Green University of Bangladesh
Department of Computer Science and Engineering (CSE)
Faculty of Sciences and Engineering
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Project
Course Title: Differential Equations And Coordinate Geometry
Course Code: MAT-201 Section: 231D2

Student Details

	Name	ID
1.	Promod Chandra Das	231002005

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Course Teacher's Name : Jakia Sultana

Lab Report Status
Marks: .... Signature:.....................
Comments:... Date:...............................

Math

Project

① The population of a city increases at a rate proportional to the present number. It has an initial population of 70000 that increases by 25% in 11 years. What will be population in 30 years?

Solution

Define $x(t)$ - population of the city at the moment t (in years)

$\frac{dx}{dt} = mx$ - reflect the fact that population of the city increases at a rate proportional to the present number. m is proportional coefficient

Let's solve the equation:

$$\frac{dx}{x} = m\,dt$$

$$\ln(x) = mt + \ln c$$

$$x = Ce^{mt} \quad \text{where } c = \text{const}$$

$$x(0) = C = 70000;$$

$$x(11) = 70000 + 0.25 \cdot 70000$$

$$= 87500$$

$$x(11) = 70000e^{11m} = 87500$$

$$e^{11m} = \frac{87500}{70000} = 1.25$$

$$11m = \ln(1.25)$$

$$m = \frac{\ln(1.25)}{11} \approx 0.020285$$

$$x(t) = 70000e^{\frac{\ln(1.25)}{11}t}$$

population 30 years

$$x(30) = 70000e^{\frac{\ln(1.25)}{11} \cdot 30}$$

$$= 70000 \times (1.25)^{2.7272}$$

$$= 128644.442$$

(2) when a chicken is removed from an oven, its Temperature is measured at 300 F. Three minutes later, its temperature is 208 F. How long will it take for the chicken to cool off to a room temperature of 70 F.

Solution

Given that

$$T_s = 70, \ T_0 = 300$$

By Newton's law of cooling

$$T = T_s + (T_0 - T_s)e^{-Kt}$$

substituting the values of T_s and T_0

$$T = 70 + (230)e^{-Kt}$$

Given that $T = 200$ when $t = 3$

$$200 = 70 + 230e^{-3K}$$

$$e^{-3K} = \frac{13}{23}$$

$$K = \frac{1}{3}\ln\left(\frac{23}{13}\right) = 0.19018$$

$$\therefore T = 70 + (270)e^{-0.19018t}$$

$T = 70$ when $T \to \infty$

Thus, the chicken cools off to room temperature after a long period of a time.

③ The differential equation $\frac{dp}{dt} = (k\cos t)\,p$, where k is a positive constant, is a mathematical model for a population p(t) that undergoes yearly seasonal fluctuations.

(i) Solve the differential equation subject to the condition $p(0) = p_0$.

(ii) Suppose $k = 0.2$ and $p(0) = 500$, what are the maximum and minimum values of these population (rounded to the nearest whole number).

Ans

(i) The given value is

$$\frac{dp}{dt} = (k\cos t)p$$

From the differential equation, we have a population p(t) for a country that experiences yearly seasonal changes.

$$\frac{dp}{dt} = (k\cos t)p$$

$$p(0) = p_0$$

we can solve this differential equation as follows because it is a first order and separable D.E

$$\frac{dP}{P} = k\cos t\,dt \quad \int \frac{1}{P}\,dP = k\int \cos t\,dt$$

$$\ln(P) = k\sin t + c_1$$

$$e^{\ln(P)} = e^{k\sin t + c_1}$$

then there

$$P = e^{c_1} e^{k\sin t}$$

$$= ce^{k\sin t}$$

After that, we must use the point of condition $(P,t) = (P_0, 0)$ into equation (1) to obtain the value of constant c.

$$P_0 = ce^{\sin 0} \quad P_0 = ce^{0}$$

then there

$$c = P_0$$

then in equation (1), substitute the value of constant c, and we have $P = P_0 e^{\sin t}$. is the population at time t.

(ii) $\frac{dp}{dt} = K\cos(t) \cdot p$

$$\frac{dp}{p} = K\cos(t) \cdot dt$$

$$\int \frac{1}{p}\, dp = \int k\cos(t)\, dt$$

$$\ln|p| = K\sin(t) + c$$

$$\ln|500| = K\sin(0) + c$$

$$\ln|500| = 0 + c$$

$$c = \ln|500|$$

$$\ln|p| = K\sin(t) + \ln|500|$$

$$|p| = e^{K\sin(t)} + \ln|500|$$

$$|p| = e^{\ln|500|} \cdot e^{K\sin(t)}$$

$$|p| = 500 \cdot e^{K\sin(t)}$$

Given $K = 0.2$

$$p = 500 \cdot e^{0.2\sin(t)}$$

Since $e^{0.2\sin(t)}$.

Maximum $\sin(t) = 1$

$\sin(t) = -1$.

Maximum value

$P_{max} = 500 \cdot e^{0.2}$

$\simeq$ ~~[illegible]~~

610.70

Minimum value

$P_{min} = 500 \cdot e^{-0.2}$

$= 409.36$

Rounding the nearest whole number

Maximum value $\simeq 610$

minimum " $\simeq 409$

main.c

```c
#include <stdio.h>
#include <math.h>
// Function to calculate population after t years
double calculatePopulation(double initialPopulation, double growthRate, double time) {
    return initialPopulation * exp(growthRate * time);
}
int main() {
    // Given data
    double initialPopulation = 70000; // Initial population
    double growthRate; // Growth rate

    // To find the growth rate, we use the fact that the population increases by 25% in 11 years
    // Using the formula: P(t) = P0 * e^(rt), where P0 is the initial population, r is the growth rate, t is the time
    // P(11) = P0 * e^(11r) = P0 * (1 + 0.25) = 70000 * 1.25
    // Therefore, e^(11r) = 1.25 => 11r = ln(1.25) => r = ln(1.25) / 11
  growthRate = log(1.25) / 11;
 // Time after which we want to find the population
    double time = 30;

    // Calculate population after 30 years
    double population = calculatePopulation(initialPopulation, growthRate, time);
    // Print the result
    printf("The population after %d years will be: %.2f\n", (int)time, population);
    return 0;
```

Console

Run — 741ms on 17:54:52, 04/16

```
The population after 30 years will be: 128646.53
```

Run — 670ms on 17:55:52, 04/16

```
The population after 30 years will be: 128646.53
```