Calculating Customer Lifetime Value using Recency, Frequency and Monetary Value with a Markov Chain Model

Background

A fundamental goal of a business marketing function is to maximize the lifetime value of the company's customers. Life time value (LTV) is usually defined as the expected present value of the net cash flows from the firm's relationship with customers over the lifetime of the relationship. LTV is a key metric used by companies not only to measure customer equity (the sum of the lifetime values of all the company's customers) but to also set an upper limit on spending to acquire customers. If the expected cash flows from the relationship with an acquired customer has a present value of v, then clearly the firm should spend no more than v to acquire the customer.

Multi-level marketing (MLM) companies have particular LTV concerns because their customers include both consumers who buy their products and the sales force that sells those products. Within multi-level marketing companies, a sales force of independent business owners (known also as "distributors") are compensated not only for the sales they generate, but also for the sales of other salespeople they recruit. The gross income of these distributors is based on a combination of retail markup on sales to their own sales force (i.e. their "downline"), other distributors and consumers, and bonuses on overall sales volume (including the volume of their downline).

Many MLM firms evaluate the performance of their distributor sales force on three variables - Recency, Frequency and Monetary value of their sales. The challenge for these firms is to combine this scoring approach, known as the RFM model, with an effective LTV program to predict and maximize the lifetime value of relationships with their independent business owners.

Using Markov Chains to model and predict customer lifetime value

Markov Chain Models are a general class of mathematical models that are well suited for modelling customer relationships and calculating LTV. In this use case, an MLM company wishes to acquire Jane Doe as an Independent Business Operator. If the relationship is successful, the company anticipates receiving NC as a net contribution to company profits from Jane's first month in which she reaches her sales goal and on each succeeding month in which she reaches that goal. Sales are summed over one month periods (using a discount rate d to account for the time value of money).

Each month Jane is an active distributor, the company will spend ME in marketing expenditures to incentivize Jane. If Jane reached her goal in the prior month, she is at recency 1 for the current period. If and when Jane reaches recency 5 (i.e. goals missed for five months), the company assumes she is no longer active and discontinues any further efforts to incentivize her. Thus there are five possible states of the company's relationship with Jane at the end of any month: recencies 1 through 4 and a fifth state r = 5, "former distributor." We see that Jane's relationship and future prospects with the company are a function only of her current state (her recency). This property is called the Markov property.

The probabilities of moving from one state to another in a single period are called transition probabilities. For Jane, the probability of moving from recency i to recency j in one period is shown in Matrix **P**. If, for example, Jane is at recency 1, the probability of her remaining at recency 1 is shown in cell $\mathbf{P}_{1,1}$ (i.e. p_1). The probability of moving to recency 2 is shown in cell $\mathbf{P}_{1,2}$ (i.e. $1-p_1$). Thus, if she is currently at recency 1, she can either remain at recency 1 by reaching the goal in the current month or, move to recency 2 by missing goal.

Since she cannot move from recency 1 to recency 3, 4, or 5 in any one period, all other probabilities in row 1 are 0. Thus, the sum of probabilities for any row of **P**=1.

P=	p_1	$1\text{-}p_1$	0	0	0
	p_2	0	$1\text{-}p_2$	0	0
	p_3	0	0	$1\text{-}p_3$	0
	p_4	0	0	0	$1\text{-}p_4$
	0	0	0	0	1

In Markov modelling, the t-step transition matrix (the probability of moving from recency i to recency j in t steps) is simply the matrix product of t one-step transition matrices. Thus $\mathbf{P}^t$ is a useful way to express the probability forecast of Jane's recency at some future period t.

Now that we have a probability forecast for Jane's future relationship with the company, we want to model the economic value of that relationship. We represent the cash flows of the company in a rewards matrix. When Jane makes quota and transitions to r = 1, the company receives net income NC and commits to continue marketing to Jane through the next period at a cost of ME. Thus the total reward to the company for Jane moving to r = 1 is NC – ME. If Jane fails to meet quota and transitions to r = 2, 3 or 4 the company's reward is –ME. If she transitions to r = 5, the company ends the relationship and the corresponding reward is 0. This is represented in the rewards matrix **R**.

R=	NC-ME
	-ME
	-ME
	-ME
	0

With these definitions of **P** and **R**, Markov decision process theory allows the expected present value of the company's relationship with Jane (i.e. her LTV) to be expressed as:

$$V^T = \sum_{t=0}^{T} \left[(1+d)^{-1} P\right]^t R(1)$$

where is the 5 X 1 column vector of expected present value over T periods.

Algorithm Implementation

Given the following transition matrix **P** for company X, we use the R statistical programming language to calculate through.

P=	.70	.30	0	0	0
	.20	0	.80	0	0
	.15	0	0	.85	0
	.05	0	0	0	.95
	0	0	0	0	1

$\boldsymbol{P^5}$ **=**	.32947	.11775	.11112	.11220	.32946
	.15276	.05472	.05472	.05304	.68476
	.07855	.02784	.02694	.03009	.83657
	.01962	.00694	.00660	.00714	.95969
	.00000	.00000	.00000	.00000	1.0000

If Jane provides net income (NC) of $150 in any period in which she reaches sales goal, and if the company incurs monthly costs (ME) of $10 to incentivize Jane, Jane's rewards matrix is as follows:

R=	$140
	-$10
	-$10
	-$10
	0

Substituting transition matrices **P** through, **R** and d = .2 into (1) gives:

$\boldsymbol{V^5}$ **=**	$267.07
	$102.70
	$58.13
	$17.41
	$0

which represents the expected present value of Jane's relationship with the company over five periods. This value consists of $150 from achieving her initial sales goal and $117.07 from future cash flows. If and when she moves to recency 2 (i.e. she achieved goal in the previous period but failed to do so in the current period), her expected lifetime value drops to $102.70.

Validation of Results

The probabilities used to calculate transition matrices are based on actual distributor performance data. The values used in d and **R** are also actual data, thus we may be confident in all input data. The predictions must be verified with actual future performance data as experience is gained. Initial results appear to be consistent with past independent business operator net contribution data.

Lessons Learned

Based on early results, the Markov Model is proving to be a very useful predictive tool. Because it is a probabilistic and not a deterministic model, it addresses the uncertainty of distributor performance and the relationships of distributors with the company while using the language and terminology of probability and expected value. It provides management with the ability to not only analyze the performance of groups or cohorts of distributors, but also to predict performance of individuals like Jane Doe. The company is not limited to evaluating average profits from a distributor segment, but can now assess expected profits from individual distributor relationships.

While certain fine-tuning is desired, such as using conditional probabilities for the values through because the Markov Chain model incorporates the language of probability and expected value, it is proving to be ideally suited for one-to-one distributer management