计算流体力学作业

194801064 郭无极

一

一维平壁非稳态热传导问题，其控制方程为：

$$\frac{\partial T}{\partial t}=\alpha\frac{\partial^2 T}{\partial x^2} \tag{1-1}$$

其中：$\alpha=\frac{\lambda}{\rho c}$为扩散系数,已知平壁初始温度为100℃，厚度$\delta=0.5m$，将其置于左表面绝热、右表面与周围流体对流换热的环境中，环境流体温度$T_\infty=20$℃，对流换热系数$h=25W/m^2K$；平壁材料$\lambda=150W/mK$，$\rho=2500kg/m^3$，$c=800J/kgK$。
平壁边界条件可表示为：

$$x=0,\left.\frac{\partial T}{\partial x}\right|_{x=0}=0 \tag{1-2}$$

$$x=\delta,\lambda\left.\frac{\partial T}{\partial x}\right|_{x=0.5}=h(T|_{x=0.5}-T\infty) \tag{1-3}$$

试选取适当的时间和空间步长，分别采用显式格式和克拉克-尼科尔森格式的差分方程，自编代码，计算导热体内的温度分布，并给出60s，300s，600s，1800s时平壁内的温度分布。

1、显式格式

求解时间选择一阶向前差分，空间选择二阶中心差分，有：

$$\frac{T_i^{n+1}-T_i^n}{\Delta t}=\alpha\frac{T_{i+1}^n-2T_i^n+T_{i-1}^n}{\Delta x^2} \tag{1-4}$$

进而有：

$$T_i^{n+1}=T_i^n+\frac{\alpha\Delta t}{\Delta x^2}(T_{i+1}^n-2T_i^n+T_{i-1}^n) \tag{1-5}$$

由Von Neumann分析方法知$\frac{\alpha\Delta t}{\Delta x^2}\le\frac{1}{2}$是离散方程收敛的CFL。计算得到$\alpha=7.5e^{-5}$故而选取$dx=0.02$，$dt=2$,所得如下$\frac{\alpha\Delta t}{\Delta x^2}=0.375\le\frac{1}{2}$，满足条件。

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
%matplotlib inline

def calculate_alpha(lamb,rho,c):
      alpha = lamb/(rho*c)
      #print(alpha)
      return alpha

length_wall = 0.5       #墙的厚度
lambda_wall = 150       #热传导系数
rho_wall = 2500          #密度
c_wall = 800              #比热容
h = 25                       #对流换热系数
T_far = 20                 #远场温度
t_step = [60,300,600,1800]     #案例求解时间
alpha = calculate_alpha(lambda_wall,rho_wall,c_wall)       #热传导速度
dt = 2        #时间步长
dx = 0.02    #网格尺寸
#pow(1.5*pow(10,-4)*dt,0.5)
#print(dt*alpha/dx**2)
n = int(length_wall/dx)+1     #n为节点数
x = np.linspace(0,0.5,n)       #划分网格
T = np.zeros(n)+100              #初始条件
#T = T.append(20)
T_result_1 =np.ndarray((len(t_step),n))       #设置输出
#print(T_result.shape)

