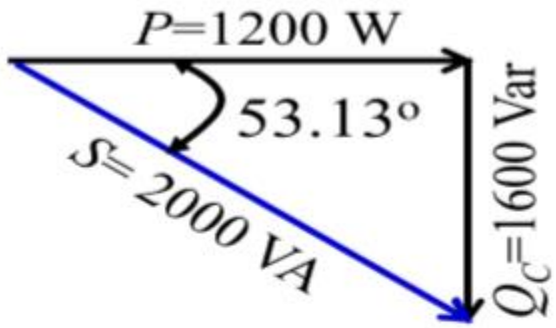
Apparent Power : $S = VI = 100\times 20 = 2000$ VA



Impedance Diagram



Phasor Diagram



Power Triangle

Resistance, Inductance and Capacitance Series Circuit

Example

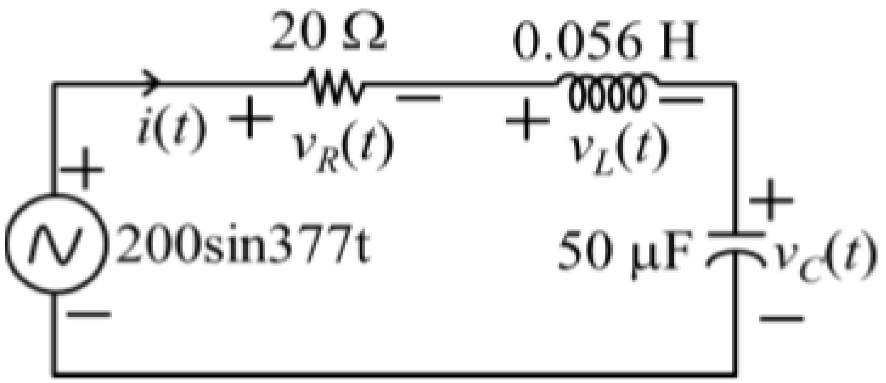
If R=20 ohms, L=0.056 Henry, C = 50 μF and applied voltage 200sin377t, (**a**) Draw the circuit diagram. (**b**) Calculate (**i**) the capacitive reactance, (**ii**) the impedance, (**iii**) the current, (**iv**) the voltage drop across the resistance and capacitance, (**v**) the power factor and reactive factor, (vi) the power, reactive power, apparent power. (**vi**) Verify the KVL. (**c**) Write the instantaneous expression of current, voltage drop across the resistance, voltage drop across the capacitance. (**d**) Draw the impedance diagram, phasor diagram, and power triangle.

Solution: $V_m = 200 \text{ V} \quad \omega = 377 \text{ rad/s} \quad R = 20 \ \Omega \quad L = 0.056 \text{ H} \quad C = 50\times10^{-6} \text{ F}$

Inductive reactance: $X_L = \omega L = 377\times0.056 = 21.1 \ \Omega$

Capacitive reactance: $X_C = \dfrac{1}{\omega C}$

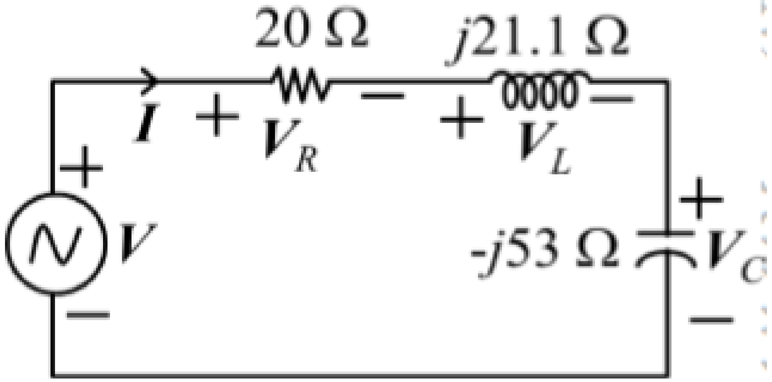
$$= \frac{1}{377\times50\times10^{-6}} = 53 \ \Omega$$



$$\boldsymbol{Z}_R = 20\angle 0° = 20\ \Omega \qquad \boldsymbol{Z}_L = 21.1\angle 90° = j21.1\ \Omega \qquad \boldsymbol{Z}_C = 53\angle -90° = -j53\ \Omega$$

$$\boldsymbol{Z} = \boldsymbol{Z}_R + \boldsymbol{Z}_L + \boldsymbol{Z}_C = 20\angle 0° + 21.1\angle 90° + 53\angle -90° = 20 + j21.1 - j53\ \Omega$$

$$\boldsymbol{Z} = 20 - j31.9 = 37.65\angle -57.9°\ \Omega$$



$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 141.4\ \text{V}$$

$$\boldsymbol{V} = 141.4\angle 0°\ \text{V}$$

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}} = \frac{141.4\angle 0°}{37.56\angle -57.9°} = 3.76\angle 57.9° = 2 + j3.2\ \text{A}$$

$$\boldsymbol{V}_R = \boldsymbol{I}\boldsymbol{Z}_R = (3.76\angle 57.9°)(20\angle 0°) = 75.1\angle 57.9° = 40 + j63.64\ \text{V}$$

$$\boldsymbol{V}_L = \boldsymbol{I}\boldsymbol{Z}_L = (3.76\angle 57.9°)(21.1\angle 90°) = 79.24\angle 148° = -67 + j42.1\ \text{V}$$

$$\boldsymbol{V}_C = \boldsymbol{I}\boldsymbol{Z}_C = (3.76\angle 57.9°)(53\angle -90°) = 199\angle -32.1° = 169 - j106\ \text{V}$$

Verification of KVL:

$$\boldsymbol{V} = 40 + j63.64 - 67 + j42.1 + 169 - j106 = 141.4\ \text{V (equal to supply voltage)}$$

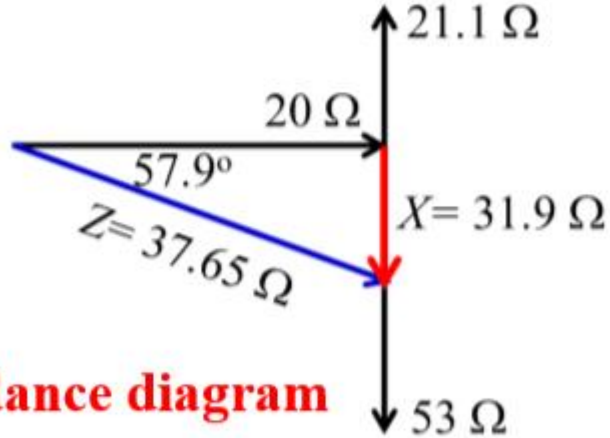
$$\theta = \theta_Z = \theta_V - \theta_i = -57.9^\circ\ \Omega \qquad pf = \cos\theta = \cos(-57.9^\circ) = 0.53$$

$$rf = \sin\theta = \sin(-57.9^\circ) = -0.85$$

$$S = VI = 141.4 \times 3.76 = 531\ \text{VA} \qquad P = VI\cos\theta_Z = 141.4 \times 3.76 \times 0.53 = 282.1\ \ \text{W}$$

