Introduction to Electrical Circuits

Sec: A, E

Finalterm
Week: 8

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Book

Introductory Circuit Analysis

Robert L. Boylestad
Eleventh Edition

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Superposition Theorem

The current through, or voltage across, any element of a network is equal to the algebraic sum of the currents or voltages produced independently by each source.

EXAMPLE 18.1 Using the superposition theorem, find the current **I** through the 4 Ω reactance (X_{L_2}) in Fig. 18.1.

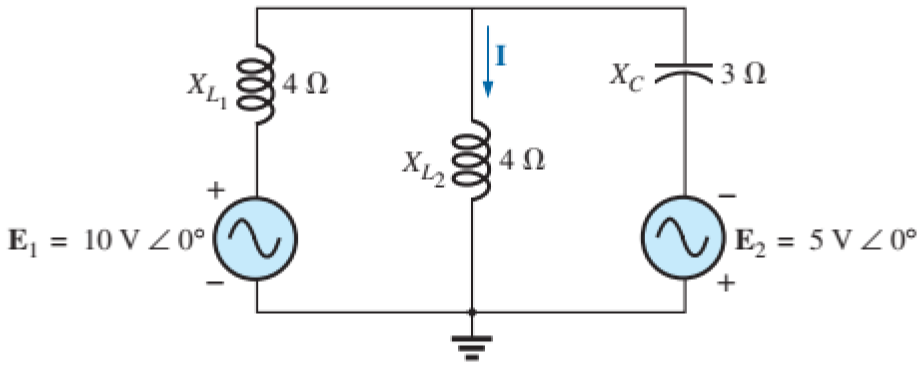
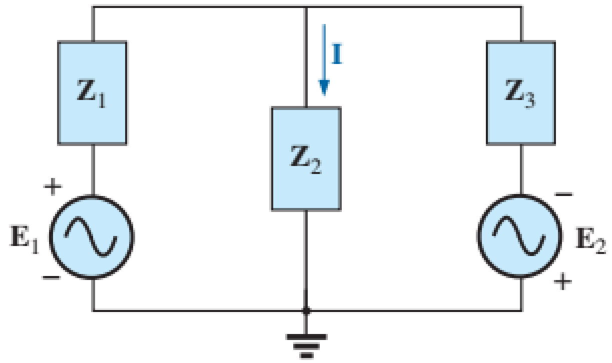


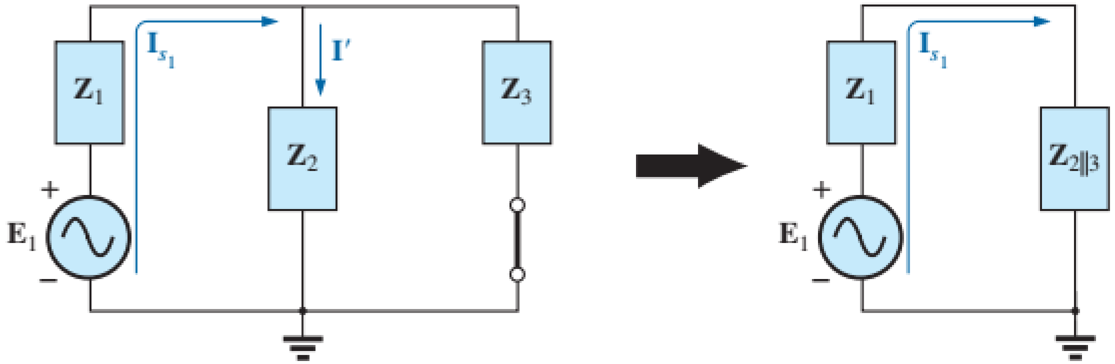
FIG. 18.1
Example 18.1.

Solution: For the redrawn circuit (Fig. 18.2),

$$\mathbf{Z}_1 = +j\,X_{L_1} = j\,4\,\Omega$$
$$\mathbf{Z}_2 = +j\,X_{L_2} = j\,4\,\Omega$$
$$\mathbf{Z}_3 = -j\,X_C = -j\,3\,\Omega$$



Considering the effects of the voltage source $\mathbf{E}_1$ (Fig. 18.3), we have

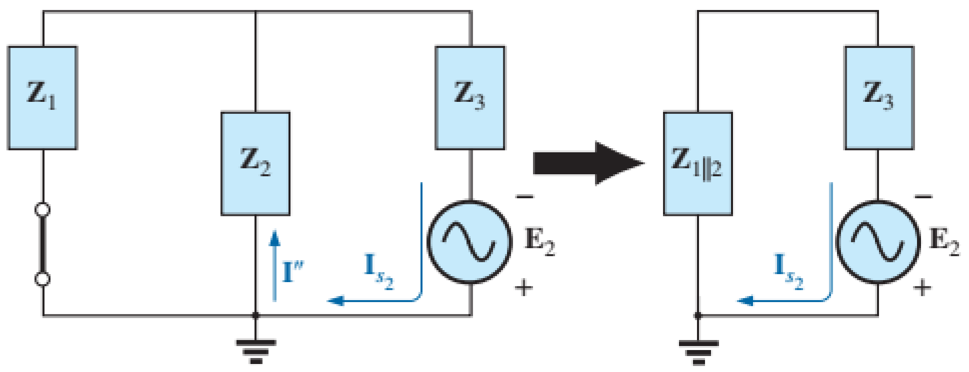


$$\mathbf{Z}_{2\|3} = \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(j\,4\,\Omega)(-j\,3\,\Omega)}{j\,4\,\Omega - j\,3\,\Omega} = \frac{12\,\Omega}{j} = -j\,12\,\Omega$$
$$= 12\,\Omega \angle -90^\circ$$