for case in range(len(t_step)):
      T_result_1[case]=T.copy()
      for ti in range(int(t_step[case]/dt)):
            #print(ti)
            T_result_1[case][1:-1] = T_result_1[case][1:-1] + alpha * dt / dx**2 * (T_result_1[case][2:] - 2 * T_result_1[case][1:-1] +
T_result_1[case][:-2])
            T_result_1[case][-1] = (-2*dx*(T_result_1[case][-1]-T_far)*h/lambda_wall+T_result_1[case][-1]+T_far)/2
            T_result_1[case][0] = T_result_1[case][1]
            #print(T_result_1[case])
np.set_printoptions(formatter={'float': '{: 0.3f}'.format})
#print(T_result_1)

```

从而得到26个节点处在60s，300s，600s，1800s时刻的温度如下表所示：

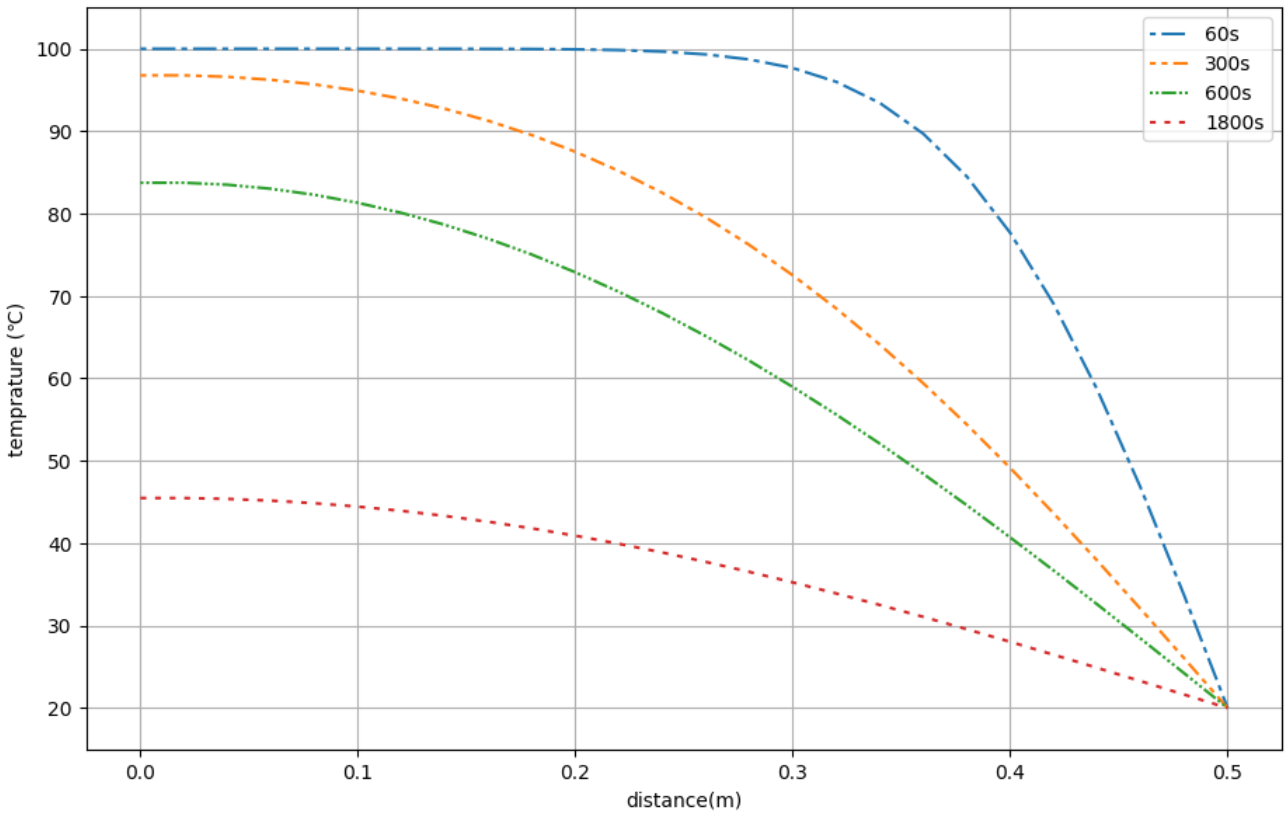
节点编号	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
60s	100.000	100.000	100.000	100.000	100.000	100.000	99.998	99.996	99.988	99.970	99.928	99.838	99.658

节点编号	0	1	2	3	4	5	6	7	8	9	10	11	12
300s	96.776	96.776	96.595	96.229	95.671	94.909	93.931	92.719	91.256	89.523	87.500	85.171	82.518
600s	83.743	83.743	83.499	83.010	82.278	81.303	80.088	78.635	76.946	75.026	72.878	70.509	67.924
1800s	45.479	45.479	45.375	45.166	44.853	44.439	43.924	43.311	42.602	41.800	40.908	39.931	38.872
节点编号	13	14	15	16	17	18	19	20	21	22	23	24	25
60s	99.317	98.706	97.667	95.993	93.430	89.695	84.516	77.669	69.039	58.659	46.742	33.678	20.000
300s	79.530	76.196	72.513	68.482	64.111	59.414	54.413	49.135	43.616	37.895	32.019	26.036	20.000
600s	65.132	62.142	58.963	55.608	52.090	48.422	44.620	40.701	36.683	32.584	28.424	24.222	20.000
1800s	37.735	36.525	35.247	33.907	32.510	31.061	29.567	28.033	26.466	24.873	23.260	21.633	20.000

绘制四个时间点温度曲线，如下图所示:

In [2]:
```
fig = plt.figure(figsize=(11, 7), dpi=100)
plt.xlabel('distance(m)')
plt.ylabel('temprature (℃)')
plt.ylim([15,105])
plt.grid(True)
line1, = plt.plot(x, T_result_1[0], dashes=[2, 2, 8, 2], label='60s')
line2, = plt.plot(x, T_result_1[1], dashes=[2,2,2,2,5,2], label='300s')
line3, = plt.plot(x, T_result_1[2], dashes=[1,1,1,1,5,1,1,1], label='600s')
line4, = plt.plot(x, T_result_1[3], dashes=[2,3], label='1800s')
plt.legend()
```

Out[2]: <matplotlib.legend.Legend at 0x2138130cf98>



2、克拉克-尼科尔森格式

求解时间选择一阶向前差分，空间选择二阶中心差分，有:

$$\frac{T_i^{n+1}-T_i^n}{\Delta t}=\alpha\frac{\frac{T_{i+1}^{n+1}+T_{i+1}^n}{2}-\frac{2T_i^{n+1}+2T_i^n}{2}+\frac{T_{i-1}^{n+1}+T_{i-1}^n}{2}}{\Delta x^2} \tag{1-6}$$

设$r=\frac{\alpha\Delta t}{2\Delta x^2}$进而有:

$$-rT_{i-1}^{n+1}+(1+2r)T_i^{n+1}-rT_{i+1}^{n+1}=rT_{i-1}^n+(1-2r)T_i^n+rT_{i+1}^n \tag{1-7}$$

继而有:

$$\begin{bmatrix}(1+2r) & -r & 0 & \cdots & & & \\ -r & (1+2r) & -r & \cdots & & & \\ & -r & (1+2r) & \cdots & & & \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ & & & \cdots & -r & (1+2r) & -r \\ & & & \cdots & 0 & -r & (1+2r)\end{bmatrix}\begin{bmatrix}T_1^{n+1}\\T_2^{n+1}\\T_3^{n+1}\\\vdots\\T_{i-1}^{n+1}\\T_i^{n+1}\end{bmatrix}=\begin{bmatrix}K_1^n\\K_2^n\\K_3^n\\\vdots\\K_{i-1}^n\\K_i^n\end{bmatrix} \tag{1-8}$$

其中$K_i^n=rT_{i-1}^n+(1-2r)T_i^n+rT_{i+1}^n$。通过追赶法求解上述三对角矩阵方程。定义求解函数:

In [3]:

```
def TDMAsolver(a, b, c, d):
      nf = len(d) # 方程数量
      ac, bc, cc, dc = map(np.array, (a, b, c, d)) # 复制矩阵
      for it in range(1, nf):
            mc = ac[it-1]/bc[it-1]
            bc[it] = bc[it] - mc*cc[it-1]
            dc[it] = dc[it] - mc*dc[it-1]

      xc = bc
      xc[-1] = dc[-1]/bc[-1]

      for il in range(nf-2, -1, -1):
            xc[il] = (dc[il]-cc[il]*xc[il+1])/bc[il]

      return xc
```

采用与显式格式同样的时间步长与网格尺寸，$dx = 0.02$，$dt = 2$。则有网格节点26个，即有26个方程需连立求解。分别迭代30次，150次，300次，900次。

In [4]:

```
r = alpha*dt/(2*dx**2) #计算r
a = np.zeros(n-1)-r       #下对角线
b = np.ones(n)+2*r        #对角线
c = np.zeros(n-1)-r       #上对角线
K = T.copy()                 #K矩阵
T_result_2 =np.ndarray((len(t_step),n))       #设置输出
for case in range(len(t_step)):
      T_result_2[case]=T.copy()
      for ti in range(int(t_step[case]/dt)):
            K[0] = T_result_2[case][1]*(1+r)
            K[1:-1]= r*T_result_2[case][:-2]+(1-2*r)*T_result_2[case][1:-1]+r*T_result_2[case][2:]
            K[-1] = (1+2*r)/2*(-2*dx*(T_result_2[case][-1]-T_far)*h/lambda_wall+T_result_2[case][-1]+T_far)-r*T_result_2[case][-2]
            T_result_2[case] = TDMAsolver(a, b, c, K)
#print(T_result_2)
```

从而得到26个节点处在$60s$，$300s$，$600s$，$1800s$时刻的温度如下表所示：

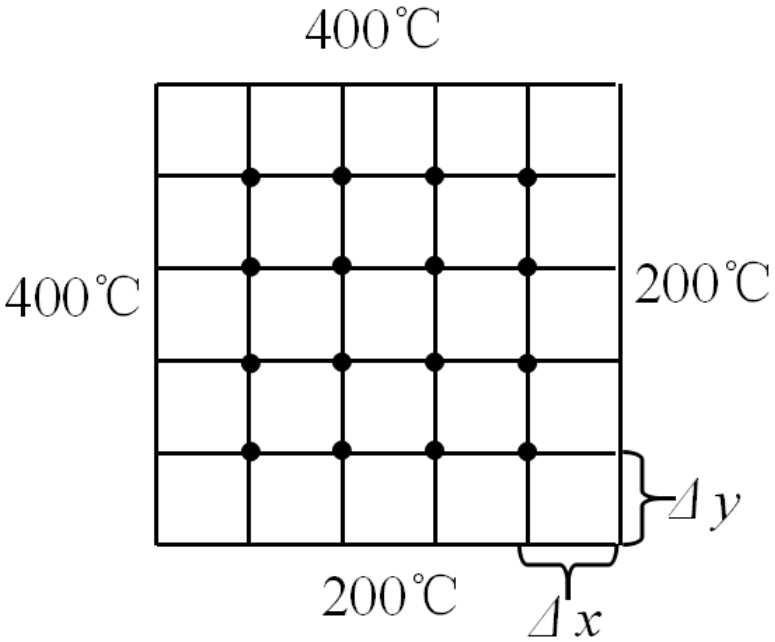
节点编号	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
60s	*100.000*	100.000	100.000	100.000	99.999	99.998	99.994	99.987	99.972	99.942	99.880	99.763	99.545
300s	96.966	96.911	96.684	96.281	95.694	94.909	93.914	92.689	91.218	89.479	87.455	85.125	82.475
600s	84.420	84.343	84.027	83.473	82.682	81.654	80.391	78.894	77.168	75.214	73.037	70.643	68.036
1800s	46.419	46.385	46.245	46.000	45.651	45.199	44.647	43.996	43.248	42.408	41.478	40.462	39.364
节点编号	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
60s	99.158	98.497	97.412	95.704	93.127	89.405	84.262	77.467	68.890	58.554	46.661	33.606	19.932
300s	79.490	76.160	72.482	68.456	64.089	59.397	54.399	49.124	43.607	37.888	32.012	26.030	19.994
600s	65.226	62.219	59.027	55.661	52.132	48.456	44.647	40.722	36.698	32.594	28.430	24.224	19.998
1800s	38.189	36.941	35.625	34.246	32.811	31.324	29.792	28.220	26.616	24.985	23.334	21.670	19.999

二

二维稳态传热问题，其控制方程为：

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{2-1}$$

二维导热体如图所示，



长宽均为$1m$, $\Delta x = \Delta = 0.2m$,四边界分别维持200°C，200°C，400°C和400℃（其中400°C于200°C交界的外角节点温度视为400°C）。试用简单迭代、G-S迭代和松弛迭代法分别求解各内部节点温度。（松弛因子取1.1，计算的收敛条件为$max\left|T_i^{k+1} - T_i^k\right| < 0.001$,至少精确到0.001°C，要求给出各迭代步值并给出各迭代收敛所需迭代步数）

解:

空间选择中心差分格式有:

$$\frac{T_{i+1,j}^n - 2T_{i,j}^n + T_{i-1,j}^n}{\Delta x^2} + \frac{T_{i,j+1}^n - 2T_{i,j}^n + T_{i,j-1}^n}{\Delta y^2} = 0 \tag{2-2}$$

因为稳态问题不存在时间项因而不存在T^{n+1},通过整理离散方程，得到 $T_{i,j}^n$:

$$T_{i,j}^n = \frac{\Delta y^2(T_{i+1,j}^n + T_{i-1,j}^n) + \Delta x^2(T_{i,j+1}^n + T_{i,j-1}^n)}{2(\Delta x^2 + \Delta y^2)} \tag{2-3}$$

对于Jacobi迭代法（即简单迭代）有:

$$JT_{i,j}^{n+1} = \frac{\Delta y^2(JT_{i+1,j}^n + JT_{i-1,j}^n) + \Delta x^2(JT_{i,j+1}^n + JT_{i,j-1}^n)}{2(\Delta x^2 + \Delta y^2)} \tag{2-4}$$

对于G-S迭代法有:

$$GT_{i,j}^{n+1} = \frac{\Delta y^2(GT_{i+1,j}^n + GT_{i-1,j}^{n+1}) + \Delta x^2(GT_{i,j+1}^n + GT_{i,j-1}^{n+1})}{2(\Delta x^2 + \Delta y^2)} \tag{2-4}$$

对于松弛迭代法有:

$$ST_{i,j}^{n+1} = ST_{i,j}^n + \omega\left(ST_{i,j}^{(n)} - \frac{\Delta y^2(ST_{i+1,j}^n + ST_{i-1,j}^{n+1}) + \Delta x^2(ST_{i,j+1}^n + ST_{i,j-1}^{n+1})}{2(\Delta x^2 + \Delta y^2)}\right) \tag{2-6}$$

其中ω为松弛因子，此处取作1.1。

In [5]:

```python
#Jacobi迭代
def Jacobi_laplace2d(T, dx, dy, llnorm_target):
      llnorm = 1
      Tn = np.empty_like(T)
      Tm = T.copy()
      flag=0
      while llnorm > llnorm_target:
            Tn = Tm.copy()
            Tm[1:-1, 1:-1] = ((dy**2 * (Tn[1:-1, 2:] + Tn[1:-1, 0:-2]) + dx**2 * (Tn[2:, 1:-1] + Tn[0:-2, 1:-1])) /(2 * (dx**2 + dy**2)))