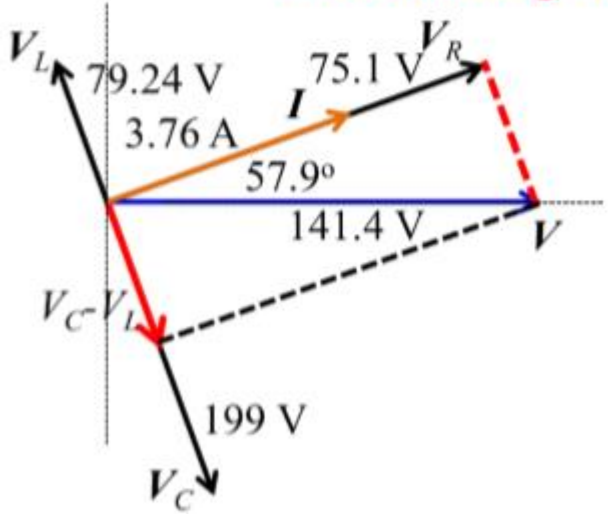
$$Q = Q_L - Q_C = VI\sin\theta_Z = 141.4 \times 3.76 \times -0.85 = -500\ \ \text{Var}$$

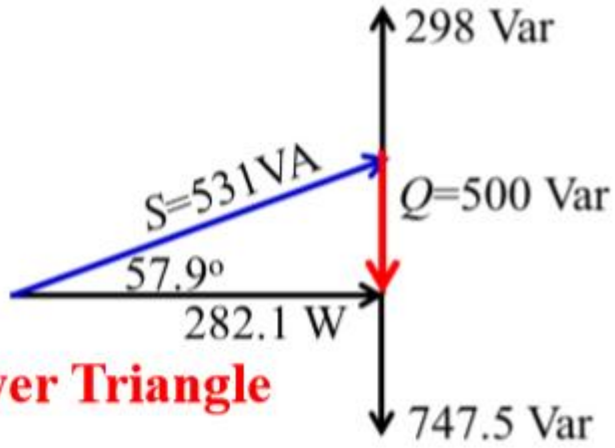
$$Q_L = I^2 X_L = 3.76^2 \times 21.1 = 298\ \text{Var} \qquad Q_C = -I^2 X_C = -3.76^2 \times 53 = -747.5\ \text{Var}$$



Impedance diagram

Phasor diagram





Power Triangle

	***RL* series Circuit**	***RC* series Circuit**	***RLC* series Circuit**
Magnitude of impedance(Z) [Ω]	$Z = \sqrt{R^2 + X_L^2}$	$Z = \sqrt{R^2 + X_C^2}$	$Z = \sqrt{R^2 + (X_L - X_C)^2}$
Angle of impedance($\theta = \theta_z$)	$\theta = \tan^{-1}\left[\dfrac{X_L}{R}\right]$	$\theta = -\tan^{-1}\left[\dfrac{X_C}{R}\right]$	$\theta = \tan^{-1}\left[\dfrac{X_L - X_C}{R}\right]$
Impedance ($\boldsymbol{Z}$) [Ω]	$\boldsymbol{Z} = Z\angle\theta = R + jX_L$	$\boldsymbol{Z} = Z\angle\theta = R - jX_C$	$\boldsymbol{Z} = Z\angle\theta = R + j\left(X_L - X_C\right)$
Phase difference between voltage and current	$\theta = \theta_v - \theta_i > 0$ Between 0° to 90°	$\theta = \theta_v - \theta_i < 0$ Between -90° to 0°	Depends on the value of X_L and X_C
Relation between voltage and current	Voltage leads current i.e. Current lags voltage	Voltage lags current i.e. Current leads voltage	Depends on the value of X_L and X_C

Voltage Divider Rule

The voltage ($\boldsymbol{V}_x$) across one or more elements in series that have total impedance $\boldsymbol{Z}_x$, can be given by:

$$\boldsymbol{V}_x = \frac{\boldsymbol{Z}_x}{\boldsymbol{Z}_T}\boldsymbol{E} = \frac{\boldsymbol{Z}_x}{\boldsymbol{Z}_T}\boldsymbol{V}$$

where, $\boldsymbol{E}$ or $\boldsymbol{V}$ is the total voltage appearing across the series circuit, and $\boldsymbol{Z}_T$ is the total impedance of the series circuit.

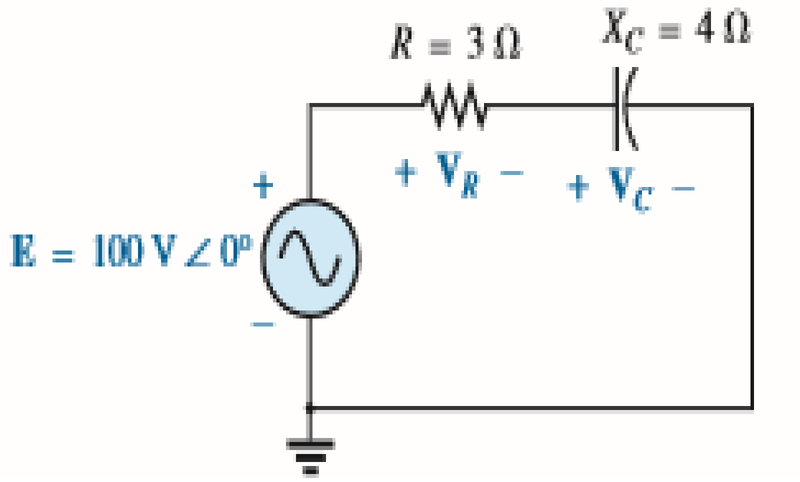


FIG. 15.40
Example 15.9.

EXAMPLE 15.9 Using the voltage divider rule, find the voltage across each element of the circuit in Fig. 15.40.

Solution:

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(4\ \Omega \angle -90°)(100\text{ V} \angle 0°)}{4\ \Omega \angle -90° + 3\ \Omega \angle 0°} = \frac{400 \angle -90°}{3 - j4}$$

$$= \frac{400 \angle -90°}{5 \angle -53.13°} = \mathbf{80\ V \angle -36.87°}$$

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(3\ \Omega \angle 0°)(100\text{ V} \angle 0°)}{5\ \Omega \angle -53.13°} = \frac{300 \angle 0°}{5 \angle -53.13°}$$

$$= \mathbf{60\ V \angle +53.13°}$$

EXAMPLE 15.11 For the circuit in Fig. 15.43:

a. Calculate **I**, $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ in phasor form.
b. Calculate the total power factor.
c. Calculate the average power delivered to the circuit.

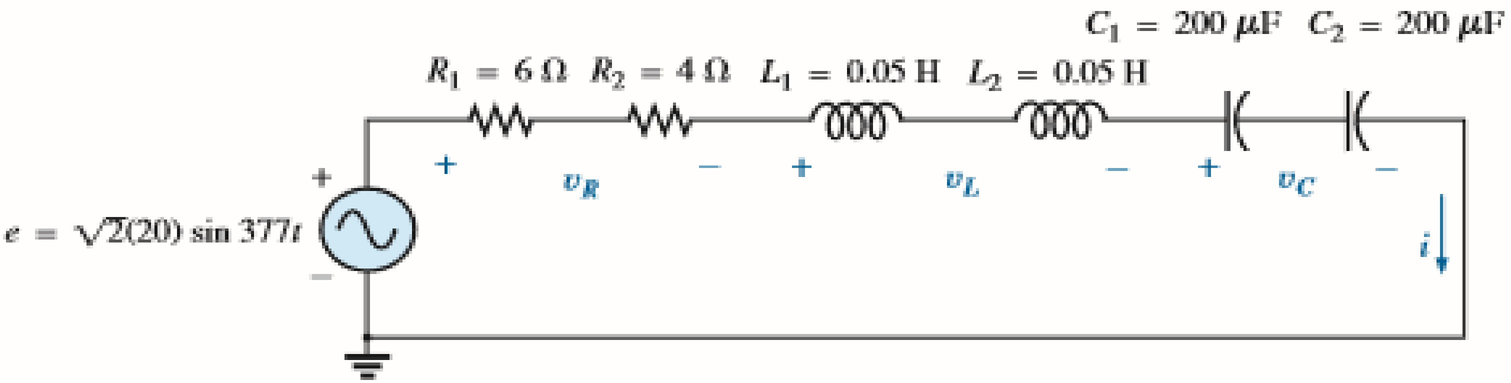


FIG. 15.43
Example 15.11.

d. Draw the phasor diagram.
e. Obtain the phasor sum of $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$, and show that it equals the input voltage **E**.
f. Find $\mathbf{V}_R$ and $\mathbf{V}_C$ using the voltage divider rule.