$$\mathbf{I}_{s_1} = \frac{\mathbf{E}_1}{\mathbf{Z}_{2\|3} + \mathbf{Z}_1} = \frac{10\text{ V} \angle 0^\circ}{-j\,12\,\Omega + j\,4\,\Omega} = \frac{10\text{ V} \angle 0^\circ}{8\,\Omega \angle -90^\circ}$$
$$= 1.25\text{ A} \angle 90^\circ$$

$$\mathbf{I}' = \frac{\mathbf{Z}_3\mathbf{I}_{s_1}}{\mathbf{Z}_2 + \mathbf{Z}_3} \qquad \text{(current divider rule)}$$
$$= \frac{(-j\,3\,\Omega)(j\,1.25\text{ A})}{j\,4\,\Omega - j\,3\,\Omega} = \frac{3.75\text{ A}}{j\,1} = 3.75\text{ A} \angle -90^\circ$$

Considering the effects of the voltage source $\mathbf{E}_2$ (Fig. 18.4), we have



$$\mathbf{Z}_{1\|2} = \frac{\mathbf{Z}_1}{N} = \frac{j\,4\ \Omega}{2} = j\,2\ \Omega$$

$$\mathbf{I}_{s_2} = \frac{\mathbf{E}_2}{\mathbf{Z}_{1\|2} + \mathbf{Z}_3} = \frac{5\ \text{V}\ \angle 0^\circ}{j\,2\ \Omega - j\,3\ \Omega} = \frac{5\ \text{V}\ \angle 0^\circ}{1\ \Omega\ \angle -90^\circ} = 5\ \text{A}\ \angle 90^\circ$$

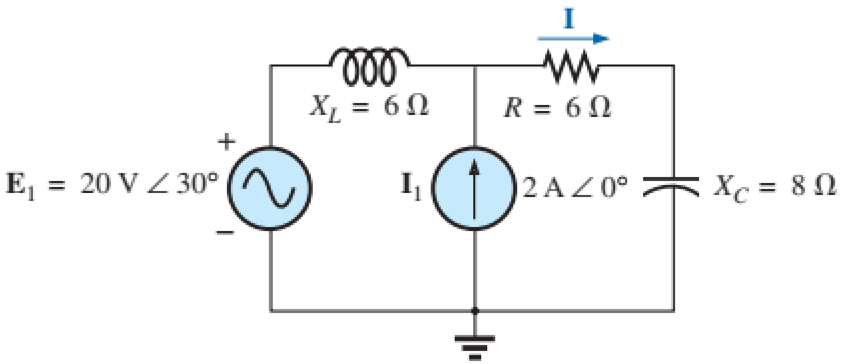
and

$$\mathbf{I}'' = \frac{\mathbf{I}_{s_2}}{2} = 2.5\ \text{A}\ \angle 90^\circ$$

The resultant current through the 4 Ω reactance X_{L_2} (Fig. 18.5) is

$$\begin{aligned}
\mathbf{I} &= \mathbf{I}' - \mathbf{I}'' \\
&= 3.75\ \text{A}\ \angle -90^\circ - 2.50\ \text{A}\ \angle 90^\circ = -j\,3.75\ \text{A} - j\,2.50\ \text{A} \\
&= -j\,6.25\ \text{A} \\
\mathbf{I} &= \mathbf{6.25\ A\ \angle -90^\circ}
\end{aligned}$$

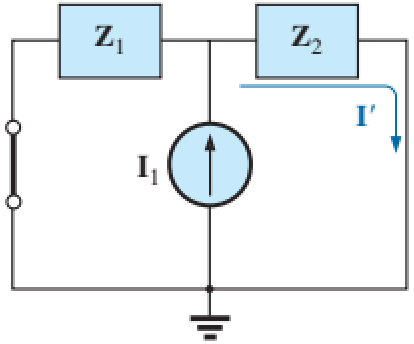
EXAMPLE 18.2 Using superposition, find the current **I** through the 6 Ω resistor in Fig. 18.6.



Solution: For the redrawn circuit (Fig. 18.7),

$$\mathbf{Z}_1 = j\,6\ \Omega \quad \mathbf{Z}_2 = 6\ \Omega - j\,8\ \Omega$$

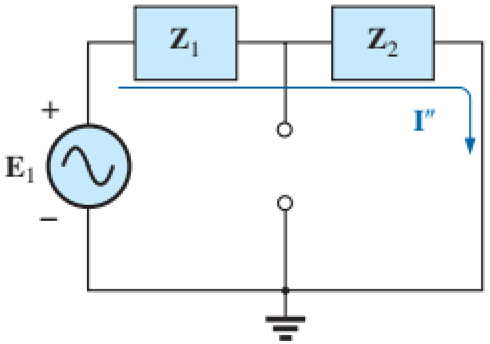
Consider the effects of the current source (Fig. 18.8). Applying the current divider rule, we have



$$\mathbf{I}' = \frac{\mathbf{Z}_1\mathbf{I}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j\,6\ \Omega)(2\ \text{A})}{j\,6\ \Omega + 6\ \Omega - j\,8\ \Omega} = \frac{j\,12\ \text{A}}{6 - j\,2}$$

$$= \frac{12\ \text{A}\ \angle 90°}{6.32\ \angle -18.43°}$$

$$\mathbf{I}' = 1.9\ \text{A}\ \angle 108.43°$$



Consider the effects of the voltage source (Fig. 18.9). Applying Ohm's law gives us

$$\mathbf{I}'' = \frac{\mathbf{E}_1}{\mathbf{Z}_T} = \frac{\mathbf{E}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{20\ \text{V}\ \angle 30°}{6.32\ \Omega\ \angle -18.43°}$$