            llnorm = np.max(np.abs(Tm[:]) - np.abs(Tn[:]))
            flag += 1
            #print("[",flag,np.reshape(Tm[1:-1,1:-1],16))
      return Tm
#G-S迭代
def GS_laplace2d(T, dx, dy, llnorm_target):
      llnorm = 1
      flag=0
      Tn = np.empty_like(T)
      Tm = T.copy()
      while llnorm > llnorm_target:
            Tn = Tm.copy()
            for i in range(1,len(T)-1):
                  for j in range(1,len(T)-1):
                        Tm[i,j]= (dy**2*(Tm[i-1,j]+Tm[i+1,j])+dx**2*(Tm[i,j-1]+Tm[i,j+1]))/(2*(dx**2+dy**2))
            llnorm = np.max(np.abs(Tm[:]) - np.abs(Tn[:]))
            flag += 1
            #print("[",flag,np.reshape(Tm[1:-1,1:-1],16))
      return Tm
#松弛迭代
def Sor_laplace2d(T, dx, dy, llnorm_target,w):
      llnorm = 1
      flag=0
      Tn = np.empty_like(T)
      Tm = T.copy()
      while llnorm > llnorm_target:
            Tn = Tm.copy()
            for i in range(1,len(T)-1):
                  for j in range(1,len(T)-1):
                        Tm[i,j]= Tm[i,j]-w*(Tm[i,j]-((Tm[i-1,j]+Tm[i+1,j])*dy**2+(Tm[i,j-1]+Tm[i,j+1])*dx**2)/(2*(dx**2+dy**2)))
            llnorm = np.max(np.abs(Tm[:]) - np.abs(Tn[:]))
            flag += 1
            #print("[",flag,np.reshape(Tm[1:-1,1:-1],16))
      return Tm
```

In [6]:

```python
dx_1 = 0.2       #设置dx
dy_1 = 0.2       #设置dy
T_1= np.zeros((6,6))+200
T_1[:, 0] = 200    # T = 200 @ x = 0
T_1[0, :] = 200    # T = 200 @ y = 0
T_1[:, -1] = 400    # T = 400 @ x = 1
T_1[-1, :] = 400    # T = 400 @ y = 1
llnorm_target = 0.001     #收敛判断条件
w = 1.1        #松弛因子
T_Jacobi = Jacobi_laplace2d(T_1, dx_1, dy_1, llnorm_target)
T_GS = GS_laplace2d(T_1, dx_1, dy_1, llnorm_target)
T_Sor = Sor_laplace2d(T_1, dx_1, dy_1, llnorm_target, w)
#print(T_1result, T_GS , T_Sor)
```

分别采用三种迭代格式求解，所得见下表：

J迭代	2-2	2-3	2-4	2-5	3-2	3-3	3-4	3-5	4-2	4-3	4-4	4-5	5-2	5-3	5-4	5-5
1	200.000	200.000	200.000	250.000	200.000	200.000	200.000	250.000	200.000	200.000	200.000	250.000	250.000	250.000	250.000	300.000
2	200.000	200.000	212.500	262.500	200.000	200.000	212.500	275.000	212.500	212.500	225.000	287.500	262.500	275.000	287.500	325.000
3	200.000	203.125	218.750	271.875	203.125	206.250	228.125	290.625	218.750	228.125	250.000	306.250	271.875	290.625	306.250	343.750
4	201.562	206.250	225.781	277.344	206.250	215.625	241.406	301.562	225.781	241.406	267.188	321.094	277.344	301.562	321.094	353.125
5	203.125	210.742	231.250	281.836	210.742	223.828	252.539	309.961	231.250	252.539	281.250	330.469	281.836	309.961	330.469	360.547
6	205.371	214.551	236.279	285.303	214.551	231.641	261.572	316.211	236.279	261.572	291.504	337.939	285.303	316.211	337.939	365.234
7	207.275	218.323	240.356	288.123	218.323	238.062	268.909	321.204	240.356	268.909	299.756	343.237	288.123	321.204	343.237	368.970
8	209.161	221.423	243.839	290.390	221.423	243.616	274.844	325.067	243.839	274.844	306.073	347.482	290.390	325.067	347.482	371.619
9	210.712	224.154	246.664	292.226	224.154	248.134	279.649	328.179	246.664	279.649	311.163	350.690	292.226	328.179	350.690	373.741
10	212.077	226.377	249.007	293.711	226.377	251.901	283.535	330.641	249.007	283.535	315.169	353.271	293.711	330.641	353.271	375.345
11	213.189	228.246	250.906	294.912	228.246	254.956	286.680	332.629	250.906	286.680	318.403	355.289	294.912	332.629	355.289	376.635
12	214.123	229.763	252.460	295.884	229.763	257.463	289.224	334.220	252.460	289.224	320.984	356.917	295.884	334.220	356.917	377.644
13	214.881	231.011	253.718	296.670	231.011	259.493	291.282	335.506	253.718	291.282	323.070	358.212	296.670	335.506	358.212	378.458
14	215.506	232.023	254.741	297.306	232.023	261.147	292.947	336.541	254.741	292.947	324.747	359.259	297.306	336.541	359.259	379.106
15	216.012	232.848	255.569	297.820	232.848	262.485	294.294	337.378	255.569	294.294	326.103	360.099	297.820	337.378	360.099	379.629
16	216.424	233.516	256.241	298.237	233.516	263.571	295.384	338.053	256.241	295.384	327.196	360.777	298.237	338.053	360.777	380.049

J迭代	2-2	2-3	2-4	2-5	3-2	3-3	3-4	3-5	4-2	4-3	4-4	4-5	5-2	5-3	5-4	5-5
17	216.758	234.059	256.784	298.573	234.059	264.450	296.265	338.599	256.784	296.265	328.081	361.325	298.573	338.599	361.325	380.389
18	217.029	234.498	257.224	298.846	234.498	265.162	296.979	339.041	257.224	296.979	328.795	361.767	298.846	339.041	361.767	380.662
19	217.249	234.854	257.581	299.066	234.854	265.738	297.556	339.398	257.581	297.556	329.373	362.125	299.066	339.398	362.125	380.884
20	217.427	235.142	257.869	299.245	235.142	266.205	298.022	339.687	257.869	298.022	329.840	362.414	299.245	339.687	362.414	381.062
21	217.571	235.375	258.102	299.389	235.375	266.582	298.400	339.920	258.102	298.400	330.218	362.647	299.389	339.920	362.647	381.207
22	217.688	235.564	258.291	299.506	235.564	266.888	298.706	340.109	258.291	298.706	330.524	362.836	299.506	340.109	362.836	381.324