Solutions:

a. Combining common elements and finding the reactance of the inductor and capacitor, we obtain

$$R_T = 6\ \Omega + 4\ \Omega = 10\ \Omega$$
$$L_T = 0.05\ \text{H} + 0.05\ \text{H} = 0.1\ \text{H}$$
$$C_T = \frac{200\ \mu\text{F}}{2} = 100\ \mu\text{F}$$
$$X_L = \omega L = (377\ \text{rad/s})(0.1\ \text{H}) = 37.70\ \Omega$$
$$X_C = \frac{1}{\omega C} = \frac{1}{(377\ \text{rad/s})(100 \times 10^{-6}\ \text{F})} = \frac{10^6\ \Omega}{37{,}700} = 26.53\ \Omega$$

Redrawing the circuit using phasor notation results in Fig. 15.44.

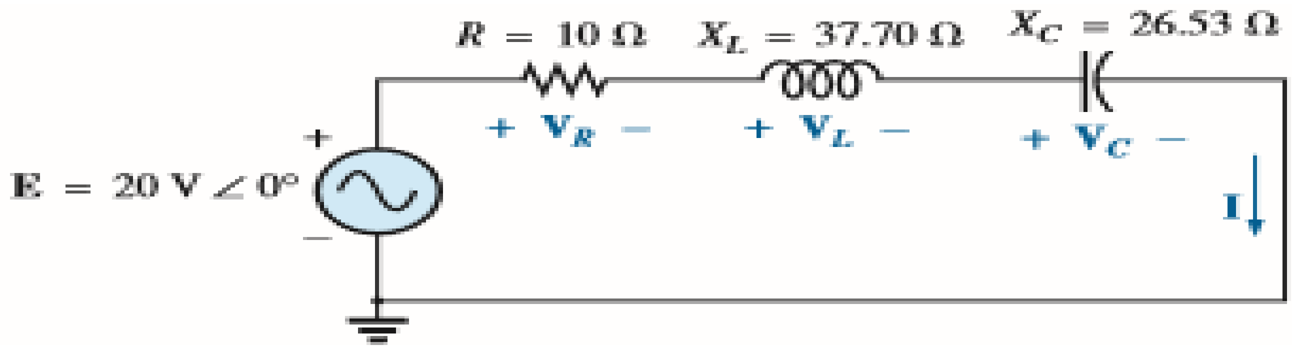


FIG. 15.44
Applying phasor notation to the circuit in Fig. 15.43.

For the circuit in Fig. 15.44,

$$\begin{aligned}\mathbf{Z}_T &= R \angle 0^\circ + X_L \angle 90^\circ + X_C \angle -90^\circ \\ &= 10\ \Omega + j37.70\ \Omega - j26.53\ \Omega \\ &= 10\ \Omega + j11.17\ \Omega = \mathbf{15\ \Omega \angle 48.16^\circ}\end{aligned}$$

The current **I** is

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{20\text{ V} \angle 0^\circ}{15\ \Omega \angle 48.16^\circ} = \mathbf{1.33\text{ A} \angle -48.16^\circ}$$

The voltage across the resistor, inductor, and capacitor can be found using Ohm's law:

$$\begin{aligned}\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (I \angle \theta)(R \angle 0^\circ) &= (1.33\text{ A} \angle -48.16^\circ)(10\ \Omega \angle 0^\circ) \\ &= \mathbf{13.30\text{ V} \angle -48.16^\circ} \\ \mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (I \angle \theta)(X_L \angle 90^\circ) &= (1.33\text{ A} \angle -48.16^\circ)(37.70\ \Omega \angle 90^\circ) \\ &= \mathbf{50.14\text{ V} \angle 41.84^\circ} \\ \mathbf{V}_C = \mathbf{I}\mathbf{Z}_C = (I \angle \theta)(X_C \angle -90^\circ) &= (1.33\text{ A} \angle -48.16^\circ)(26.53\ \Omega \angle -90^\circ) \\ &= \mathbf{35.28\text{ V} \angle -138.16^\circ}\end{aligned}$$

b. The total power factor, determined by the angle between the applied voltage **E** and the resulting current **I**, is 48.16°:

$$F_p = \cos\theta = \cos 48.16^\circ = \mathbf{0.667\ lagging}$$

or

$$F_p = \cos\theta = \frac{R}{Z_T} = \frac{10\ \Omega}{15\ \Omega} = \mathbf{0.667\ lagging}$$

c. The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta = (20\text{ V})(1.33\text{ A})(0.667) = \mathbf{17.74\ W}$$

d. The phasor diagram appears in Fig. 15.45.

e. The phasor sum of $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ is

$$\begin{aligned}\mathbf{E} &= \mathbf{V}_R + \mathbf{V}_L + \mathbf{V}_C\\ &= 13.30\text{ V}\angle -48.16^\circ + 50.14\text{ V}\angle 41.84^\circ + 35.28\text{ V}\angle -138.16^\circ\\ \mathbf{E} &= 13.30\text{ V}\angle -48.16^\circ + 14.86\text{ V}\angle 41.84^\circ\end{aligned}$$

Therefore,

$$E = \sqrt{(13.30\text{ V})^2 + (14.86\text{ V})^2} = \mathbf{20\ V}$$

and $\qquad \theta_E = \mathbf{0^\circ} \qquad$ (from phasor diagram)

and $\qquad \mathbf{E} = 20\angle 0^\circ$

f. $$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_T} = \frac{(10\ \Omega\angle 0^\circ)(20\text{ V}\angle 0^\circ)}{15\ \Omega\angle 48.16^\circ} = \frac{200\text{ V}\angle 0^\circ}{15\angle 48.16^\circ} = \mathbf{13.3\ V\angle -48.16^\circ}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(26.5\ \Omega\angle -90^\circ)(20\text{ V}\angle 0^\circ)}{15\ \Omega\angle 48.16^\circ} = \frac{530.6\text{ V}\angle -90^\circ}{15\angle 48.16^\circ} = \mathbf{35.37\ V\angle -138.16^\circ}$$

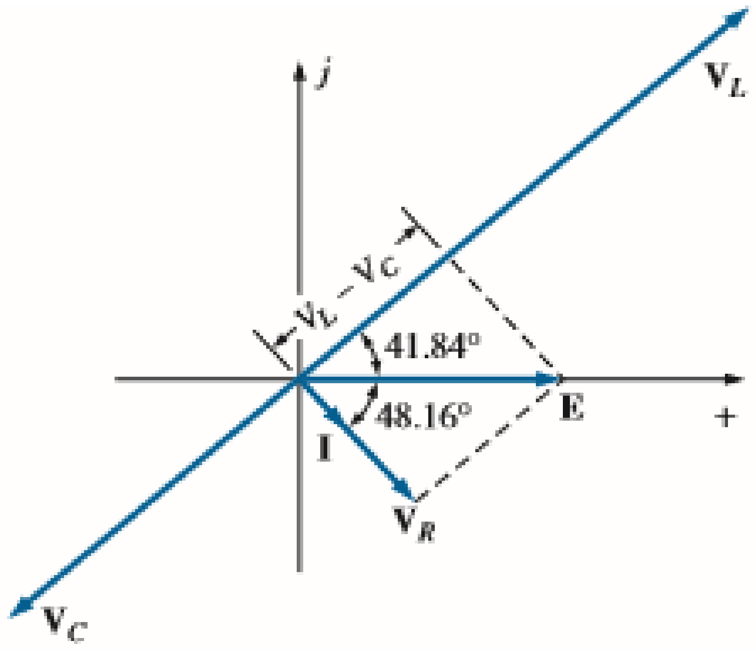


FIG. 15.45
Phasor diagram for the circuit in Fig. 15.43.

