Term End Examination - November 2014

Course	**: MAT105 - Differential and Difference Equations**	**Slot : D1+TD1**
Class NBR	**: 1366 / 1545**	
Time	**: Three Hours**	**Max.Marks:100**

PART - A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. Find the Eigen values of A , where $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -2 \\ 1 & -1 & 2 \end{bmatrix}$

2. Write the matrix of the quadratic form $x^2 + 3y^2 - 2yz + 3z^2$. Find the Eigen values and hence determine the nature of the quadratic form.

3. Determine whether the BVP $(xy')' + [x^2 + 1 + \lambda e^x]y = 0$, $y(1) + 2y'(1) = 0$, $y(2) - 3y'(2) = 0$ is a SLP.

4. Prove $J_n'(x) - \frac{n}{x} J_n(x) = -J_{n+1}(x)$.

5. Obtain the Fourier co-efficient b_n for the function $f(x) = \begin{cases} -1 & -\pi < x < 0 \\ 1 & 0 \leq x \leq \pi \end{cases}$

6. Find a_n, using Fourier series for the function $f(x) = xcosx$ from 0 to 2π .

7. Find the Z transform of $sin^2 t$.

8. Solve the following difference equation using Z transform, $y_{n+1} - 2\mathrm{y_n} = 0$, given that $y_0 = 3$.

9. A particle is executing a simple harmonic motion about the origin 0, from which the distance x of the particle is measured. Initially, $x = 20$ and velocity equal to 0 and the equation of the motion is $\frac{d^2x}{dt^2} + x = 0$. Solve for x and find the period and amplitude of the motion.

10. In a culture of Bacteria, the rate of increase is proportional to the number present. If the number doubles in 4 hours, how many may be expected at the end of 12 hours.

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. a) Reduce the quadratic form $3x_1^2 + 2x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3$ into canonical form by an orthogonal transformation. **[7]**

b) Diagonalize the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$ by means of an orthogonal transformation. **[7]**

12. a) Find the solution of the differential equation using power series method **[9]**

$y'' + x^2y = 0.$

b) Find the value of $\frac{d}{dx}[x^{-n}J_n(x)]$. **[5]**

13. a) Find the Fourier series to represent $x - x^2$ from $x = -\pi \; to \; \pi$. Hence show that **[7]**

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \; . \; . \; . = \frac{\pi^2}{12}.$$

b) The following values of y gives the displacement in inches of a certain machine part for the rotation x of the flywheel. Expand y in the form of a Fourier series up to second harmonics. **[7]**

x	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$
y	0	9.2	14.4	17.8	17.3	11.7

14. a) Find the $Z^{-1}\left[\frac{z^{-2}}{(1-z^{-1})(1-2z^{-1})(1-3z^{-1})}\right]$ by partial fraction method. **[7]**

b) Solve the difference equation using Z transform $y_{n+2} + y_n = 2^n n$. **[7]**

15. a) The differential equation of motion of a particle, which executes forced oscillations with damping is $\frac{d^2x}{dt^2} + k\frac{dx}{dt} + n^2x = n^2a\sin nt \; (k < 2n)$. If the particle starts from rest from the origin initially, find the displacement of the particle at time t. Show that the ultimate motion of the particle will be a simple harmonic oscillation of amplitude less than $\frac{3a}{4}$, provided $k > \frac{4n}{3}$. **[7]**

b) A particle of mass m is projected vertically upwards with an initial velocity v_0. The resisting force is k times to the velocity. Formulate the differential equation of motion and show that the distance s covered by the particle at time t is given by, **[7]**

$$s = \left[\frac{g}{k^2} + \frac{v_0}{k}\right](1 - e^{-kt}) - \frac{g}{k}t.$$

16. a) Verify Cayley Hamilton theorem for the matrix, $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 2 \end{bmatrix}$ **[6]**

b) Reduce the differential equation $x^2 \frac{d^2y}{dx^2} + \left(x^2 - n^2 + \frac{1}{4}\right) y = 0$, to the Bessel's equation and write its solution. **[8]**

17. a) Find the Fourier cosine series for $x(\pi - x)$ in $o < x < \pi$, and show that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$ using Parseval's identity. **[7]**

b) Find the Z transform of $r^n \cos n\,\theta$ and $r^n \sin n\,\theta$. **[7]**

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