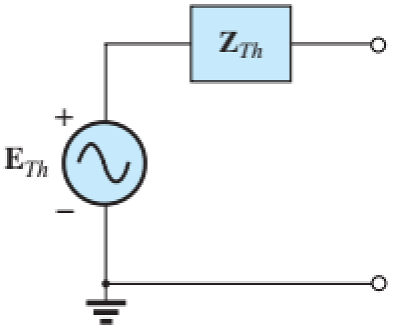
$$= 3.16\ \text{A}\ \angle 48.43°$$

The total current through the 6 Ω resistor (Fig. 18.10) is

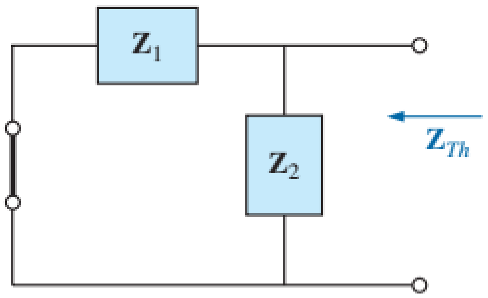
$$\begin{aligned}
\mathbf{I} &= \mathbf{I}' + \mathbf{I}'' \\
&= 1.9\ \text{A}\ \angle 108.43° + 3.16\ \text{A}\ \angle 48.43° \\
&= (-0.60\ \text{A} + j\,1.80\ \text{A}) + (2.10\ \text{A} + j\,2.36\ \text{A}) \\
&= 1.50\ \text{A} + j\,4.16\ \text{A} \\
\mathbf{I} &= \mathbf{4.42\ A\ \angle 70.2°}
\end{aligned}$$

Thevenin’s Theorem

Any two-terminal linear ac network can be replaced with an equivalent circuit consisting of a voltage source and an impedance in series.



1. ***Remove that portion of the network across which the Thévenin equivalent circuit is to be found.***
2. ***Mark (○, •, and so on) the terminals of the remaining two-terminal network.***
3. ***Calculate $\mathbf{Z}_{Th}$ by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two marked terminals.***
4. ***Calculate $\mathbf{E}_{Th}$ by first replacing the voltage and current sources and then finding the open-circuit voltage between the marked terminals.***
5. ***Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Thévenin equivalent circuit.***

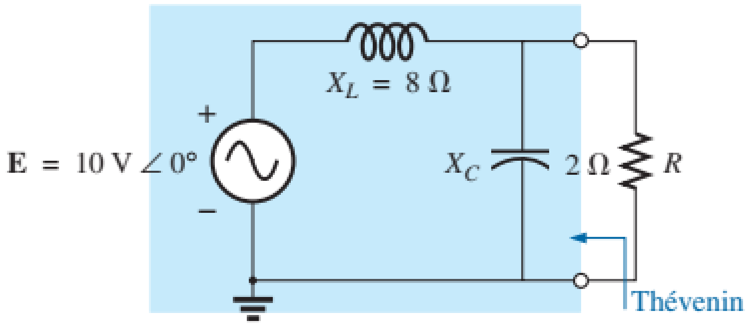


Step 3 (Fig. 18.26):

$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j\,8\,\Omega)(-j\,2\,\Omega)}{j\,8\,\Omega - j\,2\,\Omega} = \frac{-j^2\,16\,\Omega}{j\,6} = \frac{16\,\Omega}{6\,\angle 90^\circ}$$

$$= \mathbf{2.67\,\Omega\,\angle{-90^\circ}}$$

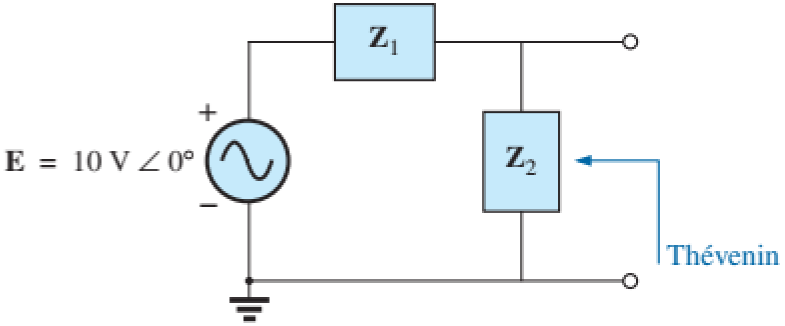
EXAMPLE 18.7 Find the Thévenin equivalent circuit for the network external to resistor R in Fig. 18.24.



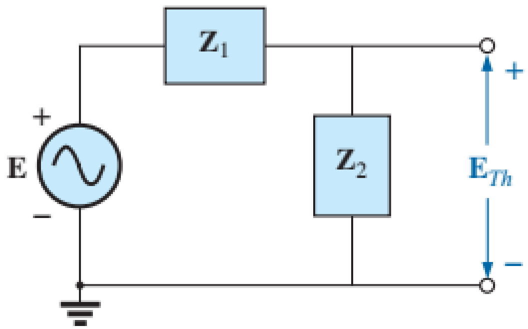
Solution:

Steps 1 and 2 (Fig. 18.25):

$$\mathbf{Z}_1 = j\,X_L = j\,8\,\Omega \quad \mathbf{Z}_2 = -j\,X_C = -j\,2\,\Omega$$



1. ***Remove that portion of the network across which the Thévenin equivalent circuit is to be found.***
2. ***Mark (○, •, and so on) the terminals of the remaining two-terminal network.***
3. ***Calculate $\mathbf{Z}_{Th}$ by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two marked terminals.***
4. ***Calculate $\mathbf{E}_{Th}$ by first replacing the voltage and current sources and then finding the open-circuit voltage between the marked terminals.***
5. ***Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Thévenin equivalent circuit.***