23	217.782	235.717	258.444	299.600	235.717	267.135	298.953	340.262	258.444	298.953	330.771	362.989	299.600	340.262	362.989	381.418
24	217.858	235.840	258.567	299.676	235.840	267.335	299.153	340.385	258.567	299.153	330.971	363.113	299.676	340.385	363.113	381.495
25	217.920	235.940	258.667	299.738	235.940	267.496	299.315	340.485	258.667	299.315	331.133	363.213	299.738	340.485	363.213	381.556
26	217.970	236.021	258.748	299.788	236.021	267.627	299.446	340.566	258.748	299.446	331.264	363.294	299.788	340.566	363.294	381.606
27	218.010	236.086	258.814	299.829	236.086	267.733	299.551	340.632	258.814	299.551	331.370	363.359	299.829	340.632	363.359	381.647
28	218.043	236.139	258.867	299.861	236.139	267.819	299.637	340.685	258.867	299.637	331.455	363.412	299.861	340.685	363.412	381.680
29	218.070	236.182	258.909	299.888	236.182	267.888	299.706	340.728	258.909	299.706	331.525	363.455	299.888	340.728	363.455	381.706
30	218.091	236.217	258.944	299.909	236.217	267.944	299.762	340.762	258.944	299.762	331.581	363.490	299.909	340.762	363.490	381.727
31	218.108	236.245	258.972	299.927	236.245	267.990	299.808	340.790	258.972	299.808	331.626	363.518	299.927	340.790	363.518	381.745
32	218.122	236.268	258.995	299.941	236.268	268.026	299.845	340.813	258.995	299.845	331.663	363.540	299.941	340.813	363.540	381.759
33	218.134	236.286	259.013	299.952	236.286	268.056	299.874	340.831	259.013	299.874	331.692	363.559	299.952	340.831	363.559	381.770
34	218.143	236.301	259.028	299.961	236.301	268.080	299.898	340.846	259.028	299.898	331.716	363.573	299.961	340.846	363.573	381.779
35	218.150	236.313	259.040	299.969	236.313	268.099	299.918	340.858	259.040	299.918	331.736	363.585	299.969	340.858	363.585	381.787
36	218.156	236.322	259.050	299.975	236.322	268.115	299.933	340.868	259.050	299.933	331.752	363.595	299.975	340.868	363.595	381.793
37	218.161	236.330	259.058	299.979	236.330	268.128	299.946	340.876	259.058	299.946	331.764	363.603	299.979	340.876	363.603	381.798
38	218.165	236.337	259.064	299.983	236.337	268.138	299.956	340.882	259.064	299.956	331.775	363.609	299.983	340.882	363.609	381.802
39	218.168	236.342	259.069	299.987	236.342	268.147	299.965	340.887	259.069	299.965	331.783	363.615	299.987	340.887	363.615	381.805
40	218.171	236.346	259.073	299.989	236.346	268.153	299.971	340.891	259.073	299.971	331.790	363.619	299.989	340.891	363.619	381.807
41	218.173	236.349	259.077	299.991	236.349	268.159	299.977	340.895	259.077	299.977	331.795	363.622	299.991	340.895	363.622	381.809
42	218.175	236.352	259.079	299.993	236.352	268.163	299.981	340.898	259.079	299.981	331.800	363.625	299.993	340.898	363.625	381.811
43	218.176	236.354	259.082	299.994	236.354	268.167	299.985	340.900	259.082	299.985	331.803	363.627	299.994	340.900	363.627	381.812
44	218.177	236.356	259.083	299.995	236.356	268.170	299.988	340.902	259.083	299.988	331.806	363.629	299.995	340.902	363.629	381.814
45	218.178	236.358	259.085	299.996	236.358	268.172	299.990	340.903	259.085	299.990	331.808	363.630	299.996	340.903	363.630	381.814
46	218.179	236.359	259.086	299.997	236.359	268.174	299.992	340.904	259.086	299.992	331.810	363.631	299.997	340.904	363.631	381.815
47	218.179	236.360	259.087	299.998	236.360	268.175	299.994	340.905	259.087	299.994	331.812	363.632	299.998	340.905	363.632	381.816
48	218.180	236.360	259.088	299.998	236.360	268.177	299.995	340.906	259.088	299.995	331.813	363.633	299.998	340.906	363.633	381.816
49	218.180	236.361	259.088	299.998	236.361	268.178	299.996	340.906	259.088	299.996	331.814	363.634	299.998	340.906	363.634	381.817

