# Individual Exercises

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## Exercise 1

## Exercise 2

a). When $n$ is very large $R$ is approaching $r$

b). Let $B$ be the set of all bids i.e.

$$B = \{b_1, b_2, ..., b_{n-1}, b_n\}$$

Where $b_i$ is the bid of player $i \in [1, n]$. The maximum bid $\max(B)$ is $b_n$ as $b_n > r > 1$ by the problem definition. Therefore $R = E[\max(B \setminus \{b_n\})]$ i.e. the expected maximum of all bids less than $b_n$.

$$\text{Let } B' = B \setminus \{b_n\}$$

So $R = E[\max(B')]$, if any $b \in B'$ bids $r$ then $\max(B')$ is necessarily $r$ as $r > 1$ as per the problem definition, else $\max(B')$ is 1.
We can now model $B'$ using a binomial distribution $B(n-1, 0.5)$ where a success is bidder $b_i$ bidding $r$. Let $X \sim B(n-1, 0.5)$ be a discrete random variable representing the number of successes in $n-1$ bidders.

$$\begin{aligned} P(X \geq 1) &= 1 - P(X = 0) \\ &= 1 - \binom{n-1}{0} \times 0.5^0 \times 0.5^{n-1} \\ &= 1 - 0.5^{n-1} \end{aligned}$$

This gives the probability that at least 1 bidder from $B'$ bids $r$ therefore by taking the limit to infinity we can determine $R$ for very large $n$.

$$\lim_{n\to\infty} P(x \geq 1) = \lim_{n\to\infty} 1 - 0.5^{n-1} = 1 - 0 = 1$$

This means as $n$ approaches $\infty$, $P(\max(B') = r) = 1$ and $P(\max(B') = 1) = 0$ therefore

$$E[\max(B')] = 1 \times r + 0 \times 1 = r$$

so $R = E[\max(B')] = r$

## Exercise 3

a). Mary's preferences $= u_M$:
$u_M(B, W) = 2, u_M(C, J) = 1, u_M(C, W) = 0, u_M(B, J) = 0$

Alice's preferences $= u_A$:
$u_A(C, J) = 2, u_A(B, W) = 1, u_A(C, W) = 0, u_A(B, J) = 0$

b).

| | | Alice | |
|---|---|---|---|
| | | W | J |
| Mary | C | $(0,0)$ | $(1,2)$ |
| | B | $(2,1)$ | $(0,0)$ |

c).