Parallel Circuits

Admittance

The reciprocal of impedance is called **admittance**.
The unit of admittance is **Siemens** [S].

$$\boldsymbol{Y} = \frac{1}{\boldsymbol{Z}} = \frac{\boldsymbol{I}}{\boldsymbol{V}} = Y\angle\theta_y = G + jB = g + jb = (\textbf{Conducatcne}) + j(\textbf{Susceptance}) \quad [S]$$

$$\textbf{Conducatcne}: G = g = Y\cos\theta_y \quad [S] \qquad \textbf{Susceptance}: B = b = Y\sin\theta_y \quad [S]$$

G or g **is called conductance**. The unit of conductance is **mho** or **Siemens**.
Conductance (*G* or *g*) is the reciprocal of resistance (*R*).

B or b **is called susceptance**. The unit of susceptance is **Siemens**.
Susceptance (*B* or *b*) is the reciprocal of reactance (*X*).

$$G = g = \frac{1}{R} \quad [S] \qquad B = b = \frac{1}{X} \quad [S] \qquad B_L = b_L = \frac{1}{X_L} \quad [S] \qquad B_C = b_C = \frac{1}{X_C} \quad [S]$$

B_L *or* b_L **is called inductive susceptance**.
B_C *or* b_C **is called capacitive susceptance**.

Admittance of a Resistance, Inductance and Capacitance

Admittance of a resistance: $$\boldsymbol{Y}_R = \frac{1}{\boldsymbol{Z}_R} = \frac{1}{R\angle 0^\circ} = G\angle 0^\circ \quad [S]$$

Admittance of an inductance: $$\boldsymbol{Y}_L = \frac{1}{\boldsymbol{Z}_L} = \frac{1}{X_L\angle 90^\circ} = B_L\angle -90^\circ \quad [S]$$

Admittance of a capacitance: $$\boldsymbol{Y}_C = \frac{1}{\boldsymbol{Z}_C} = \frac{1}{X_C\angle -90^\circ} = B_C\angle 90^\circ \quad [S]$$

Admittance of a *RL* series branch: $$\boldsymbol{Y}_{RL} = \frac{1}{\boldsymbol{Z}_{RL}} = \frac{1}{R\angle 0^\circ + X_L\angle 90^\circ} = \frac{1}{R + jX_L} \quad [S]$$

Admittance of a *RC* series branch: $$\boldsymbol{Y}_{RC} = \frac{1}{\boldsymbol{Z}_{RC}} = \frac{1}{R\angle 0^\circ + X_C\angle -90^\circ} = \frac{1}{R - jX_C} \quad [S]$$

Admittance of a *RLC* series branch:

$$\boldsymbol{Y}_{RLC} = \frac{1}{\boldsymbol{Z}_{RLC}} = \frac{1}{R\angle 0^\circ + X_L\angle 90^\circ + X_C\angle -90^\circ} = \frac{1}{R + jX_L - jX_C} \quad [S]$$

The total impedance $\mathbf{Z}_T$ of the circuit is then $1/\mathbf{Y}_T$; that is, for the network in Fig. 15.58:

$$\boxed{\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \cdots + \mathbf{Y}_N}$$

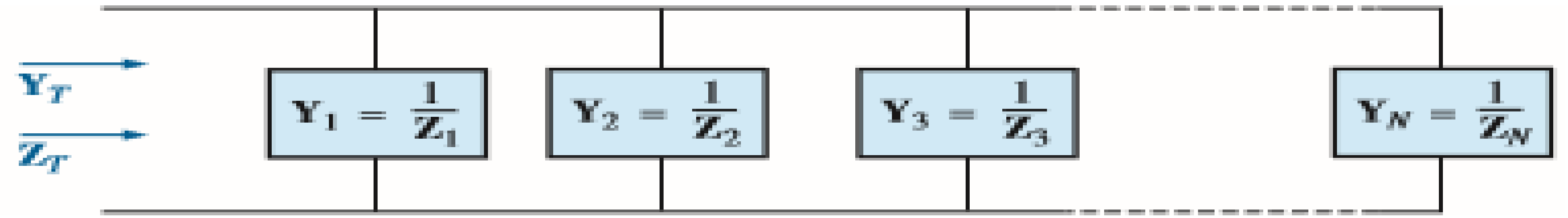


FIG. 15.58
Parallel ac network.

or, since $\mathbf{Z} = 1/\mathbf{Y}$,

$$\boxed{\frac{1}{\mathbf{Z}_T} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \cdots + \frac{1}{\mathbf{Z}_N}}$$

and

$$\boxed{\mathbf{Z}_T = \frac{1}{\dfrac{1}{\mathbf{Z}_1} + \dfrac{1}{\mathbf{Z}_2} + \dfrac{1}{\mathbf{Z}_3} + \cdots + \dfrac{1}{\mathbf{Z}_N}}}$$

For any configuration (series, parallel, series-parallel, and so on), the angle associated with the total admittance is the angle by which the source current leads the applied voltage. For inductive networks, θ_T is negative, whereas for capacitive networks, θ_T is positive.

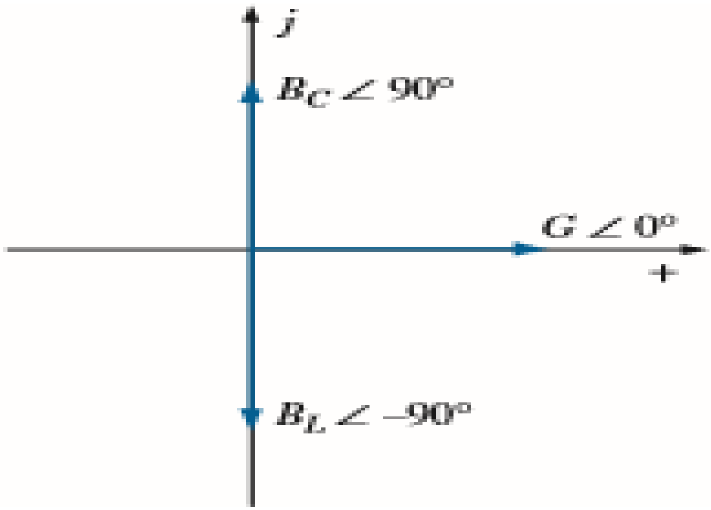


FIG. 15.59
Admittance diagram.

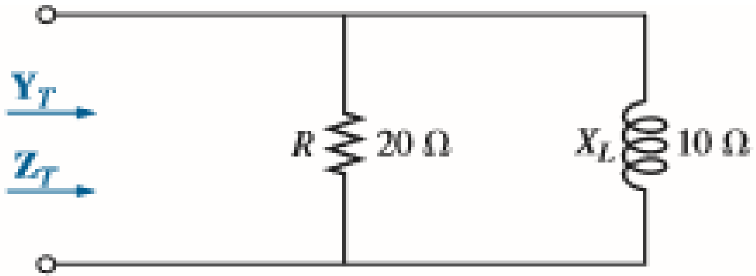


FIG. 15.60
Example 15.12.