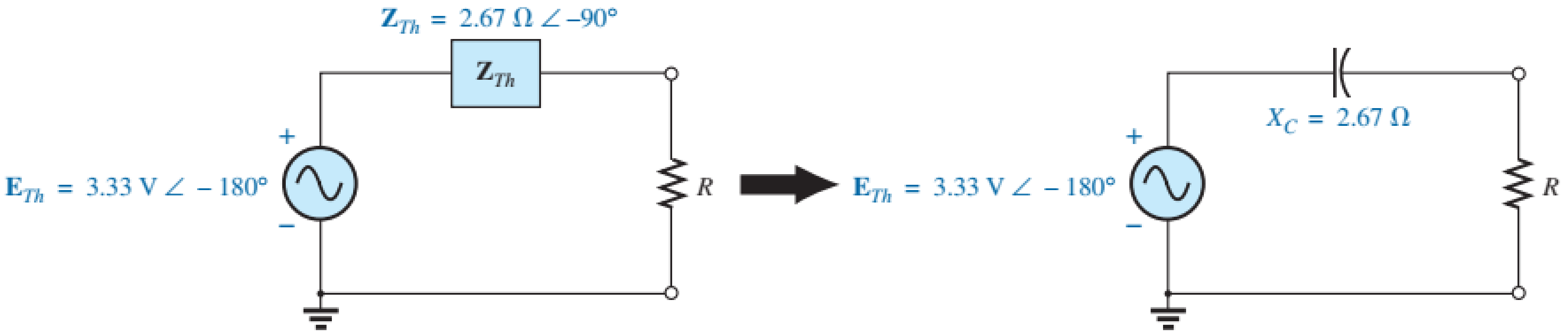


Step 4 (Fig. 18.27):

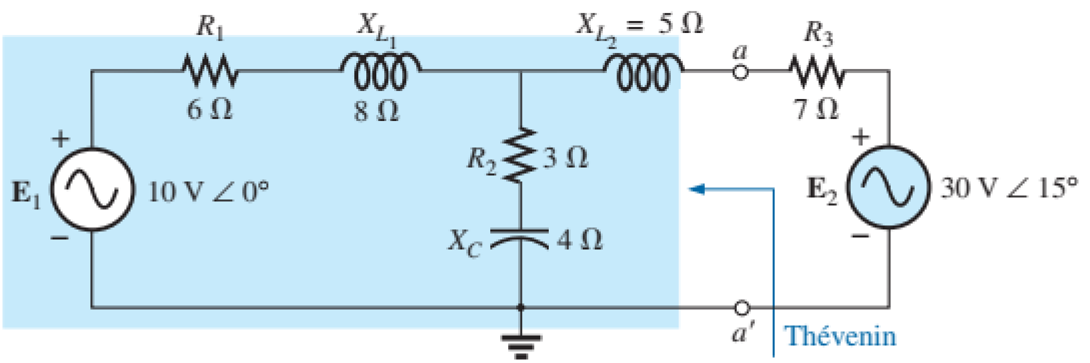
$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} \qquad \text{(voltage divider rule)}$$

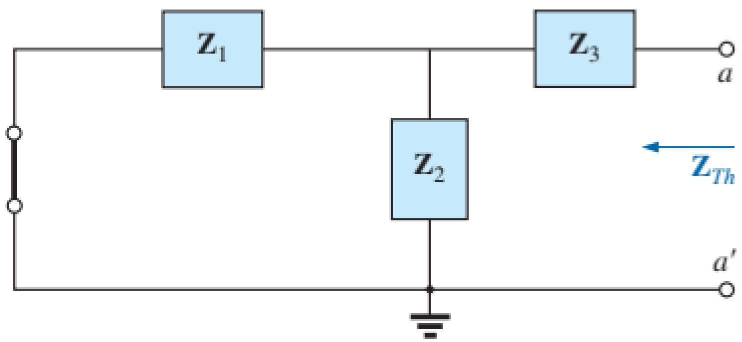
$$= \frac{(-j\,2\,\Omega)(10\text{ V})}{j\,8\,\Omega - j\,2\,\Omega} = \frac{-j\,20\text{ V}}{j\,6} = \mathbf{3.33\text{ V}\angle{-180^\circ}}$$

Step 5: The Thévenin equivalent circuit is shown in Fig. 18.28.



EXAMPLE 18.8 Find the Thévenin equivalent circuit for the network external to branch *a-a′* in Fig. 18.29.

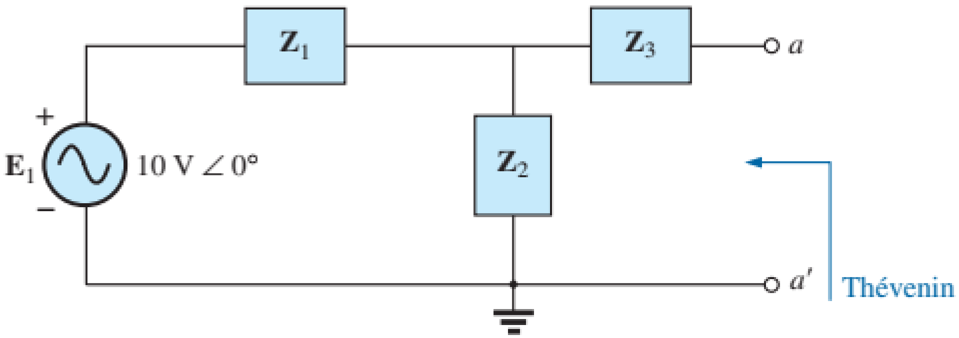




Solution:

Steps 1 and 2 (Fig. 18.30): Note the reduced complexity with subscripted impedances:

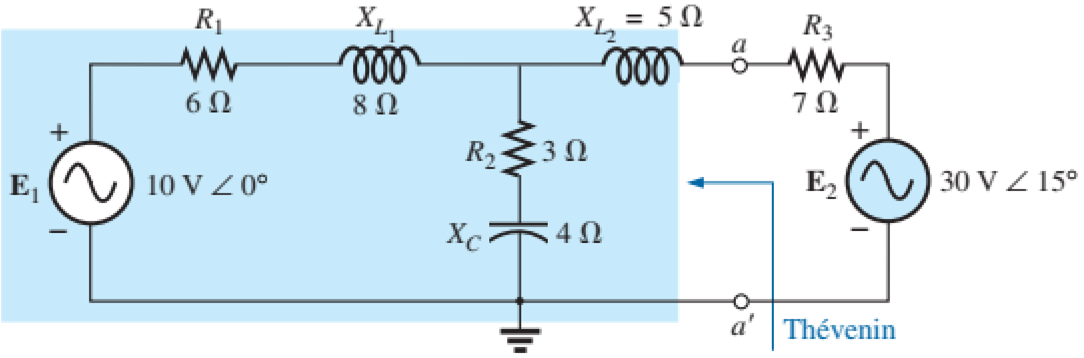
$$\mathbf{Z}_1 = R_1 + jX_{L_1} = 6\ \Omega + j\,8\ \Omega$$
$$\mathbf{Z}_2 = R_2 - jX_C = 3\ \Omega - j\,4\ \Omega$$
$$\mathbf{Z}_3 = +jX_{L_2} = j\,5\ \Omega$$



Step 3 (Fig. 18.31):

$$\mathbf{Z}_{Th} = \mathbf{Z}_3 + \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = j\,5\ \Omega + \frac{(10\ \Omega\ \angle 53.13^\circ)(5\ \Omega\ \angle -53.13^\circ)}{(6\ \Omega + j\,8\ \Omega) + (3\ \Omega - j\,4\ \Omega)}$$
$$= j\,5 + \frac{50\ \angle 0^\circ}{9 + j\,4} = j\,5 + \frac{50\ \angle 0^\circ}{9.85\ \angle 23.96^\circ}$$
$$= j\,5 + 5.08\ \angle -23.96^\circ = j\,5 + 4.64 - j\,2.06$$
$$\mathbf{Z}_{Th} = \mathbf{4.64\ \Omega + j\,2.94\ \Omega = 5.49\ \Omega\ \angle 32.36^\circ}$$

EXAMPLE 18.8 Find the Thévenin equivalent circuit for the network external to branch *a-a′* in Fig. 18.29.

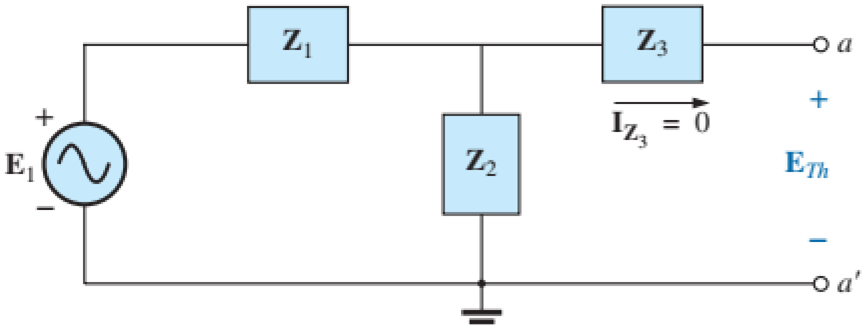


Step 4 (Fig. 18.32): Since *a-a′* is an open circuit, $\mathbf{I}_{Z_3} = 0$. Then $\mathbf{E}_{Th}$ is the voltage drop across $\mathbf{Z}_2$:

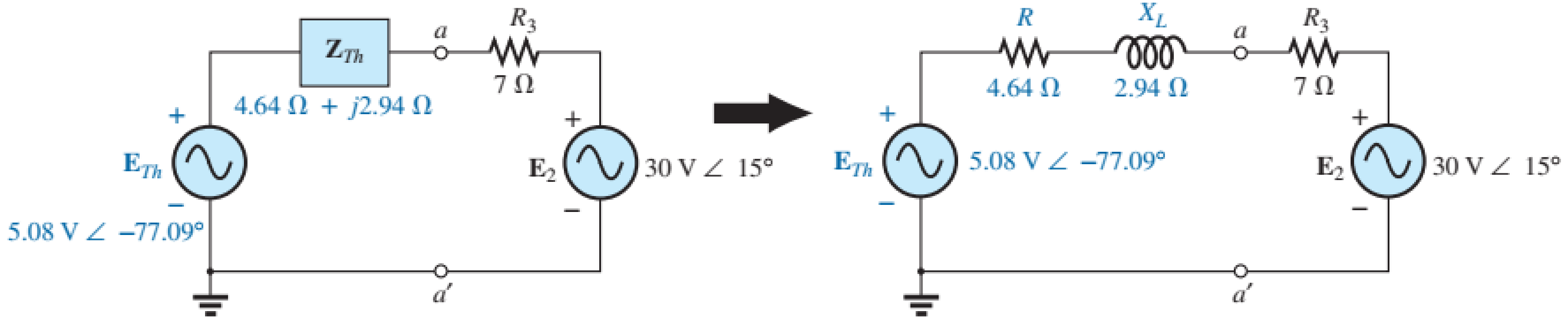
$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_2 + \mathbf{Z}_1} \qquad \text{(voltage divider rule)}$$

$$= \frac{(5\ \Omega \angle -53.13°)(10\ \text{V} \angle 0°)}{9.85\ \Omega \angle 23.96°}$$

$$\mathbf{E}_{Th} = \frac{50\ \text{V} \angle -53.13°}{9.85 \angle 23.96°} = \mathbf{5.08\ V \angle -77.09°}$$



