Second Price Sealed Bid Auctions

Second price sealed bid auctions are similar to the previous example but as the name suggests have a key difference, the winner of the auction only pays the second highest bid. This sounds counter-intuitive however if you've ever used EBay before then you have participated in such an auction, with the only difference being you can see the current winning bid. This type of auction has a very nice property in that truth-telling is a dominant strategy. This means that always bidding the amount that you personally value the item at is always the best strategy for you regardless of whether other players over or under bid. Such a property makes this a very nice and fair kind of auction to partake in as no one can gain an advantage over anyone else by employing a complex strategy. This also produces a very clean and simple pure Nash equilibrium in which all players bid their valuations as since every players action is dominant then this must be a Nash equilibrium as any other bid will give a payoff less than or equal to bidding the valuation.

Use In The Real World:

Vickrey Auctions are quite rare in the real world, the notable examples are EBay using a slightly modified Vickrey Auction, and an extended version called a Vickrey-Clarke-Groves auction being used in some network routing algorithms in order to model the opportunity cost of using certain nodes. Some reasons Vickrey Auctions being rarely used are presented in "Why are Vickrey Auctions so rare?" (Rothkopf et al, 1990), a major one being bidders being suspicious that the sealed bids may be tampered with by the auctioneer to engineer higher profits.

Questions:

Rory - Could you expand on that NE point a bit more? [TODO maybe get Ben to reword his section to have NE not be the last thing he talks about if this is going to be the question]

Yes in fact this particular type of auction has many different nash equilibria another example is if player 1 bids their valuation and all other players bid 0. In this case if player 1 were to increase their bid then they would still win the item at the same price of 0 so a payoff of $v_1 - 0$. If any other player were to increase their valuation then it is either still less than player 1's bid and their payoff is still 0 so no improvement. Or their bid is greater than player 1's and since their valuation is higher than player 1's then they will have to pay more than their valuation and obtain a negative payoff which is clearly worse than 0.

Could you expand on why truthfulness is a dominant strategy and a nash equilibrium Of course! This hinges on the fact that the amount the winner bids does not dictate how much money they spend and their payoff, that is decided

by the highest bid of the other players. As we assume all other players bids remain the same when player i deviates from their bid such a bid b' can only change whether a player wins or loses not their payoff. So if player i were to increase their bid from their valuation then this can only affect the payoff if it changes the outcome of the auction. The same is true if they decrease to below the evaluation.

- **Case 1 Change from lose to win by increasing:** For this to be the case the new bid must be greater than the current greatest bid. This current greatest bid must be greater than player i's evaluation therefore if player i's new bid is the new greatest bid then the paid price will be the old one resulting in a negative payoff thus being worse than bidding the valuation.
- **Case 2 Change from win to lose by decreasing:** For this to happen the new bid must become a losing bid so at least one bid b_j must now be greater than player i's new bid. Since player i's original bid was their valuation and b_j was the second highest bid at the time and was less than the valuation then their original payoff was $v_i - b_j$ and since $b_j < v_i$ then the payoff was greater than 0, however now since they are now losing the auction it is 0 hence no improvement.