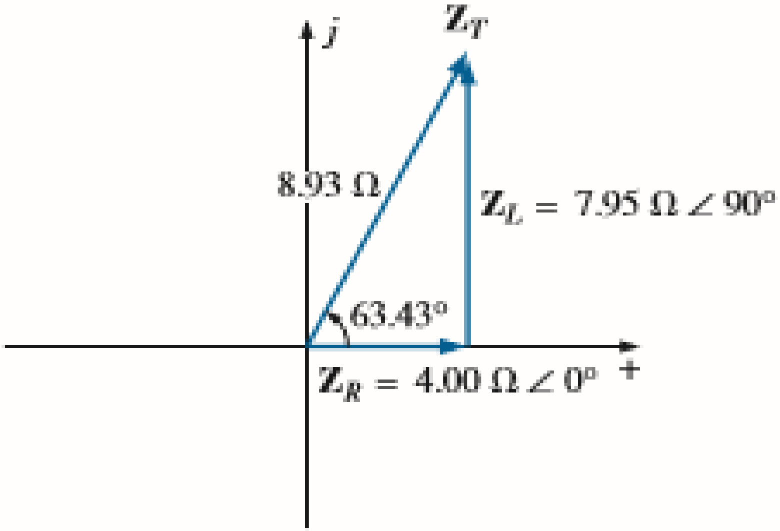


FIG. 15.61
Impedance diagram for the network in Fig. 15.60.

EXAMPLE 15.13 For the network in Fig. 15.60:

a. Calculate the input impedance.
b. Draw the impedance diagram.
c. Find the admittance of each parallel branch.
d. Determine the input admittance and draw the admittance diagram.

Solutions:

a. $$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(20\ \Omega \angle 0°)(10\ \Omega \angle 90°)}{20\ \Omega + j\,10\ \Omega}$$
$$= \frac{200\ \Omega \angle 90°}{22.361 \angle 26.57°} = \mathbf{8.93\ \Omega \angle 63.43°}$$
$$= \mathbf{4.00\ \Omega + j\,7.95\ \Omega} = R_T + j\,X_{L_T}$$

b. The impedance diagram appears in Fig. 15.61.

c. $$\mathbf{Y}_R = G\angle 0° = \frac{1}{R}\angle 0° = \frac{1}{20\ \Omega}\angle 0° = \mathbf{0.05\ S \angle 0°}$$
$$= \mathbf{0.05\ S + j\,0}$$
$$\mathbf{Y}_L = B_L\angle -90° = \frac{1}{X_L}\angle -90° = \frac{1}{10\ \Omega}\angle -90°$$
$$= \mathbf{0.1\ S \angle -90° = 0 - j\,0.1\ S}$$

d. $$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L = (0.05\ \mathrm{S} + j\,0) + (0 - j\,0.1\ \mathrm{S})$$
$$= \mathbf{0.05\ S - j\,0.1\ S} = G - jB_L$$

The admittance diagram appears in Fig. 15.62.

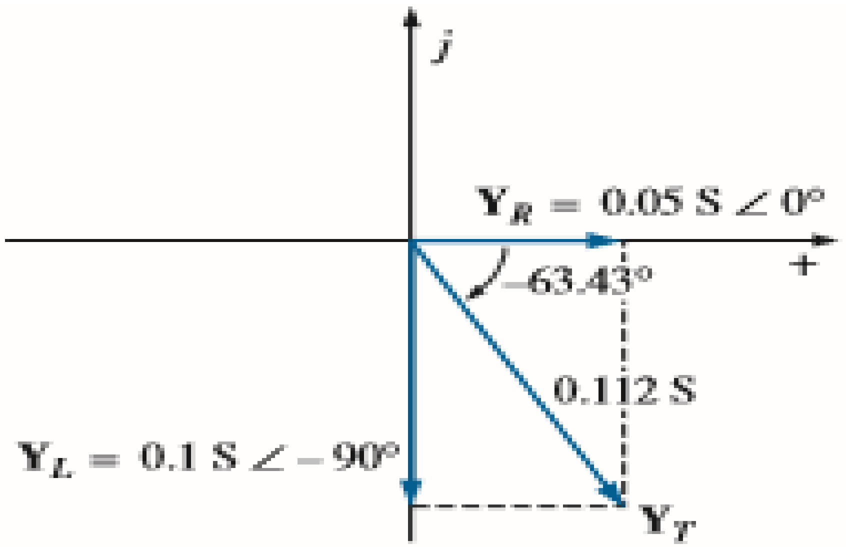


FIG. 15.62

Admittance diagram for the network in Fig. 15.60.

FIG. 15.62
Admittance diagram for the network in Fig. 15.60.

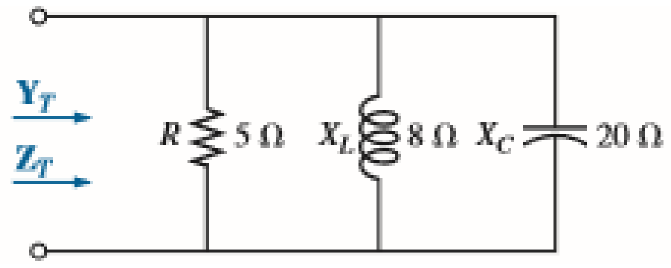


FIG. 15.63
Example 15.14.

EXAMPLE 15.14 Repeat Example 15.13 for the parallel network in Fig. 15.63.

Solutions:

a.
$$\mathbf{Z}_T = \frac{1}{\dfrac{1}{\mathbf{Z}_R} + \dfrac{1}{\mathbf{Z}_L} + \dfrac{1}{\mathbf{Z}_C}}$$
$$= \frac{1}{\dfrac{1}{5\ \Omega \angle 0^\circ} + \dfrac{1}{8\ \Omega \angle 90^\circ} + \dfrac{1}{20\ \Omega \angle -90^\circ}}$$
$$= \frac{1}{0.2\ \text{S} \angle 0^\circ + 0.125\ \text{S} \angle -90^\circ + 0.05\ \text{S} \angle 90^\circ}$$
$$= \frac{1}{0.2\ \text{S} - j\,0.075\ \text{S}} = \frac{1}{0.2136\ \text{S} \angle -20.56^\circ}$$
$$= \mathbf{4.68\ \Omega \angle 20.56^\circ}$$

or

$$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L\mathbf{Z}_C}{\mathbf{Z}_R\mathbf{Z}_L + \mathbf{Z}_L\mathbf{Z}_C + \mathbf{Z}_R\mathbf{Z}_C}$$
$$= \frac{(5\ \Omega \angle 0^\circ)(8\ \Omega \angle 90^\circ)(20\ \Omega \angle -90^\circ)}{(5\ \Omega \angle 0^\circ)(8\ \Omega \angle 90^\circ) + (8\ \Omega \angle 90^\circ)(20\ \Omega \angle -90^\circ) + (5\ \Omega \angle 0^\circ)(20\ \Omega \angle -90^\circ)}$$
$$= \frac{800\ \Omega \angle 0^\circ}{40 \angle 90^\circ + 160 \angle 0^\circ + 100 \angle -90^\circ}$$

$$= \frac{800\ \Omega}{160 + j\,40 - j\,100} = \frac{800\ \Omega}{160 - j\,60}$$

$$= \frac{800\ \Omega}{170.88\ \angle -20.56^\circ}$$

$$= \mathbf{4.68\ \Omega\ \angle 20.56^\circ = 4.38\ \Omega + \,j\,1.64\ \Omega}$$

b. The impedance diagram appears in Fig. 15.64.