GS迭代	2-2	2-3	2-4	2-5	3-2	3-3	3-4	3-5	4-2	4-3	4-4	4-5	5-2	5-3	5-4	5-5
1	200.000	200.000	200.000	250.000	200.000	200.000	200.000	262.500	200.000	200.000	200.000	265.625	250.000	262.500	265.625	332.812
2	200.000	200.000	212.500	268.750	200.000	200.000	218.750	288.281	212.500	218.750	242.188	315.820	268.750	288.281	315.820	357.910
3	200.000	203.125	222.656	277.734	203.125	210.938	241.016	308.643	222.656	241.016	278.418	336.243	277.734	308.643	336.243	368.121
4	201.562	208.789	231.885	285.132	208.789	224.902	260.962	320.584	231.885	260.962	298.602	346.827	285.132	320.584	346.827	373.413
5	204.395	215.295	240.347	290.233	215.295	238.129	274.416	327.869	240.347	274.416	310.621	352.976	290.233	327.869	352.976	376.488
6	207.648	221.531	246.545	293.603	221.531	247.973	283.252	332.458	246.545	283.252	318.114	356.765	293.603	332.458	356.765	378.382
7	210.765	226.321	250.794	295.813	226.321	254.786	289.038	335.404	250.794	289.038	322.902	359.172	295.813	335.404	359.172	379.586
8	213.160	229.685	253.634	297.260	229.685	259.362	292.825	337.314	253.634	292.825	325.999	360.725	297.260	337.314	360.725	380.362
9	214.843	231.960	255.511	298.206	231.960	262.392	295.304	338.559	255.511	295.304	328.014	361.734	298.206	338.559	361.734	380.867
10	215.980	233.471	256.745	298.826	233.471	264.387	296.926	339.372	256.745	296.926	329.330	362.392	298.826	339.372	362.392	381.196
11	216.735	234.467	257.555	299.232	234.467	265.697	297.988	339.903	257.555	297.988	330.190	362.822	299.232	339.903	362.822	381.411
12	217.234	235.121	258.085	299.497	235.121	266.555	298.683	340.251	258.085	298.683	330.753	363.104	299.497	340.251	363.104	381.552
13	217.561	235.550	258.433	299.671	235.550	267.117	299.138	340.478	258.433	299.138	331.121	363.288	299.671	340.478	363.288	381.644
14	217.775	235.831	258.660	299.785	235.831	267.485	299.436	340.627	258.660	299.436	331.362	363.408	299.785	340.627	363.408	381.704
15	217.916	236.015	258.809	299.859	236.015	267.726	299.631	340.725	258.809	299.631	331.520	363.487	299.859	340.725	363.487	381.744
16	218.008	236.135	258.906	299.908	236.135	267.883	299.758	340.788	258.906	299.758	331.623	363.539	299.908	340.788	363.539	381.769
17	218.068	236.214	258.970	299.940	236.214	267.986	299.842	340.830	258.970	299.842	331.690	363.572	299.940	340.830	363.572	381.786
18	218.107	236.266	259.012	299.960	236.266	268.054	299.896	340.857	259.012	299.896	331.734	363.594	299.960	340.857	363.594	381.797
19	218.133	236.300	259.039	299.974	236.300	268.098	299.932	340.875	259.039	299.932	331.763	363.609	299.974	340.875	363.609	381.804
20	218.150	236.322	259.057	299.983	236.322	268.127	299.956	340.887	259.057	299.956	331.782	363.618	299.983	340.887	363.618	381.809
21	218.161	236.336	259.069	299.989	236.336	268.146	299.971	340.895	259.069	299.971	331.795	363.625	299.989	340.895	363.625	381.812
22	218.168	236.346	259.076	299.993	236.346	268.158	299.981	340.900	259.076	299.981	331.803	363.629	299.993	340.900	363.629	381.814
23	218.173	236.352	259.081	299.995	236.352	268.166	299.988	340.903	259.081	299.988	331.808	363.631	299.995	340.903	363.631	381.816

GS迭代	2-2	2-3	2-4	2-5	3-2	3-3	3-4	3-5	4-2	4-3	4-4	4-5	5-2	5-3	5-4	5-5
24	218.176	236.356	259.085	299.997	236.356	268.172	299.992	340.905	259.085	299.992	331.812	363.633	299.997	340.905	363.633	381.817
25	218.178	236.359	259.087	299.998	236.359	268.175	299.995	340.906	259.087	299.995	331.814	363.634	299.998	340.906	363.634	381.817
26	218.179	236.360	259.088	299.999	236.360	268.178	299.997	340.907	259.088	299.997	331.815	363.635	299.999	340.907	363.635	381.817
