Term End Examination - November 2014

Course : MAT105 - Differential and Difference Equations **Slot : D2+TD2**

Class NBR : 1572

Time : Three Hours **Max.Marks:100**

PART - A (10 X 3 = 30 Marks)
Answer <u>ALL</u> the Questions

1. Find the eigenvalues of adj(A), if $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

2. Reduce the third order differential equation $y''' - 2y'' + 3y' - 4y = 0$, into a system of first order differential equations and express in matrix form.

3. Express the differential equation $xy'' + (1-x)y' + ny = 0, x \neq 0$ in the form of a Sturm-Liouville equation.

4. Find the singular points of the equations $x^2(x-2)^2 y'' + (x-2)y' + 3x^2 y = 0$ and determine whether they are regular singular points or not.

5. Express $f(x) = x$ as a half range sine series in $0 < x < 2$.

6. Find the root mean square value of the function x^2 in $(0,2)$.

7. Form the difference equation from $u_n = A.3^n + B.5^n$

8. Find the Z-transform of n^2.

9. Radium decomposes at a rate directly proportional to the quantity of radium present. Suppose that it is found that in 25 years approximately 1.1% of certain quantity of radium has decomposed. Determine approximately how long will take for one-half of the original amount of radium to decompose.

10. A body of temperature $80^0 F$ is placed in a room of constant temperature $50^0 F$ at a time $t = 0$. At the end of 5 minutes the body has cooled to a temperature of $70^0 F$. Find the temperature of the body at the end of 10 minutes.

Part-B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. a) Find the orthogonal transformation which transforms the quadratic form $x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$ to canonical form. **[8]**

b) Solve the following system of linear differential equations by matrix method **[6]**

$y_1' = y_1 + 2y_2$

$y_2' = 4y_1 + 3y_2$

12. a) Find the eigenvalues and eigenvectors for the Sturm-Liouville problem **[6]**

$y'' + \lambda y = 0$, $\lambda > 0$, $y(0) = 0$, $y(1) = 0$

b) Solve the differential equation in series **[8]**

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$$

13. a) Obtain the Fourier Series expansion for the function $f(x) = x^2$. $-\pi < x < \pi$. **[7]**

Hence, deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$

b) Compute the coefficient a_0, a_1 and a_2 in the Fourier Series for y from the following data. **[7]**

x	0	1	2	3	4	5
y	4	8	15	7	6	2

14. a) Solve the difference equation $y(n+2) - 3y(n+1) - 4y(n) = 4^n$ by method of undetermined coefficients. **[7]**

b) Using Z-transform, solve the difference equation $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$, $u_0 = 0$, $u_1 = 1$ **[7]**

15 a) A circuit consists of an inductance of 0.05 henrys, a resistance of 5 ohms and a condenser of capacitance 4×10^{-4} farad. If charge(Q)=current(I)$=0$ when $t = 0$, find $Q(t)$ and $I(t)$ when there is a constant emf of 110 volts. **[7]**

b) An 8lb weight is placed at one end of a spring suspended from the ceiling. The weight is raised to 5 inches above the equilibrium position and left free. Assuming the spring constant 12 lb/ft, find the equation of motion and displacement function $x(t)$. **[7]**

16. a) Verify the Cayley-Hamilton theorem for the matrix **[6]**

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}.$$

b) Solve the differential equation in series **[8]**

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$$

17. a) Find the Fourier cosine series for $f(x) = x(\pi - x)$ in $0 < x < \pi$. Hence deduce the sum of the series $\frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \cdots \infty$ **[8]**

b) Use convolution theorem to find **[6]**

$$Z^{-1}\left(\frac{z^3}{(z-2)^2(z-3)}\right)$$

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