c. $$\mathbf{Y}_R = G\angle 0^\circ = \frac{1}{R}\angle 0^\circ = \frac{1}{5\ \Omega}\angle 0^\circ$$

$$= \mathbf{0.2\ S\ \angle 0^\circ = 0.2\ S + \,j\,0}$$

$$\mathbf{Y}_L = B_L\angle -90^\circ = \frac{1}{X_L}\angle -90^\circ = \frac{1}{8\ \Omega}\angle -90^\circ$$

$$= \mathbf{0.125\ S\ \angle -90^\circ = 0 - \,j\,0.125\ S}$$

$$\mathbf{Y}_C = B_C\angle 90^\circ = \frac{1}{X_C}\angle 90^\circ = \frac{1}{20\ \Omega}\angle 90^\circ$$

$$= \mathbf{0.050\ S\ \angle +90^\circ = 0 + \,j\,0.050\ S}$$

d. $$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C$$

$$= (0.2\ S + j\,0) + (0 - j\,0.125\ S) + (0 + j\,0.050\ S)$$

$$= 0.2\ S - j\,0.075\ S = \mathbf{0.214\ S\ \angle -20.56^\circ}$$

The admittance diagram appears in Fig. 15.65.

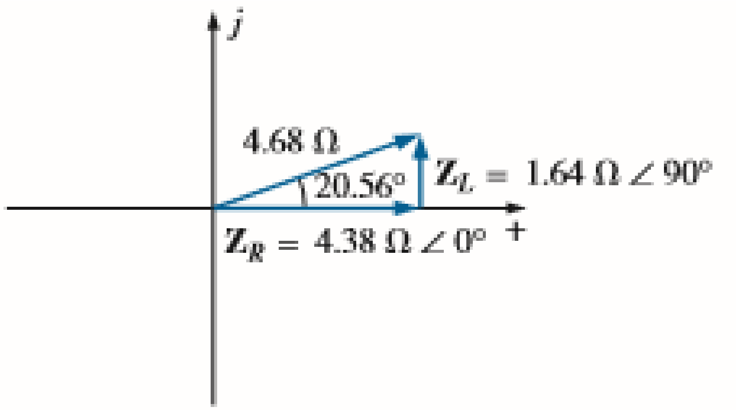


FIG. 15.64
Impedance diagram for the network in Fig. 15.63.

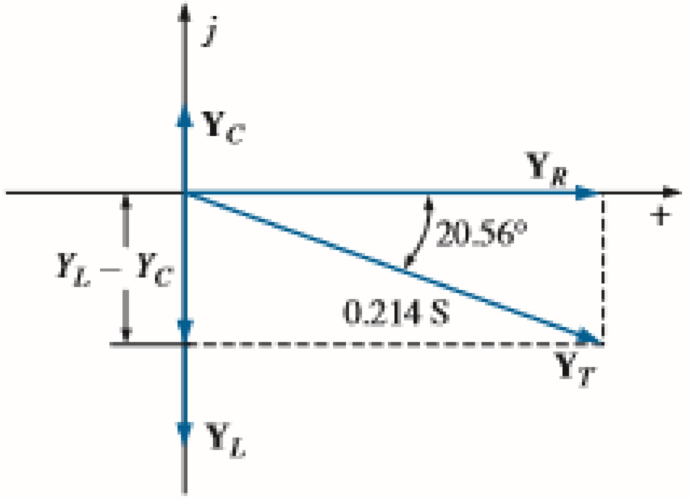
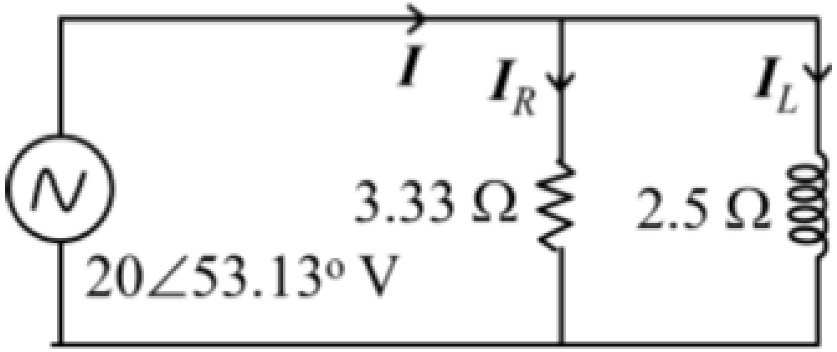


FIG. 15.65
Admittance diagram for the network in Fig. 15.63.

Example

Calculate the total admittance and current for the following circuit.

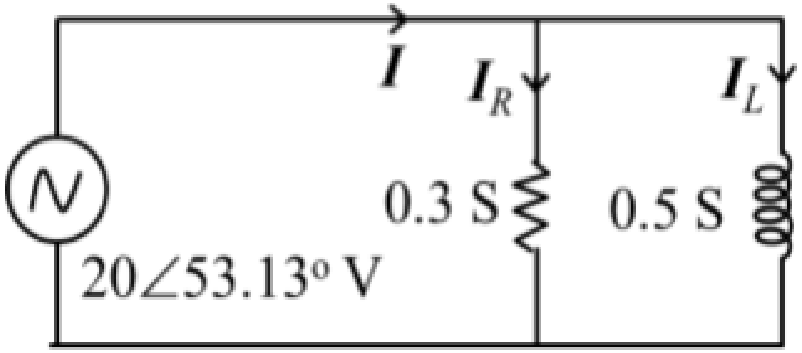


$$G = \frac{1}{3.33} = 0.3 \ [S] \qquad B_L = \frac{1}{2.5} = 0.4 \ [S]$$

$$\boldsymbol{Y}_R = 0.3\angle 0° = 0.3 \, [S] \qquad \boldsymbol{Y}_L = 0.4\angle -90° = -j0.4 \ [S]$$

$$\boldsymbol{Y} = 0.3\angle 0° + 0.4\angle -90° = 0.3 - j0.4 = 0.5\angle -53.13° \ [S]$$

$$\boldsymbol{Z} = \frac{1}{\boldsymbol{Y}} = \frac{1}{0.5\angle -53.13°} = 2\angle 53.13° \ [\Omega]$$

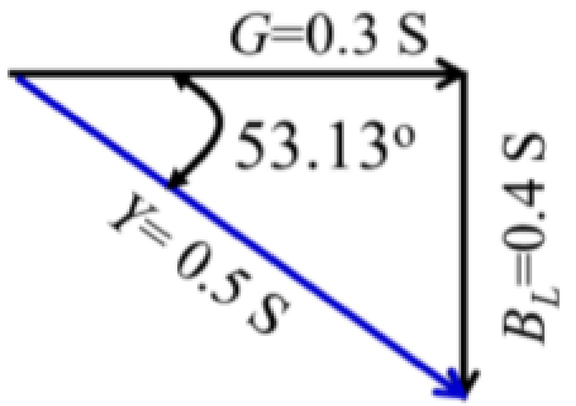


$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}} = \boldsymbol{YV} = (0.5\angle -53.13°)(20\angle 53.13°) = 10\angle 0° \ [\text{A}]$$

$$\boldsymbol{I}_R = \frac{\boldsymbol{V}}{\boldsymbol{Z}_R} = \boldsymbol{Y}_R\boldsymbol{V} = (0.3\angle 0°)(20\angle 53.13°) = 6\angle 53.13° \ [\text{A}]$$

$$\boldsymbol{I}_L = \frac{\boldsymbol{V}}{\boldsymbol{Z}_L} = \boldsymbol{Y}_L\boldsymbol{V} = (0.4\angle -90°)(20\angle 53.13°) = 8\angle -36.87° [\text{A}]$$

$$\theta = \theta_v - \theta_i = 53.13° - 0° = 53.13°$$
$$pf = \cos(53.13°) = 0.6 \ \text{lagging}$$
$$rf = \sin(53.13°) = 0.8$$