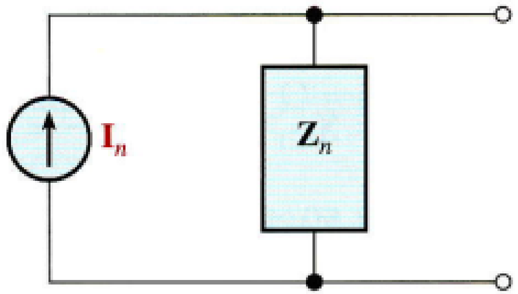
Step 5: The Thévenin equivalent circuit is shown in Fig. 18.33.



Norton’s Theorem

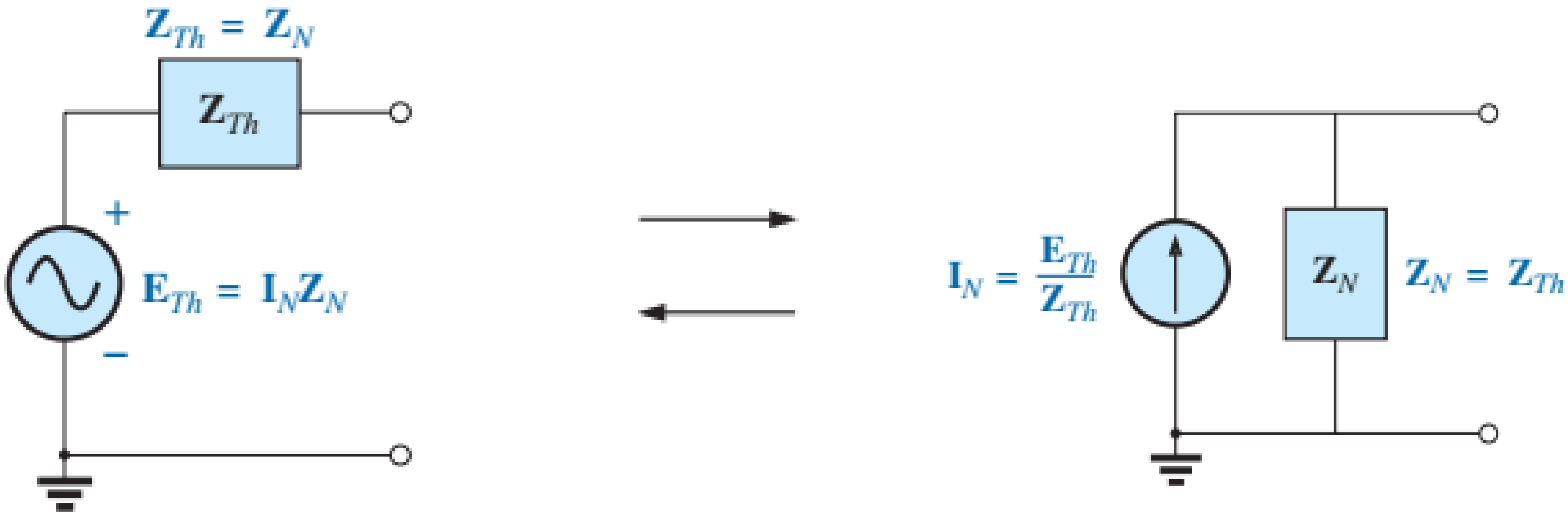
For AC:

Any two-terminal linear bilateral AC network can be replaced by an equivalent circuit consisting of a current source and a parallel impedance.



For DC:

- ***Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor***



$\mathbf{Z}_{Th} = \mathbf{Z}_N$
$\mathbf{Z}_{Th}$
+
$\mathbf{E}_{Th} = \mathbf{I}_N\mathbf{Z}_N$
–
$\mathbf{I}_N = \dfrac{\mathbf{E}_{Th}}{\mathbf{Z}_{Th}}$
$\mathbf{Z}_N$
$\mathbf{Z}_N = \mathbf{Z}_{Th}$

MAXIMUM POWER TRANSFER THEOREM

Maximum power will be delivered to a load when the load impedance is the conjugate of the Thévenin impedance across its terminals.

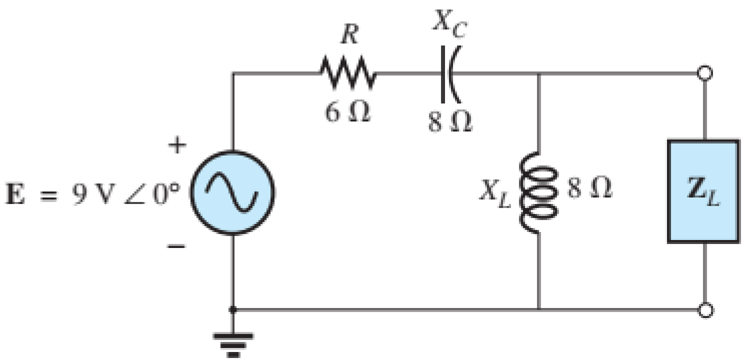
For DC:

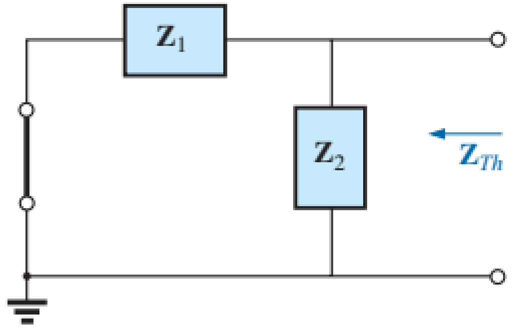
$$R_L = R_{Th}$$

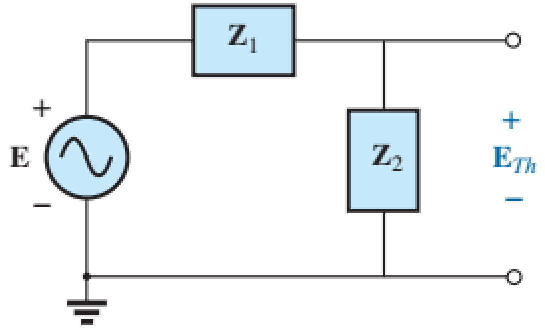
For AC:

$$Z_{Th} = (R \pm jX) \qquad Z_L = (R \mp jX)$$

EXAMPLE 18.19 Find the load impedance in Fig. 18.83 for maximum power to the load, and find the maximum power.







EXAMPLE 18.19 Find the load impedance in Fig. 18.83 for maximum power to the load, and find the maximum power.

Solution: Determine $\mathbf{Z}_{Th}$ [Fig. 18.84(a)]:

$$Z_1 = R - j\,X_C = 6\ \Omega - j\,8\ \Omega = 10\ \Omega\ \angle -53.13^\circ$$
$$Z_2 = +j\,X_L = j\,8\ \Omega$$
$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(10\ \Omega\ \angle -53.13^\circ)(8\ \Omega\ \angle 90^\circ)}{6\ \Omega - j\,8\ \Omega + j\,8\ \Omega} = \frac{80\ \Omega\ \angle 36.87^\circ}{6\ \angle 0^\circ}$$
$$= 13.33\ \Omega\ \angle 36.87^\circ = 10.66\ \Omega + j\,8\ \Omega$$

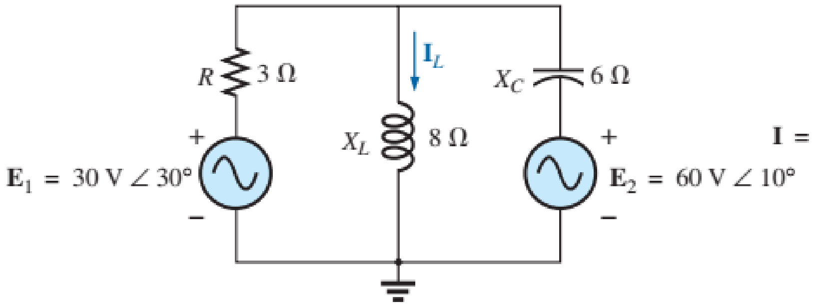
$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_2 + \mathbf{Z}_1} \qquad \text{(voltage divider rule)}$$
$$= \frac{(8\ \Omega\ \angle 90^\circ)(9\ \text{V}\ \angle 0^\circ)}{j\,8\ \Omega + 6\ \Omega - j\,8\ \Omega} = \frac{72\ \text{V}\ \angle 90^\circ}{6\ \angle 0^\circ} = 12\ \text{V}\ \angle 90^\circ$$

$$\mathbf{Z}_L = 13.3\ \Omega\ \angle -36.87^\circ = \mathbf{10.66\ \Omega - j\,8\ \Omega}$$

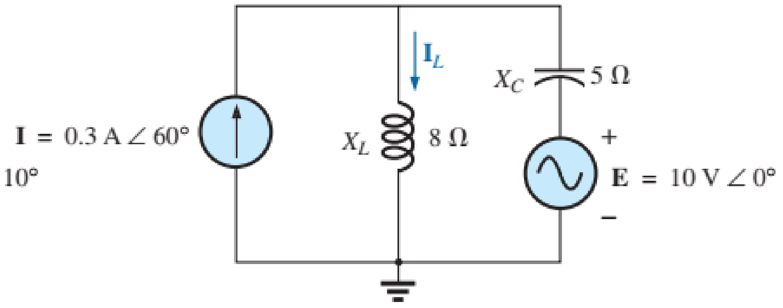
$$P_{\max} = \frac{E_{Th}^2}{4R} = \frac{(12\ \text{V})^2}{4(10.66\ \Omega)} = \frac{144}{42.64} = \mathbf{3.38\ W}$$

Exercises: Superposition Theorem

1. Using superposition, determine the current through the inductance X_L for each network in Fig. 18.107.

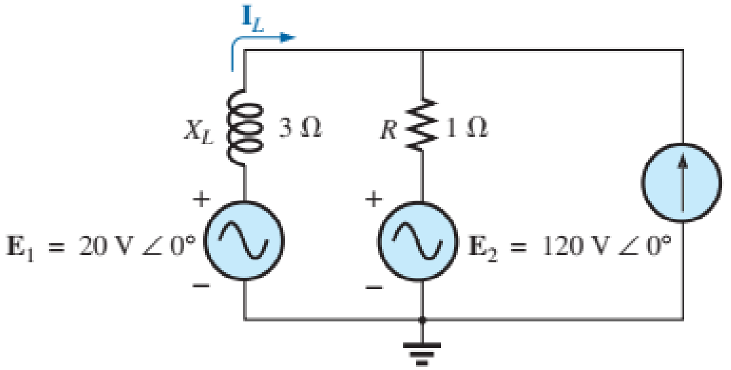


(a)