27	218.180	236.361	259.089	299.999	236.361	268.179	299.998	340.908	259.089	299.998	331.816	363.635	299.999	340.908	363.635	381.818
28	218.181	236.362	259.090	299.999	236.362	268.180	299.999	340.908	259.090	299.999	331.817	363.636	299.999	340.908	363.636	381.818

Sor迭代	2-2	2-3	2-4	2-5	3-2	3-3	3-4	3-5	4-2	4-3	4-4	4-5	5-2	5-3	5-4	5-5
1	200.000	200.000	200.000	255.000	200.000	200.000	200.000	270.125	200.000	200.000	200.000	274.284	255.000	270.125	274.284	350.856
2	200.000	200.000	215.125	272.944	200.000	200.000	223.444	294.922	215.125	223.444	253.750	329.942	272.944	294.922	329.942	366.383
3	200.000	204.159	226.138	280.997	204.159	215.182	249.904	317.240	226.138	249.904	293.540	345.725	280.997	317.240	345.725	373.511
4	202.288	211.576	236.567	289.197	211.576	232.296	271.911	327.655	236.567	271.911	310.346	353.593	289.197	327.655	353.593	377.125
5	206.138	219.468	246.002	293.836	219.468	247.029	283.843	333.334	246.002	283.843	319.555	357.895	293.836	333.334	357.895	379.130
6	210.093	226.412	251.525	296.453	226.412	255.937	290.712	336.558	251.525	290.712	324.778	360.339	296.453	336.558	360.339	380.273
7	213.517	230.628	254.741	297.962	230.628	261.144	294.664	338.410	254.741	294.664	327.774	361.742	297.962	338.410	361.742	380.931
8	215.494	233.066	256.591	298.829	233.066	264.137	296.934	339.473	256.591	296.934	329.495	362.548	298.829	339.473	362.548	381.308
9	216.637	234.469	257.655	299.327	234.469	265.858	298.239	340.084	257.655	298.239	330.483	363.011	299.327	340.084	363.011	381.525
10	217.294	235.275	258.266	299.613	235.275	266.847	298.988	340.435	258.266	298.988	331.051	363.277	299.613	340.435	363.277	381.650
11	217.672	235.738	258.617	299.778	235.738	267.415	299.419	340.637	258.617	299.419	331.377	363.430	299.778	340.637	363.430	381.721
12	217.889	236.004	258.819	299.872	236.004	267.741	299.666	340.753	258.819	299.666	331.565	363.518	299.872	340.753	363.518	381.763
13	218.013	236.157	258.934	299.927	236.157	267.929	299.808	340.819	258.934	299.808	331.673	363.568	299.927	340.819	363.568	381.786
14	218.085	236.245	259.001	299.958	236.245	268.036	299.890	340.857	259.001	299.890	331.735	363.597	299.958	340.857	363.597	381.800
15	218.126	236.295	259.039	299.976	236.295	268.098	299.937	340.879	259.039	299.937	331.770	363.614	299.976	340.879	363.614	381.808
16	218.150	236.324	259.061	299.986	236.324	268.134	299.964	340.892	259.061	299.964	331.791	363.623	299.986	340.892	363.623	381.812
17	218.163	236.341	259.074	299.992	236.341	268.154	299.979	340.899	259.074	299.979	331.802	363.629	299.992	340.899	363.629	381.815
18	218.171	236.351	259.081	299.995	236.351	268.166	299.988	340.903	259.081	299.988	331.809	363.632	299.995	340.903	363.632	381.816
19	218.176	236.356	259.085	299.997	236.356	268.173	299.993	340.906	259.085	299.993	331.813	363.634	299.997	340.906	363.634	381.817
20	218.178	236.359	259.088	299.998	236.359	268.177	299.996	340.907	259.088	299.996	331.815	363.635	299.998	340.907	363.635	381.818
21	218.180	236.361	259.089	299.999	236.361	268.179	299.998	340.908	259.089	299.998	331.816	363.636	299.999	340.908	363.636	381.818
22	218.181	236.362	259.090	299.999	236.362	268.180	299.999	340.908	259.090	299.999	331.817	363.636	299.999	340.908	363.636	381.818
23	218.181	236.363	259.090	300.000	236.363	268.181	299.999	340.909	259.090	299.999	331.818	363.636	300.000	340.909	363.636	381.818

修正内容：

1、对第一题隐式求解结果表格的写入错误进行了修正，60s时0号节点温度由1100°C改为100°C；
2、对第二题的初始化温度场选择进行了优化，由0°C改为200°C。并更新了迭代结果表格；
3、按课上内容重新编辑了公式2-4、3-5、2-6；
4、化简了In（5）第39行迭代运算。