Admittance Diagram

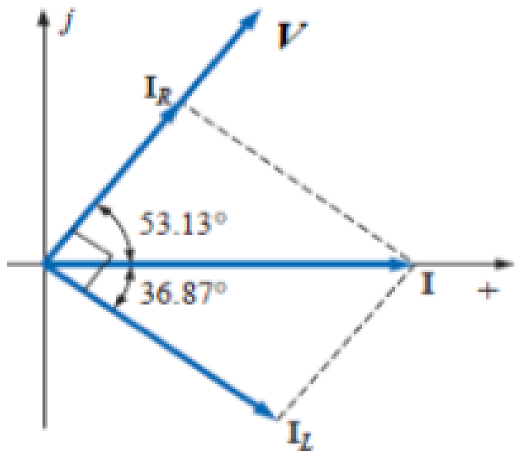
$$S = VI = 20 \times 10 = 200 \ \text{VA}$$

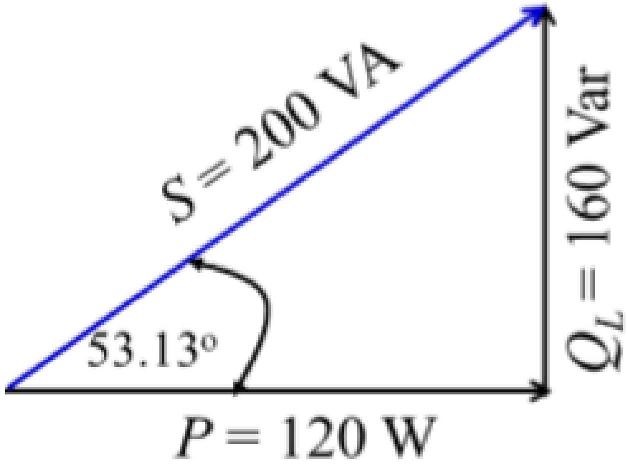
$$P = VI\cos\theta = 20 \times 10 \times 0.6 = 120 \ \text{W}$$

$$Q = VI\sin\theta = 20 \times 10 \times 0.8 = 160 \ \text{VAR}$$

$$p(t) = 120(1 - \cos 2\omega t) + 160\sin 2\omega t$$

$$Q = I_L^2 X_L = 8^2 \times 2.5 = 160 \ \text{VAR}$$

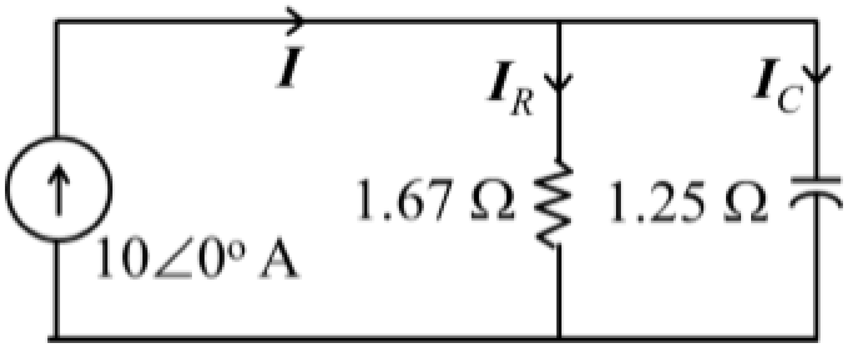




Power Triangle

Example

Calculate the total admittance and current for the following circuit.

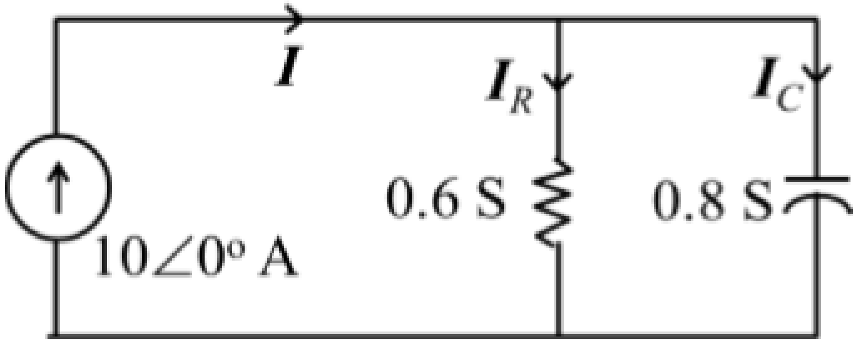


$$G = \frac{1}{1.67} = 0.6 \ [S] \qquad B_C = \frac{1}{1.25} = 0.8 \ [S]$$

$$\boldsymbol{Y}_R = 0.6\angle 0^\circ = 0.6 \, [S] \qquad \boldsymbol{Y}_C = 0.8\angle 90^\circ = j0.8 \ [S]$$

$$\boldsymbol{Y} = 0.6\angle 0^\circ + 0.8\angle 90^\circ = 0.6 + j0.8 = 1.0\angle 53.13^\circ \ [S]$$

$$\boldsymbol{Z} = \frac{1}{\boldsymbol{Y}} = \frac{1}{1\angle 53.13^\circ} = 1\angle -53.13^\circ \ [\Omega]$$



$$\boldsymbol{V} = \boldsymbol{IZ} = \frac{\boldsymbol{I}}{\boldsymbol{Y}} = \frac{10\angle 0^\circ}{1\angle 53.13^\circ} = 10\angle -53.13^\circ \ [\mathrm{V}]$$

$$\boldsymbol{I}_R = \boldsymbol{Y}_R \boldsymbol{V} = (0.6\angle 0^\circ)(10\angle -53.13^\circ) = 6\angle -53.13^\circ \ [\mathrm{A}]$$

$$\boldsymbol{I}_C = \boldsymbol{Y}_C \boldsymbol{V} = (0.8\angle 90^\circ)(10\angle -53.13^\circ) = 8\angle 36.87^\circ [\mathrm{A}]$$

$$\theta = \theta_v - \theta_i = -53.13^\circ - 0^\circ = -53.13^\circ$$

$$pf = \cos(-53.13^\circ) = 0.6 \text{ leading}$$

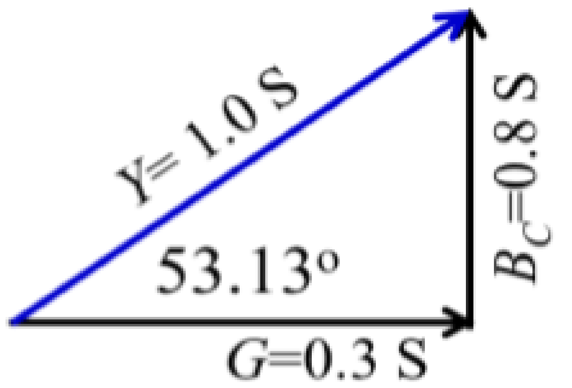
$$rf = \sin(-53.13^\circ) = -0.8$$

$$S = VI = 10 \times 10 = 100 \text{ VA}$$

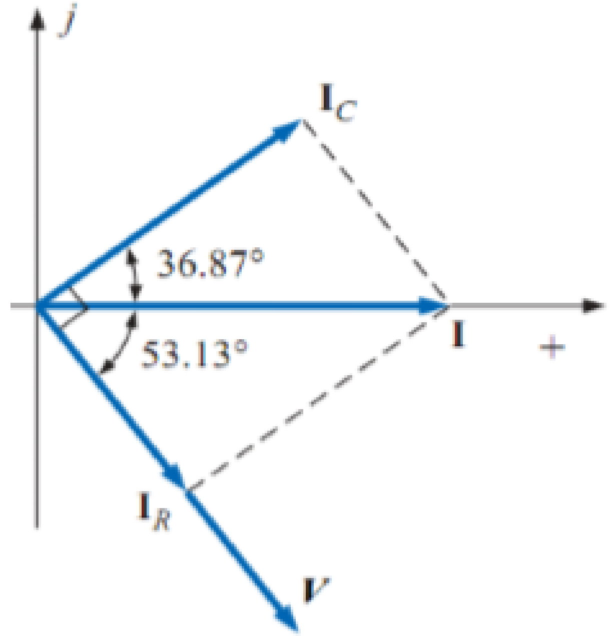
$$P = VI\cos\theta = 10 \times 10 \times 0.6 = 60 \text{ W}$$