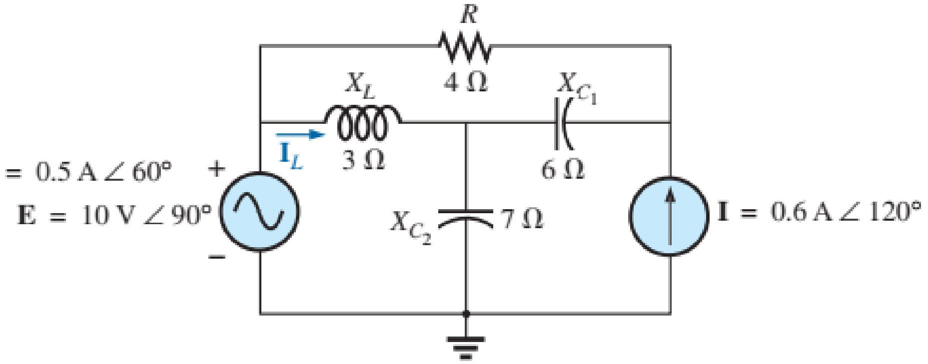


(b)

***2.** Using superposition, determine the current $\mathbf{I}_L$ for each network in Fig. 18.108.



(a)



(b)

Exercises: Thevenin & Norton

12. Find the Thévenin equivalent circuit for the portions of the networks in Fig. 18.118 external to the elements between points *a* and *b*.

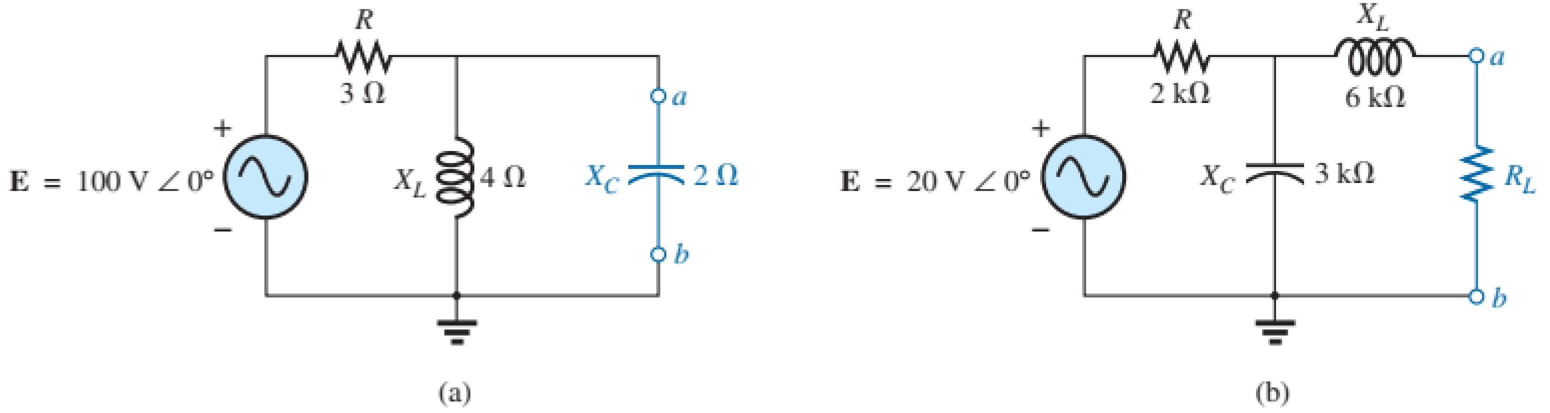
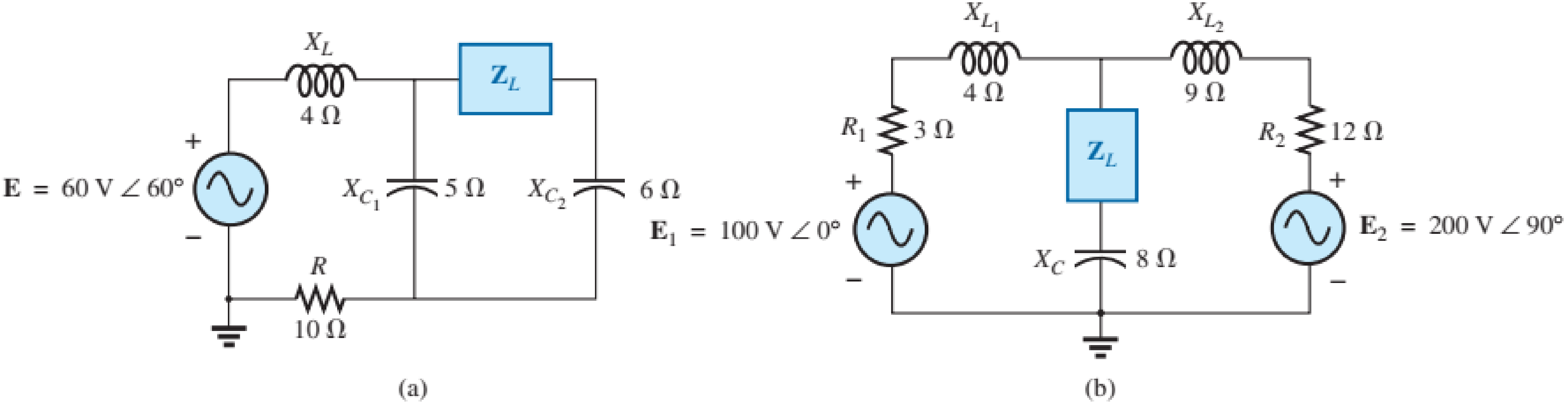


FIG. 18.118
Problems 12 and 26.

26. Find the Norton equivalent circuit for the network external to the elements between *a* and *b* for the networks in Fig. 18.118.

Exercises: Maximum Power Transfer Theorem

***40.** Find the load impedance $\mathbf{Z}_L$ for the networks in Fig. 18.127 for maximum power to the load, and find the maximum power to the load.



Thank You