Term End Examination - November 2013

Course   :   MAT105   - Differential and Difference Equations   Slot: F1+TF1

Class NBR :   2318

Time   :   Three Hours   Max.Marks:100

PART – A (10 X 3 = 30 Marks)
Answer __ALL__ Questions

1. Find the sum and product of the eigenvalues of $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$.

2. Verify Cayley-Hamilton theorem for $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$.

3. Reduce

    $e^x y'' + e^x y' + \lambda y = 0, \ y(0) = 0, y'(1) = 0$ into a Strurm Liouville problem and find its

    weight function.

4. Identify all singular points and classify as regular or irregular for the differential equation

    $\dfrac{d^2 y}{dx^2} + \dfrac{4}{x} \dfrac{dy}{dx} + \dfrac{5}{x^2} y = 0$.

5. Find the Fourier sine series of the function $f(x) = 1$ when $0 < x < \pi$.

6. Find the root mean square value of the function $f(x) = x$ in $(0, l)$

7. Solve: $y_n - y_{n-1} - y_{n-2} = 0$.

8. Find the Z - transform of $\dfrac{1}{2^n}$.

9. A body is heated to 110°C and placed in air at 10°C. After one hour its temperature is 60°C. How much additional time is required for it cool to 30°C?

10. In a circuit with resistance R, Inductance L, the current I satisfies the differential equation

    $L\dfrac{dI}{dt} + RI = E$. Taking L and R as constants and $E = kt$, solve this equation subject to the

    initial condition that $I = 0$ at $t=0$.

PART – B (5 X 14 = 70 Marks)
Answer any <u>FIVE</u> Questions

11. Reduce the quadratic form $3x^2 - 2y^2 - z^2 - 4xy + 8xz + 12yz$ into canonical form and hence find rank, signature and the nature of the quadratic form.

12. a) Show that $y'' + \lambda y = 0;\ y(0) = 0,\ y(l) = 0,\ \lambda > 0$ is a Strum-Liouville problem and hence find the eigenvalues and eigen functions. Also show that eigen functions are orthogonal. **[7]**

    b) Solve in series the equation $\dfrac{d^2 y}{dx^2} + y = 0$. **[7]**

13. a) Find the Fourier series for $f(x)$ if $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi. \end{cases}$ **[7]**

    b) The following table gives the variation of periodic function even a period T **[7]**

x	0	T/6	T/3	T/2	2T/3	5T/6
$y = f(x)$	1.98	1.3	1.05	1.3	-0.88	-0.25

    Show that $f(x) = 0.75 + 0.37\cos\theta + 1.004\sin\theta$ where $\theta = \dfrac{2\pi x}{T}$.

14. a) Solve $y_n - 3y_{n-1} - 4y_{n-2} = 4^n$. **[7]**

    b) Using convolution, find the $Z^{-1}\left[\dfrac{z^2}{(z+1)^2} \right]$. **[7]**

15. a) If half-life of uranium is 1000 years, find the percentage of original amount that will remain after 3000 years. Also find in how many years will only $\dfrac{1}{10}$ of the original amount remain. **[7]**

    b) In the LCR circuit, the charge q is given by $\dfrac{d^2 q}{dt^2} + \dfrac{1}{RC}\dfrac{dq}{dt} + \dfrac{1}{LC} q = 0$. If at t = 0, q=Q, I=0 and L = 2CR^2, prove that $q = Qe^{-kt}(\cos kt + \sin kt)$ if $2kRC = 1$. **[7]**

16. a) Use the method of diagonalization to obtain the general solution for $X'' + AX = 0$, where $X = [x_1, x_2]^T$ and $A = \begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix}$. **[7]**

    b) Solve: $(1 - x^2)\dfrac{d^2 y}{dx^2} - 2x\dfrac{dy}{dx} + n(n+1)y = 0$ using a power series about $x = 0$. **[7]**

17. a) Find the Fourier series of the function **[7]**

    $f(x) = \begin{cases} \pi + x, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$.

    b) Using Z-transform, solve $y_{n+2} + 6 y_{n+1} + 9 y_n = 2^n$ given $y_0 = y_1 = 0$. **[7]**

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