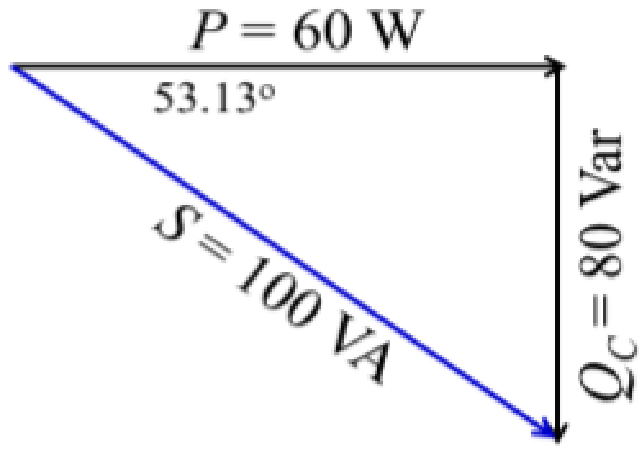
$$Q = VI\sin\theta = 10 \times 10 \times -0.8 = -80 \text{ VAR}$$

$$Q = I_C^2 X_C = 8^2 \times 1.25 = 80 \text{ VAR}$$



Admittance Diagram





Power Triangle

15.9 CURRENT DIVIDER RULE

The basic format for the **current divider rule** in ac circuits is exactly the same as that for dc circuits; that is, for two parallel branches with impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$ as shown in Fig. 15.82,

$$\boxed{\mathbf{I}_1 = \frac{\mathbf{Z}_2\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2} \quad \text{or} \quad \mathbf{I}_2 = \frac{\mathbf{Z}_1\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2}} \tag{15.35}$$

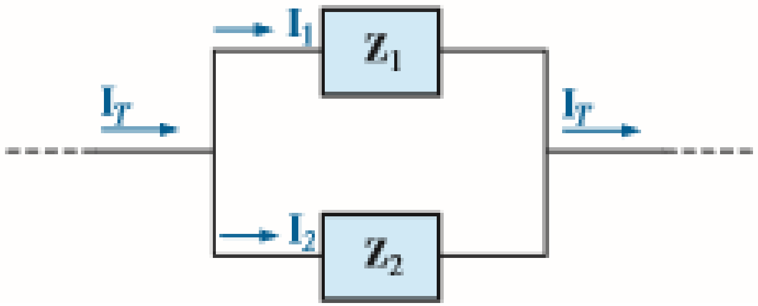


FIG. 15.82
Applying the current divider rule.

EXAMPLE 15.16 Using the current divider rule, find the current through each impedance in Fig. 15.83.

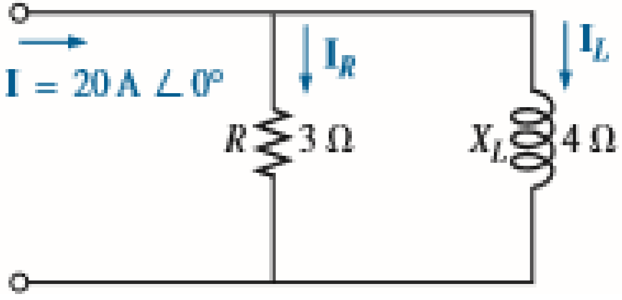


FIG. 15.83
Example 15.16.

Solution:

$$\mathbf{I}_R = \frac{\mathbf{Z}_L\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(4\ \Omega \angle 90°)(20\text{ A} \angle 0°)}{3\ \Omega \angle 0° + 4\ \Omega \angle 90°} = \frac{80\text{ A} \angle 90°}{5 \angle 53.13°}$$

$$= \mathbf{16\ A \angle 36.87°}$$

$$\mathbf{I}_L = \frac{\mathbf{Z}_R\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(3\ \Omega \angle 0°)(20\text{ A} \angle 0°)}{5\ \Omega \angle 53.13°} = \frac{60\text{ A} \angle 0°}{5 \angle 53.13°}$$

$$= \mathbf{12\ A \angle -53.13°}$$

EXAMPLE 15.17 Using the current divider rule, find the current through each parallel branch in Fig. 15.84.

Solution:

$$\mathbf{I}_{R\text{-}L} = \frac{\mathbf{Z}_C\mathbf{I}_T}{\mathbf{Z}_C + \mathbf{Z}_{R\text{-}L}} = \frac{(2\ \Omega\ \angle -90^\circ)(5\ \text{A}\ \angle 30^\circ)}{-j\,2\ \Omega + 1\ \Omega + j\,8\ \Omega} = \frac{10\ \text{A}\ \angle -60^\circ}{1 + j6}$$

$$= \frac{10\ \text{A}\ \angle -60^\circ}{6.083\ \angle 80.54^\circ} \cong \mathbf{1.64\ A\ \angle -140.54^\circ}$$

$$\mathbf{I}_C = \frac{\mathbf{Z}_{R\text{-}L}\mathbf{I}_T}{\mathbf{Z}_{R\text{-}L} + \mathbf{Z}_C} = \frac{(1\ \Omega + j\,8\ \Omega)(5\ \text{A}\ \angle 30^\circ)}{6.08\ \Omega\ \angle 80.54^\circ}$$

$$= \frac{(8.06\ \angle 82.87^\circ)(5\ \text{A}\ \angle 30^\circ)}{6.08\ \angle 80.54^\circ} = \frac{40.30\ \text{A}\ \angle 112.87^\circ}{6.083\ \angle 80.54^\circ}$$

$$= \mathbf{6.63\ A\ \angle 32.33^\circ}$$

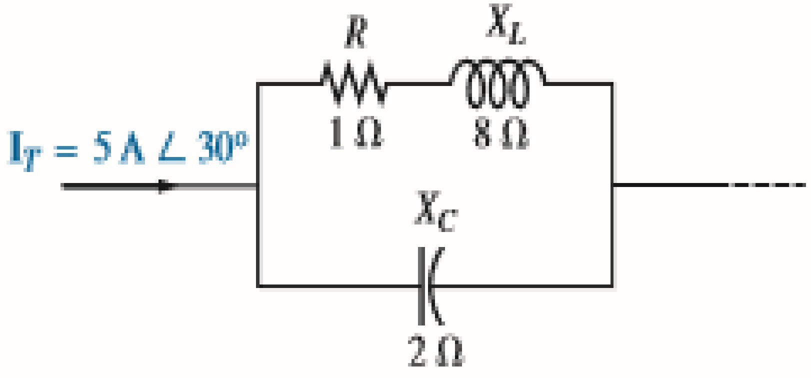


FIG. 15.84
Example 15.17.

Thank You